Homework : chapter 3.

Q 3.2

(a) $y_a(t) = \sin\Omega_0 t \xleftrightarrow{F} Y_a(j\Omega) = \frac{1}{2j}[\delta(\Omega-\Omega_0) - \delta(\Omega+\Omega_0)]$.

(b) $u_a(t) = e^{-|a|t} = e^{-at}\mu(t) + e^{at}\mu(-t)$

$u_a(t) \xleftrightarrow{F} U_a(j\Omega) = \frac{1}{a-j\Omega} + \frac{1}{a+j\Omega} = \frac{2a}{a^2+\Omega^2}$

(c) $v_a(t) = e^{j\Omega_0 t} \longleftrightarrow F(j\Omega) = \delta(\Omega-\Omega_0)$.

(d) $p_a(t) = \sum_{l=-\infty}^{\infty} \delta(t-lT)$

For $\delta(t-lT)$, $\delta(t-lT) \xleftrightarrow{F} e^{-j\Omega lT}$.

$\therefore p_a(t) \xleftrightarrow{F} \sum_{l=-\infty}^{\infty} e^{-j\Omega lT} = \frac{2\pi}{T}\sum_{l=-\infty}^{\infty}\delta(\Omega-\frac{2\pi l}{T})$

(e) $G(j\Omega) = \int_{-\infty}^{\infty} e^{-at^2} e^{-j\Omega t} dt$. Assume $\beta = \sqrt{a}\cdot t$, $d\beta = \sqrt{a}\,dt$.

$\therefore = \frac{1}{\sqrt{a}}\int e^{-\beta^2} e^{-j\Omega\frac{\beta}{\sqrt{a}}} d\beta$. Assume $\gamma = \frac{-j\Omega}{2\sqrt{a}}$

$\therefore = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} e^{-(\beta^2-2\gamma\beta)} d\beta = \frac{e^{\gamma^2}}{\sqrt{a}}\int_{-\infty}^{\infty} e^{-(\beta-\gamma)^2} d\beta = e^{\gamma^2}\sqrt{\frac{\pi}{a}}\int_{-\infty}^{\infty}\frac{e^{-(\beta-\gamma)^2}}{\sqrt{\pi}} d\beta$

$= \sqrt{\frac{\pi}{a}}\cdot e^{\gamma^2}$, as $\int_{-\infty}^{\infty}\frac{e^{-(\beta-\gamma)^2}}{\sqrt{\pi}} d\beta = 1$

$\therefore = \sqrt{\frac{\pi}{a}}\, e^{-\frac{\Omega^2}{4a}}$

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Q 3.6. For $x_a(t) \xleftrightarrow{F} X_a(j\Omega)$.

(a) Assume : $t - t_0 = \tau$

Thus, $\int_{-\infty}^{\infty} x_a(t-t_0) e^{-j\Omega t} dt$

$$= \int_{-\infty}^{\infty} x_a(\tau) e^{-j\Omega(\tau+t_0)} d\tau$$

$$= e^{-j\Omega t_0} \int_{-\infty}^{\infty} x_a(\tau) e^{-j\Omega \tau} d\tau$$

$$= e^{-j\Omega t_0} X_a(j\Omega).$$

(b) $\int_{-\infty}^{\infty} x_a(t) e^{j\Omega_0 t} e^{-j\Omega t} dt$

$$= \int_{-\infty}^{\infty} x_a(t) e^{-j(\Omega-\Omega_0)t} dt$$

$\xleftrightarrow{F} X[j(\Omega-\Omega_0)]$.

(c) As $x_a(t) \xleftrightarrow{F} X_a(j\Omega)$.

$$x_a(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_a(j\Omega) e^{j\Omega t} d\Omega.$$

$$\therefore 2\pi x_a(-t) = \cancel{\frac{1}{2\pi}} \int_{-\infty}^{\infty} X_a(j\Omega) e^{-j\Omega t} d\Omega.$$

set $t = j\Omega$,

$$2\pi x_a(-j\Omega) = \int_{-\infty}^{\infty} X_a(t) e^{-j\Omega t} d\Omega$$

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(d) Assume $at = \tau$ $(a>0)$

$$\int_{-\infty}^{\infty} x_a(at)\, e^{-j\Omega t} dt = \int_{-\infty}^{\infty} x_a(\tau) e^{-j\Omega \frac{\tau}{a}} d\frac{\tau}{a} = \frac{1}{a}\int_{-\infty}^{\infty} x_a(\tau)\, e^{-j\Omega \frac{\tau}{a}} d\tau$$

$$= \frac{1}{a} X_a(j\cdot \frac{\Omega}{a})$$

Similarly, when $a<0$

$$\int_{-\infty}^{\infty} x_a(at) e^{-j\Omega t} dt = -\frac{1}{a} X_a(j\frac{\Omega}{a})$$

$$\therefore\ x_a(at) \xleftrightarrow{F} \frac{1}{|a|} X(j\frac{\Omega}{a})$$

(e) $$x_a(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega) e^{j\Omega t} d\Omega.$$

$$\frac{dx_a(t)}{dt} = j\Omega \cdot \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega) e^{j\Omega t} d\Omega = j\Omega X_a(j\Omega)$$

