Digital Signal Processing

FIR Filter Design

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FIR Design

- No connection with the analog filter design
- Based on the magnitude response
- Linear Phase
- Described by transfer function polynomial in z^{-1}
- Guaranteed Stability
- Design Method
  - Windowed FS
  - Frequency sampling

FIR Design

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d[n]e^{-jwn} \qquad h_d[n] = \int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

Objective

Find a finite-duration $\{h_t[n]\}$ of length $2M+1$ whose DTFT $H_t(e^{j\omega})$ approximates the desired DTFT $H_d(e^{j\omega})$ *in some sense*

$$\Phi = \frac{1}{2\pi}\int_{-\pi}^{\pi}|H_d(e^{j\omega}) - H_t(e^{j\omega})|^2 d\omega = \sum_{n=-\infty}^{\infty}|h_d[n] - h_t[n]|^2$$

$$= \sum_{n=-M}^{M}|h_d[n] - h_t[n]|^2 + \Phi_M$$

Φ is minimized when $h_t[n] = h_d[n]$ for $-M \le n \le M$

$$h_t[n] = h_d[n]W_R[n]$$

FIR Design-Windowed Fourier Series

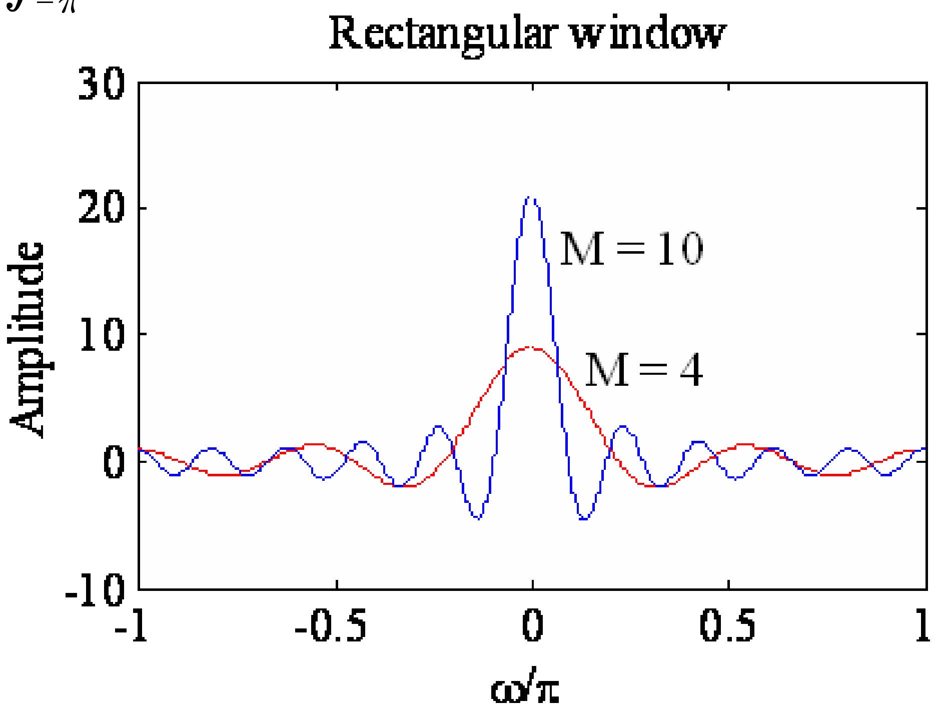
Truncating operation → windowing operation:

