E7+An picard number

September 18, 2023

```
[2]: from weilrep import *
```

```
[3]: def picard_num(l):
         m = l.gram_matrix()
         wt = 14 - (l.rank()/2)
         w = WeilRep(m)
         cusp = w.cusp_forms_dimension(wt)
         modular = w.modular_forms_dimension(wt)
         basis = w.modular_forms_basis(wt,2)
         return([cusp,modular,basis])
```

```
[4]: E = IntegralLattice('E7')
     E8 = IntegralLattice('E8')
     cusp_dim =[]
     t = list(range(1,11))
     t
```

```
[4]: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

```
[5]: A = ['A'+str(i) for i in t]
     A
```

```
[5]: ['A1', 'A2', 'A3', 'A4', 'A5', 'A6', 'A7', 'A8', 'A9', 'A10']
```

```
[6]: L = []
     for x in A:
         y = IntegralLattice(x)
         z = E.direct_sum(y)
         L.append(z)
```

```
[7]: for l in L:
         m = l.gram_matrix()
         wt = 14 - (l.rank()/2)
         w = WeilRep(m)
         cusp_dim.append(w.cusp_forms_dimension(wt))
```

```
[8]: cusp_dim
```

```
[8]: [2, 2, 3, 3, 4, 4, 5, 4, 4, 3]
```

```
[9]: modular_dim = []
     for i in list(range(10)):
         modular_dim.append(picard_num(L[i])[1])
     modular_dim
```

```
[9]: [4, 3, 4, 4, 5, 5, 7, 6, 6, 4]
```

```
[10]: picard_num(L[0])[2]
```

```
[10]: [(0, 0, 0, 0, 0, 0, 0, 0), 1 - 152*q + O(q^2)]
      [(0, 0, 0, 0, 0, 0, 0, 1/2), -20480*q^(7/4) + O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0), -1024*q^(5/4) + O(q^(9/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2), -112*q + O(q^2)]
      -----------------------------------------------------------
      [(0, 0, 0, 0, 0, 0, 0, 0), -112*q + O(q^2)]
      [(0, 0, 0, 0, 0, 0, 0, 1/2), 20480*q^(7/4) + O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0), 1024*q^(5/4) + O(q^(9/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2), 1 - 152*q + O(q^2)]
      -----------------------------------------------------------
      [(0, 0, 0, 0, 0, 0, 0, 0), -56*q + O(q^2)]
      [(0, 0, 0, 0, 0, 0, 0, 1/2), -3136*q^(7/4) + O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0), q^(1/4) + 246*q^(5/4) + O(q^(9/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2), 56*q + O(q^2)]
      -----------------------------------------------------------
      [(0, 0, 0, 0, 0, 0, 0, 0), -2*q + O(q^2)]
      [(0, 0, 0, 0, 0, 0, 0, 1/2), q^(3/4) - 14*q^(7/4) + O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0), -4*q^(5/4) + O(q^(9/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2), 2*q + O(q^2)]
```

```
[13]: picard_num(L[1])[2]
```

```
[13]: [(0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 156*q + O(q^2)]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 2/3), -54*q^(11/12) - 33156*q^(23/12) +
      O(q^(35/12))]
      [(0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), -10098*q^(5/3) + O(q^(8/3))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0), -924*q^(5/4) + O(q^(9/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), -10098*q^(5/3) + O(q^(8/3))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 1/3), -54*q^(11/12) - 33156*q^(23/12) +
      O(q^(35/12))]
      -----------------------------------------------------------
      [(0, 0, 0, 0, 0, 0, 0, 0, 0), -56*q + O(q^2)]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 2/3), 27*q^(11/12) + 3456*q^(23/12) +
      O(q^(35/12))]
      [(0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), -1512*q^(5/3) + O(q^(8/3))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0), q^(1/4) + 192*q^(5/4) + O(q^(9/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), -1512*q^(5/3) + O(q^(8/3))]
```

```
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 1/3), 27*q^(11/12) + 3456*q^(23/12) +
O(q^(35/12))]
-----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0), -6*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 2/3), 2*q^(11/12) - 20*q^(23/12) +
O(q^(35/12))]
[(0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), q^(2/3) - 8*q^(5/3) + O(q^(8/3))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0), -12*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), q^(2/3) - 8*q^(5/3) + O(q^(8/3))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 1/3), 2*q^(11/12) - 20*q^(23/12) +
O(q^(35/12))]
```

[14]: `picard_num(L[2])[2]`

[14]:
```
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 162*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/4, 1/2, 1/4), -48*q^(7/8) - 20336*q^(15/8) +
O(q^(23/8))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2), -3416*q^(3/2) + O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/4, 1/2, 3/4), -48*q^(7/8) - 20336*q^(15/8) +
O(q^(23/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0), -840*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4), -6528*q^(13/8) + O(q^(21/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2), -4*q^(3/4) - 11952*q^(7/4) +
O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4), -6528*q^(13/8) + O(q^(21/8))]
-----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -56*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/4, 1/2, 1/4), 16*q^(7/8) + 2000*q^(15/8) +
O(q^(23/8))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2), -560*q^(3/2) + O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/4, 1/2, 3/4), 16*q^(7/8) + 2000*q^(15/8) +
O(q^(23/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0), q^(1/4) + 160*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4), -896*q^(13/8) + O(q^(21/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2), 10*q^(3/4) + 1280*q^(7/4) +
O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4), -896*q^(13/8) + O(q^(21/8))]
-----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -6*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/4, 1/2, 1/4), 8*q^(7/8) - 24*q^(15/8) +
O(q^(23/8))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2), q^(1/2) - 66*q^(3/2) + O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/4, 1/2, 3/4), 8*q^(7/8) - 24*q^(15/8) +
O(q^(23/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0), -28*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4), 64*q^(13/8) + O(q^(21/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2), -6*q^(3/4) + 24*q^(7/4) +
```

```
O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4), 64*q^(13/8) + O(q^(21/8))]
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -8*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/4, 1/2, 1/4), -2*q^(7/8) + 38*q^(15/8) +
O(q^(23/8))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2), 16*q^(3/2) + O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/4, 1/2, 3/4), -2*q^(7/8) + 38*q^(15/8) +
O(q^(23/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0), -8*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4), q^(5/8) - 17*q^(13/8) + O(q^(21/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2), 4*q^(3/4) - 48*q^(7/4) + O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4), q^(5/8) - 17*q^(13/8) + O(q^(21/8))]
```

[15]: `picard_num(L[3])[2]`

[15]:
```
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 170*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/5, 2/5, 3/5, 4/5), -50*q^(17/20) -
13600*q^(37/20) + O(q^(57/20))]
[(0, 0, 0, 0, 0, 0, 0, 2/5, 4/5, 1/5, 3/5), -1700*q^(7/5) + O(q^(12/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/5, 1/5, 4/5, 2/5), -5950*q^(33/20) +
O(q^(53/20))]
[(0, 0, 0, 0, 0, 0, 0, 4/5, 3/5, 2/5, 1/5), -4675*q^(8/5) + O(q^(13/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0), -748*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/5, 2/5, 3/5, 4/5), -4675*q^(8/5) + O(q^(13/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/5, 4/5, 1/5, 3/5), -5950*q^(33/20) +
O(q^(53/20))]
[(0, 0, 0, 0, 0, 0, 0, 3/5, 1/5, 4/5, 2/5), -1700*q^(7/5) + O(q^(12/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/5, 3/5, 2/5, 1/5), -50*q^(17/20) -
13600*q^(37/20) + O(q^(57/20))]
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -56*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/5, 2/5, 3/5, 4/5), 10*q^(17/20) + 1225*q^(37/20)
+ O(q^(57/20))]
[(0, 0, 0, 0, 0, 0, 0, 2/5, 4/5, 1/5, 3/5), -280*q^(7/5) + O(q^(12/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/5, 1/5, 4/5, 2/5), 5*q^(13/20) + 630*q^(33/20) +
O(q^(53/20))]
[(0, 0, 0, 0, 0, 0, 0, 4/5, 3/5, 2/5, 1/5), -560*q^(8/5) + O(q^(13/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0), q^(1/4) + 140*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/5, 2/5, 3/5, 4/5), -560*q^(8/5) + O(q^(13/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/5, 4/5, 1/5, 3/5), 5*q^(13/20) + 630*q^(33/20) +
O(q^(53/20))]
[(0, 0, 0, 0, 0, 0, 0, 3/5, 1/5, 4/5, 2/5), -280*q^(7/5) + O(q^(12/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/5, 3/5, 2/5, 1/5), 10*q^(17/20) + 1225*q^(37/20)
+ O(q^(57/20))]
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -20*q + O(q^2)]
```

4

```
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/5, 2/5, 3/5, 4/5), 18*q^(17/20) + 112*q^(37/20) +
O(q^(57/20))]
[(0, 0, 0, 0, 0, 0, 0, 2/5, 4/5, 1/5, 3/5), q^(2/5) - 58*q^(7/5) + O(q^(12/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/5, 1/5, 4/5, 2/5), -4*q^(13/20) - 18*q^(33/20) +
O(q^(53/20))]
[(0, 0, 0, 0, 0, 0, 0, 4/5, 3/5, 2/5, 1/5), 188*q^(8/5) + O(q^(13/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0), -100*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/5, 2/5, 3/5, 4/5), 188*q^(8/5) + O(q^(13/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/5, 4/5, 1/5, 3/5), -4*q^(13/20) - 18*q^(33/20) +
O(q^(53/20))]
[(0, 0, 0, 0, 0, 0, 0, 3/5, 1/5, 4/5, 2/5), q^(2/5) - 58*q^(7/5) + O(q^(12/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/5, 3/5, 2/5, 1/5), 18*q^(17/20) + 112*q^(37/20) +
O(q^(57/20))]
--------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -10*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/5, 2/5, 3/5, 4/5), -4*q^(17/20) + 56*q^(37/20) +
O(q^(57/20))]
[(0, 0, 0, 0, 0, 0, 0, 2/5, 4/5, 1/5, 3/5), 7*q^(7/5) + O(q^(12/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/5, 1/5, 4/5, 2/5), 2*q^(13/20) - 20*q^(33/20) +
O(q^(53/20))]
[(0, 0, 0, 0, 0, 0, 0, 4/5, 3/5, 2/5, 1/5), q^(3/5) - 14*q^(8/5) + O(q^(13/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0), O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/5, 2/5, 3/5, 4/5), q^(3/5) - 14*q^(8/5) + O(q^(13/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/5, 4/5, 1/5, 3/5), 2*q^(13/20) - 20*q^(33/20) +
O(q^(53/20))]
[(0, 0, 0, 0, 0, 0, 0, 3/5, 1/5, 4/5, 2/5), 7*q^(7/5) + O(q^(12/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/5, 3/5, 2/5, 1/5), -4*q^(17/20) + 56*q^(37/20) +
O(q^(57/20))]
```

[16]: `picard_num(L[4])[2]`

```
[16]:  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 156*q + O(q^2)]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/6, 1/3, 1/2, 2/3, 5/6), -44*q^(5/6) -
       9536*q^(11/6) + O(q^(17/6))]
       [(0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 1/3, 2/3), -1045*q^(4/3) + O(q^(7/3))]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2), -2424*q^(3/2) + O(q^(5/2))]
       [(0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 2/3, 1/3), -1045*q^(4/3) + O(q^(7/3))]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/6, 2/3, 1/2, 1/3, 1/6), -44*q^(5/6) -
       9536*q^(11/6) + O(q^(17/6))]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0), -660*q^(5/4) + O(q^(9/4))]
       [(0, 0, 0, 0, 0, 0, 0, 1/6, 1/3, 1/2, 2/3, 5/6), -2*q^(7/12) - 3460*q^(19/12) +
       O(q^(31/12))]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 2/3, 0, 1/3, 2/3), -2*q^(7/12) -
       3460*q^(19/12) + O(q^(31/12))]
       [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2), -660*q^(5/4) + O(q^(9/4))]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 1/3, 0, 2/3, 1/3), -2*q^(7/12) -
       3460*q^(19/12) + O(q^(31/12))]
```

```
[(0, 0, 0, 0, 0, 0, 0, 5/6, 2/3, 1/2, 1/3, 1/6), -2*q^(7/12) - 3460*q^(19/12) +
O(q^(31/12))]
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/6, 1/3, 1/2, 2/3, 5/6), 16*q^(5/6) + 640*q^(11/6)
+ O(q^(17/6))]
[(0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 1/3, 2/3), -160*q^(4/3) + O(q^(7/3))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2), 288*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 2/3, 1/3), -160*q^(4/3) + O(q^(7/3))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/6, 2/3, 1/2, 1/3, 1/6), 16*q^(5/6) + 640*q^(11/6)
+ O(q^(17/6))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0), q^(1/4) + 108*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/6, 1/3, 1/2, 2/3, 5/6), -2*q^(7/12) - 340*q^(19/12) +
O(q^(31/12))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 2/3, 0, 1/3, 2/3), q^(7/12) + 365*q^(19/12) +
O(q^(31/12))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2), -132*q^(5/4) + O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 1/3, 0, 2/3, 1/3), q^(7/12) + 365*q^(19/12) +
O(q^(31/12))]
[(0, 0, 0, 0, 0, 0, 0, 5/6, 2/3, 1/2, 1/3, 1/6), -2*q^(7/12) - 340*q^(19/12) +
O(q^(31/12))]
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/6, 1/3, 1/2, 2/3, 5/6), 16*q^(5/6) + 640*q^(11/6)
+ O(q^(17/6))]
[(0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 1/3, 2/3), -160*q^(4/3) + O(q^(7/3))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2), 288*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 2/3, 1/3), -160*q^(4/3) + O(q^(7/3))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/6, 2/3, 1/2, 1/3, 1/6), 16*q^(5/6) + 640*q^(11/6)
+ O(q^(17/6))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0), -132*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/6, 1/3, 1/2, 2/3, 5/6), q^(7/12) + 365*q^(19/12) +
O(q^(31/12))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 2/3, 0, 1/3, 2/3), -2*q^(7/12) - 340*q^(19/12)
+ O(q^(31/12))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2), q^(1/4) + 108*q^(5/4) + O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 1/3, 0, 2/3, 1/3), -2*q^(7/12) - 340*q^(19/12)
+ O(q^(31/12))]
[(0, 0, 0, 0, 0, 0, 0, 5/6, 2/3, 1/2, 1/3, 1/6), q^(7/12) + 365*q^(19/12) +
O(q^(31/12))]
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -54*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/6, 1/3, 1/2, 2/3, 5/6), 4*q^(5/6) + 256*q^(11/6)
+ O(q^(17/6))]
[(0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 1/3, 2/3), q^(1/3) + 68*q^(4/3) +
O(q^(7/3))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2), -216*q^(3/2) + O(q^(5/2))]
```

```
[(0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 2/3, 1/3), q^(1/3) + 68*q^(4/3) +
O(q^(7/3))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/6, 2/3, 1/2, 1/3, 1/6), 4*q^(5/6) + 256*q^(11/6)
+ O(q^(17/6))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0), -108*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/6, 1/3, 1/2, 2/3, 5/6), 2*q^(7/12) + 132*q^(19/12) +
O(q^(31/12))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 2/3, 0, 1/3, 2/3), 2*q^(7/12) + 132*q^(19/12)
+ O(q^(31/12))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2), -108*q^(5/4) + O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 1/3, 0, 2/3, 1/3), 2*q^(7/12) + 132*q^(19/12)
+ O(q^(31/12))]
[(0, 0, 0, 0, 0, 0, 0, 5/6, 2/3, 1/2, 1/3, 1/6), 2*q^(7/12) + 132*q^(19/12) +
O(q^(31/12))]
---------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -12*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/6, 1/3, 1/2, 2/3, 5/6), -5*q^(5/6) + 43*q^(11/6)
+ O(q^(17/6))]
[(0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 1/3, 2/3), -4*q^(4/3) + O(q^(7/3))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2), q^(1/2) - 18*q^(3/2) +
O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 2/3, 1/3), -4*q^(4/3) + O(q^(7/3))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/6, 2/3, 1/2, 1/3, 1/6), -5*q^(5/6) + 43*q^(11/6)
+ O(q^(17/6))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0), 10*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/6, 1/3, 1/2, 2/3, 5/6), q^(7/12) + 2*q^(19/12) +
O(q^(31/12))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 2/3, 0, 1/3, 2/3), q^(7/12) + 2*q^(19/12) +
O(q^(31/12))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2), 10*q^(5/4) + O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 1/3, 0, 2/3, 1/3), q^(7/12) + 2*q^(19/12) +
O(q^(31/12))]
[(0, 0, 0, 0, 0, 0, 0, 5/6, 2/3, 1/2, 1/3, 1/6), q^(7/12) + 2*q^(19/12) +
O(q^(31/12))]
```

[17]: `picard_num(L[5])[2]`

[17]:
```
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 150*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7), -42*q^(23/28) -
6752*q^(51/28) + O(q^(79/28))]
[(0, 0, 0, 0, 0, 0, 0, 2/7, 4/7, 6/7, 1/7, 3/7, 5/7), -712*q^(9/7) +
O(q^(16/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/7, 6/7, 2/7, 5/7, 1/7, 4/7), -1210*q^(39/28) +
O(q^(67/28))]
[(0, 0, 0, 0, 0, 0, 0, 4/7, 1/7, 5/7, 2/7, 6/7, 3/7), -330*q^(8/7) +
O(q^(15/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/7, 3/7, 1/7, 6/7, 4/7, 2/7), -2*q^(15/28) -
```

2208*q^(43/28) + O(q^(71/28))]
[(0, 0, 0, 0, 0, 0, 0, 6/7, 5/7, 4/7, 3/7, 2/7, 1/7), -3*q^(4/7) - 2598*q^(11/7)
+ O(q^(18/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0), -576*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7), -3*q^(4/7) - 2598*q^(11/7)
+ O(q^(18/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/7, 4/7, 6/7, 1/7, 3/7, 5/7), -2*q^(15/28) -
2208*q^(43/28) + O(q^(71/28))]
[(0, 0, 0, 0, 0, 0, 0, 3/7, 6/7, 2/7, 5/7, 1/7, 4/7), -330*q^(8/7) +
O(q^(15/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/7, 1/7, 5/7, 2/7, 6/7, 3/7), -1210*q^(39/28) +
O(q^(67/28))]
[(0, 0, 0, 0, 0, 0, 0, 5/7, 3/7, 1/7, 6/7, 4/7, 2/7), -712*q^(9/7) +
O(q^(16/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 6/7, 5/7, 4/7, 3/7, 2/7, 1/7), -42*q^(23/28) -
6752*q^(51/28) + O(q^(79/28))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -126*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7), 38*q^(23/28) +
2752*q^(51/28) + O(q^(79/28))]
[(0, 0, 0, 0, 0, 0, 0, 2/7, 4/7, 6/7, 1/7, 3/7, 5/7), -311*q^(9/7) +
O(q^(16/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/7, 6/7, 2/7, 5/7, 1/7, 4/7), 182*q^(39/28) +
O(q^(67/28))]
[(0, 0, 0, 0, 0, 0, 0, 4/7, 1/7, 5/7, 2/7, 6/7, 3/7), q^(1/7) + 54*q^(8/7) +
O(q^(15/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/7, 3/7, 1/7, 6/7, 4/7, 2/7), -2*q^(15/28) -
768*q^(43/28) + O(q^(71/28))]
[(0, 0, 0, 0, 0, 0, 0, 6/7, 5/7, 4/7, 3/7, 2/7, 1/7), 4*q^(4/7) + 1298*q^(11/7)
+ O(q^(18/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0), -448*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7), 4*q^(4/7) + 1298*q^(11/7)
+ O(q^(18/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/7, 4/7, 6/7, 1/7, 3/7, 5/7), -2*q^(15/28) -
768*q^(43/28) + O(q^(71/28))]
[(0, 0, 0, 0, 0, 0, 0, 3/7, 6/7, 2/7, 5/7, 1/7, 4/7), q^(1/7) + 54*q^(8/7) +
O(q^(15/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/7, 1/7, 5/7, 2/7, 6/7, 3/7), 182*q^(39/28) +
O(q^(67/28))]
[(0, 0, 0, 0, 0, 0, 0, 5/7, 3/7, 1/7, 6/7, 4/7, 2/7), -311*q^(9/7) +
O(q^(16/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 6/7, 5/7, 4/7, 3/7, 2/7, 1/7), 38*q^(23/28) +
2752*q^(51/28) + O(q^(79/28))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -28*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7), 14*q^(23/28) +
352*q^(51/28) + O(q^(79/28))]

[(0, 0, 0, 0, 0, 0, 0, 2/7, 4/7, 6/7, 1/7, 3/7, 5/7), -92*q^(9/7) + O(q^(16/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/7, 6/7, 2/7, 5/7, 1/7, 4/7), 143*q^(39/28) +
O(q^(67/28))]
[(0, 0, 0, 0, 0, 0, 0, 4/7, 1/7, 5/7, 2/7, 6/7, 3/7), -66*q^(8/7) + O(q^(15/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/7, 3/7, 1/7, 6/7, 4/7, 2/7), q^(15/28) +
208*q^(43/28) + O(q^(71/28))]
[(0, 0, 0, 0, 0, 0, 0, 6/7, 5/7, 4/7, 3/7, 2/7, 1/7), -2*q^(4/7) - 220*q^(11/7)
+ O(q^(18/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0), q^(1/4) + 80*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7), -2*q^(4/7) - 220*q^(11/7)
+ O(q^(18/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/7, 4/7, 6/7, 1/7, 3/7, 5/7), q^(15/28) +
208*q^(43/28) + O(q^(71/28))]
[(0, 0, 0, 0, 0, 0, 0, 3/7, 6/7, 2/7, 5/7, 1/7, 4/7), -66*q^(8/7) + O(q^(15/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/7, 1/7, 5/7, 2/7, 6/7, 3/7), 143*q^(39/28) +
O(q^(67/28))]
[(0, 0, 0, 0, 0, 0, 0, 5/7, 3/7, 1/7, 6/7, 4/7, 2/7), -92*q^(9/7) + O(q^(16/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 6/7, 5/7, 4/7, 3/7, 2/7, 1/7), 14*q^(23/28) +
352*q^(51/28) + O(q^(79/28))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -56*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7), 2*q^(23/28) +
128*q^(51/28) + O(q^(79/28))]
[(0, 0, 0, 0, 0, 0, 0, 2/7, 4/7, 6/7, 1/7, 3/7, 5/7), q^(2/7) + 122*q^(9/7) +
O(q^(16/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/7, 6/7, 2/7, 5/7, 1/7, 4/7), -110*q^(39/28) +
O(q^(67/28))]
[(0, 0, 0, 0, 0, 0, 0, 4/7, 1/7, 5/7, 2/7, 6/7, 3/7), -55*q^(8/7) + O(q^(15/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/7, 3/7, 1/7, 6/7, 4/7, 2/7), 2*q^(15/28) +
240*q^(43/28) + O(q^(71/28))]
[(0, 0, 0, 0, 0, 0, 0, 6/7, 5/7, 4/7, 3/7, 2/7, 1/7), q^(4/7) + 66*q^(11/7) +
O(q^(18/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0), -112*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7), q^(4/7) + 66*q^(11/7) +
O(q^(18/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/7, 4/7, 6/7, 1/7, 3/7, 5/7), 2*q^(15/28) +
240*q^(43/28) + O(q^(71/28))]
[(0, 0, 0, 0, 0, 0, 0, 3/7, 6/7, 2/7, 5/7, 1/7, 4/7), -55*q^(8/7) + O(q^(15/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/7, 1/7, 5/7, 2/7, 6/7, 3/7), -110*q^(39/28) +
O(q^(67/28))]
[(0, 0, 0, 0, 0, 0, 0, 5/7, 3/7, 1/7, 6/7, 4/7, 2/7), q^(2/7) + 122*q^(9/7) +
O(q^(16/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 6/7, 5/7, 4/7, 3/7, 2/7, 1/7), 2*q^(23/28) +
128*q^(51/28) + O(q^(79/28))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -28*q + O(q^2)]

