UoG-UESTC Joint School

Degrees of BEng

Electrical Circuit Analysis

December 2015

Attempt all questions.

Total 100 marks.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

Electrical Circuit Analysis Formula List.

$$For\ an\ inductor: v(t) = L\frac{di}{dt},\quad i(t) = \frac{1}{L}\int vdt;\quad For\ a\ capacitor: v(t) = \frac{1}{C}\int idt,\quad i(t) = C\frac{dv}{dt}$$

$$V_{Ave} = \frac{1}{T}\int_0^T v(t)dt,\quad V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt};\qquad Energy = \frac{1}{2}CV^2 = \frac{1}{2}LI^2 = Power \times time;$$

$$Power = \frac{V^2}{R} = I^2R = V \times I = Force \times velocity = Torque \times angular\ velocity$$

$$Power\ \ Factor = \cos(\varphi)\ \ or\ \frac{\cos(\varphi)}{\sqrt{1+THD^2}} = \frac{R}{Z} = \frac{P}{S};\qquad Three\ \ phase\ \ power: P = \sqrt{3}V_L I_L \cos(\varphi)$$

1st Order DE: $\tau\frac{dx(t)}{dt} + x(t) = F \quad t \ge 0$, Solution: $x(t) = x(\infty) + \left[x(0) - x(\infty)\right]e^{-t/\tau}$

2nd Order DE: $\frac{1}{\omega_n^2}\frac{d^2x(t)}{dt^2} + \frac{2\zeta}{\omega_n}\frac{dx(t)}{dt} + x(t) = K_S f(t)$,

Solution: $x(t) = A_1 e^{\left(-\zeta\omega_n + \omega_n\sqrt{\zeta^2-1}\right)t} + A_2 e^{\left(-\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}\right)t} + x(\infty) \quad t \ge 0$

$\left(\sin\left(ax\right)\right)' = a\cos x,\quad \left(\cos\left(ax\right)\right)' = -a\sin x$

$\int\sin\left(ax\right)dx = -\frac{1}{a}\cos\left(ax\right),\quad \int\cos\left(ax\right)dx = \frac{1}{a}\sin\left(ax\right)$

$\cos\left(A \pm B\right) = \cos A\cos B \mp \sin A\sin B$

$\sin\left(A \pm B\right) = \sin A\cos B \pm \cos A\sin B$

$\cos x = \sin\left(x + \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2} - x\right)$

$\sin x = \cos\left(\frac{\pi}{2} - x\right) = -\cos\left(\frac{\pi}{2} + x\right)$

$e^{j\theta} = \cos\theta + j\sin\theta$

$\mathbf{V}(j\omega) = \mathbf{V} = V_m e^{j\phi} \equiv V_m\angle\phi$

$\mathbf{Z} = R + jX = |Z|\angle\varphi,\ |Z| = \sqrt{R^2 + X^2}$

Q1 Consider the series resistor and inductor combination shown in Figure Q1.

(a) The current $i(t)$ can be written in the form $i(t) = I_m \sin(\omega t + \varphi)$ A, determine the values of I_m and φ . [5]

(b) Calculate the average power dissipation P_{avg} in the circuit, and explain where the power is dissipated. [5]

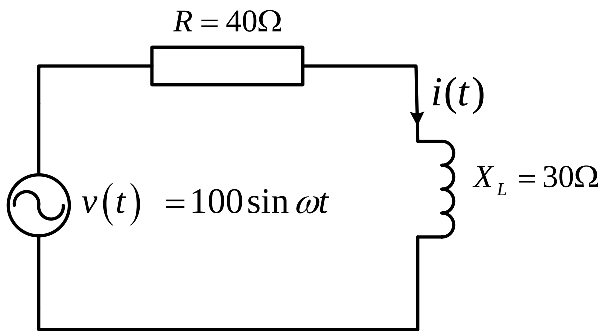

Figure Q1

Q2 The circuit below is used to analyze the charging and discharging of an inductor of value *L* in series with resistors of value R_1 and R_2. The resistor R_1 and inductor L have been connected to the battery of voltage V_S for a long time before the switch is moved at time t = 0 to connect them to R_2 instead of the battery.

