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Assignment Title:

**Characterization and Principles of Analogue Amplitude and Frequency Modulation Techniques**

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# REPORT FOR LAB2-3

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## INTRODUCTION

This report explores the property of the AM and the FM modulation.

Amplitude Modulation hides the baseband message signal in the amplitude and shifts by a sinusoidal carrier which frequency fall in the passband over which we wish to communicate. This report calculates the bandwidth and plots the spectrum of the message signal and DSB AM signal. For Conventional AM, we calculate  $A_{c,\min}$ , analysis and plot the envelop of the receiving signal. We understand the reason why  $A_c$  need to be bigger than  $A_{c,\min}$ .

Angle Modulation hides the baseband message signal in the phase or the frequency of a sinusoidal carrier, which forms the FM and PM. This helps to let frequency fall in the passband shifting to the place which we wish to communicate. Theoretically, FM is the same as PM. This report calculates the bandwidth and plots the spectrum of the message signal, FM signal. We use the Carson's formula to calculate the approximation bandwidth of FM signal. We analysis the range of  $K_f$  for letting passband around the carrier frequency or extracting the correct message signal.

From the analysis, for Bandwidth, we can find the bandwidth consumption of the Amplitude Modulation is less than the Angle Modulation. We can also say that the best Angle Modulation consumes the same as the worst Amplitude Modulation. For power of the signal, we can find the energy consumption of Amplitude Modulation is worse than the Angle Modulation. Because Amplitude is correlated with power, Modulate the signal in Phase and Frequency would not influence the power of the signal.

Compare the Angle Modulation with Amplitude Modulation, Angle Modulation has stronger anti-interference ability, higer power utilization rate but occupies more bandwidth. Both of them have their own Pros and cons.

## 1 THEORETICAL ANALYSIS

### 1.1 Amplitude Modulation

#### DSB AM

For the baseband message signal  $m(t)$ , We do the modulation:

$$y_{DSM}(t) = m(t)\cos(2\pi f_c t)$$

We can calculate the spectrum of the transfer signal.

$$u(t) = m(t)\cos(2\pi f_c t) \xrightarrow{\mathcal{F}} U(f) = \frac{1}{2}M(f + f_c) + \frac{1}{2}M(f - f_c)$$

We would apply the following steps to do the demodulation:

$$2u(t)\cos(2\pi f_c t) = m(t)\cos(2\pi f_c t)^2 \xrightarrow{\mathcal{F}} M_{de}(f) = M(f) + \frac{1}{2}M(f + 2f_c) + \frac{1}{2}M(f - 2f_c)$$

$$M(f) = LPF\{M_{de}(f)\}$$

$$\text{Envelopdetection}\{\text{IFFT}(M(f))\} = m(t)$$

#### Conventional AM

For the baseband message signal  $m(t)$ , We do the modulation:

$$y_{AM}(t) = (A_c + m(t))\cos(2\pi f_c t)$$

We can calculate the spectrum of the transfer signal.

$$u(t) = [A_c + m(t)]\cos(2\pi f_c t) \xrightarrow{\mathcal{F}} U(f) = \pi A_c \delta(|2\pi f| - 2\pi f_c) + \frac{1}{2}M(f + f_c) + \frac{1}{2}M(f - f_c)$$

We would apply the following steps to do the demodulation:

$$2u(t)\cos(2\pi f_c t) = m(t)\cos(2\pi f_c t)^2 \xrightarrow{\mathcal{F}} M_{withA_c}(f)$$

$$M_{withA_c}(f) = M(f) + 2\pi A_c \delta(f) + \frac{1}{2}M(f + 2f_c) + \frac{1}{2}M(f - 2f_c) + \pi A_c \delta(|2\pi f| - 2\pi f_c)$$

$$\text{Envelopdetection}\{\text{IFFT}(LPF(M_{withA_c}(f)))\} = m(t)$$

We also have the modulation index:

$$a_{mod} = \frac{AM_0}{A_c} = \frac{A|\min_t m(t)|}{A_c}$$

Envelop can be detect correctly if  $a_{mod} < 1$

## 1.2 Angle Modulation

### FM

For the baseband message signal  $m(t)$ , We do the modulation:

$$v(t) = \cos[2\pi f_c t + \theta(t)]$$

$$\theta(t) = 2\pi k_f \int_0^t m(\tau) d\tau$$

We can calculate the spectrum of the transfer signal (narrowband).

$$U_p(f) \approx \frac{A_c}{2} (\delta(f - f_c) + \delta(f + f_c)) - \frac{A_c}{2} (|\Theta(f - f_c)| + |\Theta(f + f_c)|)$$

We would apply the following steps to do the demodulation:

$$\frac{dv_r(t)}{dt} = \sin[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau] (2\pi f_c + 2\pi k_f m(t))$$

$$\text{Envelop}[\frac{dv_r(t)}{dt}] = 2\pi f_c + 2\pi k_f m(t)$$

$$\text{DC - Block}[\text{Envelop}[\frac{dv_r(t)}{dt}]] = 2\pi k_f m(t)$$

We also ahve some other indexs: Frequency deviation and the Modulation index:

$$\Delta f_{\max} = k_f \max_t |m(t)|$$

$$\beta = \frac{\Delta f_{\max}}{B} = \frac{k_f \max_t |m(t)|}{B}$$

For analysis the bandwidth, we can use the Carson's formula:

$$B_{FM} \approx 2B + 2k_f \max_t |m(t)|$$

FM also have the property:

If  $\beta < 1$ , it belongs to narrowband FM. Under this condition, bandwidth equals to  $2B$ .

If  $\beta > 1$ , it belongs to wideband FM. Under this condition, bandwidth equals to  $2\Delta f_{\max}$

## 2 RESULTS AND CONCLUSION

### Symbol and Abbreviation

The following Table includes the symbol and abbreviation used in this report.

Symbol or Abbreviation	Meaning
$m(t)$	Message signal
$M(f)$	Fourier transform of message signal
$u(t)$	AM signal
$U(f)$	Fourier transform of AM signal
$v(t)$	FM signal
$V(f)$	Fourier transform of FM signal
$B_{FM}$	Bandwidth of FM signal
$B$	Bandwidth of message signal
$k_f$	Frequency shift index
$2\Delta f_{max}$	Estimate bandwidth of wideband FM
$2B$	Estimate bandwidth of narrowband FM
Envelop	Envelop detection
LPF	Low pass filter
IFFT	Inverse fourier transform
$a_{mod}$	AM Modulation index
$A_{c,min}$	The minimum $A_c$ ensuring correct envelop detection
$\beta$	FM Modulation index
$\xrightarrow{\mathcal{F}}$	Fourier Transform

## 2.1 Task A

Consider the message signal  $m(t) = (1/T)\text{sinc}(t/T)$  with  $T = 1\text{msec}$ .

