Chapter 6.

6.1. (a) $g[n] = nr^n \cos(\omega_0 n)\mu[n]$

As we know, the z-transform of $r^n \cos(\omega_0 n)\mu[n]$ is $\frac{1-(r\cos\omega_0)z^{-1}}{1-(2r\cos\omega_0)z^{-1}+r^2z^{-2}}$

Apply z-transform theorems: ~~assume $b[n]=n\alpha^n$~~ $a[n] = n\,(r^n \cos(\omega_0 n)\mu[n])$

So $G(z) = -z\frac{d}{dz}\left(\frac{1-(r\cos\omega_0)z^{-1}}{1-(2r\cos\omega_0)z^{-1}+r^2z^{-2}}\right) = -z\,\frac{-r\cos\omega_0 z^{-2}+2r^2z^{-3}-r^3\cos\omega_0 z^{-4}}{(1-(2r\cos\omega_0)z^{-1}+r^2z^{-2})^2}$

$= \frac{r\cos\omega_0 z^{-1}-2r^2z^{-2}+r^3\cos\omega_0 z^{-3}}{(1-(2r\cos\omega_0)z^{-1}+r^2z^{-2})^2}, \quad |z|>r.$

(b). $g[n] = nr^n\sin(\omega_0 n)\mu[n].$

As we know, the z-transform of $(r^n \sin\omega_0 n)\mu[n]$ is $\frac{(r\sin\omega_0)z^{-1}}{1-(2r\cos\omega_0)z^{-1}+r^2z^{-2}}$

Apply z-transform theorems

$G(z) = -z\frac{d}{dz}\left(\frac{(r\sin\omega_0)z^{-1}}{1-(2r\cos\omega_0)z^{-1}+r^2z^{-2}}\right)$

$= -z\,\frac{-r\sin\omega_0 z^{-2}(1-(2r\cos\omega_0)z^{-1}+r^2z^{-2}) - (2r\cos\omega_0 z^{-2} - \frac{2r^2}{z^3})\,r\sin\omega_0 z^{-1}}{(1-(2r\cos\omega_0)z^{-1}+r^2z^{-2})^2}$

$= -z\,\frac{-r\sin\omega_0 z^{-2}+2r^2\cos\omega_0\sin\omega_0 z^{-3}-r^3\sin\omega_0 z^{-4}-2r^2\sin\omega_0\cos\omega_0 z^{-3}+2r^3\sin\omega_0 z^{-4}}{(1-(2r\cos\omega_0)z^{-1}+r^2z^{-2})^2}$

$= -z\,\frac{-r\sin\omega_0 z^{-2}+r^3\sin\omega_0 z^{-4}}{(1-(2r\cos\omega_0)z^{-1}+r^2z^{-2})^2}$

$= \frac{r\sin\omega_0 z^{-1}-r^3\sin\omega_0 z^{-3}}{(1-(2r\cos\omega_0)z^{-1}+r^2z^{-2})^2}, \quad |z|>r$

6.3. $x[n] = \frac{1}{n!}\mu[n]$

$X(z) = \sum_{n=-\infty}^{\infty}\frac{1}{n!}\mu[n]z^{-n} = \sum_{n=0}^{\infty}\frac{z^{-n}}{n!}$

According to Taylor Expansion

$e^z = 1+z+\frac{z^2}{2!}+\frac{z^3}{3!}+\cdots+\frac{z^n}{n!}+\cdots$

Let $z=z^{-1}$

$e^{z^{-1}} = \sum_{n=0}^{\infty}\frac{z^{-n}}{n!}$

So $X(z) = e^{z^{-1}}$ So the ROC of $X(z)$ is $z\neq 0$.

