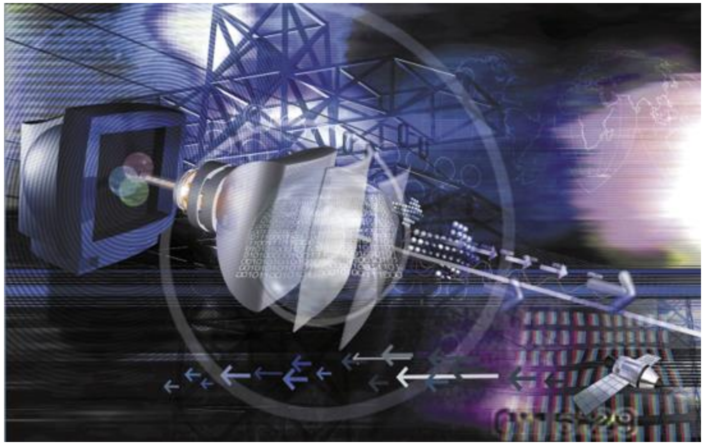
电子科技大学格拉斯哥学院
Glasgow College, UESTC

Communication Circuits Design

Academic year 2018/2019 – Semester 2 – Week 2
Lecture 2.1: Mixers

Francesco Fioranelli, Wasim Ahmad, Faisal Tariq

Outline



- **Mixers: basic concept and usage**
- **Some simple implementations**
- **Some metrics of quality**

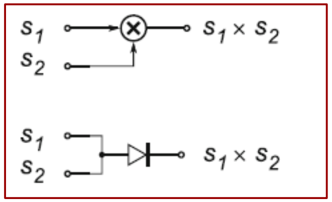
References:

- *J. Beasley, G. Miller, "Modern Electronic Communication", Pearson, 9th ed. – Pages 136-138 & 332-334*
- *R. Sobot, "Wireless Communication Electronics", Springer, at UoG Library online – Chapter 9*
- *B. Razavi, "RF Microelectronics", Prentice Hall, 2nd ed. – Chapter 6*

Principle of mixing signals

A **mixer** is an electronic circuit that can multiply two AC signals to provide an output (3 ports component). This operation always requires a <u>non-linear</u> component, typically diode and/or transistors.

Mathematically, mixing two signals means multiplying them. This produces in first approximation outputs containing the **sum** ω1+ ω2 and **difference** |ω1- ω2| of the original input frequencies ω1 and ω2.



$$\sin(\omega_1 t)\times\sin(\omega_2 t)=\frac{1}{2}\left[\cos(|\omega_1-\omega_2|t)-\cos(\omega_1+\omega_2)t\right]$$

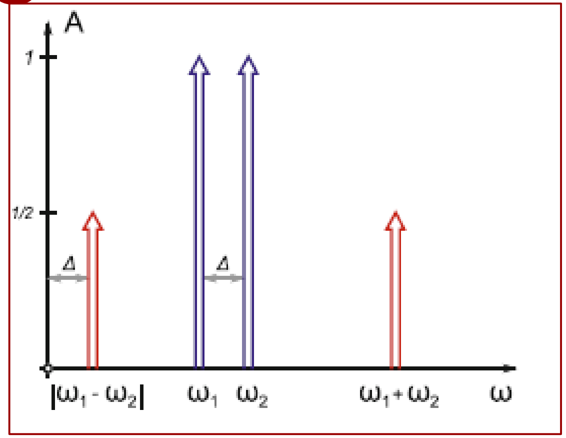
$$\cos(\omega_1 t)\times\cos(\omega_2 t)=\frac{1}{2}\left[\cos(|\omega_1-\omega_2|t)+\cos(\omega_1+\omega_2)t\right]$$

The original signals can be sine or cosine. If they are different, the output of the mixing process will have a tone (sum) with higher frequency than the original highest input AND a tone (difference) with lower frequency than the original lowest input.

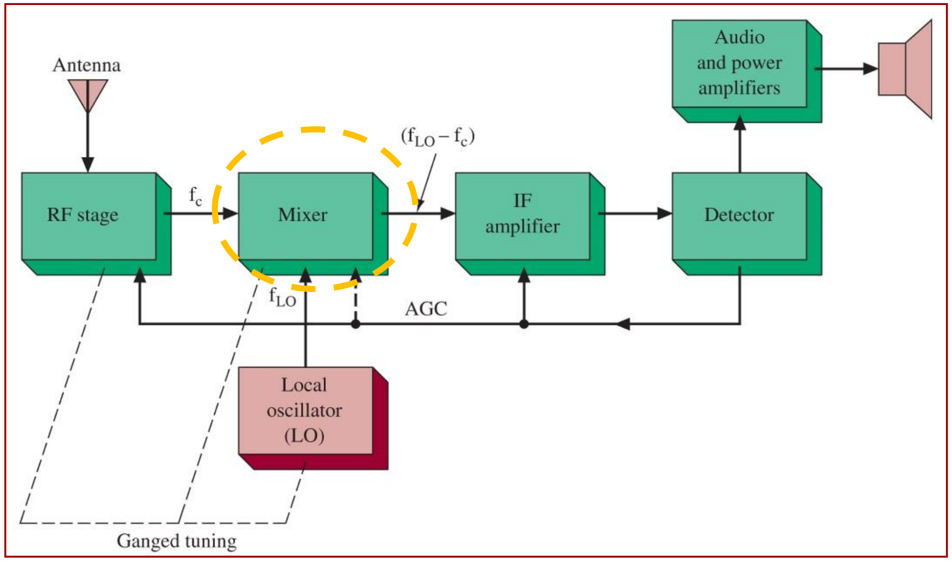
Much easier to see in the spectral/frequency domain.