### 2.1.1 Question i for Task A

**Question:** Determine the bandwidth of  $m(t)$  theoretically.

**Answer:** The bandwidth of the signal  $m(t)$  is 500Hz.

We have the fourier transfer as follow:

$$x(t) = \text{sinc}(t) \xrightarrow{\mathcal{F}} X(jw) = \begin{cases} 1 & |w| \leq \pi \\ 0 & \text{Otherwise} \end{cases}$$

$$m(t) = 1000\text{sinc}(1000t) \xrightarrow{\mathcal{F}} M(f) = \begin{cases} 1 & |f| \leq 500 \\ 0 & \text{Otherwise} \end{cases}$$

$$|f| \leq 500\text{Hz}$$

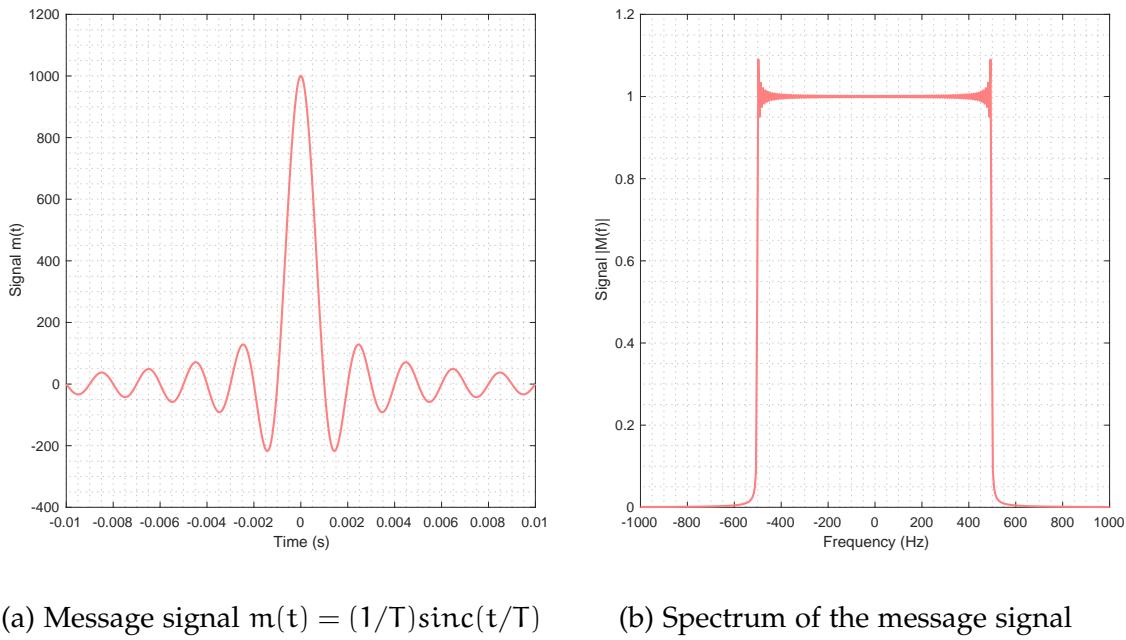
From  $w = 2\pi f$  and the above fourier analysis, we can derive the bandwidth of the signal  $m(t)$  is 500Hz.

### 2.1.2 Question ii for Task A

**Question:** Determine the bandwidth of  $m(t)$  computationally by determining and plotting the magnitude of the Fourier transform of  $m(t)$ .

**Answer:** The bandwidth of the signal  $m(t)$  is 500Hz.

By programming on the MATLAB, we can get the following graphs, as shown in Figure 1. The plotting result is the same as the theoretical analysis.



**Figure 1:** Graphs of the message signal

### Appendix for the MATLAB program of Task A Question ii

```

1 % AUTHOR: Changgang Zheng
2 % UoG: 2289258Z
3 clear
4 clc
5 %% Problem 1
6 T=0.001;
7 Ts=0.00001;
8 time=[-0.1000000001:Ts:0.1000000001];
9 fs=1/Ts;
10 f=(0:length(time)-1)*fs/length(time)-fs/2;
11
12 syms t
13 m=(1/T)*sinc(t/T);
14 m=subs(m,t,time);

```

```

15 figure;
16 set(gcf, 'PaperPosition', [0 0 6 6]);
17 set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
18 Lab2_11=plot(time,m,'LineWidth',1.5,'color',[1,0.5,0.5]);
19 axis([-0.01 0.01 -400 1200])
20 % title('Message Signal m(t)')
21 xlabel('Time (s)')
22 ylabel('Signal m(t)')
23 grid minor
24 saveas(Lab2_11,'/Users/changgang/Desktop/Lab2-11.pdf')
25
26 M=fftshift(fft(double(m)))\fs;
27 figure;
28 set(gcf, 'PaperPosition', [0 0 6 6]);
29 set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
30 Lab2_12=plot(f,abs(M),'LineWidth',1.5,'color',[1,0.5,0.5]);
31 axis([-1000 1000 -10000 120000])
32 % title('Fourier Transform of |M(f)|')
33 xlabel('Frequency (Hz)')
34 ylabel('Signal |M(f)|')
35 grid minor
36 saveas(Lab2_12,'/Users/changgang/Desktop/Lab2-12.pdf')

```

The above is the code we used to generate the graphs.

## 2.2 Task B

**Consider the message signal  $m(t) = (1/T)\text{sinc}(t/T)$  with  $T = 1\text{msec}$ .**

### 2.2.1 Question i for Task B

**Question:** Consider the DSB AM signal  $u(t) = m(t)\cos(2\pi f_c t)$ , where  $f_c = 10/T$ . Determine the bandwidth  $u(t)$  theoretically.

