



Communication Circuits Design

Academic year 2018/2019 – Semester 2 – Week 2
Tutorials

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Exercise set 1 – Mixers (Tue 26th)

1. Two signals $A\cos(2\pi 30,000t)$ and $B\cos(2\pi 70,000t)$ are fed into the input ports of a mixer diode. Indicate the frequency components (up to 2nd order included) at the output of the mixer and sketch the spectrum (disregard amplitude differences between them).
2. A GSM RF signal at 800 MHz is down-converted to an IF signal of 200 MHz. What are the suitable LO frequencies? How many mW of power should the LO generate if the mixer is a level 13 model?
3. Find the image frequency for a TV receiver where the RF signal at 433 MHz is down-converted to a 10 MHz IF signal with a 443 MHz LO. What would be the image attenuation in dB if a filter with $Q=300$ is used?
4. Discuss the advantages of a BJT implementation of mixers vs an implementation using diodes.

Exercise set 1 – Mixers (Tue 26th) - Solutions

1. There are two input signals at the mixer with frequency 30 kHz and 70 kHz respectively. The mixer is a diode mixer, so the output of the mixer up to the second order component will have the original signals (30 and 70 kHz), their second harmonic 60 and 140 kHz, and the sum and difference signals 40 and 100 kHz (f_1+f_2 and f_2-f_1).

Because they disregard amplitude differences, the sketch of the spectrum can just be an X axis with frequency and a vertical line (Dirac delta) for each frequency at the output.

2. Essentially a mixer takes the RF and the LO signal to give you an IF so that the IF can be either the sum or the difference of the RF and LO signal. To go from RF 800 down to IF 200 you can use an LO which is either 1000 MHz ($1000-800=200$) or 600 MHz ($800-600=200$).

The level of the mixer means the required amount of power of the LO in dBm – In this case, $13=10*\log_{10}(X/1\text{mW}) \rightarrow$ Find X which is basically 20 mW

3. The image frequency is an undesired RF signal that mixed with the LO can give you a frequency overlapping with the desired IF generated by the original RF. In this case, RF at 433 MHz and IF 10 MHz with LO 443 MHz for the desired channel – the image is therefore 453 MHz (because 453 at RF with LO 443 can give you 10 MHz at IF). The image rejection A_r has a formula below where ω_0 has to be calculated first (ω_0 is the frequency you want, 433 in this case, and ω is the image 453 in this case), then calculate A_r in dB

4. Advantages of BJT mixers vs diode mixers are:

there is a conversion gain, meaning that the mixer can be active and amplify the signals internally so that the output can be larger than the input;

both diode and BJT mixers still need small input signals otherwise they will generate a lot of spurious harmonics. This comes to the approximation of the exponential relation between current and voltage that is true for both diodes and BJT, so when you expand that in Taylor series you can take only first two terms IF the input signals are small.

Exercise set 1 – Mixers

Taken from Sobot book, chapter 9

9.2] Starting from $s_1(t) = \sin(2\pi f t)$ where $f=10\text{MHz}$, find two other single tones that could be used to generate a single tone at $f=1\text{kHz}$. Explain the process and the result. Can you also think of the spectra of these waveforms?

9.3] A large number of radio stations transmit their programs at various carrier frequencies. A radio receiver is tuned to receive an AM signal transmitted at a carrier frequency of $f_{\text{RF}}=980\text{ kHz}$. The LO inside the receiver is tuned at 1.435 MHz . Find:

- A. The frequencies at the output of the receiver mixer
- B. What frequency is the IF (intermediate signal) frequency
- C. The frequency of a radio station that would represent an image frequency to the desired radio station
- D. The frequency graph (spectrum) of the frequencies involved

9.4] An RF amplifier placed after a mixer has an LC tank filter with $Q=20$ and is tuned at frequency f_0 . Estimate the attenuation of the image signal if the image frequency f is 10% higher than the RF signal. What would change if the Q factor was 200?

Exercise set 1 – Solutions

Taken from Sobot book, chapter 9

9.2]

9.2 The trigonometric identity $\sin x \times \sin y = \frac{1}{2}[\cos(|x - y|) - \cos(x + y)]$ shows that the frequency spectrum of two multiplied tones contains another two tones (and not the original ones): a low-frequency tone with frequency $f_{LF} = |f_1 - f_2|$ and a high-frequency tone $f_{HF} = f_1 + f_2$.

