Digital Signal Processing

Discrete Time Signal in Time Domain

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Outline

- Time Domain Representation
- Typical Sequence & Sequence Representation
- Operation on Sequences
- Operation on Finite Length Sequences
- Sampling Process
- Correlation of Signals

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Time Domain Representation

Discrete Time Signal: a sequences of numbers

$$x = \{x[n]\}, \quad -\infty < n < \infty$$

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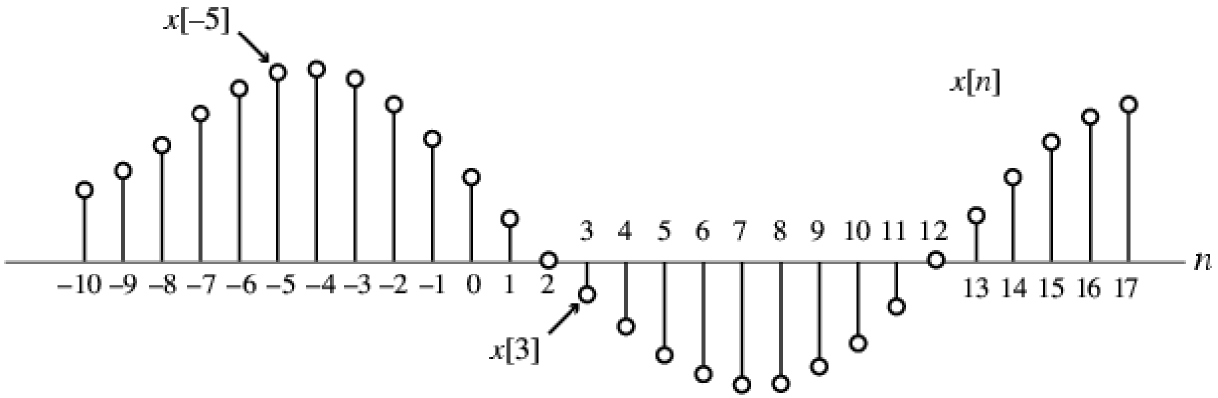
  - Graph



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Time Domain Representation

Length of a discrete time signal

- Finite-length (finite-duration)
  - sequence defined only for interval: $N_1 \leq n \leq N_2$
  - Length: $N = N_2 - N_1 + 1$

$$\mathbf{x} = [x[0], x[1] \cdots x[N-1]]^T$$

Zero-padding: append zero valued samples

Time Domain Representation

Types of a discrete time signal

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  - Two-sided sequence: n for positive & negative

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Typical Sequence—Basic Sequence

Unit Sample Sequence(impulse)

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Unit Step

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Exponential sequence

$$x[n] = A\alpha^{n}$$

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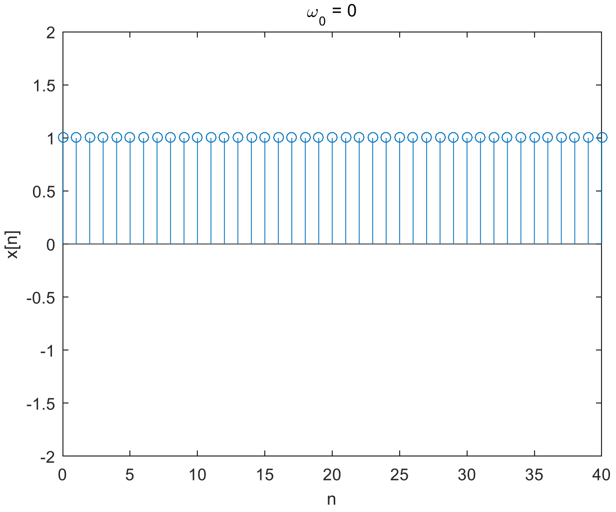

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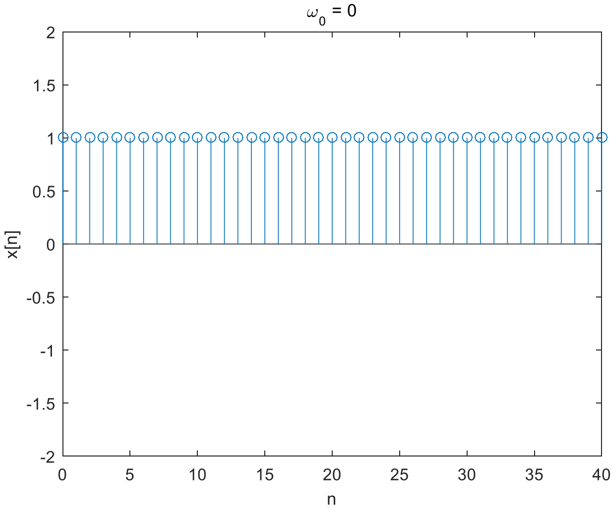

