CS182 Spring 2016: Homework 5

Due date: Friday, April 1, 2016 (before class).

1. Using induction, verify that for all $n \geq 1$, the sum of the squares of the first 2n positive integers is given by the formula:

$$1^2 + 2^2 + 3^2 + ... + (2n)^2 = \frac{n(2n+1)(4n+1)}{3}$$

2. Consider the sequence of real numbers defined by the relations:

$$x_1 = 1, x_{n+1} = \sqrt{1 + 2x_n}$$

for $n \geq 1$.
Use the Principle of Mathematical Induction to show that $x_n < 4$ for all $n \geq 1$.

3. Show that $n! > 3^n$ for $n \geq 7$ via induction.

4. Let $p_0 = 1, p_1 = cos\theta$ (for θ some fixed constant) and $p_{n+1} = 2p_1 p_n - p_{n-1}$ for $n \geq 1$.
Use Principle of Mathematical Induction to prove that $p_n = cos(n\theta)$ for $n \geq 0$.

5. Using strong induction, prove that the Fibonacci sequence:
$a_0 = 1, a_1 = 1$, $a_{k+1} = a_k + a_{k+1}$ for $k \geq 1$:

$$a_k \geq (\frac{3}{2})^{k-2}.$$