CS182 Spring 2016: Solution to Homework 1

Due date: Tuesday, September 4th, 2012 (before class).

1. (8pts) Show that each of these implications is a tautology by using truth tables.

   (a) $[q \wedge (p \to q)] \to q$.

p	q	$p \to q$	$q \wedge (p \to q)$	$[q \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	T

   (b) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$.

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$	$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

   (c) $[\neg p \wedge (p \vee q)] \to q$.

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

   (d) $[(p \to q) \wedge (q \to r)] \to (p \to r)$.

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

2. (2pts) Determine whether $(\neg p \wedge (p \to q)) \to \neg q$ is a tautology.

p	q	$\neg p$	$p \to q$	$\neg p \wedge (p \to q)$	$\neg q$	$(\neg p \wedge (p \to q)) \to \neg q$
T	T	F	T	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

Thus $(\neg p \wedge (p \to q)) \to \neg q$ is not a tautology.

3. (2pts) Show that $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$ are logically equivalent.

p	q	r	$p \to q$	$p \to r$	$(p \to q) \vee (p \to r)$	$q \vee r$	$p \to (q \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

4. (4pts) Use truth tables to verify the associative laws

(a) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$.

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	F	T	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

(b) $(p \vee q) \vee r \equiv p \vee (q \vee r)$.

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$q \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	T	F	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

5. (4pts) Suppose you are given the following truth table

p	q	r	
F	F	F	T
F	F	T	F
F	T	F	T
F	T	T	F
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	F

Now give the **disjunctive normal form** corresponding to this table.
$(\neg p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg q \wedge r)$.

6. (2pts) Show that $[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$ is a tautology.

p	q	r	$p \vee q$	$\neg p \vee r$	$(p \vee q) \wedge (\neg p \vee r)$	$(q \vee r)$	$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	T	T	T	T	T
T	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	F	T	T
F	F	F	F	T	F	F	T

7. (8pts) Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." For each of the following sentences, choose the correct statements expressed in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the universe of disclosure consist of all students in your class.

(a) Some student in your class has a cat and a ferret, but not a dog.
   i. $\exists x(C(x) \land D(x) \land F(x))$.
   ii. $\exists x(C(x) \land \neg D(x) \land F(x))$. (Correct)

(b) No student in your class has a cat, a dog, and a ferret.
   i. $\neg\exists x(C(x) \land D(x) \land F(x))$. (Correct)
   ii. $\forall x(\neg C(x) \land \neg D(x) \land \neg F(x))$.

(c) A student in your class has a cat, a dog, and a ferret.
   i. $\forall x(C(x) \land D(x) \land F(x))$.
   ii. $\exists x(C(x) \land D(x) \land F(x))$. (Correct)

(d) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has one of these animals as a pet.
   i. $\exists x(C(x)) \land \exists y(D(y)) \land \exists z(F(z))$. (Correct)
   ii. $\exists x(C(x) \land D(x) \land F(x))$.

8. (6pts) Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the universe of disclosure consists of all animals.

   (a) $\exists x(R(x) \land H(x))$.
   There exists an animal that is a rabbit and hops. Alternatively, some rabbits hop. Or, some hopping animals are rabbits.

   (b) $\forall x(R(x) \to H(x))$.
   If an animal is a rabbit, then that animal hops. Alternatively, every rabbit hops.

   (c) $\forall x(R(x) \land H(x))$.
   Every animal is a rabbit and hops.

9. (6pts) Suppose the universe of disclosure of the propositional function $P(x)$ consists of the integers -2, -1, 0, 1, and 2. Write out each of these propositions using disjunctions, conjunctions, and negations.

   (a) $\exists x P(x)$.
   $P(-2) \lor P(-1) \lor P(0) \lor P(1) \lor P(2)$.

   (b) $\forall x P(x)$.
   $P(-2) \land P(-1) \land P(0) \land P(1) \land P(2)$.

   (c) $\exists x \neg P(x)$.
   $\neg P(-2) \lor \neg P(-1) \lor \neg P(0) \lor \neg P(1) \lor \neg P(2)$.

   (d) $\forall x \neg P(x)$.
   $\neg P(-2) \land \neg P(-1) \land \neg P(0) \land \neg P(1) \land \neg P(2)$.

(e) $\neg \exists x P(x)$.
$\neg(P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2))$. (Same as (d))

(f) $\neg \forall x P(x)$.
$\neg(P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2))$. (Same as (c))

10. (8pts) Determine the truth value of each of these statements if the universe of disclosure of each variable consists of all real numbers.

 (a) $\exists x(x^2 = 3)$.
 True. ($x = \sqrt{3}$)

 (b) $\exists x(x^2 = -1)$.
 False. ($x = \sqrt{-1}$ is not a real number)

 (c) $\forall x(x^3 + 1 \geq 1)$.
 False. (consider the case when $x < 0$)

 (d) $\forall x(x^2 \neq x)$.
 False. (not true for $x = 1$ or $x = 0$)

11. (6pts) Find a counterexample, if possible, to these universally quantified statements, where the universe of disclosure of all variables consists of all real numbers.

 (a) $\forall x(x^2 \neq x)$.
 False since $1^2 = 1$.

 (b) $\forall x(x^2 \neq 2)$.
 False since $x = \sqrt{2}$ is a counter example.

 (c) $\forall x(|x| > 0)$.
 False since $|0| = 0$.

12. (8pts) Let $R(x)$ be "x is in the correct place," $E(x)$ be "x is in excellent condition," $T(x)$ be "x is a tool," and the universe of disclosure be all things. For each of the following sentences, choose the correct logical expressions.

 (a) All tools are in the correct place and are in excellent condition.

  i. $\forall x(T(x) \rightarrow (R(x) \wedge E(x)))$. (Correct)
  ii. $\forall x(R(x) \wedge E(x))$.

 (b) Everything is in the correct place and in excellent condition.

  i. $\exists x(R(x) \wedge E(x))$.
  ii. $\forall x(R(x) \wedge E(x))$. (Correct)

 (c) No tool is in the correct place.

  i. $\forall x(T(x) \rightarrow \neg R(x))$. (Correct)
  ii. $\forall x(T(x) \wedge \neg R(x))$.

(d) Nothing is in the correct place and is in excellent condition.

i. $\exists x \neg(R(x) \wedge E(x))$.

ii. $\forall x \neg(R(x) \wedge E(x))$. (Correct)