CS182 Spring 2016: Homework 2

Due date: Friday, February 5, 2016 (before class).

1. Let the universe of discourse be the set of all real numbers. Let $P(x)$ be the statement "x is an integer", $Q(x)$ be the statement "x is a rational number", and $R(x)$ be the statement "x is greater than zero".

   (a) Translate the following English sentence to a logical expression using the statements above:
   "Every real number is rational and greater than zero"

   (b) What is the negation of the expression $\exists x(R(x) \to [P(x) \wedge Q(x)])$? (Choose 1)

      i. $\forall x(\neg R(x) \vee [(P(x) \wedge Q(x)])$

      ii. $\forall x(R(x) \wedge [\neg P(x) \vee \neg Q(x)])$

   (c) Translate the following logical expression to English: $\forall x(Q(x) \to P(x))$.

   (d) Translate the following English sentence to a logical expression: "All irrational numbers are not integers"

   (e) Translate the following logical expression to English: $\exists x(\neg Q(x) \wedge R(x))$

   (f) What is the negation of the expression $\forall x([P(x) \vee Q(x)] \to \neg R(x))$? (Choose 1)

      i. $\exists x([P(x) \vee Q(x)] \wedge R(x))$

      ii. $\exists x([\neg P(x) \wedge \neg Q(x)] \vee \neg R(x))$

2. Let the universe of discourse be the set of all positive integers. let $P(x, y)$ be the statement "$x + y < 2$"

   (a) Translate the following logical expression to English: $\forall x \exists y P(x, y)$

   (b) What is $\neg P(x, y)$? (Choose 1)

      i. $x + y \geq 2$

      ii. $x + y > 2$

   (c) Is expression in part (a) true? Justify your answer.

   (d) Translate the following logical expression to English: $\exists x \forall y \neg P(x, y)$

   (e) Translate the following English sentence to a logical expression: "There are two distinct positive integers such that their sum is less than two"

3. Let the universe of discourse be all students at Purdue. Let $P(x, y)$ be the statement "x is friends with y". Let $C(x, y)$ be the statement "x shares a class with y". Let $Q(x)$ be the statement "x is in CS 182".

   (a) Translate the following English sentence to a logical expression: "If a student is in CS 182, then there is different student in CS 182 that is friends with them"

(b) Translate the following logical expression to English: $\forall x \forall y (P(x, y) \to C(x, y))$

(c) What is the negation of $\forall x \forall y (P(x, y) \to C(x, y))$?

i. $\exists x \exists y (P(x, y) \wedge \neg C(x, y))$

ii. $\exists x \forall y (P(x, y) \to C(x, y))$

(d) Translate the following logical expression to English: $\exists x \forall y (C(x, y) \to P(x, y))$

(e) What is the negation of $\exists x \forall y (C(x, y) \to P(x, y))$? (Choose 1)

i. $\forall x \exists y (C(x, y) \wedge \neg P(x, y))$

ii. $\forall x \exists y (\neg C(x, y) \vee P(x, y))$

(f) Translate the following English sentence to a logical expression: "There is a student not in CS 182 such that for every student not friends with them, they share a class together"

4. Let P be the proposition "you baked some cookies". Let Q be the proposition "you went to the store". Let R be the proposition "it was snowing outside". Let S be the proposition "your oven was broken".

(a) Given the premises "If you baked come cookies, then you went to the store" and "If it was snowing outside, then you bakes some cookies", which of the following can be concluded? (Choose 1)

i. "If you went to the store, then it was snowing outside"

ii. "You went to the store or it wasn't snowing outside"

(b) Given the premises $(\neg P \wedge \neg Q) \to S$, $\neg P$, $\neg Q$, and $S \to R$, which of the following can be concluded? (Choose 1)

i. $\neg S$

ii. R

(c) Translate the following argument to English:

$\neg P \vee \neg S$

P

$\therefore \neg S$

(d) Translate the following argument to English:

$\neg R \to Q$

$Q \to P$

$\therefore \neg R \to P$

(e) In the following argument, label which rules of inference are used:

i. Q (Premise)
ii. $R \rightarrow \neg Q$ (Premise)
iii. $\neg R$ (a. ___________ of i and ii)
iv. $P \rightarrow R$ (Premise)
v. $R \rightarrow Q$ (Premise)
vi. $P \rightarrow Q$ (b. ___________ of iv and v)
vii. $\neg R \vee S$ (c. ___________ of iii with S)
viii. $R \vee (P \rightarrow Q)$ (d. ___________ of vi with R)
ix. $\therefore S \vee (P \rightarrow Q)$ (e. ___________ of vii and viii)

(f) Write each premise and the conclusion from part (e) in English.

(g) Let $A(x)$, $B(x)$, and $C(x)$ be propositional functions. Use rules of inference to show if $\forall x(A(x) \vee B(x))$ and $\forall x((\neg A(x) \wedge B(x)) \rightarrow C(x))$ are true, then $\forall x(\neg C(x) \rightarrow A(x))$ is also true, where the domains of all the quantifiers are the same.

5. For each of the following questions, if the proof method is not specified, you are free to choose a proof method.

(a) Use a direct proof to show that the difference of two rational numbers is rational.

(b) Use a proof by contradiction to show that if n is an integer and $3n + 2$ is even, then n is even.

(c) Use a proof by contrapositive to show that if n is an integer and n^2 is odd, then n is odd.

(d) Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even.

(e) Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$, where m and n are any real number.