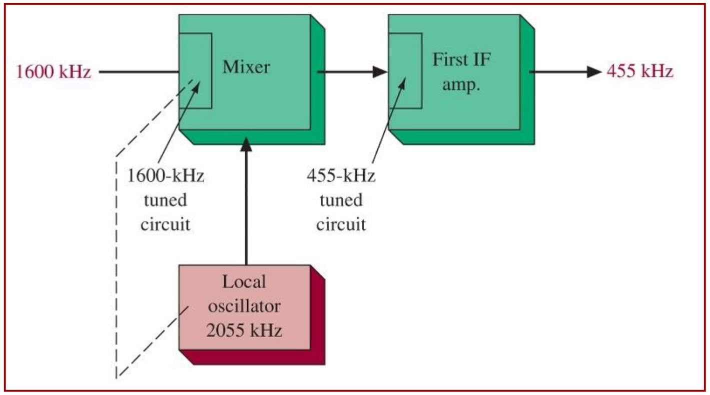
Principle of mixing signals

The output of the mixer has 2 frequency components



However, in many cases, we are only interested in one of the two components, hence one needs to be filtered out. So it is not unusual to see a combination of **mixer+filter**





In the example (superheterodyne receiver) just look at the mixing stage and see how the 2 input signals (1600 and 2055 kHz) are mixed and then the output is filtered to capture the 455 kHz (difference) component.

Principle of mixing signals

Why do we mix signals?

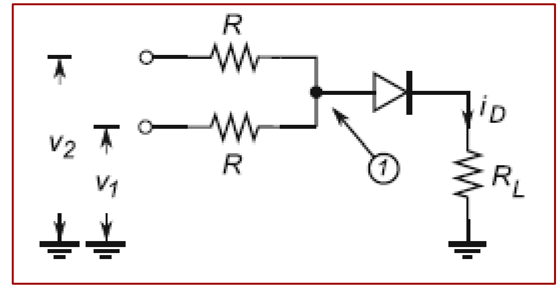
At the ***transmitter***, you may consider one input of the mixer as the carrier frequency generated by the local oscillator, and the other as the information (baseband) signal. If you keep at the output the sum component, you have perform an **up-conversion** of your baseband signal up to RF band.

At the ***receiver***, you may consider one input of the mixer still as the carrier generated to the local oscillator, and the other as the signal at RF (high) frequencies received from a transmitter. If you keep at the output the difference component, you have performed a **down-conversion** of your RF signal down to base band.

This result is also true if one of the inputs to the mixer is not a pure sinusoidal signal, but has a bandwidth, a certain range of frequencies. Question for you: *How would the spectrum of the output look like?*

Diode mixer

A very simple mixer. Two signals v_1 and v_2 are added together to generate v_D then used as input of the mixer.



$v_1 = V_1\cos(\omega_1 t)$ & $v_2 = V_2\cos(\omega_2 t)$

$v_D = \frac{1}{2}(v_1 + v_2)$ this is the signal at node 1

$$i_D = I_s\{\exp\left(\frac{v_D}{V_T}\right) - 1\}$$

IS and VT are diode structural parameters – Now assume the current through the diode is small so the exponential can be approximated as a Taylor series and the terms of order higher than 2 can be discarded

$$i_D = I_S\left\{\left[1 + \frac{v_D}{V_t} + \frac{1}{2}\left(\frac{v_D}{V_t}\right)^2 + \frac{1}{6}\left(\frac{v_D}{V_t}\right)^3 + \frac{1}{24}\left(\frac{v_D}{V_t}\right)^4 + \cdots\right] - 1\right\}$$

Linear term has exactly the original frequencies from the two input signals v_1 and v_2

$$\frac{v_D}{V_t} = \frac{1}{2V_t}[V_1 \cdot \cos(\omega_1 t) + V_2 \cdot \cos(\omega_2 t)] = f(\omega_1, \omega_2).$$

