Seminar on hyper-Kähler manifolds

Some suggestions on preprint

1. On product identities and the Chow rings of holomorphic symplectic varieties, Ignacio Barros, L. Flapan, A. Marian, and R. Silversmith [1].

2. This is continuation of our last semester seminar with emphasis of very recent developments.

   Hilbert schemes of K3 surfaces, generalized Kummer, and cobordism classes of hyper-Kähler manifolds, Georg Oberdieck, Jieao Song, Claire Voisin. [14]

   Justin Sawon, Topological bounds on hyperkähler manifolds.

   Second Chern class and Fujiki constants of hyperkähler manifolds, Beckmann, Thorsten ; Song, Jieao [5]

   Computing Riemann-Roch polynomials and classifying hyper-Kähler fourfolds, Olivier Debarre, Daniel Huybrechts, Emanuele Macrì, Claire Voisin

Schedules:

- Lecture 1 (Guo, 3.7): Origins of the BV conjecture. We will cover [1]
  - The statement of Bloch-Beilinson conjecture, refer to [19].
  - Beauville's weak splitting conjecture [3]. For an irreducible symplectic variety X, the partial cycle class map

    $$cl_X : \ DCH^*(X) \to H^*(X)$$

    is injective.
  - The development of the BV conjecture, know and unknown results, applications. References [4], [20], [8], [17].
- Lecture 2 (Zhang, 3.14): Chow-Künneth decomposition [12] [11]
  - Introduce the concept of Chow-Künneth decomposition due to J.P. Murre in the category of Chow motives.
  - Relations with the conjectural Bloch-Beilinsen filtration.

[1] Maybe we should also cover the Beauvile's decomposition of Chow ring of Ableian variety[2]

  - The Chow-Künneth decompositions on the Chow group of curves, surface, abelian varieties.

- Lecture 3 (Shen, 3.21): Fourier decomposition for hyperkähler fourfolds [18]

  - Review the decomposition and the Fourier transforms on the Chow ring of abelian varieties originated by Beauville [2].
  - Explain the main idea and results of the Fourier transform for hyperkähler fourfolds of $K3^{[2]}$-type.

- Lecture 4 (Lyu, 3.28): Chow-Künneth decomposition and Fourier-decomposition on the double EPW sextics. [7] [15].

- Lecture 5 (Wang, 4.11): Cone structure of hyperkähler variety.

- Lecture 6 (Zhou, 4.18): On fibrations and measures of irrationality of hyper-Kähler manifolds.

- Lecture 7 (Guo/Si, 4.25): Computing Riemann-Roch polynomials and classifying hyper-Kähler fourfolds, D-H-M-C.

- Lecture 8 (Guo/Si, 5.9): Computing Riemann-Roch polynomials and classifying hyper-Kähler fourfolds, D-H-M-C.

Other interesting topics

- Introduction to the Franchetta conjecture and the status of the art . We will survey the results of [16] [6] [9].

- New methods on BV conjecture and Chow–Künneth decomposition:

  Maulik-Neguţ's proof of weak version of BV conjecture for $X = S^{[n]}$ using Lehn's formula [10] and NOY's work on refinement of Chow–Künneth decomposition [13].

Time:

Monday, 6:30pm. 2h including 0.5h for free discussion.

Place:

QuanZhai, PKU

References

[1] Ignacio Barros, Laure Flapan, Alina Marian, and Rob Silversmith. On product identities and the Chow rings of holomorphic symplectic varieties. *arXiv e-prints*, page arXiv:1912.13419, December 2019.

[2] Arnaud Beauville. Sur l'anneau de Chow d'une variété abélienne. *Math. Ann.*, 273(4):647–651, 1986.

[3] Arnaud Beauville. On the splitting of the Bloch-Beilinson filtration. In *Algebraic cycles and motives. Vol. 2*, volume 344 of *London Math. Soc. Lecture Note Ser.*, pages 38–53. Cambridge Univ. Press, Cambridge, 2007.

[4] Arnaud Beauville and Claire Voisin. On the Chow ring of a $K3$ surface. *J. Algebraic Geom.*, 13(3):417–426, 2004.

[5] Thorsten Beckmann and Jieao Song. Second Chern class and Fujiki constants of hyperk\"ahler manifolds. *arXiv e-prints*, page arXiv:2201.07767, January 2022.

[6] Nicolas Bergeron and Zhiyuan Li. Tautological classes on moduli spaces of hyper-Kähler manifolds. *Duke Math. J.*, 168(7):1179–1230, 2019.

[7] Andrea Ferretti. The Chow ring of double EPW sextics. *Rend. Mat. Appl. (7)*, 31(3-4):69–217, 2011.

[8] Lie Fu. Beauville-Voisin conjecture for generalized Kummer varieties. *Int. Math. Res. Not. IMRN*, (12):3878–3898, 2015.

[9] Lie Fu, Robert Laterveer, and Charles Vial. The generalized Franchetta conjecture for some hyper-Kähler varieties. *J. Math. Pures Appl. (9)*, 130:1–35, 2019. With an appendix by the authors and Mingmin Shen.

[10] Davesh Maulik and Andrei Neguţ. Lehn's formula in Chow and Conjectures of Beauville and Voisin. *arXiv e-prints*, page arXiv:1904.05262, April 2019.

[11] J. P. Murre. On the motive of an algebraic surface. *J. Reine Angew. Math.*, 409:190–204, 1990.

[12] J. P. Murre. On a conjectural filtration on the Chow groups of an algebraic variety. I. The general conjectures and some examples. *Indag. Math. (N.S.)*, 4(2):177–188, 1993.

[13] Andrei Neguţ, Georg Oberdieck, and Qizheng Yin. Motivic decompositions for the Hilbert scheme of points of a K3 surface. *J. Reine Angew. Math.*, 778:65–95, 2021.

[14] Georg Oberdieck, Jieao Song, and Claire Voisin. Hilbert schemes of K3 surfaces, generalized Kummer, and cobordism classes of hyper-Kähler manifolds. *arXiv e-prints*, page arXiv:2110.02211, October 2021.

[15] Kieran G. O'Grady. Irreducible symplectic 4-folds and Eisenbud-Popescu-Walter sextics. *Duke Math. J.*, 134(1):99–137, 2006.

[16] Nebojsa Pavic, Junliang Shen, and Qizheng Yin. On O'Grady's generalized Franchetta conjecture. *Int. Math. Res. Not. IMRN*, (16):4971–4983, 2017.

[17] Ulrike Rieß. On Beauville's conjectural weak splitting property. *Int. Math. Res. Not. IMRN*, (20):6133–6150, 2016.

[18] Mingmin Shen and Charles Vial. The Fourier transform for certain hyperkähler fourfolds. *Mem. Amer. Math. Soc.*, 240(1139):vii+163, 2016.

[19] Claire Voisin. Remarks on filtrations on Chow groups and the Bloch conjecture. *Ann. Mat. Pura Appl. (4)*, 183(3):421–438, 2004.

[20] Claire Voisin. On the Chow ring of certain algebraic hyper-Kähler manifolds. *Pure Appl. Math. Q.*, 4(3, Special Issue: In honor of Fedor Bogomolov. Part 2):613–649, 2008.