Communication Circuits Design

Academic year 2018/2019 – Semester 2 – Week 2

Lecture 2.2: LC circuits and matching networks

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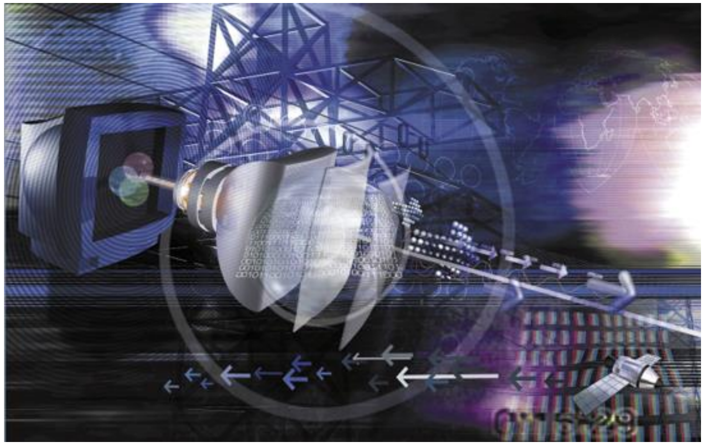
Outline



- **LC blocks and properties**
- **The "tank circuit"**
- **Parasitic parameters**
- **Matching networks**

References:

•*J. Beasley, G. Miller, "Modern Electronic Communication", Pearson, 9th ed. – Chapter 1.7*

•*B. Razavi, "RF Microelectronics", Prentice Hall, 2nd ed. – Chapter 2*

•*R. Sobot, "Wireless Communication Electronics", Springer, at UoG Library online – Chapter 5-6*

Practical Capacitors and Inductors

You will remember from Circuits Analysis in year 2 that

-capacitors store energy in the electric field and their capacitance is measured in Farad [F]

-inductors store energy in the magnetic field and their inductance is measured in Henry [H]

At higher frequencies for communication applications, these components have also some ***losses*** of energy, in the leakage between conductors plate in the capacitor and in the winding wire resistance in the inductor.

These losses are modelled by a parameter **Q quality factor** defined below. High Q means good quality components.

$$\boldsymbol{Q_L} = \frac{reactance}{resistance} = \frac{\omega L}{R} \quad \text{and} \quad \boldsymbol{Q_C} = \frac{susceptance}{conductance} = \frac{\omega C}{G} = \omega C R$$

Values up to about 500 are typically available for inductors, about 1000 for capacitors in typical radio circuits.

The **dissipation D** is the inverse of the Q factor, D=1/Q

Resonance and notch filter

When capacitors and inductors are used together, the circuit can reach the so-called status of **resonance**. This is when the inductive and capacitive reactance are balanced and equal ($X_L=X_C$), and happens at a specific frequency, the **resonant frequency f_r** or **f_0**.

For an <u>RLC series</u> circuit we can have $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$

At the resonance:

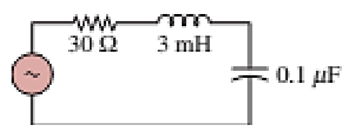


FIGURE 14 Series *RLC* circuit.

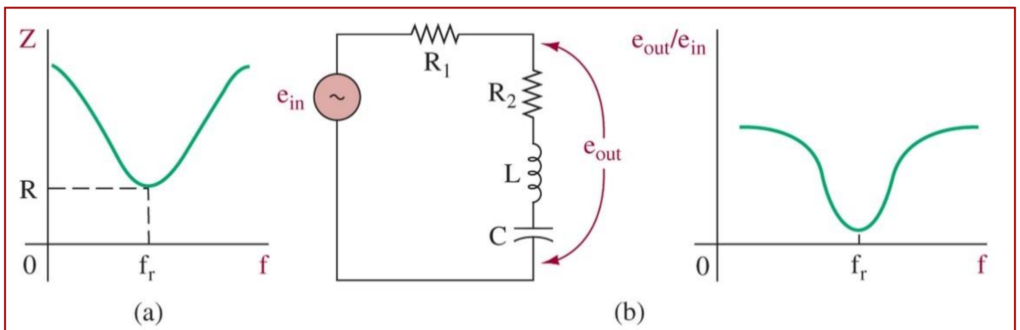
$$X_L = X_C$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad \text{(Hz)}$$

In this case the impedance of the RLC series at the resonance is minimal, and this can be used to implement a <u>notch filter</u>.