Square term has additional frequency components, $2\omega_1$ $2\omega_2$ $\omega_1 \pm \omega_2$

$$\frac{1}{2}\left(\frac{v_D}{V_t}\right)^2 = \frac{1}{2V_t^2}\left\{\frac{1}{2}[V_1 \cdot \cos(\omega_1 t) + V_2 \cdot \cos(\omega_2 t)]\right\}^2$$

$$= \frac{1}{8V_t^2}[V_1^2\cos^2(\omega_1 t) + 2V_1V_2\cos(\omega_1 t)\cos(\omega_2 t) + V_2^2\cos^2(\omega_2 t)]$$

$$= \frac{1}{8V_t^2}\left[V_1^2\,\frac{1}{2}(1 + \cos(2\omega_1 t)) + V_1V_2(\cos(|\omega_1 - \omega_2|t) + \cos((\omega_1 + \omega_2)t)) + V_2^2\,\frac{1}{2}(1 + \cos(2\omega_2 t))\right],$$

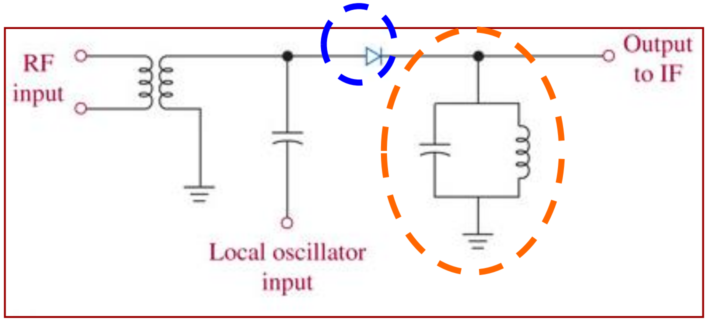
Diode mixer

So a diode-based mixer produces the ideal outputs (sum and difference $\omega_1 \pm \omega_2$) but also two undesired components $2\omega_1$ and $2\omega_2$

$$\boxed{\frac{1}{2}\left(\frac{v_D}{V_t}\right)^2 = f\left[(\omega_1-\omega_2), 2\omega_1, 2\omega_2, (\omega_1+\omega_2)\right]}$$

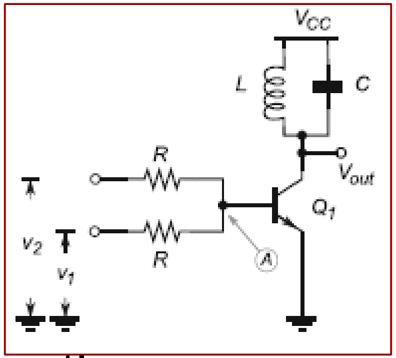
This means that the diode mixer is simple, but not very efficient because several unwanted components are generated wasting some power and furthermore they need to be filtered out.

Note that if the small signal approximation is not valid, even more higher terms of the series are present with more undesired components, hence diode mixer is practical only with low input signal levels.



In the example above the diode mixer (blue) is followed by an LC band-pass filter (which we will discuss more in detail later on)

BJT mixer



Mixers can be made also using transistors, for example BJT.

The two input signal are fed to the base of the BJT

$v_1 = V_1\cos(\omega_1 t)$ & $v_2 = V_2\cos(\omega_2 t)$

$v_{BE} = \frac{1}{2}(v_1 + v_2)$ and $i_C = I_s\{\exp\left(\frac{v_{BE}}{V_T}\right) - 1\}$ for BJT formulae

If we expand the exponential into Taylor series and focus on the **square term**

$$I_{Cs} = \beta I_S \frac{V_1 V_2}{8V_t^2}\left[\cos(|\omega_1 - \omega_2|t) + \cos((\omega_1 + \omega_2)t)\right].$$

Compared to diode mixer:

-there is a "***conversion gain***" β which means that the output term can be larger the input (= the mixer can internally amplify the signal, or if you want the mixer has also an internal amplifier, ***active mixer***) -> this is important if the block after the mixer needs a certain power level to work

-still small signal approximation necessary to disregard higher order terms

Note that there is again an LC circuit at the collector (output) of the BJT to filter out the unwanted components generated by the mixing process

JFET mixer

If the BJT is replaced by a JFET transistor, the analysis is very similar as that in the previous slide. Inputs: $v_1 = V_1 \cos(\omega_1 t)$ & $v_2 = V_2 \cos(\omega_2 t)$

However, the voltage-current relationship is different (no exponential):

$$I_D = I_{DSS}(1 - \frac{v_{GS}}{V_P})^2$$

for JFET formulae, where I_{DSS} is the saturation drain current, V_{GS} is the gate-source voltage, and V_P is the pinch-off voltage.