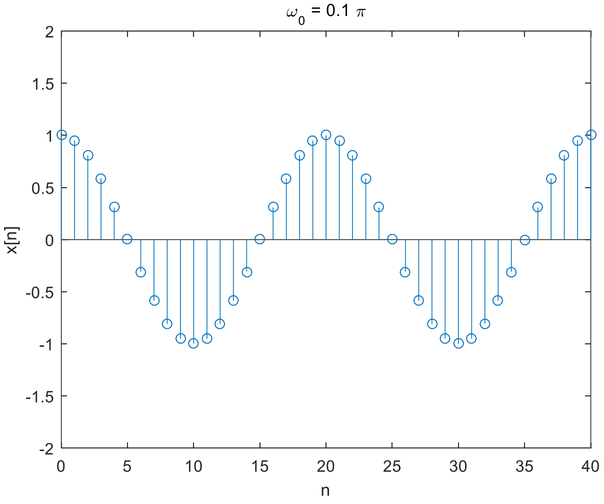

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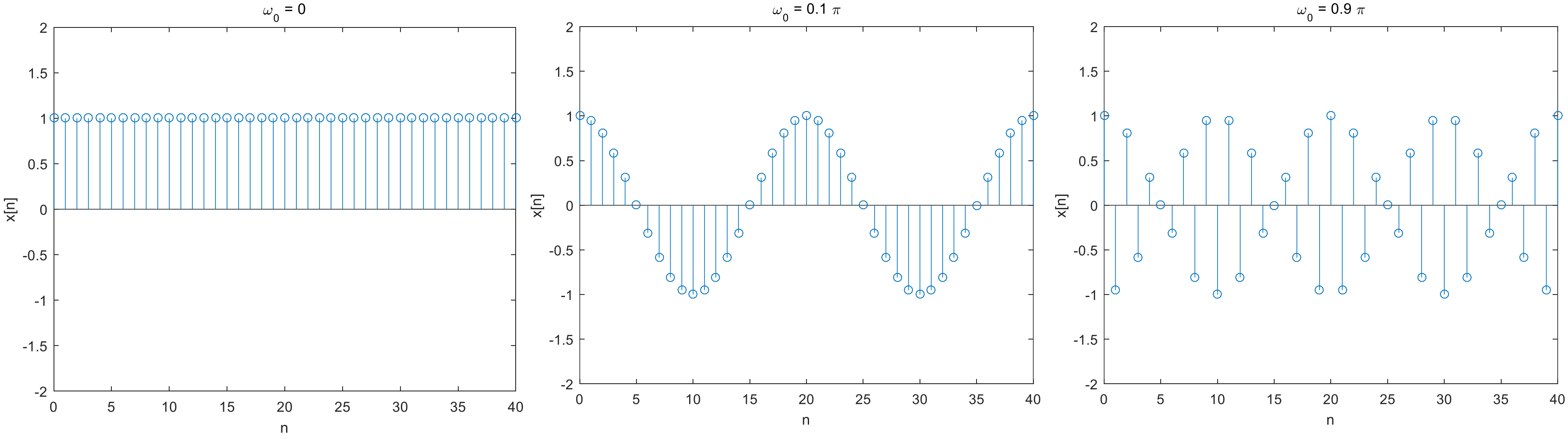

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$$x[n+N] = x[n]$$

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$$x[n+N] = x[n] \Rightarrow x[n+N] = A\cos(\omega_0(n+N)+\phi)$$

Periodic if $\omega_0 N = 2r\pi$ $\Rightarrow$ $N = 2r\pi/\omega_0 \in \mathbb{Z}$

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Not All Sequences are Periodic

Discrete Time Signals—Basic Sequence

Example: Combining Basic Sequence

$$
\begin{aligned}
u[n] &= \delta[n] + \delta[n-1] + \cdots \\
&= \sum_{k=0}^{\infty} \delta[n-k]
\end{aligned}
$$