```
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7), -11*q^(23/28) +
10*q^(51/28) + O(q^(79/28))]
[(0, 0, 0, 0, 0, 0, 0, 2/7, 4/7, 6/7, 1/7, 3/7, 5/7), -20*q^(9/7) + O(q^(16/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/7, 6/7, 2/7, 5/7, 1/7, 4/7), q^(11/28) -
19*q^(39/28) + O(q^(67/28))]
[(0, 0, 0, 0, 0, 0, 0, 4/7, 1/7, 5/7, 2/7, 6/7, 3/7), 10*q^(8/7) + O(q^(15/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/7, 3/7, 1/7, 6/7, 4/7, 2/7), q^(15/28) +
33*q^(43/28) + O(q^(71/28))]
[(0, 0, 0, 0, 0, 0, 0, 6/7, 5/7, 4/7, 3/7, 2/7, 1/7), 2*q^(4/7) + 52*q^(11/7) +
O(q^(18/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0), 42*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7), 2*q^(4/7) + 52*q^(11/7) +
O(q^(18/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/7, 4/7, 6/7, 1/7, 3/7, 5/7), q^(15/28) +
33*q^(43/28) + O(q^(71/28))]
[(0, 0, 0, 0, 0, 0, 0, 3/7, 6/7, 2/7, 5/7, 1/7, 4/7), 10*q^(8/7) + O(q^(15/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/7, 1/7, 5/7, 2/7, 6/7, 3/7), q^(11/28) -
19*q^(39/28) + O(q^(67/28))]
[(0, 0, 0, 0, 0, 0, 0, 5/7, 3/7, 1/7, 6/7, 4/7, 2/7), -20*q^(9/7) + O(q^(16/7))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 6/7, 5/7, 4/7, 3/7, 2/7, 1/7), -11*q^(23/28) +
10*q^(51/28) + O(q^(79/28))]
```

[18]: `picard_num(L[6])[2]`

[18]:
```
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 142*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8), -40*q^(13/16) -
4768*q^(29/16) + O(q^(45/16))]
[(0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4, 0, 1/4, 1/2, 3/4), -512*q^(5/4) +
O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/8, 3/4, 1/8, 1/2, 7/8, 1/4, 5/8), -680*q^(21/16)
+ O(q^(37/16))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -134*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/8, 1/4, 7/8, 1/2, 1/8, 3/4, 3/8), -680*q^(21/16)
+ O(q^(37/16))]
[(0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4, 0, 3/4, 1/2, 1/4), -512*q^(5/4) +
O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8), -40*q^(13/16) -
4768*q^(29/16) + O(q^(45/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), -496*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8), -4*q^(9/16) -
1944*q^(25/16) + O(q^(41/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/4, 1/2, 3/4, 0, 1/4, 1/2, 3/4), -2*q^(1/2) -
1488*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 3/8, 3/4, 1/8, 1/2, 7/8, 1/4, 5/8), -196*q^(17/16) +
O(q^(33/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -528*q^(5/4) +
O(q^(9/4))]
```

[(0, 0, 0, 0, 0, 0, 0, 5/8, 1/4, 7/8, 1/2, 1/8, 3/4, 3/8), -196*q^(17/16) + O(q^(33/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/4, 1/2, 1/4, 0, 3/4, 1/2, 1/4), -2*q^(1/2) - 1488*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8), -4*q^(9/16) - 1944*q^(25/16) + O(q^(41/16))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -134*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8), 40*q^(13/16) + 4768*q^(29/16) + O(q^(45/16))]
[(0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4, 0, 1/4, 1/2, 3/4), -512*q^(5/4) + O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/8, 3/4, 1/8, 1/2, 7/8, 1/4, 5/8), 680*q^(21/16) + O(q^(37/16))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), 1 - 142*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/8, 1/4, 7/8, 1/2, 1/8, 3/4, 3/8), 680*q^(21/16) + O(q^(37/16))]
[(0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4, 0, 3/4, 1/2, 1/4), -512*q^(5/4) + O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8), 40*q^(13/16) + 4768*q^(29/16) + O(q^(45/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), -528*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8), 4*q^(9/16) + 1944*q^(25/16) + O(q^(41/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/4, 1/2, 3/4, 0, 1/4, 1/2, 3/4), -2*q^(1/2) - 1488*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 3/8, 3/4, 1/8, 1/2, 7/8, 1/4, 5/8), 196*q^(17/16) + O(q^(33/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -496*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 5/8, 1/4, 7/8, 1/2, 1/8, 3/4, 3/8), 196*q^(17/16) + O(q^(33/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/4, 1/2, 1/4, 0, 3/4, 1/2, 1/4), -2*q^(1/2) - 1488*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8), 4*q^(9/16) + 1944*q^(25/16) + O(q^(41/16))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -192*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8), 40*q^(13/16) + 3520*q^(29/16) + O(q^(45/16))]
[(0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4, 0, 1/4, 1/2, 3/4), O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/8, 3/4, 1/8, 1/2, 7/8, 1/4, 5/8), -568*q^(21/16) + O(q^(37/16))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), 192*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/8, 1/4, 7/8, 1/2, 1/8, 3/4, 3/8), -568*q^(21/16) + O(q^(37/16))]
[(0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4, 0, 3/4, 1/2, 1/4), O(q^(9/4))]

[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8), 40*q^(13/16) +
3520*q^(29/16) + O(q^(45/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), -640*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8), 5*q^(9/16) +
1555*q^(25/16) + O(q^(41/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/4, 1/2, 3/4, 0, 1/4, 1/2, 3/4), O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 3/8, 3/4, 1/8, 1/2, 7/8, 1/4, 5/8), q^(1/16) -
194*q^(17/16) + O(q^(33/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), 640*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 5/8, 1/4, 7/8, 1/2, 1/8, 3/4, 3/8), q^(1/16) -
194*q^(17/16) + O(q^(33/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/4, 1/2, 1/4, 0, 3/4, 1/2, 1/4), O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8), 5*q^(9/16) +
1555*q^(25/16) + O(q^(41/16))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -56*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8), O(q^(45/16))]
[(0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4, 0, 1/4, 1/2, 3/4), q^(1/4) + 122*q^(5/4) +
O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/8, 3/4, 1/8, 1/2, 7/8, 1/4, 5/8), O(q^(37/16))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -56*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/8, 1/4, 7/8, 1/2, 1/8, 3/4, 3/8), O(q^(37/16))]
[(0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4, 0, 3/4, 1/2, 1/4), q^(1/4) + 122*q^(5/4) +
O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8), O(q^(45/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), -112*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8), O(q^(41/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/4, 1/2, 3/4, 0, 1/4, 1/2, 3/4), 2*q^(1/2) +
240*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 3/8, 3/4, 1/8, 1/2, 7/8, 1/4, 5/8), O(q^(33/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -112*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 5/8, 1/4, 7/8, 1/2, 1/8, 3/4, 3/8), O(q^(33/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/4, 1/2, 1/4, 0, 3/4, 1/2, 1/4), 2*q^(1/2) +
240*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8), O(q^(41/16))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -24*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8), 12*q^(13/16) +
176*q^(29/16) + O(q^(45/16))]
[(0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4, 0, 1/4, 1/2, 3/4), -56*q^(5/4) +
O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/8, 3/4, 1/8, 1/2, 7/8, 1/4, 5/8), 76*q^(21/16) +
O(q^(37/16))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/8, 1/4, 7/8, 1/2, 1/8, 3/4, 3/8), 76*q^(21/16) +

O(q^(37/16))]
[(0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4, 0, 3/4, 1/2, 1/4), -56*q^(5/4) +
O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8), 12*q^(13/16) +
176*q^(29/16) + O(q^(45/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), q^(1/4) + 56*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8), -2*q^(9/16) -
140*q^(25/16) + O(q^(41/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/4, 1/2, 3/4, 0, 1/4, 1/2, 3/4), q^(1/2) +
120*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 3/8, 3/4, 1/8, 1/2, 7/8, 1/4, 5/8), -34*q^(17/16) +
O(q^(33/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), 66*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 5/8, 1/4, 7/8, 1/2, 1/8, 3/4, 3/8), -34*q^(17/16) +
O(q^(33/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/4, 1/2, 1/4, 0, 3/4, 1/2, 1/4), q^(1/2) +
120*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8), -2*q^(9/16) -
140*q^(25/16) + O(q^(41/16))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8), -12*q^(13/16) -
176*q^(29/16) + O(q^(45/16))]
[(0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4, 0, 1/4, 1/2, 3/4), -56*q^(5/4) +
O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/8, 3/4, 1/8, 1/2, 7/8, 1/4, 5/8), -76*q^(21/16) +
O(q^(37/16))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -24*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/8, 1/4, 7/8, 1/2, 1/8, 3/4, 3/8), -76*q^(21/16) +
O(q^(37/16))]
[(0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4, 0, 3/4, 1/2, 1/4), -56*q^(5/4) +
O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8), -12*q^(13/16) -
176*q^(29/16) + O(q^(45/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), 66*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8), 2*q^(9/16) +
140*q^(25/16) + O(q^(41/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/4, 1/2, 3/4, 0, 1/4, 1/2, 3/4), q^(1/2) +
120*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 3/8, 3/4, 1/8, 1/2, 7/8, 1/4, 5/8), 34*q^(17/16) +
O(q^(33/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), q^(1/4) + 56*q^(5/4)
+ O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 5/8, 1/4, 7/8, 1/2, 1/8, 3/4, 3/8), 34*q^(17/16) +
O(q^(33/16))]

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[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/4, 1/2, 1/4, 0, 3/4, 1/2, 1/4), q^(1/2) +
120*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8), 2*q^(9/16) +
140*q^(25/16) + O(q^(41/16))]
------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8), -11*q^(13/16) -
55*q^(29/16) + O(q^(45/16))]
[(0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4, 0, 1/4, 1/2, 3/4), O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/8, 3/4, 1/8, 1/2, 7/8, 1/4, 5/8), q^(5/16) +
44*q^(21/16) + O(q^(37/16))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), 32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/8, 1/4, 7/8, 1/2, 1/8, 3/4, 3/8), q^(5/16) +
44*q^(21/16) + O(q^(37/16))]
[(0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4, 0, 3/4, 1/2, 1/4), O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8), -11*q^(13/16) -
55*q^(29/16) + O(q^(45/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), 64*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8), 2*q^(9/16) +
84*q^(25/16) + O(q^(41/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/4, 1/2, 3/4, 0, 1/4, 1/2, 3/4), O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 3/8, 3/4, 1/8, 1/2, 7/8, 1/4, 5/8), -22*q^(17/16) +
O(q^(33/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -64*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 5/8, 1/4, 7/8, 1/2, 1/8, 3/4, 3/8), -22*q^(17/16) +
O(q^(33/16))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/4, 1/2, 1/4, 0, 3/4, 1/2, 1/4), O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 7/8, 3/4, 5/8, 1/2, 3/8, 1/4, 1/8), 2*q^(9/16) +
84*q^(25/16) + O(q^(41/16))]
```

[19]: `picard_num(L[7])[2]`

[19]:
```
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 132*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/9, 2/9, 1/3, 4/9, 5/9, 2/3, 7/9, 8/9),
-38*q^(29/36) - 3312*q^(65/36) + O(q^(101/36))]
[(0, 0, 0, 0, 0, 0, 0, 2/9, 4/9, 2/3, 8/9, 1/9, 1/3, 5/9, 7/9), -378*q^(11/9) +
O(q^(20/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 2/3, 0, 1/3, 2/3, 0, 1/3, 2/3), -414*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 4/9, 8/9, 1/3, 7/9, 2/9, 2/3, 1/9, 5/9), -67*q^(8/9) -
4181*q^(17/9) + O(q^(26/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/9, 1/9, 2/3, 2/9, 7/9, 1/3, 8/9, 4/9),
-264*q^(41/36) + O(q^(77/36))]
[(0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 2/3, 1/3, 0, 2/3, 1/3), -129*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/9, 5/9, 1/3, 1/9, 8/9, 2/3, 4/9, 2/9),
-2*q^(17/36) - 1036*q^(53/36) + O(q^(89/36))]
```

[(0, 0, 0, 0, 0, 0, 0, 8/9, 7/9, 2/3, 5/9, 4/9, 1/3, 2/9, 1/9), -5*q^(5/9) -
1434*q^(14/9) + O(q^(23/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0), -420*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/9, 2/9, 1/3, 4/9, 5/9, 2/3, 7/9, 8/9), -5*q^(5/9) -
1434*q^(14/9) + O(q^(23/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/9, 4/9, 2/3, 8/9, 1/9, 1/3, 5/9, 7/9),
-2*q^(17/36) - 1036*q^(53/36) + O(q^(89/36))]
[(0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 1/3, 2/3, 0, 1/3, 2/3), -129*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/9, 8/9, 1/3, 7/9, 2/9, 2/3, 1/9, 5/9),
-264*q^(41/36) + O(q^(77/36))]
[(0, 0, 0, 0, 0, 0, 0, 5/9, 1/9, 2/3, 2/9, 7/9, 1/3, 8/9, 4/9), -67*q^(8/9) -
4181*q^(17/9) + O(q^(26/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 1/3, 0, 2/3, 1/3, 0, 2/3, 1/3), -414*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 7/9, 5/9, 1/3, 1/9, 8/9, 2/3, 4/9, 2/9), -378*q^(11/9) +
O(q^(20/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 8/9, 7/9, 2/3, 5/9, 4/9, 1/3, 2/9, 1/9),
-38*q^(29/36) - 3312*q^(65/36) + O(q^(101/36))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -258*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/9, 2/9, 1/3, 4/9, 5/9, 2/3, 7/9, 8/9),
38*q^(29/36) + 3312*q^(65/36) + O(q^(101/36))]
[(0, 0, 0, 0, 0, 0, 0, 2/9, 4/9, 2/3, 8/9, 1/9, 1/3, 5/9, 7/9), 378*q^(11/9) +
O(q^(20/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 2/3, 0, 1/3, 2/3, 0, 1/3, 2/3), -834*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 4/9, 8/9, 1/3, 7/9, 2/9, 2/3, 1/9, 5/9), 67*q^(8/9) +
4181*q^(17/9) + O(q^(26/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/9, 1/9, 2/3, 2/9, 7/9, 1/3, 8/9, 4/9),
264*q^(41/36) + O(q^(77/36))]
[(0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 2/3, 1/3, 0, 2/3, 1/3), 1 - 261*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/9, 5/9, 1/3, 1/9, 8/9, 2/3, 4/9, 2/9),
2*q^(17/36) + 1036*q^(53/36) + O(q^(89/36))]
[(0, 0, 0, 0, 0, 0, 0, 8/9, 7/9, 2/3, 5/9, 4/9, 1/3, 2/9, 1/9), 5*q^(5/9) +
1434*q^(14/9) + O(q^(23/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0), -828*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/9, 2/9, 1/3, 4/9, 5/9, 2/3, 7/9, 8/9), 5*q^(5/9) +
1434*q^(14/9) + O(q^(23/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/9, 4/9, 2/3, 8/9, 1/9, 1/3, 5/9, 7/9),
2*q^(17/36) + 1036*q^(53/36) + O(q^(89/36))]
[(0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 1/3, 2/3, 0, 1/3, 2/3), 1 - 261*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/9, 8/9, 1/3, 7/9, 2/9, 2/3, 1/9, 5/9),
264*q^(41/36) + O(q^(77/36))]
[(0, 0, 0, 0, 0, 0, 0, 5/9, 1/9, 2/3, 2/9, 7/9, 1/3, 8/9, 4/9), 67*q^(8/9) +
4181*q^(17/9) + O(q^(26/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 1/3, 0, 2/3, 1/3, 0, 2/3, 1/3), -834*q^(5/4) +
O(q^(9/4))]

[(0, 0, 0, 0, 0, 0, 0, 7/9, 5/9, 1/3, 1/9, 8/9, 2/3, 4/9, 2/9), 378*q^(11/9) +
O(q^(20/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 8/9, 7/9, 2/3, 5/9, 4/9, 1/3, 2/9, 1/9),
38*q^(29/36) + 3312*q^(65/36) + O(q^(101/36))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -108*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/9, 2/9, 1/3, 4/9, 5/9, 2/3, 7/9, 8/9),
-37*q^(29/36) - 1088*q^(65/36) + O(q^(101/36))]
[(0, 0, 0, 0, 0, 0, 0, 2/9, 4/9, 2/3, 8/9, 1/9, 1/3, 5/9, 7/9), -172*q^(11/9) +
O(q^(20/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 2/3, 0, 1/3, 2/3, 0, 1/3, 2/3), -135*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 4/9, 8/9, 1/3, 7/9, 2/9, 2/3, 1/9, 5/9), -8*q^(8/9) -
294*q^(17/9) + O(q^(26/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/9, 1/9, 2/3, 2/9, 7/9, 1/3, 8/9, 4/9), q^(5/36) +
24*q^(41/36) + O(q^(77/36))]
[(0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 2/3, 1/3, 0, 2/3, 1/3), 54*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/9, 5/9, 1/3, 1/9, 8/9, 2/3, 4/9, 2/9),
2*q^(17/36) + 411*q^(53/36) + O(q^(89/36))]
[(0, 0, 0, 0, 0, 0, 0, 8/9, 7/9, 2/3, 5/9, 4/9, 1/3, 2/9, 1/9), 6*q^(5/9) +
644*q^(14/9) + O(q^(23/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0), 270*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/9, 2/9, 1/3, 4/9, 5/9, 2/3, 7/9, 8/9), 6*q^(5/9) +
644*q^(14/9) + O(q^(23/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/9, 4/9, 2/3, 8/9, 1/9, 1/3, 5/9, 7/9),
2*q^(17/36) + 411*q^(53/36) + O(q^(89/36))]
[(0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 1/3, 2/3, 0, 1/3, 2/3), 54*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/9, 8/9, 1/3, 7/9, 2/9, 2/3, 1/9, 5/9), q^(5/36) +
24*q^(41/36) + O(q^(77/36))]
[(0, 0, 0, 0, 0, 0, 0, 5/9, 1/9, 2/3, 2/9, 7/9, 1/3, 8/9, 4/9), -8*q^(8/9) -
294*q^(17/9) + O(q^(26/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 1/3, 0, 2/3, 1/3, 0, 2/3, 1/3), -135*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 7/9, 5/9, 1/3, 1/9, 8/9, 2/3, 4/9, 2/9), -172*q^(11/9) +
O(q^(20/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 8/9, 7/9, 2/3, 5/9, 4/9, 1/3, 2/9, 1/9),
-37*q^(29/36) - 1088*q^(65/36) + O(q^(101/36))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -54*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/9, 2/9, 1/3, 4/9, 5/9, 2/3, 7/9, 8/9),
-2*q^(29/36) - 80*q^(65/36) + O(q^(101/36))]
[(0, 0, 0, 0, 0, 0, 0, 2/9, 4/9, 2/3, 8/9, 1/9, 1/3, 5/9, 7/9), q^(2/9) +
96*q^(11/9) + O(q^(20/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 2/3, 0, 1/3, 2/3, 0, 1/3, 2/3), 54*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 4/9, 8/9, 1/3, 7/9, 2/9, 2/3, 1/9, 5/9), -28*q^(8/9) -
203*q^(17/9) + O(q^(26/9))]

[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/9, 1/9, 2/3, 2/9, 7/9, 1/3, 8/9, 4/9),
-56*q^(41/36) + O(q^(77/36))]
[(0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 2/3, 1/3, 0, 2/3, 1/3), 27*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/9, 5/9, 1/3, 1/9, 8/9, 2/3, 4/9, 2/9),
2*q^(17/36) + 188*q^(53/36) + O(q^(89/36))]
[(0, 0, 0, 0, 0, 0, 0, 8/9, 7/9, 2/3, 5/9, 4/9, 1/3, 2/9, 1/9), -q^(5/9) -
42*q^(14/9) + O(q^(23/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0), -108*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/9, 2/9, 1/3, 4/9, 5/9, 2/3, 7/9, 8/9), -q^(5/9) -
42*q^(14/9) + O(q^(23/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/9, 4/9, 2/3, 8/9, 1/9, 1/3, 5/9, 7/9),
2*q^(17/36) + 188*q^(53/36) + O(q^(89/36))]
[(0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 1/3, 2/3, 0, 1/3, 2/3), 27*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/9, 8/9, 1/3, 7/9, 2/9, 2/3, 1/9, 5/9),
-56*q^(41/36) + O(q^(77/36))]
[(0, 0, 0, 0, 0, 0, 0, 5/9, 1/9, 2/3, 2/9, 7/9, 1/3, 8/9, 4/9), -28*q^(8/9) -
203*q^(17/9) + O(q^(26/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 1/3, 0, 2/3, 1/3, 0, 2/3, 1/3), 54*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 7/9, 5/9, 1/3, 1/9, 8/9, 2/3, 4/9, 2/9), q^(2/9) +
96*q^(11/9) + O(q^(20/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 8/9, 7/9, 2/3, 5/9, 4/9, 1/3, 2/9, 1/9),
-2*q^(29/36) - 80*q^(65/36) + O(q^(101/36))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -36*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/9, 2/9, 1/3, 4/9, 5/9, 2/3, 7/9, 8/9),
-10*q^(29/36) - 72*q^(65/36) + O(q^(101/36))]
[(0, 0, 0, 0, 0, 0, 0, 2/9, 4/9, 2/3, 8/9, 1/9, 1/3, 5/9, 7/9), 36*q^(11/9) +
O(q^(20/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 2/3, 0, 1/3, 2/3, 0, 1/3, 2/3), q^(1/4) +
78*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 4/9, 8/9, 1/3, 7/9, 2/9, 2/3, 1/9, 5/9), 16*q^(8/9) +
38*q^(17/9) + O(q^(26/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/9, 1/9, 2/3, 2/9, 7/9, 1/3, 8/9, 4/9),
-33*q^(41/36) + O(q^(77/36))]
[(0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 2/3, 1/3, 0, 2/3, 1/3), -38*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/9, 5/9, 1/3, 1/9, 8/9, 2/3, 4/9, 2/9), -q^(17/36)
- 68*q^(53/36) + O(q^(89/36))]
[(0, 0, 0, 0, 0, 0, 0, 8/9, 7/9, 2/3, 5/9, 4/9, 1/3, 2/9, 1/9), 2*q^(5/9) +
84*q^(14/9) + O(q^(23/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0), 84*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/9, 2/9, 1/3, 4/9, 5/9, 2/3, 7/9, 8/9), 2*q^(5/9) +
84*q^(14/9) + O(q^(23/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/9, 4/9, 2/3, 8/9, 1/9, 1/3, 5/9, 7/9), -q^(17/36)
- 68*q^(53/36) + O(q^(89/36))]
[(0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 1/3, 2/3, 0, 1/3, 2/3), -38*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/9, 8/9, 1/3, 7/9, 2/9, 2/3, 1/9, 5/9),