**Answer:** The bandwidth of  $u_{AM}(t)$  is 1000Hz

$$x(t) = \cos(2\pi f_c t) \xrightarrow{\mathcal{F}} X(f) = \pi[\delta(f - f_c) + \delta(f + f_c)]$$

$$u(t) = m(t)\cos(2\pi f_c t) \xrightarrow{\mathcal{F}} U(f) = \begin{cases} \frac{1}{2} & |f \pm f_c| \leq 500 \\ 0 & \text{Otherwise} \end{cases}$$

The bandwidth of this signal is:

$$|f \pm f_c| \leq 500$$

$$f_c - 500 \leq f \leq f_c + 500 \quad \text{and} \quad -f_c - 500 \leq f \leq -f_c + 500$$

$$9500 \leq f \leq 10500 \quad \text{and} \quad -10500 \leq f \leq -9500$$

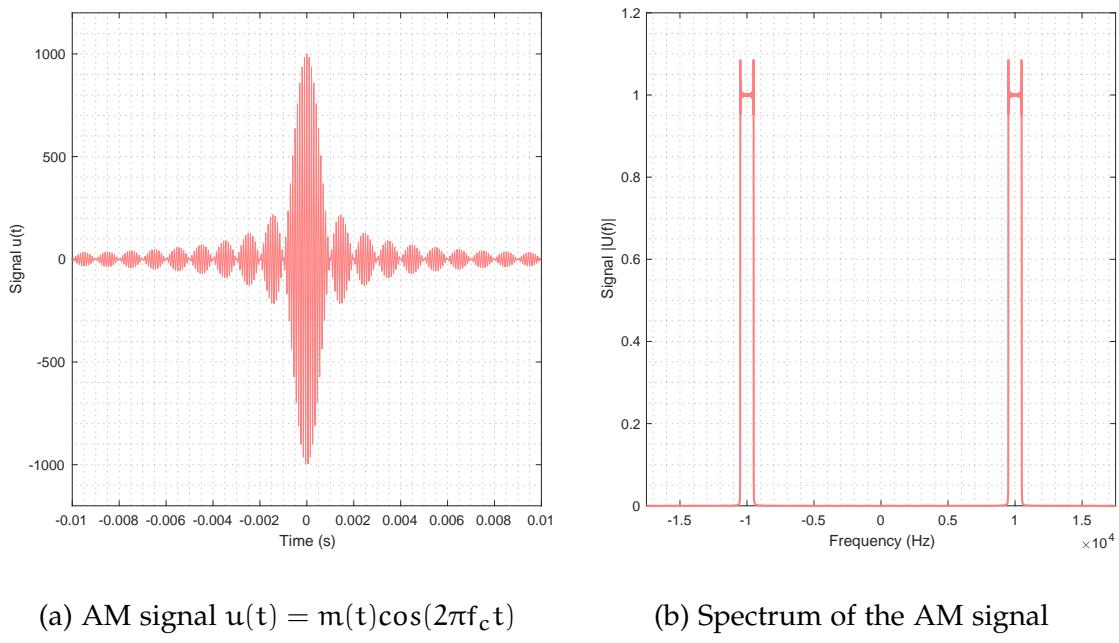
**So the bandwidth of  $u_{AM}(t)$  is 1000Hz**

### 2.2.2 Question ii for Task B

**Question:** Consider the DSB AM signal  $u(t) = m(t)\cos(2\pi f_c t)$ , where  $f_c = 10/T$ . Determine the bandwidth  $u(t)$  computationally by determining and plotting the magnitude of the Fourier transform of  $u(t)$ .

**Answer:** The bandwidth of  $u_{AM}(t)$  is 1000Hz.

By programming on the MATLAB, we can get the following graphs, as shown in Figure 2. The plotting result is the same as the theoretical analysis.



**Figure 2:** The graphs of the DSB AM signal

### Appendix for the MATLAB program of Task B Question ii

```

1 % AUTHOR: Changgang Zheng
2 % UoG: 2289258Z
3 clear
4 clc
5 %% Problem 2
6 T=0.001;
7 fc=10/T;
8 Ts=0.00001;
9 time=[-0.1000000001:Ts:0.1000000001];
10 fs=1/Ts;
11 f=(0:length(time)-1)*fs/length(time)-fs/2;
12
13 syms t

```

```

14 m=(1/T)*sinc(t/T);
15 u=m*cos(2*pi*fc*t);
16 u=subs(u,t,time);
17
18 figure;
19 set(gcf, 'PaperPosition', [0 0 6 6]);
20 set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
21 Lab2_21=plot(time,u,'LineWidth',0.5,'color',[1,0.5,0.5]);
22 axis([-0.01 0.01 -1200 1200])
23 %title('Message Signal u(t)')
24 xlabel('Time (s)')
25 ylabel('Signal u(t)')
26 grid minor
27 saveas(Lab2_21,'/Users/changgang/Desktop/Lab2-21.pdf')
28
29 U=fftshift(fft(double(u)))\fs;
30 figure;
31 set(gcf, 'PaperPosition', [0 0 6 6]);
32 set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
33 Lab2_22=plot(f,abs(U),'LineWidth',1,'color',[1,0.5,0.5]);
34 axis([-17500 17500 -10000 60000])
35 %title('Fourier Transform of |U(f)|')
36 xlabel('Frequency (Hz)')
37 ylabel('Signal |U(f)|')
38 grid minor
39 saveas(Lab2_22,'/Users/changgang/Desktop/Lab2-22.pdf')

```

The above is the code we used to generate the graphs.

### 2.2.3 *Question iii for Task B*

**Question:** Compare the spectra of  $m(t)$  and  $u(t)$ .

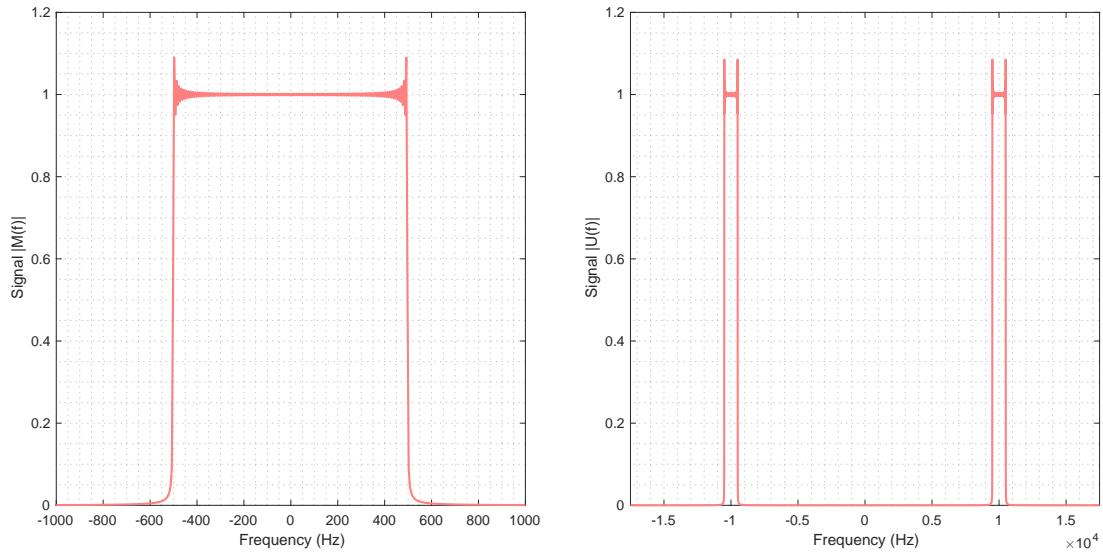
**Answer:** The spectrum of  $u(t)$  is the double side moving of the  $m(t)$ . Because it equals to message signal multiply strong 'cos' carrier signal. The bandwidth of the  $m(t)$  is 500Hz and the bandwidth of the  $u(t)$  is 1000Hz. The  $m(t)$  is the message signal and the  $u(t)$  is the DSB of the message signal, which double its bandwidth. This result is verified by both theoretical calculation and programming on the MATLAB, as shown in Figure 3.