$$\therefore\ \frac{dx_a(t)}{dt} \xleftrightarrow{F} j\Omega X_a(j\Omega)$$

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Q 3.18

(a) $y_1[n] = \begin{cases} 1 & -N \le n \le N \\ 0 & \text{others} \end{cases}$

$$\therefore Y_1(j\Omega) = \sum_{n=-\infty}^{\infty} y_1[n] e^{-j\Omega n}$$

$$= \sum_{n=-N}^{N} e^{-j\Omega n}$$

$$= \frac{e^{j\Omega N}(1 - e^{-j\Omega(2N+1)})}{1 - e^{-j\Omega}}$$

$$= \frac{e^{j\frac{1}{2}\Omega}(e^{j\Omega N} - e^{-j\Omega(N+1)})}{e^{j\frac{1}{2}\Omega}(1 - e^{-j\Omega})}$$

$$= \frac{\sin[\Omega(N+\frac{1}{2})]}{\sin(\frac{1}{2}\Omega)}$$

(b) $y_2[n] = \begin{cases} 1 & 0 \le n \le N \\ 0 & \text{others} \end{cases}$

$$Y(j\Omega) = \sum_{n=-\infty}^{\infty} y_2[n] e^{-j\Omega n}$$

$$= \sum_{n=0}^{N} e^{-j\Omega n}$$

$$= \frac{1 - e^{-j\Omega(N+1)}}{1 - e^{-j\Omega}}$$

$$= \frac{e^{-j\Omega\frac{N+1}{2}}(e^{j\Omega\frac{N+1}{2}} - e^{-j\Omega\frac{N+1}{2}})}{e^{-j\frac{\Omega}{2}}(e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}})}$$

$$= e^{-j\Omega\frac{N}{2}} \frac{\sin[w(\frac{N+1}{2})]}{\sin(\frac{1}{2}\Omega)}$$

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(c) $y_3[n] = \begin{cases} 1 - \frac{|n|}{N} & -N \le n \le N \\ 0 & \text{others} \end{cases}$

$\therefore N y_3[n] = \begin{cases} N - |n| & -N \le n \le N \\ 0 & \text{others.} \end{cases}$

consider conclusion in (a) that $x[n] = u[n+N] - u[n-N]$

$$\xleftrightarrow{F} \frac{\sin[w(N+\frac{1}{2})]}{\sin(\frac{w}{2})}$$

we find that $N y_3[n] = x_3[n] \circledast x_3[n]$

where $x_3[n] = \begin{cases} 1 & -\frac{N-1}{2} \le n \le \frac{N-1}{2} \\ 0 & \text{others.} \end{cases} \longleftrightarrow \frac{\sin(\frac{N}{2}w)}{\sin(\frac{w}{2})}$

$$\therefore Y_3(e^{jw}) = \frac{1}{N} \frac{\sin^2(\frac{N}{2}w)}{\sin^2(\frac{w}{2})}$$

(d) consider $y_4[n] = N y_3[n] + y_1[n]$

$$Y_4(e^{jw}) = \frac{\sin[(N+\frac{1}{2})w]}{\sin(\frac{w}{2})} + \frac{\sin^2(w(\frac{N}{2}))}{\sin^2(\frac{w}{2})}$$

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(e) $y_s[n] = \begin{cases} \cos(\pi n/2N) & -N \le n \le N \\ 0 & \text{others} \end{cases}$

$$= \frac{1}{2}\begin{cases} e^{j(\frac{\pi n}{2N})} + e^{-j(\frac{\pi n}{2N})} & -N \le n \le N \\ 0 & \text{others} \end{cases}$$

$$= \frac{1}{2} e^{j\pi/2N \cdot n} y_1[n] + \frac{1}{2} e^{-j\frac{\pi}{2N} \cdot n} y_1[n]$$

using a frequency shift property:

$$y_s = \frac{1}{2}\left[\frac{\sin[(w-\frac{\pi}{2N})(N+\frac{1}{2})]}{\sin(\frac{w-\frac{\pi}{2N}}{2})}\right] + \frac{1}{2}\left[\frac{\sin[(w+\frac{\pi}{2N})(N+\frac{1}{2})]}{\sin(\frac{w-\frac{\pi}{2N}}{2})}\right]$$

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Q2.23

(a) $H_1(e^{jw}) = -4 + 3\cos w + 4\cos 2w$

$$= -4 + \frac{3}{2}e^{jw} + \frac{3}{2}e^{-jw} + 2e^{j2w} + 2e^{-j2w}$$

$$h[n] = \{2,\ \frac{3}{2},\ \underset{\uparrow}{-4},\ \frac{3}{2},\ 2\}$$

(b) $H_2(e^{jw}) = (-4 + 3\cos w + 4\cos 2w)\cos(\frac{w}{2})e^{-j\frac{w}{2}}$

$$= (-4 + \frac{3}{2}e^{jw} + \frac{3}{2}e^{-jw} + 2e^{j2w} + 2e^{-j2w})(\frac{1}{2}e^{j\frac{w}{2}} + \frac{1}{2}e^{-j\frac{w}{2}})e^{-j\frac{w}{2}}$$