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-33*q^(41/36) + O(q^(77/36))]
[(0, 0, 0, 0, 0, 0, 0, 5/9, 1/9, 2/3, 2/9, 7/9, 1/3, 8/9, 4/9), 16*q^(8/9) +
38*q^(17/9) + O(q^(26/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 1/3, 0, 2/3, 1/3, 0, 2/3, 1/3), q^(1/4) +
78*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 7/9, 5/9, 1/3, 1/9, 8/9, 2/3, 4/9, 2/9), 36*q^(11/9) +
O(q^(20/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 8/9, 7/9, 2/3, 5/9, 4/9, 1/3, 2/9, 1/9),
-10*q^(29/36) - 72*q^(65/36) + O(q^(101/36))]
--------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -20*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/9, 2/9, 1/3, 4/9, 5/9, 2/3, 7/9, 8/9),
10*q^(29/36) + 72*q^(65/36) + O(q^(101/36))]
[(0, 0, 0, 0, 0, 0, 0, 2/9, 4/9, 2/3, 8/9, 1/9, 1/3, 5/9, 7/9), -36*q^(11/9) +
O(q^(20/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 2/3, 0, 1/3, 2/3, 0, 1/3, 2/3), 42*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 4/9, 8/9, 1/3, 7/9, 2/9, 2/3, 1/9, 5/9), -16*q^(8/9) -
38*q^(17/9) + O(q^(26/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/9, 1/9, 2/3, 2/9, 7/9, 1/3, 8/9, 4/9),
33*q^(41/36) + O(q^(77/36))]
[(0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 2/3, 1/3, 0, 2/3, 1/3), -18*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/9, 5/9, 1/3, 1/9, 8/9, 2/3, 4/9, 2/9), q^(17/36)
+ 68*q^(53/36) + O(q^(89/36))]
[(0, 0, 0, 0, 0, 0, 0, 8/9, 7/9, 2/3, 5/9, 4/9, 1/3, 2/9, 1/9), -2*q^(5/9) -
84*q^(14/9) + O(q^(23/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0), q^(1/4) + 36*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/9, 2/9, 1/3, 4/9, 5/9, 2/3, 7/9, 8/9), -2*q^(5/9) -
84*q^(14/9) + O(q^(23/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/9, 4/9, 2/3, 8/9, 1/9, 1/3, 5/9, 7/9), q^(17/36)
+ 68*q^(53/36) + O(q^(89/36))]
[(0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 1/3, 2/3, 0, 1/3, 2/3), -18*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/9, 8/9, 1/3, 7/9, 2/9, 2/3, 1/9, 5/9),
33*q^(41/36) + O(q^(77/36))]
[(0, 0, 0, 0, 0, 0, 0, 5/9, 1/9, 2/3, 2/9, 7/9, 1/3, 8/9, 4/9), -16*q^(8/9) -
38*q^(17/9) + O(q^(26/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 1/3, 0, 2/3, 1/3, 0, 2/3, 1/3), 42*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 7/9, 5/9, 1/3, 1/9, 8/9, 2/3, 4/9, 2/9), -36*q^(11/9) +
O(q^(20/9))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 8/9, 7/9, 2/3, 5/9, 4/9, 1/3, 2/9, 1/9),
10*q^(29/36) + 72*q^(65/36) + O(q^(101/36))]
```

[21]: `picard_num(L[8])[2]`

```
[21]: [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 120*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/10, 1/5, 3/10, 2/5, 1/2, 3/5, 7/10, 4/5, 9/10),
-36*q^(4/5) - 2232*q^(9/5) + O(q^(14/5))]
[(0, 0, 0, 0, 0, 0, 0, 1/5, 2/5, 3/5, 4/5, 0, 1/5, 2/5, 3/5, 4/5), -282*q^(6/5)
+ O(q^(11/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/10, 3/5, 9/10, 1/5, 1/2, 4/5, 1/10, 2/5, 7/10),
-268*q^(6/5) + O(q^(11/5))]
[(0, 0, 0, 0, 0, 0, 0, 2/5, 4/5, 1/5, 3/5, 0, 2/5, 4/5, 1/5, 3/5), -39*q^(4/5) -
2168*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -120*q +
O(q^2)]
[(0, 0, 0, 0, 0, 0, 0, 3/5, 1/5, 4/5, 2/5, 0, 3/5, 1/5, 4/5, 2/5), -39*q^(4/5) -
2168*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/10, 2/5, 1/10, 4/5, 1/2, 1/5, 9/10, 3/5, 3/10),
-268*q^(6/5) + O(q^(11/5))]
[(0, 0, 0, 0, 0, 0, 0, 4/5, 3/5, 2/5, 1/5, 0, 4/5, 3/5, 2/5, 1/5), -282*q^(6/5)
+ O(q^(11/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 9/10, 4/5, 7/10, 3/5, 1/2, 2/5, 3/10, 1/5, 1/10),
-36*q^(4/5) - 2232*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0), -348*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/10, 1/5, 3/10, 2/5, 1/2, 3/5, 7/10, 4/5, 9/10),
-6*q^(11/20) - 1032*q^(31/20) + O(q^(51/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/5, 2/5, 3/5, 4/5, 0, 1/5, 2/5, 3/5, 4/5),
-2*q^(9/20) - 732*q^(29/20) + O(q^(49/20))]
[(0, 0, 0, 0, 0, 0, 0, 3/10, 3/5, 9/10, 1/5, 1/2, 4/5, 1/10, 2/5, 7/10),
-90*q^(19/20) - 3228*q^(39/20) + O(q^(59/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/5, 4/5, 1/5, 3/5, 0, 2/5, 4/5, 1/5, 3/5),
-144*q^(21/20) + O(q^(41/20))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -28*q^(3/4) -
1884*q^(7/4) + O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/5, 1/5, 4/5, 2/5, 0, 3/5, 1/5, 4/5, 2/5),
-144*q^(21/20) + O(q^(41/20))]
[(0, 0, 0, 0, 0, 0, 0, 7/10, 2/5, 1/10, 4/5, 1/2, 1/5, 9/10, 3/5, 3/10),
-90*q^(19/20) - 3228*q^(39/20) + O(q^(59/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/5, 3/5, 2/5, 1/5, 0, 4/5, 3/5, 2/5, 1/5),
-2*q^(9/20) - 732*q^(29/20) + O(q^(49/20))]
[(0, 0, 0, 0, 0, 0, 0, 9/10, 4/5, 7/10, 3/5, 1/2, 2/5, 3/10, 1/5, 1/10),
-6*q^(11/20) - 1032*q^(31/20) + O(q^(51/20))]
-----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -120*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/10, 1/5, 3/10, 2/5, 1/2, 3/5, 7/10, 4/5, 9/10),
-39*q^(4/5) - 2168*q^(9/5) + O(q^(14/5))]
[(0, 0, 0, 0, 0, 0, 0, 1/5, 2/5, 3/5, 4/5, 0, 1/5, 2/5, 3/5, 4/5), -268*q^(6/5)
+ O(q^(11/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/10, 3/5, 9/10, 1/5, 1/2, 4/5, 1/10, 2/5, 7/10),
-282*q^(6/5) + O(q^(11/5))]
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[(0, 0, 0, 0, 0, 0, 0, 2/5, 4/5, 1/5, 3/5, 0, 2/5, 4/5, 1/5, 3/5), -36*q^(4/5) -
2232*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), 1 - 120*q +
O(q^2)]
[(0, 0, 0, 0, 0, 0, 0, 3/5, 1/5, 4/5, 2/5, 0, 3/5, 1/5, 4/5, 2/5), -36*q^(4/5) -
2232*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/10, 2/5, 1/10, 4/5, 1/2, 1/5, 9/10, 3/5, 3/10),
-282*q^(6/5) + O(q^(11/5))]
[(0, 0, 0, 0, 0, 0, 0, 4/5, 3/5, 2/5, 1/5, 0, 4/5, 3/5, 2/5, 1/5), -268*q^(6/5)
+ O(q^(11/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 9/10, 4/5, 7/10, 3/5, 1/2, 2/5, 3/10, 1/5, 1/10),
-39*q^(4/5) - 2168*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0), 348*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/10, 1/5, 3/10, 2/5, 1/2, 3/5, 7/10, 4/5, 9/10),
6*q^(11/20) + 1032*q^(31/20) + O(q^(51/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/5, 2/5, 3/5, 4/5, 0, 1/5, 2/5, 3/5, 4/5),
2*q^(9/20) + 732*q^(29/20) + O(q^(49/20))]
[(0, 0, 0, 0, 0, 0, 0, 3/10, 3/5, 9/10, 1/5, 1/2, 4/5, 1/10, 2/5, 7/10),
90*q^(19/20) + 3228*q^(39/20) + O(q^(59/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/5, 4/5, 1/5, 3/5, 0, 2/5, 4/5, 1/5, 3/5),
144*q^(21/20) + O(q^(41/20))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), 28*q^(3/4) +
1884*q^(7/4) + O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/5, 1/5, 4/5, 2/5, 0, 3/5, 1/5, 4/5, 2/5),
144*q^(21/20) + O(q^(41/20))]
[(0, 0, 0, 0, 0, 0, 0, 7/10, 2/5, 1/10, 4/5, 1/2, 1/5, 9/10, 3/5, 3/10),
90*q^(19/20) + 3228*q^(39/20) + O(q^(59/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/5, 3/5, 2/5, 1/5, 0, 4/5, 3/5, 2/5, 1/5),
2*q^(9/20) + 732*q^(29/20) + O(q^(49/20))]
[(0, 0, 0, 0, 0, 0, 0, 9/10, 4/5, 7/10, 3/5, 1/2, 2/5, 3/10, 1/5, 1/10),
6*q^(11/20) + 1032*q^(31/20) + O(q^(51/20))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -160*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/10, 1/5, 3/10, 2/5, 1/2, 3/5, 7/10, 4/5, 9/10),
-48*q^(4/5) - 1888*q^(9/5) + O(q^(14/5))]
[(0, 0, 0, 0, 0, 0, 0, 1/5, 2/5, 3/5, 4/5, 0, 1/5, 2/5, 3/5, 4/5), -112*q^(6/5)
+ O(q^(11/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/10, 3/5, 9/10, 1/5, 1/2, 4/5, 1/10, 2/5, 7/10),
112*q^(6/5) + O(q^(11/5))]
[(0, 0, 0, 0, 0, 0, 0, 2/5, 4/5, 1/5, 3/5, 0, 2/5, 4/5, 1/5, 3/5), 48*q^(4/5) +
1888*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), 160*q +
O(q^2)]
[(0, 0, 0, 0, 0, 0, 0, 3/5, 1/5, 4/5, 2/5, 0, 3/5, 1/5, 4/5, 2/5), 48*q^(4/5) +
1888*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/10, 2/5, 1/10, 4/5, 1/2, 1/5, 9/10, 3/5, 3/10),

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112*q^(6/5) + O(q^(11/5))]
[(0, 0, 0, 0, 0, 0, 0, 4/5, 3/5, 2/5, 1/5, 0, 4/5, 3/5, 2/5, 1/5), -112*q^(6/5)
+ O(q^(11/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 9/10, 4/5, 7/10, 3/5, 1/2, 2/5, 3/10, 1/5, 1/10),
-48*q^(4/5) - 1888*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0), 450*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/10, 1/5, 3/10, 2/5, 1/2, 3/5, 7/10, 4/5, 9/10),
8*q^(11/20) + 1000*q^(31/20) + O(q^(51/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/5, 2/5, 3/5, 4/5, 0, 1/5, 2/5, 3/5, 4/5),
q^(9/20) + 278*q^(29/20) + O(q^(49/20))]
[(0, 0, 0, 0, 0, 0, 0, 3/10, 3/5, 9/10, 1/5, 1/2, 4/5, 1/10, 2/5, 7/10),
-32*q^(19/20) - 1136*q^(39/20) + O(q^(59/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/5, 4/5, 1/5, 3/5, 0, 2/5, 4/5, 1/5, 3/5),
q^(1/20) - 170*q^(21/20) + O(q^(41/20))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -40*q^(3/4) -
2160*q^(7/4) + O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/5, 1/5, 4/5, 2/5, 0, 3/5, 1/5, 4/5, 2/5),
q^(1/20) - 170*q^(21/20) + O(q^(41/20))]
[(0, 0, 0, 0, 0, 0, 0, 7/10, 2/5, 1/10, 4/5, 1/2, 1/5, 9/10, 3/5, 3/10),
-32*q^(19/20) - 1136*q^(39/20) + O(q^(59/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/5, 3/5, 2/5, 1/5, 0, 4/5, 3/5, 2/5, 1/5),
q^(9/20) + 278*q^(29/20) + O(q^(49/20))]
[(0, 0, 0, 0, 0, 0, 0, 9/10, 4/5, 7/10, 3/5, 1/2, 2/5, 3/10, 1/5, 1/10),
8*q^(11/20) + 1000*q^(31/20) + O(q^(51/20))]
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[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -50*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/10, 1/5, 3/10, 2/5, 1/2, 3/5, 7/10, 4/5, 9/10),
-4*q^(4/5) - 104*q^(9/5) + O(q^(14/5))]
[(0, 0, 0, 0, 0, 0, 0, 1/5, 2/5, 3/5, 4/5, 0, 1/5, 2/5, 3/5, 4/5), q^(1/5) +
64*q^(6/5) + O(q^(11/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/10, 3/5, 9/10, 1/5, 1/2, 4/5, 1/10, 2/5, 7/10),
68*q^(6/5) + O(q^(11/5))]
[(0, 0, 0, 0, 0, 0, 0, 2/5, 4/5, 1/5, 3/5, 0, 2/5, 4/5, 1/5, 3/5), -8*q^(4/5) -
49*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -40*q +
O(q^2)]
[(0, 0, 0, 0, 0, 0, 0, 3/5, 1/5, 4/5, 2/5, 0, 3/5, 1/5, 4/5, 2/5), -8*q^(4/5) -
49*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/10, 2/5, 1/10, 4/5, 1/2, 1/5, 9/10, 3/5, 3/10),
68*q^(6/5) + O(q^(11/5))]
[(0, 0, 0, 0, 0, 0, 0, 4/5, 3/5, 2/5, 1/5, 0, 4/5, 3/5, 2/5, 1/5), q^(1/5) +
64*q^(6/5) + O(q^(11/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 9/10, 4/5, 7/10, 3/5, 1/2, 2/5, 3/10, 1/5, 1/10),
-4*q^(4/5) - 104*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0), -100*q^(5/4) +
O(q^(9/4))]
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[(0, 0, 0, 0, 0, 0, 0, 1/10, 1/5, 3/10, 2/5, 1/2, 3/5, 7/10, 4/5, 9/10),
-2*q^(11/20) - 56*q^(31/20) + O(q^(51/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/5, 2/5, 3/5, 4/5, 0, 1/5, 2/5, 3/5, 4/5),
2*q^(9/20) + 124*q^(29/20) + O(q^(49/20))]
[(0, 0, 0, 0, 0, 0, 0, 3/10, 3/5, 9/10, 1/5, 1/2, 4/5, 1/10, 2/5, 7/10),
34*q^(19/20) + 140*q^(39/20) + O(q^(59/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/5, 4/5, 1/5, 3/5, 0, 2/5, 4/5, 1/5, 3/5),
-16*q^(21/20) + O(q^(41/20))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -20*q^(3/4) -
180*q^(7/4) + O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/5, 1/5, 4/5, 2/5, 0, 3/5, 1/5, 4/5, 2/5),
-16*q^(21/20) + O(q^(41/20))]
[(0, 0, 0, 0, 0, 0, 0, 7/10, 2/5, 1/10, 4/5, 1/2, 1/5, 9/10, 3/5, 3/10),
34*q^(19/20) + 140*q^(39/20) + O(q^(59/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/5, 3/5, 2/5, 1/5, 0, 4/5, 3/5, 2/5, 1/5),
2*q^(9/20) + 124*q^(29/20) + O(q^(49/20))]
[(0, 0, 0, 0, 0, 0, 0, 9/10, 4/5, 7/10, 3/5, 1/2, 2/5, 3/10, 1/5, 1/10),
-2*q^(11/20) - 56*q^(31/20) + O(q^(51/20))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -40*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/10, 1/5, 3/10, 2/5, 1/2, 3/5, 7/10, 4/5, 9/10),
-8*q^(4/5) - 49*q^(9/5) + O(q^(14/5))]
[(0, 0, 0, 0, 0, 0, 0, 1/5, 2/5, 3/5, 4/5, 0, 1/5, 2/5, 3/5, 4/5), 68*q^(6/5) +
O(q^(11/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/10, 3/5, 9/10, 1/5, 1/2, 4/5, 1/10, 2/5, 7/10),
q^(1/5) + 64*q^(6/5) + O(q^(11/5))]
[(0, 0, 0, 0, 0, 0, 0, 2/5, 4/5, 1/5, 3/5, 0, 2/5, 4/5, 1/5, 3/5), -4*q^(4/5) -
104*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -50*q +
O(q^2)]
[(0, 0, 0, 0, 0, 0, 0, 3/5, 1/5, 4/5, 2/5, 0, 3/5, 1/5, 4/5, 2/5), -4*q^(4/5) -
104*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/10, 2/5, 1/10, 4/5, 1/2, 1/5, 9/10, 3/5, 3/10),
q^(1/5) + 64*q^(6/5) + O(q^(11/5))]
[(0, 0, 0, 0, 0, 0, 0, 4/5, 3/5, 2/5, 1/5, 0, 4/5, 3/5, 2/5, 1/5), 68*q^(6/5) +
O(q^(11/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 9/10, 4/5, 7/10, 3/5, 1/2, 2/5, 3/10, 1/5, 1/10),
-8*q^(4/5) - 49*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0), 100*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/10, 1/5, 3/10, 2/5, 1/2, 3/5, 7/10, 4/5, 9/10),
2*q^(11/20) + 56*q^(31/20) + O(q^(51/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/5, 2/5, 3/5, 4/5, 0, 1/5, 2/5, 3/5, 4/5),
-2*q^(9/20) - 124*q^(29/20) + O(q^(49/20))]
[(0, 0, 0, 0, 0, 0, 0, 3/10, 3/5, 9/10, 1/5, 1/2, 4/5, 1/10, 2/5, 7/10),
-34*q^(19/20) - 140*q^(39/20) + O(q^(59/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/5, 4/5, 1/5, 3/5, 0, 2/5, 4/5, 1/5, 3/5),