Resonance as band-pass filter

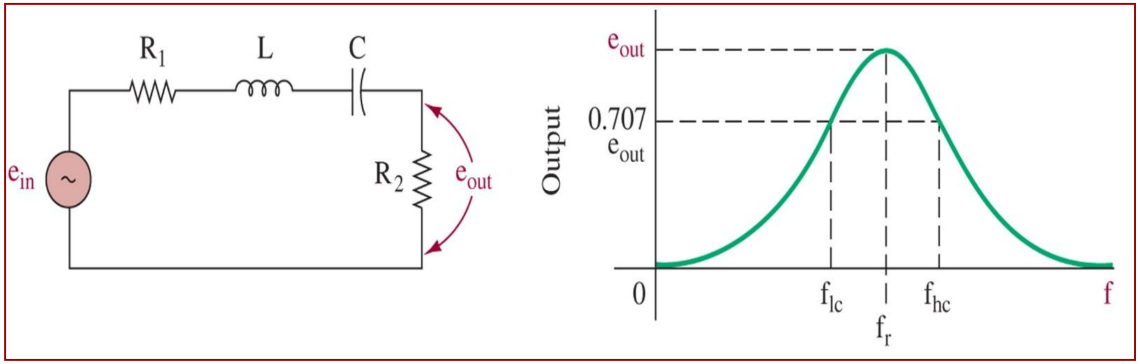
If we change the position of the RLC series and the other resistor, we can implement a band-pass filter as below.

The peak of the bell shape is at the resonance $\mathbf{f_r}$, whereas the bandwidth **BW** of the filter can be defined as the range of frequencies between the two values where the output voltage has fallen to $\sqrt{2} = 0.707$ times its maximum.

$$\boldsymbol{BW} = \frac{R}{2\pi L}$$

$$\boldsymbol{Q_s} = \frac{\omega L}{R} = \frac{1}{\omega RC} = \frac{1}{R}\sqrt{L/_C}$$

$$\boldsymbol{Q_s} = \frac{\boldsymbol{f_r}}{\boldsymbol{BW}}$$



It can be demonstrated that the relationships above exist, where the quality factor Q compares how narrow (=selective) the BPF is compared to its resonant (center) frequency. Higher Q = more selective filter.

Higher Q implies lower R in a series RLC but this is always limited by the internal resistance of the wires of the inductor.

The “tank” circuit

We now consider a parallel LC circuit. Note that the resistor in this case represents the intrinsic resistance of the inductor winding wires – in other words there is always some R even if you do not have a physical resistor.

In this case one can analyse the total admittance Y and impose that at resonance the imaginary part is equal to zero.

$$Y(\omega) = \frac{1}{R + j\omega L} + j\omega C = \frac{R - j\omega L}{R^2 + (\omega L)^2} + j\omega C$$
$$= \frac{R}{R^2 + (\omega L)^2} + j\left(\omega C - \frac{\omega L}{R^2 + (\omega L)^2}\right)$$

The resonant frequency will be equal to

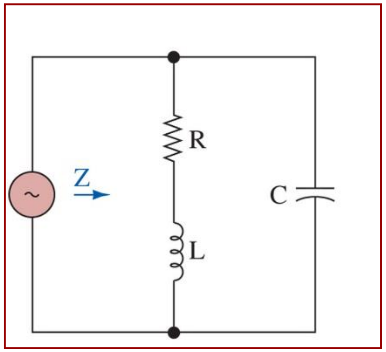
$$\boxed{\boldsymbol{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}}$$ **resonant frequency for RLC parallel**

However, for typical configurations (Q>10) the R can be neglected and the formulae from the series RLC can be used.

$$\boldsymbol{Q_p} = \omega RC = R\sqrt{C/L} \qquad \boldsymbol{Q_p = \frac{f_r}{BW}}$$