$$= \frac{1}{2}(-4 + \frac{3}{2}e^{jw} + \frac{3}{2}e^{-jw} + 2e^{j2w} + 2e^{-j2w})(1 + e^{-jw})$$

$$= (-2 + \frac{3}{4}e^{jw} + \frac{3}{4}e^{-jw} + e^{j2w} + e^{-j2w}) + (-2e^{-jw} + \frac{3}{4} + \frac{3}{4}e^{-j2w}$$
$$+ e^{jw} + e^{-j3w})$$

$$= e^{-j3w} + \frac{7}{4}e^{-j2w} - \frac{5}{4}e^{-jw} - \frac{5}{4} + \frac{7}{4}e^{jw} + e^{j2w}$$

$$h_2[n] = \{1,\ \frac{7}{4},\ -\frac{5}{4},\ \underset{\uparrow}{-\frac{5}{4}},\ \frac{7}{4},\ 1\}$$

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(c). $H_3(e^{jw}) = j(-4+3\cos w+4\cos 2w)\sin w$

$= \frac{1}{2}(-4+\frac{3}{2}e^{jw}+\frac{3}{2}e^{-jw}+2e^{j2w}+2e^{-j2w})(e^{jw}-e^{-jw})$

$= (-2e^{jw}+\frac{3}{4}e^{j2w}+\frac{3}{4}\cancel{e^{j2w}}+e^{j3w}+e^{-jw})$

$-(-2e^{-jw}+\frac{3}{4}+\frac{3}{4}e^{-j2w}+e^{jw}+e^{-j3w})$

$= -e^{-j3w}-\frac{3}{4}e^{-j2w}+3e^{-jw}+0-3e^{jw}+\frac{3}{4}e^{j2w}$

$h_3[n] = \{-1, -\frac{3}{4}, 3, \underset{\uparrow}{0}, -3, \frac{3}{4}, 1\}$

(d) $H_4(e^{jw}) = j(-4+3\cos w+4\cos 2w)(\sin\frac{w}{2})e^{jw/2}$

$= \frac{1}{2}(-4+\frac{3}{2}e^{jw}+\frac{3}{2}e^{-jw}+2e^{j2w}+2e^{-j2w})(e^{j\frac{w}{2}}-e^{-j\frac{w}{2}})e^{jw/2}$

$= (-2+\frac{3}{4}e^{jw}+\frac{3}{4}e^{-jw}+e^{j2w}+e^{-j2w})(e^{jw}-1)$

$= (-2e^{jw}+\frac{3}{4}e^{j2w}+\frac{3}{4}+e^{j3w}+e^{-jw})$

$-(-2+\frac{3}{4}e^{jw}+\frac{3}{4}e^{-jw}+e^{j2w}+e^{-j2w})$

$= -e^{-j2w}+\frac{1}{4}e^{-jw}+\frac{11}{4}-\frac{11}{4}e^{jw}-\frac{1}{4}e^{j2w}+e^{j3w}$

$h_4[n] = \{-1, \frac{1}{4}, \underset{\uparrow}{\frac{11}{4}}, -\frac{11}{4}, -\frac{1}{4}, 1\}$

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Q3.43

According to Parseval's Theorem, suppose $g[n] = h^*[n]$

$$\sum_{n=-\infty}^{\infty} |g[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |G(e^{jw})|^2 dw$$

So we want that: $|G(e^{jw})|^2 = \frac{a}{b+c\cos w}$

This reminds us that:

$$\begin{cases} g_1[n] = \alpha^n \mu[n] \xleftrightarrow{F} \frac{1}{1-\alpha e^{-jw}}, & |\alpha|<1 \\ g_2[n] = -\alpha^n \mu[-n-1] \xleftrightarrow{F} \frac{1}{1-\alpha e^{-jw}}, & |\alpha|>1 \end{cases}$$

(a) if $|G_1(e^{jw})|^2 = \frac{4}{5\cancel{4}+4\cos w}$, $\alpha = -2$, $|\alpha|>1$

$$g_1[n] = -2\cdot(-2)^n \mu[-n-1] \xleftrightarrow{F} \frac{2}{1+2e^{-jw}}$$

$$\therefore \sum_{n=-\infty}^{\infty} |g_1[n]|^2 = 4\sum_{n=-\infty}^{-1} 4^n = \overset{\frac{4}{3}}{\cancel{4}} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |G(e^{jw})|^2 dw$$