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16*q^(21/20) + O(q^(41/20))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), 20*q^(3/4) +
180*q^(7/4) + O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/5, 1/5, 4/5, 2/5, 0, 3/5, 1/5, 4/5, 2/5),
16*q^(21/20) + O(q^(41/20))]
[(0, 0, 0, 0, 0, 0, 0, 7/10, 2/5, 1/10, 4/5, 1/2, 1/5, 9/10, 3/5, 3/10),
-34*q^(19/20) - 140*q^(39/20) + O(q^(59/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/5, 3/5, 2/5, 1/5, 0, 4/5, 3/5, 2/5, 1/5),
-2*q^(9/20) - 124*q^(29/20) + O(q^(49/20))]
[(0, 0, 0, 0, 0, 0, 0, 9/10, 4/5, 7/10, 3/5, 1/2, 2/5, 3/10, 1/5, 1/10),
2*q^(11/20) + 56*q^(31/20) + O(q^(51/20))]
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -16*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/10, 1/5, 3/10, 2/5, 1/2, 3/5, 7/10, 4/5, 9/10),
8*q^(4/5) + 16*q^(9/5) + O(q^(14/5))]
[(0, 0, 0, 0, 0, 0, 0, 1/5, 2/5, 3/5, 4/5, 0, 1/5, 2/5, 3/5, 4/5), -24*q^(6/5) +
O(q^(11/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/10, 3/5, 9/10, 1/5, 1/2, 4/5, 1/10, 2/5, 7/10),
24*q^(6/5) + O(q^(11/5))]
[(0, 0, 0, 0, 0, 0, 0, 2/5, 4/5, 1/5, 3/5, 0, 2/5, 4/5, 1/5, 3/5), -8*q^(4/5) -
16*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), 16*q +
O(q^2)]
[(0, 0, 0, 0, 0, 0, 0, 3/5, 1/5, 4/5, 2/5, 0, 3/5, 1/5, 4/5, 2/5), -8*q^(4/5) -
16*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/10, 2/5, 1/10, 4/5, 1/2, 1/5, 9/10, 3/5, 3/10),
24*q^(6/5) + O(q^(11/5))]
[(0, 0, 0, 0, 0, 0, 0, 4/5, 3/5, 2/5, 1/5, 0, 4/5, 3/5, 2/5, 1/5), -24*q^(6/5) +
O(q^(11/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 9/10, 4/5, 7/10, 3/5, 1/2, 2/5, 3/10, 1/5, 1/10),
8*q^(4/5) + 16*q^(9/5) + O(q^(14/5))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0), q^(1/4) + 20*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/10, 1/5, 3/10, 2/5, 1/2, 3/5, 7/10, 4/5, 9/10),
-2*q^(11/20) - 44*q^(31/20) + O(q^(51/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/5, 2/5, 3/5, 4/5, 0, 1/5, 2/5, 3/5, 4/5),
q^(9/20) + 36*q^(29/20) + O(q^(49/20))]
[(0, 0, 0, 0, 0, 0, 0, 3/10, 3/5, 9/10, 1/5, 1/2, 4/5, 1/10, 2/5, 7/10),
-10*q^(19/20) - 36*q^(39/20) + O(q^(59/20))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/5, 4/5, 1/5, 3/5, 0, 2/5, 4/5, 1/5, 3/5),
17*q^(21/20) + O(q^(41/20))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -8*q^(3/4) -
4*q^(7/4) + O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/5, 1/5, 4/5, 2/5, 0, 3/5, 1/5, 4/5, 2/5),
17*q^(21/20) + O(q^(41/20))]
[(0, 0, 0, 0, 0, 0, 0, 7/10, 2/5, 1/10, 4/5, 1/2, 1/5, 9/10, 3/5, 3/10),
-10*q^(19/20) - 36*q^(39/20) + O(q^(59/20))]
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      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/5, 3/5, 2/5, 1/5, 0, 4/5, 3/5, 2/5, 1/5),
      q^(9/20) + 36*q^(29/20) + O(q^(49/20))]
      [(0, 0, 0, 0, 0, 0, 0, 9/10, 4/5, 7/10, 3/5, 1/2, 2/5, 3/10, 1/5, 1/10),
      -2*q^(11/20) - 44*q^(31/20) + O(q^(51/20))]
```

[20]: `picard_num(L[9])[2]`

[20]:
```
      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 106*q + O(q^2)]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/11, 2/11, 3/11, 4/11, 5/11, 6/11, 7/11, 8/11,
      9/11, 10/11), -34*q^(35/44) - 1436*q^(79/44) + O(q^(123/44))]
      [(0, 0, 0, 0, 0, 0, 0, 2/11, 4/11, 6/11, 8/11, 10/11, 1/11, 3/11, 5/11, 7/11,
      9/11), -210*q^(13/11) + O(q^(24/11))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/11, 6/11, 9/11, 1/11, 4/11, 7/11, 10/11, 2/11,
      5/11, 8/11), -182*q^(51/44) + O(q^(95/44))]
      [(0, 0, 0, 0, 0, 0, 0, 4/11, 8/11, 1/11, 5/11, 9/11, 2/11, 6/11, 10/11, 3/11,
      7/11), -25*q^(8/11) - 1172*q^(19/11) + O(q^(30/11))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/11, 10/11, 4/11, 9/11, 3/11, 8/11, 2/11, 7/11,
      1/11, 6/11), -60*q^(39/44) - 1704*q^(83/44) + O(q^(127/44))]
      [(0, 0, 0, 0, 0, 0, 0, 6/11, 1/11, 7/11, 2/11, 8/11, 3/11, 9/11, 4/11, 10/11,
      5/11), -14*q^(7/11) - 917*q^(18/11) + O(q^(29/11))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/11, 3/11, 10/11, 6/11, 2/11, 9/11, 5/11, 1/11,
      8/11, 4/11), -84*q^(43/44) - 2212*q^(87/44) + O(q^(131/44))]
      [(0, 0, 0, 0, 0, 0, 0, 8/11, 5/11, 2/11, 10/11, 7/11, 4/11, 1/11, 9/11, 6/11,
      3/11), -65*q^(10/11) - 1832*q^(21/11) + O(q^(32/11))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 9/11, 7/11, 5/11, 3/11, 1/11, 10/11, 8/11, 6/11,
      4/11, 2/11), -2*q^(19/44) - 516*q^(63/44) + O(q^(107/44))]
      [(0, 0, 0, 0, 0, 0, 0, 10/11, 9/11, 8/11, 7/11, 6/11, 5/11, 4/11, 3/11, 2/11,
      1/11), -7*q^(6/11) - 716*q^(17/11) + O(q^(28/11))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -280*q^(5/4) +
      O(q^(9/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/11, 2/11, 3/11, 4/11, 5/11, 6/11, 7/11, 8/11, 9/11,
      10/11), -7*q^(6/11) - 716*q^(17/11) + O(q^(28/11))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/11, 4/11, 6/11, 8/11, 10/11, 1/11, 3/11, 5/11,
      7/11, 9/11), -2*q^(19/44) - 516*q^(63/44) + O(q^(107/44))]
      [(0, 0, 0, 0, 0, 0, 0, 3/11, 6/11, 9/11, 1/11, 4/11, 7/11, 10/11, 2/11, 5/11,
      8/11), -65*q^(10/11) - 1832*q^(21/11) + O(q^(32/11))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/11, 8/11, 1/11, 5/11, 9/11, 2/11, 6/11, 10/11,
      3/11, 7/11), -84*q^(43/44) - 2212*q^(87/44) + O(q^(131/44))]
      [(0, 0, 0, 0, 0, 0, 0, 5/11, 10/11, 4/11, 9/11, 3/11, 8/11, 2/11, 7/11, 1/11,
      6/11), -14*q^(7/11) - 917*q^(18/11) + O(q^(29/11))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 6/11, 1/11, 7/11, 2/11, 8/11, 3/11, 9/11, 4/11,
      10/11, 5/11), -60*q^(39/44) - 1704*q^(83/44) + O(q^(127/44))]
      [(0, 0, 0, 0, 0, 0, 0, 7/11, 3/11, 10/11, 6/11, 2/11, 9/11, 5/11, 1/11, 8/11,
      4/11), -25*q^(8/11) - 1172*q^(19/11) + O(q^(30/11))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 8/11, 5/11, 2/11, 10/11, 7/11, 4/11, 1/11, 9/11,
      6/11, 3/11), -182*q^(51/44) + O(q^(95/44))]
      [(0, 0, 0, 0, 0, 0, 0, 9/11, 7/11, 5/11, 3/11, 1/11, 10/11, 8/11, 6/11, 4/11,
```

2/11), -210*q^(13/11) + O(q^(24/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 10/11, 9/11, 8/11, 7/11, 6/11, 5/11, 4/11, 3/11,
2/11, 1/11), -34*q^(35/44) - 1436*q^(79/44) + O(q^(123/44))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -44*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/11, 2/11, 3/11, 4/11, 5/11, 6/11, 7/11, 8/11,
9/11, 10/11), -5*q^(35/44) - 14*q^(79/44) + O(q^(123/44))]
[(0, 0, 0, 0, 0, 0, 0, 2/11, 4/11, 6/11, 8/11, 10/11, 1/11, 3/11, 5/11, 7/11,
9/11), 86*q^(13/11) + O(q^(24/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/11, 6/11, 9/11, 1/11, 4/11, 7/11, 10/11, 2/11,
5/11, 8/11), q^(7/44) + 27*q^(51/44) + O(q^(95/44))]
[(0, 0, 0, 0, 0, 0, 0, 4/11, 8/11, 1/11, 5/11, 9/11, 2/11, 6/11, 10/11, 3/11,
7/11), -14*q^(8/11) - 198*q^(19/11) + O(q^(30/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/11, 10/11, 4/11, 9/11, 3/11, 8/11, 2/11, 7/11,
1/11, 6/11), -25*q^(39/44) - 120*q^(83/44) + O(q^(127/44))]
[(0, 0, 0, 0, 0, 0, 0, 6/11, 1/11, 7/11, 2/11, 8/11, 3/11, 9/11, 4/11, 10/11,
5/11), 10*q^(7/11) + 104*q^(18/11) + O(q^(29/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/11, 3/11, 10/11, 6/11, 2/11, 9/11, 5/11, 1/11,
8/11, 4/11), 40*q^(43/44) + 184*q^(87/44) + O(q^(131/44))]
[(0, 0, 0, 0, 0, 0, 0, 8/11, 5/11, 2/11, 10/11, 7/11, 4/11, 1/11, 9/11, 6/11,
3/11), -20*q^(10/11) - 38*q^(21/11) + O(q^(32/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 9/11, 7/11, 5/11, 3/11, 1/11, 10/11, 8/11, 6/11,
4/11, 2/11), -3*q^(19/44) - 146*q^(63/44) + O(q^(107/44))]
[(0, 0, 0, 0, 0, 0, 0, 10/11, 9/11, 8/11, 7/11, 6/11, 5/11, 4/11, 3/11, 2/11,
1/11), 2*q^(6/11) + 14*q^(17/11) + O(q^(28/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 110*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/11, 2/11, 3/11, 4/11, 5/11, 6/11, 7/11, 8/11, 9/11,
10/11), 2*q^(6/11) + 14*q^(17/11) + O(q^(28/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/11, 4/11, 6/11, 8/11, 10/11, 1/11, 3/11, 5/11,
7/11, 9/11), -3*q^(19/44) - 146*q^(63/44) + O(q^(107/44))]
[(0, 0, 0, 0, 0, 0, 0, 3/11, 6/11, 9/11, 1/11, 4/11, 7/11, 10/11, 2/11, 5/11,
8/11), -20*q^(10/11) - 38*q^(21/11) + O(q^(32/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/11, 8/11, 1/11, 5/11, 9/11, 2/11, 6/11, 10/11,
3/11, 7/11), 40*q^(43/44) + 184*q^(87/44) + O(q^(131/44))]
[(0, 0, 0, 0, 0, 0, 0, 5/11, 10/11, 4/11, 9/11, 3/11, 8/11, 2/11, 7/11, 1/11,
6/11), 10*q^(7/11) + 104*q^(18/11) + O(q^(29/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 6/11, 1/11, 7/11, 2/11, 8/11, 3/11, 9/11, 4/11,
10/11, 5/11), -25*q^(39/44) - 120*q^(83/44) + O(q^(127/44))]
[(0, 0, 0, 0, 0, 0, 0, 7/11, 3/11, 10/11, 6/11, 2/11, 9/11, 5/11, 1/11, 8/11,
4/11), -14*q^(8/11) - 198*q^(19/11) + O(q^(30/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 8/11, 5/11, 2/11, 10/11, 7/11, 4/11, 1/11, 9/11,
6/11, 3/11), q^(7/44) + 27*q^(51/44) + O(q^(95/44))]
[(0, 0, 0, 0, 0, 0, 0, 9/11, 7/11, 5/11, 3/11, 1/11, 10/11, 8/11, 6/11, 4/11,
2/11), 86*q^(13/11) + O(q^(24/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 10/11, 9/11, 8/11, 7/11, 6/11, 5/11, 4/11, 3/11,
2/11, 1/11), -5*q^(35/44) - 14*q^(79/44) + O(q^(123/44))]

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[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -44*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/11, 2/11, 3/11, 4/11, 5/11, 6/11, 7/11, 8/11,
9/11, 10/11), -6*q^(35/44) - 84*q^(79/44) + O(q^(123/44))]
[(0, 0, 0, 0, 0, 0, 0, 2/11, 4/11, 6/11, 8/11, 10/11, 1/11, 3/11, 5/11, 7/11,
9/11), q^(2/11) + 36*q^(13/11) + O(q^(24/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/11, 6/11, 9/11, 1/11, 4/11, 7/11, 10/11, 2/11,
5/11, 8/11), 62*q^(51/44) + O(q^(95/44))]
[(0, 0, 0, 0, 0, 0, 0, 4/11, 8/11, 1/11, 5/11, 9/11, 2/11, 6/11, 10/11, 3/11,
7/11), 2*q^(8/11) + 22*q^(19/11) + O(q^(30/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/11, 10/11, 4/11, 9/11, 3/11, 8/11, 2/11, 7/11,
1/11, 6/11), -20*q^(39/44) - 144*q^(83/44) + O(q^(127/44))]
[(0, 0, 0, 0, 0, 0, 0, 6/11, 1/11, 7/11, 2/11, 8/11, 3/11, 9/11, 4/11, 10/11,
5/11), -10*q^(7/11) - 92*q^(18/11) + O(q^(29/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/11, 3/11, 10/11, 6/11, 2/11, 9/11, 5/11, 1/11,
8/11, 4/11), 4*q^(43/44) + 36*q^(87/44) + O(q^(131/44))]
[(0, 0, 0, 0, 0, 0, 0, 8/11, 5/11, 2/11, 10/11, 7/11, 4/11, 1/11, 9/11, 6/11,
3/11), 31*q^(10/11) + 92*q^(21/11) + O(q^(32/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 9/11, 7/11, 5/11, 3/11, 1/11, 10/11, 8/11, 6/11,
4/11, 2/11), 2*q^(19/44) + 68*q^(63/44) + O(q^(107/44))]
[(0, 0, 0, 0, 0, 0, 0, 10/11, 9/11, 8/11, 7/11, 6/11, 5/11, 4/11, 3/11, 2/11,
1/11), -3*q^(6/11) - 48*q^(17/11) + O(q^(28/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -88*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/11, 2/11, 3/11, 4/11, 5/11, 6/11, 7/11, 8/11, 9/11,
10/11), -3*q^(6/11) - 48*q^(17/11) + O(q^(28/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/11, 4/11, 6/11, 8/11, 10/11, 1/11, 3/11, 5/11,
7/11, 9/11), 2*q^(19/44) + 68*q^(63/44) + O(q^(107/44))]
[(0, 0, 0, 0, 0, 0, 0, 3/11, 6/11, 9/11, 1/11, 4/11, 7/11, 10/11, 2/11, 5/11,
8/11), 31*q^(10/11) + 92*q^(21/11) + O(q^(32/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/11, 8/11, 1/11, 5/11, 9/11, 2/11, 6/11, 10/11,
3/11, 7/11), 4*q^(43/44) + 36*q^(87/44) + O(q^(131/44))]
[(0, 0, 0, 0, 0, 0, 0, 5/11, 10/11, 4/11, 9/11, 3/11, 8/11, 2/11, 7/11, 1/11,
6/11), -10*q^(7/11) - 92*q^(18/11) + O(q^(29/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 6/11, 1/11, 7/11, 2/11, 8/11, 3/11, 9/11, 4/11,
10/11, 5/11), -20*q^(39/44) - 144*q^(83/44) + O(q^(127/44))]
[(0, 0, 0, 0, 0, 0, 0, 7/11, 3/11, 10/11, 6/11, 2/11, 9/11, 5/11, 1/11, 8/11,
4/11), 2*q^(8/11) + 22*q^(19/11) + O(q^(30/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 8/11, 5/11, 2/11, 10/11, 7/11, 4/11, 1/11, 9/11,
6/11, 3/11), 62*q^(51/44) + O(q^(95/44))]
[(0, 0, 0, 0, 0, 0, 0, 9/11, 7/11, 5/11, 3/11, 1/11, 10/11, 8/11, 6/11, 4/11,
2/11), q^(2/11) + 36*q^(13/11) + O(q^(24/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 10/11, 9/11, 8/11, 7/11, 6/11, 5/11, 4/11, 3/11,
2/11, 1/11), -6*q^(35/44) - 84*q^(79/44) + O(q^(123/44))]
-------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -12*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/11, 2/11, 3/11, 4/11, 5/11, 6/11, 7/11, 8/11,
```

9/11, 10/11), 6*q^(35/44) - 8*q^(79/44) + O(q^(123/44))]
[(0, 0, 0, 0, 0, 0, 0, 2/11, 4/11, 6/11, 8/11, 10/11, 1/11, 3/11, 5/11, 7/11,
9/11), -16*q^(13/11) + O(q^(24/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 3/11, 6/11, 9/11, 1/11, 4/11, 7/11, 10/11, 2/11,
5/11, 8/11), 14*q^(51/44) + O(q^(95/44))]
[(0, 0, 0, 0, 0, 0, 0, 4/11, 8/11, 1/11, 5/11, 9/11, 2/11, 6/11, 10/11, 3/11,
7/11), -4*q^(8/11) - 8*q^(19/11) + O(q^(30/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 5/11, 10/11, 4/11, 9/11, 3/11, 8/11, 2/11, 7/11,
1/11, 6/11), 8*q^(39/44) - 17*q^(83/44) + O(q^(127/44))]
[(0, 0, 0, 0, 0, 0, 0, 6/11, 1/11, 7/11, 2/11, 8/11, 3/11, 9/11, 4/11, 10/11,
5/11), -4*q^(7/11) + 2*q^(18/11) + O(q^(29/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 7/11, 3/11, 10/11, 6/11, 2/11, 9/11, 5/11, 1/11,
8/11, 4/11), 9*q^(43/44) - 16*q^(87/44) + O(q^(131/44))]
[(0, 0, 0, 0, 0, 0, 0, 8/11, 5/11, 2/11, 10/11, 7/11, 4/11, 1/11, 9/11, 6/11,
3/11), -6*q^(10/11) - 8*q^(21/11) + O(q^(32/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 9/11, 7/11, 5/11, 3/11, 1/11, 10/11, 8/11, 6/11,
4/11, 2/11), q^(19/44) + 16*q^(63/44) + O(q^(107/44))]
[(0, 0, 0, 0, 0, 0, 0, 10/11, 9/11, 8/11, 7/11, 6/11, 5/11, 4/11, 3/11, 2/11,
1/11), -2*q^(6/11) - 16*q^(17/11) + O(q^(28/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), q^(1/4) + 8*q^(5/4)
+ O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/11, 2/11, 3/11, 4/11, 5/11, 6/11, 7/11, 8/11, 9/11,
10/11), -2*q^(6/11) - 16*q^(17/11) + O(q^(28/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/11, 4/11, 6/11, 8/11, 10/11, 1/11, 3/11, 5/11,
7/11, 9/11), q^(19/44) + 16*q^(63/44) + O(q^(107/44))]
[(0, 0, 0, 0, 0, 0, 0, 3/11, 6/11, 9/11, 1/11, 4/11, 7/11, 10/11, 2/11, 5/11,
8/11), -6*q^(10/11) - 8*q^(21/11) + O(q^(32/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 4/11, 8/11, 1/11, 5/11, 9/11, 2/11, 6/11, 10/11,
3/11, 7/11), 9*q^(43/44) - 16*q^(87/44) + O(q^(131/44))]
[(0, 0, 0, 0, 0, 0, 0, 5/11, 10/11, 4/11, 9/11, 3/11, 8/11, 2/11, 7/11, 1/11,
6/11), -4*q^(7/11) + 2*q^(18/11) + O(q^(29/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 6/11, 1/11, 7/11, 2/11, 8/11, 3/11, 9/11, 4/11,
10/11, 5/11), 8*q^(39/44) - 17*q^(83/44) + O(q^(127/44))]
[(0, 0, 0, 0, 0, 0, 0, 7/11, 3/11, 10/11, 6/11, 2/11, 9/11, 5/11, 1/11, 8/11,
4/11), -4*q^(8/11) - 8*q^(19/11) + O(q^(30/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 8/11, 5/11, 2/11, 10/11, 7/11, 4/11, 1/11, 9/11,
6/11, 3/11), 14*q^(51/44) + O(q^(95/44))]
[(0, 0, 0, 0, 0, 0, 0, 9/11, 7/11, 5/11, 3/11, 1/11, 10/11, 8/11, 6/11, 4/11,
2/11), -16*q^(13/11) + O(q^(24/11))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 10/11, 9/11, 8/11, 7/11, 6/11, 5/11, 4/11, 3/11,
2/11, 1/11), 6*q^(35/44) - 8*q^(79/44) + O(q^(123/44))]