<u>Higher Q implies higher </u>R in a parallel RLC

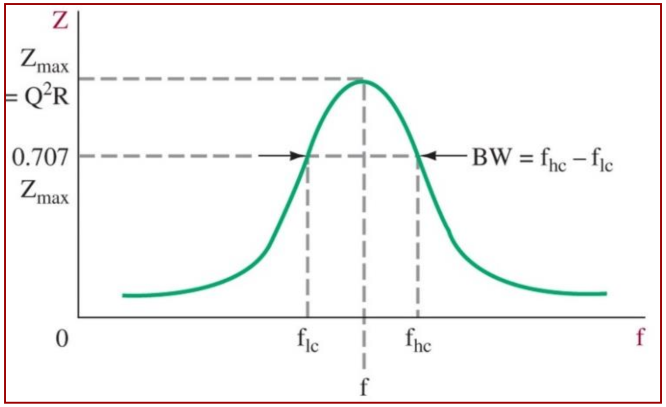
Note that at resonance the admittance Y is minimal, hence the impedance Z is maximal.



The “tank” circuit

So at resonance the RLC has maximum impedance $Z=Q^2R$ – if we use it as a load, it can implement a band-pass filter.

The parallel LC circuit is commonly called “**tank circuit**” as it can store energy alternatively in each reactive element, L and C, by releasing and passing this energy between each other.



So RLC blocks have frequency selectivity properties and therefore can be used for the implementation of **filters**. At frequencies below 100 kHz RC blocks can be used (inductors would be too bulky), above RLC blocks are preferable.

The number of RLC blocks/sections is sometimes called the order of the filter or number of poles of the filter.

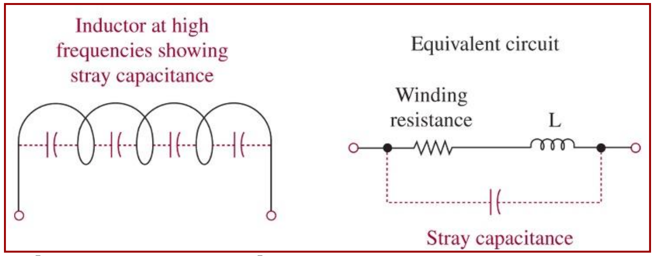
You may come across several configurations named after the researchers who developed them: Butterworth, Chebyshev, Cauer or elliptical, and Bessel or Thomson

High frequency effects

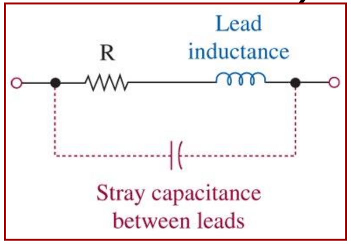
So far we are used to consider an inductor just as an inductor with a certain inductance L. However, at high frequencies used in communication applications, <u>the inductor behaves as an RLC circuit</u>. There is:

-a resistance generated by the winding wires

-a stray capacitance generated between charges between the wires



Similarly, <u>even in a resistor</u> we have a stray capacitance between the leads (the higher, the closer the leads are) and an inductive effect in the wire (the higher, the longer the wire is). These effects increase with frequency!



Take home message: physically you may have more RLC circuits that you think on your circuit due to these **parasitic parameters** & minimise/shorten all lead lengths on a PCB for high RF frequencies