(a) Determine the values of voltage $v_L(t)$ and current $i(t)$ in the circuit for t=0-, just before the switch is moved, and explain your analysis. [4]

(b) Determine the values of the voltage $v_L(t)$ and current $i(t)$ at t=0+, just after the switch is moved, and explain your analysis. [4]

(c) Determine the values of voltage $v_L(t)$ and current $i(t)$ for large times, $t \rightarrow \infty$, when a new steady state is reached, and explain your analysis. [4]

(d) Use Kirchoff's laws to set up a differential equation for $i(t)$ for t> 0, find the solution to the differential equation. [8]

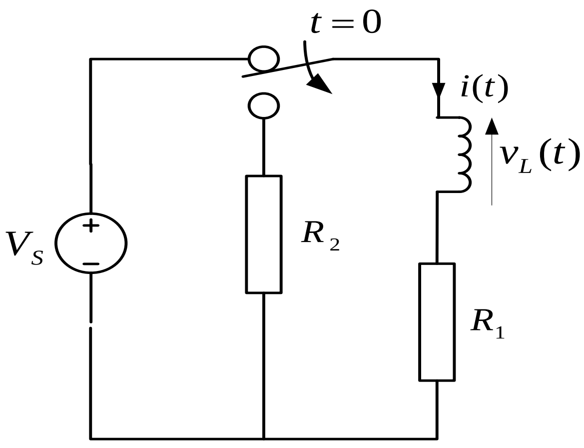

Figure Q2

Q3 In the circuit of Figure Q3, the switch S has been closed for a long time, and is opened at time t = 0 seconds.

(a) Calculate the values of the current $i_R(t)$ through the resistor R, the current $i_L(t)$ through the inductor L and the current $i_C(t)$ through the capacitor C in the circuit for t =0+ immediately after the switch is opened, and explain your analysis [6]

(b) Construct a differential equation for the LCR circuit for $t > 0$. You may choose a convenient variable to be the subject of the equation. [6]

(c) Determine whether the circuit is underdamped, critically damped or overdamped. [8]

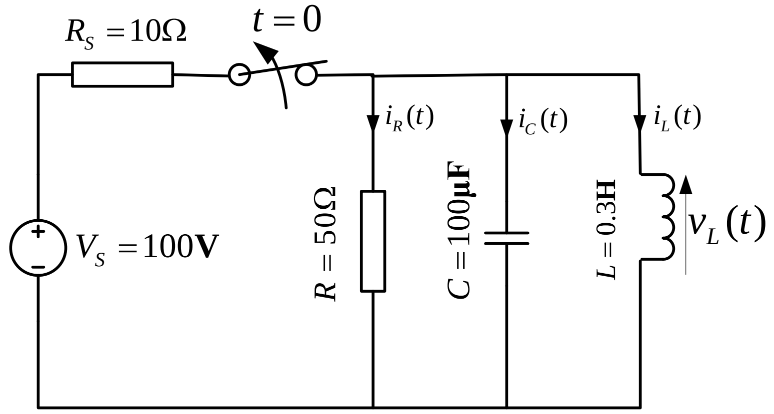

Figure Q3

Q4 A circuit is shown in Figure Q4, where $i_s(t) = 6\cos 2t$, $L_1 = 0.5\mathbf{H}$, $L_2 = 1\mathbf{H}$, $R_1 = 10\Omega$, $R_2 = 5\Omega$, $C = 0.5\mathbf{F}$. Determine

(a) The impedance of the resistor R_1 in series with the Capacitor C, the impedance of the resistor R_2 in series with the inductor L_2, and the total impedance of the circuit connected to the current source $i_s(t)$. [12]

(b) The current i in both phasor form and time-dependant expression which flows through the resistor R_2 and the inductor L_2 . [8]