E7+An' modification

September 18, 2023

```
[2]: from weilrep import *
```

```
[9]: def dis(L):
         m = L.gram_matrix()
         w = WeilRep(m)
         discrim = w.ds()
         return(discrim)
```

```
[1]: def picard_num(l):
         m = l.gram_matrix()
         wt = 11 - (l.rank()/2)
         w = WeilRep(m)
         cusp = w.cusp_forms_dimension(wt)
         modular = w.modular_forms_dimension(wt)
         basis = w.modular_forms_basis(wt,2)
         c_basis = w.cusp_forms_basis(wt,2)
         return([cusp,modular,basis])
```

```
[4]: E = IntegralLattice('E7')
     E8 = IntegralLattice('E8')
     N5 =␣
      ↪Matrix(ZZ,[[2,0,0,0,0,0,1],[0,-2,1,0,0,0,0],[0,1,-2,1,0,0,0],[0,0,1,-2,1,0,1],[0,0,0,1,-2,1
     N6 =␣
      ↪Matrix(ZZ,[[2,0,0,0,0,0,0,1],[0,-2,1,0,0,0,0,0],[0,1,-2,1,0,0,0,0],[0,0,1,-2,1,0,0,0],[0,0,

     L5 = IntegralLattice(N5).twist(-1)
     L6 = IntegralLattice(N6).twist(-1)
```

```
[10]: N5
```

```
[10]: [ 2  0  0  0  0  0  1]
      [ 0 -2  1  0  0  0  0]
      [ 0  1 -2  1  0  0  0]
      [ 0  0  1 -2  1  0  1]
      [ 0  0  0  1 -2  1  0]
      [ 0  0  0  0  1 -2  0]
      [ 1  0  0  1  0  0 -2]
```

```
[11]: dis(L5)
```

```
[11]: [(0, 0, 0, 0, 0, 0, 0),
       (1/4, 7/12, 1/6, 3/4, 5/6, 11/12, 1/2),
       (1/2, 1/6, 1/3, 1/2, 2/3, 5/6, 0),
       (3/4, 3/4, 1/2, 1/4, 1/2, 3/4, 1/2),
       (0, 1/3, 2/3, 0, 1/3, 2/3, 0),
       (1/4, 11/12, 5/6, 3/4, 1/6, 7/12, 1/2),
       (1/2, 1/2, 0, 1/2, 0, 1/2, 0),
       (3/4, 1/12, 1/6, 1/4, 5/6, 5/12, 1/2),
       (0, 2/3, 1/3, 0, 2/3, 1/3, 0),
       (1/4, 1/4, 1/2, 3/4, 1/2, 1/4, 1/2),
       (1/2, 5/6, 2/3, 1/2, 1/3, 1/6, 0),
       (3/4, 5/12, 5/6, 1/4, 1/6, 1/12, 1/2)]
```

```
[6]: picard_num(L5)
```

```
[6]: [3,
      4,
      [(0, 0, 0, 0, 0, 0, 0), 1 - 180*q + O(q^2)]
      [(1/4, 7/12, 1/6, 3/4, 5/6, 11/12, 1/2), -1728*q^(35/24) + O(q^(59/24))]
      [(1/2, 1/6, 1/3, 1/2, 2/3, 5/6, 0), -54*q^(5/6) - 7560*q^(11/6) + O(q^(17/6))]
      [(3/4, 3/4, 1/2, 1/4, 1/2, 3/4, 1/2), -320*q^(9/8) + O(q^(17/8))]
      [(0, 1/3, 2/3, 0, 1/3, 2/3, 0), -945*q^(4/3) + O(q^(7/3))]
      [(1/4, 11/12, 5/6, 3/4, 1/6, 7/12, 1/2), -1728*q^(35/24) + O(q^(59/24))]
      [(1/2, 1/2, 0, 1/2, 0, 1/2, 0), -2100*q^(3/2) + O(q^(5/2))]
      [(3/4, 1/12, 1/6, 1/4, 5/6, 5/12, 1/2), -1728*q^(35/24) + O(q^(59/24))]
      [(0, 2/3, 1/3, 0, 2/3, 1/3, 0), -945*q^(4/3) + O(q^(7/3))]
      [(1/4, 1/4, 1/2, 3/4, 1/2, 1/4, 1/2), -320*q^(9/8) + O(q^(17/8))]
      [(1/2, 5/6, 2/3, 1/2, 1/3, 1/6, 0), -54*q^(5/6) - 7560*q^(11/6) + O(q^(17/6))]
      [(3/4, 5/12, 5/6, 1/4, 1/6, 1/12, 1/2), -1728*q^(35/24) + O(q^(59/24))]
      ----------------------------------------------------------
      [(0, 0, 0, 0, 0, 0, 0), -156*q + O(q^2)]
      [(1/4, 7/12, 1/6, 3/4, 5/6, 11/12, 1/2), 27*q^(35/24) + O(q^(59/24))]
      [(1/2, 1/6, 1/3, 1/2, 2/3, 5/6, 0), 54*q^(5/6) + 4158*q^(11/6) + O(q^(17/6))]
      [(3/4, 3/4, 1/2, 1/4, 1/2, 3/4, 1/2), q^(1/8) - 15*q^(9/8) + O(q^(17/8))]
      [(0, 1/3, 2/3, 0, 1/3, 2/3, 0), -756*q^(4/3) + O(q^(7/3))]
      [(1/4, 11/12, 5/6, 3/4, 1/6, 7/12, 1/2), 27*q^(35/24) + O(q^(59/24))]
      [(1/2, 1/2, 0, 1/2, 0, 1/2, 0), 2*q^(1/2) + 1428*q^(3/2) + O(q^(5/2))]
      [(3/4, 1/12, 1/6, 1/4, 5/6, 5/12, 1/2), 27*q^(35/24) + O(q^(59/24))]
      [(0, 2/3, 1/3, 0, 2/3, 1/3, 0), -756*q^(4/3) + O(q^(7/3))]
      [(1/4, 1/4, 1/2, 3/4, 1/2, 1/4, 1/2), q^(1/8) - 15*q^(9/8) + O(q^(17/8))]
      [(1/2, 5/6, 2/3, 1/2, 1/3, 1/6, 0), 54*q^(5/6) + 4158*q^(11/6) + O(q^(17/6))]
      [(3/4, 5/12, 5/6, 1/4, 1/6, 1/12, 1/2), 27*q^(35/24) + O(q^(59/24))]
      ----------------------------------------------------------
      [(0, 0, 0, 0, 0, 0, 0), -26*q + O(q^2)]
      [(1/4, 7/12, 1/6, 3/4, 5/6, 11/12, 1/2), 64*q^(35/24) + O(q^(59/24))]
```

```
[(1/2, 1/6, 1/3, 1/2, 2/3, 5/6, 0), 12*q^(5/6) + 66*q^(11/6) + O(q^(17/6))]
[(3/4, 3/4, 1/2, 1/4, 1/2, 3/4, 1/2), -64*q^(9/8) + O(q^(17/8))]
[(0, 1/3, 2/3, 0, 1/3, 2/3, 0), q^(1/3) + 52*q^(4/3) + O(q^(7/3))]
[(1/4, 11/12, 5/6, 3/4, 1/6, 7/12, 1/2), 64*q^(35/24) + O(q^(59/24))]
[(1/2, 1/2, 0, 1/2, 0, 1/2, 0), -2*q^(1/2) - 128*q^(3/2) + O(q^(5/2))]
[(3/4, 1/12, 1/6, 1/4, 5/6, 5/12, 1/2), 64*q^(35/24) + O(q^(59/24))]
[(0, 2/3, 1/3, 0, 2/3, 1/3, 0), q^(1/3) + 52*q^(4/3) + O(q^(7/3))]
[(1/4, 1/4, 1/2, 3/4, 1/2, 1/4, 1/2), -64*q^(9/8) + O(q^(17/8))]
[(1/2, 5/6, 2/3, 1/2, 1/3, 1/6, 0), 12*q^(5/6) + 66*q^(11/6) + O(q^(17/6))]
[(3/4, 5/12, 5/6, 1/4, 1/6, 1/12, 1/2), 64*q^(35/24) + O(q^(59/24))]
-----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0), -28*q + O(q^2)]
[(1/4, 7/12, 1/6, 3/4, 5/6, 11/12, 1/2), q^(11/24) + q^(35/24) + O(q^(59/24))]
[(1/2, 1/6, 1/3, 1/2, 2/3, 5/6, 0), -10*q^(5/6) + 62*q^(11/6) + O(q^(17/6))]
[(3/4, 3/4, 1/2, 1/4, 1/2, 3/4, 1/2), 10*q^(9/8) + O(q^(17/8))]
[(0, 1/3, 2/3, 0, 1/3, 2/3, 0), 12*q^(4/3) + O(q^(7/3))]
[(1/4, 11/12, 5/6, 3/4, 1/6, 7/12, 1/2), q^(11/24) + q^(35/24) + O(q^(59/24))]
[(1/2, 1/2, 0, 1/2, 0, 1/2, 0), 2*q^(1/2) + 20*q^(3/2) + O(q^(5/2))]
[(3/4, 1/12, 1/6, 1/4, 5/6, 5/12, 1/2), q^(11/24) + q^(35/24) + O(q^(59/24))]
[(0, 2/3, 1/3, 0, 2/3, 1/3, 0), 12*q^(4/3) + O(q^(7/3))]
[(1/4, 1/4, 1/2, 3/4, 1/2, 1/4, 1/2), 10*q^(9/8) + O(q^(17/8))]
[(1/2, 5/6, 2/3, 1/2, 1/3, 1/6, 0), -10*q^(5/6) + 62*q^(11/6) + O(q^(17/6))]
[(3/4, 5/12, 5/6, 1/4, 1/6, 1/12, 1/2), q^(11/24) + q^(35/24) + O(q^(59/24))]]
```

[7]: `N6`

[7]:
```
[ 2  0  0  0  0  0  0  1]
[ 0 -2  1  0  0  0  0  0]
[ 0  1 -2  1  0  0  0  0]
[ 0  0  1 -2  1  0  0  0]
[ 0  0  0  1 -2  1  0  1]
[ 0  0  0  0  1 -2  1  0]
[ 0  0  0  0  0  1 -2  0]
[ 1  0  0  0  1  0  0 -2]
```

[8]: `picard_num(L6)`

[8]:
```
[3,
 4,
 [(0, 0, 0, 0, 0, 0, 0, 0), 1 - 178*q + O(q^2)]
 [(4/11, 6/11, 1/11, 7/11, 2/11, 5/11, 8/11, 3/11), -1050*q^(15/11) +
O(q^(26/11))]
 [(8/11, 1/11, 2/11, 3/11, 4/11, 10/11, 5/11, 6/11), -q^(5/11) - 1497*q^(16/11)
+ O(q^(27/11))]
 [(1/11, 7/11, 3/11, 10/11, 6/11, 4/11, 2/11, 9/11), -678*q^(14/11) +
O(q^(25/11))]
 [(5/11, 2/11, 4/11, 6/11, 8/11, 9/11, 10/11, 1/11), -53*q^(9/11) -
```

```
5797*q^(20/11) + O(q^(31/11))]
  [(9/11, 8/11, 5/11, 2/11, 10/11, 3/11, 7/11, 4/11), -267*q^(12/11) +
O(q^(23/11))]
  [(2/11, 3/11, 6/11, 9/11, 1/11, 8/11, 4/11, 7/11), -267*q^(12/11) +
O(q^(23/11))]
  [(6/11, 9/11, 7/11, 5/11, 3/11, 2/11, 1/11, 10/11), -53*q^(9/11) -
5797*q^(20/11) + O(q^(31/11))]
  [(10/11, 4/11, 8/11, 1/11, 5/11, 7/11, 9/11, 2/11), -678*q^(14/11) +
O(q^(25/11))]
  [(3/11, 10/11, 9/11, 8/11, 7/11, 1/11, 6/11, 5/11), -q^(5/11) - 1497*q^(16/11)
+ O(q^(27/11))]
  [(7/11, 5/11, 10/11, 4/11, 9/11, 6/11, 3/11, 8/11), -1050*q^(15/11) +
O(q^(26/11))]
  ------------------------------------------------------
  [(0, 0, 0, 0, 0, 0, 0, 0), -182*q + O(q^2)]
  [(4/11, 6/11, 1/11, 7/11, 2/11, 5/11, 8/11, 3/11), 650*q^(15/11) +
O(q^(26/11))]
  [(8/11, 1/11, 2/11, 3/11, 4/11, 10/11, 5/11, 6/11), 208*q^(16/11) +
O(q^(27/11))]
  [(1/11, 7/11, 3/11, 10/11, 6/11, 4/11, 2/11, 9/11), -560*q^(14/11) +
O(q^(25/11))]
  [(5/11, 2/11, 4/11, 6/11, 8/11, 9/11, 10/11, 1/11), 65*q^(9/11) +
4160*q^(20/11) + O(q^(31/11))]
  [(9/11, 8/11, 5/11, 2/11, 10/11, 3/11, 7/11, 4/11), q^(1/11) - 144*q^(12/11) +
O(q^(23/11))]
  [(2/11, 3/11, 6/11, 9/11, 1/11, 8/11, 4/11, 7/11), q^(1/11) - 144*q^(12/11) +
O(q^(23/11))]
  [(6/11, 9/11, 7/11, 5/11, 3/11, 2/11, 1/11, 10/11), 65*q^(9/11) +
4160*q^(20/11) + O(q^(31/11))]
  [(10/11, 4/11, 8/11, 1/11, 5/11, 7/11, 9/11, 2/11), -560*q^(14/11) +
O(q^(25/11))]
  [(3/11, 10/11, 9/11, 8/11, 7/11, 1/11, 6/11, 5/11), 208*q^(16/11) +
O(q^(27/11))]
  [(7/11, 5/11, 10/11, 4/11, 9/11, 6/11, 3/11, 8/11), 650*q^(15/11) +
O(q^(26/11))]
  ------------------------------------------------------
  [(0, 0, 0, 0, 0, 0, 0, 0), -56*q + O(q^2)]
  [(4/11, 6/11, 1/11, 7/11, 2/11, 5/11, 8/11, 3/11), -55*q^(15/11) +
O(q^(26/11))]
  [(8/11, 1/11, 2/11, 3/11, 4/11, 10/11, 5/11, 6/11), q^(5/11) + 120*q^(16/11) +
O(q^(27/11))]
  [(1/11, 7/11, 3/11, 10/11, 6/11, 4/11, 2/11, 9/11), q^(3/11) + 120*q^(14/11) +
O(q^(25/11))]
  [(5/11, 2/11, 4/11, 6/11, 8/11, 9/11, 10/11, 1/11), q^(9/11) + 64*q^(20/11) +
O(q^(31/11))]
  [(9/11, 8/11, 5/11, 2/11, 10/11, 3/11, 7/11, 4/11), -56*q^(12/11) +
O(q^(23/11))]
```

```
  [(2/11, 3/11, 6/11, 9/11, 1/11, 8/11, 4/11, 7/11), -56*q^(12/11) +
O(q^(23/11))]
  [(6/11, 9/11, 7/11, 5/11, 3/11, 2/11, 1/11, 10/11), q^(9/11) + 64*q^(20/11) +
O(q^(31/11))]
  [(10/11, 4/11, 8/11, 1/11, 5/11, 7/11, 9/11, 2/11), q^(3/11) + 120*q^(14/11) +
O(q^(25/11))]
  [(3/11, 10/11, 9/11, 8/11, 7/11, 1/11, 6/11, 5/11), q^(5/11) + 120*q^(16/11) +
O(q^(27/11))]
  [(7/11, 5/11, 10/11, 4/11, 9/11, 6/11, 3/11, 8/11), -55*q^(15/11) +
O(q^(26/11))]
  ----------------------------------------------------------
  [(0, 0, 0, 0, 0, 0, 0, 0), -30*q + O(q^2)]
  [(4/11, 6/11, 1/11, 7/11, 2/11, 5/11, 8/11, 3/11), q^(4/11) + 10*q^(15/11) +
O(q^(26/11))]
  [(8/11, 1/11, 2/11, 3/11, 4/11, 10/11, 5/11, 6/11), q^(5/11) + 42*q^(16/11) +
O(q^(27/11))]
  [(1/11, 7/11, 3/11, 10/11, 6/11, 4/11, 2/11, 9/11), -10*q^(14/11) +
O(q^(25/11))]
  [(5/11, 2/11, 4/11, 6/11, 8/11, 9/11, 10/11, 1/11), -11*q^(9/11) - q^(20/11) +
O(q^(31/11))]
  [(9/11, 8/11, 5/11, 2/11, 10/11, 3/11, 7/11, 4/11), 21*q^(12/11) +
O(q^(23/11))]
  [(2/11, 3/11, 6/11, 9/11, 1/11, 8/11, 4/11, 7/11), 21*q^(12/11) + O(q^(23/11))]
  [(6/11, 9/11, 7/11, 5/11, 3/11, 2/11, 1/11, 10/11), -11*q^(9/11) - q^(20/11) +
O(q^(31/11))]
  [(10/11, 4/11, 8/11, 1/11, 5/11, 7/11, 9/11, 2/11), -10*q^(14/11) +
O(q^(25/11))]
  [(3/11, 10/11, 9/11, 8/11, 7/11, 1/11, 6/11, 5/11), q^(5/11) + 42*q^(16/11) +
O(q^(27/11))]
  [(7/11, 5/11, 10/11, 4/11, 9/11, 6/11, 3/11, 8/11), q^(4/11) + 10*q^(15/11) +
O(q^(26/11))]]
```

[ ]:

[ ]:

E7+Dn picard number

September 18, 2023

```python
[2]: from weilrep import *
```

```python
[3]: E = IntegralLattice('E7')
     cusp_dim =[]
     t = list(range(4,11))
     t
```

```
[3]: [4, 5, 6, 7, 8, 9, 10]
```

```python
[4]: D = ['D'+str(i) for i in t]
     D
```

```
[4]: ['D4', 'D5', 'D6', 'D7', 'D8', 'D9', 'D10']
```

```python
[5]: L = []
     for x in D:
         y = IntegralLattice(x)
         z = E.direct_sum(y)
         L.append(z)
```

```python
[6]: for l in L:
         m = l.gram_matrix()
         wt = 14 - (l.rank()/2)
         w = WeilRep(m)
         cusp_dim.append(w.cusp_forms_dimension(wt))
```

```python
[7]: cusp_dim
```

```
[7]: [4, 3, 4, 3, 3, 2, 1]
```

```python
[8]: def picard_num(l):
         m = l.gram_matrix()
         wt = 14 - (l.rank()/2)
         w = WeilRep(m)
         cusp = w.cusp_forms_dimension(wt)
         modular = w.modular_forms_dimension(wt)
         basis = w.modular_forms_basis(wt,2)
         return([cusp,modular,basis])
```

```
[9]: modular_dim = []
     for i in list(range(7)):
         modular_dim.append(picard_num(L[i])[1])
     modular_dim
```

[9]: [5, 4, 5, 4, 6, 3, 4]

```
[10]: picard_num(L[0])[2]
```

[10]: [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 174*q + O(q^2)]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0), -16*q^(3/4) - 10176*q^(7/4) +
      O(q^(11/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2), -3200*q^(3/2) + O(q^(5/2))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 1/2, 1/2), -16*q^(3/4) - 10176*q^(7/4) +
      O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0), -864*q^(5/4) + O(q^(9/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0), -3200*q^(3/2) + O(q^(5/2))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 0, 1/2), -16*q^(3/4) - 10176*q^(7/4) +
      O(q^(11/4))]
      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -3200*q^(3/2) + O(q^(5/2))]

      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -56*q + O(q^2)]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0), 8*q^(3/4) + 992*q^(7/4) +
      O(q^(11/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2), -448*q^(3/2) + O(q^(5/2))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 1/2, 1/2), 8*q^(3/4) + 992*q^(7/4) +
      O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0), q^(1/4) + 144*q^(5/4) + O(q^(9/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0), -448*q^(3/2) + O(q^(5/2))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 0, 1/2), 8*q^(3/4) + 992*q^(7/4) +
      O(q^(11/4))]
      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -448*q^(3/2) + O(q^(5/2))]

      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -8*q + O(q^2)]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0), 4*q^(3/4) - 16*q^(7/4) +
      O(q^(11/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2), q^(1/2) - 50*q^(3/2) + O(q^(5/2))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 1/2, 1/2), 4*q^(3/4) - 16*q^(7/4) +
      O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0), -16*q^(5/4) + O(q^(9/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0), 32*q^(3/2) + O(q^(5/2))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 0, 1/2), -6*q^(3/4) + 56*q^(7/4) +
      O(q^(11/4))]
      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 32*q^(3/2) + O(q^(5/2))]

      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -8*q + O(q^2)]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0), -6*q^(3/4) + 56*q^(7/4) +
```

```
O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2), 32*q^(3/2) + O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 1/2, 1/2), 4*q^(3/4) - 16*q^(7/4) +
O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0), -16*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0), q^(1/2) - 50*q^(3/2) + O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 0, 1/2), 4*q^(3/4) - 16*q^(7/4) +
O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 32*q^(3/2) + O(q^(5/2))]
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -8*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0), 4*q^(3/4) - 16*q^(7/4) +
O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2), 32*q^(3/2) + O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 1/2, 1/2), -6*q^(3/4) + 56*q^(7/4) +
O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0), -16*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0), 32*q^(3/2) + O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 0, 1/2), 4*q^(3/4) - 16*q^(7/4) +
O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), q^(1/2) - 50*q^(3/2) + O(q^(5/2))]
```

[11]: `picard_num(L[1])[2]`

```
[11]: [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 190*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 3/4, 1/4), -4*q^(5/8) - 5100*q^(13/8)
+ O(q^(21/8))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -2920*q^(3/2) + O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 1/4, 3/4), -4*q^(5/8) - 5100*q^(13/8)
+ O(q^(21/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0), -840*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 3/4, 1/4), -1568*q^(11/8) + O(q^(19/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 1/2, 1/2), -28*q^(3/4) - 8384*q^(7/4) +
O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 1/4, 3/4), -1568*q^(11/8) + O(q^(19/8))]
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -56*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 3/4, 1/4), 4*q^(5/8) + 492*q^(13/8) +
O(q^(21/8))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -336*q^(3/2) + O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 1/4, 3/4), 4*q^(5/8) + 492*q^(13/8) +
O(q^(21/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0), q^(1/4) + 132*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 3/4, 1/4), -224*q^(11/8) + O(q^(19/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 1/2, 1/2), 6*q^(3/4) + 728*q^(7/4) +
O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 1/4, 3/4), -224*q^(11/8) + O(q^(19/8))]
```

```
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 3/4, 1/4), -2*q^(5/8) + 42*q^(13/8) +
O(q^(21/8))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 192*q^(3/2) + O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 1/4, 3/4), -2*q^(5/8) + 42*q^(13/8) +
O(q^(21/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0), -96*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 3/4, 1/4), q^(3/8) - 19*q^(11/8) +
O(q^(19/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 1/2, 1/2), 16*q^(3/4) + 128*q^(7/4) +
O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 1/4, 3/4), q^(3/8) - 19*q^(11/8) +
O(q^(19/8))]
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -10*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 3/4, 1/4), 2*q^(5/8) - 10*q^(13/8) +
O(q^(21/8))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), q^(1/2) - 30*q^(3/2) + O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 1/4, 3/4), 2*q^(5/8) - 10*q^(13/8) +
O(q^(21/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0), -4*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 3/4, 1/4), 16*q^(11/8) + O(q^(19/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 1/2, 1/2), -6*q^(3/4) + 64*q^(7/4) +
O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 1/4, 3/4), 16*q^(11/8) + O(q^(19/8))]
```

[11]: `picard_num(L[2])[2]`

```
[11]: [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 210*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 0, 1/2), -2560*q^(3/2) +
O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0), -768*q^(5/4) + O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 1/2, 1/2), -40*q^(3/4) - 6720*q^(7/4) +
O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0), -768*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 0, 1/2), -768*q^(5/4) + O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0), -2560*q^(3/2) +
O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -2560*q^(3/2) + O(q^(5/2))]
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -56*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 0, 1/2), -224*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0), q^(1/4) + 124*q^(5/4) +
O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 1/2, 1/2), 4*q^(3/4) + 480*q^(7/4) +
O(q^(11/4))]
```

```
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0), -112*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 0, 1/2), -112*q^(5/4) + O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0), 2*q^(1/2) + 244*q^(3/2) +
O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 2*q^(1/2) + 244*q^(3/2) +
O(q^(5/2))]
------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 0, 1/2), 256*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0), -128*q^(5/4) + O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 1/2, 1/2), 16*q^(3/4) + 384*q^(7/4) +
O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0), q^(1/4) + 108*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 0, 1/2), -128*q^(5/4) + O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0), 256*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -2*q^(1/2) - 212*q^(3/2) +
O(q^(5/2))]
------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 0, 1/2), 256*q^(3/2) + O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0), -128*q^(5/4) + O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 1/2, 1/2), 16*q^(3/4) + 384*q^(7/4) +
O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0), -128*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 0, 1/2), q^(1/4) + 108*q^(5/4) +
O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0), -2*q^(1/2) - 212*q^(3/2) +
O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 256*q^(3/2) + O(q^(5/2))]
------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -12*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 0, 1/2), q^(1/2) - 6*q^(3/2) +
O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0), 8*q^(5/4) + O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 1/2, 1/2), -6*q^(3/4) + 48*q^(7/4) +
O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0), 8*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 0, 1/2), 8*q^(5/4) + O(q^(9/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0), q^(1/2) - 6*q^(3/2) +
O(q^(5/2))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), q^(1/2) - 6*q^(3/2) + O(q^(5/2))]
```

[12]: `picard_num(L[3])[2]`

[12]:
```
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 206*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 1/4, 3/4), -1280*q^(11/8) +
```

O(q^(19/8))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -2*q^(1/2) - 2148*q^(3/2) +
O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 3/4, 1/4), -1280*q^(11/8) +
O(q^(19/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), -688*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 1/4, 3/4), -384*q^(9/8) +
O(q^(17/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 1/2, 1/2), -40*q^(3/4) -
5344*q^(7/4) + O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 3/4, 1/4), -384*q^(9/8) +
O(q^(17/8))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -184*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 1/4, 3/4), 32*q^(11/8) +
O(q^(19/8))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 4*q^(1/2) + 1368*q^(3/2) +
O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 3/4, 1/4), 32*q^(11/8) +
O(q^(19/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), -560*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 1/4, 3/4), q^(1/8) - 5*q^(9/8) +
O(q^(17/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 1/2, 1/2), 40*q^(3/4) + 2848*q^(7/4)
+ O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 3/4, 1/4), q^(1/8) - 5*q^(9/8) +
O(q^(17/8))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -28*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 1/4, 3/4), 128*q^(11/8) +
O(q^(19/8))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -2*q^(1/2) - 156*q^(3/2) +
O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 3/4, 1/4), 128*q^(11/8) +
O(q^(19/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), q^(1/4) + 80*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 1/4, 3/4), -64*q^(9/8) +
O(q^(17/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 1/2, 1/2), 14*q^(3/4) + 224*q^(7/4)
+ O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 3/4, 1/4), -64*q^(9/8) +
O(q^(17/8))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -28*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 1/4, 3/4), q^(3/8) -
7*q^(11/8) + O(q^(19/8))]

