Chapter 10
FIR Digital Filter Design

Zhiliang LIU

Zhiliang_Liu@uestc.edu.cn

5/17/2019

10.1 Preliminary Considerations

10.1.1 Basic Approaches to FIR Filter Design

- Unlike IIR digital filter design, FIR filter design have no connection with the design of analog filters.
- The design of FIR filters is based on a direct approximation of the specified magnitude response, with added requirement that the phase response be linear.

10.1.1 Basic Approaches to FIR Filter Design

The real-coefficient FIR filters are described by a transfer function that is a polynomial in z^{-1}.

Always guaranteed stable!

Two Basic Approaches:

The first method is based on *Windowed Fourier Series*.
The second method is *Frequency Sampling Approach* (based on inversed DFT to the frequency samples).

10.2 FIR Filter Design Based on Windowed Fourier Series

10.2.1 Least Integral-Squared Error Design of FIR Filters

Let $H_d(e^{j\omega})$ denote the desired frequency response

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d[n]e^{-j\omega n} \cdots (10.7)$$

where

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega, \quad -\infty \le n \le \infty \cdots (10.8)$$

- In general, $H_d(e^{j\omega})$ is piecewise constant with sharp transitions between bands, in which case, $\{h_d[n]\}$ is of infinite length and noncausal.

- <u>**Objective**</u>

  Find a finite-duration $\{h_t[n]\}$ of length $2M+1$ whose DTFT $H_t(e^{j\omega})$ approximates the desired DTFT $H_d(e^{j\omega})$ <u>*in some sense.*</u>

$$H_d(e^{j\omega}) \xrightarrow{IDTFT} h_d[n] \xrightarrow{Truncation} h_t[n] = h_d[n]w_R[n] \xrightarrow{DTFT} H_t(e^{j\omega})$$

- **Commonly used approximation criterion: minimize the integral-squared error**

$$\Phi = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| H_t(e^{j\omega}) - H_d(e^{j\omega}) \right|^2 d\omega \cdots (10.9)$$

where

$$H_t(e^{j\omega}) = \sum_{n=-M}^{M} h_t[n] e^{-j\omega n} \cdots (10.10)$$

- **Using Parseval’s relation we can write**

$$\Phi = \sum_{n=-\infty}^{\infty} \left| h_t[n] - h_d[n] \right|^2$$

$$= \sum_{n=-M}^{M} \left| h_t[n] - h_d[n] \right|^2 + \sum_{n=-\infty}^{-M-1} h_d^2[n] + \sum_{n=M+1}^{\infty} h_d^2[n]$$

It follows from the above that Φ is minimum when $h_t[n] = h_d[n]$ for $-M \le n \le M$

$\Rightarrow$ Best finite-length approximation to ideal infinite-length impulse response in the mean-square sense is obtained by *truncation*

$$h_t[n] = h_d[n] \cdot w_R[n]$$

$$w_R[n] = \begin{cases} 1, & 0 \le |n| \le M \\ 0, & \text{otherwise} \end{cases}$$

Rectangular Window

- **A causal FIR filter with an impulse response *h*[*n*] can be derived from $h_t[n]$ by delaying:**

$$h[n] = h_t[n - M]$$

- **FIR filter *h*[*n*]---linear phase**

  $h_t[n]$---zero phase

- **They have the same magnitude response.**

$$H(e^{j\omega}) = H_t(e^{j\omega})e^{-j\omega M}$$

10.2.2 Gibbs Phenomenon (10.2.3)

- **Truncating operation → windowing operation:**

$$h_t[n] = h_d[n] \cdot w[n]$$

Periodic continuous convolution

In the frequency domain

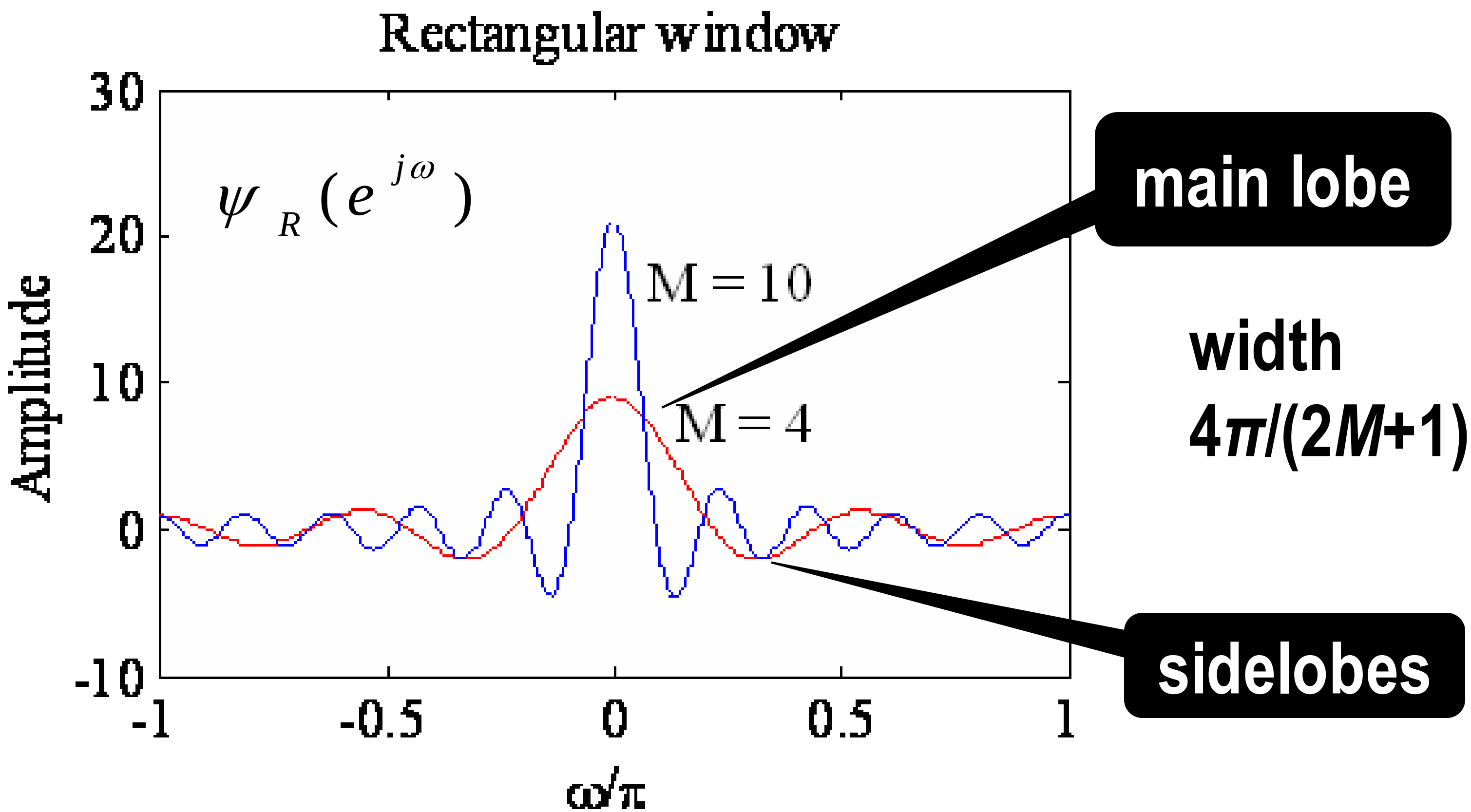
$$H_t(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\varphi}) \Psi(e^{j(\omega-\varphi)}) \, d\varphi$$

Where $H_t(e^{j\omega})$ and $\psi(e^{j\omega})$ are the DTFTs of $h_t[n]$ and $w[n]$, respectively

- **As an example, the LPF design is studied**
- **A rectangular window is used to achieve simple truncation:**

$$w_R[n] = \begin{cases} 1, & 0 \le |n| \le M \\ 0, & \text{otherwise} \end{cases}$$

$$\psi_R(e^{j\omega}) = \frac{\sin(\omega(2M+1)/2)}{\sin(\omega/2)}$$

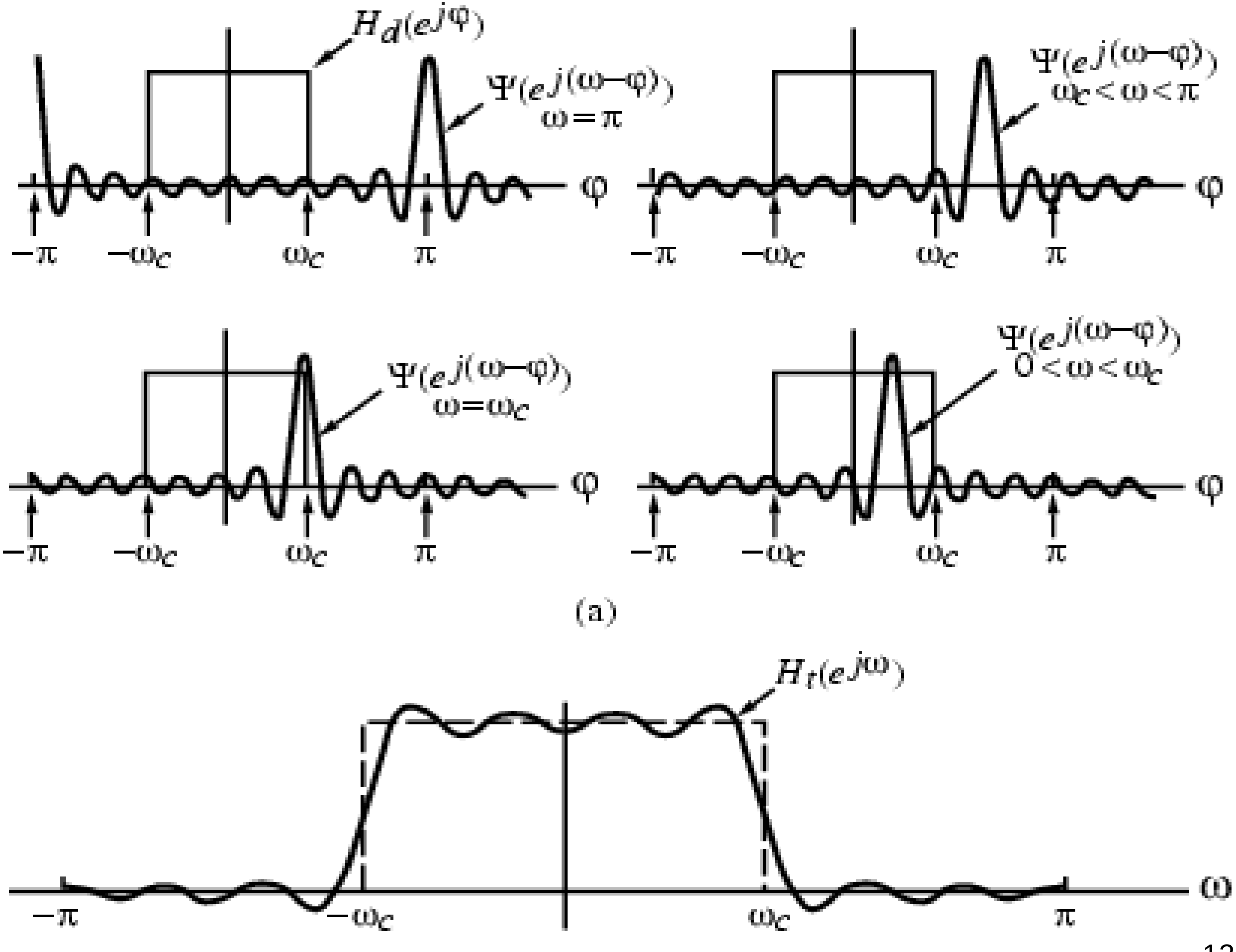


$\uparrow M \rightarrow$ width of main lobe $\downarrow$

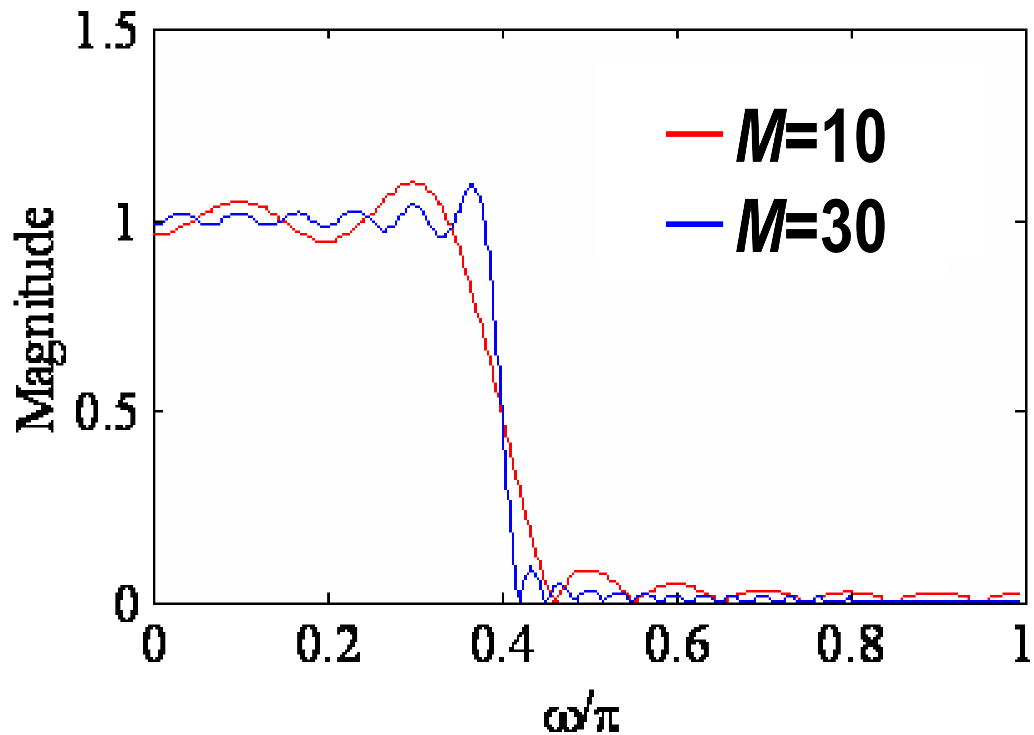
Area under each lobe remains constant while width of each lobe decreases with an increase in *M*.

- **Thus $H_t(e^{j\omega})$ is obtained by a periodic continuous convolution of $H_d(e^{j\omega})$ with $\psi(e^{j\omega})$.**

- **The LPF amplitude response will have oscillatory behavior shown in next slide.**


$H_d(e^{j\varphi})$
$\Psi(e^{j(\omega-\varphi)})$
$\omega=\pi$
φ
$-\pi$
$-\omega_c$
ω_c
π
$\Psi(e^{j(\omega-\varphi)})$
$\omega_c<\omega<\pi$
$\Psi(e^{j(\omega-\varphi)})$
$\omega=\omega_c$
$\Psi(e^{j(\omega-\varphi)})$
$0<\omega<\omega_c$
(a)
$H_t(e^{j\omega})$
ω
$-\pi$
$-\omega_c$
ω_c
π

(b)



oscillatory behavior

- **Observation:**
- ↑M (length of the lowpass filter) → ↑ number of ripples (both passband and stopband) with corresponding ripple widths↓
- Height of the largest ripples remain the same independent of length

- **Ripples in $H_t(e^{j\omega})$ around the point of discontinuity occur more closely but with no decrease in amplitude as M increases**

---Gibbs phenomenon

- **Gibbs phenomenon - Oscillatory behavior in the magnitude responses of causal FIR LPFs obtained by truncating the impulse response coefficients of ideal filter.**

- **Why Gibbs phenomenon?**

Rectangular window has an abrupt transition to zero outside the range $-M \le n \le M$, which results in Gibbs phenomenon in $H_t(e^{j\omega})$.