If we only focus now on the **non-linear square term**

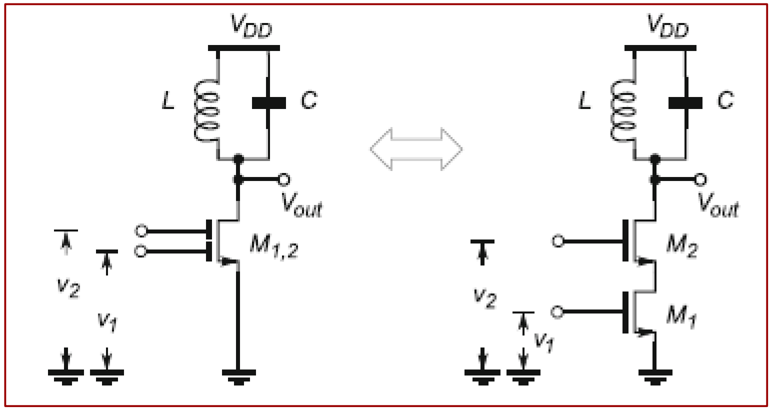
$$\begin{aligned} I_D &\sim -I_{DSS}\,\frac{1}{4}\,\frac{[V_1 \cdot \cos(\omega_1 t) + V_2 \cdot \cos(\omega_2 t)]^2}{V_P^2} \\ &\sim -I_{DSS}\,\frac{V_1 V_2}{2 V_P^2}\,[\cos(|\omega_1 - \omega_2| t) + \cos((\omega_1 + \omega_2) t)] \end{aligned}$$

--There are no higher order terms here because there is no exponential formula. This allows the mixer to work more efficiently (no power wasted in higher order components) and to accept higher signal levels at input.

--Still, there is a conversion gain provided by the term I_{DSS}

Dual-gate MOSFET mixer

A dual gate MOSFET transistor can be used to implement a mixer as well. This is equivalent to two twin MOSFETs fed by two input signals at two different frequencies $v_1 = V_{DC1} + V_1 \sin(\omega_1 t)$ & $v_2 = V_{DC2} + V_2 \sin(\omega_2 t)$



After some calculations (see Sobot's book chapter 9 if interested) we get

$$
\begin{aligned}
V_{\text{out}} &= [g_{\text{m}}{}' + g_{\text{m}\Delta} \sin(\omega_2 t)]\, R_{\text{D}} V_1 \sin(\omega_1 t) \\
&= g_{\text{m}}{}' R_{\text{D}} V_1 \sin(\omega_1 t) + g_{\text{m}\Delta} R_{\text{D}} V_1 \sin(\omega_2 t) \sin(\omega_1 t) \\
&\sim g_{\text{m}\Delta} R_{\text{D}} V_1 \left[\cos(|\omega_1 - \omega_2| t) + \cos((\omega_1 + \omega_2) t)\right],
\end{aligned}
$$

Similarly to the JFET case, the output has the two $\omega_1 \pm \omega_2$ components and a conversion gain given by $g_{m\Delta}$. Note that this depends on the amplitude of v_2 so there is interest in maximising amplitude of v_2 to get larger mixer output.

Image frequency

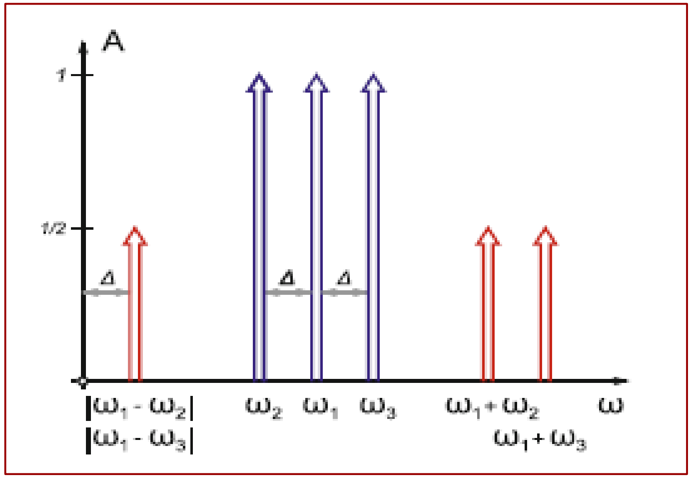
A less obvious but significant problem of the mixing operation is the image frequencies. If we look back at the multiplication formulae

$$\sin(\omega_1 t) \times \sin(\omega_2 t) = \frac{1}{2}\left[\cos(|\omega_1 - \omega_2|t) - \cos(\omega_1 + \omega_2)t\right]$$

$$\cos(\omega_1 t) \times \cos(\omega_2 t) = \frac{1}{2}\left[\cos(|\omega_1 - \omega_2|t) + \cos(\omega_1 + \omega_2)t\right]$$

We can see that given a certain (angular) frequency ω_1, there are two separate frequencies ω_2 and ω_3 that can provide the same difference, so $|\omega_1 - \omega_3| = |\omega_1 - \omega_2|$, whereas the sum components are easy to separate.