As $|G(e^{jw})|^2$ is an even function:

$$\int_0^{\pi} \frac{4}{5+4\cos w} dw = \frac{4\pi}{3}$$

(b) if $|G_1(e^{jw})|^2 = \frac{1}{3.25-3\cos w}$, $\alpha = 1.5$ $|\alpha|>1$

$$\therefore g_2[n] = -(1.5)^n \mu[-n-1] \xleftrightarrow{F} \frac{1}{1-1.5e^{-jw}}$$

$$\therefore \sum_{n=-\infty}^{\infty} [g_2[n]]^2 = \sum_{n=-\infty}^{-1} [(1.5)^n]^2 = \sum_{n=1}^{\infty} (\frac{4}{9})^n = \frac{4}{5}$$

$$\therefore \int_0^{\pi} \frac{1}{3.25-3\cos w} dw = \frac{4}{5}\pi$$

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(c) This remind us the form $g_3[n] = -n\alpha^n u[-n-1]$. $|\alpha|>1$

$$g_3[n] \xleftrightarrow{F} \frac{1}{(1-\alpha e^{-jw})^2} = G_3(e^{jw})$$

and: $$|G_3(e^{jw})|^2 = \frac{1}{(1+\alpha^2-2\alpha\cos w)^2}$$

consider $\frac{4}{(5-4\cos w)^2}$, $\alpha = 2$

$\therefore$ $g_4[n] = -2n \cdot 2^n u[-n-1]$

$$\therefore \sum_{n=-\infty}^{\infty} |g_4[n]|^2 = \sum_{n=-\infty}^{-1} n^2 \cdot 2^{2n+2} = 4\sum_{n=1}^{\infty} \frac{1}{n^2 2^n}$$

$$\therefore \int_{-\infty}^{\infty} \frac{4}{(5-4\cos w)^2} = 4\pi \sum_{n=1}^{\infty} \frac{1}{n^2 2^n}$$

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Q 3.53

consider:

$$g_2[n] = g_1[n] + g_1[n-4]$$

$$g_3[n] = g_1[n] + g_1[-n+7]$$

$$g_4[n] = g_1[-n+3] + g_1[n-4]$$

$$\therefore\ G_2(e^{jw}) = G_1(e^{jw}) + e^{-j4w} G_1(e^{jw})$$

$$G_2(e^{jw}) = G_1[e^{jw}) + e^{-j7w} G_1(e^{-jw})$$

$$G_3(e^{jw}) = e^{-j3w} G_1(e^{-jw}) + e^{-j4w} G_1(e^{jw})$$

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Q3.61

AS sampling frequency is 3KHz, low-pass filter to ~~[illegible]~~ keep ~~300~~ the signal under 900Hz.

Thus, 300Hz, 500Hz is kept naturally;

1200 HZ is excluded as 3000-1200=1800, both 1200 and 1800 is larger than 900HZ.

Similarly, we keep 850HZ as 3000-2150=850HZ.

Finally, 500HZ is ~~kee~~ kept, as 3500-3000=500HZ.

So, 300HZ, 500HZ, 850 HZ are kept.

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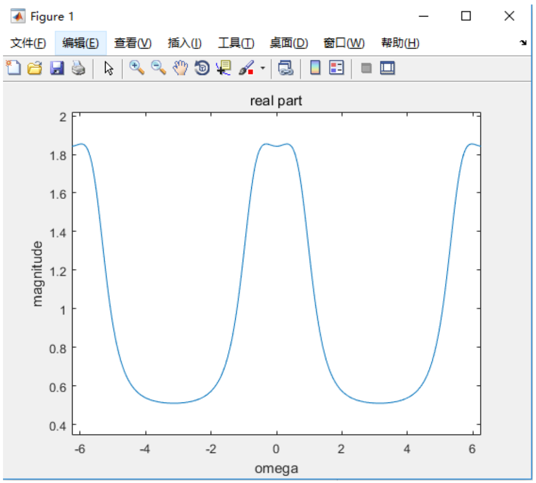
MATLAB practice:

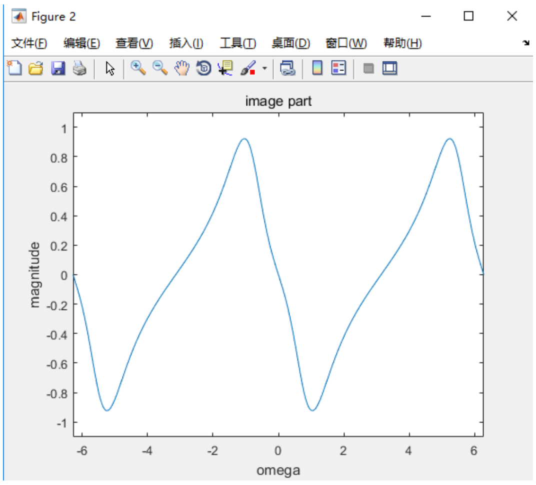
M3.1:

Here we experiment (r, θ) pairs as follow: (0.5, π /4), (0. 7, π /7), (0. 1, π /9), and all the results are shown as follow:

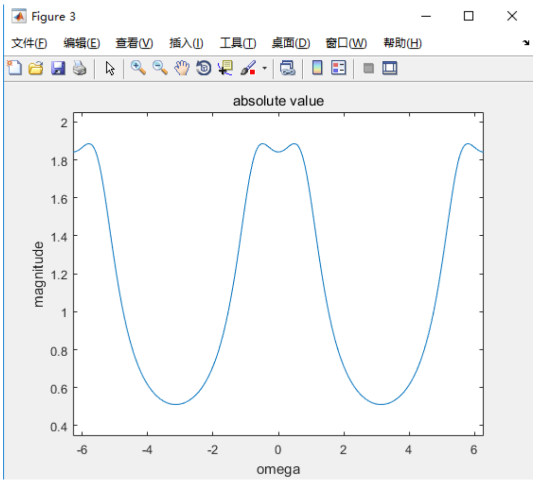
Group 1: r=0.5, θ = π /4:



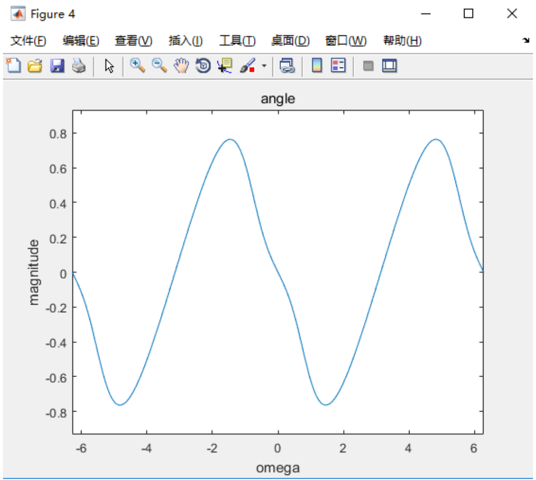
(a)



(b)



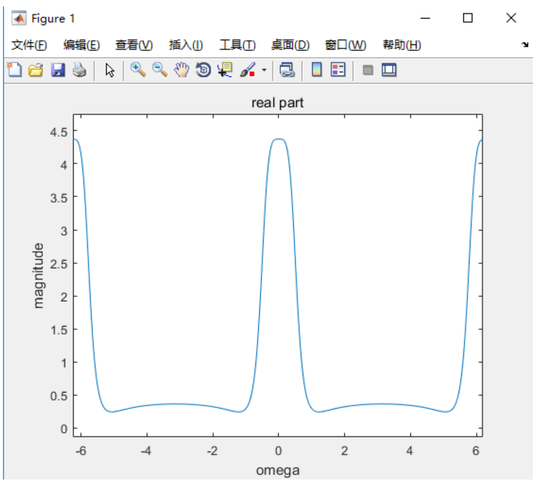
(c)

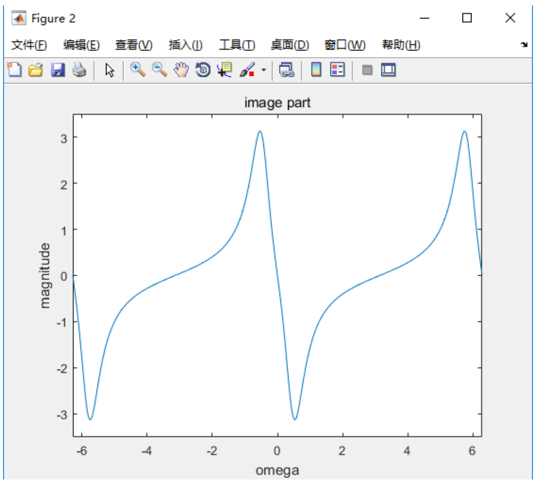


(d)

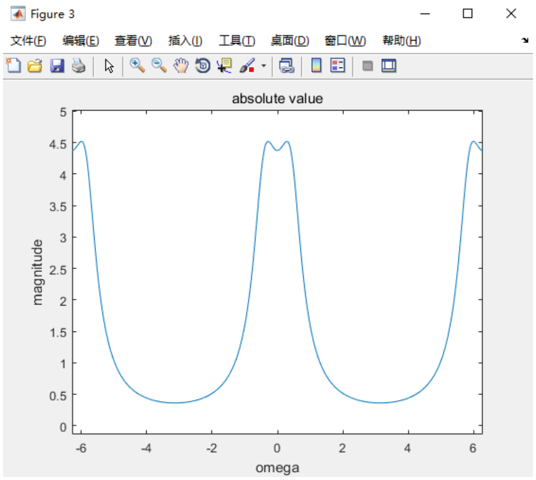
Figure : (a) real part; (b)image part; (c)absolute value; (d) phase (angle)

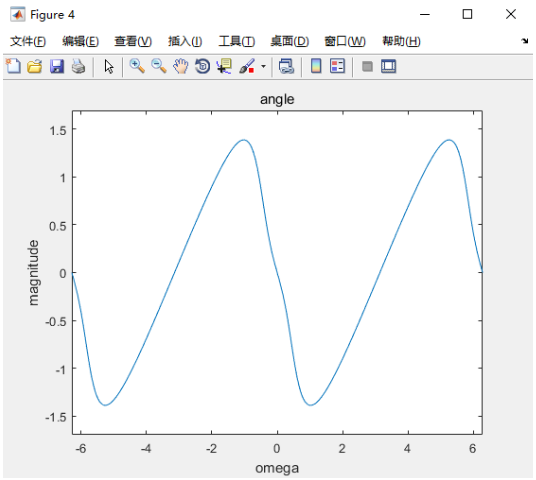
Group 2: r=0.7, θ = π /7:





(a) (b)

