Glasgow College, UESTC

Digital Signal Processing

Homework 5

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HOMEWORK 4

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CONTENTS

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LIST OF FIGURES

INTRODUCTION

This report is the homework that should be finished on the MATLAB, there are four questions about Digital Signal Processing. Which is about generate the complex exponential functions, explore their properties, understand the sampling theory and understand the true meaning of the autocorrelation.

1 PROBLEM-6.1

1.1 Question(a)

$$Z\{nr^n\cos(\omega_o n)\,u[n]\} = -z\frac{d}{dz}\left[\frac{1-r\cos(\omega_o)z^{-1}}{1-2r\cos(\omega_o)+r^2z^{-2}}\right]$$

$$= \frac{r\cos(\omega_o)z^{-1}-2r^2\cos(\omega_o)z^{-2}+r^3\cos(\omega_o)z^{-3}}{\left(1-2r\cos(\omega_o)z^{-1}+r^2z^{-2}\right)^2}$$

$$\text{ROC} : |z| > r$$

1.2 Question(b)

$$Z\{nr^n\sin(\omega_o n)\,u[n]\} = -z\frac{d}{dz}\left[\frac{r\sin(\omega_o)z^{-1}}{1-2r\cos(\omega_o)z^{-1}+r^2z^{-2}}\right]$$

$$= \frac{r\sin(\omega_o)^{-1}-r^3\sin(\omega_o)^{-3}}{\left(1-2r\cos(\omega_o)z^{-1}+r^2z^{-2}\right)^2}$$

$$\text{ROC} : |z| > r$$

2 PROBLEM-6.3

The obvious argument is using Taylor Series expansion, we cazn conclude:

$$X(z) = \sum_{n=0}^{\infty}\frac{z^{-n}}{n!} = \exp\left(z^{-1}\right), \text{ROC} : z \neq 0$$

3 PROBLEM-6.5

3.1 Question(a)

$$Z\{\delta[n]\} = \sum_{n=-\infty}^{\infty}\delta[n]z^{-n} = \delta[0] = 1 \text{ This is converge at any point.}$$

3.2 Question(b)

$$x\lfloor n\rfloor = \alpha^n\mu|n| <= ZT => Z\{x[n]\} = X(z) = \sum_{n=-\infty}^{\infty}x[n]z^{-n} = \frac{1}{1-\alpha z^{-1}}, |z| > |\alpha|$$

$$g[n] = nx[n] <= ZT => Z\{g[n]\} = G(z) = \sum_{n=-\infty}^{\infty}nx[n]z^{-n}$$

So we can conclude that:

$$G(z) = -z\frac{dX(z)}{dz} = \frac{\alpha z^{-1}}{\left(1-\alpha z^{-1}\right)^2}, |z| > |\alpha|$$

3.3 Question(c)

$$x[n] = r^n \sin(\omega_o n)\,\mu[n] = \frac{r^n}{2j}\left(e^{j\omega_o n} - e^{-j\omega_o n}\right)\mu[n]$$

$$Z\{r^n \sin(\omega_o n)\,\mu[n]\} = \frac{1}{2j}\left(\frac{1}{1-re^{j\omega_o}z^{-1}}\right) - \frac{1}{2j}\left(\frac{1}{1-re^{-j\omega_o}z^{-1}}\right)$$

$$= \frac{\frac{r}{2j}\left(e^{j\omega_o}-e^{-j\omega_o}\right)z^{-1}}{1-r\left(e^{j\omega_o}+e^{-j\omega_o}\right)z^{-1}+r^2z^{-2}}$$

$$= \frac{r\sin(\omega_o)z^{-1}}{1-2r\cos(\omega_o)z^{-1}+r^2z^{-2}}, \quad \text{ROC} : |z| > |r|$$

4 PROBLEM-6.7

None of them have the same Z-transform.