Discrete Time Signals—Basic Sequence

Example: Combining Basic Sequence

$$u[n] = \delta[n] + \delta[n-1] + \cdots$$

$$= \sum_{k=0}^{\infty} \delta[n-k]$$

$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

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$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

$$x[n] = \begin{cases} A\alpha^{n} & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$= A\alpha^{n} u[n]$$

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$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

Any sequence can be represented by impulse

$$\boxed{x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]}$$

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Operation on Sequence

Elementary Operation

Product

$$w[n] = x[n] \cdot y[n]$$

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$$w[n] = x[n] \cdot y[n]$$

Add

$$w[n] = x[n] + y[n]$$

Operation on Sequence

Elementary Operation

Product

$$w[n] = x[n] \cdot y[n]$$

Add

$$w[n] = x[n] + y[n]$$

Multiplication

$$w[n] = Ax[n]$$

Operation on Sequence

Elementary Operation

Product

$$w[n] = x[n] \cdot y[n]$$

Add

$$w[n] = x[n] + y[n]$$

Multiplication

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Delay/Advance

$$w[n] = x[n - n_d]$$

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Multiplication

$$w[n] = Ax[n]$$

Delay/Advance

$$w[n] = x[n - n_d]$$

Time Reversal

$$w[n] = x[-n]$$

Operation on Sequence

Convolution Sum

$$y[n] = x[n] \circledast h[n]$$

$$\begin{aligned} y(t) &= x(t) \circledast h(t) \\ &= \int x(\tau) y(t-\tau) d\tau \end{aligned}$$

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$$y[n] = x[n] \circledast h[n]$$
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$$\begin{aligned} y[n] &= x[n] \circledast h[n] \\ &= \sum_{k=-\infty}^{\infty} x[k] h[n-k] \end{aligned}$$

- The computation is simply the sum of products
- Only delays, additions, and multiplications

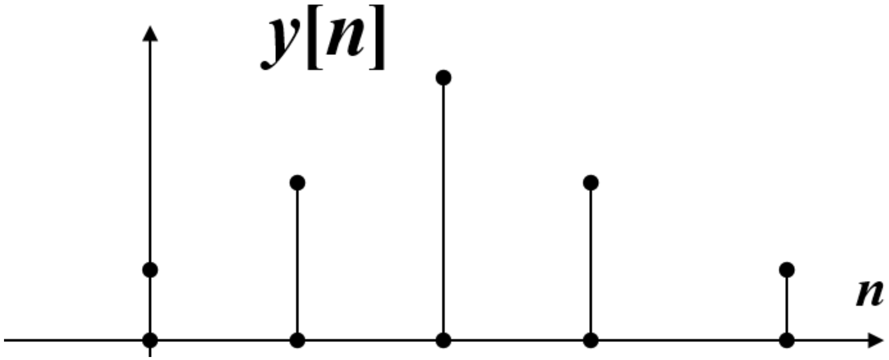
Operation on Sequence

Computation of Convolutional Sum

Eg.1 $x[n] = h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4]$$



A sequence of length *M* convolved with a sequence of length *N*, then resultant sequence is of length *M*+*N*-1

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Operation on Sequence

Computation of Convolutional Sum

Eg.2

$$x[n] = (1/2)^{n-1} u[n-2]$$

$$h[n] = u[n+2]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{k-2} u[k-2] u[(n-k)+2]$$

$$= \sum_{k=2}^{n+2} \left(\frac{1}{2}\right)^{k-2} = \left[2 - \left(\frac{1}{2}\right)^{n}\right] u[n]$$