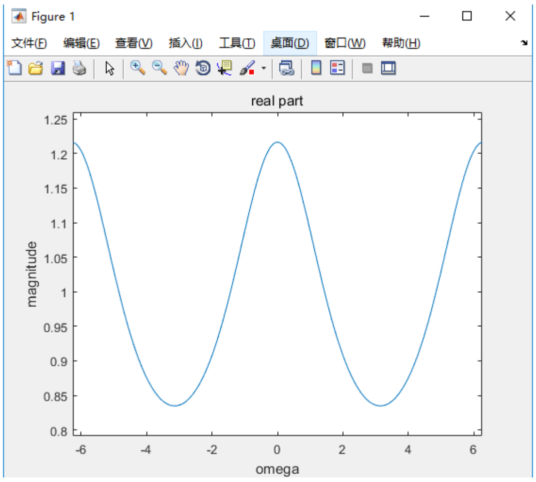


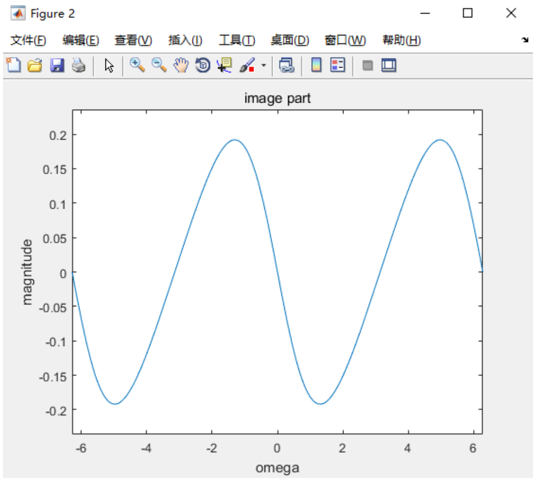


(c) (d)

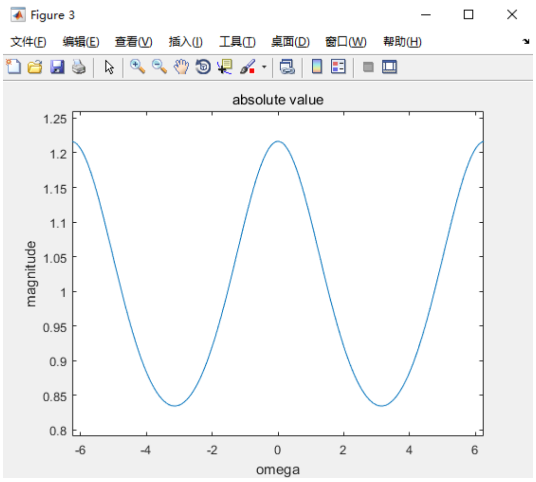
Figure : (a) real part; (b)image part; (c)absolute value; (d) phase (angle)

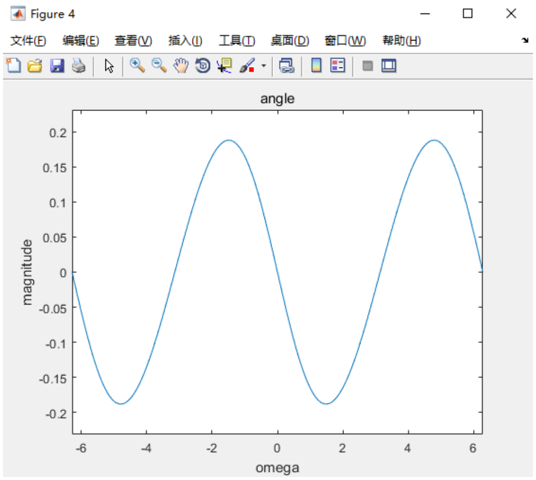
Group 3: r=0.7, θ = π /9:





(a) (b)



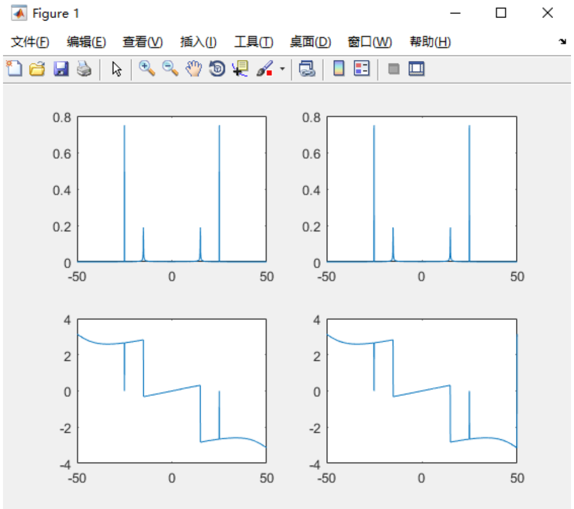


(c) (d)