4.1 Question(a)

$$x_1[n] = (0.6)^n\mu[n] + (-0.8)^n\mu[n]$$

$$Z\{x_1[n]\} = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}}, \ \text{ROC} : \ |z| > 0.8$$

4.2 Question(b)

$$x_1[n] = (0.6)^n\mu[n] - (-0.8)^n\mu[-n-1]$$

$$Z\{x_2[n]\} = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}}, \ \text{ROC} : \ 0.6 < |z| < 0.8$$

4.3 Question(c)

$$x_1[n] = -(0.6)^n\mu - n - 1 - (-0.8)^n\mu[-n-1]$$

$$Z\{x_3[n]\} = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}}, \ \text{ROC} : \ |z| < 0.6$$

4.4 Question(d)

$$x_1[n] = -(0.6)^n\mu[-n-1] + (-0.8)^n\mu[n]$$

$$Z\{x_4|n]\} = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}}$$

$$\text{ROC} : \ |z| < 0.6 \ \mathit{and} \ |z| > 0.8 \ \mathit{but\ all\ not\ converge}$$

5 PROBLEM-6.10

5.1 Question(a)

$$x_1[n] = \alpha^n u[n+2] + \beta^n \mu[n+2] \text{ with } |\beta| > |\alpha|$$

$$Z\{\alpha^n \mu[n+2] + \beta^n \mu[n+2]\} = \sum_{n=-2}^{\infty} X_1[n] z^{-n}$$

$$= \frac{1}{\alpha z^{-1}(1-\alpha z^{-1})} + \frac{1}{\beta^2 z^{-2}(1-\beta^1 z^{-1})}$$

$$\text{ROC} : |Z| > \beta$$

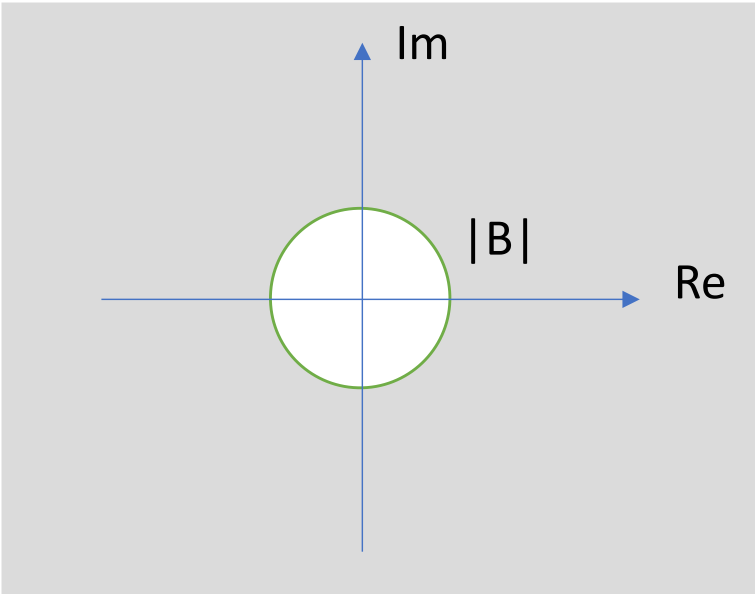

Figure 1: ROC for Q1

5.2 Question(b)

$$x_2[n] = \alpha^n u[n-2] + \beta^n \mu[-n-1] \text{ with } |\beta| > |\alpha|$$

$$Z\{x_2[n]\} = Z\{\alpha^n \mu[n-2]\} + Z\{\beta^n \mu[-n-1]\}$$

$$= \frac{\alpha^2 z^{-2}}{1-\alpha z^{-1}} - \frac{1}{1-\beta z^{-1}}$$

$$\text{ROC} : \alpha < |Z| < \beta$$

5.3 Question(c)

$$x_1[n] = \alpha^n u[n+2] + \beta^n \mu[-n-1] \text{ with } |\beta| > |Z| > |\alpha|$$