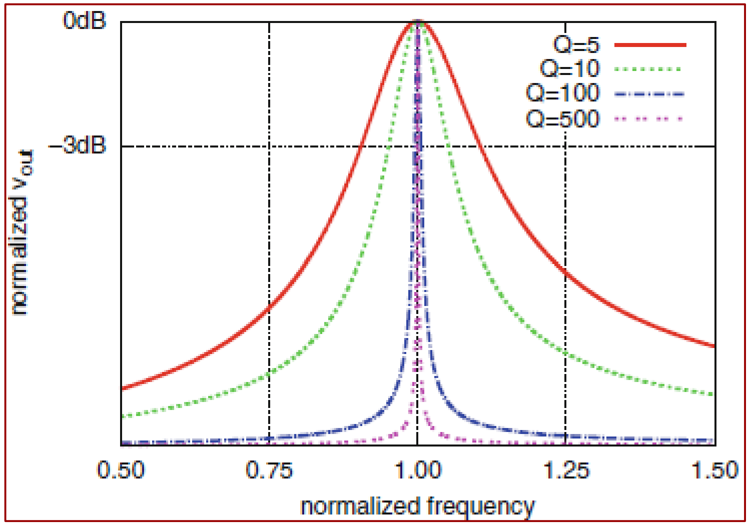
Summary

So far we have seen that RLC series and parallel circuits resonates at a specific frequency (resonance). This means that voltages at that frequency can pass through the circuit, whereas others are attenuated.

This selective behaviour in frequency is practically a filter, either notch or band-pass depending on the configuration. The selectivity of the circuit is modelled by its quality factor Q (the highest the better).

In practical terms, the quality factor Q cannot be infinite (there are always parasitic resistances in the circuit), hence the RLC block will have a certain bandwidth (range of frequencies).

See as an example the voltage across an inductor L for different Q – the frequency is normalised to the resonant one



Serial to Parallel Conversion

It is often useful to transform serial RLC networks into parallel and vice versa. This is typically done at a single frequency and does not affect the quality factor of the network, hence the two Q are assumed to be equal $Q_S = Q_P = Q$ where $Q_S = \frac{X_S}{R_S}$ and $Q_P = \frac{R_P}{X_P}$

With some mathematical processing shown here one can obtain these formulae:

$R_P = R_S(1 + Q^2)$ and $X_P = X_S(1 + \frac{1}{Q^2})$

For typical Q>10, $\boldsymbol{R_P = R_S Q^2}$ and $\boldsymbol{X_P = X_S}$

$$Z_s = R_s + jX_s = R_s + jQ_sR_s = R_s(1 + jQ_s),$$

$$\begin{aligned} Y_p &= \frac{1}{Z_s} = \frac{1}{R_s(1+jQ)} = \frac{1}{R_s(1+jQ)}\frac{1-jQ}{1-jQ} \\ &= \frac{1}{R_s(1+Q^2)} - j\frac{Q}{R_s(1+Q^2)} \\ &= \frac{1}{R_s(1+Q^2)} - j\frac{Q}{\frac{X_s}{Q}(1+Q^2)} \\ &= \frac{1}{R_p} - j\frac{1}{X_p}, \end{aligned}$$

Note that series to parallel increases the resistance and retains reactance; parallel to series reduces the resistance and retains the reactance.

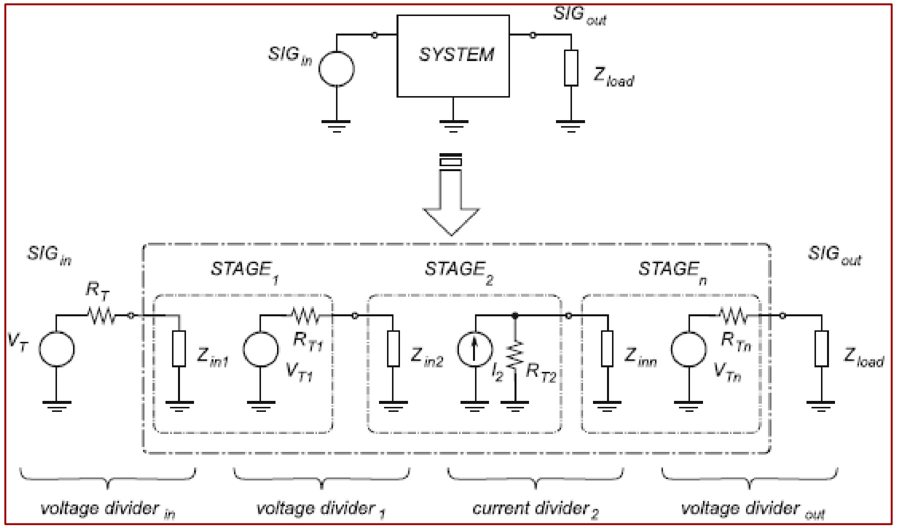
So we can use these transformations to **change the "perceived" resistance** but why should we do this?