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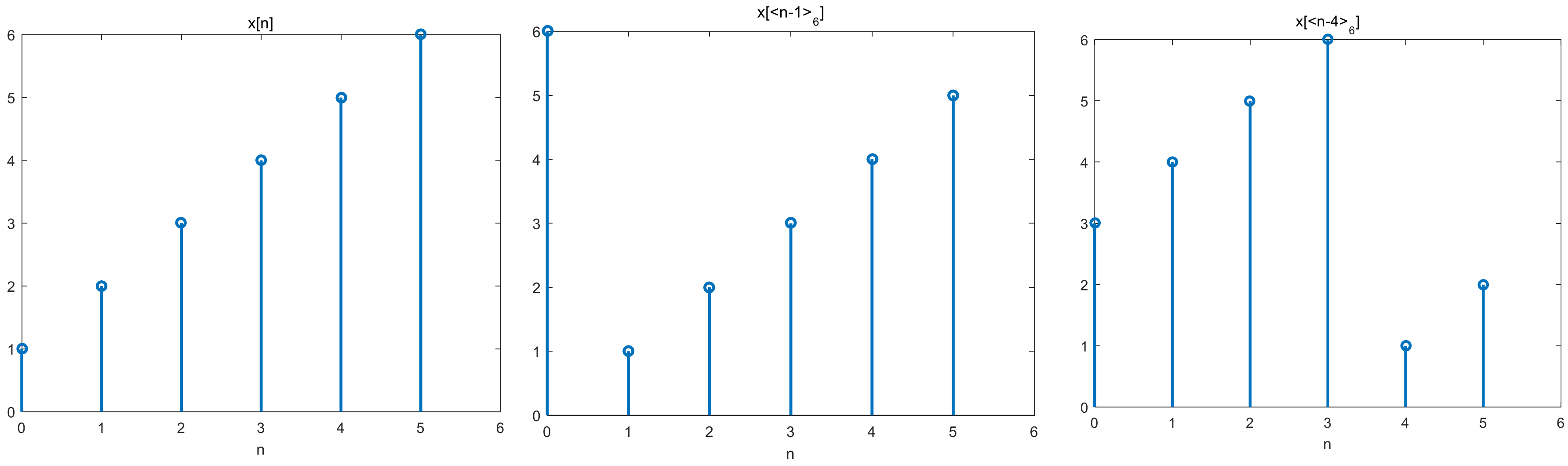
Operation on Finite Length Sequence

Circular Shift

$$x_c[n-n_0] = x[\langle n-n_0\rangle_N]$$

$$= \begin{cases} x[n-n_0] & \text{for } n_0 < n \le N-1 \\ x[N+n-n_0] & \text{for } 0 < n \le n_0 \end{cases}$$



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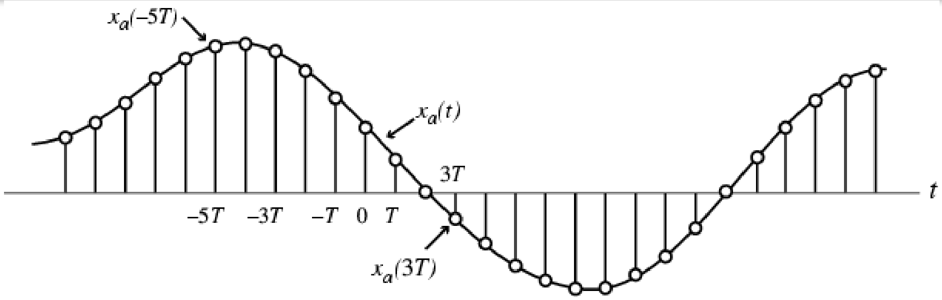
$$x[n] = x_a(nT)$$



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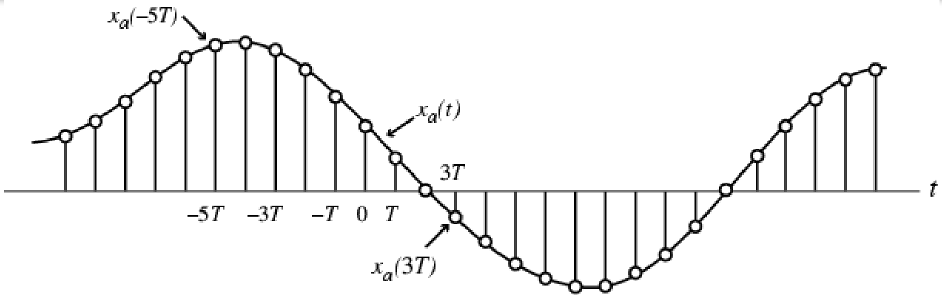