6.5. (a). $X(z) = \sum_{n=-\infty}^{\infty}\delta[n]z^{-n} = 1$, ROC: All values of z

(b) $X(z) = \sum_{n=-\infty}^{\infty} n\alpha^n\mu[n]z^{-n} = S = \sum_{n=0}^{\infty} n\alpha^n z^{-n} = 0+(\alpha z^{-1})^1+2(\alpha z^{-1})^2+\cdots+n(\alpha z^{-1})^n$

$\alpha z^{-1}S = (\alpha z^{-1})^2+2(\alpha z^{-1})^3+\cdots+n(\alpha z^{-1})^{n+1}$

$(1-\alpha z^{-1})S = (\alpha z^{-1})^1+(\alpha z^{-1})^2+(\alpha z^{-1})^3+\cdots+(\alpha z^{-1})^n - n(\alpha z^{-1})^{n+1}$

$(1-\alpha z^{-1})S = \frac{\alpha z^{-1}(1-(\alpha z^{-1})^n)}{1-\alpha z^{-1}} - n(\alpha z^{-1})^{n+1}$

$S = \frac{\alpha z^{-1}-(\alpha z^{-1})^{n+1}}{(1-\alpha z^{-1})^2} - n(\alpha z^{-1})^{n+1}$

As S must converge, $\alpha z^{-1}<1$ $\quad |z|>|\alpha|$

$X(z) = S = \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}, \quad |z|>|\alpha|$

(c). $x[n] = (r^n \sin w_0 n)\mu[n]$.

$$X(z) = \sum_{n=0}^{+\infty} r^n \sin w_0 n \, z^{-n} = \sum_{n=0}^{+\infty} r^n z^{-n}\left(\frac{e^{jw_0 n} - e^{-jw_0 n}}{2j}\right)$$

$$= \frac{1}{2j}\sum_{n=0}^{+\infty}(rz^{-1}e^{jw_0})^n - \frac{1}{2j}\sum_{n=0}^{+\infty}(rz^{-1}e^{-jw_0})^n$$

$$= \frac{1}{2j}\frac{1}{1-rz^{-1}e^{jw_0}} - \frac{1}{2j}\frac{1}{1-rz^{-1}e^{-jw_0}} \qquad rz^{-1}e^{jw_0} < 1 \quad rz^{-1}e^{-jw_0} < 1$$

$$= \frac{1}{2j}\frac{2 - rz^{-1}(e^{jw_0} + e^{-jw_0})}{1 - rz^{-1}(e^{jw_0}+e^{-jw_0}) + r^2 z^{-2}}$$

$$= \frac{1}{2j}\frac{rz^{-1}(e^{jw_0} - e^{-jw_0})}{1 - rz^{-1}2\cos w_0 + r^2 z^{-2}} = \frac{rz^{-1}\sin w_0}{1-(2r\cos w_0)z^{-1} + r^2 z^{-2}} = \frac{(r\sin w_0)z^{-1}}{1-(2r\cos w_0)z^{-1}+r^2z^{-2}}$$

For ROC:

$|z| > |re^{jw_0}|$ So the ROC is $|z| > |r|$

$|z| > |re^{-jw_0}|$

6.7. (a). $x_1[n] = (0.6)^n \mu[n] + (-0.8)^n \mu[n]$

$$X_1(z) = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}} \qquad |z| > 0.6,\ |z| > 0.8. \Rightarrow |z| > 0.8.$$

(b) $x_2[n] = (0.6)^n \mu[n] - (-0.8)^n \mu[-n-1]$.

$$X_2(z) = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}} \qquad |z| > 0.6,\ |z| < 0.8. \Rightarrow 0.6 < |z| < 0.8$$

As $X_1(z)$ and $X_2(z)$ have different ROC, so they don't have the same z-transform

(c) $x_3[n] = -(0.6)^n \mu[-n-1] - (-0.8)^n \mu[-n-1]$

$$X_3(z) = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}} \qquad |z| < 0.6.$$

(d). $x_4[n] = -(0.6)^n \mu[-n-1] + (-0.8)^n \mu[n]$.

$$= \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}} \qquad |z| < 0.6 \ \& \ |z| > 0.8$$