```
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 2*q^(1/2) + 44*q^(3/2) +
O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 3/4, 1/4), q^(3/8) -
7*q^(11/8) + O(q^(19/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), 40*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 1/4, 3/4), 8*q^(9/8) + O(q^(17/8))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 1/2, 1/2), -12*q^(3/4) + 16*q^(7/4)
+ O(q^(11/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 3/4, 1/4), 8*q^(9/8) + O(q^(17/8))]
```

[13]: `picard_num(L[4])[2]`

[13]:
```
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 198*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), -640*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), -192*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -40*q^(3/4) -
4128*q^(7/4) + O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0), -608*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), -192*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), -640*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -4*q^(1/2) - 1752*q^(3/2) +
O(q^(5/2))]
------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -192*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), 640*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), 1 - 198*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 40*q^(3/4) +
4128*q^(7/4) + O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0), -640*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), 192*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), -608*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 4*q^(1/2) + 1752*q^(3/2) +
O(q^(5/2))]
------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -192*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), -608*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), 192*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 40*q^(3/4) +
4128*q^(7/4) + O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0), -640*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), 1 - 198*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), 640*q^(5/4) +
```

```
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 4*q^(1/2) + 1752*q^(3/2) +
O(q^(5/2))]
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), q^(1/4) +
56*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), 32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -12*q^(3/4) -
112*q^(7/4) + O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0), 64*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), -24*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), -64*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 2*q^(1/2) + 108*q^(3/2) +
O(q^(5/2))]
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -24*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), 64*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), -32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 12*q^(3/4) +
112*q^(7/4) + O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0), q^(1/4) + 56*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), -32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), 64*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -2*q^(1/2) - 108*q^(3/2) +
O(q^(5/2))]
----------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), -64*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), -24*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -12*q^(3/4) -
112*q^(7/4) + O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0), 64*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), 32*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), q^(1/4) +
56*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 2*q^(1/2) + 108*q^(3/2) +
O(q^(5/2))]
```

```
[14]: picard_num(L[5])[2]
```

```
[14]: [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 186*q + O(q^2)]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 3/4, 1/4),
      -320*q^(9/8) + O(q^(17/8))]
      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -6*q^(1/2) - 1380*q^(3/2)
      + O(q^(5/2))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 1/4, 3/4),
      -320*q^(9/8) + O(q^(17/8))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0), -528*q^(5/4) +
      O(q^(9/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 3/4, 1/4), -96*q^(7/8) -
      4192*q^(15/8) + O(q^(23/8))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -40*q^(3/4) -
      3072*q^(7/4) + O(q^(11/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 1/4, 3/4), -96*q^(7/8) -
      4192*q^(15/8) + O(q^(23/8))]
      -------------------------------------------------------------
      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -144*q + O(q^2)]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 3/4, 1/4), q^(1/8) -
      9*q^(9/8) + O(q^(17/8))]
      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 8*q^(1/2) + 720*q^(3/2) +
      O(q^(5/2))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 1/4, 3/4), q^(1/8) -
      9*q^(9/8) + O(q^(17/8))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0), 352*q^(5/4) +
      O(q^(9/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 3/4, 1/4), 8*q^(7/8) -
      120*q^(15/8) + O(q^(23/8))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -48*q^(3/4) -
      1152*q^(7/4) + O(q^(11/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 1/4, 3/4), 8*q^(7/8) -
      120*q^(15/8) + O(q^(23/8))]
      -------------------------------------------------------------
      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -20*q + O(q^2)]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 3/4, 1/4), 32*q^(9/8)
      + O(q^(17/8))]
      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -2*q^(1/2) - 68*q^(3/2) +
      O(q^(5/2))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 1/4, 3/4), 32*q^(9/8)
      + O(q^(17/8))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0), q^(1/4) + 36*q^(5/4) +
      O(q^(9/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 3/4, 1/4), -16*q^(7/8) -
      16*q^(15/8) + O(q^(23/8))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 10*q^(3/4) +
      40*q^(7/4) + O(q^(11/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 1/4, 3/4), -16*q^(7/8) -
      16*q^(15/8) + O(q^(23/8))]
```

```
[10]: picard_num(L[6])[2]
```

```
[10]: [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 170*q + O(q^2)]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), -160*q +
      O(q^2)]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), -48*q^(3/4) -
      2048*q^(7/4) + O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -40*q^(3/4) -
      2176*q^(7/4) + O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -448*q^(5/4) +
      O(q^(9/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), -48*q^(3/4) -
      2048*q^(7/4) + O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), -160*q +
      O(q^2)]
      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -8*q^(1/2) -
      1040*q^(3/2) + O(q^(5/2))]
      ------------------------------------------------------------
      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -160*q + O(q^2)]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), 1 - 170*q
      + O(q^2)]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), -40*q^(3/4) -
      2176*q^(7/4) + O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -48*q^(3/4) -
      2048*q^(7/4) + O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 448*q^(5/4) +
      O(q^(9/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), 48*q^(3/4) +
      2048*q^(7/4) + O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), 160*q +
      O(q^2)]
      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 8*q^(1/2) +
      1040*q^(3/2) + O(q^(5/2))]
      ------------------------------------------------------------
      [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -160*q + O(q^2)]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), 160*q +
      O(q^2)]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), 48*q^(3/4) +
      2048*q^(7/4) + O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -48*q^(3/4) -
      2048*q^(7/4) + O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 448*q^(5/4) +
      O(q^(9/4))]
      [(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), -40*q^(3/4) -
      2176*q^(7/4) + O(q^(11/4))]
      [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), 1 - 170*q
      + O(q^2)]
```

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 8*q^(1/2) +
1040*q^(3/2) + O(q^(5/2))]

[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -16*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), 16*q +
O(q^2)]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), -8*q^(3/4) +
O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), 8*q^(3/4) +
O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0), q^(1/4) + 20*q^(5/4)
+ O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2), -8*q^(3/4) +
O(q^(11/4))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2, 0), 16*q +
O(q^2)]
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -2*q^(1/2) -
36*q^(3/2) + O(q^(5/2))]

E7+Dn' picard number

September 18, 2023

```
[1]: from weilrep import *
```

```
[2]: def picard_num(l):
         m = l.gram_matrix()
         wt = 11 - (l.rank()/2)
         w = WeilRep(m)
         cusp = w.cusp_forms_dimension(wt)
         modular = w.modular_forms_dimension(wt)
         basis = w.modular_forms_basis(wt,2)
         c_basis = w.cusp_forms_basis(wt,2)
         return([cusp,modular,basis])
```

```
[3]: E = IntegralLattice('E7')
     E8 = IntegralLattice('E8')
     N6 =␣
      ↪Matrix(ZZ,[[2,1,0,0,0,0,0,0],[1,-2,1,0,0,0,0,0],[0,1,-2,0,1,0,0,0],[0,0,0,-2,1,0,0,0],[0,0,
     N7 =␣
      ↪Matrix(ZZ,[[2,1,0,0,0,0,0,0,0],[1,-2,1,0,0,0,0,0,0],[0,1,-2,0,1,0,0,0,0],[0,0,0,-2,1,0,0,0,
     N8 =␣
      ↪Matrix(ZZ,[[2,1,0,0,0,0,0,0,0,0],[1,-2,1,0,0,0,0,0,0,0],[0,1,-2,0,1,0,0,0,0,0],[0,0,0,-2,1,
     N9 =␣
      ↪Matrix(ZZ,[[2,1,0,0,0,0,0,0,0,0,0],[1,-2,1,0,0,0,0,0,0,0,0],[0,1,-2,0,1,0,0,0,0,0,0],[0,0,0
     L6 = IntegralLattice(N6).twist(-1)
     L7 = IntegralLattice(N7).twist(-1)
     L8 = IntegralLattice(N8).twist(-1)
     L9 = IntegralLattice(N9).twist(-1)
```

```
[6]: N6
```

```
[6]: [ 2  1  0  0  0  0  0  0]
     [ 1 -2  1  0  0  0  0  0]
     [ 0  1 -2  0  1  0  0  0]
     [ 0  0  0 -2  1  0  0  0]
     [ 0  0  1  1 -2  1  0  0]
     [ 0  0  0  0  1 -2  1  0]
     [ 0  0  0  0  0  1 -2  1]
     [ 0  0  0  0  0  0  1 -2]
```

```
[4]: det(N6)
```

```
[4]: -8
```

```
[5]: det(N7)
```

```
[5]: 6
```

```
[7]: det(N8)
```

```
[7]: -4
```

```
[8]: det(N9)
```

```
[8]: 2
```

$L6$ corresponds to D_6'

```
[12]: picard_num(L6)
```

```
[12]: [3,
      4,
      [(0, 0, 0, 0, 0, 0, 0, 0), 1 - 206*q + O(q^2)]
      [(3/4, 1/2, 1/4, 1/2, 0, 1/4, 1/2, 3/4), -384*q^(9/8) + O(q^(17/8))]
      [(1/2, 0, 1/2, 0, 0, 1/2, 0, 1/2), -2*q^(1/2) - 2148*q^(3/2) + O(q^(5/2))]
      [(1/4, 1/2, 3/4, 1/2, 0, 3/4, 1/2, 1/4), -384*q^(9/8) + O(q^(17/8))]
      [(1/2, 0, 1/2, 1/2, 0, 0, 0, 0), -40*q^(3/4) - 5344*q^(7/4) + O(q^(11/4))]
      [(1/4, 1/2, 3/4, 0, 0, 1/4, 1/2, 3/4), -1280*q^(11/8) + O(q^(19/8))]
      [(0, 0, 0, 1/2, 0, 1/2, 0, 1/2), -688*q^(5/4) + O(q^(9/4))]
      [(3/4, 1/2, 1/4, 0, 0, 3/4, 1/2, 1/4), -1280*q^(11/8) + O(q^(19/8))]
      ------------------------------------------------------------
      [(0, 0, 0, 0, 0, 0, 0, 0), -184*q + O(q^2)]
      [(3/4, 1/2, 1/4, 1/2, 0, 1/4, 1/2, 3/4), q^(1/8) - 5*q^(9/8) + O(q^(17/8))]
      [(1/2, 0, 1/2, 0, 0, 1/2, 0, 1/2), 4*q^(1/2) + 1368*q^(3/2) + O(q^(5/2))]
      [(1/4, 1/2, 3/4, 1/2, 0, 3/4, 1/2, 1/4), q^(1/8) - 5*q^(9/8) + O(q^(17/8))]
      [(1/2, 0, 1/2, 1/2, 0, 0, 0, 0), 40*q^(3/4) + 2848*q^(7/4) + O(q^(11/4))]
      [(1/4, 1/2, 3/4, 0, 0, 1/4, 1/2, 3/4), 32*q^(11/8) + O(q^(19/8))]
      [(0, 0, 0, 1/2, 0, 1/2, 0, 1/2), -560*q^(5/4) + O(q^(9/4))]
      [(3/4, 1/2, 1/4, 0, 0, 3/4, 1/2, 1/4), 32*q^(11/8) + O(q^(19/8))]
      ------------------------------------------------------------
      [(0, 0, 0, 0, 0, 0, 0, 0), -28*q + O(q^2)]
      [(3/4, 1/2, 1/4, 1/2, 0, 1/4, 1/2, 3/4), -64*q^(9/8) + O(q^(17/8))]
      [(1/2, 0, 1/2, 0, 0, 1/2, 0, 1/2), -2*q^(1/2) - 156*q^(3/2) + O(q^(5/2))]
      [(1/4, 1/2, 3/4, 1/2, 0, 3/4, 1/2, 1/4), -64*q^(9/8) + O(q^(17/8))]
      [(1/2, 0, 1/2, 1/2, 0, 0, 0, 0), 14*q^(3/4) + 224*q^(7/4) + O(q^(11/4))]
      [(1/4, 1/2, 3/4, 0, 0, 1/4, 1/2, 3/4), 128*q^(11/8) + O(q^(19/8))]
      [(0, 0, 0, 1/2, 0, 1/2, 0, 1/2), q^(1/4) + 80*q^(5/4) + O(q^(9/4))]
      [(3/4, 1/2, 1/4, 0, 0, 3/4, 1/2, 1/4), 128*q^(11/8) + O(q^(19/8))]
```

```
              ----------------------------------------------------------
              [(0, 0, 0, 0, 0, 0, 0, 0), -28*q + O(q^2)]
              [(3/4, 1/2, 1/4, 1/2, 0, 1/4, 1/2, 3/4), 8*q^(9/8) + O(q^(17/8))]
              [(1/2, 0, 1/2, 0, 0, 1/2, 0, 1/2), 2*q^(1/2) + 44*q^(3/2) + O(q^(5/2))]
              [(1/4, 1/2, 3/4, 1/2, 0, 3/4, 1/2, 1/4), 8*q^(9/8) + O(q^(17/8))]
              [(1/2, 0, 1/2, 1/2, 0, 0, 0, 0), -12*q^(3/4) + 16*q^(7/4) + O(q^(11/4))]
              [(1/4, 1/2, 3/4, 0, 0, 1/4, 1/2, 3/4), q^(3/8) - 7*q^(11/8) + O(q^(19/8))]
              [(0, 0, 0, 1/2, 0, 1/2, 0, 1/2), 40*q^(5/4) + O(q^(9/4))]
              [(3/4, 1/2, 1/4, 0, 0, 3/4, 1/2, 1/4), q^(3/8) - 7*q^(11/8) + O(q^(19/8))]]
```

[13]: `picard_num(L7)`

[13]:
```
        [2,
         3,
         [(0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 234*q + O(q^2)]
         [(5/6, 1/3, 5/6, 1/6, 1/3, 2/3, 0, 1/3, 2/3), -324*q^(13/12) + O(q^(25/12))]
         [(2/3, 2/3, 2/3, 1/3, 2/3, 1/3, 0, 2/3, 1/3), -1053*q^(4/3) + O(q^(7/3))]
         [(1/2, 0, 1/2, 1/2, 0, 0, 0, 0, 0), -52*q^(3/4) - 4680*q^(7/4) + O(q^(11/4))]
         [(1/3, 1/3, 1/3, 2/3, 1/3, 2/3, 0, 1/3, 2/3), -1053*q^(4/3) + O(q^(7/3))]
         [(1/6, 2/3, 1/6, 5/6, 2/3, 1/3, 0, 2/3, 1/3), -324*q^(13/12) + O(q^(25/12))]
         ----------------------------------------------------------
         [(0, 0, 0, 0, 0, 0, 0, 0, 0), -240*q + O(q^2)]
         [(5/6, 1/3, 5/6, 1/6, 1/3, 2/3, 0, 1/3, 2/3), q^(1/12) - 200*q^(13/12) +
        O(q^(25/12))]
         [(2/3, 2/3, 2/3, 1/3, 2/3, 1/3, 0, 2/3, 1/3), 528*q^(4/3) + O(q^(7/3))]
         [(1/2, 0, 1/2, 1/2, 0, 0, 0, 0, 0), 66*q^(3/4) + 3696*q^(7/4) + O(q^(11/4))]
         [(1/3, 1/3, 1/3, 2/3, 1/3, 2/3, 0, 1/3, 2/3), 528*q^(4/3) + O(q^(7/3))]
         [(1/6, 2/3, 1/6, 5/6, 2/3, 1/3, 0, 2/3, 1/3), q^(1/12) - 200*q^(13/12) +
        O(q^(25/12))]
         ----------------------------------------------------------
         [(0, 0, 0, 0, 0, 0, 0, 0, 0), -30*q + O(q^2)]
         [(5/6, 1/3, 5/6, 1/6, 1/3, 2/3, 0, 1/3, 2/3), 20*q^(13/12) + O(q^(25/12))]
         [(2/3, 2/3, 2/3, 1/3, 2/3, 1/3, 0, 2/3, 1/3), q^(1/3) + 22*q^(4/3) +
        O(q^(7/3))]
         [(1/2, 0, 1/2, 1/2, 0, 0, 0, 0, 0), -12*q^(3/4) - 24*q^(7/4) + O(q^(11/4))]
         [(1/3, 1/3, 1/3, 2/3, 1/3, 2/3, 0, 1/3, 2/3), q^(1/3) + 22*q^(4/3) +
        O(q^(7/3))]
         [(1/6, 2/3, 1/6, 5/6, 2/3, 1/3, 0, 2/3, 1/3), 20*q^(13/12) + O(q^(25/12))]]
```

[14]: `picard_num(L8)`

[14]:
```
        [1,
         3,
         [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 264*q + O(q^2)]
         [(0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), -240*q + O(q^2)]
         [(1/2, 0, 1/2, 1/2, 0, 0, 0, 0, 0, 0), -64*q^(3/4) - 4224*q^(7/4) +
        O(q^(11/4))]
```

```
[(1/2, 0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 1/2), -768*q^(5/4) + O(q^(9/4))]
-------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -240*q + O(q^2)]
[(0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), 1 - 264*q + O(q^2)]
[(1/2, 0, 1/2, 1/2, 0, 0, 0, 0, 0, 0), 64*q^(3/4) + 4224*q^(7/4) + O(q^(11/4))]
[(1/2, 0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 1/2), 768*q^(5/4) + O(q^(9/4))]
-------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -32*q + O(q^2)]
[(0, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 1/2), 32*q + O(q^2)]
[(1/2, 0, 1/2, 1/2, 0, 0, 0, 0, 0, 0), -12*q^(3/4) - 88*q^(7/4) + O(q^(11/4))]
[(1/2, 0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 1/2), q^(1/4) + 54*q^(5/4) + O(q^(9/4))]]
```

[16]: `picard_num(L9)`

[16]:
```
[0,
 1,
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 330*q + O(q^2)]
 [(1/2, 0, 1/2, 1/2, 0, 0, 0, 0, 0, 0, 0), -88*q^(3/4) - 4224*q^(7/4) +
O(q^(11/4))]]
```

E7+En picard number

September 18, 2023

```
[2]: from weilrep import *
```

```
[10]: def picard_num(l):
          m = l.gram_matrix()
          wt = 14 - (l.rank()/2)
          w = WeilRep(m)
          cusp = w.cusp_forms_dimension(wt)
          modular = w.modular_forms_dimension(wt)
          basis = w.modular_forms_basis(wt,2)
          return([cusp,modular,basis])
```

```
[3]: E7 = IntegralLattice('E7')
     E8 = IntegralLattice('E8')
     E6 = IntegralLattice('E6')
     cusp_dim =[]
     t = list(range(6,9))
     t
```

```
[3]: [6, 7, 8]
```

```
[4]: E = ['E'+str(i) for i in t]
     E
```

```
[4]: ['E6', 'E7', 'E8']
```

```
[6]: L = []
     for x in E:
         y = IntegralLattice(x)
         z = E7.direct_sum(y)
         L.append(z)
```

```
[7]: for l in L:
         m = l.gram_matrix()
         wt = 14 - (l.rank()/2)
         w = WeilRep(m)
         cusp_dim.append(w.cusp_forms_dimension(wt))
```

```
[8]: cusp_dim
```

```
[8]:  [2, 2, 1]
```

```
[11]:  modular_dim = []
       for i in list(range(len(L))):
           modular_dim.append(picard_num(L[i])[1])
       modular_dim
```

```
[11]:  [3, 3, 2]
```

```
[12]:  picard_num(L[0])[2]
```

```
[12]:  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 222*q + O(q^2)]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 0, 1/3, 0, 2/3, 1/3), -6*q^(7/12) -
       4128*q^(19/12) + O(q^(31/12))]
       [(0, 0, 0, 0, 0, 0, 0, 1/3, 0, 2/3, 0, 1/3, 2/3), -1365*q^(4/3) + O(q^(7/3))]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0), -924*q^(5/4) + O(q^(9/4))]
       [(0, 0, 0, 0, 0, 0, 0, 2/3, 0, 1/3, 0, 2/3, 1/3), -1365*q^(4/3) + O(q^(7/3))]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 0, 2/3, 0, 1/3, 2/3), -6*q^(7/12) -
       4128*q^(19/12) + O(q^(31/12))]
       -----------------------------------------------------------
       [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -56*q + O(q^2)]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 0, 1/3, 0, 2/3, 1/3), 3*q^(7/12) +
       363*q^(19/12) + O(q^(31/12))]
       [(0, 0, 0, 0, 0, 0, 0, 1/3, 0, 2/3, 0, 1/3, 2/3), -168*q^(4/3) + O(q^(7/3))]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0), q^(1/4) + 126*q^(5/4) +
       O(q^(9/4))]
       [(0, 0, 0, 0, 0, 0, 0, 2/3, 0, 1/3, 0, 2/3, 1/3), -168*q^(4/3) + O(q^(7/3))]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 0, 2/3, 0, 1/3, 2/3), 3*q^(7/12) +
       363*q^(19/12) + O(q^(31/12))]
       -----------------------------------------------------------
       [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -54*q + O(q^2)]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 2/3, 0, 1/3, 0, 2/3, 1/3), 2*q^(7/12) +
       128*q^(19/12) + O(q^(31/12))]
       [(0, 0, 0, 0, 0, 0, 0, 1/3, 0, 2/3, 0, 1/3, 2/3), q^(1/3) + 66*q^(4/3) +
       O(q^(7/3))]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0), -108*q^(5/4) + O(q^(9/4))]
       [(0, 0, 0, 0, 0, 0, 0, 2/3, 0, 1/3, 0, 2/3, 1/3), q^(1/3) + 66*q^(4/3) +
       O(q^(7/3))]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 1/3, 0, 2/3, 0, 1/3, 2/3), 2*q^(7/12) +
       128*q^(19/12) + O(q^(31/12))]
```

```
[13]:  picard_num(L[1])[2]
```

```
[13]:  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 276*q + O(q^2)]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2, 0, 1/2), -4*q^(1/2) -
       2976*q^(3/2) + O(q^(5/2))]
       [(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), -1024*q^(5/4) + O(q^(9/4))]
       [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 0, 1/2), -1024*q^(5/4) + O(q^(9/4))]
```

```
-----------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -56*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2, 0, 1/2), 2*q^(1/2) + 240*q^(3/2)
+ O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), q^(1/4) + 122*q^(5/4) +
O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 0, 1/2), -112*q^(5/4) + O(q^(9/4))]
-----------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -56*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 1/2, 0, 0, 1/2, 0, 1/2), 2*q^(1/2) + 240*q^(3/2)
+ O(q^(5/2))]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), -112*q^(5/4) + O(q^(9/4))]
[(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 0, 1/2), q^(1/4) + 122*q^(5/4) +
O(q^(9/4))]
```

[14]: `picard_num(L[2])[2]`

[14]:
```
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 390*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), -1248*q^(5/4) +
O(q^(9/4))]
-----------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -56*q + O(q^2)]
[(0, 1/2, 0, 0, 1/2, 0, 1/2, 0, 0, 0, 0, 0, 0, 0), q^(1/4) + 120*q^(5/4) +
O(q^(9/4))]
```

E8+A1 and so on unigonal picard number

September 18, 2023