Maximum Power Transfer

Before answering that question we need to think of a complicated communication system. Often the aim of the design is to **maximise the power** from a generator to an antenna for transmission AND from an antenna to a digitiser for reception.

In order to model a complex system made of hundreds of components, we can think in terms of blocks/sections represented by voltage dividers thanks to Thevenin theorem.

Each block will feel a source voltage V_T with source impedance R_T from the previous block and a load impedance Z_{IN} which is the input impedance of the following block. This series of blocks is easier to analyse.



Maximum Power Transfer

Now let us consider only one stage with a source voltage V_0 with source impedance Z_0 and load impedance Z_L. We can calculate the average power PL dissipated at the resistive part of the load as

$$P_L = I_{RMS}^2 R_L = \frac{1}{2}|I|^2 R_L = \frac{1}{2}\left(\frac{|V_0|}{|Z_0 + Z_L|}\right)^2 R_L$$

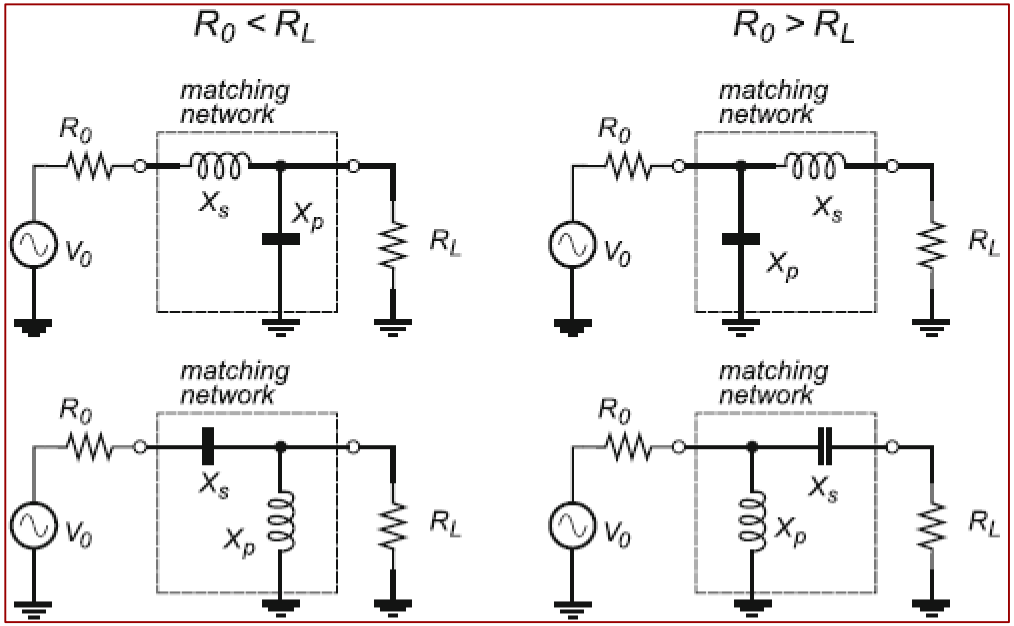
One can demonstrate (see Sobot's book chapter 6) that to maximise the power P_L, two conditions are necessary, which are resistance $\mathbf{R_0=R_L}$ and reactance $\mathbf{X_0=-X_L}$ -> this can be summarised by $\boldsymbol{Z_0 = Z_L^*}$

This is called **conjugate matching** and is the condition that guarantees the most efficient power transfer (note that this is calculated only at a specific frequency, so for signals across a range of frequencies the efficiency can be different).

If the impedances are not matched some power is reflected back with two potential problems: waste of useful power and undesirable/unexpected effects generate by this reflected signals.

Matching networks

So ideally we want to match our impedances, but what if two of those are inherently not the same? Then we need a **matching network**. This is a block of components that will transform the perceived impedance of 2 components to match them.



Match source R_0 to load R_L when they are different.

Source R_0 smaller than load R_L on the left, vice versa on the right.