As ROC doesn't exist, so this sequence doesn't exist a z transform

6.10. (a) $x_1[n] = \alpha^n \mu[n+1] + \beta^n \mu[n+2]$

As we know $\alpha^n \mu[n] \longleftrightarrow \frac{1}{1-\alpha z^{-1}}$ $\quad |z| > |\alpha|$

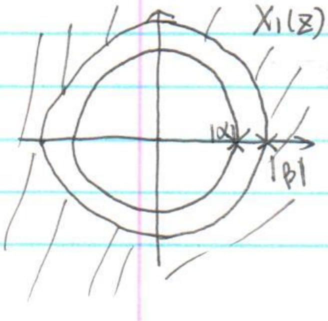
Apply Time-shifting theorem $\alpha^{n+1}\mu[n+1] \longleftrightarrow z\frac{1}{1-\alpha z^{-1}} = \frac{z}{1-\alpha z^{-1}}$

So the z-transform of $\alpha^n \mu[n+1]$ is $\frac{\alpha^{-1}z}{1-\alpha z^{-1}}$

Similarly: $\beta^n \mu[n] \longleftrightarrow \frac{1}{1-\beta z^{-1}}$ $\quad |z| > |\beta|$

$\beta^{n+2}\mu[n+2] \longleftrightarrow \frac{z^2}{1-\beta z^{-1}}$

So the z-transform of $\beta^n \mu[n+2]$ is $\frac{\beta^{-2}z^2}{1-\beta z^{-1}}$



$$X_1(z) = \frac{\alpha^{-1}z}{1-\alpha z^{-1}} + \frac{\beta^{-2}z^2}{1-\beta z^{-1}} \qquad |z| > |\beta|$$

$$= \frac{\alpha^{-1}z - \alpha^{-1}\beta + \beta^{-2}z^2 - \alpha\beta^{-2}z}{(1-\alpha z^{-1})(1-\beta z^{-1})} = \frac{(\alpha^{-1} - \alpha\beta^{-2})z + \beta^{-2}z^2 - \alpha^{-1}\beta}{(1-\alpha z^{-1})(1-\beta z^{-1})} \qquad |z| > \beta.$$

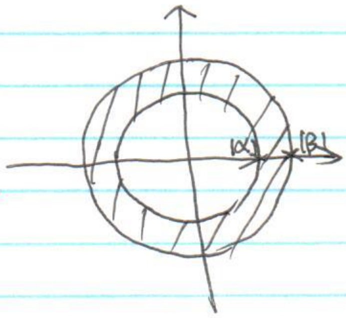
(b) $x_2[n] = \alpha^n \mu[n-2] + \beta^n \mu[-n-1]$

Solution: $\alpha^n \mu[n] \longleftrightarrow \frac{1}{1-\alpha z^{-1}}$ $\quad |z| > |\alpha|$.

$\alpha^{n-2}\mu[n-2] \longleftrightarrow \frac{z^{-2}}{1-\alpha z^{-1}}$

$\alpha^{n}\mu[n-2] \longleftrightarrow \frac{(\alpha z^{-1})^2}{1-\alpha z^{-1}}$

$-\beta^n \mu[-n-1] \longleftrightarrow \frac{1}{1-\beta z^{-1}}$ $\quad |z| < |\beta|$



$$X_2(z) = \frac{(\alpha z^{-1})^2}{1-\alpha z^{-1}} - \frac{1}{1-\beta z^{-1}} \quad |\alpha| < |z| < |\beta|$$

$$= \frac{\alpha^2 z^{-2} - \alpha^2\beta z^{-3} - 1 + \alpha z^{-1}}{(1-\alpha z^{-1})(1-\beta z^{-1})}$$

$$= \frac{-1 + \alpha z^{-1} + \alpha^2 z^{-2} - \alpha^2 \beta z^{-3}}{(1-\alpha z^{-1})(1-\beta z^{-1})} \quad |\alpha| < |z| < |\beta|$$