```
[1]: from weilrep import *
```

```
[2]: def dis(L):
         m = L.gram_matrix()
         w = WeilRep(m)
         discrim = w.ds()
         return(discrim)
```

```
[3]: def picard_num(l):
         m = l.gram_matrix()
         wt = 14 - (l.rank()/2)
         w = WeilRep(m)
         cusp = w.cusp_forms_dimension(wt)
         modular = w.modular_forms_dimension(wt)
         basis = w.modular_forms_basis(wt,2)
         return([cusp,modular,basis])
```

```
[4]: def picard_num_NS(l):
         m = l.gram_matrix()
         wt = 11 - (l.rank()/2)
         w = WeilRep(m)
         cusp = w.cusp_forms_dimension(wt)
         modular = w.modular_forms_dimension(wt)
         basis = w.modular_forms_basis(wt,2)
         return([cusp,modular,basis])
```

```
[5]: E = IntegralLattice('E8')
     cusp_dim =[]
     t = list(range(0,9))
     t
```

```
[5]: [0, 1, 2, 3, 4, 5, 6, 7, 8]
```

```
[6]: A = [IntegralLattice('A'+str(i)) for i in t]
     A
```

```
[6]: [Lattice of degree 0 and rank 0 over Integer Ring
      Standard basis
```

1

```
Standard scalar product,
Lattice of degree 1 and rank 1 over Integer Ring
Standard basis
Inner product matrix:
[2],
Lattice of degree 2 and rank 2 over Integer Ring
Standard basis
Inner product matrix:
[ 2 -1]
[-1  2],
Lattice of degree 3 and rank 3 over Integer Ring
Standard basis
Inner product matrix:
[ 2 -1  0]
[-1  2 -1]
[ 0 -1  2],
Lattice of degree 4 and rank 4 over Integer Ring
Standard basis
Inner product matrix:
[ 2 -1  0  0]
[-1  2 -1  0]
[ 0 -1  2 -1]
[ 0  0 -1  2],
Lattice of degree 5 and rank 5 over Integer Ring
Standard basis
Inner product matrix:
[ 2 -1  0  0  0]
[-1  2 -1  0  0]
[ 0 -1  2 -1  0]
[ 0  0 -1  2 -1]
[ 0  0  0 -1  2],
Lattice of degree 6 and rank 6 over Integer Ring
Standard basis
Inner product matrix:
[ 2 -1  0  0  0  0]
[-1  2 -1  0  0  0]
[ 0 -1  2 -1  0  0]
[ 0  0 -1  2 -1  0]
[ 0  0  0 -1  2 -1]
[ 0  0  0  0 -1  2],
Lattice of degree 7 and rank 7 over Integer Ring
Standard basis
Inner product matrix:
[ 2 -1  0  0  0  0  0]
[-1  2 -1  0  0  0  0]
[ 0 -1  2 -1  0  0  0]
[ 0  0 -1  2 -1  0  0]
```

```
[ 0  0  0 -1  2 -1  0]
[ 0  0  0  0 -1  2 -1]
[ 0  0  0  0  0 -1  2],
Lattice of degree 8 and rank 8 over Integer Ring
Standard basis
Inner product matrix:
[ 2 -1  0  0  0  0  0  0]
[-1  2 -1  0  0  0  0  0]
[ 0 -1  2 -1  0  0  0  0]
[ 0  0 -1  2 -1  0  0  0]
[ 0  0  0 -1  2 -1  0  0]
[ 0  0  0  0 -1  2 -1  0]
[ 0  0  0  0  0 -1  2 -1]
[ 0  0  0  0  0  0 -1  2]]
```

[7]:
```
A1 = IntegralLattice('A1')
A2 = IntegralLattice('A2')
A2
```

[7]:
```
Lattice of degree 2 and rank 2 over Integer Ring
Standard basis
Inner product matrix:
[ 2 -1]
[-1  2]
```

[30]:
```
picard_num(E.direct_sum(A1))
```

[30]:
```
[1,
 2,
 [(0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 266*q + O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2), -26752*q^(7/4) + O(q^(11/4))]
 ----------------------------------------------------------
 [(0, 0, 0, 0, 0, 0, 0, 0, 0), -2*q + O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2), q^(3/4) - 16*q^(7/4) + O(q^(11/4))]]
```

[8]:
```
L = E.direct_sum(A1)
L = L.direct_sum(A1)
L
```

[8]:
```
Lattice of degree 10 and rank 10 over Integer Ring
Standard basis
Inner product matrix:
[ 2  0 -1  0  0  0  0  0  0  0]
[ 0  2  0 -1  0  0  0  0  0  0]
[-1  0  2 -1  0  0  0  0  0  0]
[ 0 -1 -1  2 -1  0  0  0  0  0]
[ 0  0  0 -1  2 -1  0  0  0  0]
```

```
[ 0  0  0  0 -1  2 -1  0  0  0]
[ 0  0  0  0  0 -1  2 -1  0  0]
[ 0  0  0  0  0  0 -1  2  0  0]
[ 0  0  0  0  0  0  0  0  2  0]
[ 0  0  0  0  0  0  0  0  0  2]
```

[9]: `picard_num(L)`

[9]:
```
[2,
 3,
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 204*q + O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2), -16896*q^(7/4) + O(q^(11/4))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0), -32*q^(3/4) - 16320*q^(7/4) + O(q^(11/4))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), -4928*q^(3/2) + O(q^(5/2))]
 ----------------------------------------------------------------
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -20*q + O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2), 128*q^(7/4) + O(q^(11/4))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0), 8*q^(3/4) - 16*q^(7/4) + O(q^(11/4))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), q^(1/2) - 80*q^(3/2) + O(q^(5/2))]
 ----------------------------------------------------------------
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0), O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2), q^(3/4) - 18*q^(7/4) + O(q^(11/4))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0), -q^(3/4) + 18*q^(7/4) + O(q^(11/4))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2), O(q^(5/2))]]
```

[10]:
```
M4 = Matrix(ZZ,[[-2,0,2,2],[0,-2,1,0],[2,1,-2,1],[2,0,1,14]])
M4.determinant()
```

[10]: `-4`

[11]: `M4`

[11]:
```
[-2  0  2  2]
[ 0 -2  1  0]
[ 2  1 -2  1]
[ 2  0  1 14]
```

[12]:
```
L4 = IntegralLattice(M4).twist(-1)
picard_num_NS(L4)
```

[12]:
```
[2,
 3,
 [(0, 0, 0, 0), 1 - 204*q + O(q^2)]
 [(1/2, 0, 0, 0), -16896*q^(7/4) + O(q^(11/4))]
 [(0, 1/2, 0, 1/2), -4928*q^(3/2) + O(q^(5/2))]
 [(1/2, 1/2, 0, 1/2), -32*q^(3/4) - 16320*q^(7/4) + O(q^(11/4))]
 ----------------------------------------------------------------
 [(0, 0, 0, 0), -20*q + O(q^2)]
```

4

```
     [(1/2, 0, 0, 0), 128*q^(7/4) + O(q^(11/4))]
     [(0, 1/2, 0, 1/2), q^(1/2) - 80*q^(3/2) + O(q^(5/2))]
     [(1/2, 1/2, 0, 1/2), 8*q^(3/4) - 16*q^(7/4) + O(q^(11/4))]
     ---------------------------------------------------------
     [(0, 0, 0, 0), O(q^2)]
     [(1/2, 0, 0, 0), q^(3/4) - 18*q^(7/4) + O(q^(11/4))]
     [(0, 1/2, 0, 1/2), O(q^(5/2))]
     [(1/2, 1/2, 0, 1/2), -q^(3/4) + 18*q^(7/4) + O(q^(11/4))]]
```

[13]: `dis(L4)`

[13]: `[(0, 0, 0, 0), (1/2, 0, 0, 0), (0, 1/2, 0, 1/2), (1/2, 1/2, 0, 1/2)]`

[14]:
```
K = E.direct_sum(A1)
K = K.direct_sum(A2)
K
```

[14]:
```
Lattice of degree 11 and rank 11 over Integer Ring
Standard basis
Inner product matrix:
[ 2  0 -1  0  0  0  0  0  0  0  0]
[ 0  2  0 -1  0  0  0  0  0  0  0]
[-1  0  2 -1  0  0  0  0  0  0  0]
[ 0 -1 -1  2 -1  0  0  0  0  0  0]
[ 0  0  0 -1  2 -1  0  0  0  0  0]
[ 0  0  0  0 -1  2 -1  0  0  0  0]
[ 0  0  0  0  0 -1  2 -1  0  0  0]
[ 0  0  0  0  0  0 -1  2  0  0  0]
[ 0  0  0  0  0  0  0  0  2  0  0]
[ 0  0  0  0  0  0  0  0  0  2 -1]
[ 0  0  0  0  0  0  0  0  0 -1  2]
```

[15]: `picard_num(K)`

[15]:
```
[2,
 3,
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 204*q + O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/3, 2/3), -2430*q^(17/12) + O(q^(29/12))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), -8262*q^(5/3) + O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0), -34*q^(3/4) - 11424*q^(7/4) +
O(q^(11/4))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), -8262*q^(5/3) + O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 2/3, 1/3), -2430*q^(17/12) + O(q^(29/12))]
 ---------------------------------------------------------
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -40*q + O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/3, 2/3), q^(5/12) - 94*q^(17/12) +
O(q^(29/12))]
```

```
    [(0, 0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), 168*q^(5/3) + O(q^(8/3))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0), 14*q^(3/4) + 154*q^(7/4) + O(q^(11/4))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), 168*q^(5/3) + O(q^(8/3))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 2/3, 1/3), q^(5/12) - 94*q^(17/12) +
    O(q^(29/12))]
    -----------------------------------------------------------
    [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -2*q + O(q^2)]
    [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/3, 2/3), 2*q^(17/12) + O(q^(29/12))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), q^(2/3) - 16*q^(5/3) + O(q^(8/3))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0), -2*q^(3/4) + 32*q^(7/4) + O(q^(11/4))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), q^(2/3) - 16*q^(5/3) + O(q^(8/3))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 2/3, 1/3), 2*q^(17/12) + O(q^(29/12))]]
```

[16]:
```
M5 =
    Matrix(ZZ,[[-2,0,2,2,2],[0,-2,1,0,0],[2,1,-2,1,1],[2,0,1,14,15],[2,0,1,15,14]])
M5
```

[16]:
```
[-2  0  2  2  2]
[ 0 -2  1  0  0]
[ 2  1 -2  1  1]
[ 2  0  1 14 15]
[ 2  0  1 15 14]
```

[17]:
```
L5 = IntegralLattice(M5).twist(-1)
picard_num_NS(L5)
```

[17]:
```
[2,
 3,
 [(0, 0, 0, 0, 0), 1 - 204*q + O(q^2)]
 [(5/6, 0, 0, 2/3, 2/3), -2430*q^(17/12) + O(q^(29/12))]
 [(2/3, 0, 0, 1/3, 1/3), -8262*q^(5/3) + O(q^(8/3))]
 [(1/2, 0, 0, 0, 0), -34*q^(3/4) - 11424*q^(7/4) + O(q^(11/4))]
 [(1/3, 0, 0, 2/3, 2/3), -8262*q^(5/3) + O(q^(8/3))]
 [(1/6, 0, 0, 1/3, 1/3), -2430*q^(17/12) + O(q^(29/12))]
 -----------------------------------------------------------
 [(0, 0, 0, 0, 0), -40*q + O(q^2)]
 [(5/6, 0, 0, 2/3, 2/3), q^(5/12) - 94*q^(17/12) + O(q^(29/12))]
 [(2/3, 0, 0, 1/3, 1/3), 168*q^(5/3) + O(q^(8/3))]
 [(1/2, 0, 0, 0, 0), 14*q^(3/4) + 154*q^(7/4) + O(q^(11/4))]
 [(1/3, 0, 0, 2/3, 2/3), 168*q^(5/3) + O(q^(8/3))]
 [(1/6, 0, 0, 1/3, 1/3), q^(5/12) - 94*q^(17/12) + O(q^(29/12))]
 -----------------------------------------------------------
 [(0, 0, 0, 0, 0), -2*q + O(q^2)]
 [(5/6, 0, 0, 2/3, 2/3), 2*q^(17/12) + O(q^(29/12))]
 [(2/3, 0, 0, 1/3, 1/3), q^(2/3) - 16*q^(5/3) + O(q^(8/3))]
 [(1/2, 0, 0, 0, 0), -2*q^(3/4) + 32*q^(7/4) + O(q^(11/4))]
 [(1/3, 0, 0, 2/3, 2/3), q^(2/3) - 16*q^(5/3) + O(q^(8/3))]
```

6

```
        [(1/6, 0, 0, 1/3, 1/3), 2*q^(17/12) + O(q^(29/12))]]
```

[18]: `dis(L5)`

[18]:
```
[(0, 0, 0, 0, 0),
 (5/6, 0, 0, 2/3, 2/3),
 (2/3, 0, 0, 1/3, 1/3),
 (1/2, 0, 0, 0, 0),
 (1/3, 0, 0, 2/3, 2/3),
 (1/6, 0, 0, 1/3, 1/3)]
```

[24]:
```
Bor6 = E.direct_sum(A[2])
Bor6 = Bor6.direct_sum(A[2])
Bor6
```

[24]:
```
Lattice of degree 12 and rank 12 over Integer Ring
Standard basis
Inner product matrix:
[ 2  0 -1  0  0  0  0  0  0  0  0  0]
[ 0  2  0 -1  0  0  0  0  0  0  0  0]
[-1  0  2 -1  0  0  0  0  0  0  0  0]
[ 0 -1 -1  2 -1  0  0  0  0  0  0  0]
[ 0  0  0 -1  2 -1  0  0  0  0  0  0]
[ 0  0  0  0 -1  2 -1  0  0  0  0  0]
[ 0  0  0  0  0 -1  2 -1  0  0  0  0]
[ 0  0  0  0  0  0 -1  2  0  0  0  0]
[ 0  0  0  0  0  0  0  0  2 -1  0  0]
[ 0  0  0  0  0  0  0  0 -1  2  0  0]
[ 0  0  0  0  0  0  0  0  0  0  2 -1]
[ 0  0  0  0  0  0  0  0  0  0 -1  2]
```

[25]: `picard_num(Bor6)`

[25]:
```
[3,
 4,
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 168*q + O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 1/3, 2/3), -1215*q^(4/3) + O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 2/3, 1/3), -1215*q^(4/3) + O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 0), -5832*q^(5/3) + O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), -18*q^(2/3) - 5544*q^(5/3) +
O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 2/3, 1/3), -1215*q^(4/3) + O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 0), -5832*q^(5/3) + O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 1/3, 2/3), -1215*q^(4/3) + O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), -18*q^(2/3) - 5544*q^(5/3) +
O(q^(8/3))]
 ----------------------------------------------------------------
```

```
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -54*q + O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 1/3, 2/3), q^(1/3) + 70*q^(4/3) +
O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 2/3, 1/3), q^(1/3) + 70*q^(4/3) +
O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 0), 243*q^(5/3) + O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), 6*q^(2/3) + 147*q^(5/3) +
O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 2/3, 1/3), -162*q^(4/3) + O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 0), 243*q^(5/3) + O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 1/3, 2/3), -162*q^(4/3) + O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), 6*q^(2/3) + 147*q^(5/3) +
O(q^(8/3))]
 -----------------------------------------------------------
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -54*q + O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 1/3, 2/3), -162*q^(4/3) + O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 2/3, 1/3), -162*q^(4/3) + O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 0), 243*q^(5/3) + O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), 6*q^(2/3) + 147*q^(5/3) +
O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 2/3, 1/3), q^(1/3) + 70*q^(4/3) +
O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 0), 243*q^(5/3) + O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 1/3, 2/3), q^(1/3) + 70*q^(4/3) +
O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), 6*q^(2/3) + 147*q^(5/3) +
O(q^(8/3))]
 -----------------------------------------------------------
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 1/3, 2/3), O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 2/3, 1/3), O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 0), q^(2/3) - 16*q^(5/3) + O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), -q^(2/3) + 16*q^(5/3) + O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 2/3, 1/3), O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 0), q^(2/3) - 16*q^(5/3) + O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 1/3, 2/3), O(q^(7/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), -q^(2/3) + 16*q^(5/3) + O(q^(8/3))]]
```

```
[19]: M6 =␣
      ↪Matrix(ZZ,[[-2,1,0,1,1,1],[1,-2,0,1,1,1],[0,0,-2,1,0,0],[1,1,1,-2,1,1],[1,1,0,1,14,15],[1,1
      M6
```

```
[19]: [-2  1  0  1  1  1]
      [ 1 -2  0  1  1  1]
      [ 0  0 -2  1  0  0]
      [ 1  1  1 -2  1  1]
      [ 1  1  0  1 14 15]
```

```
[ 1   1   0   1 15 14]
```

```
[20]: L6 = IntegralLattice(M6).twist(-1)
      picard_num_NS(L6)
```

[20]: [3,
       4,
       [(0, 0, 0, 0, 0, 0), 1 - 168*q + O(q^2)]
       [(1/3, 0, 0, 0, 1/3, 1/3), -1215*q^(4/3) + O(q^(7/3))]
       [(2/3, 0, 0, 0, 2/3, 2/3), -1215*q^(4/3) + O(q^(7/3))]
       [(0, 1/3, 0, 0, 1/3, 1/3), -1215*q^(4/3) + O(q^(7/3))]
       [(1/3, 1/3, 0, 0, 2/3, 2/3), -5832*q^(5/3) + O(q^(8/3))]
       [(2/3, 1/3, 0, 0, 0, 0), -18*q^(2/3) - 5544*q^(5/3) + O(q^(8/3))]
       [(0, 2/3, 0, 0, 2/3, 2/3), -1215*q^(4/3) + O(q^(7/3))]
       [(1/3, 2/3, 0, 0, 0, 0), -18*q^(2/3) - 5544*q^(5/3) + O(q^(8/3))]
       [(2/3, 2/3, 0, 0, 1/3, 1/3), -5832*q^(5/3) + O(q^(8/3))]

       [(0, 0, 0, 0, 0, 0), -54*q + O(q^2)]
       [(1/3, 0, 0, 0, 1/3, 1/3), q^(1/3) + 70*q^(4/3) + O(q^(7/3))]
       [(2/3, 0, 0, 0, 2/3, 2/3), q^(1/3) + 70*q^(4/3) + O(q^(7/3))]
       [(0, 1/3, 0, 0, 1/3, 1/3), -162*q^(4/3) + O(q^(7/3))]
       [(1/3, 1/3, 0, 0, 2/3, 2/3), 243*q^(5/3) + O(q^(8/3))]
       [(2/3, 1/3, 0, 0, 0, 0), 6*q^(2/3) + 147*q^(5/3) + O(q^(8/3))]
       [(0, 2/3, 0, 0, 2/3, 2/3), -162*q^(4/3) + O(q^(7/3))]
       [(1/3, 2/3, 0, 0, 0, 0), 6*q^(2/3) + 147*q^(5/3) + O(q^(8/3))]
       [(2/3, 2/3, 0, 0, 1/3, 1/3), 243*q^(5/3) + O(q^(8/3))]

       [(0, 0, 0, 0, 0, 0), -54*q + O(q^2)]
       [(1/3, 0, 0, 0, 1/3, 1/3), -162*q^(4/3) + O(q^(7/3))]
       [(2/3, 0, 0, 0, 2/3, 2/3), -162*q^(4/3) + O(q^(7/3))]
       [(0, 1/3, 0, 0, 1/3, 1/3), q^(1/3) + 70*q^(4/3) + O(q^(7/3))]
       [(1/3, 1/3, 0, 0, 2/3, 2/3), 243*q^(5/3) + O(q^(8/3))]
       [(2/3, 1/3, 0, 0, 0, 0), 6*q^(2/3) + 147*q^(5/3) + O(q^(8/3))]
       [(0, 2/3, 0, 0, 2/3, 2/3), q^(1/3) + 70*q^(4/3) + O(q^(7/3))]
       [(1/3, 2/3, 0, 0, 0, 0), 6*q^(2/3) + 147*q^(5/3) + O(q^(8/3))]
       [(2/3, 2/3, 0, 0, 1/3, 1/3), 243*q^(5/3) + O(q^(8/3))]

       [(0, 0, 0, 0, 0, 0), O(q^2)]
       [(1/3, 0, 0, 0, 1/3, 1/3), O(q^(7/3))]
       [(2/3, 0, 0, 0, 2/3, 2/3), O(q^(7/3))]
       [(0, 1/3, 0, 0, 1/3, 1/3), O(q^(7/3))]
       [(1/3, 1/3, 0, 0, 2/3, 2/3), q^(2/3) - 16*q^(5/3) + O(q^(8/3))]
       [(2/3, 1/3, 0, 0, 0, 0), -q^(2/3) + 16*q^(5/3) + O(q^(8/3))]
       [(0, 2/3, 0, 0, 2/3, 2/3), O(q^(7/3))]
       [(1/3, 2/3, 0, 0, 0, 0), -q^(2/3) + 16*q^(5/3) + O(q^(8/3))]
       [(2/3, 2/3, 0, 0, 1/3, 1/3), q^(2/3) - 16*q^(5/3) + O(q^(8/3))]]

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```
[21]: dis(L6)
```

```
[21]: [(0, 0, 0, 0, 0, 0),
      (1/3, 0, 0, 0, 1/3, 1/3),
      (2/3, 0, 0, 0, 2/3, 2/3),
      (0, 1/3, 0, 0, 1/3, 1/3),
      (1/3, 1/3, 0, 0, 2/3, 2/3),
      (2/3, 1/3, 0, 0, 0, 0),
      (0, 2/3, 0, 0, 2/3, 2/3),
      (1/3, 2/3, 0, 0, 0, 0),
      (2/3, 2/3, 0, 0, 1/3, 1/3)]
```

```
[24]: A3 = IntegralLattice('A3')
      L7 = E.direct_sum(A2)
      L7 = L7.direct_sum(A3)
      L7
```

```
[24]: Lattice of degree 13 and rank 13 over Integer Ring
      Standard basis
      Inner product matrix:
      [ 2  0 -1  0  0  0  0  0  0  0  0  0  0]
      [ 0  2  0 -1  0  0  0  0  0  0  0  0  0]
      [-1  0  2 -1  0  0  0  0  0  0  0  0  0]
      [ 0 -1 -1  2 -1  0  0  0  0  0  0  0  0]
      [ 0  0  0 -1  2 -1  0  0  0  0  0  0  0]
      [ 0  0  0  0 -1  2 -1  0  0  0  0  0  0]
      [ 0  0  0  0  0 -1  2 -1  0  0  0  0  0]
      [ 0  0  0  0  0  0 -1  2  0  0  0  0  0]
      [ 0  0  0  0  0  0  0  0  2 -1  0  0  0]
      [ 0  0  0  0  0  0  0  0 -1  2  0  0  0]
      [ 0  0  0  0  0  0  0  0  0  0  2 -1  0]
      [ 0  0  0  0  0  0  0  0  0  0 -1  2 -1]
      [ 0  0  0  0  0  0  0  0  0  0  0 -1  2]
```

```
[25]: picard_num(L7)
```

```
[25]: [4,
       5,
       [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 160*q + O(q^2)]
       [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 1/4, 1/2, 3/4), -780*q^(31/24) +
      O(q^(55/24))]
       [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 1/2, 0, 1/2), -420*q^(7/6) + O(q^(13/6))]
       [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4), -4*q^(5/8) - 3540*q^(13/8) +
      O(q^(21/8))]
       [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 0, 0), -15*q^(2/3) - 4032*q^(5/3) +
      O(q^(8/3))]
       [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 1/4, 1/2, 3/4), -780*q^(31/24) +
```