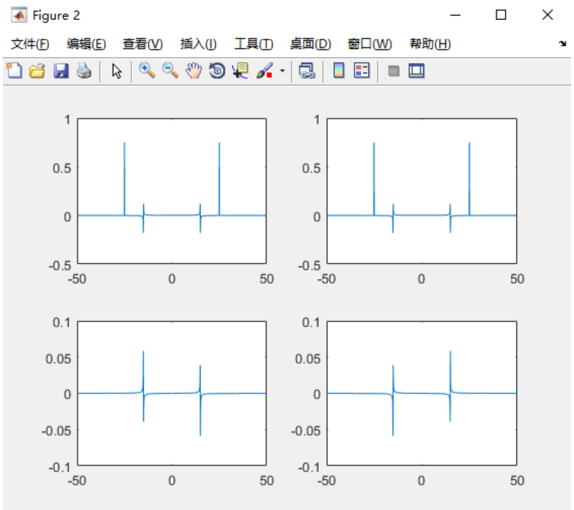
Figure : (a) real part; (b)image part; (c)absolute value; (d) phase (angle)

M3.4:

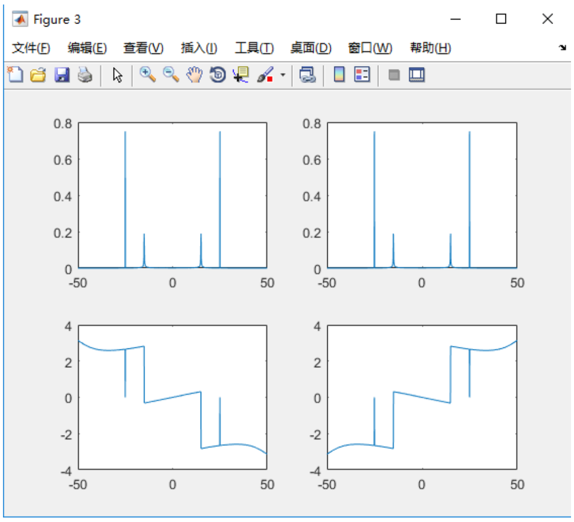
Here we test a real signal to prove that it satisfies the properties in table 3.1



In figure 1, we prove $X(e^{jw}) = X^*(e^{-jw})$. In first row, their magnitude are shown and they are the same; in the second row, their phase are shown and they are the same. Prove over;



In figure 2, we prove $X_{re}(e^{jw}) = X_{re}(e^{-jw})$ and $X_{im}(e^{jw}) = -X_{im}(e^{-jw})$. The first conclusion is proved in first row as their real part are the same; the second conclusion is proved in second row as their image part are inverse.

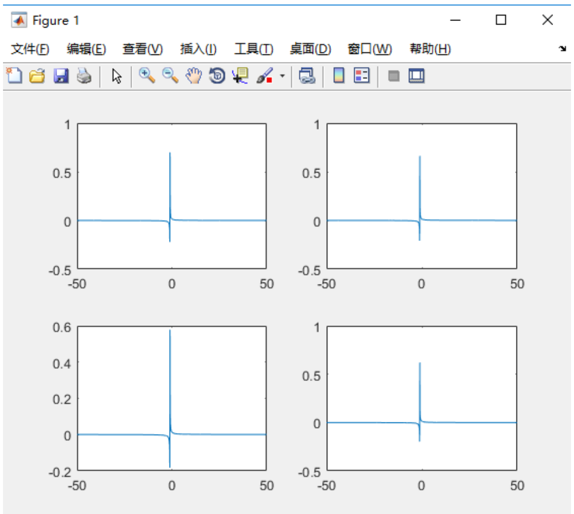


In figure 3, we prove $|X(e^{jw})| = |X(e^{-jw})|$ and $\arg\{X(e^{jw})\} = -\arg\{X(e^{-jw})\}$. The first conclusion is proved in first row as the absolute value are the same; the second conclusion is proved in the second row as their angle are inverse.

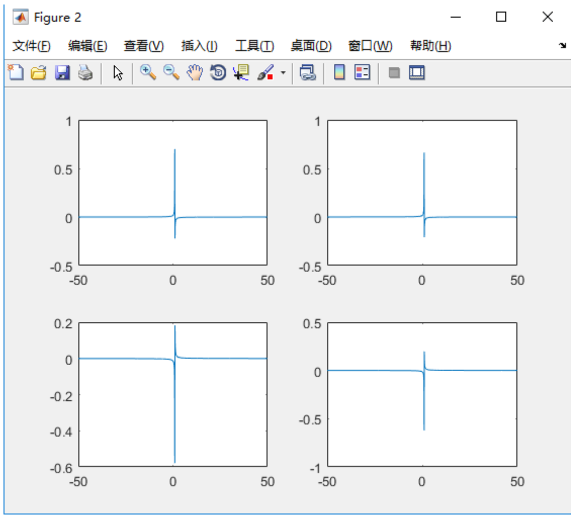
Note that the sequence tested here can be modified in the MATLAB file, while the conclusion stay solid.