$$Z\{x_2[n]\} = Z\{\alpha^n \mu[n+2]\} + Z\{\beta^n \mu[-n-1]\}$$

$$= \frac{\alpha^{-2} z^2}{1-\alpha z^{-1}} - \frac{1}{1-\beta z^{-1}}$$

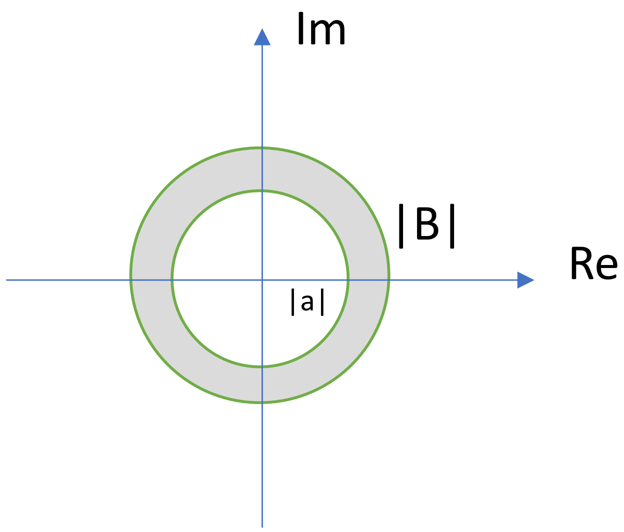

Figure 2: ROC for Q2

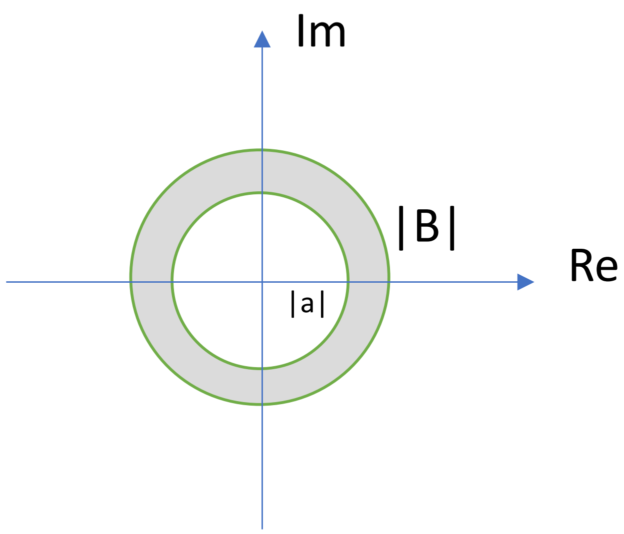

Figure 3: ROC for Q3

6 PROBLEM-6.13

6.1 Question(a)

$$X_a(z) = \frac{7+3.6z^{-1}}{1+0.9z^{-1}+0.18z^{-2}} = \frac{2}{1+0.6z^{-1}} + \frac{5}{1+0.3z^{-1}}$$

$$\text{left-sided: } |z| < 0.3, x_a[n] = -2(-0.6)^{-n-1}\mu[-n-1] - 5(-0.3)^n\mu[-n-1]$$

$$\text{right-sided: } 0.6 < |z|, x_c[n] = 2(-0.6)^n\mu[n] + 5(-0.3)^n\mu[n]$$

$$\text{two-sided: } 0.3 < |z| < 0.6, x_b[n] = -2(-0.6)^n\mu[-n-1] + 5(-0.3)^n\mu[n]$$

6.2 Question(b)

$$X_b(z) = \frac{3-2z^{-1}}{1-0.6z^{-1}+0.08z^{-2}} = \frac{-4}{1-0.4z^{-1}} + \frac{7}{1-0.2z^{-1}}$$

$$\text{left-sided: } |z| < 0.2, x_a[n] = 4(0.4)^n\mu[-n-1] - 7(0.2)^n\mu[-n-1]$$

$$\text{two-sided: } 0.4 < |z|, x_c[n] = 4(0.4)^n\mu[-n-1] + 7(0.2)^n u[n]$$

$$\text{right-sided: } 0.2 < |z| < 0.4, x_b[n] = -4(0.4)^n u[n] + 7(0.2)^n\mu[n]$$

6.3 Question(c)

$$X_C(z) = \frac{4-1.6z^{-1}-0.4z^{-2}}{\left(1+0.6z^{-1}\right)\left(1-0.4z^{-1}\right)^2} = \frac{2}{1+0.6z^{-1}} + \frac{3}{1-0.4z^{-1}} + \frac{-1}{\left(1-0.4z^{-1}\right)^2}$$

$$\text{left-sided: } |z| < 0.4, x_a[n] =$$
$$-2(-0.6)^n\mu[-n-1] - 3(0.4)^n\mu[-n-1] + (n+1)(0.4)^n\mu[-n-1]$$

$$\text{right-sided: } 0.5 < |z|, \quad x_c[n] = 2(-0.6)^n\mu[n] + 3(0.4)^n\mu[n] - (n+1)(0.4)^n\mu[n]$$

$$\text{two-sided: } 0.4 < |z| < 0.6, x_b[n] =$$
$$-2(-0.6)^n\mu[-n-1] + 3(0.4)^n\mu[n] - (n+1)(0.4)^n\mu[n]$$

7 PROBLEM-6.81

$$H(Z) = \frac{3(Z+1.8)(z-4)}{(z+0.3)(Z-0.6)(z+5)}$$

7.1 Question(a)

Since H(z) has poles at 0.3, 0.6, and 5, ROC of a two-sided sequence would be 0.6 < |z| < 5. The ROC contains the unit circle, which means H exist.

7.2 Question(b)

The system can be stable if the ROC is $0.6 < |z| < 5$. But it cannot be causal because this ROC means it is two-sided.

7.3 Question(c)

$$h[n] = A(-0.3)^n \mu[n] + B(0.6)^n \mu[n] + C(-5)^n \mu[-n]$$

$$where\ A\ B\ and\ C\ are\ constant.$$

8 PROBLEM-M6.1

8.1 Question(a)

```
num = [3 -2.4 15.36 3.84 9];
den = [5 -8.5 17.6 4.7 -6];
% num = input('Type in the numerator coefficients = ');
% den = input('Type in the denominator coefficients = ');
K = num(1)/den(1);
Numfactors = factorize(num);
Denfactors = factorize(den);
p1 = figure;
disp('Numerator factors');
disp(Numfactors);
disp('Denominator factors');
disp(Denfactors);
disp('Gain constant');
disp(K);
zplane(num,den)
grid on;
set(p1, 'PaperPosition', [0.05 0.05 7 7]);
set(p1, 'PaperSize', [7.05 7.05]); %Keep the same paper size
saveas(p1,['6_1.pdf'],'pdf')
```

```
>> problem_6_1
Numerator factors
   1.000000000000000  -1.200000000000001   5.000000000000004
   1.000000000000000   0.400000000000001   0.599999999999997

Denominator factors
   1.000000000000000  -1.799999999999999   4.000000000000002
   1.000000000000000   0.600000000000002                   0
   1.000000000000000  -0.500000000000003                   0

Gain constant
```

```
0.600000000000000
```

$$G_1(z) = \frac{3z^4 - 2.4z^3 + 15.36z^2 + 3.84z + 9}{5z^4 - 8.5z^3 + 17.6z^2 + 4.7z - 6}$$

$$G_1(z) = \frac{0.6(1 - 1.2z^{-1} + 5z^{-2})(1 + 0.4z^{-1} + 0.6z^{-2})}{(1 - 1.8z^{-1} + 4z^{-2})(1 + 0.6z^{-1})(1 - 0.5z^{-1})}$$

which implies that it has three ROCs :

$$\begin{aligned}
&|z| < 0.5, \text{ the inverse } z \text{ -transform is left-sided} \\
&0.5 < |z| < 0.6, \text{ the inverse } z \text{ -transform is two-sided} \\
&0.6 < |z| < 2, \text{ the inverse } z \text{ -transform is two-sided} \\
&|z| > 2, \text{ the inverse } z \text{ -transform is right-sided}
\end{aligned}$$

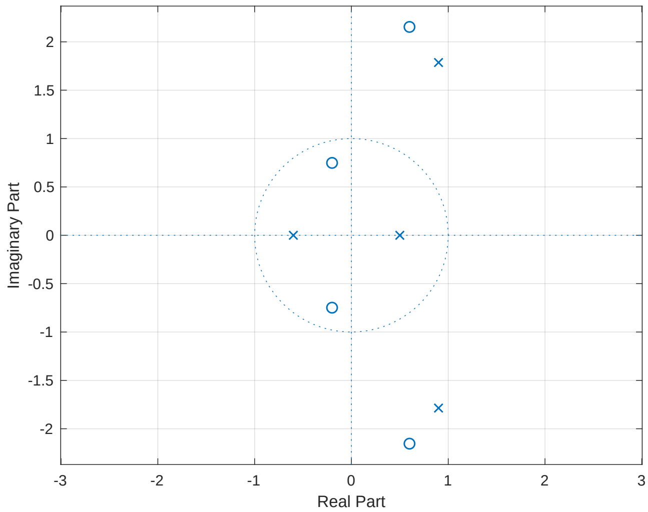

Figure 4: Poles and Zeros

8.2 Question(b)

```
num = [2 0.2 6.4 4.6 2.4];
den = [5 1 6.6 0.42 24];
% num = input('Type in the numerator coefficients = ');
% den = input('Type in the denominator coefficients = ');
K = num(1)/den(1);
Numfactors = factorize(num);
Denfactors = factorize(den);
```

```
p1b = figure;
disp('Numerator factors');
disp(Numfactors);
disp('Denominator factors');
disp(Denfactors);
disp('Gain constant');
disp(K);
zplane(num,den)
grid on;
set(p1b, 'PaperPosition', [0.05 0.05 7 7]);
set(p1b, 'PaperSize', [7.05 7.05]); %Keep the same paper size
saveas(p1b,['6_1b.pdf'],'pdf')
```

```
>> problem_6_1
Numerator factors
   1.000000000000000  -0.659202286947652   3.341329391493415
   1.000000000000000   0.759202286947652   0.359138492318365

Denominator factors
   1.000000000000000   1.853979199237185   2.294318582077409
   1.000000000000000  -1.653979199237185   2.092124449279308

Gain constant
   0.400000000000000
```

$$G_2(z) = \frac{2z^4+0.2z^3+6.4z^2+4.6z+2.4}{5z^4+z^3+6.6z^2+0.42z+24}$$

$$G_2(z) = \frac{0.4\left(1-0.6592z^{-1}+3.3413z^{-2}\right)\left(1+0.7592z^{-1}+0.3591z^{-2}\right)}{\left(1+1.8540z^{-1}+2.2943z^{-2}\right)\left(1-1.6540z^{-1}+2.0921z^{-2}\right)}$$

Which implies that it has fourROCs :

$$\begin{aligned}
&|z| < 1.4464, \text{ the inverse } z\text{ -transform is left-sided}\\
&1.4464 < |z| < 1.5147, \text{ the inverse } z\text{ -transform is two-sided}\\
&|z| > 1.5147, \text{ the inverse } z\text{ -transform is right-sided}
\end{aligned}$$

9 PROBLEM-M6.2

```
%Program 6_3
% Partial-Fraction Expansion of Rational z-Transform
%
num1 = [7];
den1 = [1 0.3 -0.1];
[r1,p1,k1] = residuez(num1,den1);
disp('Residues');
disp(r1')
disp('Poles');
disp(p1')
```

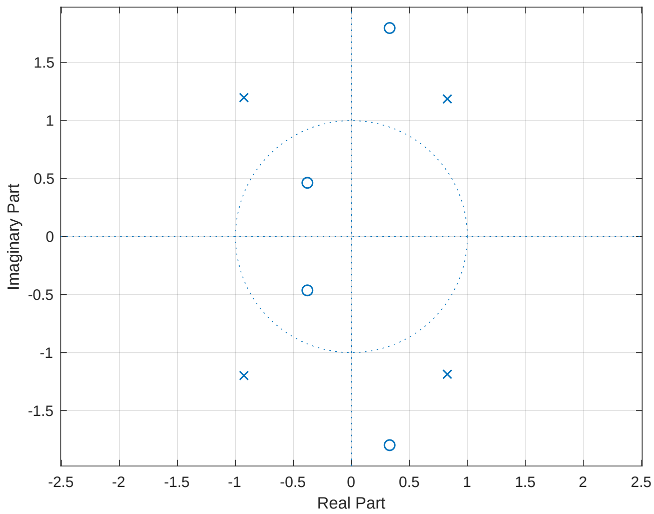

Figure 5: Poles and Zeros

```
disp('Constants');
disp(k1)

num2 = [0 3 1.8 1.28];
den2 = [1 0.3 -0.24 -0.08];
[r2,p2,k2] = residuez(num2,den2);
disp('Residues');
disp(r2')
disp('Poles');
disp(p2')
disp('Constants');
disp(k2)
```

```
>> problem_6_2
Residues
     5     2

Poles
  -0.500000000000000   0.200000000000000

Constants
Residues
  7.234567901234565 + 0.000000000000000i 15.987654320987652 + ...
     0.000002839715556i -7.222222222222222 - 0.000002839715556i

Poles
```

```
  0.500000000000000 + 0.000000000000000i -0.400000000000000 - ...
     0.000000007104771i -0.400000000000000 + 0.000000007104771i

Constants
   -16
```

9.1 Question(a)

According to the results shown below, the partial-fraction expansion is $x_a(z) = \frac{5}{1+0.5z^{-1}} + \frac{2}{1-0.2z^{-1}}$, since the poles are at $z_1 = -0.5, z_2 = 0.2$