Example : $x_a(t) = A\cos(2\pi f_0 t)$

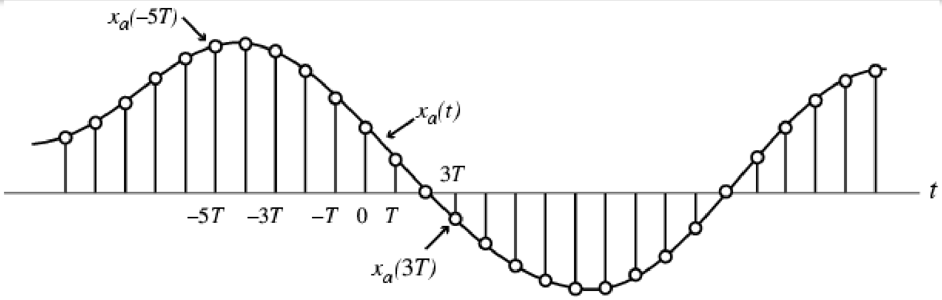
$$x[n] = A\cos(2\pi f_0 nT) = A\cos\left(\frac{2\pi\Omega_0}{\Omega_T}n\right)$$

$$nT = \frac{1}{F_T}n = \frac{2\pi}{\Omega_T}n$$

$$\Omega_0 = 2\pi f_0$$

Sampling Process

$$x[n]=x_a(nT)$$



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$F_T=1/T$ Sampling Frequency (samples/sec)

$$\Omega_0=2\pi f_0$$

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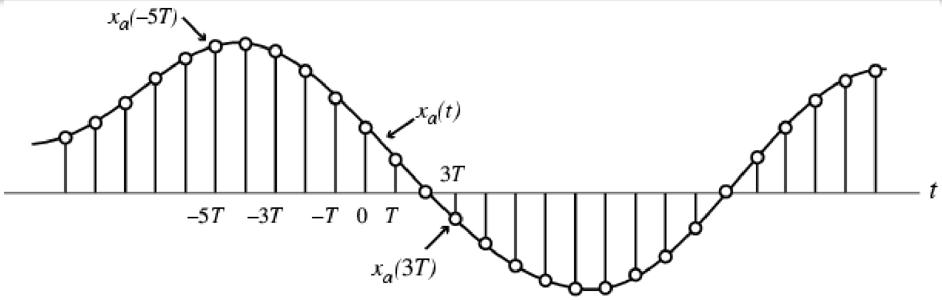

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$F_T = 1/T$ Sampling Frequency (samples/sec)

$$\Omega_0 = 2\pi f_0$$

$\Omega_T = 2\pi F_t$ Sampling angular Frequency(radians/sample)

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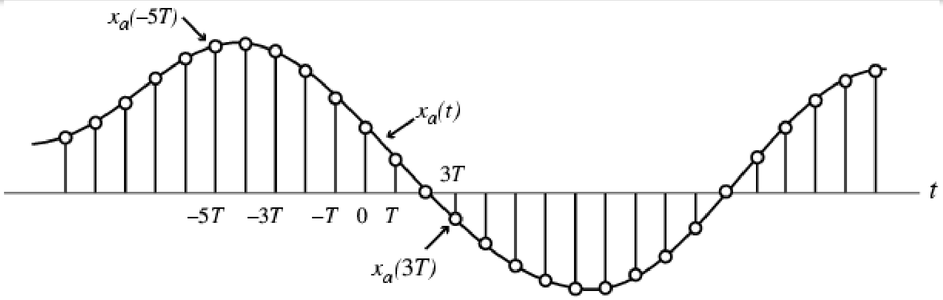

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$\Omega_T = 2\pi F_t$ Sampling angular Frequency(radians/sample)

$\omega_0 = \dfrac{2\pi f_0}{\Omega_T}$ normalized angular Frequency(radians/sample)

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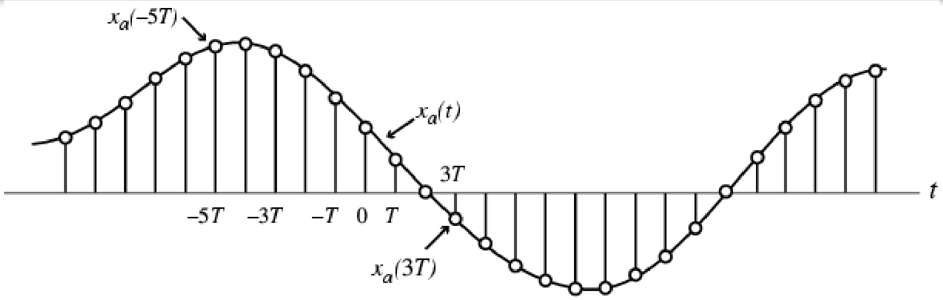