## 2.3 Task C

**Consider the message signal**  $m(t) = (1/T)\text{sinc}(t/T)$  **with**  $T = 1\text{msec}$ .

### 2.3.1 *Question i for Task C*

**Question:** Consider the conventional AM signal  $u(t) = [A_c + m(t)]\cos(2\pi f_c t)$  with  $f_c = 10/T$  and choose  $A_c$  to have the smallest possible value  $A_{c,\min}$  for



(a) Spectrum of message signal

(b) Spectrum of AM signal

Figure 3: Spectrum of message signal and DSB AM signal

correct envelop detection. Determine the bandwidth of  $u_{AM}(t)$  theoretically.

**Answer:** The  $A_{c,min} = 217.2335$ , the bandwidth of  $u_{AM}(t)$  is 1000Hz

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\frac{dsinc(t)}{dt} = \frac{\cos(\pi t)}{t} - \frac{\sin(\pi t)}{\pi t^2}$$

Let  $dsinc(t)/dt = 0$  and from the property of the sinc function:

$$\frac{dsinc(t)}{dt} = \frac{\cos(\pi t)}{t} - \frac{\sin(\pi t)}{\pi t^2} = 0$$

$$\pi t - \tan(\pi t) = 0$$

We know the lowest value of the sinc function is located at first zero derivate point close to 0, so we have:

$$\pi t \approx 4.4931$$

From the above, we can find when  $\pi t \approx 4.4931$  and  $t \approx 1.4302$ , sinc function has the lowest values  $-0.2172$ . When receive the message  $m(t) = (1/T)\text{sinc}(t/T)$  with  $T = 1\text{msec}$ , we can find when  $t \approx 0.00143$ , sinc function has the lowest values  $-217.2335$ . **Thus, we can find  $A_{c,min}$ :**

$$A_{c,min} = |\min(m(t))| = 217.2335$$

After we calculate the  $A_{c,min}$ , we can also find the bandwidth of the signal  $u_{AM}(t)$ :

$$x(t) = 1 \xrightarrow{\mathcal{F}} X(f) = 2\pi\delta(2\pi f)$$

$$[A_c + m(t)] \cos(2\pi f_c t) \xrightarrow{\mathcal{F}} U(f) = \begin{cases} \frac{1}{2} & |f \pm f_c| \leq 500, |f| \neq f_c \\ \pi A_c \delta(|2\pi f| - 2\pi f_c) + \frac{1}{2} & |f| = f_c \\ 0 & \text{Otherwise} \end{cases}$$

The bandwidth of this signal is:

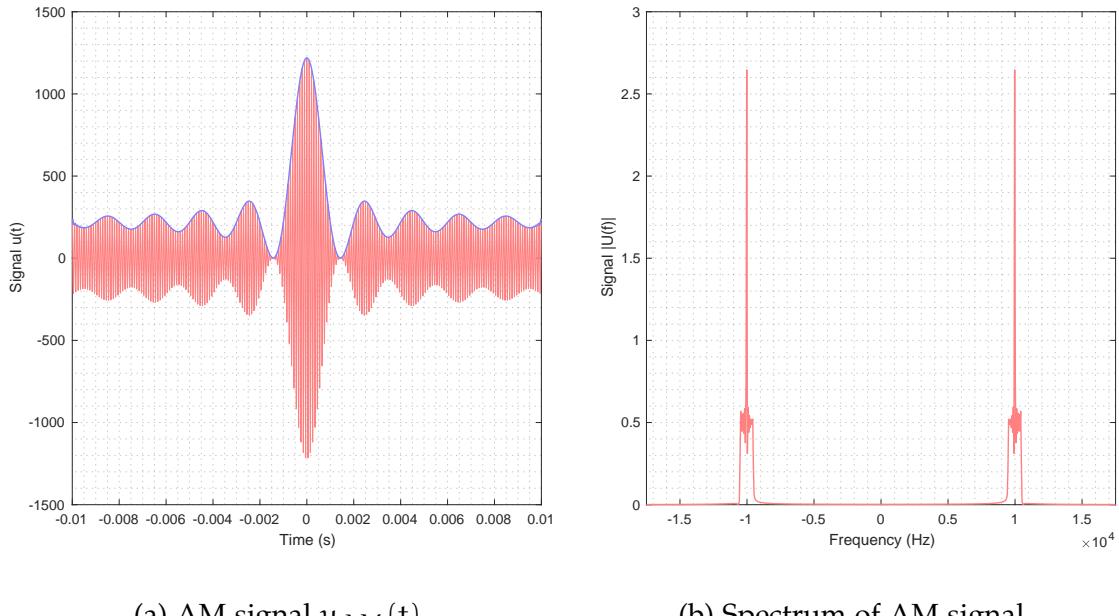
$$\begin{aligned} |f \pm f_c| &\leq 500 \\ f_c - 500 \leq f &\leq f_c + 500 \quad \text{and} \quad -f_c - 500 \leq f \leq -f_c + 500 \\ 9500 \leq f &\leq 10500 \quad \text{and} \quad -10500 \leq f \leq -9500 \end{aligned}$$

**So the bandwidth of  $u_{AM}(t)$  is 1000Hz**

### 2.3.2 Question ii for Task C

**Question:** Determine the bandwidth of  $u_{AM}(t)$  computationally by determining and plotting the magnitude of the Fourier transform of  $u_{AM}(t)$ .

