CS182 Spring 2016: Homework 4

Due date: Friday, March 4, 2016 (before class).

1. Prove that $\mathbb{Z}^+ \times \mathbb{Z}^+$ is countably infinite.

2. Consider the set $\mathbb{R}$ consisting of all real numbers. Give an example of a subset S of $\mathbb{R}$ with $S \neq \mathbb{R}$ such that:

   (a) S is countably infinite

   (b) S is finite

   (c) S is uncountably infinite

3. Let B be the set of all possible bit strings. That is, $B = \{b_0 b_1 \ldots b_n \mid n \in \mathbb{Z}^+ \cup \{0\} \text{ and } b_i \in \{0, 1\}\}$. Show that B is countably infinite.

4. For each of the following sets, state the cardinality (finite, countably infinite, or uncountably infinite) (you do not need to justify your answer):

   (a) The set $\mathbb{R} - \mathbb{Q}$

   (b) The set $\mathbb{Z} \cap \mathbb{Q}$

   (c) The set $\mathcal{P}(\mathbb{Z})$

   (d) The set $\mathbb{Z} \cap \{r \mid r \in \mathbb{R}, 0 \leq r \leq 1\}$

5. List the next 5 terms in each of the following sequences:

   (a) $a_0 = 7$, $a_n = 2 * a_{n-1} - 5$

   (b) $b_0 = 1$, $b_1 = 2$, $b_n = 2 * b_{n-1} + b_{n-2}$

   (c) $c_0 = 2$, $c_n = (c_{n-1})^n$

   (d) $a_0 = 2$, $a_1 = 3$, $a_n = \dfrac{a_{n-1}}{a_{n-2}}$

6. Evaluate the following sums (note: page 166 in the book can be helpful for these):

   (a) $\sum_{i=1}^{m} i^2 + i$

   (b) $\sum_{j=17}^{n} 6j^2$

   (c) $\sum_{k=20}^{\infty} (\frac{1}{2})^k$

   (d) $\sum_{h=10}^{20} h^3 - 2h^2$

7. Find a solution to each of these recurrence relations with their given initial conditions:

(a) $a_n = -a_{n-1}$, $a_0 = 6$

(b) $a_n = (n+1)a_{n-1}$, $a_0 = 2$

(c) $a_n = a_{n-1} - n$, $a_0 = 3$

8. Let $\mathbf{A} = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 4 & -1 \end{bmatrix}$ and let $\mathbf{B} = \begin{bmatrix} 2 & 3 \\ -2 & 5 \\ -1 & 7 \end{bmatrix}$

(a) What is the size of $\mathbf{A}$?

(b) What is the size of $\mathbf{B}$?

(c) What is $\mathbf{A}^T + \mathbf{B}$?

(d) What is $\mathbf{AB}$?

(e) What is $\mathbf{BA}$?