(c) $x_3[n] = \alpha^n \mu[n+2] + \beta^n \mu[-n-1]$

Solution: $\alpha^n \mu[n] \longleftrightarrow \frac{1}{1-\alpha z^{-1}}$

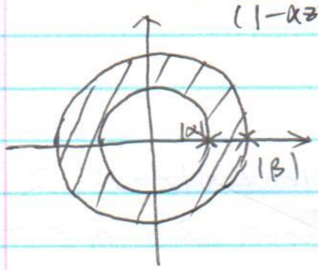
$\alpha^{n+2}\mu[n+2] \longleftrightarrow \frac{z^{2}}{1-\alpha z^{-1}}$

$\alpha^{n}\mu[n+2] \longleftrightarrow \frac{\alpha^{-2} z^{-2}}{1-\alpha z^{-1}}$. $\quad |z| > |\alpha|$

$-\beta^n \mu[-n-1] \longleftrightarrow \frac{1}{1-\beta z^{-1}}$ $\quad |z| < |\beta|$

$$X_3(z) = \frac{\alpha^{-2} z^{2}}{1-\alpha z^{-1}} - \frac{1}{1-\beta z^{-1}}$$

$$= \frac{\alpha^{-2} z^2 - \alpha^{-2} z - 1 + \alpha z^{-1}}{(1-\alpha z^{-1})(1-\beta z^{-1})} = \frac{\alpha^{-2} z(z-1) - 1 + \alpha z^{-1}}{(1-\alpha z^{-1})(1-\beta z^{-1})} \quad |\alpha| < |z| < |\beta|$$



6.13 (a) $X_a(z) = \frac{7 + 3.6 z^{-1}}{1 + 0.9 z^{-1} + 0.18 z^{-2}}$.

$$= \frac{7 + 3.6 z^{-1}}{(1+0.3z^{-1})(1+0.6z^{-1})} = \frac{A}{1+0.3z^{-1}} + \frac{B}{1+0.6z^{-1}}.$$

The poles is: $z_1 = 0.3$, $z_2 = 0.6$. $\quad = \frac{A + B + (0.3B + 0.6A)z^{-1}}{(1+0.3z^{-1})(1+0.6z^{-1})}$

So $\begin{cases} A + B = 7 \\ 0.3B + 0.6A = 3.6. \end{cases}$ $\quad$ So $\begin{cases} A = 5 \\ B = 2 \end{cases}$

So $X_a(z) = \frac{5}{1+0.3z^{-1}} + \frac{2}{1+0.6z^{-1}}$

For $|z| < 0.3$, $x_a[n] = -5(-0.3)^n \mu[-n-1] - 2(-0.6)^n \mu[-n-1]$.

For $0.3 < |z| < 0.6$, $x_a[n] = 5(-0.3)^n \mu[n] - 2(-0.6)^n \mu[-n-1]$

For $|z| > 0.6$, $x_a[n] = 5(-0.3)^n \mu[n] + 2(-0.6)^n \mu[n]$

(b) $X_b(z) = \dfrac{3-2z^{-1}}{1-0.6z^{-1}+0.08z^{-2}} = \dfrac{3-2z^{-1}}{(1-0.4z^{-1})(1-0.2z^{-1})}$

$= \dfrac{A}{1-0.4z^{-1}} + \dfrac{B}{1-0.2z^{-1}} = \dfrac{A+B-(0.2A+0.4B)z^{-1}}{(1-0.4z^{-1})(1-0.2z^{-1})}$

$\begin{cases} A+B=3 \\ 0.2A+0.4B=2 \end{cases}$ So $\begin{cases} A=-4 \\ B=7 \end{cases}$

So $X_b(z) = -\dfrac{4}{1-0.4z^{-1}} + \dfrac{7}{1-0.2z^{-1}}$

For $|z|<0.2$, $x_b[n]=4(0.4)^n\mu[-n-1]-7(0.2)^n\mu[-n-1]$.