It is easier to see in the spectral domain------>



Given ω_1, if the purpose of the circuit is mixing with ω_2 then ω_3 is considered the "ghost" or "image" frequency; vice versa if the ideal mixer is for ω_1 and ω_3 then ω_2 is the image.

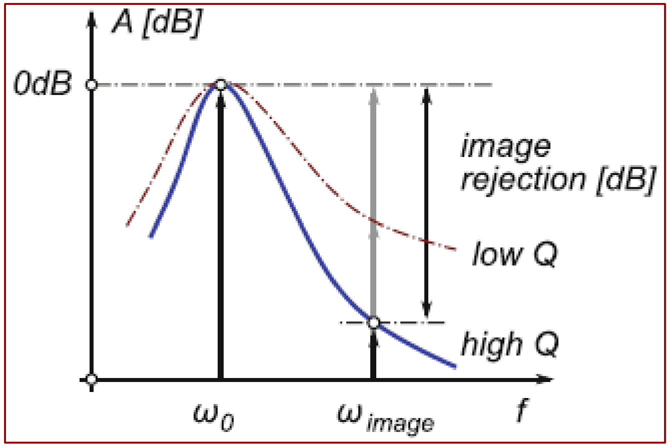
Image frequency rejection

How to deal with images?

First, communication **standards** and regulations are defined so that some frequencies cannot be transmitted as they would act as images for those in use for important applications.

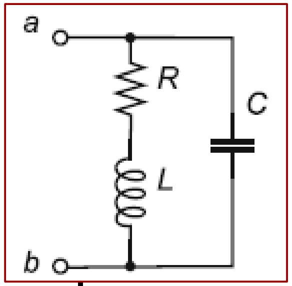
Second, communication circuits must be designed to guarantee a given level of rejection (attenuation) of ghost images. Given the desired frequency (or band of frequency) a **band-pass filter** is used to reject the images.

Band-pass filters can be implemented using LC parallel circuits (we will analyse them in detail), but here is an example of their transfer function at the desired ω_0 (or resonant frequency) and at an ω_{image} to be rejected



Q measures the quality of the filter (if high then the function shape is narrow, and vice versa)

Image frequency rejection



A simple circuit of a BP filter for image rejection is an LC parallel

The output voltage of this filter can be calculated dividing the current by the admittance Y of the filter so $|V_{out}| = \frac{|I|}{|Y|}$

It can be demonstrated that the absolute value of the admittance close to the desired/resonant frequency ω_0 is equal to

$$|Y| = Y_0\sqrt{1 + (\delta Q)^2}$$

Where Q is the quality factor of the filter, Y_0 its admittance at the resonance and $\delta = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$

If now we compare the voltage output near the resonance with the one at the resonance we get a relative measure of how much the filter attenuates the image $|\boldsymbol{A_r}| = \frac{|\boldsymbol{V_0}|}{\boldsymbol{V_0}(\boldsymbol{\omega_0})} = \frac{\mathbf{1}}{\sqrt{\mathbf{1}+(\boldsymbol{\delta Q})^2}}$

This can also be measured in dB as $20\log_{10}\frac{\mathbf{1}}{\sqrt{\mathbf{1}+(\boldsymbol{\delta Q})^2}}$

Image frequency rejection

$$|\boldsymbol{A_r}| = \frac{|\boldsymbol{V_0}|}{\boldsymbol{V_0(\omega_0)}} = \frac{\mathbf{1}}{\sqrt{\mathbf{1+(\delta Q)^2}}}$$

A numerical example for you.

Example 9.1. An AM broadcast receiver is tuned to 500 kHz with an LC resonator whose $Q = 50$. Calculate the signal rejection in dB of unwanted signal being transmitted at 1,430 kHz.

First you need to calculate $\boldsymbol{\delta}$

Then calculate $\boldsymbol{A_r}$ and then apply the dB scale

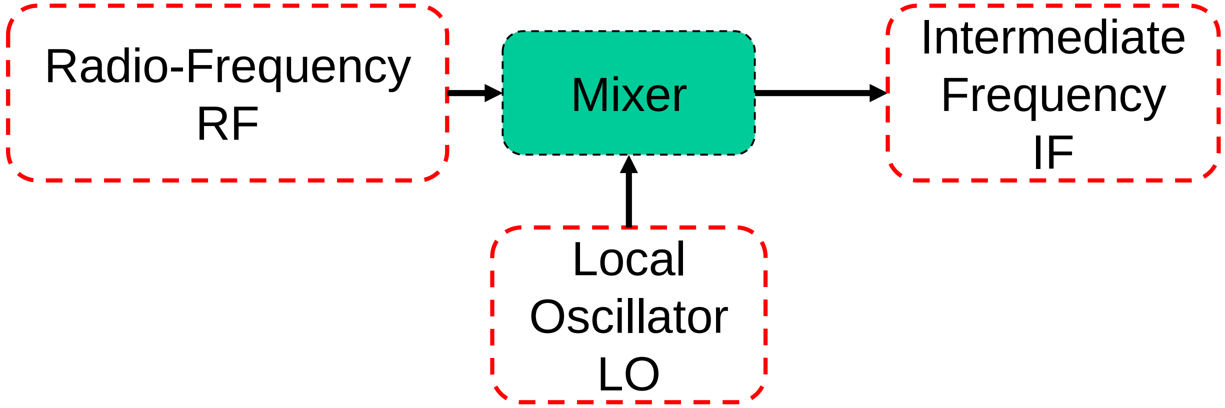
You should get something near -42dB as a final result