In order to find the frequency of the unknown signals that, after multiplication with a 10 MHz tone produces a 1 kHz signal, we need to look at two possible differences, i.e., $10.001 \text{ MHz} - 10.000 \text{ MHz} = 1 \text{ kHz}$ and $|9.999 \text{ MHz} - 10.000 \text{ MHz}| = 1 \text{ kHz}$. Hence, multiplication of a 10 MHz tone with these two tones, i.e., 9.999 MHz and 10.001 MHz, results in two overlapping 1 kHz LF tones in the output frequency spectrum. (Note: the HF tones in the output spectrum are not identical: one is at 20.001 MHz and the other is at 19.999 MHz—their difference is double the LF tone.)

9.3]

9.3

- (a) The received signal and LO frequencies are mixed at the receiver's mixer, therefore the output is the sum and the difference of the two:

$$\text{sum: } 1435 \text{ kHz} + 980 \text{ kHz} = 2415 \text{ kHz,}$$

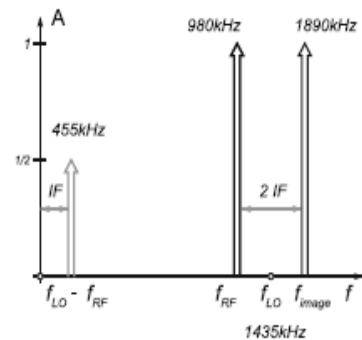
$$\text{difference: } 1435 \text{ kHz} - 980 \text{ kHz} = 455 \text{ kHz.}$$

- (b) The receiver is supposed to down-convert the incoming waveform, so IF is $f_{IF} = 455 \text{ kHz}$.
 (c) Working backwards from the mixer output, it is straightforward to find the frequencies with the same f_{IF} for the given local oscillator f_{LO} :

$$\text{sum: } 1435 \text{ kHz} + 455 \text{ kHz} = 1890 \text{ kHz,}$$

$$\text{difference: } 1435 \text{ kHz} - 455 \text{ kHz} = 980 \text{ kHz.}$$

Fig. 7 Graph for Solution 9.3



In other words, a station operating at 1890 kHz will result in the same IF as the wanted station operating at 980 kHz. It would not be possible to separate the two, which means that the other frequency is the image of the wanted frequency.

A practical solution to this problem is to simultaneously tune the tuned RF amplifier at the receiver's input to the same frequency as the wanted frequency.

- (d) A graphical representation for the case in which $f_{LO} > f_{RF}$ is shown in Fig. 7.

Exercise set 1 – Solutions

Taken from Sobot book, chapter 9

9.4]

9.4 The formula for relative amplitudes A_r of LC tank frequencies is:

$$A_r = \frac{1}{\sqrt{1 + Q^2 \left(\frac{f}{f_0} - \frac{f_0}{f} \right)^2}}.$$

Therefore, for $Q = 20$, $f/f_0 = 1.1/1 = 1.1$ and $f_0/f = 1/1.1 = 0.9090\dots$. We find that $A_r = 0.253 = -11.93$ dB. For comparison, if $Q = 200$, then $A_r = -31.64$ dB.

Exercise set 2 – LC networks

Taken from Beasley-Miller book, chapter 1

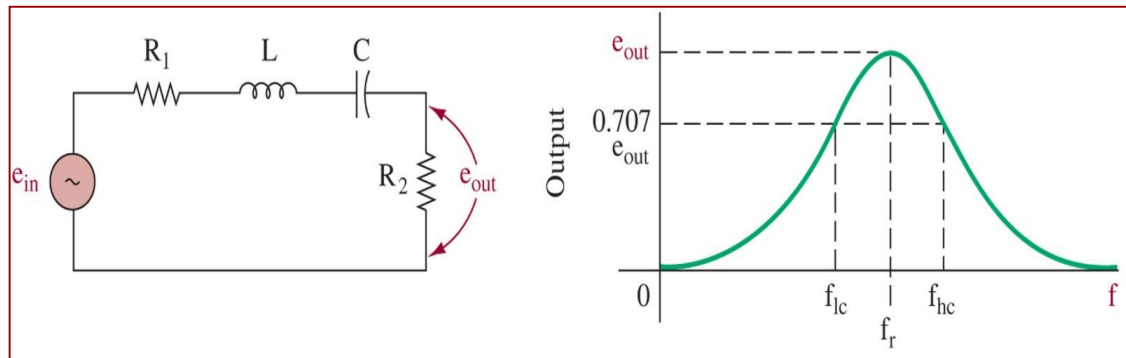
50] Calculate an inductor Q at 100 MHz if the inductance is 6mH and the series resistance 1.2k Ω . Determine its dissipation factor D as well.