For $0.2<|z|<0.4$, $x_b[n]=4(0.4)^n\mu[-n-1]+7(0.2)^n\mu[n]$

For $|z|>0.4$ $x_b[n]=7(0.2)^n\mu[n]-4(0.4)^n\mu[n]$.

(c) $X_c(z) = \dfrac{4-1.6z^{-1}-0.4z^{-2}}{(1+0.6z^{-1})(1-0.4z^{-1})^2} = \dfrac{A}{1+0.6z^{-1}} + \dfrac{B}{1-0.4z^{-1}} + \dfrac{C}{(1-0.4z^{-1})^2}$

$= \dfrac{A(1-0.8z^{-1}+0.16z^{-2})+B(1+0.2z^{-1}-0.24z^{-2})+C+0.6Cz^{-1}}{(1+0.6z^{-1})(1-0.4z^{-1})^2}$

$= \dfrac{A+B+C+(0.2B+0.6C-0.8A)z^{-1}+(0.16A-0.24B)z^{-2}}{(1+0.6z^{-1})(1-0.4z^{-1})^2}$

$\begin{cases} A+B+C=4 \\ 0.2B+0.6C-0.8A=-1.6 \\ 0.16A-0.24B=-0.4 \end{cases}$ $\begin{cases} A=2 \\ B=3 \\ C=-1 \end{cases}$

$X_c(z) = \dfrac{2}{1+0.6z^{-1}} + \dfrac{3}{1-0.4z^{-1}} - \dfrac{1}{(1-0.4z^{-1})^2}$

For $|z|<0.4$, $x_c[n]=-2(-0.6)^n\mu[-n-1]-3(0.4)^n\mu[-n-1]+(n+1)(0.4)^n\mu[-n-1]$

For $0.4<|z|<0.6$, $x_c[n]=-2(-0.6)^n\mu[-n-1]+3(0.4)^n\mu[n]-(n+1)(0.4)^n\mu[n]$

For $|z|>0.6$, $x_c[n]=2(-0.6)^n\mu[n]+3(0.4)^n\mu[n]-(n+1)(0.4)^n\mu[n]$

6.81. (a) Poles are $z=-0.3, 0.6, -5$. So if the ROC is $0.6<|z|<5$, then the frequency response exist

(b) If ROC contain unit circle, so the system is stable.

So the system can be stable if its ROC is $0.6<|z|<5$.

But under this circumstance, it's not causal because it's a two-sided signal.

(c) $H(z) = \dfrac{3(z+1.8)(z-4)}{(z+0.3)(z-0.6)(z+5)} = \dfrac{A}{z+0.3} + \dfrac{B}{z-0.6} + \dfrac{C}{z+5}$

So $h[n]=A(-0.3)^n\mu[n]+B(0.6)^n\mu[n]-C(-5)^n\mu[-n-1]$.

A+

Matlab Homework for Chapter 6

周润珂 2016200104026

M6.1

a) Code:

```
% Program 6_1
% Determination of the Factored Form
% of a Rational z-Transform
%
num = input('Type in the numerator coefficients = ');
den = input('Type in the denominator coefficients = ');
K = num(1)/den(1);
Numfactors = factorize(num)
Denfactors = factorize(den)
disp('Numerator factors');disp(Numfactors);
disp('Denominator factors');disp(Denfactors);
disp('Gain constant');disp(K);
zplane(num, den)
```

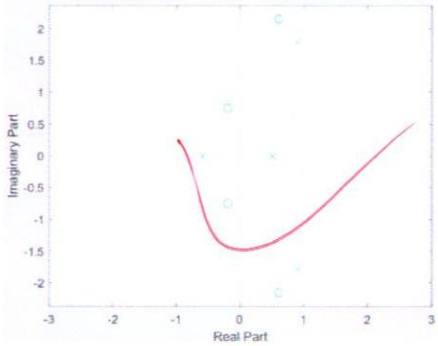
```
Type in the numerator coefficients = [3 -2.4 15.36 3.84 9]
Type in the denominator coefficients = [5 -8.5 17.6 4.7 -6]
```

