CS182 Spring 2016: Homework 8

Due date: Wednesday, April 27th 2016 (before class).

1. Pigeon Hole Principle and Permutations and Combinations

   (a) Show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs of those integers with sum equal to 11.

   (b) How many bit strings contain exactly five 0s and 14 1s if every 0 must be immediately followed by two 1s?

2. Binomial Coefficients and Identities and Generalized Permutations and Combinations

   (a) Give a formula for the coefficient of x^k in the expansion of $(x + 1/x)^{100}$, where k is a nonnegative integer.

   (b) How many ways are there to distribute five distinguishable objects into three indistinguishable boxes?

3. Discrete Probability

   (a) What is the probability that a player of a lottery wins the prize offered for correctly choosing five (but not six) numbers out of six integers chosen at random from the integers between 1 and 40, inclusive?

   (b) What is the probability that a five-card poker hand contains a flush (five cards of the same suit)?

4. Probability Theory

   (a) What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up heads?

   (b) Find the probability of each outcome when a loaded die is rolled if a 3 is twice as likely to appear as every other number on the die.

5. Bayes' Theorem

   (a) When a test for steroids is given to soccer players, 98% of the players taking steroids test positive and 12% of the players not taking steroids test positive. Suppose that 5% of soccer players take steroids. What is the probability that a soccer player who tests positive takes steroids?

   (b) A space probe near Neptune communicates with Earth using bit strings. Suppose that in its transmissions it sends a 1 one-third of the time and a 0 two-thirds of the time. When a 0 is sent, the probability that it is received correctly is 0.9, and the probability that it is received incorrectly (as a 1) is 0.1. When a 1 is sent, the probability that it is received correctly is 0.8, and the probability that it is received incorrectly (as a 0) is 0.2. What's the probability a 0 was received? What's the probability that a 0 was transmitted given that a 0 was received? (Use Bayes' Theorem)

6. Expected Value and Variance

   (a) What is the expected sum of the numbers that appear when three fair dice are rolled?

   (b) What is the variance of the number of times a 6 appears when a fair die is rolled 10 times?