Matching network is an L-shaped circuit – parallel reactance next to higher R, series reactance next to lower R

Note that you could build the matching network with just resistors, not reactive components, but that would create losses.

Also note that the choice of C or L for each element may depend on DC blocking. If you need to block the DC then use capacitors.

Matching networks

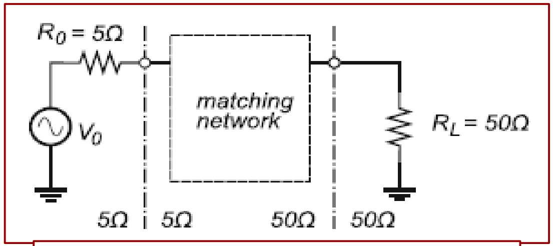
An example. Match a 50Ω resistive source at 100 MHz to a load which is a serial connection of a 50Ω resistor and 22 pF capacitor. Demonstrate that you can do this with only one inductor L (should be of value approximately 110 nH).

-Can you draw the circuit?

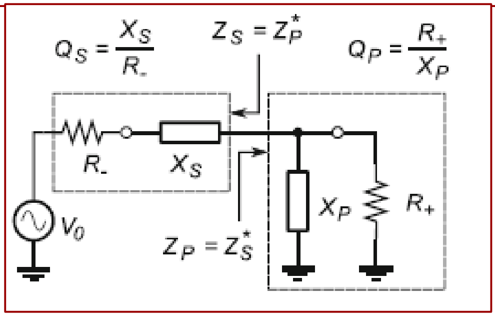
-Where do you place the inductor?

Matching networks

Another example. Design a matching network for the case below where there is a source resistor of 5Ω and a load of 50Ω.



Assume frequency of 100 MHz.



1) Need to add a series element to the lower R and a parallel element to the higher R

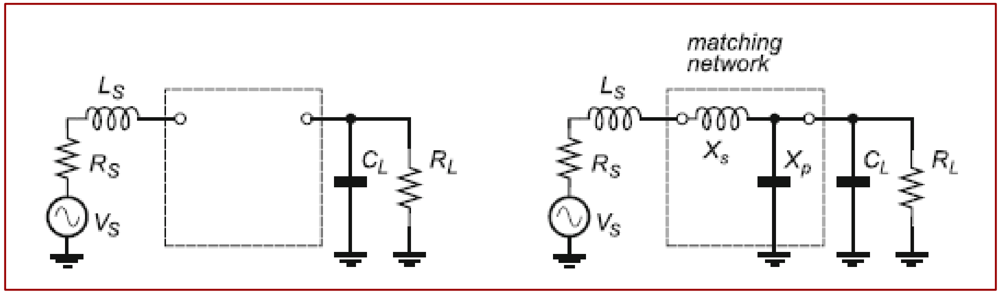
2) At the design frequency the two subnetworks must present complex conjugate impedances to each other, hence having same Q factor. $Q_S = \frac{X_S}{R_-}$ and $Q_P = \frac{R_+}{X_P}$ and recalling that serial and parallel can be transformed into each other by $R_P = R_S(1 + Q^2)$ we obtain

$$Q_S = Q_P = \sqrt{\frac{R_+}{R_-} - 1}$$

3) Finally calculate the value of L and C needed – Should be in the range of 23.85 nH (series L) and 95.49 pF (parallel C)

Matching networks

The sources and load can also be complex, so complex matching is needed. See the example below where there are L_S and C_L as parasitic parameters.



In this case, two options:

-Absorb the parasitic into the matching network (first design the matching network considering only resistances at source and load; then tweak the values to balance out the reactance)

-Resonate out the reactance at source and load (useful if these are larger than the reactance designed to match the resistive part)

Chapter 6 of Sobot's book shows some examples of this. One example will be shown at the tutorials.

Summary

In this set of slides we have discussed:

-LC resonant circuits and how they can be used to implement filters

-explored the concept of resonance of RLC circuits and usage for BPF

-introduced the concept of parasitic parameters at high frequencies

-introduced series to parallel transformation and impedance matching

-discussed the usage of these concepts in matching networks