51] Calculate a capacitor Q at 100 MHz if the capacitance is 0.001 μ F and the leakage resistance is 0.7M Ω . Again determine also the dissipation factor D .

52] What would be the impedance at 100 MHz of the inductor and capacitor at point 50-51 if they are placed in series? What would be their resonant frequency? And what would be the impedance at the resonant frequency?

56] An FM radio receiver uses an LC bandpass filter with resonant frequency equal to 10.7 MHz and requires 200 kHz of bandwidth BW. What would be the required Q of the filter?

57] If the circuit at question 56 is used as in the figure below, and $C=0.1$ nF, calculate the required inductor value L and the value of R_1 (ignore the load R_2).



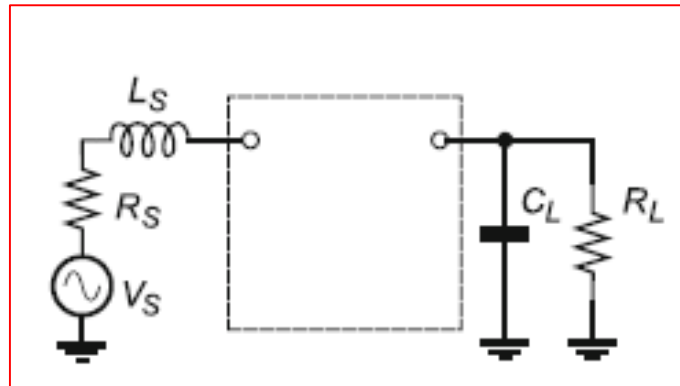
Exercise set 2 – LC networks

Taken from Sobot book, chapter 5-6

5.8] Calculate the resonant frequency of a serial RLC network with $R=30\Omega$, $L=3\text{mH}$, and $C=100\text{nF}$. Calculate its impedance at $f=10\text{ kHz}$ and $f=5\text{kHz}$.

5.11] A serial RC branch consists of $R_s=10\Omega$ and $C_s=7.95\text{pF}$. Convert it into its equivalent parallel network form at $f=1\text{GHz}$.

Example 6.3] Design a single stage LC matching network at $f=100\text{MHz}$ for a source V_s presenting an impedance made of the series of $R=5\Omega$ and $L_s=13.85\text{nH}$, and driving a load resistance $R_L=50\Omega$ with $C_L=45.49\text{nF}$ in parallel. Note that the matching network must preserve a DC path between source and load.



Exercise set 2 - Solutions

50]

$$f=100\text{MHz} \quad L=6\text{mH} \quad R=1.2\text{k}$$

$$Q = \frac{\omega L}{R} = \frac{2\pi(100\text{MHz})(6 \times 10^{-3})}{1.2 \times 10^3} = 3.14 \times 10^3$$

$$D = \frac{1}{Q} = 0.318 \times 10^{-3}$$

51]

$$f=100\text{MHz} \quad C=.001\mu\text{F} \quad R=.7 \times 10^6$$

$$Q = \frac{\omega C}{G} = \frac{2\pi(100 \times 10^6)}{\frac{1}{.7 \times 10^{-6}}} = 4.398 \times 10^5 \quad D = \frac{1}{Q} = 2.27 \times 10^{-6}$$

52]

Z of inductor is its R plus its reactance

$$Z_L = 1200 + j\omega L = 1200 + j37.7 \times 10^5 \Omega$$

Z of capacitor is its R plus its reactance

$$Z_C = 0.7 \times 10^6 - j1.592 \Omega$$

Total Z is the sum of those above (series impedance); note that the book gives you an approximation ignoring the smaller component (if the reactance is very large with respect to the resistance, ignore the resistance) -> best to keep both of them, the full complex number

Resonant frequency is the usual formula; the impedance at resonance is the only resistive part (of the inductor, as it is a series circuit)