**Answer:** By programming on the MATLAB, we can get the following graphs, as shown in Figure 4. The plotting result is the same as the theoretical analysis.



**Figure 4:** The graphs of the AM signal

**Appendix for the MATLAB program of Task B Question ii**

```

1 % AUTHOR: Changgang Zheng
2 % UoG: 2289258Z
3 %% Problem 3
4 clear
5 clc
6
7 T=0.001;
8 fc=10/T;
9 Ts=0.00001;
10 time=[-0.010000000001:Ts:0.010000000001];
11 fs=1/Ts;
12 f=(0:length(time)-1)*fs/length(time)-fs/2;
13
14 syms t
15 m=(1/T)*sinc(t/T);
16 u=(m+217.2335)*cos(2*pi*fc*t);
17 u=subs(u,t,time);
18
19 figure;
20 set(gcf, 'PaperPosition', [0 0 6 6]);
21 set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
22 Lab2_31=plot(time,u,'LineWidth',0.5,'color',[1,0.5,0.5]);
23 %title('Message Signal u(t)')
24 xlabel('Time (s)')
25 ylabel('Signal u(t)')
26
27 %% envelop detection
28 hold on
29 y = abs(hilbert(double(u)));
30 plot(time,y,'LineWidth',1,'color',[0.5,0.5,1]);
31 xlim([-0.01 0.01])
32 grid minor
33 saveas(Lab2_31,'/Users/changgang/Desktop/Lab2-31.pdf')
34 %%
35 U=fftshift(fft(double(u)))\fs;
36 figure;
37 set(gcf, 'PaperPosition', [0 0 6 6]);
38 set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
39 Lab2_32=plot(f,abs(U),'LineWidth',1,'color',[1,0.5,0.5]);
40 xlim([-17500 17500])
41 %title('Fourier Transform of |U(f)|')
42 xlabel('Frequency (Hz)')
43 ylabel('Signal |U(f)|')
44 grid minor
45 saveas(Lab2_32,'/Users/changgang/Desktop/Lab2-32.pdf')

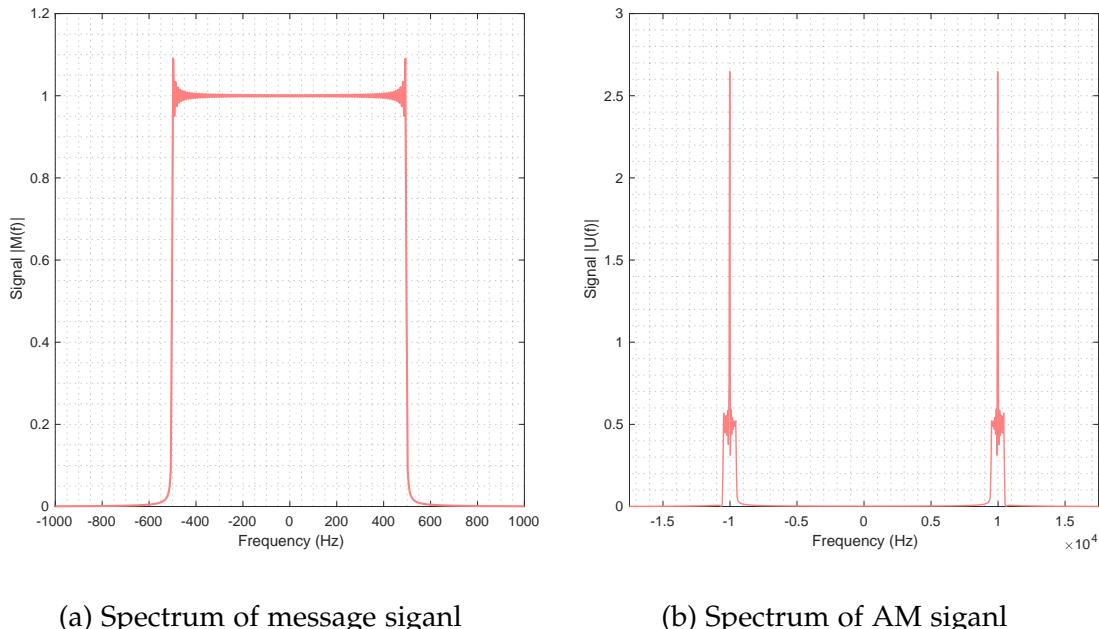
```

The above is the code we used to generate the graphs.

### 2.3.3 Question iii for Task C

**Question:** Compare the spectra of  $u_{AM}(t)$  and  $m(t)$

**Answer:** The spectrum of the  $u_{AM}(t)$  is the double side shifting of  $M(f)$  and  $\pi A_c \delta(f)$ .  $M(f)$  and  $\pi A_c \delta(f)$  are shifted to  $\pm f_c$  by the carrier signal. The bandwidth of the  $m(t)$  is 500Hz and the bandwidth of the  $u_{AM}(t)$  is 1000Hz, which double its bandwidth. This result is verified by both theoretical calculation and programming on the MATLAB, as shown in Figure 5.



(a) Spectrum of message signal

(b) Spectrum of AM signal

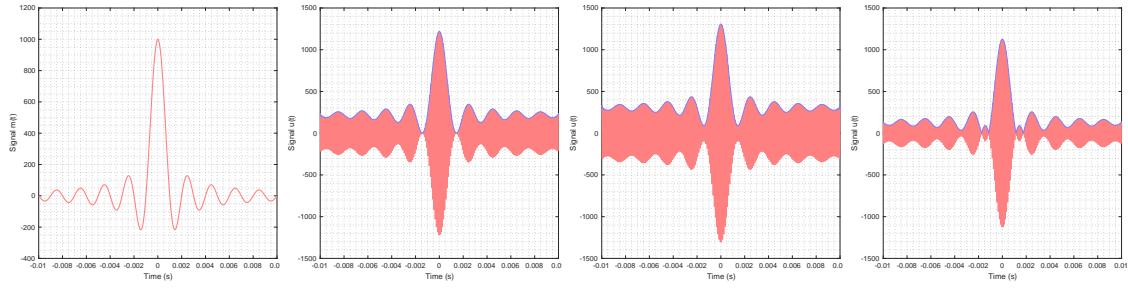
Figure 5: Spectrum of message signal and AM signal

### 2.3.4 Question iv for Task C

**Question:** Plot  $u_{AM}(t)$  for  $A_c = A_{c,min}$  and for the following two cases of  $A_c$  (choose your own values) such that  $A_c > A_{c,min}$ ,  $A_c < A_{c,min}$ .