$$h_t[n] = h_d[n] W_R[n]$$

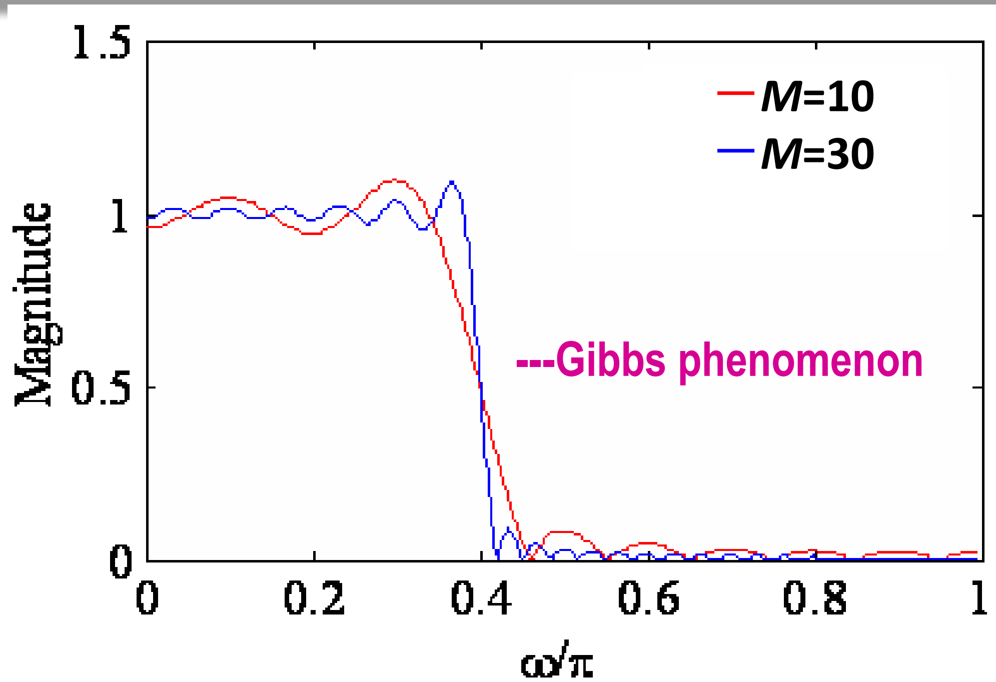
$$H_t(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\tau})\Psi_R(e^{j(\omega-\tau)})\,d\tau$$

$$w_R[n] = \begin{cases} 1, & 0 \le |n| \le M \\ 0, & \text{otherwise} \end{cases}$$

$$\psi_R(e^{j\omega}) = \frac{\sin(\omega(2M+1)/2)}{\sin(\omega/2)}$$



FIR Design-Windowed Fourier Series



•**Observation**: ↑M (length of the lowpass filter) →↑ number of ripples (both passband and stopband) with corresponding ripple widths↓

•Height of the largest ripples remain the same independent of length

FIR Design-Windowed Fourier Series

- Gibbs phenomenon—Oscillatory of the magnitude due to truncation
- Rectangular window—Zero outside $-M \le n \le M$
- To Reduce Gibbs phenomenon
  - Using a window that tapers smoothly to zero
  - smooth transition from passband to stopband in the magnitude specifications