Note! Multiple tuning stage (BP filters) can be used one after the other to increase the image rejection capabilities. It is typical to have one tuning stage at the output of the mixer and one at the input of the following amplifier.

Image frequency rejection

Another example. *Determine the image frequency for a standard broadcast band receiver using a 455 kHz IF signal and tuned to a radio station at 620 kHz.*

Remember that every mixer has 3 ports. At the RX typically the mixer down-converts the signal, so in this case the IF (with frequency < RF) is 455 kHz and the RF is the original radio station at 620 kHz.



So you need first to find the LO frequency. [should be 1.075 MHz]

Then find the frequency of the signal which mixed with the LO would still give you 455 kHz. [should be 1.53 MHz]

Mixer example

A possible mixer from Minicircuits ZP-3
https://www.minicircuits.com/pdfs/ZP-3.pdf

Parameters

-frequency range at 3 ports IF, LO, RF
-conversion loss
-isolation
-level
-VSWR
-max RF power
-max IF current
-schematics and drawing (*this tells you which port is which between RF IF LO*).

Coaxial

Frequency Mixer

ZP-3+
ZP-3

Level 7 (LO Power +7 dBm) 0.15 to 400 MHz

BNC version shown
CASE STYLE: GG60

Connectors	Model
BNC	ZP-3+
SMA	ZP-3-S(+)

+RoHS Compliant
The +Suffix identifies RoHS Compliance. See our web site for RoHS Compliance methodologies and qualifications

Maximum Ratings

Operating Temperature	-55°C to 100°C
Storage Temperature	-55°C to 100°C
RF Power	50mW
IF Current	40mA

Permanent damage may occur if any of these limits are exceeded.

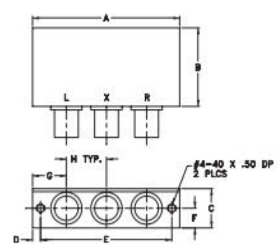
Coaxial Connections

LO	L
RF	R
IF	X

Features
- low conversion loss, 4.7 dB typ.
- IF response to DC
- excellent L-R isolation, 46 dB typ., L-I, 47 dB typ.
- rugged shielded case

Applications
- VHF/UHF
- cellular
- instrumentation

Outline Drawing



Outline Dimensions (inch / mm)

A	B	C	D	E	F	G	H	wt
2.31	1.20	.60	.125	2.062	.30	.53	.63	grams
58.67	30.48	15.24	3.18	52.37	7.62	13.46	16.00	75.0

Electrical Specifications

FREQUENCY (MHz)		CONVERSION LOSS (dB)				LO-RF ISOLATION (dB)						LO-IF ISOLATION (dB)					
LO/RF	IF	Mid-Band m			Total Range	L		M		U		L		M		U	
f_L-f_U		$\overline{X}$	σ	Max.	Max.	Typ.	Min.	Typ.	Min.	Typ.	Min.	Typ.	Min.	Typ.	Min.	Typ.	Min.
0.15-400	DC-400	4.7	0.10	7.0	8.0	60	50	46	30	35	25	60	40	47	25	35	20

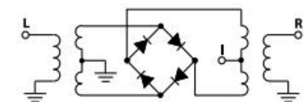
1 dB COMP.: +1 dBm typ.
For phase detection, DC output has positive polarity with in-phase LO and RF signals.

L = low range [f_L to 10 f_L] M = mid range [10 f_L to f_U/2] U = upper range [f_U/2 to f_U]
m= mid band [2f_L to f_U/2]