**Answer:** The following Figure 6 shows the  $u_{AM}(t)$  for  $A_c = A_{c,min}$ ,  $A_c > A_{c,min}$  and  $A_c < A_{c,min}$ . From the plot, we can find for  $a_{mod} \leq 0$ , which means  $A_c \geq A_{c,min}$ , the message signal is pushed upon zero. Under this condition, the envelop can be detected correctly. When  $A_c < A_{c,min}$ , as shown in Figure 6 (d), the envelop cannot be detected correctly.

*Appendix for the MATLAB program of Task B Question iv*

(a) Message signal      (b)  $A_c = A_{c,\min}$       (c)  $A_c > A_{c,\min}$       (d)  $A_c < A_{c,\min}$ **Figure 6:** AM signal and their envelop with different  $A_c$ 

```

1 % AUTHOR: Changgang Zheng
2 % UoG: 2289258Z
3 %% Problem 3
4 clear
5 clc
6
7 T=0.001;Ts=0.00001;fs=1/Ts;fc=10/T;
8 time=[-0.010000000001:Ts:0.010000000001];
9 f=(0:length(time)-1)*fs/length(time)-fs/2;
10 syms t
11 m=(1/T)*sinc(t/T);
12
13 %% ===== upper shift ...
14 u=(m+307.2335)*cos(2*pi*fc*t);
15 u=subs(u,t,time);
16 figure;
17 set(gcf, 'PaperPosition', [0 0 6 6]);
18 set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
19 Lab2_31up=plot(time,u,'LineWidth',0.5,'color',[1,0.5,0.5]);
20 %title('Message Signal u(t)')
21 xlabel('Time (s)')
22 ylabel('Signal u(t)')
23
24 %% envelop detection
25 hold on
26 yu = abs(hilbert(double(u)));
27 plot(time,yu,'LineWidth',1,'color',[0.5,0.5,1]);
28 xlim([-0.01 0.01])
29 grid minor
30 saveas(Lab2_31up,'/Users/changgang/Desktop/Lab2-31up.pdf')
31 %%
32 Uu=fftshift(fft(double(u)))\fs;
33 figure;
34 set(gcf, 'PaperPosition', [0 0 6 6]);
35 set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
36 Lab2_32up=plot(f,abs(Uu),'LineWidth',1,'color',[1,0.5,0.5]);
37 xlim([-17500 17500])

```

```

38 %title('Fourier Transform of |U(f)|')
39 xlabel('Frequency (Hz)')
40 ylabel('Signal |U(f)|')
41 grid minor
42 saveas(Lab2_32up,'/Users/changgang/Desktop/Lab2-32up.pdf')
43
44 %% ===== lower shift ...
45 u=(m+127.2335)*cos(2*pi*fc*t);
46 u=subs(u,t,time);
47 figure;
48 set(gcf, 'PaperPosition', [0 0 6 6]);
49 set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
50 Lab2_31low=plot(time,u,'LineWidth',0.5,'color',[1,0.5,0.5]);
51 %title('Message Signal u(t)')
52 xlabel('Time (s)')
53 ylabel('Signal u(t)')
54
55 %% envelop detection
56 hold on
57 yl = abs(hilbert(double(u)));
58 plot(time,yl,'LineWidth',1,'color',[0.5,0.5,1]);
59 xlim([-0.01 0.01])
60 grid minor
61 saveas(Lab2_31low,'/Users/changgang/Desktop/Lab2-31low.pdf')
62 %%
63 Ul=fftshift(fft(double(u)))\fs;
64 figure;
65 set(gcf, 'PaperPosition', [0 0 6 6]);
66 set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
67 Lab2_32low=plot(f,abs(Ul),'LineWidth',1,'color',[1,0.5,0.5]);
68 xlim([-17500 17500])
69 %title('Fourier Transform of |U(f)|')
70 xlabel('Frequency (Hz)')
71 ylabel('Signal |U(f)|')
72 grid minor
73 saveas(Lab2_32low,'/Users/changgang/Desktop/Lab2-32low.pdf')

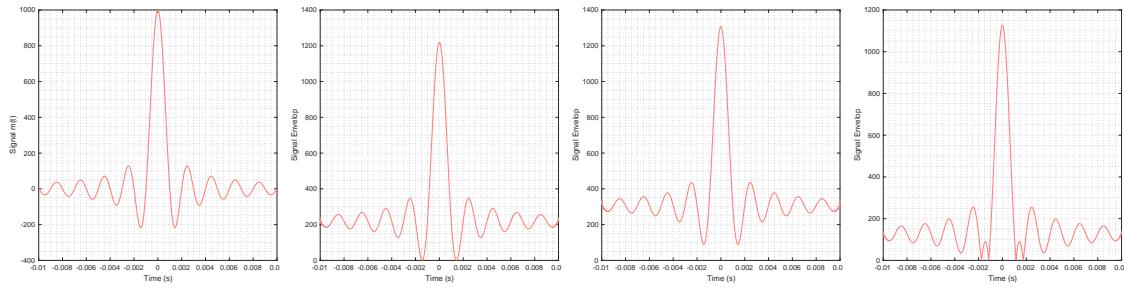
```

The aboving is the code we used to generate the graphs.

### 2.3.5 Question v for Task C

**Question:** Compare the time domain plots of  $u_{AM}(t)$  and  $m(t)$  for all choices of  $A_c$  in Question iv when deriving  $m(t)$  from  $u_{AM}(t)$  through envelop detection.

**Answer:** From the analysis, if  $A_c \geq A_{c,\min}$ , the envelop can be detected correctly. If  $A_c < A_{c,\min}$ , some part of message  $m(t) + A_c$  would under zero. However, when mutiply it with the carrier signal  $\cos(2\pi f_c t)$  with high frequency, the under zero part of message would turnover, as shown in Figure 7 (d). This would influence the shape of the envelop and lead to incorrect envelop detaction.