- **Gibbs phenomenon can be reduced :**

(1) $w[n]$ or $\psi(e^{j\omega})$: Using a window that tapers smoothly to zero at each end, or

(2) $h_d[n]$ or $H_d(e^{j\omega})$: Providing a smooth transition from passband to stopband in the magnitude specifications

- **Similar oscillatory behavior observed in the magnitude responses of the truncated versions of other types of ideal filters (HP, BP, BS).**

10.2.3 Fixed Window Functions(10.2.4)

- **Using a tapered window causes the height of the sidelobes to diminish, with a increased main lobe width resulting in a wider transition at the discontinuity**
- **Rectangular:**

$$w[n] = 1, -M \le n \le M$$

- **Hann:**

$$w[n] = 0.5 + 0.5\cos[2\pi n/(2M+1)], -M \le n \le M$$

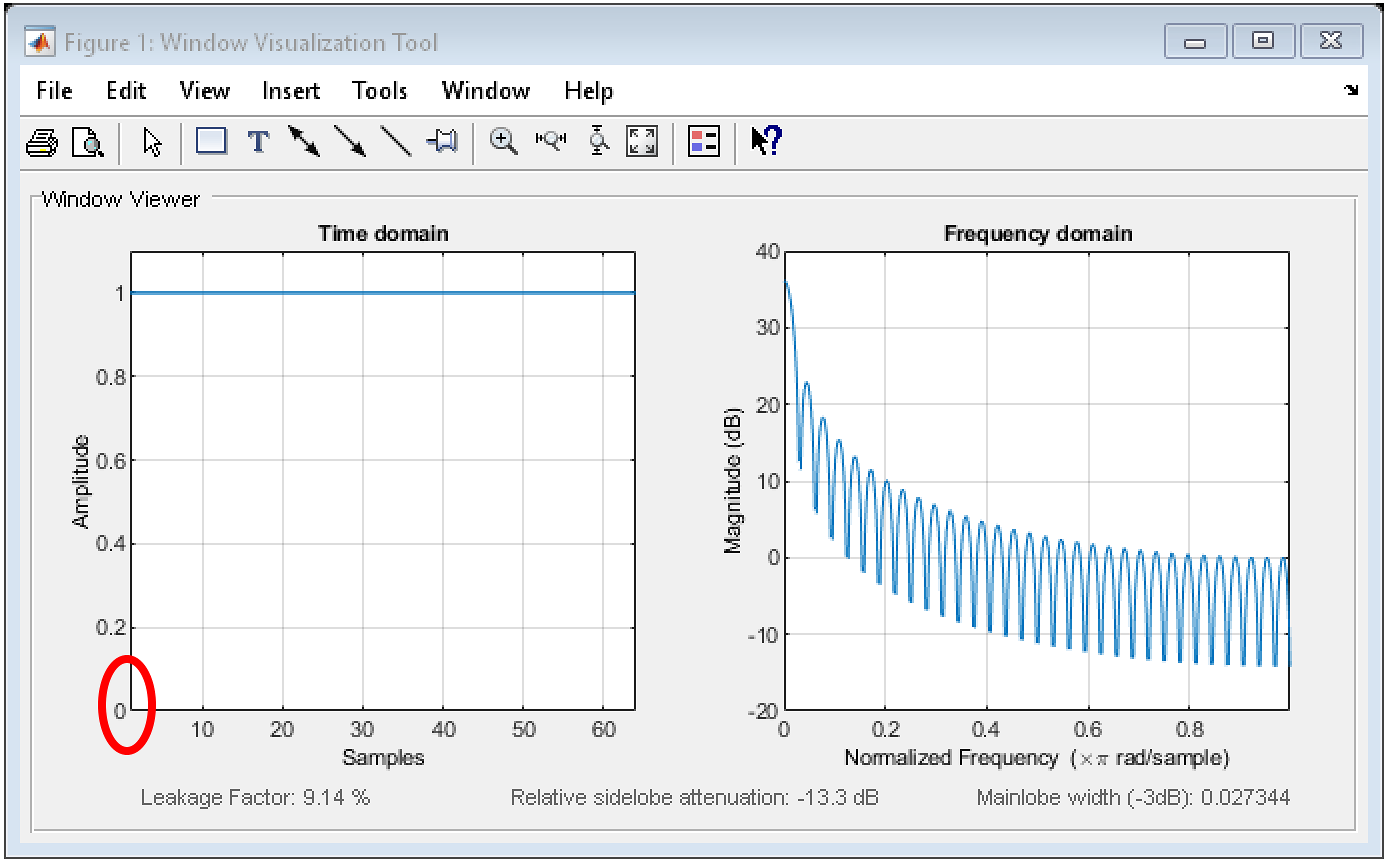
- **Hamming:**

$$w[n] = 0.54 + 0.46\cos[2\pi n/(2M+1)], -M \le n \le M$$

- **Blackman:**

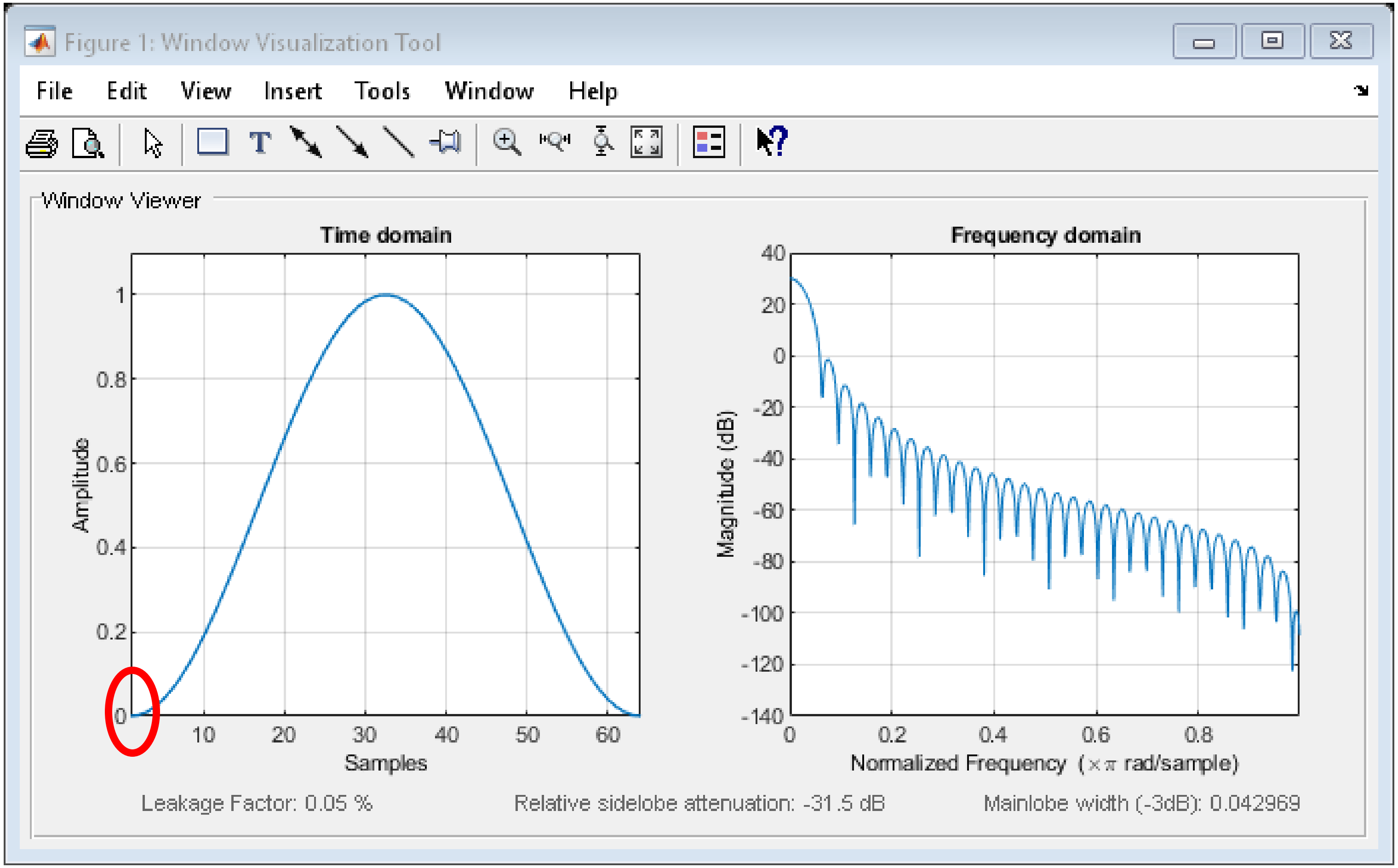
$$w[n] = 0.42 + 0.5\cos[2\pi n/(2M+1)] + 0.08\cos[4\pi n/(2M+1)]$$