FIR Design-Windowed Fourier Series

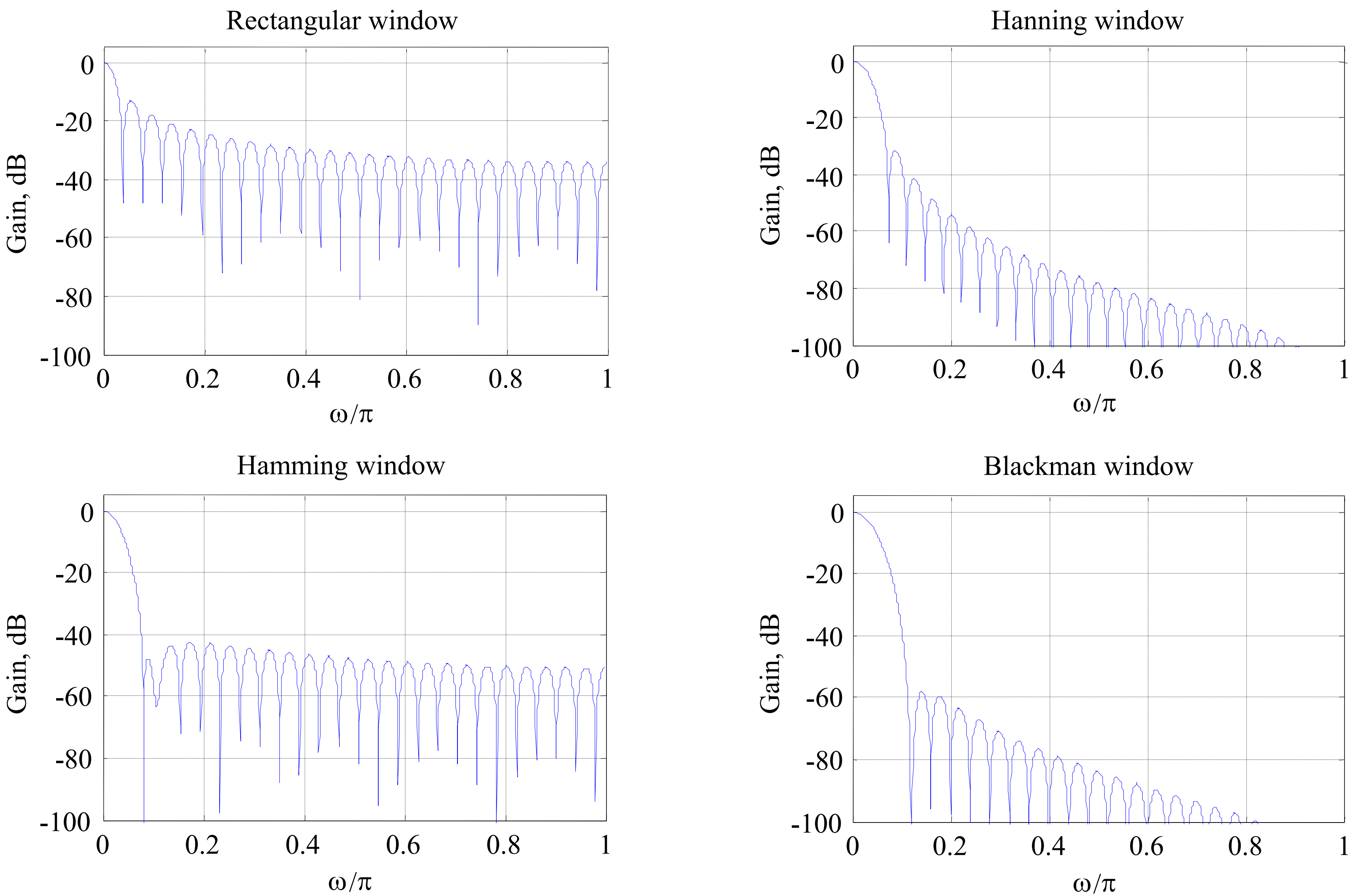
- Windowing: causes the height of the sidelobes to diminish, with a increased main lobe width
- Hann:

  $$w[n] = 0.5 + 0.5\cos[2\pi n/(2M+1)],\ -M \leq n \leq M$$
- Hamming:

  $$w[n] = 0.54 + 0.46\cos[2\pi n/(2M+1)],\ -M \leq n \leq M$$
- Blackman:

  $$w[n] = 0.42 + 0.5\cos[2\pi n/(2M+1)] + 0.08\cos[4\pi n/(2M+1)]$$

FIR Design-Windowed Fourier Series



FIR Design-Windowed Fourier Series

Type of windows	Main lobe width Δ_{ML}	Relative sidelobe level A_{sl}	Minimum stopband attenuation	Transition bandwidth $\Delta\omega$
Rectangular	$4\pi/(2M+1)$	13.3dB	20.9dB	$0.92\pi/M$
Hann	$8\pi/(2M+1)$	31.5dB	43.9dB	$3.11\pi/M$
Hamming	$8\pi/(2M+1)$	42.7dB	54.5dB	$3.32\pi/M$
Blackman	$12\pi/(2M+1)$	75.3dB	75.3dB	$5.56\pi/M$

FIR Design-Windowed Fourier Series

Adjustable window Function: *Kaiser window*.

$$w[n]=\frac{I_0\left(\beta\sqrt{1-\left[\frac{n}{M}\right]^2}\right)}{I_0(\beta)},\quad -M\le n\le M$$

$$N\cong\frac{-20\log_{10}\left(\sqrt{\delta_P\delta_S}\right)-13}{14.6(\omega_S-\omega_P)/(2\pi)}$$

FIR Design-Windowed Fourier Series

(i) All above formulas are used in case $\omega_S > \omega_P$.

(ii) Interchange δ_S, δ_P, the order of FIR are same.

(iii) Ripples δ_S, δ_P are decreased, the order of FIR is increased.

(iv) Transition bandwidth $\omega_S - \omega_P$ is decreased, the order of FIR is increased.

FIR Design-Windowed Fourier Series

Design Steps for Windowed Low Pass FIR Filters

(1) Choose a pass band edge frequency in Hz:

$$f_c = (f_p + f_s)/2$$

(2) Calculate $\omega_c = 2\pi f_c / f_T$, the infinite impulse response for an ideal low pass filter:

$$h_d[n] = \sin(n\omega_c)/n\pi$$

(3) Choose a window based on the specifications

(4) Calculate FIR from

$h_t[n] = h_d[n]w[n]$, notice it is noncausal.

(5) Shift $h_t[n]$ to the right by M to make the filter causal

$$h[n] = h_t[n - M]$$

FIR Design-Windowed Fourier Series

FIR Filter Design Example 1st

The specifications of a low pass filter:

Pass band edge	f_p	2kHz
Stop band edge	f_s	3kHz
Stop band attenuation		40dB
Sampling frequency	f_T	10kHz

Design

(1) Transition width $\Delta f = 3 - 2 = 1$ kHz

$$f_c = (2000+3000)/2 = 2500\text{Hz}$$

$$\omega_c = 2\pi f_1/f_T = 2\pi 2500/10000 = 0.5\pi$$

FIR Design-Windowed Fourier Series

- (2) $h_d[n] = \sin(n\omega_c)/n\pi = \sin(0.5\pi n)/n\pi$
- (3) Choose Hanning window (Table 10.2, 40dB required)

Transition Width $\Delta\omega = 2\pi\Delta f / f_T = 0.2\pi$

$M = 3.11\pi/\Delta\omega = 15.55$, the least $M = 16$,
choose order $N = 2M = 32$, length $N+1 = 33$

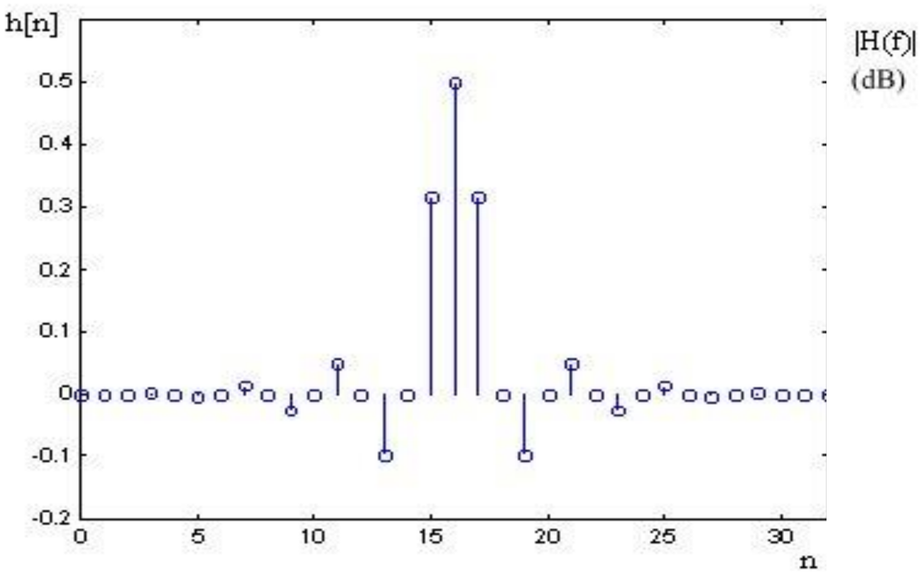
$w[n] = 0.5 + 0.5\cos(2\pi n/33)$

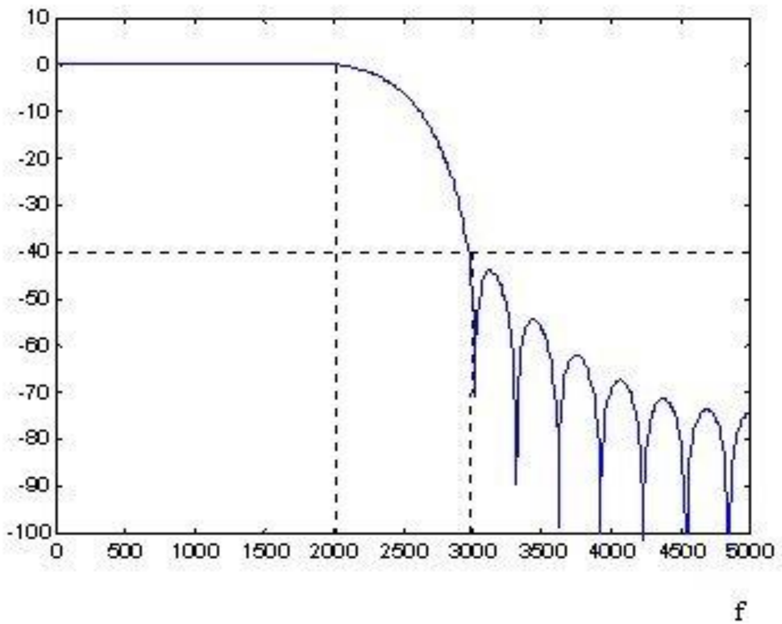
(4) $h_t[n] = h_d[n]w[n] = [\sin(0.5\pi n)/n]\, w[n]$,

(5) $h[n] = h_t[n-M] = h_t[n-16]$

Type I linear-phase FIR LPF

FIR Design-Windowed Fourier Series





Impulse Response

frequency Response

- $G(j\omega_p) = -0.06$ dB at $\omega_p = 2.013$kHz
- $G(j\omega_s) = -40$ dB at $\omega_s = 2.976$kHz

FIR Design-Frequency Sampling

From Frequency Specification samples to h[n]

$$H(k) = H\left(e^{j\frac{2\pi}{N+1}k}\right)$$

The impulse response *h*[*n*] can be derived from IDFT of *H*(*k*).

***H*(*k*) must be linear phase FIR.**

The Design Steps:

(1)From ideal filter, the values of *H*(*k*) are computed (linear phase).

(2)From the IDFT of *H*(*k*), *h*[*n*] is derived.

FIR Design-Frequency Sampling

Design a linear phase FIR BPF, with the passband edge frequencies at 500Hz and 700Hz.

The sampling frequency is f_s = 3.3KHz, and the order N=32(length 33).

Design

(1)Determination of $H(k)$. N = 32, f_s = 3.3 KHz,

$$\Delta f = \frac{fs}{N+1} = 0.1\mathrm{KHz}$$

we choose Type 1 FIR.

$$H(\omega) = H(2\pi - \omega)$$

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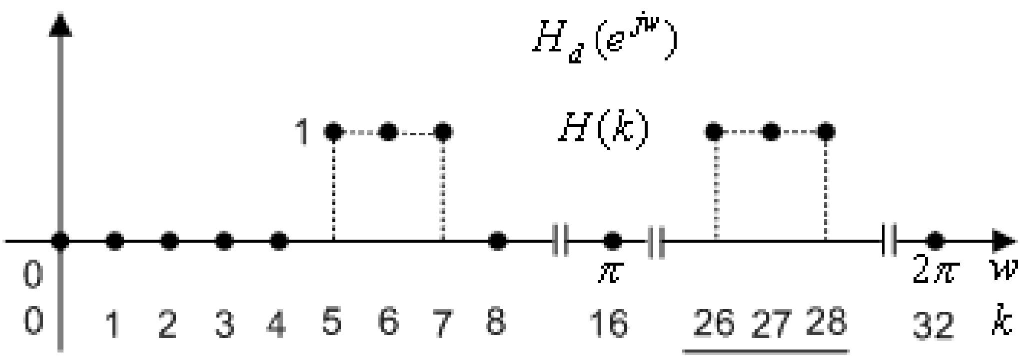
FIR Design-Frequency Sampling

$$\theta(\omega) = -M\omega = -16\omega \quad \omega_k = 2\pi/33$$

$$H(k) = H\left(e^{j\frac{2\pi}{N+1}k}\right) = H\left(e^{j\frac{2\pi\Delta f}{f_s}k}\right)$$

$$H(k) = \begin{cases} 1 \cdot e^{j\frac{32\pi}{33}k} & k = 5, 67, 26, 27, 28 \\ 0 & \text{otherwise} \end{cases}$$

$$h[n] = \frac{1}{N+1} \sum_{k=0}^{N} H(k) W_{N+1}^{-kn}$$



$$< N+1-k >_{N+1} = < 33-k >_{33}$$