Results:

```
Numerator factors
   1.000000000000000  -1.199999999999999   4.999999999999989
   1.000000000000000   0.399999999999999   0.600000000000011

Denominator factors
   1.000000000000000  -1.800000000000000   4.000000000000000
   1.000000000000000   0.600000000000000                   0
   1.000000000000000  -0.500000000000000                   0

Gain constant
   0.600000000000000
```



The factored form of the z-transform is:

$$G1(z) = 0.6 \cdot \frac{(1 - 1.199z^{-1} + 4.99z^{-2})(1 + 0.399z^{-1} + 0.6z^{-2})}{(1 - 1.8z^{-1} + 4z^{-2})(1 + 0.6z^{-1})(1 - 0.5z^{-1})}$$

There are ROCs associated with G1(z):

$$R1: |z| < 0.5, R2: 0.5 < |z| < 0.6, R3: 0.6 < |z| < 2, R4: |z| >> 2$$

The inverse z-transform of R1 is left-sided. The inverse z-transform of R2 is two-sided. The inverse z-transform of R3 is two-sided. The inverse z-transform of R4 is right-sided.

b) Codes:

```
Type in the numerator coefficients = [2 0.2 6.4 4.6 2.4]
Type in the denominator coefficients = [5 1 6.6 0.42 24]
```

The factored form of the z-transform is: $G2(z) = 0.6 \cdot \frac{(1-0.659z^{-1}+3.34z^{-2})(1+0.759z^{-1}+0.359z^{-2})}{(1+1.854z^{-1}+2.294z^{-2})(1-1.654z^{-1}+2.092z^{-2})}$

There are ROCs associated with G1(z):

$$R1: |z| < 1.446, R2: 1.446 < |z| < 1.515$$
$$R3: |z| >> 1.515$$

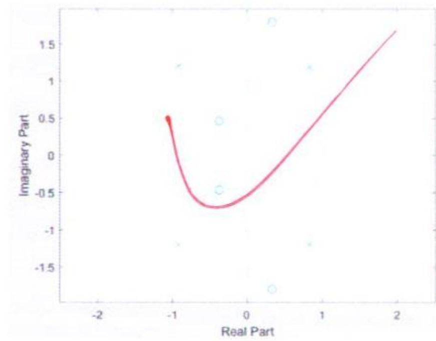
The inverse z-transform of R1 is left-sided. The inverse z-transform of R2 is two-sided. The inverse z-transform of R3 is right-sided.

Results:

```
Numerator factors
   1.000000000000000  -0.659202286947652   3.341329391493416
   1.000000000000000   0.759202286947652   0.359138492318365

Denominator factors
   1.000000000000000   1.853979199237186   2.294318582077411
   1.000000000000000  -1.653979199237186   2.092124449279311

Gain constant
   0.400000000000000
```



M6.2

a) Codes:

```
num = input('Type in numerator coefficients = ');
den = input('Type in denominator coefficients = ');
[r,p,k] = residuez(num,den);
disp('Residues');disp(r')
disp('Poles');disp(p')
disp('Constants');disp(k)

Type in numerator coefficients = [7]
Type in denominator coefficients = [1 0.3 -0.1]
```

Results:

```
Residues
     5     2

Poles
  -0.500000000000000   0.200000000000000
```

The partial-fraction expansions is:$X_a(z) = \frac{5}{1+0.5z^{-1}} + \frac{2}{1-0.2z^{-1}}$

So the inverse z-transform of Xa(z) is:

If $|z| < 0.2, x_a[n] = -5(-0.5)^n\mu[-n-1] - 2(0.2)^n\mu[-n-1]$

If $0.2 < |z| < 0.5, x_a[n] = -5(-0.5)^n\mu[-n-1] + 2(0.2)^n\mu[n]$

If $|z| > 0.5, x_a[n] = 5(-0.5)^n\mu[n] + 2(0.2)^n\mu[n]$

b) Codes:

```
Type in numerator coefficients = [0 3 1.8 1.28]
Type in denominator coefficients = [1 0.3 -0.24 -0.08]
```