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi(6 \times 10^{-3} \times 0.001 \times 10^{-6})^{1/2}} = 65\text{kHz} \quad \text{At } f_r, Z = R_{ind} = 1200 \Omega$$

56-57]

$$Q = \frac{f_r}{BW} = \frac{10.7 \times 10^6}{200 \times 10^3} = 53.5$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = 10.7 \times 10^6 = \frac{1}{2\pi(L \times 0.1 \times 10^{-9})^{1/2}}$$

$$\therefore L = 2.21\mu\text{H}$$

$$Q = \frac{X_L}{R} = \frac{2\pi \times 10.7 \times 10^6 \times 2.21 \times 10^{-6}}{R} = 53.5$$

↑ from Prob.51

$$R = \frac{2\pi \times 10.7 \times 10^6 \times 2.21 \times 10^{-6}}{53.5} = 2.78 \Omega$$

Note the formulae from the RLC series

Exercise set 2 - Solutions

5.8]

5.8 At $f = 10 \text{ kHz}$, we have,

$$X_L = 2\pi 10 \text{ kHz } 3 \text{ mH} = 188.5 \Omega,$$

$$X_C = \frac{1}{2\pi 10 \text{ kHz } 100 \text{ nF}} = 159.2 \Omega,$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{30^2 + (188.5 - 159.2)^2} \Omega = 41.9 \Omega.$$

We note that the serial RLC circuit looks more inductive at 10 kHz.

At $f = 5 \text{ kHz}$, we have,

$$X_L = 2\pi 5 \text{ kHz } 3 \text{ mH} = 94.2 \Omega,$$

$$X_C = \frac{1}{2\pi 5 \text{ kHz } 100 \text{ nF}} = 318.3 \Omega,$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{30^2 + (94.2 - 318.3)^2} \Omega = 226.1 \Omega.$$

We note that the serial RLC circuit looks more capacitive at 5 kHz.

5.11]

5.11 The two networks must have the same Q factor, which is found by definition as

$$Q_S = \frac{X_S}{R_S} = \frac{1}{2\pi C_S R_S} = \frac{1}{2\pi 7.95 \text{ pF } 10 \Omega} = 2.$$

Using (5.77) and (5.78), we write

$$R_p = R_s(1 + Q^2) = 10 \Omega(1 + 2^2) = 50 \Omega,$$

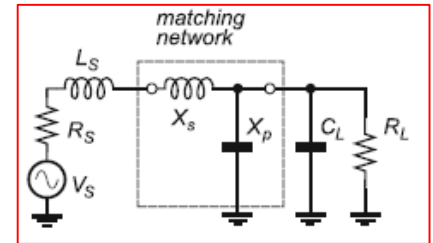
$$X_p = X_s \left(1 + \frac{1}{Q^2}\right) = \frac{1}{2\pi 1 \text{ GHz } 7.95 \text{ pF}} \left(1 + \frac{1}{2^2}\right) = 25.024 \Omega \quad \therefore \quad C_p = 6.36 \text{ pF}.$$

Exercise set 2 - Solutions

6.3] As the network has to preserve DC connection between source and load, you need an inductor in series (and C in parallel), vice versa not acceptable.

First always match the real parts of the impedances.

As the source impedance is 5Ω and the load 50Ω , you have already solved this in slide 15 when discussing matching networks -> the results were a series inductor of $\sim 23.85\text{nH}$ and parallel capacitor of $\sim 95.49\text{nF}$



However, in this case the source has already an inductance L_S in series equal to 13.85nH , so in order to achieve $\sim 23.85\text{nH}$ you only need an additional 10nH for your X_s . So $X_s = 10\text{nH}$.

Similarly for the capacitor, you need $\sim 95.49\text{nF}$. However there is already a parallel capacitor $C_L = 45.49\text{nF}$, so an additional 50nF capacitor is needed. So $X_p = 50\text{nF}$.

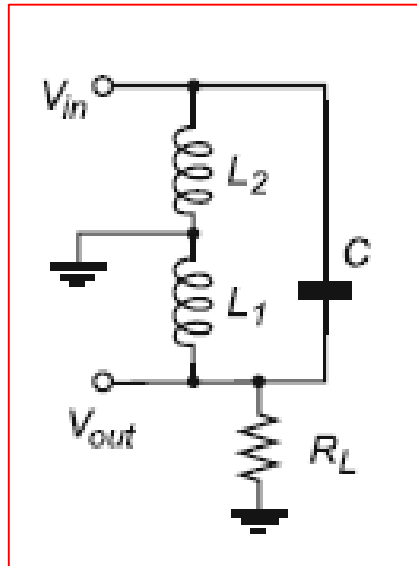
Exercise set 3 – oscillators

Taken from Sobot book, chapter 8

8.3-5] Estimate the resonant frequency ω_0 of an oscillator whose feedback network is shown below. Note that $L_1=0.5\mu\text{H}$, $L_2=1.5\mu\text{H}$, and $C=126.65\text{pF}$

Using the same data generated above, estimate the feedback network gain β .

Finally using the same data above, estimate the effective resistance R_{eff} that this feedback network would present to the output of the amplifier part of the oscillator. Assume that the input impedance of the amplifier is $R_{\text{in}}=10\text{k}\Omega$ and the Q factor of the inductor is $Q=50$.

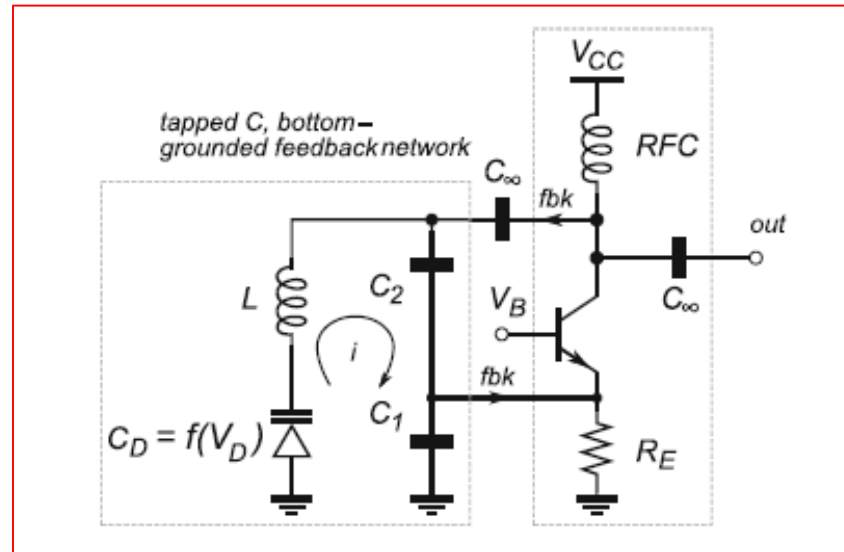


Exercise set 3 – oscillators

Taken from Sobot book, chapter 8

8.8] For the Clapp oscillator shown below, calculate the frequency of oscillation when the bias on the varicap diode is 0V and when this is chosen to be $V_D = -7V$.

$L = 100\mu H$ – $C_1 = C_2 = 300pF$ – $C_0 = 20pF$ (varicap diode at no bias applied)



Exercise set 3 – solutions

8.3-5]

8.3 The circulating resonant current i_c in a tapped L, centre-grounded network perceives the L_1 , L_2 , and C components in series, therefore $L_{\text{eff}} = L_1 + L_2 = 2\mu\text{H}$. The resonant frequency is calculated as

$$f_0 = \frac{1}{2\pi\sqrt{L_{\text{eff}}C}} = 10\text{MHz}.$$

8.4 By inspection, the input voltage is distributed across the L_2 inductor, while the output voltage is distributed across the L_1 inductor. Since the same resonating current i_c is circulating through both components, it is straightforward to write

$$\begin{aligned} v_{\text{in}} &= i_c j\omega L_2, & \beta &= \frac{v_{\text{out}}}{v_{\text{in}}} = -\frac{i_c j\omega L_1}{i_c j\omega L_2} = -\frac{L_1}{L_2} = -\frac{0.5\mu\text{H}}{1.5\mu\text{H}} = -0.333. \\ v_{\text{out}} &= -i_c j\omega L_1, \end{aligned}$$

8.5 Using the formula provided in the textbook and knowing that the R_L resistor of the feedback network (Fig. 8.5) is in fact the input resistance of the amplifier, a straightforward implementation of the formula yields

$$\begin{aligned} R_{\text{eff}} &= R_L \left(\frac{L_2}{L_1} \right)^2 \parallel \frac{Q\omega_0 L_2^2}{L_1 + L_2} \\ &= 10\text{k}\Omega \left(\frac{1.5\mu\text{H}}{0.5\mu\text{H}} \right)^2 \parallel \frac{50 \times 2\pi \times 10\text{MHz} \times (1.5\mu\text{H})^2}{2\mu\text{H}} \\ &= 90\text{k}\Omega \parallel 3.534\text{k}\Omega = 3.4\text{k}\Omega. \end{aligned} \tag{48}$$

Once the correct type of feedback network is identified, it is pretty much a straightforward application of the formulae studied in the lectures.