, which implies that it has three ROCs: $|Z| < 0.2, 0.2 < |Z| < 0.5, |2| > 0.5$

$$\text{ROCs} : |Z| < 0.2, 0.2 < |Z| < 0.5, |2| > 0.5$$

$$\text{When } |Z| < 0.2 : x_a[n] = -5(-0.5)^n u[-n-1] - 2(0.2)^n u[-n-1]$$

$$\text{When } 0.2 < |Z| < 0.5 : x_a[n] = -5(-0.5)^n u[-n-1] + 2(0.2)^n u[n]$$

$$\text{When } |Z| > 0.5 : x_a[n] = 5(-0.5)^n u[n] + 2(0.2)^n u[n]$$

9.2 Question(b)

According to the results shown below, the partial-fraction expansion is:

$$X_b(z) = -16 + \frac{7.2346}{1-0.5z^{-1}} + \frac{15.9877}{1+0.4z^{-1}} - \frac{7.2222}{(1+0.4z^{-1})^2}$$

which implies that it has three ROCs : $|Z| < 0.4, 0.4 < |Z| < 0.5, |Z| > 0.$

$$\text{When } |Z| < 0.4 : x_b[n] = -7.2346(0.5)^n u[-n-1] - 15.9877(-0.4)^n u[-n-1] + 7.2222(n+1)(-0.4)^n u[-n-1]$$

$$\text{When } 0.4 < |Z| < 0.5 : x_b[n] = -7.2346(0.5)^n u[-n-1] + 15.9877(-0.4)^n u[n] + 7.2222(n+1)(-0.4)^n u[n]$$

$$\text{When } |Z| > 0.5 : x_b[n] = 7.2346(0.5)^n u[n] + 15.9877(-0.4)^n u[n] - 7.2222(n+1)(-0.4)^n u[n]$$

10 PROBLEM-M6.4

```
%Program 6_4
clc
clear
format short
L = 30;
```

```

%% Problem a
num1 = [-17.5 -2];
den1 = [5 0.5 -1];
[y, t] = impz(num1,den1,L);
disp('Question a');
disp('coefficient of the power serious expension');
disp(y')


%% Problem b
num1 = [15 8.4 2.64 2.88];
den1 = [5 2 1.8 0.72];
[y, t] = impz(num1,den1,L);
disp('Question b');
disp('coefficient of the power serious expension');
disp(y')


%% Problem c
num1 = [256 228 64 9];
den1 = [48 48 24 12 3];
[y, t] = impz(num1,den1,L);
disp('Question c');
disp('coefficient of the power serious expension');
disp(y')


%% Problem d
num1 = [-54 -48 -12.8 -3.2];
den1 = [18 15 5.4 0.8];
[y, t] = impz(num1,den1,L);
disp('Question d');
disp('coefficient of the power serious expension');
disp(y')
```

10.1 Question(a)

```
Question a
coefficient of the power serious expension
Columns 1 through 14

 -3.5000   -0.0500   -0.6950    0.0595   -0.1450    0.0264   ...
    -0.0316    0.0084   -0.0072    0.0024   -0.0017    0.0006   ...
    -0.0004    0.0002

Columns 15 through 28

 -0.0001    0.0000   -0.0000    0.0000   -0.0000    0.0000   ...
    -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000   ...
    -0.0000    0.0000

```

```
Columns 29 through 30

 -0.0000    0.0000
```

$$X_1(z) = 2 + \frac{6}{2+z^{-1}} - \frac{12.5}{2.5-z^{-1}}$$
$$= 2 + \frac{3}{1+(1/2)z^{-1}} - \frac{5}{1-(2/5)z^{-1}}, \quad |z| > 0.5$$

$$x_1[n] = 2\delta[n] + 3\left(-\tfrac{1}{2}\right)^n \mu[n] - 5\left(\tfrac{2}{5}\right)^n \mu[n]$$

Comparing to the first 30 samples, they are identical to each other.