(a) Message signal

(b)  $A_c = A_{c,\min}$ (c)  $A_c > A_{c,\min}$ (d)  $A_c < A_{c,\min}$ **Figure 7:** Envelop with different  $A_c$ **Appendix for the MATLAB program of Task C Question v**

```

1 % AUTHOR: Changgang Zheng
2 % UoG: 2289258Z
3 %% Connect with code for Question iv
4 figure;
5 set(gcf, 'PaperPosition', [0 0 6 6]);
6 set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
7 e1=plot(time,subs(m,t,time),'LineWidth',1,'color',[1,0.5,0.5]);
8 xlabel('Time (s)')
9 ylabel('Signal m(t)')
10 grid minor
11 saveas(e1,'/Users/changgang/Desktop/e1.pdf')
12
13 figure;
14 set(gcf, 'PaperPosition', [0 0 6 6]);
15 set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
16 e2=plot(time,y,'LineWidth',1,'color',[1,0.5,0.5]);
17 xlabel('Time (s)')
18 ylabel('Signal Envelop')
19 grid minor
20 saveas(e2,'/Users/changgang/Desktop/e2.pdf')
21
22 figure;
23 set(gcf, 'PaperPosition', [0 0 6 6]);
24 set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
25 e3=plot(time,yu,'LineWidth',1,'color',[1,0.5,0.5]);
26 xlabel('Time (s)')
27 ylabel('Signal Envelop')
28 grid minor
29 saveas(e3,'/Users/changgang/Desktop/e3.pdf')
30
31 figure;
32 set(gcf, 'PaperPosition', [0 0 6 6]);
33 set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
34 e4=plot(time,y1,'LineWidth',1,'color',[1,0.5,0.5]);
35 xlabel('Time (s)')

```

```

36 ylabel('Signal Envelop')
37 grid minor
38 saveas(e4, '/Users/changgang/Desktop/e4.pdf')

```

The above is the code we used to generate the graphs.

## 2.4 Task D

**Consider the message signal  $m(t) = (1/T)\text{sinc}(t/T)$  with  $T = 1\text{msec}$ .**

### 2.4.1 Question i for Task D

**Question:** Consider the FM signal  $v(t) = \cos[2\pi f_c t + \theta(t)]$ , where  $\theta(t) = 2\pi k_f \int_0^t m(\tau) d\tau$  and  $f_c = 10/T$ . Determine the bandwidth of  $v(t)$  computationally for  $k_f = 1/4$  and  $k_f = 4$  by determining and plotting the magnitude of the Fourier transform of  $v(t)$ .

**Answer:** When  $k_f = 1/4$ , bandwidth equals to 1000Hz. When  $k_f = 4$ , bandwidth equals to 8000Hz

We can also get this result by calculation: If we try the narrow band FM modulation, From:

$$v(t) = \cos[2\pi f_c t + \theta(t)]$$

$$\theta(t) = 2\pi k_f \int_0^t m(\tau) d\tau$$

For  $\theta \approx 0$ , which means  $\cos[\theta(t)] \approx 1$  and  $\sin[\theta(t)] \approx \theta(t)$  we can get:

$$v(t) = A_c e^{j\theta(t)} = A_c \cos[\theta(t)] + j A_c \sin[\theta(t)]$$

$$v(t) \approx A_c + j A_c \theta(t)$$

$$v_p(t) \approx A_c \cos(2\pi f_c t) - \theta(t) A_c \sin(2\pi f_c t)$$

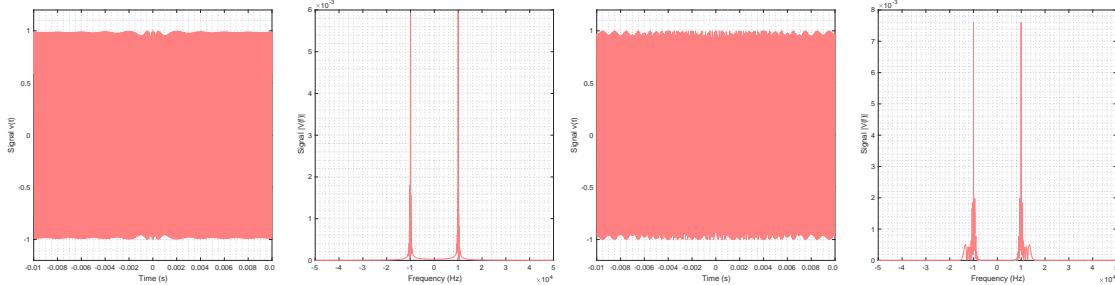
$$U_p(f) \approx \frac{A_c}{2} (\delta(f - f_c) + \delta(f + f_c)) - \frac{A_c}{2} (|\Theta(f - f_c)| + |\Theta(f + f_c)|)$$

From the above equations, we can see it is hard to get the result. From the text book, we can find:

$$\beta = \frac{\Delta f_{\max}}{B} = \frac{k_f \max_t |m(t)|}{B}$$

If  $\beta < 1$ , it belongs to narrowband FM. Under this condition, bandwidth equals to  $2B$ . If  $\beta > 1$ , it belongs to wideband FM. Under this condition, bandwidth equals to  $2\Delta f_{\max}$ . For  $k_f = 1/4$  we have  $\beta = 1/2$  which belongs narrowband FM. For  $k_f = 4$  we have  $\beta = 8$  which belongs wideband FM.

**So, when  $k_f = 1/4$ , bandwidth equals to 1000Hz. When  $k_f = 4$ , bandwidth equals to 8000Hz.**

(a) FM signal  $k = 1/4$  (b) Spectrum  $k = 1/4$  (c) FM signal  $k = 4$  (d) Spectrum  $k = 4$ **Figure 8:** FM signal and Spectrum with  $k = 4$  and  $k = 1/4$ 

From the plot Figure 8 by MATLAB, we can get the bandwidth of the signal  $v(t)$  with  $k_f = 1/4$  is around 1000Hz, the bandwidth of the signal  $v(t)$  with  $k_f = 4$  is 8000Hz.

### *Appendix for the MATLAB program of Task D Question i*

```

1 % AUTHOR: Changgang Zheng
2 % UoG: 2289258Z
3 function problem_2nd
4     clear;
5     clc;
6     for k_f=[1/4, 4]
7         T=0.001;
8         f_c=10/T;
9         Ts=0.00001;
10        time=[-0.010000000001:Ts:0.010000000001];
11        fs=1/Ts;
12        f=(0:length(time)-1)*fs/length(time)-fs/2;
13        syms t tt;
14        m=1/T*sinc(tt/T);
15        %seita=2*pi*k_f*running_sum(m);
16        seita=2*pi*k_f*int(m,tt,[0,t]);
17        if k_f==1/4
18            v1=subs(cos(2*pi*f_c*t+seita),t,time);
19        else
20            v2=subs(cos(2*pi*f_c*t+seita),t,time);
21        end
22    end
23
24    V1=fftshift(fft(double(v1)))/fs;
25    figure;
26    set(gcf, 'PaperPosition', [0 0 6 6]);
27    set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
28    Lab3_21=plot(f,abs(V1),'LineWidth',1,'color',[1,0.5,0.5]);
29    xlim([-50000 50000])
30    xlabel('Frequency (Hz)')

```

```

31     ylabel('Signal |V(f)|')
32     grid minor
33     saveas(Lab3_21, '/Users/changgang/Desktop/Lab3-21.pdf')
34
35     V2=fftshift(fft(double(v2)))/fs;
36     figure;
37     set(gcf, 'PaperPosition', [0 0 6 6]);
38     set(gcf, 'PaperSize', [6 6]); %Keep the same paper size
39     Lab3_22=plot(f,abs(V2), 'LineWidth',1,'color',[1,0.5,0.5]);
40     xlim([-50000 50000])
41     xlabel('Frequency (Hz)')
42     ylabel('Signal |V(f)|')
43     grid minor
44     saveas(Lab3_22, '/Users/changgang/Desktop/Lab3-22.pdf')

```

The above is the code we used to generate the graphs.

#### 2.4.2 Question ii for Task D

**Question:** Verify the correctness of the results in i) by comparing them with the Carson's formula  $B_{FM}$  of the FM signals and verify if the results are consistent with the Carson's formula  $B_{FM} \approx 2B + 2\Delta f_{max} = 2B + 2k_f \max_t |m(t)|$ , where  $B = 1/(2T)$  is the bandwidth of the message signal.

**Answer:**

$$B_{FM} \approx 2B + 2\Delta f_{max}$$

Where  $\Delta f_{max} = k_f \max_t |m(t)|$ ,  $\max_t |m(t)| = 1000$  and  $B = 500$ .

For  $k_f = \frac{1}{4}$  we have:

$$B_{FM} \approx 2B + 2k_f \max_t |m(t)| = 2 \times 500 + 2 \times \frac{1}{4} \times 1000 = 1500 \text{ (Hz)}$$

For  $k_f = 4$  we have:

$$B_{FM} \approx 2B + 2k_f \max_t |m(t)| = 2 \times 500 + 2 \times 4 \times 1000 = 9000 \text{ (Hz)}$$

The result we get is close to the result we get from question i. This means the answer from the first question is correct.

#### 2.4.3 Question iii for Task D

**Question:** Compare the bandwidths of the FM signals for  $k_f = 1/4$  and  $k_f = 4$  and also compare them with the bandwidths of the AM signals in B) and C)

**Answer:** From the Question ii, if we apply the frequency modulation, the bandwidth of the signal  $v(t)$  with  $k_f = 1/4$  is 1500Hz, the bandwidth of the signal  $v(t)$  with  $k_f = 4$  is 9000Hz. From the Task B and C, The AM signal  $u(t)$  have the

bandwidth 1000Hz. Compare the bandwidth, we can conclude the bandwidth of AM modulation is higher than the FM modulation.

$$B_{FM} \approx 2B + 2k_f \max_t |m(t)|$$

For this equation, to different  $k_f$ , the variable  $B$  is depend on the message signal, which do not change. The  $\max_t |m(t)|$  also do not change. So, the higher  $k_f$  will result in the bigger  $B_{FM}$ .

#### 2.4.4 Question iv for Task D

**Question:** Derive an approximation for the maximum theoretical value  $k_{(f,max)}$  of  $k_f$  in order the FM signal  $v(t)$  to remain passband around the carrier frequency  $f_c$ .

**Answer:** The maximum of  $K_f$  is  $k_{(f,max)} = 9.5$

For letting the signal remain passband, we need let  $B_{FM}/2 \leq f_c$

$$B_{FM} \approx 2B + 2k_f \max_t |m(t)| \leq 2f_c$$

$$500 + k_f 1000 \leq 10000$$

$$k_f \leq 9.5$$

So, the maximum of  $K_f$  is  $k_{(f,max)} = 9.5$  in order that signal  $v(t)$  remain passband around the carrier frequency  $f_c$

#### 2.4.5 Question v for Task D

**Question:** Compare  $k_{f,max}$  of iv) with the maximum value  $k_{(fd,max)}$  of  $k_f$  for correct message detection at the output of the discriminator demodulator.

**Answer:** The maximum value of the  $k_f d$  is 46.0355. Compare with  $k_f = 9.5$ , it is much bigger. Which means before the signal cannot be detected correctly, the signal spectrum would have overlap

The method to calculate  $k_f d$ : When we want to extract the message from the received signal  $v_r(t)$ , we need to have the following four steps:

$$v_r(t) = \cos[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau]$$

$$\frac{dv_r(t)}{dt} = \sin[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau] (2\pi f_c + 2\pi k_f m(t))$$

$$\text{Envelop}[\frac{dv_r(t)}{dt}] = 2\pi f_c + 2\pi k_f m(t)$$

$$\text{DC - Block}\{\text{Envelop}[\frac{dv_r(t)}{dt}]\} = 2\pi k_f m(t)$$

For the aboving steps, we can find that one step could lead to an error. When doing the envelop detection, we need to make sure  $a_{\text{mod}} \geq 0$  which means  $2\pi(f_c + k_f \min(m(t))) \geq 0$  this implies

$$\begin{aligned} f_c + k_f \min(m(t)) &\geq 0 \\ k_{fd,\max} &\leq -\frac{f_c}{\min(m(t))} \\ k_{fd,\max} &\leq \frac{10000}{217.2235} \approx 46.0355 \end{aligned}$$

**The maximum value of the  $k_f d$  is 46.0355**

### 3 SUMMARY

This report implements the modulation process and analyzes the spectrum and bandwidth of the DSB AM, Conventional AM and FM signal. From this, we can understand how to do these modulation and what should focuse when doing the modulation. For instance, the smallest  $A_c$  to detect envelop of AM signal correctly, the maximum  $k_f$  to ensure the correct demodulation of FM signal.

All these help me to have deeper understanding on the Angle modulation and Amplitude Modulation. Compare the Angle Modulation with Amplitude Modulation, Angle Modulation has stronger anti-interference ability, higer power utilization rate but occupies more bandwidth. Both of them have their own Pros and cons.

### REFERENCE

Upamanyu Madhow, Introduction to Communication Systems, January 17, 2014, University of California, Santa Barbara.