Exercise set 3 – solutions

8.8] Remember the formula of the varicap diode capacitance as a function of bias voltage V_D . For example at -7V

$$C_D = \frac{C_0}{\sqrt{1 + \frac{|V_D|}{0.5}}} = \frac{20 \text{ pF}}{\sqrt{1 + \frac{|-0.7|}{0.5}}} = 5.16 \text{ pF}$$

From there the following analysis can be derived

- (a) The three capacitances, C_1 , C_2 , and C_D are perceived by the resonating loop as being in series, hence the total loop capacitance C at zero bias is,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_0} = 0.05671/\text{pF} \quad \therefore \quad C = 17.65 \text{ pF},$$

which means that the zero biasing frequency is

$$f_0 = \frac{1}{2\pi\sqrt{17.65 \text{ pF} \times 100 \mu\text{H}}} \approx 3.789 \text{ MHz}.$$

- (b) At $V_D = -7\text{V}$, the total loop capacitance is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_D} \quad \therefore \quad C = 4.998 \text{ pF},$$

which means that the zero biasing frequency is

$$f_0 = \frac{1}{2\pi\sqrt{4.998 \text{ pF} \times 100 \mu\text{H}}} \approx 7.126 \text{ MHz}.$$

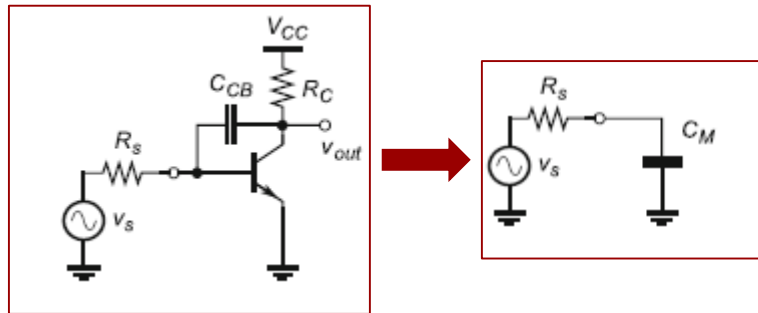
Exercise set 4 – amplifiers

Inspired from Sobot book, chapter 7.6

A] Consider an ideal single-stage CE amplifier with infinite input resistance and voltage gain of 49 (its absolute value). The amplifier is driven by a voltage source with output resistance $R_s = 75\Omega$. An additional capacitance is present between base and collector of the BJT, $C_{cb} = 2\text{pF}$.

Estimate the useful range of input frequencies for the amplifier without considering Miller capacitance effect, and then taking this into account.

Tip: recheck the theory for Miller capacitance on the slides -> the amplifier behaves as an RC low-pass filter whose C includes Miller capacitance effect. The range of useful frequencies is given by the cut-off of the filter



B] Explain the meaning of the VSWR and 1dB compression point parameters for an amplifier

Exercise set 4 – solutions

A] If Miller capacitance is neglected, then the cut-off frequency is

$$f_c = \frac{1}{2\pi R_s C_{CB}} \cong 1.06 \text{ GHz}$$

However, if we count Miller capacitance -> $C_m = C_{cb}(A_v + 1) = 100 \text{ pF}$

So the new cut-off is

$$f_c = \frac{1}{2\pi R_s C_M} \cong 21.22 \text{ MHz}$$

Bear in mind that a typical exam question at this stage might ask you also to
-comment, describe the effect of Miller capacitance (reduction of bandwidth...)
-evaluate the effect on the bandwidth of possible tolerances on the source resistance R_s (± 5 or 10%).

B] This is explained in the slides (slide 36 and 37) in details. *This question expects you to explain the meaning of 2 parameters, and you can also give max/min ranges for VSWR and a graph to explain the 1dB compression point.*