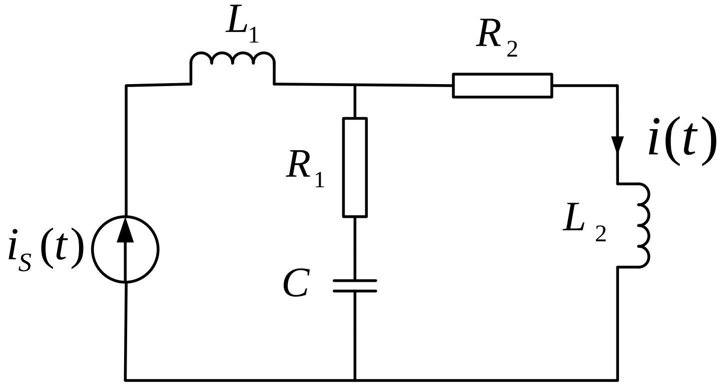

Figure Q4

Q5 The circuits shown in Figure Q5 are two filters with $R_L = 10\mathbf{k}\Omega$, $R_1 = R_F = 10\mathbf{k}\Omega$, $C_1 = C_F = 1\mathbf{mF}$, $R_{in} = 1\mathbf{k}\Omega$. Determine

(a) The respective voltage frequency response function for each filter circuit

$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} \quad [8]$$

(b) What type (high- or low- pass) of filter is each filter circuit? [4]

(c) The respective cutoff frequency for each filter circuit. [4]

(d) The respective pass-band gain for each filter circuit. [4]

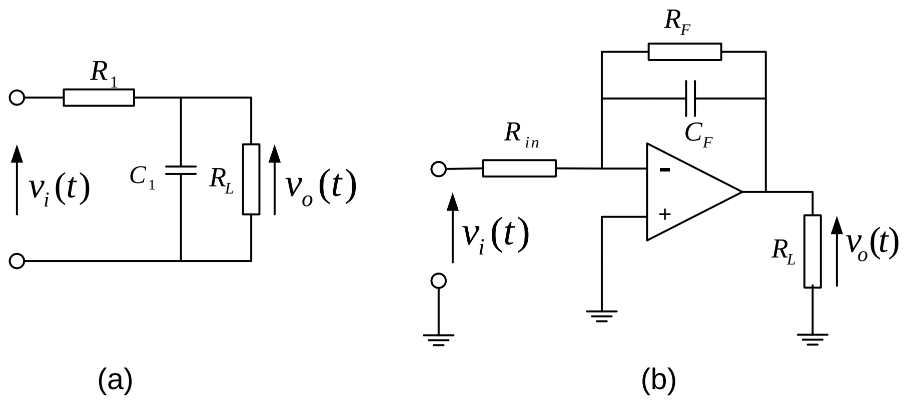

Figure Q5

Q6 Figure Q6 shows an analog computer circuit with R_1=0.4MΩ, R_2=R_3=R_5=1MΩ, R_4=2.5kΩ, C_1=C_2=1μF, where $f(t)$ is the input, $x(t)$ is the output, and y and z are intermediate variables. Determine

(a) The differential equation for the sub-circuit from y to z. [3]

(b) The differential equation for the sub-circuit from z to $x(t)$. [3]

(c) The differential equation for the whole circuit from $f(t)$ to $x(t)$. [4]

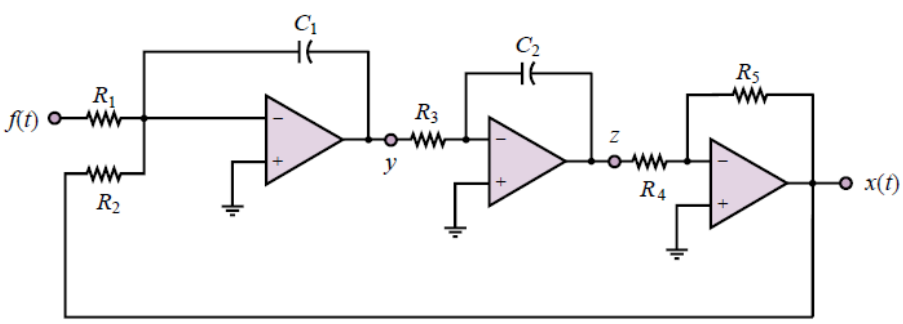

Figure Q6

End of Question Paper