10.2 Question(b)

```
Question b
coefficient of the power serious expension
  Columns 1 through 14

    3.0000    0.4800   -0.7440    0.2688    0.0912   -0.0261   ...
       -0.0611    0.0207    0.0175   -0.0056   -0.0070    0.0023 ...
          0.0024   -0.0008

  Columns 15 through 28

   -0.0009    0.0003    0.0003   -0.0001   -0.0001    0.0000    ...
      0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000    ...
      -0.0000    0.0000

  Columns 29 through 30

    0.0000   -0.0000
```

$$X_2(z) = 4 - \frac{10}{5+2z^{-1}} + \frac{1-0.48z^{-1}}{1+0.36z-2}$$
$$= 4 - \frac{2}{1+\frac{2}{5}z^{-1}} + \frac{0.5-j0.4}{1+j0.6z^{-1}} + \frac{0.5+j0.4}{1-j0.6z^{-1}}, \quad |z| > 1$$

$$x_2[n] = 4\delta[n] - 2\left(-\tfrac{2}{5}\right)^n \mu[n] + (0.5-j0.4)(-j0.6)^n\mu[n] + (0.5+j0.4)(j0.6)^n\mu[n]$$

Comparing to the first 30 samples, they are identical to each other.

10.3 Question(c)

```
Question c
coefficient of the power serious expension
```

```
  Columns 1 through 14

    5.3333   -0.5833   -0.7500   -0.1042    0.2917   -0.0156   ...
       -0.0573   -0.0013    0.0156    0.0003   -0.0042    0.0002 ...
          0.0008    0.0001

  Columns 15 through 28

   -0.0003    0.0000    0.0000    0.0000   -0.0000    0.0000    ...
       0.0000    0.0000   -0.0000    0.0000    0.0000    0.0000    ...
      -0.0000    0.0000

  Columns 29 through 30

    0.0000    0.0000
```

$$\begin{aligned} X_3(z) &= \frac{-6}{(6+3z^{-1})^2} + \frac{9}{6+3z^{-1}} + \frac{4}{1+0.25z^{-2}} \\ &= \frac{-1/6}{(1+0.5z^{-1})^2} + \frac{3/2}{1+0.5z^{-1}} + \frac{2}{1+j0.5z^{-1}} + \frac{2}{1-j0.5z^{-1}}, \quad |z|>1 \end{aligned}$$

$$x_3[n] = -\frac{1}{6}(n+1)(-0.5)^n\mu[n] + \frac{3}{2}(-0.5)^n\mu[n] + 2(-j0.5)^n\mu[n] + 2(j0.5)^n\mu[n]$$

Comparing to the first 30 samples, they are identical to each other.

10.4 Question(d)

```
Question d
coefficient of the power serious expension
  Columns 1 through 14

   -3.0000   -0.1667    0.3278   -0.2676    0.1321   -0.0443    ...
       0.0092   -0.0003   -0.0006    0.0002    0.0001   -0.0001    ...
       0.0000   -0.0000

  Columns 15 through 28

    0.0000    0.0000   -0.0000    0.0000    0.0000   -0.0000    ...
       0.0000   -0.0000   -0.0000    0.0000   -0.0000    0.0000    ...
       0.0000   -0.0000

  Columns 29 through 30

    0.0000   -0.0000
```

$$X_4(z) = -4 + \frac{6}{6+2z^{-1}} + \frac{z^{-1}}{6+3z^{-1}+0.8z^{-2}}$$

$$
\begin{aligned}
x_4(z) &= -4 + \frac{6}{6+2z^{-1}} + \frac{z^{-1}}{6+3z^{-1}+0.8z^{-2}} \\
&= -4 + \frac{1}{1+\frac{1}{3}z^{-1}} + \frac{j0.3131}{1+\left(\frac{1}{4}+j0.3485\right)z^{-1}} - \frac{j0.3131}{1+\left(\frac{1}{4}-j0.3485\right)z^{-1}}, \quad |z| > 0.3651
\end{aligned}
$$

$$
x_4[n] = -4\delta[n] + \left(-\tfrac{1}{3}\right)^n \mu[n] + j0.3131\left(-\tfrac{1}{4}-j0.2661\right)^n \mu[n] - j0.3131\left(-\tfrac{1}{4}+j0.2661\right)^n \mu[n]
$$

Comparing to the first 30 samples, they are identical to each other.

11 SUMMARY

For this Homework, I understand more about Digital Signal Processing, as well as how to use the MATLAB to plot and analysis series and how to smooth the signals. I also understand what will be the effect if we use the filter to smooth the signal. I also know more about the property of the filter and how to generate the system to help us impliment some operation like square root.

REFERENCES

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