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Sampling Process

Example: $x_a(t) = A\cos(2\pi f_i t)$

$f_0 = 3\,\mathrm{Hz}$ $f_1 = 7\,\mathrm{Hz}$

$f_2 = 13\,\mathrm{Hz}$

$F_T = 10$ samples/sec

$$x_0[n] = \cos(0.6\pi n)$$

$$x_1[n] = \cos(1.4\pi n) = \cos((-2\pi + 1.4\pi)n) = \cos(-0.6\pi n)$$

$$x_2[n] = \cos(2.6\pi n) = \cos((2\pi + 0.6\pi)n) = \cos(0.6\pi n)$$

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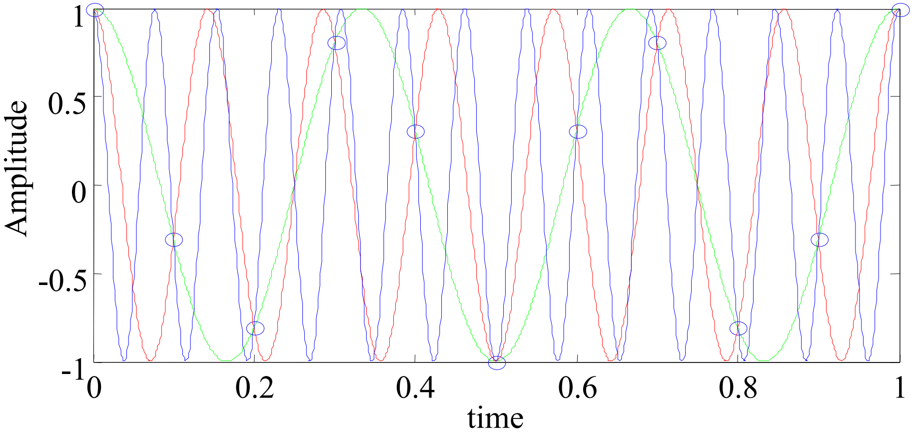

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$$x_1[n] = \cos(1.4\pi n) = \cos((-2\pi + 1.4\pi)n) = \cos(-0.6\pi n)$$

$$x_2[n] = \cos(2.6\pi n) = \cos((2\pi + 0.6\pi)n) = \cos(0.6\pi n)$$

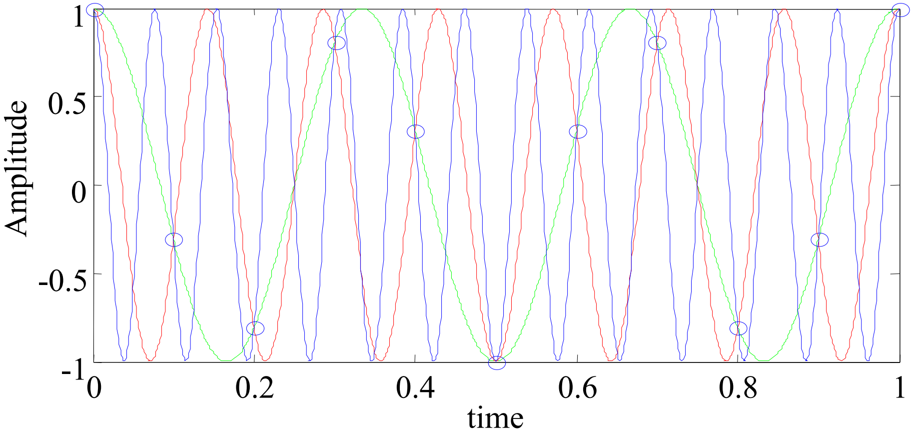
Sampling Process

Example: $x_a(t) = A\cos(2\pi f_i t)$

$f_0 = 3\,\mathrm{Hz}$ $f_1 = 7\,\mathrm{Hz}$

$f_2 = 13\,\mathrm{Hz}$

$F_T = 10 \text{ samples/sec}$



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Alias

Outline

- Time Domain Representation
- Typical Sequence & Sequence Representation
- Operation on Sequences
- Operation on Finite Length Sequences
- Sampling Process
- Correlation of Signals

Correlation of Signals

Correlation

$$r_{xy}[l] = \sum_{n=-\infty}^{\infty} x[n]y[n-l]$$

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$$= x[l] \circledast y[-l]$$

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