Typical Performance Data

Frequency (MHz) RF	Frequency (MHz) LO	Conversion Loss (dB) LO +7dBm	VSWR RF Port (:1) LO +7dBm	Frequency (MHz) LO	Isolation L-R (dB) LO +7dBm	Isolation L-I (dB) LO +7dBm	VSWR LO Port (:1) LO +7dBm
0.15	30.15	5.37	1.57	10.00	68.68	61.84	2.59
0.23	30.23	5.27	1.41	20.00	65.36	56.87	2.60
0.30	30.30	5.21	1.33	30.00	63.22	54.20	2.59
0.50	30.50	5.16	1.25	40.00	61.75	52.09	2.58
1.00	31.00	5.08	1.21	76.00	57.56	47.59	2.54
2.80	32.80	4.91	1.21	94.00	56.48	45.97	2.50
6.40	36.40	4.91	1.21	112.00	54.90	44.70	2.50
10.00	40.00	4.73	1.21	149.00	52.63	42.36	2.57
28.00	58.00	4.71	1.21	168.00	54.13	42.02	2.55
64.00	94.00	4.75	1.17	206.00	49.62	38.81	2.62
100.00	130.00	4.83	1.14	225.00	48.10	38.56	2.66
138.00	168.00	4.85	1.13	244.00	48.03	37.82	2.68
157.00	187.00	4.88	1.10	282.00	53.65	37.79	2.67
195.00	225.00	4.92	1.08	301.00	55.10	38.07	2.76
233.00	263.00	4.97	1.10	320.00	54.03	37.59	2.82
252.00	282.00	5.10	1.12	340.00	52.86	36.62	2.76
271.00	301.00	5.17	1.14	360.00	51.53	35.44	2.69
290.00	320.00	5.15	1.17	390.00	47.44	33.11	2.86
370.00	340.00	5.38	1.10	410.00	45.39	32.24	3.05
400.00	370.00	5.41	1.05	430.00	44.42	32.17	3.06

Electrical Schematic



Mixer selection metrics

Choosing your mixer for your design -> need to understand the specs and the metrics to make “your shopping list”.

Typical vendor: Minicircuits

Send to a Friend

- [ ] Surface Mount
- [x] Connector
- [ ] Plug-In
- [ ] Die

- [ ] Level 3-4
- [ ] Level 7
- [ ] Level 10
- [ ] Level 13
- [ ] Level 17
- [ ] Level 23
- [ ] Upconverter

All ezsamples

Frequency Mixers, Coaxial, Level 3-4, 2.0 to 7000 MHz

Model Number	Case Style	LO Level (dBm)	RF in @ 1 dB Comp. (dBm) Typ.	RF/LO Freq. Low (MHz)	RF/LO Freq. High (MHz)	IF Freq. Low (MHz)	IF Freq. High (MHz)	Conv. Loss (dB) Typ.	Conv. Loss (dB) (σ)	Conv. Loss (dB) Max.	LO-RF Isolation (dB) Typ.	LO-IF Isolation (dB) Typ.	IP3 (dBm) Center Band Typ.
ZX05-1L-S+	FL905	3.0	0.0	2.0	500.0	DC	500.0	5.2	0.1	7.2	55.0	45.0	16.0
ZX05-10L-S+	FL905	4.0	1.0	10.0	1000.0	DC	800.0	7.2	0.1	8.2	60.0	37.0	16.0
ZX05-73L-S+	FL905	4.0	1.0	2400.0	7000.0	DC	3000.0	6.5	0.1	8.3	33.0	20.0	10.0

Frequency Mixers, Coaxial, Level 7, 500 Hz to 15 GHz

Model Number	Case Style	RF/LO Freq. Low (MHz)	RF/LO Freq. High (MHz)	IF Freq. Low (MHz)	IF Freq. High (MHz)	Conv. Loss (dB) Typ.	Conv. Loss (dB) Std. Dev.	Conv. Loss (dB) Max.	LO-RF Isolation (dB) Typ.	LO-IF Isolation (dB) Typ.
ZAD-1+	M22	0.5	500.0	DC	500.0	5.24	0.1	8.5	45.0	40.0
ZAD-1-1+	M22	0.1	500.0	DC	500.0	4.83	0.04	8.5	45.0	40.0
ZAD-2+	M22	1.0	1000.0	0.5	500.0	5.66	0.07	8.5	35.0	35.0
ZAD-3+	M22	0.025	200.0	DC	200.0	4.61	0.06	8.5	45.0	40.0
ZAD-6+	M22	0.003	100.0	DC	100.0	4.65	0.08	8.5	45.0	40.0
ZAD-8+	M22	5.0E-4	10.0	DC	10.0	5.79	0.05	8.5	50.0	50.0
ZAD-11+	M22	5.0	2000.0	10.0	600.0	7.12	0.12	9.0	35.0	30.0
ZAM-42	F14	1500.0	4200.0	DC	500.0	5.67	0.11	8.5	25.0	18.0

Mixer metrics

We can look at some of the metrics of interest – you may also read these application notes here
https://www.minicircuits.com/app/AN00-010.pdf

Frequency range.
Self-explanatory. It is the minimum and maximum frequency between which the mixer can operate properly at each three ports. Note that these ranges are given (typically) for RF/LO port and for IF port.

Conversion loss.
If the mixer is passive (no amplifiers inside) the loss indicates the reduction in signal level from the RF signal to the IF signal. It is typically measured in dB.
It is normally expressed with an average value, and with its maximum value and standard deviation.

Mixer metrics

Isolation.

Takes into account how well isolated, separated the LO port is from the RF and the IF port respectively. Ideally no signal from the LO should leak into the RF and IF port; practically there will be some leakage measured in dB with respect to the LO original signal.

Typical and minimum (worst case) values are typically given.

Max ratings.

The maximum power acceptable at the RF port of the mixer in order not to damage the mixer itself.

Level.

This indicates the require power of the signal at the Local Oscillator for the mixer to work properly. Typically expressed in dBm.

If you go back 2 slides on the Minicircuit slides you can see on the left level 3-4, 7, 10 mixers and so on.

Mixers can still operate if the LO level is a bit lower (underdriven) or higher (overdriven) but the performances must be checked very carefully from the datasheet.

Mixer metrics

VSWR.

It stands for Voltage Standing Wave Ratio. It can be also expressed with another parameter, the **Return Loss (RL)**, which is the ratio of the reflected power to the incident power at the specific port of a mixer

$$RL = -20\log_{10}\left|\frac{Vref}{Vinc}\right|; VSWR = \frac{1+10^{-\frac{RL}{20}}}{1-10^{-\frac{RL}{20}}}; \left|\frac{Vref}{Vinc}\right| = \frac{VSWR-1}{VSWR+1}$$

Minimum reflections (extreme case of no reflections at all) -> theoretical value for the RL is infinity, leading to VSWR = 1.

Maximum reflections (total, specular reflections) -> theoretical value for the RL is 0, leading to VSWR to infinity.

Practically you want values as close to 1 as possible. Rule of thumbs.

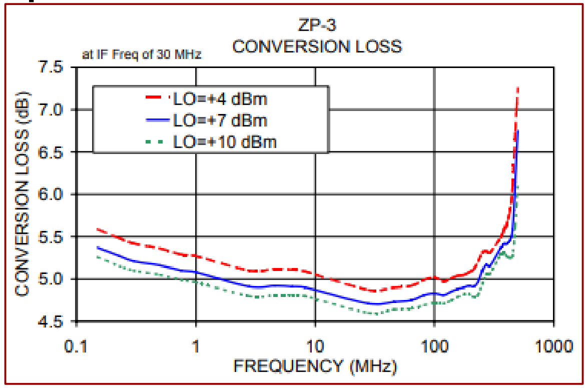
Half voltage reflected -> RL~-13 dB, VSWR ~ 1.5

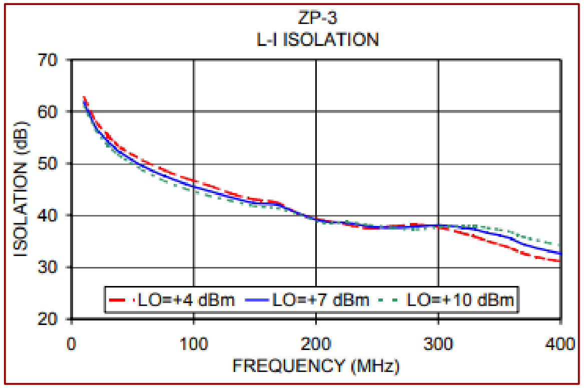
75% voltage reflected -> RL~-5.7 dB, VSWR ~ 3.13

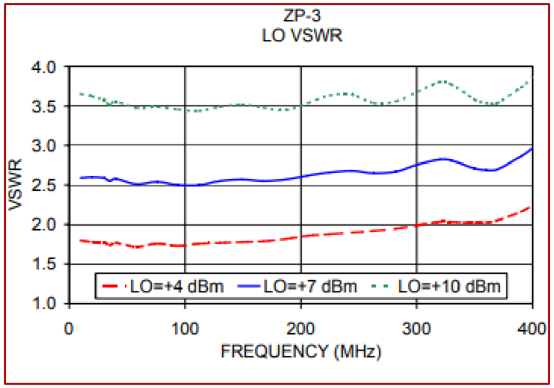
The mixer has 3 ports so VSWR is defined for each one of them.

Mixer metrics

Mixer parameters also depend on frequency, hence some of them expressed in terms of average/max/min values within certain segments of the frequency band of interest. In any case, the datasheets tend to provide plots like those below.







Note the effect of overdriving or underdriving the mixer with different LO power levels (the plots refer to the level 7 mixer shown a few slides before) – it can be very different for different metrics.

Finally mixers (especially active mixers) may also have a parameter to take into account the ***distortion*** of higher order harmonics, typically the ***third order intermodulation products*** $2f_1 \pm f_2$ and $2f_2 \pm f_1$ generated by mixing f_1 and f_2 –We will discuss this later on when presenting amplifiers

Summary

In this slides we have:

-defined the principle of mixing as multiplication in time domain

-understood the implication of mixing signals in the frequency domain (Up & Down conversion)

-investigated some simple implementations of mixers using diode, BJT, JFET, MOSFET

-presented the concept of image frequency and discussed how to reject/attenuate them

-discussed some of the key metrics of quality for mixers with reference to realistic examples and datasheets