Rectangular Window



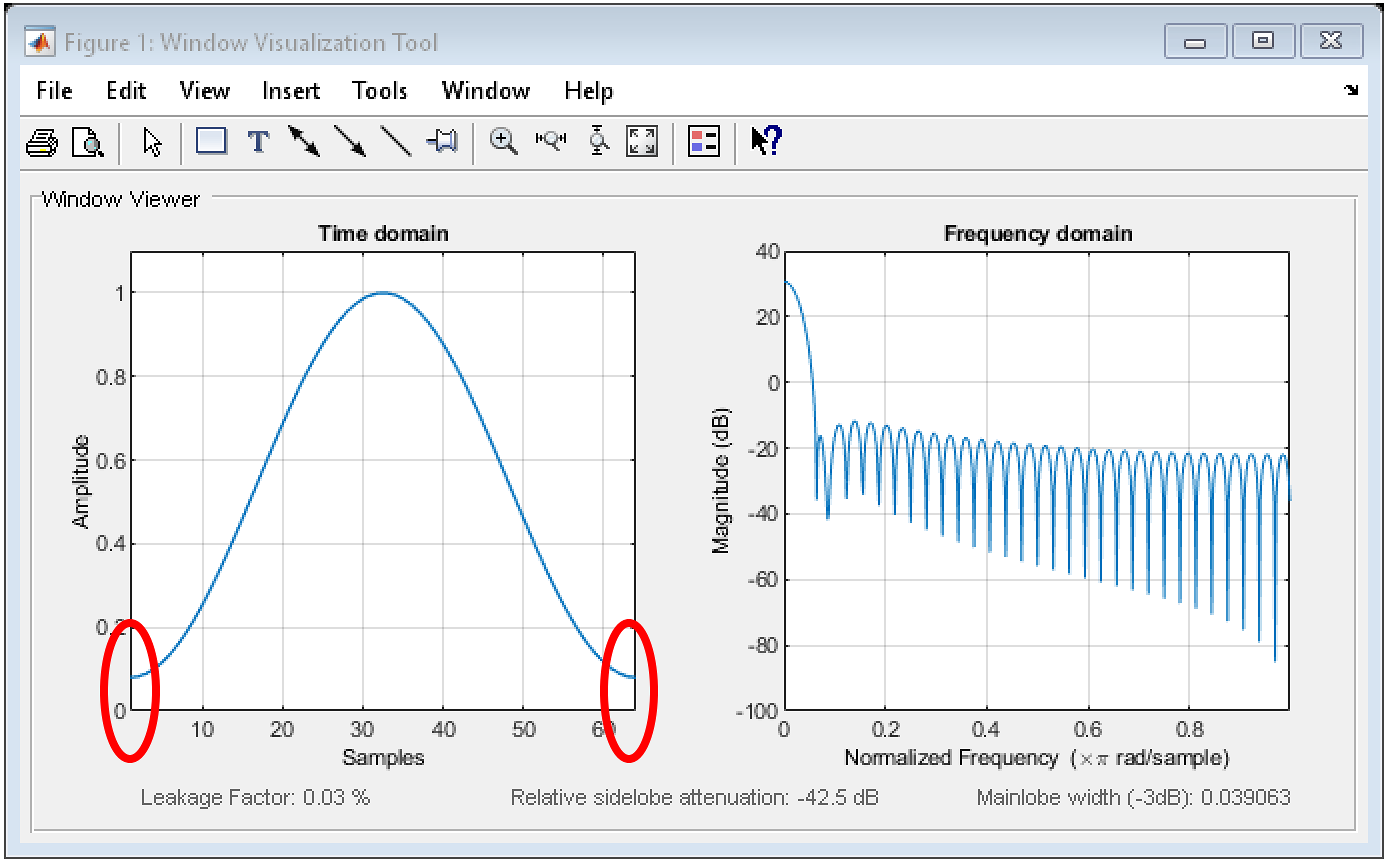
Hanning Window

$$w[n] = 0.5 + 0.5\cos[2\pi n/(2M+1)], \quad -M \le n \le M$$



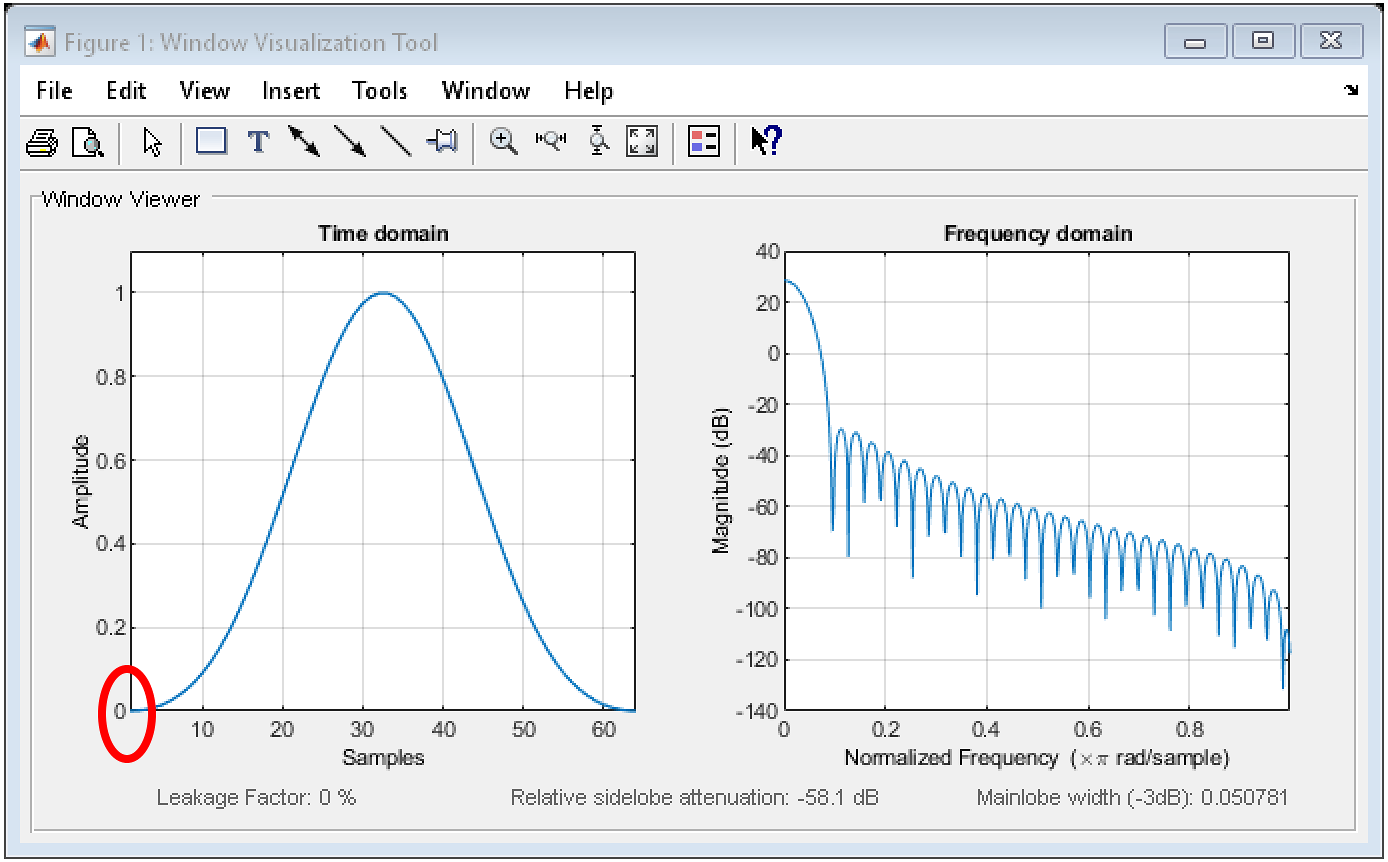
Hamming Window

$$w[n] = 0.54 + 0.46\cos[2\pi n/(2M+1)],\ -M \le n \le M$$

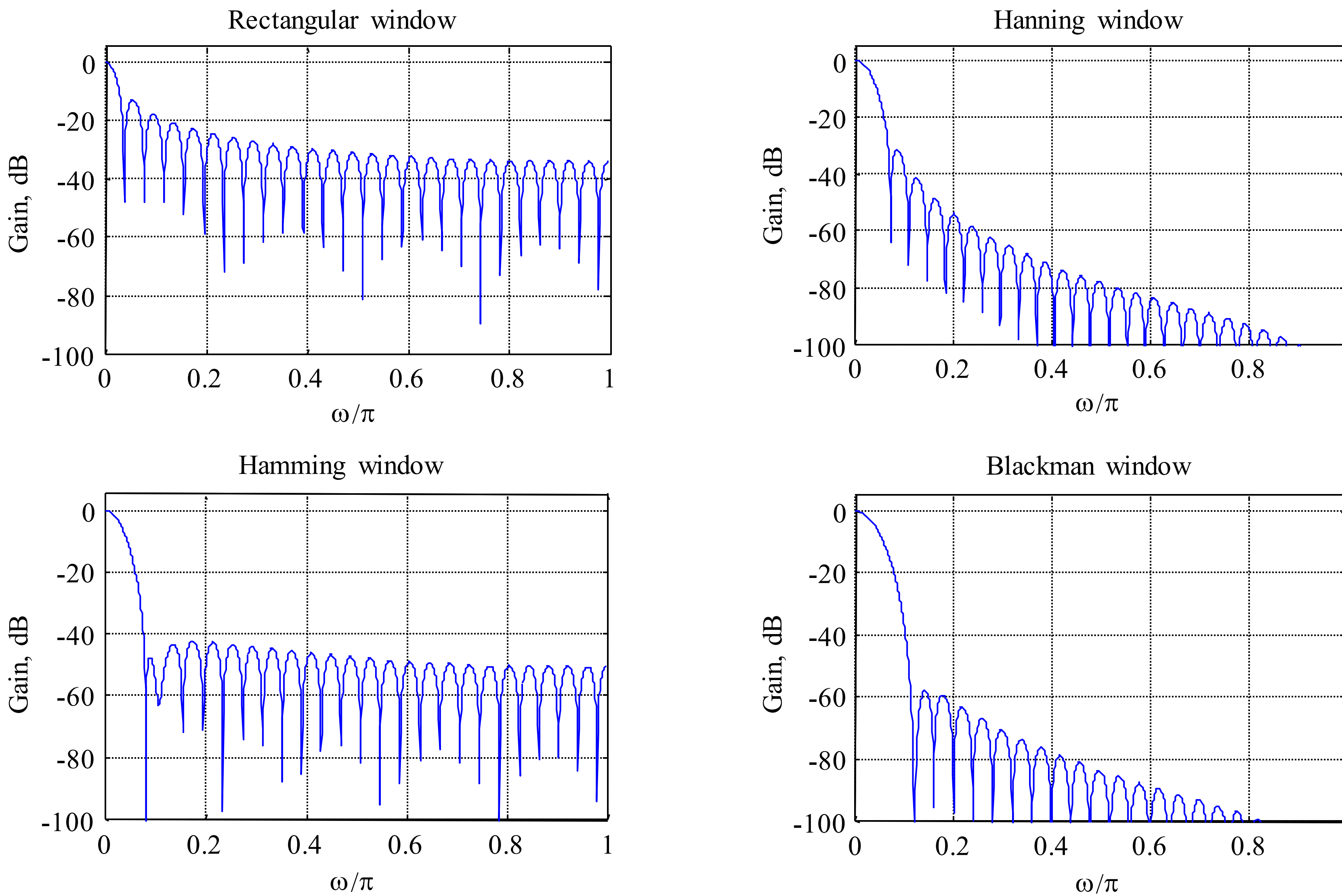


Blackman Window

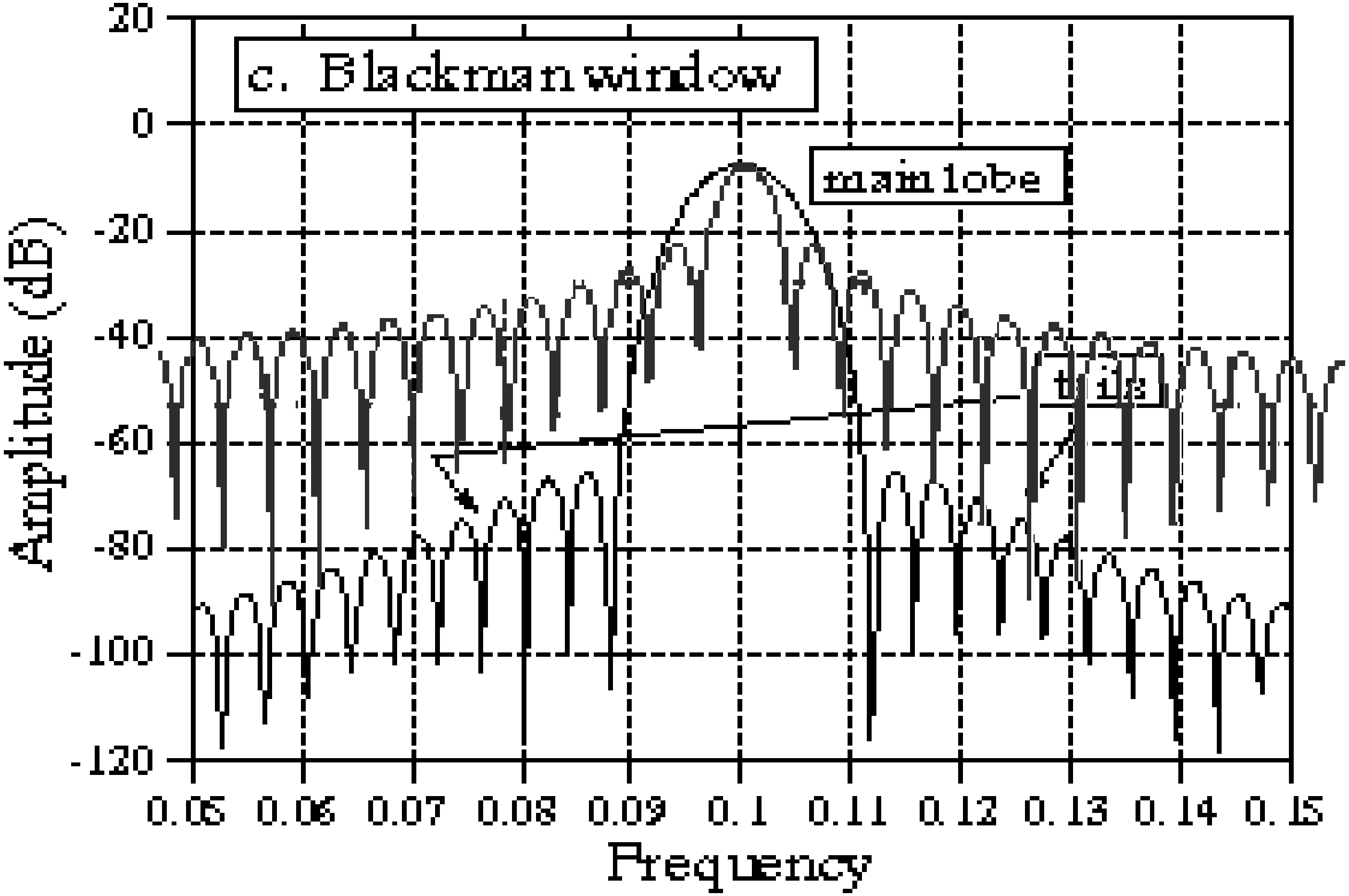
$$w[n] = 0.42 + 0.5\cos[2\pi n/(2M{+}1)] + 0.08\cos[4\pi n/(2M{+}1)], \; -M \le n \le M$$



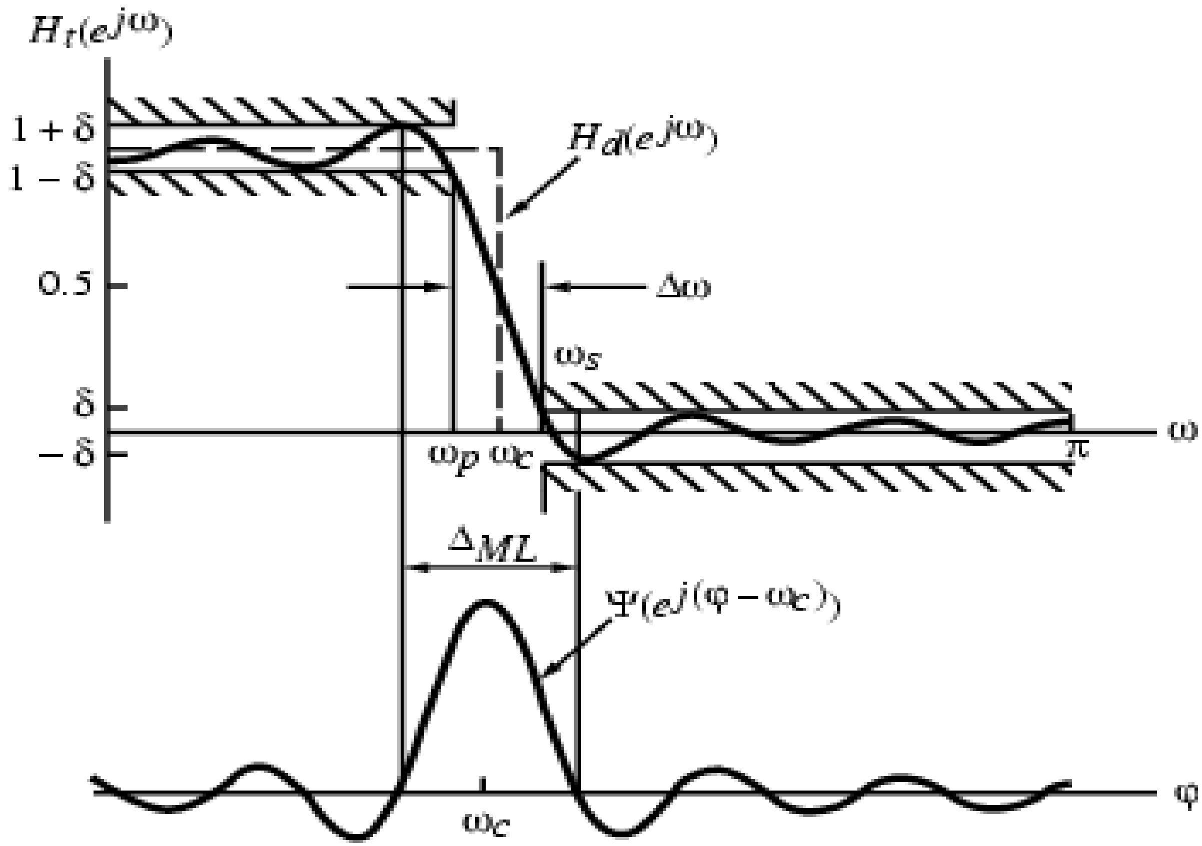
- **Plots of magnitudes of the DTFTs of these windows for *M* = 25 are shown below**



Rectangular and Blackman windows



Performance of a window function in LPF design



Observe

$$H_t(e^{j(\omega_c+\Delta\omega)}) + H_t(e^{j(\omega_c-\Delta\omega)}) \cong 1$$

$$H_t(e^{j\omega_c}) \cong 0.5$$

- **Distance between the locations of the maximum passband deviation and minimum stopband value $\cong \Delta_{ML}$**
- **Width of transition band**

$$\Delta\,\omega = \omega_s - \omega_p < \Delta_{ML}$$

In addition,

$$\Delta\,\omega \approx c\,/\,M$$

where *c* is a constant for most practical purposes

- **To ensure a fast transition from passband to stopband, window should have a very small main lobe width---↑*M***
- **To reduce the passband and stopband ripple, the area under the sidelobes should be very small---↓*M* (very small *M*)**
- **Unfortunately, these two requirements are contradictory.**

We have to trade-off between the specifications. As a result Table 10.2 is shown in next slide.

Table 10.2 Properties of some fixed window functions

Correlated

Type of windows	Main lobe width Δ_{ML}	Relative sidelobe level A_{sl}	Minimum stopband attenuation	Transition bandwidth $\Delta\omega$
Rectangular	$4\pi/(2M+1)$	13.3dB	20.9dB	$0.92\pi/M$
Hann	$8\pi/(2M+1)$	31.5dB	43.9dB	$3.11\pi/M$
Hamming	$8\pi/(2M+1)$	42.7dB	54.5dB	$3.32\pi/M$
Blackman	$12\pi/(2M+1)$	75.3dB	75.3dB	$5.56\pi/M$

Correlated

10.2.4 Adjustable Window Functions (10.2.5)

There are two adjustable window functions.

One of them is *Dolph-Chebyshev* window (Eqs. (10.34) ~ (10.38)).

Another is the most widely used *Kaiser window*.

Kaiser window:

$$w[n] = \frac{I_0\left(\beta\sqrt{1-\left[\frac{n}{M}\right]^2}\right)}{I_0(\beta)}, \quad -M \le n \le M$$

where,

$$I_0(u) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{ju\sin y}\,dy \approx 1 + \sum_{k=1}^{\infty}\left[\frac{\left(\frac{u}{2}\right)^k}{k!}\right]^2$$

(zero-order Bessel function)

We can adjust β to modify the window function. So, it is called as *adjustable window function*. Eqs. (10.41) and (10.42) give how to get β and N.

10.2.5 Estimation of the Filter Order(10.1.2)

For the design of FIR LPF, the order *N* is directly estimated from the digital filter specification. Several authors have advanced formulas. For example,

Kaiser's formula:

$$N \cong \frac{-20\log_{10}(\sqrt{\delta_P \delta_S}) - 13}{14.6(\omega_S - \omega_P)/(2\pi)} \qquad \text{.....(10.3)}$$

Bellanger's formula:

$$N \cong \frac{2\log_{10}(10\delta_P \delta_S)}{3(\omega_S - \omega_P)/(2\pi)} - 1 \qquad \text{.....(10.4)}$$

Hermann's formula:

$$N \cong \frac{D_\infty(\delta_P,\delta_S) - F(\delta_P,\delta_S)[(\omega_S-\omega_P)/2\pi]^2}{(\omega_S-\omega_P)/(2\pi)} \quad \text{.....(10.5)}$$

Read P462~465 and understand the situations of these formulas.

Table 10.1 Comparison of FIR filter orders

Filter No.	Actual Order	Kaiser's Formula	Bellanger's Formula	Hermann's Formula
1	159	158	163	151
2	38	34	37	37
3	14	12	13	12

Common points of FIR filter order formulas

(i) All above formulas are used in case $\omega_S > \omega_P$.

(ii) Interchange δ_S, δ_P, the order of FIR are same.

(iii) Ripples δ_S, δ_P are decreased, the order of FIR is increased.

(iv) Transition bandwidth $\omega_S - \omega_P$ is decreased, the order of FIR is increased.