O(q^(55/24))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2), -2080*q^(3/2) + O(q^(5/2))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 3/4, 1/2, 1/4), -780*q^(31/24) +
O(q^(55/24))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 0, 0), -15*q^(2/3) - 4032*q^(5/3) +
O(q^(8/3))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4), -4*q^(5/8) - 3540*q^(13/8) +
O(q^(21/8))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 1/2, 0, 1/2), -420*q^(7/6) + O(q^(13/6))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 3/4, 1/2, 1/4), -780*q^(31/24) +
O(q^(55/24))]

  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -132*q + O(q^2)]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 1/4, 1/2, 3/4), -220*q^(31/24) +
O(q^(55/24))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 1/2, 0, 1/2), q^(1/6) + 128*q^(7/6) +
O(q^(13/6))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4), 12*q^(5/8) + 1404*q^(13/8) +
O(q^(21/8))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 0, 0), 924*q^(5/3) + O(q^(8/3))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 1/4, 1/2, 3/4), -220*q^(31/24) +
O(q^(55/24))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2), -990*q^(3/2) + O(q^(5/2))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 3/4, 1/2, 1/4), -220*q^(31/24) +
O(q^(55/24))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 0, 0), 924*q^(5/3) + O(q^(8/3))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4), 12*q^(5/8) + 1404*q^(13/8) +
O(q^(21/8))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 1/2, 0, 1/2), q^(1/6) + 128*q^(7/6) +
O(q^(13/6))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 3/4, 1/2, 1/4), -220*q^(31/24) +
O(q^(55/24))]

  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -48*q + O(q^2)]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 1/4, 1/2, 3/4), q^(7/24) + 110*q^(31/24) +
O(q^(55/24))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 1/2, 0, 1/2), -48*q^(7/6) + O(q^(13/6))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4), -3*q^(5/8) + 54*q^(13/8) +
O(q^(21/8))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 0, 0), 8*q^(2/3) + 112*q^(5/3) +
O(q^(8/3))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 1/4, 1/2, 3/4), -123*q^(31/24) +
O(q^(55/24))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2), 288*q^(3/2) + O(q^(5/2))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 3/4, 1/2, 1/4), -123*q^(31/24) +
O(q^(55/24))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 0, 0), 8*q^(2/3) + 112*q^(5/3) +

O(q^(8/3))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4), -3*q^(5/8) + 54*q^(13/8) +
O(q^(21/8))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 1/2, 0, 1/2), -48*q^(7/6) + O(q^(13/6))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 3/4, 1/2, 1/4), q^(7/24) + 110*q^(31/24) +
O(q^(55/24))]

  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -48*q + O(q^2)]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 1/4, 1/2, 3/4), -123*q^(31/24) +
O(q^(55/24))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 1/2, 0, 1/2), -48*q^(7/6) + O(q^(13/6))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4), -3*q^(5/8) + 54*q^(13/8) +
O(q^(21/8))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 0, 0), 8*q^(2/3) + 112*q^(5/3) +
O(q^(8/3))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 1/4, 1/2, 3/4), q^(7/24) + 110*q^(31/24) +
O(q^(55/24))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2), 288*q^(3/2) + O(q^(5/2))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 3/4, 1/2, 1/4), q^(7/24) + 110*q^(31/24) +
O(q^(55/24))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 0, 0), 8*q^(2/3) + 112*q^(5/3) +
O(q^(8/3))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4), -3*q^(5/8) + 54*q^(13/8) +
O(q^(21/8))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 1/2, 0, 1/2), -48*q^(7/6) + O(q^(13/6))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 3/4, 1/2, 1/4), -123*q^(31/24) +
O(q^(55/24))]

  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -4*q + O(q^2)]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 1/4, 1/2, 3/4), 6*q^(31/24) + O(q^(55/24))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 1/2, 0, 1/2), -3*q^(7/6) + O(q^(13/6))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3/4, 1/2, 1/4), 2*q^(5/8) - 22*q^(13/8) +
O(q^(21/8))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 0, 0, 0), -3*q^(2/3) + 36*q^(5/3) +
O(q^(8/3))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 1/4, 1/2, 3/4), 6*q^(31/24) + O(q^(55/24))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2), q^(1/2) - 24*q^(3/2) +
O(q^(5/2))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 3/4, 1/2, 1/4), 6*q^(31/24) + O(q^(55/24))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 0, 0, 0), -3*q^(2/3) + 36*q^(5/3) +
O(q^(8/3))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/4, 1/2, 3/4), 2*q^(5/8) - 22*q^(13/8) +
O(q^(21/8))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3, 1/2, 0, 1/2), -3*q^(7/6) + O(q^(13/6))]
  [(0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3, 3/4, 1/2, 1/4), 6*q^(31/24) +
O(q^(55/24))]]

```
[17]: M8 =␣
      ↪Matrix(ZZ,[[-2,0,1,1,1,1,1,1],[0,-2,1,0,0,0,0,0],[1,1,-2,1,1,0,0,0],[1,0,1,14,15,0,0,0],[1,
      M8
```

```
[17]: [-2  0  1  1  1  1  1  1]
      [ 0 -2  1  0  0  0  0  0]
      [ 1  1 -2  1  1  0  0  0]
      [ 1  0  1 14 15  0  0  0]
      [ 1  0  1 15 14  0  0  0]
      [ 1  0  0  0  0 -2  0  0]
      [ 1  0  0  0  0  0 -2  0]
      [ 1  0  0  0  0  0  0 -2]
```

```
[18]: L8 = IntegralLattice(M8).twist(-1)
      picard_num_NS(L8)
```

```
[18]: [4,
       5,
       [(0, 0, 0, 0, 0, 0, 0, 0), 1 - 168*q + O(q^2)]
       [(1/3, 0, 0, 1/3, 1/3, 1/6, 1/6, 2/3), -384*q^(7/6) + O(q^(13/6))]
       [(2/3, 0, 0, 2/3, 2/3, 1/3, 1/3, 1/3), -21*q^(2/3) - 3192*q^(5/3) + O(q^(8/3))]
       [(0, 0, 0, 0, 0, 1/2, 1/2, 0), -1792*q^(3/2) + O(q^(5/2))]
       [(1/3, 0, 0, 1/3, 1/3, 2/3, 2/3, 2/3), -21*q^(2/3) - 3192*q^(5/3) + O(q^(8/3))]
       [(2/3, 0, 0, 2/3, 2/3, 5/6, 5/6, 1/3), -384*q^(7/6) + O(q^(13/6))]
       [(0, 0, 0, 0, 0, 1/2, 0, 1/2), -1792*q^(3/2) + O(q^(5/2))]
       [(1/3, 0, 0, 1/3, 1/3, 2/3, 1/6, 1/6), -384*q^(7/6) + O(q^(13/6))]
       [(2/3, 0, 0, 2/3, 2/3, 5/6, 1/3, 5/6), -384*q^(7/6) + O(q^(13/6))]
       [(0, 0, 0, 0, 0, 0, 1/2, 1/2), -1792*q^(3/2) + O(q^(5/2))]
       [(1/3, 0, 0, 1/3, 1/3, 1/6, 2/3, 1/6), -384*q^(7/6) + O(q^(13/6))]
       [(2/3, 0, 0, 2/3, 2/3, 1/3, 5/6, 5/6), -384*q^(7/6) + O(q^(13/6))]
       ------------------------------------------------------------
       [(0, 0, 0, 0, 0, 0, 0, 0), -132*q + O(q^2)]
       [(1/3, 0, 0, 1/3, 1/3, 1/6, 1/6, 2/3), q^(1/6) + 132*q^(7/6) + O(q^(13/6))]
       [(2/3, 0, 0, 2/3, 2/3, 1/3, 1/3, 1/3), -2*q^(2/3) + 660*q^(5/3) + O(q^(8/3))]
       [(0, 0, 0, 0, 0, 1/2, 1/2, 0), -726*q^(3/2) + O(q^(5/2))]
       [(1/3, 0, 0, 1/3, 1/3, 2/3, 2/3, 2/3), -2*q^(2/3) + 660*q^(5/3) + O(q^(8/3))]
       [(2/3, 0, 0, 2/3, 2/3, 5/6, 5/6, 1/3), q^(1/6) + 132*q^(7/6) + O(q^(13/6))]
       [(0, 0, 0, 0, 0, 1/2, 0, 1/2), 6*q^(1/2) + 696*q^(3/2) + O(q^(5/2))]
       [(1/3, 0, 0, 1/3, 1/3, 2/3, 1/6, 1/6), -110*q^(7/6) + O(q^(13/6))]
       [(2/3, 0, 0, 2/3, 2/3, 5/6, 1/3, 5/6), -110*q^(7/6) + O(q^(13/6))]
       [(0, 0, 0, 0, 0, 0, 1/2, 1/2), 6*q^(1/2) + 696*q^(3/2) + O(q^(5/2))]
       [(1/3, 0, 0, 1/3, 1/3, 1/6, 2/3, 1/6), -110*q^(7/6) + O(q^(13/6))]
       [(2/3, 0, 0, 2/3, 2/3, 1/3, 5/6, 5/6), -110*q^(7/6) + O(q^(13/6))]
       ------------------------------------------------------------
       [(0, 0, 0, 0, 0, 0, 0, 0), -96*q + O(q^2)]
       [(1/3, 0, 0, 1/3, 1/3, 1/6, 1/6, 2/3), -128*q^(7/6) + O(q^(13/6))]
       [(2/3, 0, 0, 2/3, 2/3, 1/3, 1/3, 1/3), 16*q^(2/3) + 480*q^(5/3) + O(q^(8/3))]
```

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```
[(0, 0, 0, 0, 0, 1/2, 1/2, 0), 768*q^(3/2) + O(q^(5/2))]
[(1/3, 0, 0, 1/3, 1/3, 2/3, 2/3, 2/3), 16*q^(2/3) + 480*q^(5/3) + O(q^(8/3))]
[(2/3, 0, 0, 2/3, 2/3, 5/6, 5/6, 1/3), -128*q^(7/6) + O(q^(13/6))]
[(0, 0, 0, 0, 0, 1/2, 0, 1/2), 768*q^(3/2) + O(q^(5/2))]
[(1/3, 0, 0, 1/3, 1/3, 2/3, 1/6, 1/6), q^(1/6) + 114*q^(7/6) + O(q^(13/6))]
[(2/3, 0, 0, 2/3, 2/3, 5/6, 1/3, 5/6), -128*q^(7/6) + O(q^(13/6))]
[(0, 0, 0, 0, 0, 0, 1/2, 1/2), -6*q^(1/2) - 654*q^(3/2) + O(q^(5/2))]
[(1/3, 0, 0, 1/3, 1/3, 1/6, 2/3, 1/6), -128*q^(7/6) + O(q^(13/6))]
[(2/3, 0, 0, 2/3, 2/3, 1/3, 5/6, 5/6), q^(1/6) + 114*q^(7/6) + O(q^(13/6))]
--------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0), -96*q + O(q^2)]
[(1/3, 0, 0, 1/3, 1/3, 1/6, 1/6, 2/3), -128*q^(7/6) + O(q^(13/6))]
[(2/3, 0, 0, 2/3, 2/3, 1/3, 1/3, 1/3), 16*q^(2/3) + 480*q^(5/3) + O(q^(8/3))]
[(0, 0, 0, 0, 0, 1/2, 1/2, 0), 768*q^(3/2) + O(q^(5/2))]
[(1/3, 0, 0, 1/3, 1/3, 2/3, 2/3, 2/3), 16*q^(2/3) + 480*q^(5/3) + O(q^(8/3))]
[(2/3, 0, 0, 2/3, 2/3, 5/6, 5/6, 1/3), -128*q^(7/6) + O(q^(13/6))]
[(0, 0, 0, 0, 0, 1/2, 0, 1/2), -6*q^(1/2) - 654*q^(3/2) + O(q^(5/2))]
[(1/3, 0, 0, 1/3, 1/3, 2/3, 1/6, 1/6), -128*q^(7/6) + O(q^(13/6))]
[(2/3, 0, 0, 2/3, 2/3, 5/6, 1/3, 5/6), q^(1/6) + 114*q^(7/6) + O(q^(13/6))]
[(0, 0, 0, 0, 0, 0, 1/2, 1/2), 768*q^(3/2) + O(q^(5/2))]
[(1/3, 0, 0, 1/3, 1/3, 1/6, 2/3, 1/6), q^(1/6) + 114*q^(7/6) + O(q^(13/6))]
[(2/3, 0, 0, 2/3, 2/3, 1/3, 5/6, 5/6), -128*q^(7/6) + O(q^(13/6))]
--------------------------------------------------------------
[(0, 0, 0, 0, 0, 0, 0, 0), -6*q + O(q^2)]
[(1/3, 0, 0, 1/3, 1/3, 1/6, 1/6, 2/3), 3*q^(7/6) + O(q^(13/6))]
[(2/3, 0, 0, 2/3, 2/3, 1/3, 1/3, 1/3), -3*q^(2/3) + 30*q^(5/3) + O(q^(8/3))]
[(0, 0, 0, 0, 0, 1/2, 1/2, 0), q^(1/2) - 12*q^(3/2) + O(q^(5/2))]
[(1/3, 0, 0, 1/3, 1/3, 2/3, 2/3, 2/3), -3*q^(2/3) + 30*q^(5/3) + O(q^(8/3))]
[(2/3, 0, 0, 2/3, 2/3, 5/6, 5/6, 1/3), 3*q^(7/6) + O(q^(13/6))]
[(0, 0, 0, 0, 0, 1/2, 0, 1/2), q^(1/2) - 12*q^(3/2) + O(q^(5/2))]
[(1/3, 0, 0, 1/3, 1/3, 2/3, 1/6, 1/6), 3*q^(7/6) + O(q^(13/6))]
[(2/3, 0, 0, 2/3, 2/3, 5/6, 1/3, 5/6), 3*q^(7/6) + O(q^(13/6))]
[(0, 0, 0, 0, 0, 0, 1/2, 1/2), q^(1/2) - 12*q^(3/2) + O(q^(5/2))]
[(1/3, 0, 0, 1/3, 1/3, 1/6, 2/3, 1/6), 3*q^(7/6) + O(q^(13/6))]
[(2/3, 0, 0, 2/3, 2/3, 1/3, 5/6, 5/6), 3*q^(7/6) + O(q^(13/6))]]
```

[26]:
```
D4 = IntegralLattice('D4')
Bor8 = E.direct_sum(D4)
Bor8 = Bor8.direct_sum(A2)
Bor8
```

[26]:
```
Lattice of degree 14 and rank 14 over Integer Ring
Standard basis
Inner product matrix:
[ 2  0 -1  0  0  0  0  0  0  0  0  0  0  0]
[ 0  2  0 -1  0  0  0  0  0  0  0  0  0  0]
[-1  0  2 -1  0  0  0  0  0  0  0  0  0  0]
```

```
[ 0 -1 -1  2 -1  0  0  0  0  0  0  0  0  0]
[ 0  0  0 -1  2 -1  0  0  0  0  0  0  0  0]
[ 0  0  0  0 -1  2 -1  0  0  0  0  0  0  0]
[ 0  0  0  0  0 -1  2 -1  0  0  0  0  0  0]
[ 0  0  0  0  0  0 -1  2  0  0  0  0  0  0]
[ 0  0  0  0  0  0  0  0  2 -1  0  0  0  0]
[ 0  0  0  0  0  0  0  0 -1  2 -1 -1  0  0]
[ 0  0  0  0  0  0  0  0  0 -1  2  0  0  0]
[ 0  0  0  0  0  0  0  0  0 -1  0  2  0  0]
[ 0  0  0  0  0  0  0  0  0  0  0  0  2 -1]
[ 0  0  0  0  0  0  0  0  0  0  0  0 -1  2]
```

[27]: `picard_num(Bor8)`

[27]: ```
[4,
 5,
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), 1 - 168*q + O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 1/3, 2/3), -384*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), -21*q^(2/3) - 3192*q^(5/3) +
O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 0, 0), -1792*q^(3/2) + O(q^(5/2))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), -21*q^(2/3) - 3192*q^(5/3) +
O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 2/3, 1/3), -384*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 0, 0), -1792*q^(3/2) + O(q^(5/2))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/3, 2/3), -384*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 2/3, 1/3), -384*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 0, 0), -1792*q^(3/2) + O(q^(5/2))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 1/3, 2/3), -384*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 2/3, 1/3), -384*q^(7/6) +
O(q^(13/6))]

 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -132*q + O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 1/3, 2/3), q^(1/6) + 132*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), -2*q^(2/3) + 660*q^(5/3) +
O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 0, 0), -726*q^(3/2) + O(q^(5/2))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), -2*q^(2/3) + 660*q^(5/3) +
O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 2/3, 1/3), q^(1/6) + 132*q^(7/6) +
O(q^(13/6))]
```

```
[(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 0, 0), 6*q^(1/2) + 696*q^(3/2) +
O(q^(5/2))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/3, 2/3), -110*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 2/3, 1/3), -110*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 0, 0), 6*q^(1/2) + 696*q^(3/2) +
O(q^(5/2))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 1/3, 2/3), -110*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 2/3, 1/3), -110*q^(7/6) +
O(q^(13/6))]
 ------------------------------------------------------------
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -96*q + O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 1/3, 2/3), -128*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), 16*q^(2/3) + 480*q^(5/3) +
O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 0, 0), 768*q^(3/2) + O(q^(5/2))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), 16*q^(2/3) + 480*q^(5/3) +
O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 2/3, 1/3), -128*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 0, 0), 768*q^(3/2) + O(q^(5/2))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/3, 2/3), q^(1/6) + 114*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 2/3, 1/3), -128*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 0, 0), -6*q^(1/2) - 654*q^(3/2) +
O(q^(5/2))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 1/3, 2/3), -128*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 2/3, 1/3), q^(1/6) + 114*q^(7/6) +
O(q^(13/6))]
 ------------------------------------------------------------
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -96*q + O(q^2)]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 1/3, 2/3), -128*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), 16*q^(2/3) + 480*q^(5/3) +
O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 0, 0), 768*q^(3/2) + O(q^(5/2))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), 16*q^(2/3) + 480*q^(5/3) +
O(q^(8/3))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 2/3, 1/3), -128*q^(7/6) +
O(q^(13/6))]
 [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 0, 0), -6*q^(1/2) - 654*q^(3/2) +
O(q^(5/2))]
```

```
    [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/3, 2/3), -128*q^(7/6) +
O(q^(13/6))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 2/3, 1/3), q^(1/6) + 114*q^(7/6) +
O(q^(13/6))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 0, 0), 768*q^(3/2) + O(q^(5/2))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 1/3, 2/3), q^(1/6) + 114*q^(7/6) +
O(q^(13/6))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 2/3, 1/3), -128*q^(7/6) +
O(q^(13/6))]
    ------------------------------------------------------------
    [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), -6*q + O(q^2)]
    [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 1/3, 2/3), 3*q^(7/6) + O(q^(13/6))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2/3, 1/3), -3*q^(2/3) + 30*q^(5/3) +
O(q^(8/3))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 0, 0), q^(1/2) - 12*q^(3/2) +
O(q^(5/2))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/3, 2/3), -3*q^(2/3) + 30*q^(5/3) +
O(q^(8/3))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 1/2, 2/3, 1/3), 3*q^(7/6) + O(q^(13/6))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 0, 0), q^(1/2) - 12*q^(3/2) +
O(q^(5/2))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 1/3, 2/3), 3*q^(7/6) + O(q^(13/6))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 2/3, 1/3), 3*q^(7/6) + O(q^(13/6))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 0, 0), q^(1/2) - 12*q^(3/2) +
O(q^(5/2))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 1/2, 1/3, 2/3), 3*q^(7/6) + O(q^(13/6))]
    [(0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 1/2, 0, 2/3, 1/3), 3*q^(7/6) + O(q^(13/6))]]
```

[29]: `picard_num(D4)`

[29]:
```
[3,
 4,
 [(0, 0, 0, 0), 1 - 48*q + O(q^2)]
 [(1/2, 0, 1/2, 0), -4096*q^(3/2) + O(q^(5/2))]
 [(1/2, 0, 0, 1/2), -4096*q^(3/2) + O(q^(5/2))]
 [(0, 0, 1/2, 1/2), -4096*q^(3/2) + O(q^(5/2))]
 ------------------------------------------------------------
 [(0, 0, 0, 0), -8*q + O(q^2)]
 [(1/2, 0, 1/2, 0), q^(1/2) - 260*q^(3/2) + O(q^(5/2))]
 [(1/2, 0, 0, 1/2), 256*q^(3/2) + O(q^(5/2))]
 [(0, 0, 1/2, 1/2), 256*q^(3/2) + O(q^(5/2))]
 ------------------------------------------------------------
 [(0, 0, 0, 0), -8*q + O(q^2)]
 [(1/2, 0, 1/2, 0), 256*q^(3/2) + O(q^(5/2))]
 [(1/2, 0, 0, 1/2), q^(1/2) - 260*q^(3/2) + O(q^(5/2))]
 [(0, 0, 1/2, 1/2), 256*q^(3/2) + O(q^(5/2))]
 ------------------------------------------------------------
```

```
[(0, 0, 0, 0), -8*q + O(q^2)]
[(1/2, 0, 1/2, 0), 256*q^(3/2) + O(q^(5/2))]
[(1/2, 0, 0, 1/2), 256*q^(3/2) + O(q^(5/2))]
[(0, 0, 1/2, 1/2), q^(1/2) - 260*q^(3/2) + O(q^(5/2))]]
```