Results:

```
Residues
  7.2346 + 0.0000i   15.9877 + 0.0000i   -7.2222 - 0.0000i

Poles
  0.5000 + 0.0000i   -0.4000 - 0.0000i   -0.4000 + 0.0000i

Constants
  -16
```

The partial-fraction expansions is:$X_a(z) = -16 + \frac{7.2346}{1-0.5z^{-1}} - \frac{7.2222}{1+0.4z^{-1}} + \frac{15.9877}{(1+0.4z^{-1})^2}$

So the inverse z-transform of Xa(z) is:

If $|z| < 0.4$,

$x_a[n] = -16\delta[n] - 7.2346(0.5)^n\mu[-n-1] + 7.2222(-0.4)^n\mu[-n-1] - 15.9877(n+1)(-0.4)^n\mu[-n-1]$

If $0.4 < |z| < 0.5$,

$x_a[n] = -16\delta[n] - 7.2346(0.5)^n\mu[-n-1] - 7.2222(-0.4)^n\mu[n] + 15.9877(\mathrm{n}+1)(-0.4)^n\mu[n]$

If $|z| > 0.5$,

$x_a[n] = -16\delta[n] + 7.2346(0.5)^n\mu[n] - 7.2222(-0.4)^n\mu[n] + 15.9877(\mathrm{n}+1)(-0.4)^n\mu[n]$

M6.4:

a) $X_1(z) = 2 + \frac{6}{2+z^{-1}} - \frac{12.5}{2.5-z^{-1}} = \frac{-17.5z^{-1}-2z^{-2}}{5+0.5z^{-1}-z^{-2}}$

The inverse z-transform of X1(z) is:$x_1[n] = 2\delta[n] + 3(-0.5)^n\mu[n] - 5(0.4)^n\mu[n]$

Codes:

```
L = input('Type in the length of output vector = ');
% Read in the numerator and denominator coefficients of
% the z-transform
num = input('Type in the numerator coefficients = ');
den = input('Type in the denominator coefficients = ');
% Compute the desired number of inverse transform coefficients
[y,t] = impz(num,den,L);
disp('Coefficients of the power series expansion');
disp(y')
```

Results:

As we can see, the inverse z-transform we calculated using MATLAB is identical to the result we derive from the equation.

b) $X_2(z) = 4 - \frac{10}{5+2z^{-1}} + \frac{1-0.48z^{-1}}{1+0.36z^{-2}} = \frac{15+7.6z^{-1}+2.64z^{-2}+2.88z^{-3}}{5+2z^{-1}+1.8z^{-2}+0.72z^{-3}}$

Codes:

```
Type in the length of output vector = 30
Type in the numerator coefficients = [15 7.6 2.64 2.88]
Type in the denominator coefficients = [5 2 1.8 0.72]
```

Results:

c) $X_3(z) = \frac{-6}{(6+3z^{-1})^2} + \frac{9}{6+3z^{-1}} + \frac{4}{1+0.25z^{-2}} = \frac{256+228z^{-1}+64z^{-2}+9z^{-3}}{48+48z^{-1}+24z^{-2}+12z^{-3}+3z^{-4}}$

Codes:

```
Type in the length of output vector = 30
Type in the numerator coefficients = [256 228 64 9]
Type in the denominator coefficients = [48 48 24 12 3]
```

Results:

d) $X_4(z) = -4 + \frac{6}{6+2z^{-1}} + \frac{z^{-1}}{6+3z^{-1}+0.8z^{-2}} = -\frac{108+96z^{-1}+36.4z^{-2}+6.4z^{-3}}{36+30z^{-1}+10.8z^{-2}+1.6z^{-3}}$

Codes:

```
Type in the length of output vector = 30
Type in the numerator coefficients = [-108 -96 -36.4 -6.4]
Type in the denominator coefficients = [36 30 10.8 1.6]
```

Results: