UoG-UESTC Joint School

Degrees of BEng

Electrical Circuit Analysis

Mid-term Test

Nov 2015

Attempt all questions.

Total 100 marks.

The numbers in square brackets in the right-hand margin indicate the marks allotted to the part of the question against which the mark is shown. These marks are for guidance only.

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

End of Question Paper

<u>Electrical Circuit Analysis formula list.</u>

$$For\ an\ inductor: v(t)=L\frac{di}{dt},\quad i(t)=\frac{1}{L}\int vdt;\quad For\ a\ capacitor: v(t)=\frac{1}{C}\int idt,\quad i(t)=C\frac{dv}{dt}$$

$$V_{Ave}=\frac{1}{T}\int_0^T v(t)dt,\quad V_{RMS}=\sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt};\qquad Energy=\frac{1}{2}CV^2=\frac{1}{2}LI^2=Power\times time;$$

$$Power=\frac{V^2}{R}=I^2R=V\times I=Force\times velocity=Torque\times angular\ velocity$$

$$Power\ \ Factor=\cos(\varphi)\ \ or\ \frac{\cos(\varphi)}{\sqrt{1+THD^2}}=\frac{R}{Z}=\frac{P}{S};\qquad Three\ \ phase\ \ power: P=\sqrt{3}V_LI_L\cos(\varphi)$$

1st Order DE: $\tau\dfrac{dx(t)}{dt}+x(t)=F\quad t\geq 0$, Solution: $x(t)=x(\infty)+\left[x(0)-x(\infty)\right]e^{-t/\tau}$

2nd Order DE: $\dfrac{1}{\omega_n^2}\dfrac{d^2x(t)}{dt^2}+\dfrac{2\zeta}{\omega_n}\dfrac{dx(t)}{dt}+x(t)=K_S\,f(t)$,

Solution: $x(t)=A_1e^{\left(-\zeta\omega_n+\omega_n\sqrt{\zeta^2-1}\right)t}+A_2e^{\left(-\zeta\omega_n-\omega_n\sqrt{\zeta^2-1}\right)t}+x(\infty)\quad t\geq 0$

$$\left(\sin\left(ax\right)\right)'=a\cos x,\quad \left(\cos\left(ax\right)\right)'=-a\sin x$$

$$\int\sin\left(ax\right)dx=-\frac{1}{a}\cos\left(ax\right),\quad \int\cos\left(ax\right)dx=\frac{1}{a}\sin\left(ax\right)$$

$$\cos\left(A\pm B\right)=\cos A\cos B\mp\sin A\sin B$$

$$\sin\left(A\pm B\right)=\sin A\cos B\pm\cos A\sin B$$

$$\cos x=\sin\left(x+\frac{\pi}{2}\right)=\sin\left(\frac{\pi}{2}-x\right)$$

$$\sin x=\cos\left(\frac{\pi}{2}-x\right)=-\cos\left(\frac{\pi}{2}+x\right)$$

$$e^{j\theta}=\cos\theta+j\sin\theta$$

$$\mathbf{V}(j\omega)=\mathbf{V}=V_me^{j\phi}\equiv V_m\angle\phi$$

$$\mathbf{Z}=R+jX=\left|Z\right|\angle\varphi,\ \left|Z\right|=\sqrt{R^2+X^2}$$

Q1 If the plots shown in Figure Q1 are the voltage across and the current through an ideal capacitor, determine the capacitance. [10]

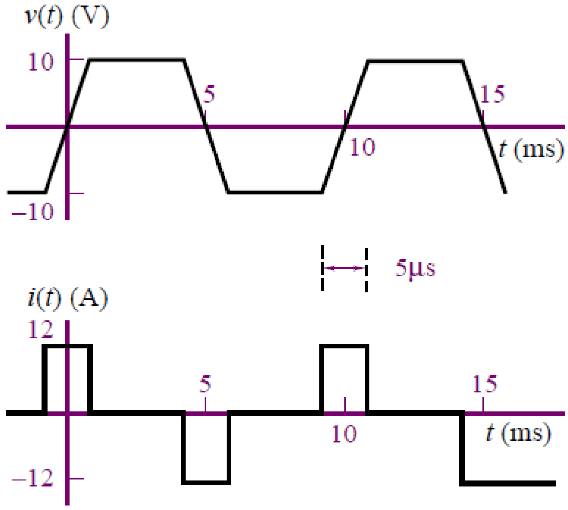

Figure Q1

Solution:

$$i_C = C\frac{dv_C}{dt} \Rightarrow 12 = C\frac{10-(-10)}{5\times10^{-6}} \Rightarrow C = 3\times10^{-6}F$$

Q2 The circuit below is used to analyse the charging and discharging of an inductor of value L in series with a resistor of value R. The resistor and inductor have been connected to the battery of voltage V_s for a long time before the switch is moved at time t = 0 to disconnect them.

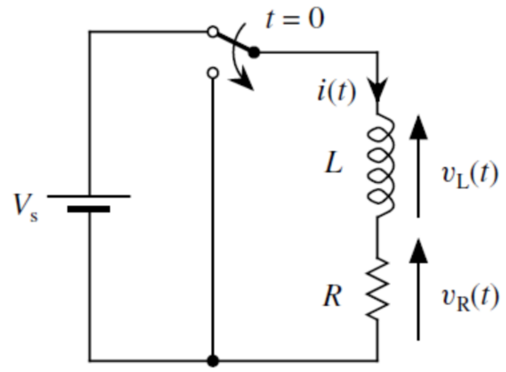

Figure Q2

(a) Calculate the voltages v_L and v_R and current i in the circuit for t =0-, just before the switch is moved. [5]

(b) What are the values of the voltages v_L, v_R and current i at t=0+, just after the switch is moved? [5]

(c) What are the values of v_L, v_R and i for large times, t →∞, when a new steady state is reached? [5]

(d) Use Kirchoff's laws to set up a differential equation for i(t) for t > 0, find the solution to the differential equation. [5]

(e) Sketch plots of v_L(t), v_R(t) and i(t) from t=0_- to large times. [5]

Solution:

(a) $v_L(0\_) = 0$, $i(0\_) = i_L(0\_) = \mathrm{V}_S / R_1$ [4]

(b) $v_L(0_+) = -\mathrm{V}_S$, $i(0_+) = i_L(0_+) = \mathrm{V}_S / R_1$ [4]

(c) $v_L(\infty) = 0,\quad i(\infty) = i_L(\infty) = 0$ [4]

(d) KVL gives $v_L + v_R = 0$, as already used. Expressing this in terms of current gives Standard 1st order differential equation (DE)

$$L\frac{di}{dt} + iR = 0,\quad t > 0 \Rightarrow \quad \frac{L}{R}\frac{di}{dt} + i = 0 \quad t > 0$$ [4]

Remember for standard 1st order DE

$$\tau\frac{dx(t)}{dt} + x(t) = F \quad t \geq 0 \qquad solution: \quad x(t) = x(\infty) + \left[x(0) - x(\infty)\right]e^{-t/\tau}$$

$$\Rightarrow i(t) = i(\infty) + \left[i(0) - i(\infty)\right]e^{-t/\tau} = \frac{V_s}{R}e^{-t/\tau} \qquad where \quad I_0 = \frac{V_s}{R},\quad \tau = \frac{L}{R}$$ [4]

Q3 The circuit below has been in steady state for a long time before the switch opens at t = 0.

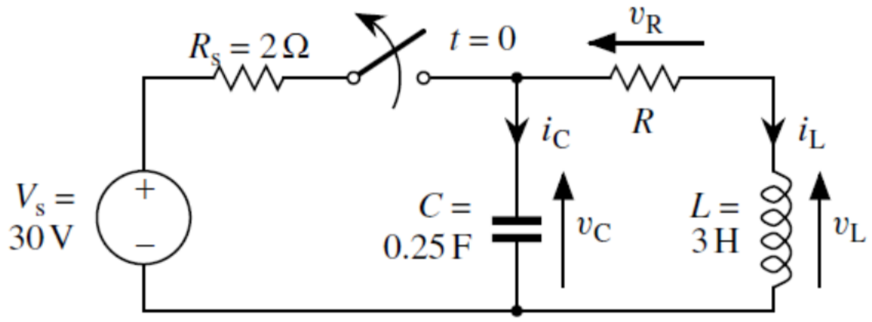

Figure Q3

(a) Find the value of R so that the voltage v_C across the capacitor is 20V before the switch opens. [5]

(b) Tabulate the conditions in the LCR circuit for t = 0-, t = 0+ and t →∞. [10]

(c) Construct a differential equation for the LCR circuit for t > 0. You may choose a convenient variable to be the subject of the equation. Is the transient under, critically or over-damped? [10]

Solution:

(a) In the steady state before the switch opens, the capacitor can be treated as an open circuit and the inductor as a short circuit. The 30V source therefore sees a 2Ω resistor and R in series. The voltage across the capacitor is the same as that across R and L because they are in parallel, but there is no voltage across the inductor so the voltages across the capacitor and resistor are the same. We therefore need 20V across the resistor, which means 10V across the 2Ω resistor, so the current is 5 A. This shows that R = 4Ω, which could instead be found from the potential divider formula (if you can remember it reliably).

(b) The table below shows the voltages across all components and the currents through the two branches, using the notation shown in the circuit. The signs are critical!

time	v_L	v_C	v_R	$i_L = i_R$	i_C
$t = 0_-$	0 V	20 V	20 V	5 A	0 A
$t = 0_+$	0 V	20 V	20 V	5 A	–5 A
$t \to \infty$	0 V	0 V	0 V	0 A	0 A

The only row of the table that requires work is for t = 0+. Neither v_C nor i_L are allowed to change instantaneously. The voltage v_R does not change because its current is the same as i_L. There is nothing to stop v_L changing but KVL requires $v_C = v_L + v_R$ and this keeps v_L the same. The physical reason why v_L remains the same is that the RL branch of the circuit does not 'notice' immediately that the switch has changed. Its current remains constant because of the inductor and the voltage across it remains constant because of the capacitor so the switch has no instant effect. The current i_C was zero before the switch is opened but must becomes $i_C = -i_L$ by KCL after the switch is opened.

(c) Start from KVL to construct the differential equation. This is the same as in the lectures on series resonant circuits except for the signs of voltages and currents, which is an unavoidable nuisance. The components are in series so the current is the same through each of them and the strategy is therefore to write the voltages in terms of this current.

$$\begin{aligned} v_C - v_R - v_L &= 0 \\ v_C - R i_R - L\frac{di_L}{dt} &= 0 \\ v_C + R i_C + L\frac{di_C}{dt} &= 0 \end{aligned}$$

It is awkward to write vc in terms of current so instead write the current in terms of vc Using (the circuit is one kind of RCL in series)

$$i_C = C\frac{dv_C}{dt}.$$

These gives

$$v_C + RC\frac{dv_C}{dt} + LC\frac{d^2 v_C}{dt^2} = 0.$$

This is the standard differential equation for a series LCR circuit.

(d) The differential equation is homogeneous (its RHS is zero), without forced input. Its characteristics equation is

$$LCs^2 + RCs + 1 = 0,$$

With

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -\zeta\omega_n \pm \sqrt{(\zeta\omega_n)^2 - \omega_n^2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\omega_n = \frac{1}{\sqrt{LC}}, \quad \zeta = \frac{RC}{2\sqrt{LC}}, \quad underpamped: \omega_d = \omega_n\sqrt{1-\zeta^2}$$

Then

$$\omega_n = \frac{1}{\sqrt{3\times 0.25}} = 1.15 rad/s, \quad \zeta = \frac{4\times 0.25}{2\sqrt{3\times 0.25}} = 0.575 < 1,$$

$$underpamped: \omega_d = \omega_n\sqrt{1-\zeta^2} = 0.94 rad/s$$

General Solution

$$v_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad t \ge 0$$

$$s_1 = -0.66 + j0.94, \quad s_2 = -0.66 - j0.94$$

(d) $v_C(0_+) = 20\mathbf{V} \quad \dot{v}_C(0_+) = \frac{i_C(0_+)}{C} = \frac{-5\mathbf{A}}{0.25\mathbf{F}} = -20\mathbf{V}/s$

Q4 A circuit is shown in Figure Q4, where $v_s(t) = 10\cos(100t + \pi/6)$, $R_1 = 1\mathbf{k\Omega}$, $R_2 = 2\mathbf{k\Omega}$, $L = 1\mathbf{mH}$, $C = 1\mathbf{mF}$.

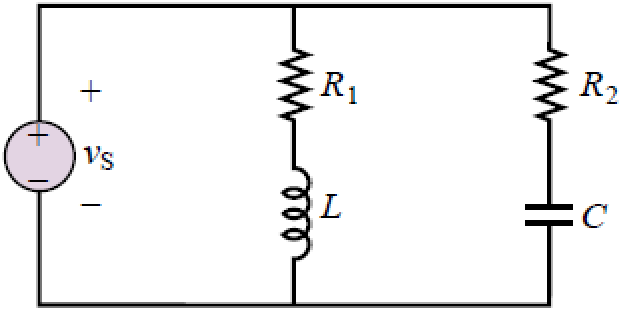

Figure Q4

(a) Determine the equivalent impedance in the circuit shown in Figure Q4.[10]

(b) Calculate the current $i_s(t)$ flowing through the voltage source. [10]

Solution:

(a)

$$Z_1 = R_1 + j\omega L = 1000 + j100 \times 0.001 = 1000 + j0.1$$

$$Z_2 = R_2 + \frac{1}{j\omega C} = 2000 + \frac{1}{j100 \times 0.001} = 2000 - j10$$

$$Z = Z_1 \parallel Z_2 = \frac{(1000 + j0.1)(2000 - j10)}{3000 - j9.9} = \frac{2\times 10^6 + 1 - j9800}{3000 - j9.9}$$

$$\approx \frac{2\times 10^3 - j9.8}{3 - j0.0099} \approx 667\angle\left[-\tan^{-1}(4.9\times 10^{-3}) + \tan^{-1}(3.3\times 10^{-3})\right]$$

$$= 667\angle(-0.28° + 0.19°) = 667\angle -0.07°$$

(b)

$$V = 10\angle \pi/6 = 10\angle 30°$$

$$Z \approx 667\angle -0.07°$$

$$I = V/Z = 10\angle 30° / 667\angle -0.07° = 0.015\angle 30.07°\mathbf{A}$$

$$i(t) = 0.015\cos(\omega t + 30.07°)\mathbf{A}$$

Q5 A circuit is shown in Figure Q5, where $R_S = 500\Omega$, $R_L = 5\mathbf{k}\Omega$, $L = 1\mathbf{mH}$, $C = 5\mathbf{pF}$.

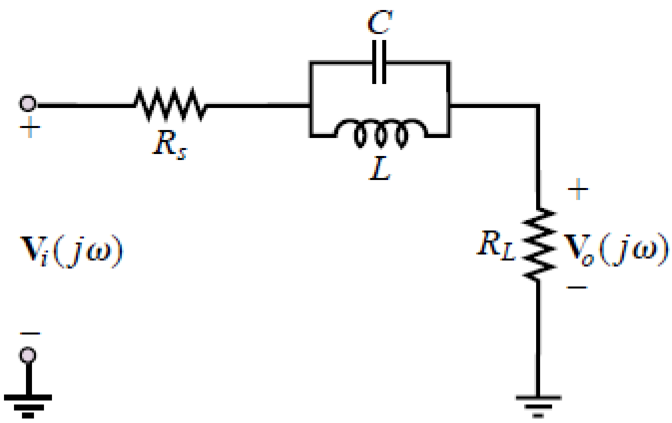

Figure Q5

(a) Compute the voltage frequency response function

$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} \qquad [10]$$

(b) What type of filter (high-pass, low-pass, band-pass, or notch) is this? [10]

Solution:

(a)

$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{R_L}{R_S + R_L + j\omega L \,\|\, 1/(j\omega C)}$$

$$= \frac{R_L}{R_S + R_L + \dfrac{j\omega L \times 1/(j\omega C)}{j\omega L + 1/(j\omega C)}} = \frac{R_L}{R_S + R_L + \dfrac{j\omega L}{j^2\omega^2 LC + 1}}$$

$$= \frac{R_L}{R_S + R_L} \frac{(j^2\omega^2 LC + 1)}{j^2\omega^2 LC + j\omega \dfrac{L}{R_S + R_L} + 1} = \frac{5000}{5500} \frac{(j^2\omega^2 LC + 1)}{j^2\omega^2 LC + \dfrac{j\omega L}{5500} + 1}$$

(B)

It is a notch filter…….explain…..plot …..