M3.5

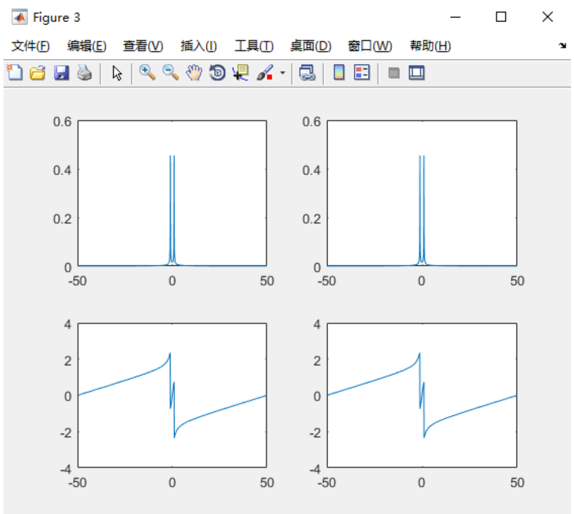
Here we test a complex signal to prove that it satisfies the properties in table 3.2. Note that some slight error exist because of fft-algorithm. Check more details in the code.



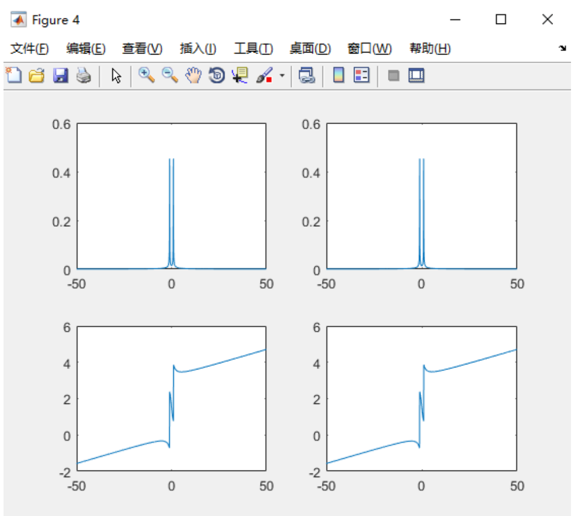
In figure 1, we prove that $\mathcal{F}\{x[-n]\} = X(e^{-jw})$. Both the real and image parts of the two are shown and compared in different row and the are very close. Prove over.



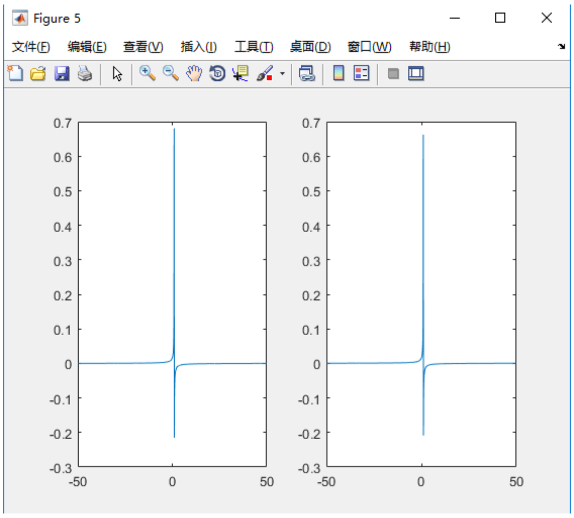
In figure 2, we prove that $\mathcal{F}\{\mathrm{x}*[-\mathrm{n}]\} = \mathrm{X}*\left(e^{jw}\right)$. Both the real and image parts of the two are shown and compared in different row and the are very close. Prove over.



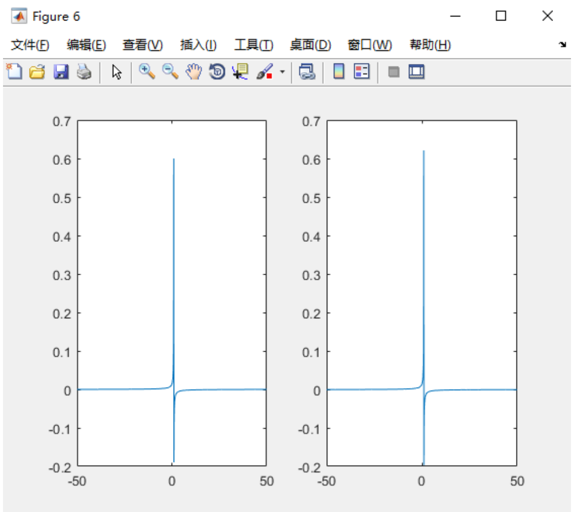
In figure 3, we prove that $\mathcal{F}\{\mathrm{x}_{re}[\mathrm{n}]\} = \frac{1}{2}\{\mathrm{X}\left(e^{jw}\right) + \mathrm{X}*\left(e^{-jw}\right)\}$. Both the magnitude and phase parts of the two are shown and compared in different rows and the are very close. Prove over.



In figure 4, we prove that $\mathcal{F}\{\mathrm{j}x_{im}[n]\} = \frac{1}{2}\{X(e^{jw}) - X*(e^{-jw})\}$. Both the magnitude and phase parts of the two are shown and compared in different rows and the are very close. Prove over.



In figure 5, we prove that $\mathcal{F}\{x_{cs}[n]\} = X_{re}(e^{jw})$. The real part of the two are shown and compared in different rows and the are very close. Prove over.



In figure 6, we prove that $\mathcal{F}\{x_{ca}[n]\} = \mathrm{j}X_{im}(e^{jw})$. The real part of the two are shown and compared in different rows and the are very close. Prove over.