10.2.6 Low pass FIR filter design

Design Steps for Windowed Low Pass FIR Filters

(1) Choose a pass band edge frequency in Hz:

$$f_c = (f_p + f_s)/2$$

(2) Calculate $\omega_c = 2\pi f_c / f_T$, the infinite impulse response for an ideal low pass filter:

$$h_d[n] = \sin(n\omega_c)/n\pi$$

(3) Choose a window based on the specifications (Table 10.2).

(4) Calculate FIR from $h_t[n] = h_d[n]w[n]$, notice it is noncausal.

(5) Shift $h_t[n]$ to the right by M to make the filter causal

$$h[n] = h_t[n - M]$$

The performance of designed filter should be testified!

FIR Filter Design Example

The specifications of a low pass filter:

Pass band edge	f_p	2kHz
Stop band edge	f_s	3kHz
Minimum stop band attenuation		40dB
Sampling frequency	f_T	10kHz

Design

(1) Transition width $\Delta f = 3 - 2 = 1$ kHz

$$f_c = (2000+3000)/2 = 2500\text{Hz}$$

$$\omega_c = 2\pi f_c / f_T = 2\pi 2500/10000 = 0.5\pi$$

- (2) $h_d[n] = \sin(n\omega_c)/n\pi = \sin(0.5\pi n)/n\pi$
- (3) Choose Hanning window (Table 10.2, 40dB required)

Transition Width $\Delta\omega = 2\pi\Delta f / f_T = 2\pi 1000/10000 = 0.2\pi$

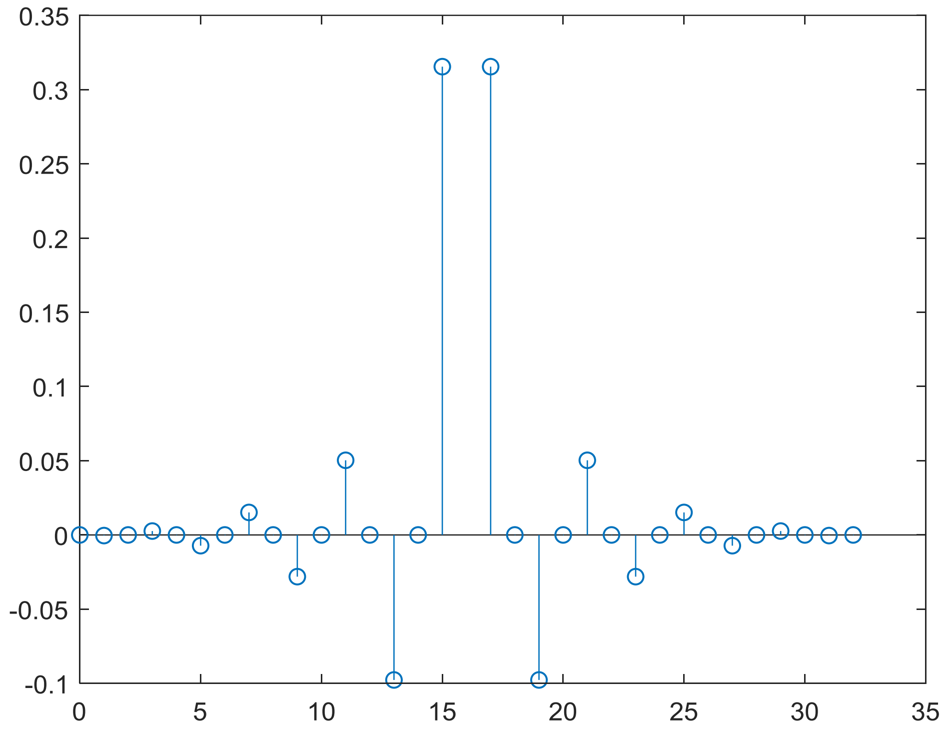
$M = 3.11\pi/\Delta\omega = 15.55$, the least $M = 16$,
choose order $N = 2M = 32$, length $N+1 = 33$

$w[n] = 0.5 + 0.5\cos(2\pi n/33)$

(4) $h_t[n] = h_d[n]w[n] = [\sin(0.5\pi n)/n]\ w[n]$,

(5) $h[n] = h_t[n-M] = h_t[n-16]$

Type I linear-phase FIR LPF

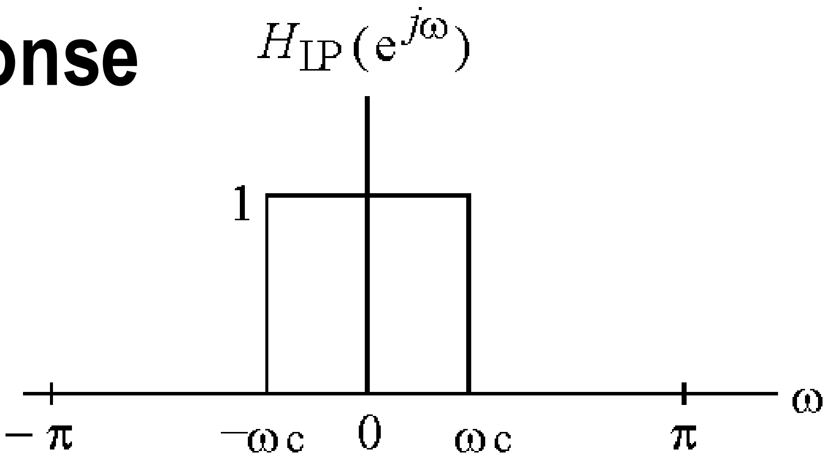


- $H(e^{j\omega p}) = -0.06$ dB at $\omega_p = 2.013$kHz
- $H(e^{j\omega s}) = -40$ dB at $\omega_s = 2.976$kHz

n	$h_t[n]$	$w[n]$	$h[n]$	New n
−16	(0. 0000)	0. 0023	(0. 0000)	0
−15	(0. 0004)	0. 0203	(0. 0212)	1
−14	0. 0000	0. 0556	0. 0000	2
−13	0. 0026	0. 1070	0. 0245	3
−12	(0. 0000)	0. 1726	(0. 0000)	4

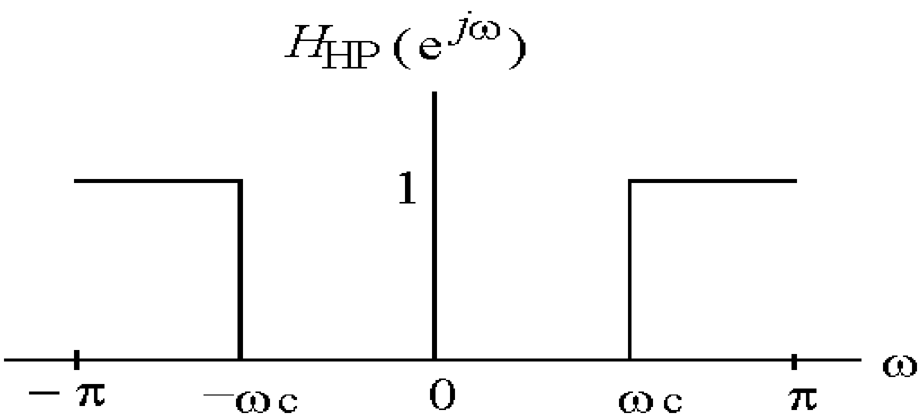
10.2.7 Impulse Response of Ideal Filters (10.2.2)

Zero-phase frequency response



Ideal lowpass filter

$$h_{LP}[n] = \frac{\sin \omega_c n}{\pi n}, \quad -\infty \le n \le \infty$$

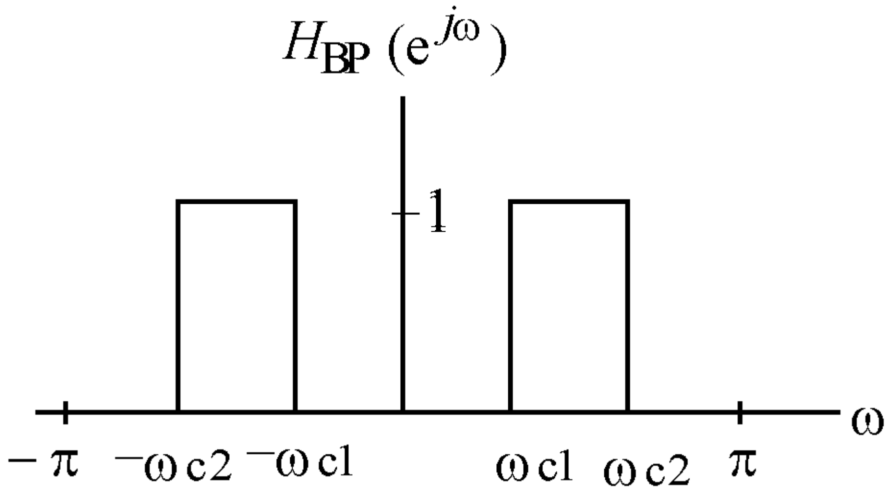


Ideal highpass filter

$$h_{HP}[n] = \begin{cases} 1 - \dfrac{\omega_c}{\pi}, & n = 0 \\ -\dfrac{\sin \omega_c n}{\pi n}, & n \neq 0 \end{cases}$$

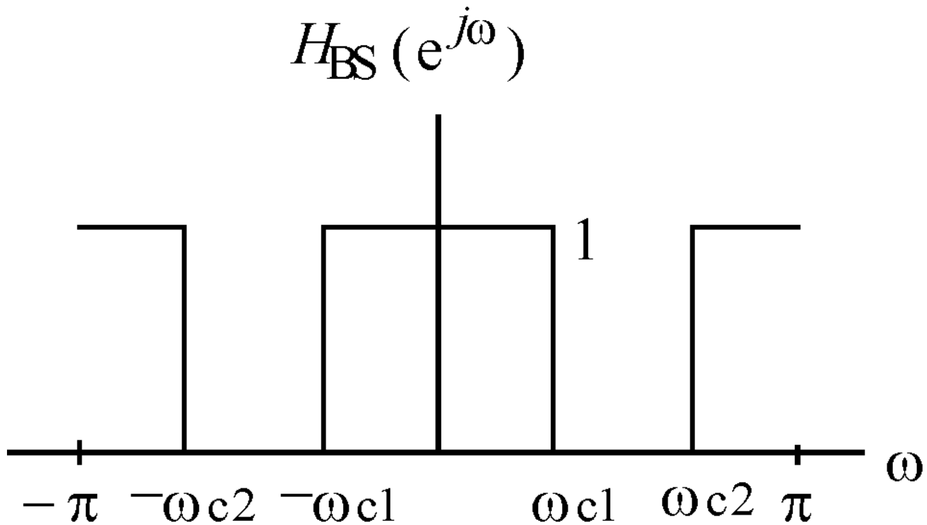
How to get ?

- **Ideal bandpass filter**



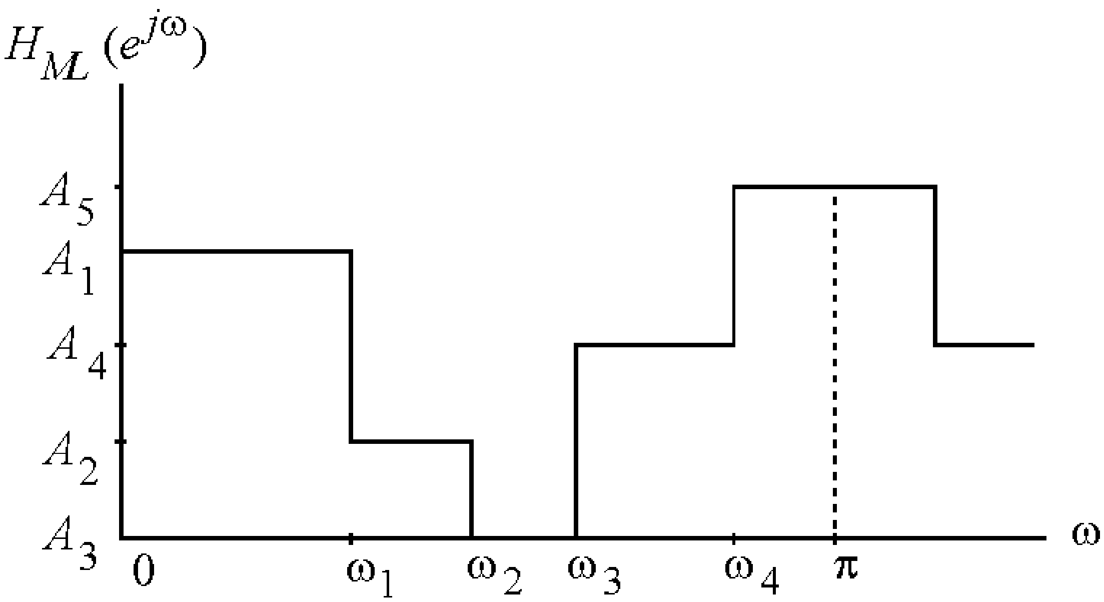
$$h_{BP}[n] = \begin{cases} \dfrac{\sin \omega_{c2} n}{\pi n} - \dfrac{\sin \omega_{c1} n}{\pi n}, & n \neq 0 \\ \dfrac{\omega_{c2}}{\pi} - \dfrac{\omega_{c1}}{\pi}, & n = 0 \end{cases}$$

- **Ideal bandstop filter**



$$h_{BS}[n] = \begin{cases} 1 - \dfrac{\omega_{c2} - \omega_{c1}}{\pi}, & n = 0 \\ \dfrac{\sin \omega_{c1} n}{\pi n} - \dfrac{\sin \omega_{c2} n}{\pi n}, & n \neq 0 \end{cases}$$

- **Ideal multi-band filter**



$$H_{ML}(e^{j\omega}) = A_k,$$

$$\omega_{k-1} \le \omega \le \omega_k,$$

$$\omega_0 = 0, \omega_L = \pi$$

$$k = 1, 2, \ldots, L$$

$$A_{L+1} = 0$$

$$h_{ML}[n] = \sum_{\ell=1}^{L} (A_\ell - A_{\ell+1}) \frac{\sin(\omega_\ell n)}{\pi n}$$

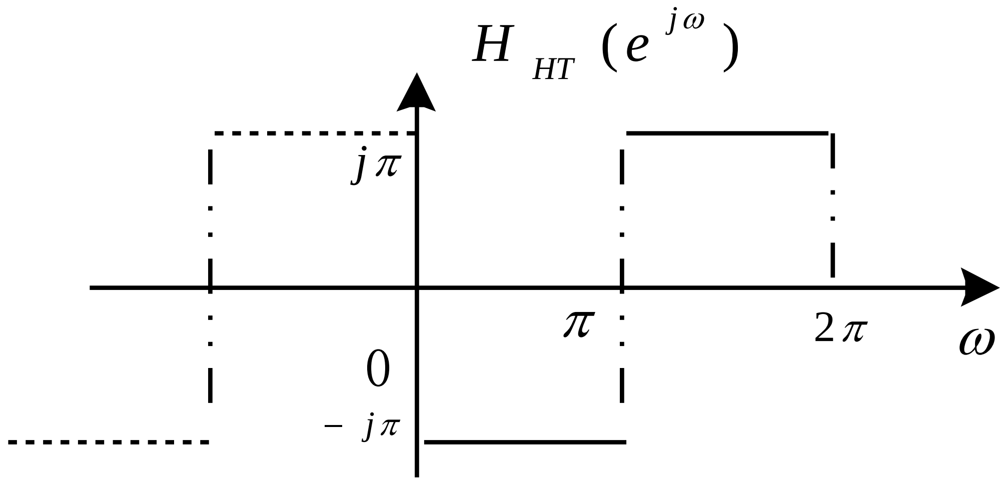
Ideal discrete-time Hilbert transformer

$$H_{HT}(e^{j\omega}) = \begin{cases} j, & -\pi < \omega < 0 \\ -j, & 0 < \omega < \pi \end{cases}$$

$$h_{HT}[n] = \begin{cases} 0, & \text{for } n \text{ even} \\ 2/\pi \mathrm{n}, & \text{for } n \text{ odd} \end{cases}$$

How to get ?

Hilbert transformer (90-degree phase shifter)



$$h_{HT}[n] = \frac{1}{2\pi}\int_{-\pi}^{0} je^{j\omega n} d\omega + \frac{1}{2\pi}\int_{0}^{\pi}(-j)e^{j\omega n} d\omega = \frac{2\sin^2(\frac{n\pi}{2})}{n\pi}$$

$$n \neq 0, and \ , h_{HT}[0] = 0 \qquad h_{HT}[n] = \frac{1}{n\pi}[1-(-1)^n]$$

Ideal discrete-time differentiator

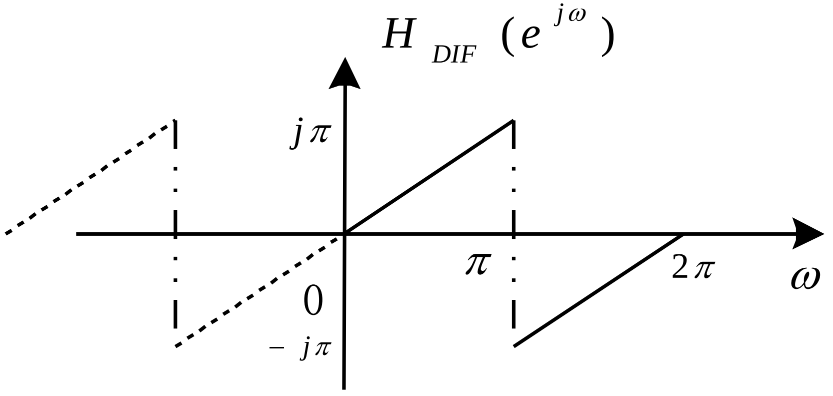
$$H_{DIF}(e^{j\omega}) = j\omega, \quad 0 \le |\omega| \le \pi$$

$$h_{DIF}[n] = \begin{cases} 0, & n = 0 \\ \dfrac{\cos \pi n}{n}, & n \ne 0 \end{cases}$$

How to get ?

Differentiator

$$H_{DIF}(e^{j\omega}) = j\omega, 0 \le |\omega| \le \pi$$



$$h_{DIF}[n] = \frac{1}{2\pi}\int_0^{\pi} j\omega e^{j\omega n} d\omega$$

$$+\frac{1}{2\pi}\int_{\pi}^{2\pi} j(\omega - 2\pi)e^{j\omega n} d\omega = \frac{\cos(n\pi)}{n} - \frac{\sin(n\pi)}{\pi n^2}$$

$$= \left\{ \begin{matrix} 0, n = 0 \\ \dfrac{\cos(n\pi)}{n}, |n| \ne 0 \end{matrix} \right\}$$

In another way, let $H_a(j\Omega) = j\Omega P_{2\pi}(\Omega)$

and using inverse FT, we get

$$h_a(t) = \frac{\sin(\pi t)}{\pi t} * \delta'(t) = \frac{\cos(\pi t)}{t} - \frac{\sin(\pi t)}{\pi t^2}$$

$t \rightarrow n$, **we get**

$$h_{DIF}[n] = \frac{\cos(n\pi)}{n} - \frac{\sin(n\pi)}{\pi n^2}$$

The 2M+1-point linear phase FIR

$$H_{diff}(e^{j\omega}) = (j\omega)e^{-jM\omega}, (-\pi < \omega < \pi)$$

$$h_{diff}[n] = \frac{\cos\pi(n-M)}{(n-M)} - \frac{\sin\pi(n-M)}{\pi(n-M)}$$

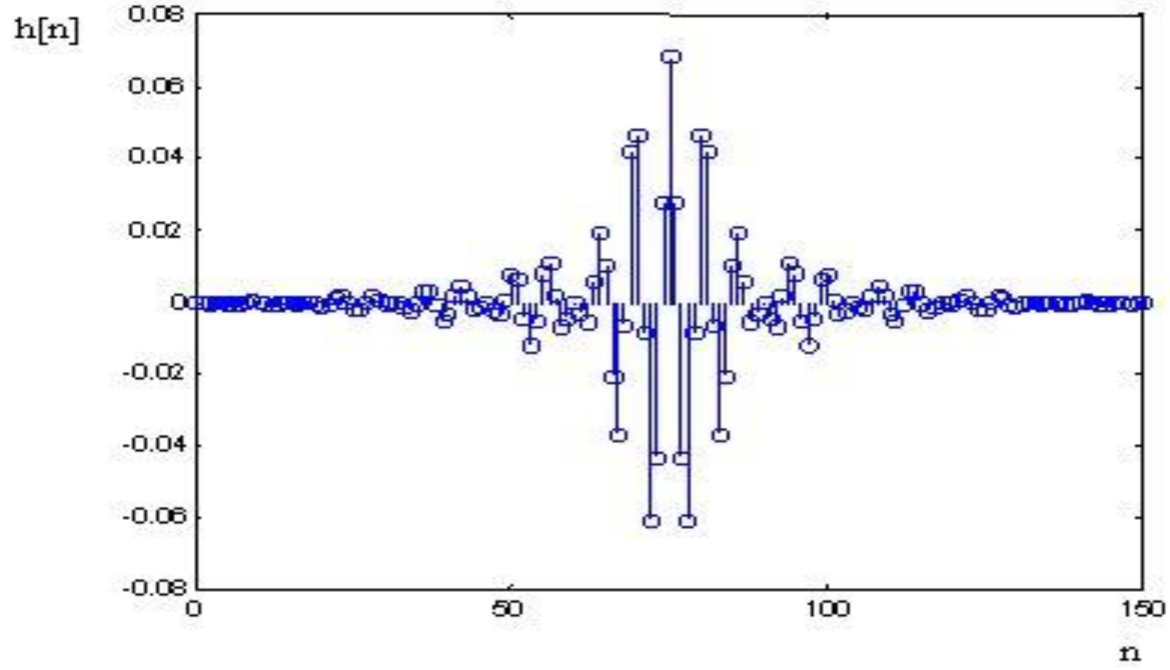
10.2.8 FIR Filter Design Example (appended)

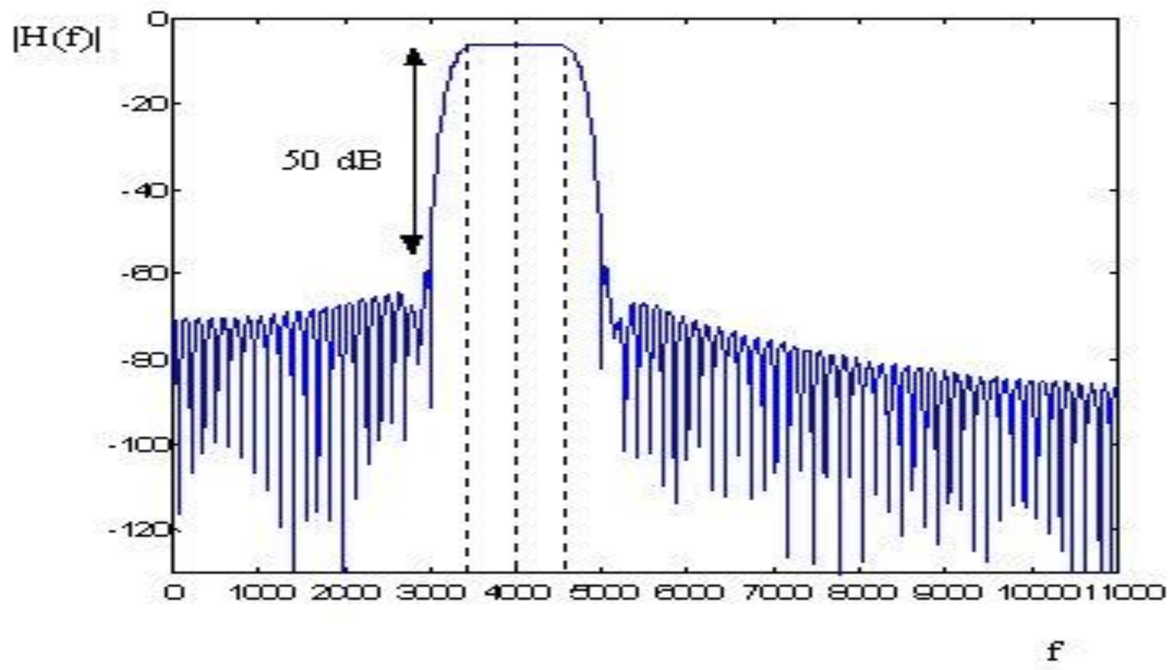
Example: Bandpass FIR Filter Design

- FIR bandpass filter :
- center frequency 4kHz
- pass band edges 3.5 and 4.5 kHz
- sampling frequency f_T=22kHz
- transition width Δf =500Hz
- Minimum stop band attenuation 50 dB

Design

Equivalent LPF filter :


h[n]
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
0
50
100
150
n


|H(f)|
0
-20
-40
-60
-80
-100
-120
50 dB
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
f

Example 3: FIR LPF Filter Design

A linear phase FIR LPF (Type 1) designed Using Kaiser window with the following specifications:

$$\omega_p = 0.3\pi ,$$

$$\omega_s = 0.5\pi ,$$

Minimum stopband attenuation α_s=40dB

Design

$$\omega_c = (\omega_p + \omega_s)/2 = 0.4\pi$$

From Eq. (10.44) ($21 \le \alpha_s \le 50$), we get

$$\beta = 0.5842\ (\alpha_s - 21)^4 + 0.07886\ (\alpha_s - 21)$$

$$\beta = 0.5842(19)^{0.4} + 0.07886 \times 19 = 3.3953$$

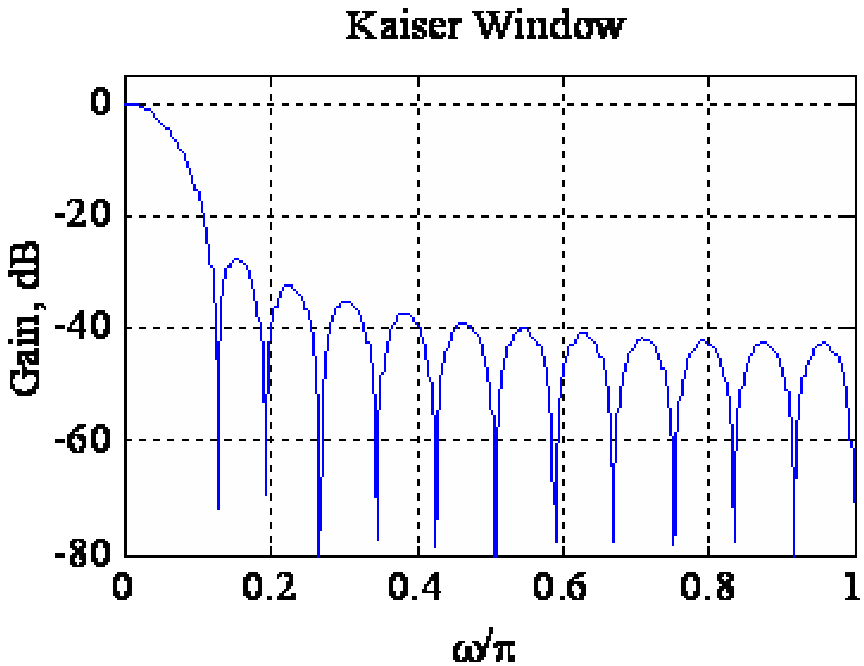
From Eq. (10.45), we get

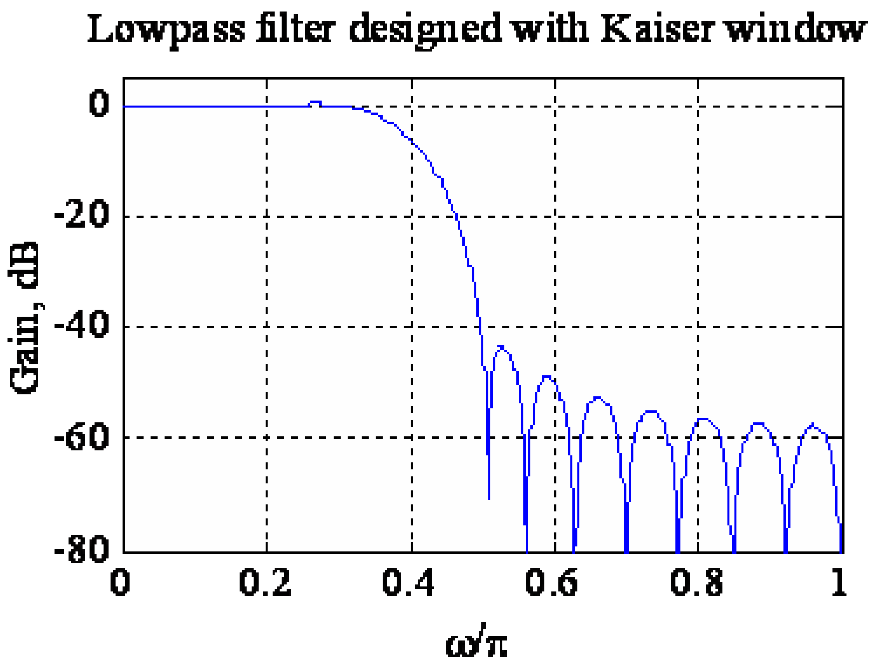
$$N = \frac{40 - 8}{2.285(0.2\pi)} = 22.2886$$

- **Choose N = 24 (Type 1), implying M =12**
- **Hence $h_t[n] = \{\sin(0.4\pi n)/\pi n\}w[n], -12 \le n \le 12$**

where $w[n]$ is the n-th coefficient of a length-25 Kaiser window with $\beta = 3.3953$

$$h[n] = h_t[n - M] = h_t[n-12]$$





10.3 FIR Filter Design Based on Frequncy Sampling Approach (Problem 10.31)

The method of using window is an approximation in time-domain. The idea can be used in frequency-Domain.

10.3.1 The method

$$H_d(e^{j\omega}) \xrightarrow{Sampling} H(k) \xrightarrow{IDFT} h[n] \xrightarrow{DTFT} H(e^{j\omega})$$

10.3.1 The method

Let $H(k) = H(e^{j\frac{2\pi}{N+1}k})$ **denote the sampling values of *N*+1-point FIR filter $H(e^{j\omega})$ designed. We do**

$$H(k) = H_d(e^{e^{j\frac{2\pi}{N+1}k}}) \xrightarrow{approximation} H_d(e^{j\omega})$$

$$\downarrow$$

$$H(e^{j\omega}) \xrightarrow{approximation} H_d(e^{j\omega})$$

Normally, $H(k) = H_d(k)$. The impulse response $h[n]$ can be derived from IDFT of $H(k)$.

It is noticed that $H(k)$ must satisfy the constraint to the linear phase FIR.

The Design Steps:

(1) Determine $H(e^{j\omega})$.

(2) From ideal filter, the values of $H(k)$ are computed (linear phase).

(3) From the IDFT of $H(k)$, $h[n]$ is derived.

10.3.2 Design Examples

Design a linear phase FIR BPF, with the passband edge frequencies at 500Hz and 700Hz. The sampling frequency is f_s = 3.3KHz, and the order N=32 (length 33).

Design

(1)Determination of $H(k)$. N = 32, f_s = 3.3 KHz,

$$\Delta f = \frac{fs}{N+1} = 0.1\text{KHz}$$ **(discrimination)**

BPF and linear phase are both required. The length is 33 (odd).

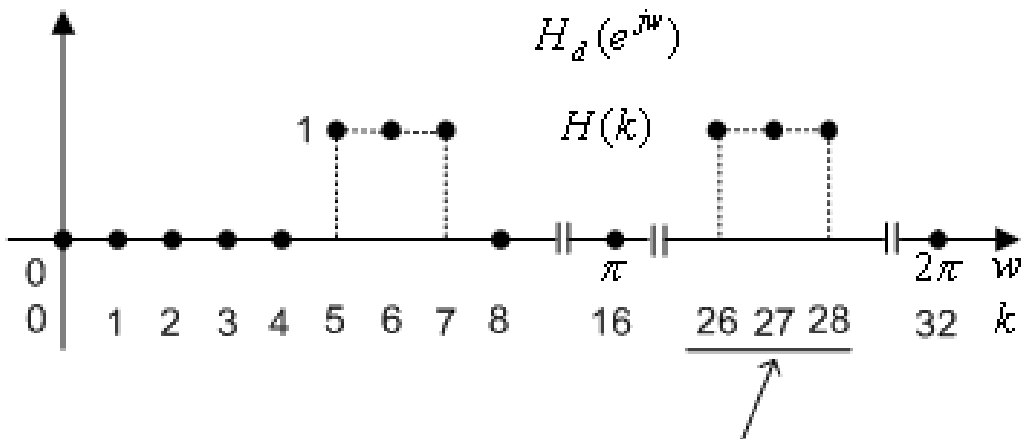
So, we choose Type 1 FIR.

That means:

$$H(\omega) = H(2\pi - \omega)$$

$$\theta(\omega) = -M\omega = -16\omega \ and, \omega_k = \frac{2\pi}{33}k$$

$$H(k) = H(e^{j\frac{2\pi}{N+1}k}) = \begin{cases} 1 \cdot e^{-j\frac{32\pi}{33}k}, k = 5.6.7, 26.27.28 \\ 0, \quad others \end{cases}$$



$$< N+1-k >_{N+1} = < 33-k >_{33}$$

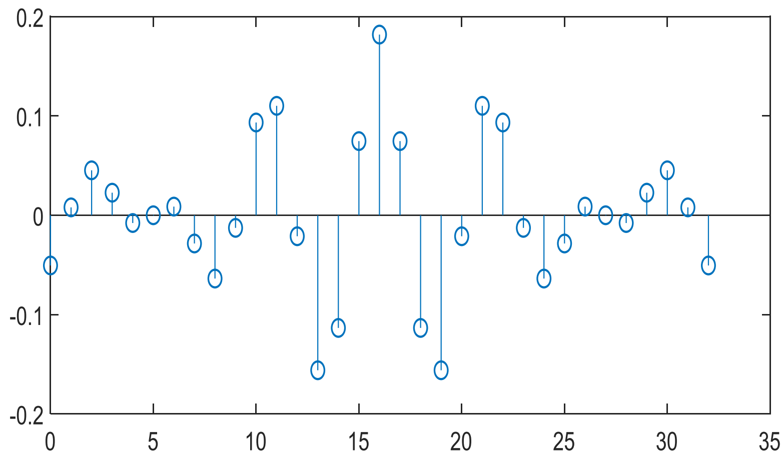
N+1-points IDFT

$$h[n] = \frac{1}{N+1}\sum_{k=0}^{N} H(k) W_{N+1}^{-kn} = \frac{1}{33}\sum_{k=0}^{32} H(k) e^{j\frac{2\pi}{33}kn}$$

$$= \frac{1}{33}\{e^{-j\frac{32\pi}{33}5} e^{j\frac{2\pi}{33}5n} + e^{-j\frac{32\pi}{33}28} e^{j\frac{2\pi}{33}28n} + e^{-j\frac{32\pi}{33}6} e^{j\frac{2\pi}{33}6n} + e^{-j\frac{32\pi}{33}27} e^{j\frac{2\pi}{33}27n}$$

$$+ e^{-j\frac{32\pi}{33}7} e^{j\frac{2\pi}{33}7n} + e^{-j\frac{32\pi}{33}26} e^{j\frac{2\pi}{33}26n}\}$$

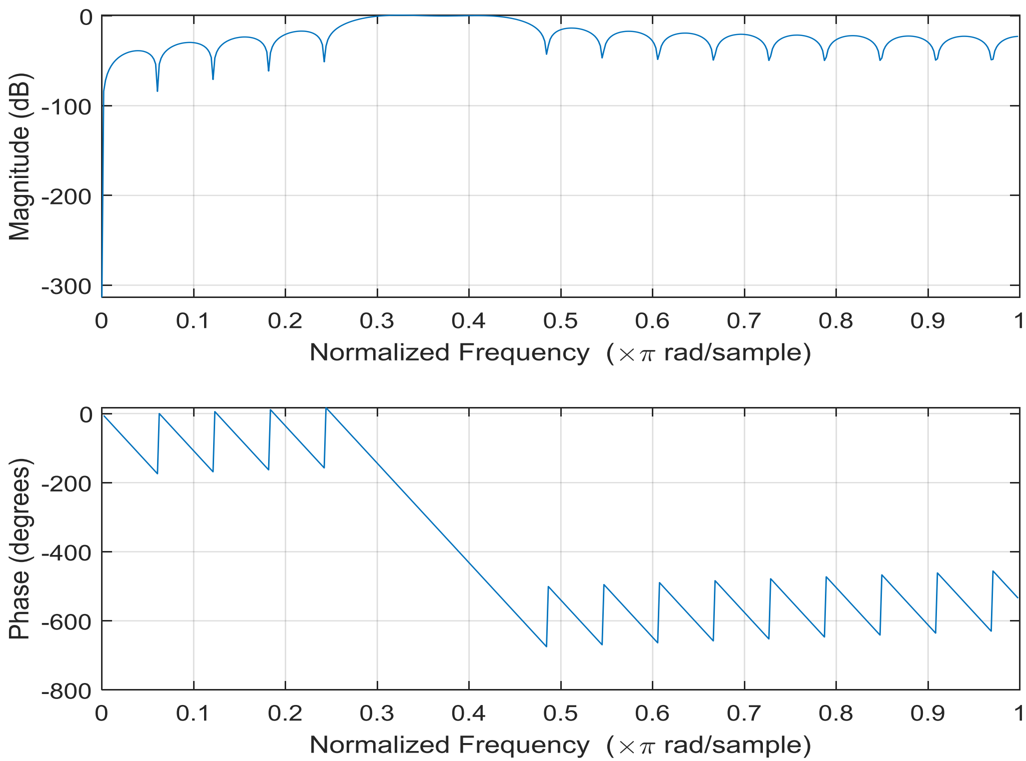
$$= \frac{1}{33}\{2\cos(\frac{2\pi}{33}5(n-16)) + 2\cos(\frac{2\pi}{33}6(n-16)) + 2\cos(\frac{2\pi}{33}7(n-16))\}$$



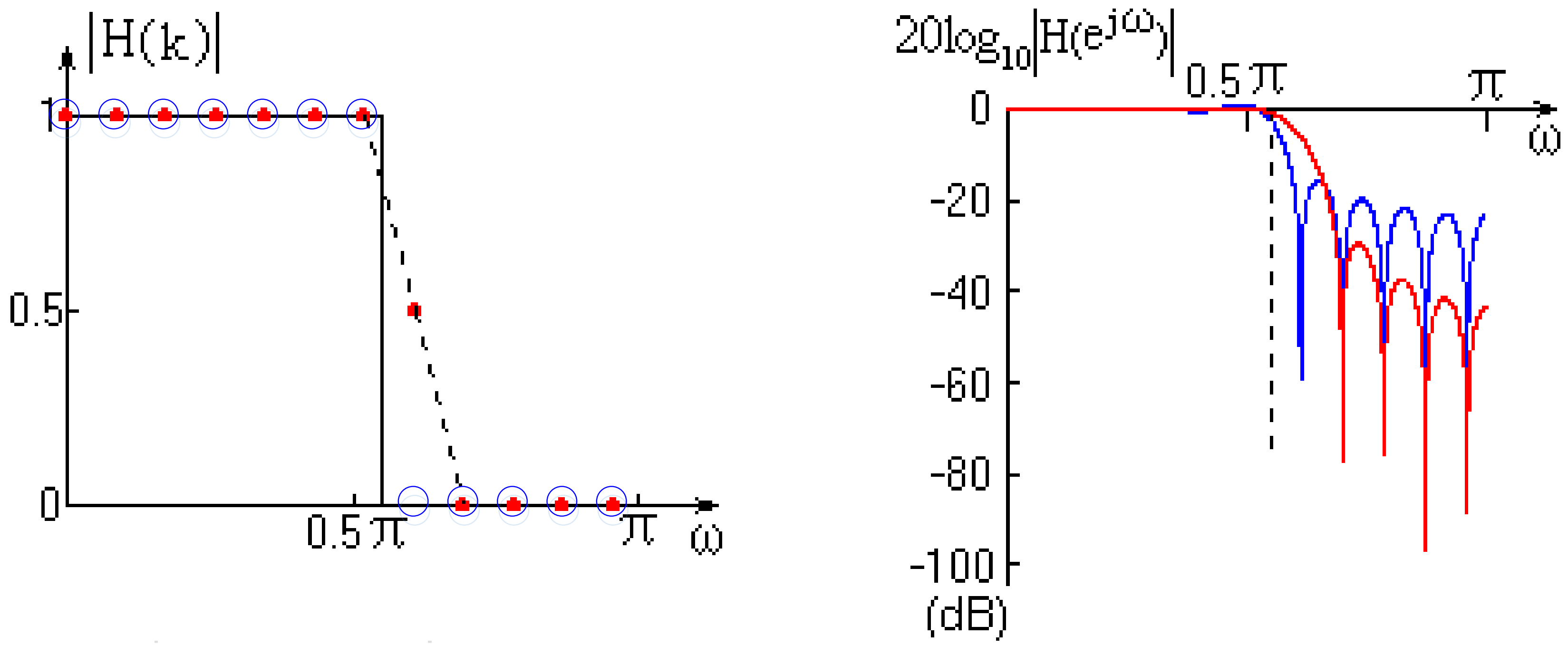
DTFT

$$H(e^{j\omega}) = e^{-j\frac{16}{33}\omega}\left[\cos 16\omega \cos 17\omega\right]\cdot\left[\frac{1.78}{0.58-\cos\omega}+\frac{1.68}{0.42-\cos\omega}\right]$$

The better design is to append one or more samples of magnitude on the transition band.



For example, a LPF with one sample of magnitude on the transition band is shown bellow.



It should be noted that the frequency sampling approach is convenient to some narrow filters.

Example: **Design a linear phase FIR LPF by using frequency sampling approach. The length is N+1 = 15.The magnitude values sampled are**

$$H_k = \begin{cases} 1, & k = 0 \\ 0.5, & k = 1,14 \\ 0, & k = 2,3,\ldots 13 \end{cases}$$

(1)Determination of $H(k)$. N=14, **and linear phase**

$$\theta(\omega) = -\frac{N}{2}\omega = -M\omega = -7\omega \quad \textbf{（Type 1）}$$

So,

$$H(k) = H_k \cdot e^{-j\frac{N}{2}\cdot\frac{2\pi}{N+1}k} = \begin{cases} 1, & k = 0 \\ 0.5e^{-j\frac{14}{15}\pi}, & k = 1 \\ 0, & k = 2 \sim 13 \\ 0.5e^{-j\frac{14}{15}\times 14\pi}, & k = 14 \end{cases}$$

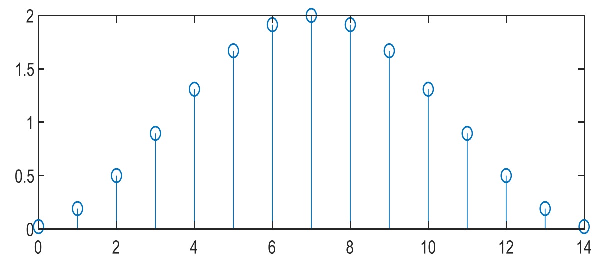
(2) Computation of *h*[*n*]

N+1-points IDFT

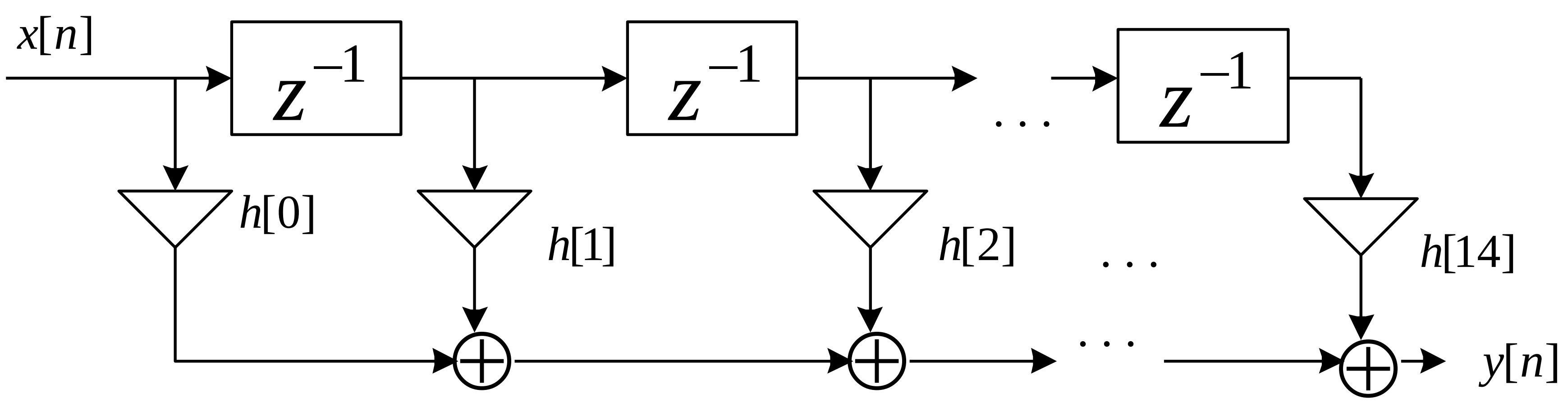
$$h[n] = \frac{1}{N+1}\sum_{k=0}^{N} H(k) W_{N+1}^{-kn} = \frac{1}{15}[1 + \frac{1}{2}e^{-j\frac{14}{15}\pi} \cdot e^{j\frac{2\pi}{15}n} + \frac{1}{2}e^{-j\frac{14\times 14}{15}\pi} \cdot e^{j\frac{2\pi}{15}\times 14n}]$$

$$h[n] = \frac{1}{15}\left[1 + \cos\left(\frac{2n\pi}{15} - \frac{14\pi}{15}\right)\right] = \frac{1}{15}\left[1 + \cos\frac{2\pi}{15}(n-7)\right]$$

$$n = 0, 1, \cdots 14$$



(2) The Structure



Direct-Form

The computational complexity:15 multiplications (*N*+1), and 14 additions (*N*).

Other structure?

10.4 window-based FIR Filter Design Using MATLAB(10.5.4)

Window Generation

```
w = hann(L)
w = hamming(L)
w = blackman(L)
w = kaiser(L, beta)
```

FIR Filter Summary

FIR Filters

Filter Design Method	Description	Filter Functions
Windowing	Apply window to truncated inverse Fourier transform of specified "brick wall" filter	`fir1`, `fir2`, `kaiserord`
Multiband with Transition Bands	Equiripple or least squares approach over sub-bands of the frequency range	`firls`, `firpm`, `firpmord`
Constrained Least Squares	Minimize squared integral error over entire frequency range subject to maximum error constraints	`fircls`, `fircls1`
Arbitrary Response	Arbitrary responses, including nonlinear phase and complex filters	`cfirpm`
Raised Cosine	Lowpass response with smooth, sinusoidal transition	`rcosdesign`

Example 10.25

```
% Lowpass Filter Design Using the Kaiser Window
fpts = input('Type the band edges = ');
mag  = input('Type the desired magnitude values
= ');
dev = input('Type the ripples in each band = ');
[N,Wn,beta,ftype] = kaiserord(fpts, mag, dev);
kw = kaiser(N+1, beta);
b = fir1(N, Wn, kw);
[h,omega] = freqz(b,1,512);
plot(omega/pi,20*log10(abs(h)));
grid;
xlabel('\omega/\pi');
ylabel('Gain, dB');
```

Type in the bandedges = [0.3 0.4]
Type in the desired magnitude values = [1 0]
Type in the ripples in each band = [0.003162 0.003162]

N =

59

Wn =

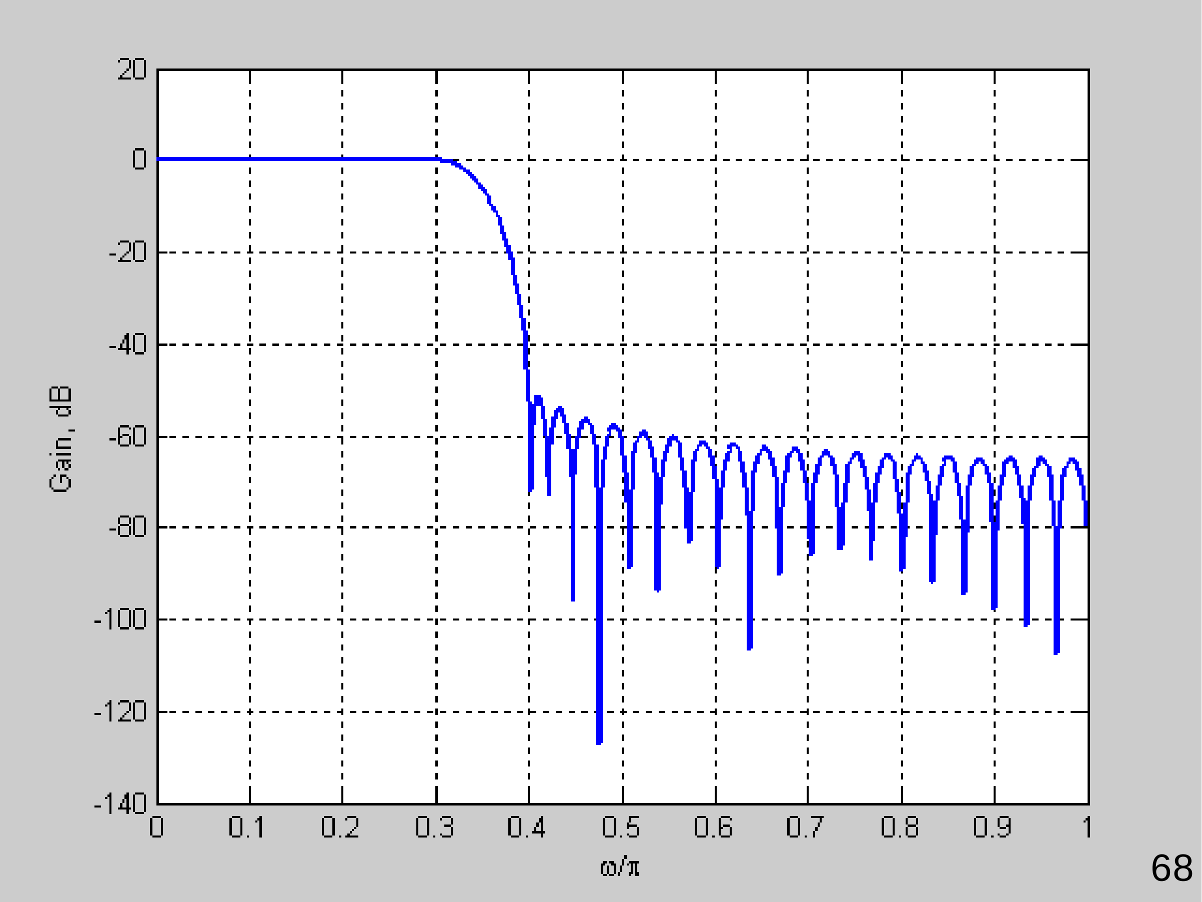
0.3500

beta =

4.5513

ftype =

low



Example 10.26

Use the same program as example 10.25

By modifying `b = fir1(N,Wn,'high',kw);`

Type in the bandedges = [0.4 0.55]

Type in the desired magnitude values = [0 1]

Type in the ripples in each band = [0.02 0.02]

N =

26

Wn =

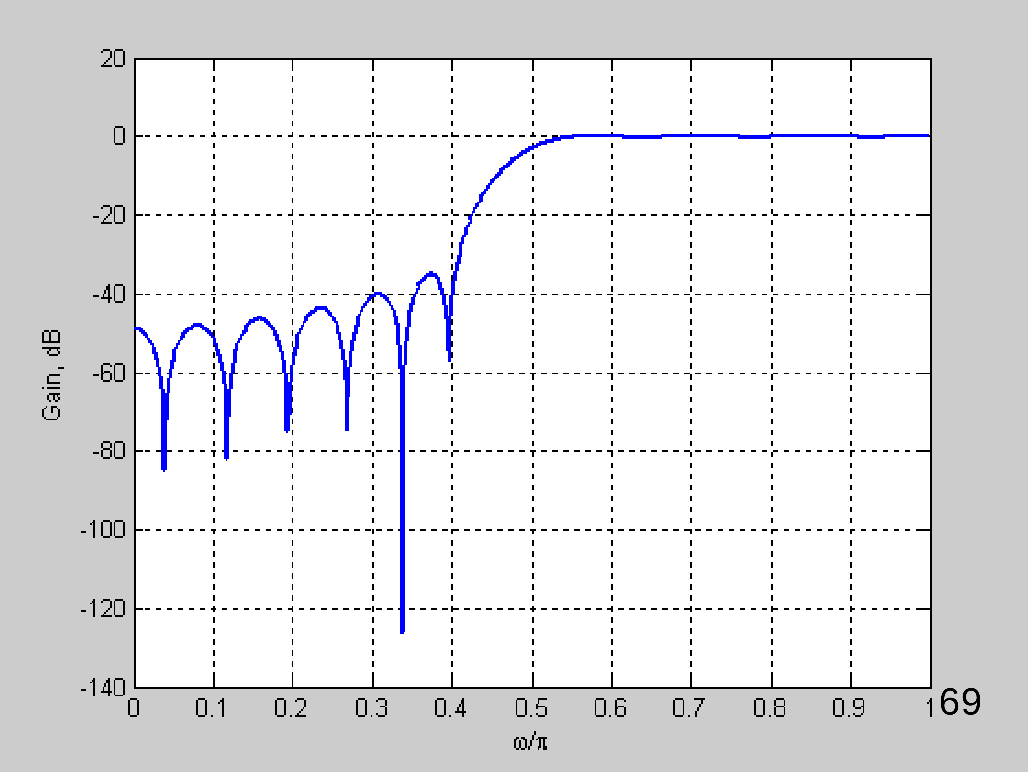
0.4750

beta =

2.6523

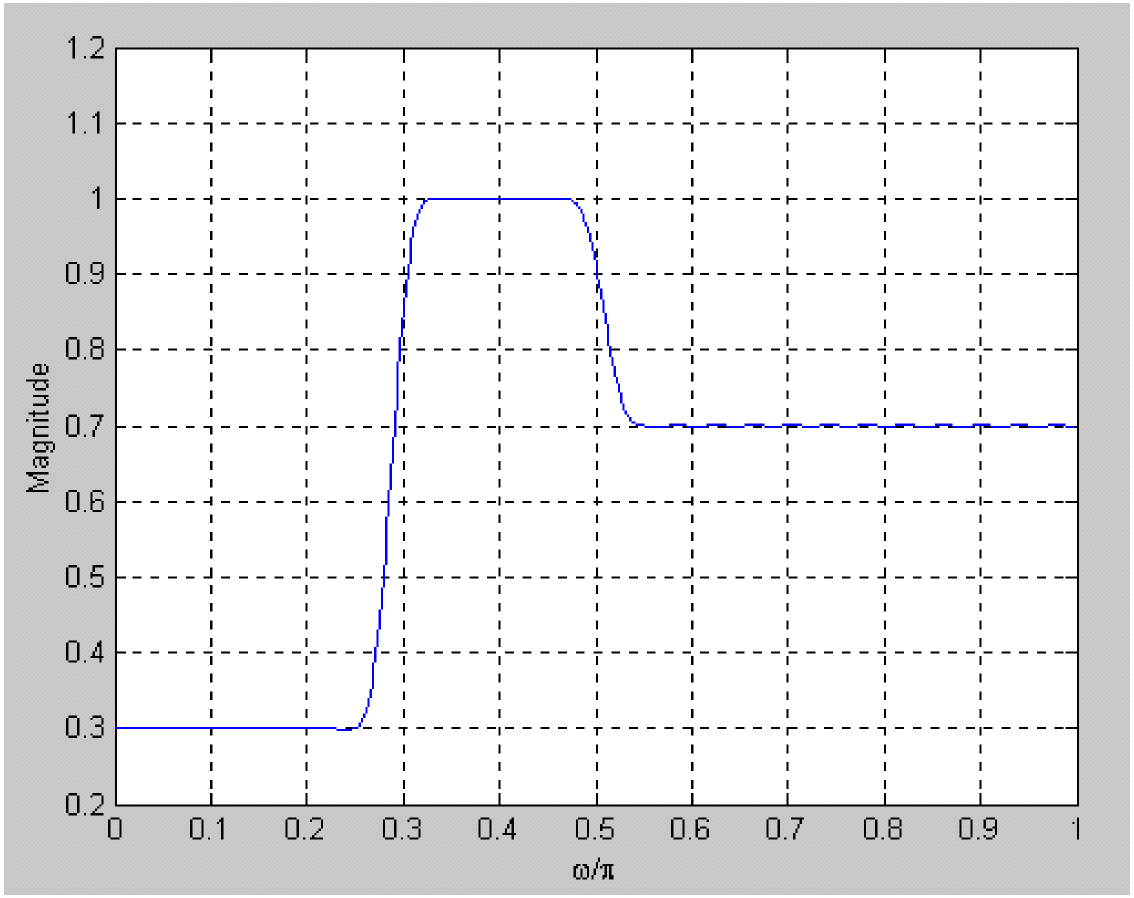
ftype =

high



Multilevel Filter Design

N=100, f = [0 0.28 0.3 0.5 0.52 1] ,
m = [0.3 0.3 1.0 1.0 0.7 0.7]

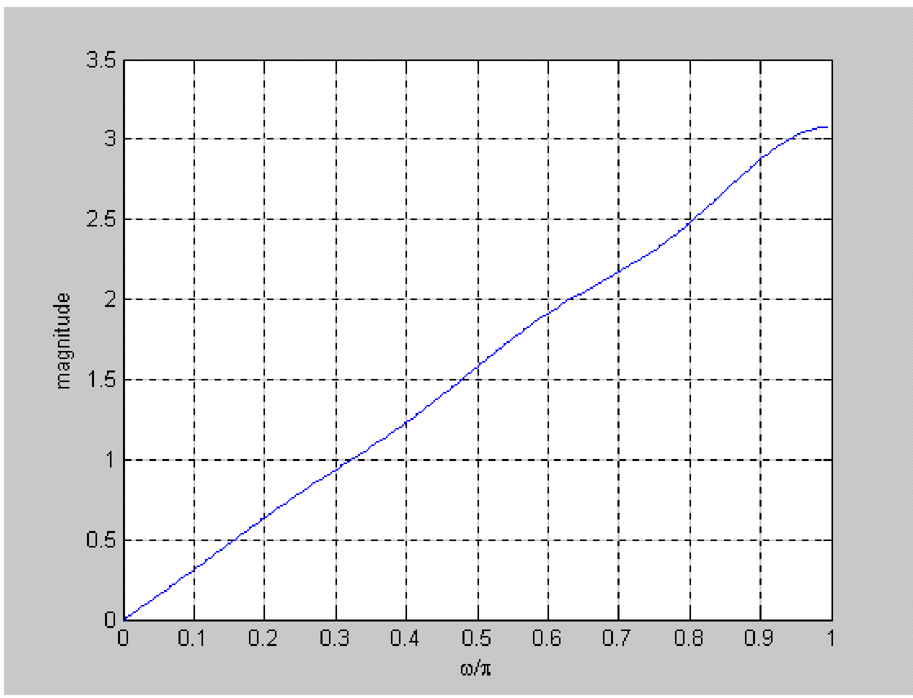


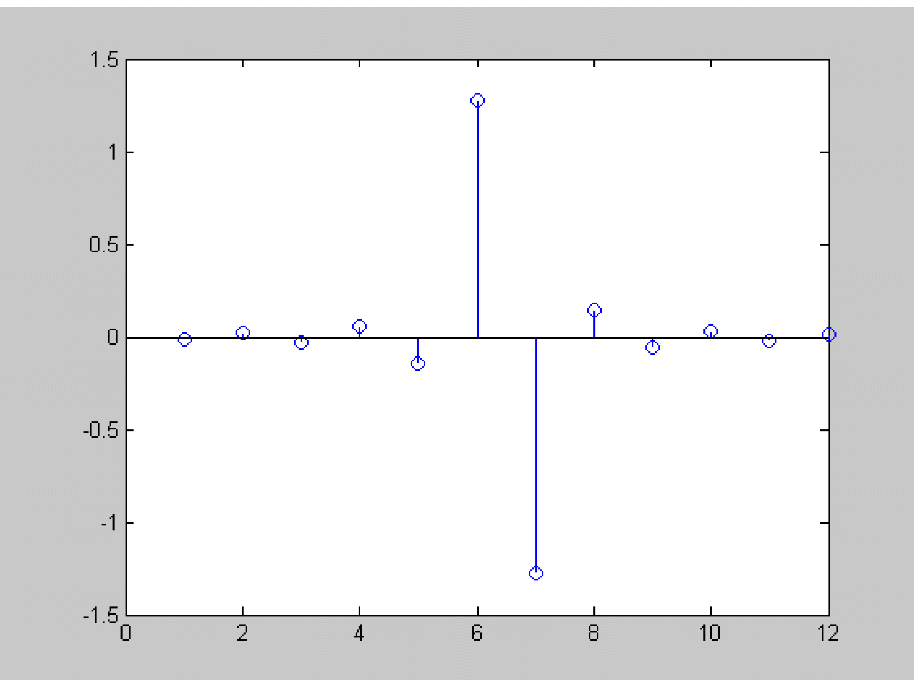

```
b = fir2(N,f,m)
```

Fig.10.31

Example (differentiator)

(length = 12, *N* = 11, Type 4)





Example (differentiator) (length=51, *N*=50, type3)

Note: Type 3 or 4 can be used in practice for a low pass differentiator.

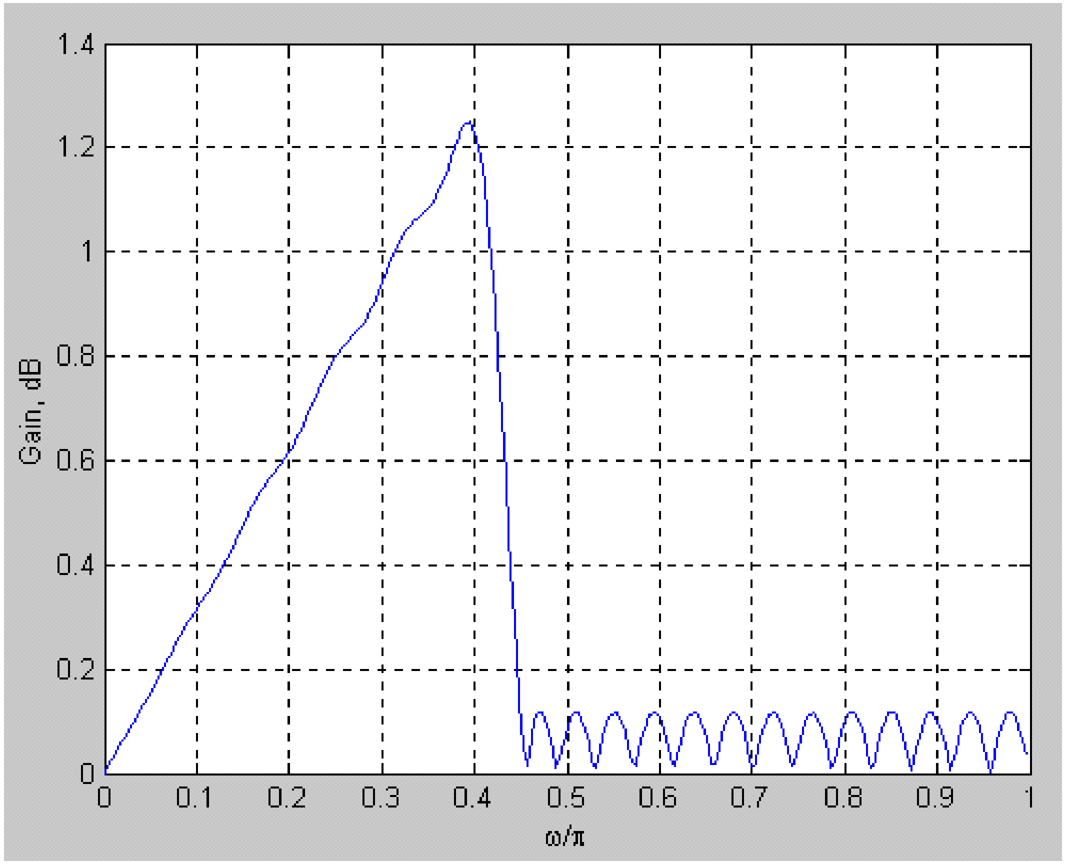
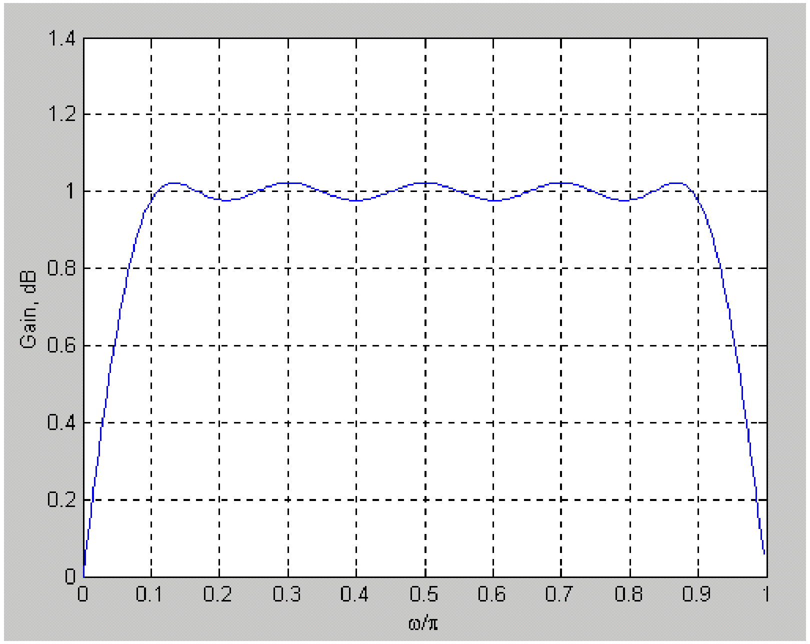


Fig.10.23(a)

Example (FIR Hilbert transformer) (length = 21, *N* = 20, type3) Note: This is a real



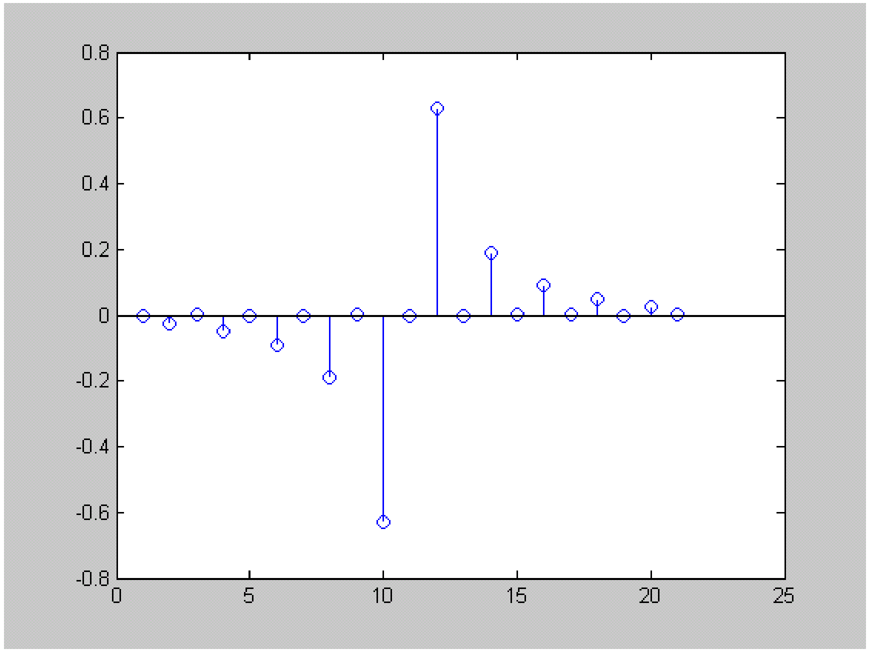
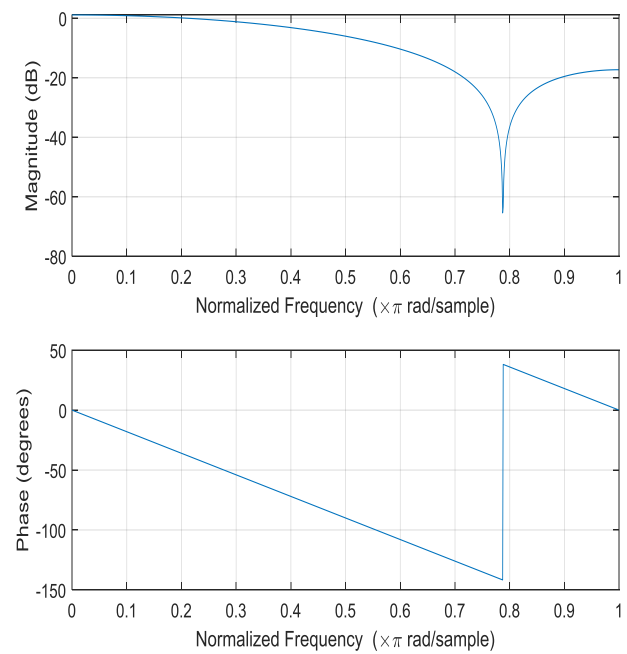


Fig.10.24(a)

Test 1

- Design a low pass FIR digital filter with $M = 1$, ω_c=0.5π, and rectangular window.

$$h[n] = \{0.5\pi, \underset{\uparrow}{0.5}, 0.5\pi\}$$



Test 2 (From Chapter 7)

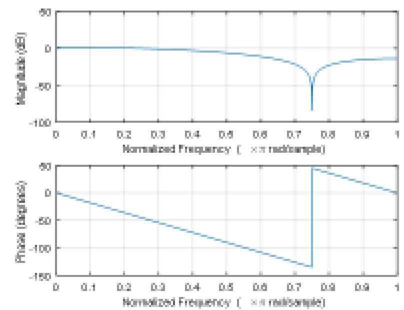
Test 1

- Design a linear-phase (Type 1), FIR (length 3), low-pass filter (LPF). Let $\omega_1=\pi/4$ in the passband, $\omega_2=3\pi/4$ in the stopband.

95

Test

- $h[n] = [0.3536, 0.5, 0.3536]$
- $H(e^{j\omega}) = 0.7072\cos\omega+0.5$
- $\theta(\omega) = -\omega$



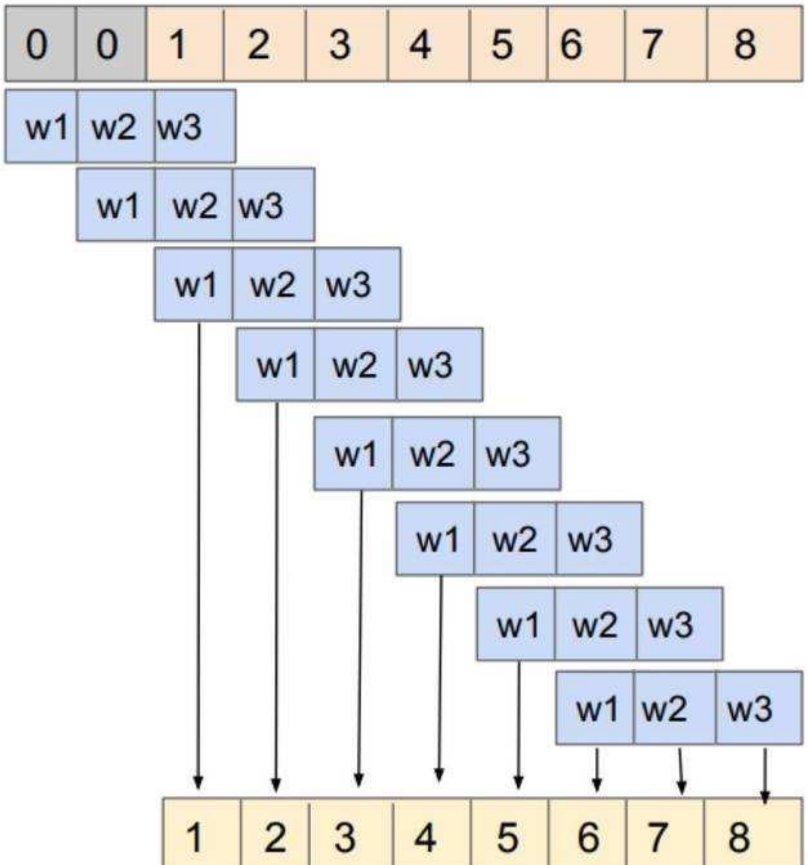
96

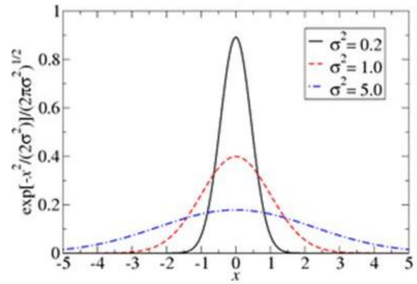
Similar to Test 1, but using a different solution.

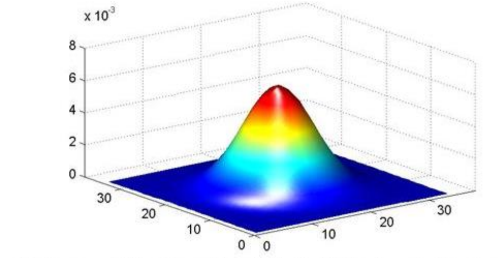
Test 3

- 1D Gaussian filter: $f(x) = e^{-\frac{x^2}{2\sigma^2}}$

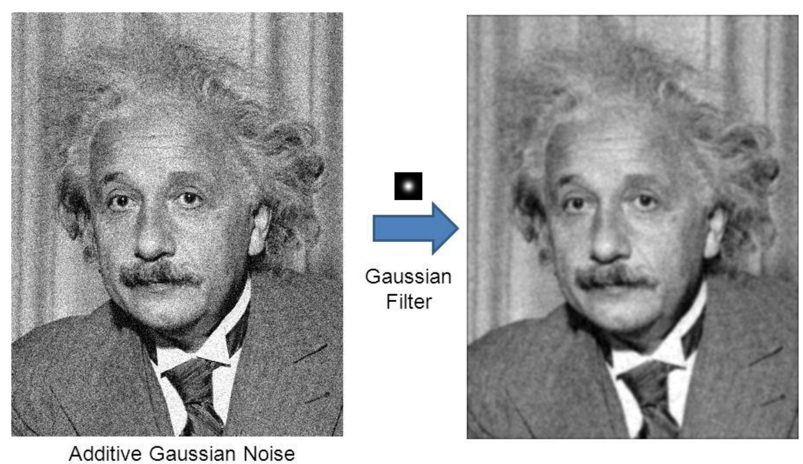
- 2D Gaussian filter: $f(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$





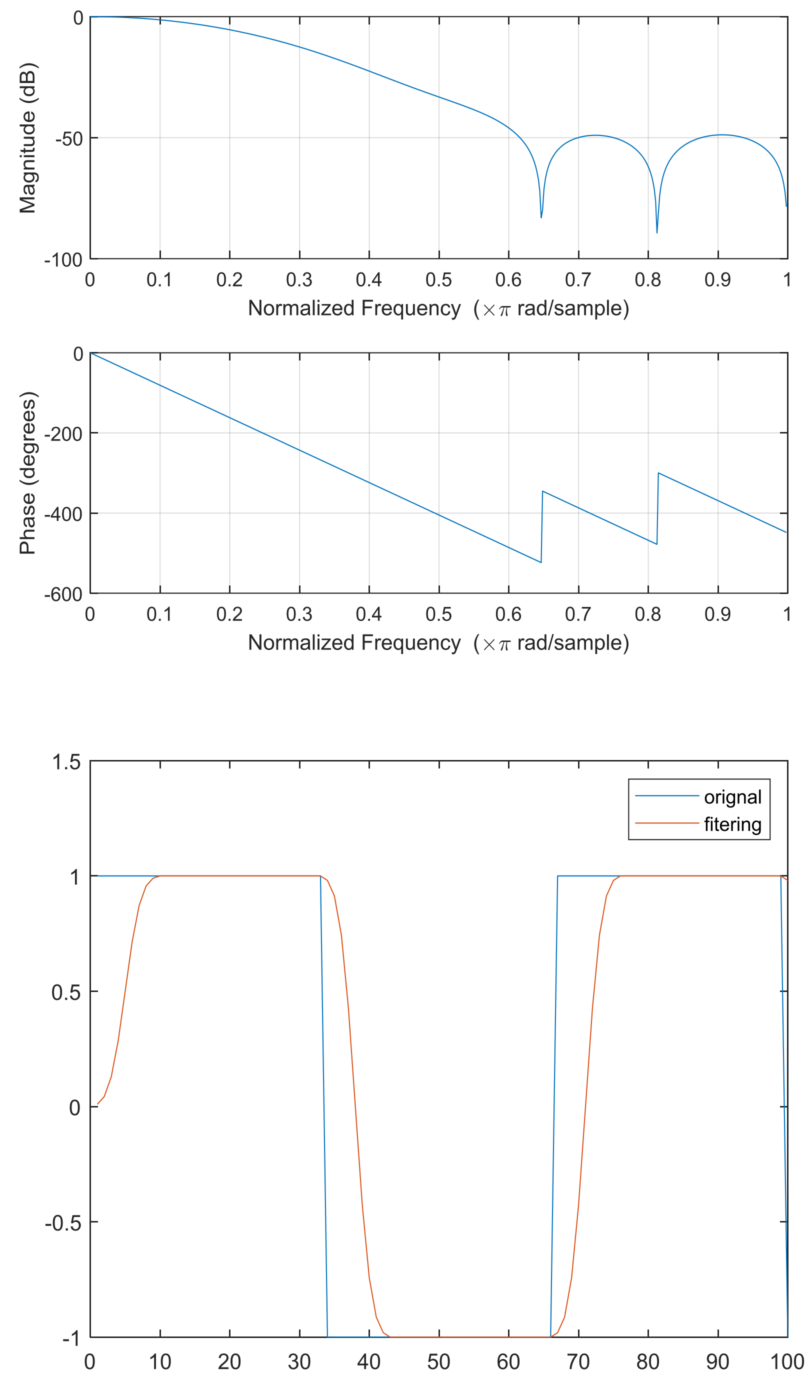


Denoising



Test 3

```
b = gausswin(10);
b = b/sum(b);
% freqz(b);
t = linspace(0,3*pi)';
x = square(t);
x_filtering = filter(b,1,x);
plot([x,x_filtering]);
```



Thanks!

Any questions?