电子科技大学
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Digital Signal Processing

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Second Assignment	
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3.2

(a) $y_a(t) = \sin(\Omega_0 t) = \frac{1}{2j}(e^{j\Omega_0 t} - e^{-j\Omega_0 t})$

As we have the following FT pair: $e^{j\Omega_0 t} \leftrightarrow 2\pi\delta(\Omega-\Omega_0)$

So by linearity property:

$$y_a(t) \longleftrightarrow \frac{1}{2j}\left[2\pi\delta(\Omega-\Omega_0) - 2\pi\,\delta(\Omega+\Omega_0)\right] = \frac{\pi}{j}\left[\delta(\Omega-\Omega_0) - \delta(\Omega+\Omega_0)\right]$$

(b) By the definition of CTFT:

$$X(j\Omega) = \int_{-\infty}^{\infty} u_a(t)\, e^{-j\Omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-\alpha|t|} e^{-j\Omega t}\, dt = \int_{-\infty}^{0} e^{\alpha t} e^{-j\Omega t}\, dt + \int_{0}^{\infty} e^{-\alpha t} e^{-j\Omega t}\, dt$$

$$= \frac{1}{\alpha - j\Omega} e^{(\alpha - j\Omega)t}\Big|_{-\infty}^{0} + \frac{1}{-\alpha - j\Omega} e^{-(\alpha + j\Omega)t}\Big|_{0}^{\infty}$$

When $\alpha > 0$,

$$X(j\Omega) = \frac{1}{\alpha - j\Omega}(1-0) - \frac{1}{\alpha + j\Omega}(0-1)$$

$$= \frac{2\alpha}{\alpha^2 + \Omega^2}$$

(c) By the definition of CTFT:

$$X(j\Omega) = \int_{-\infty}^{\infty} e^{j\Omega_0 t} e^{-j\Omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{j(\Omega_0 - \Omega)t}\, dt = 2\pi\delta(\Omega - \Omega_0)$$

(d) By the definition:

$$X(j\Omega) = \int_{-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \delta(t - lT)\, e^{-j\Omega t}\, dt$$

$$= \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(t - lT)\, e^{-j\Omega t}\, dt$$

$$= \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(t - lT)\, e^{-j\Omega lT}\, dt$$

$$= \sum_{l=-\infty}^{\infty} e^{-j\Omega lT}$$

(e) By definition, $X(j\Omega) = \int_{-\infty}^{\infty} e^{-\alpha t^2} e^{-j\Omega t}\, dt$

$$= \int_{-\infty}^{\infty} e^{-(\alpha t^2 + j\Omega t + (\frac{j\Omega}{2\sqrt{\alpha}})^2)} \cdot e^{(\frac{j\Omega}{2\sqrt{\alpha}})^2}\, dt = e^{(\frac{j\Omega}{2\sqrt{\alpha}})^2} \int_{-\infty}^{\infty} e^{-(\sqrt{\alpha}t + \frac{j\Omega}{2\sqrt{\alpha}})^2}\, dt$$

Let $u = \sqrt{\alpha}t + \frac{j\Omega}{2\sqrt{\alpha}}$, $du = \sqrt{\alpha}\,dt$, so $X(j\Omega) = e^{-\frac{\Omega^2}{4\alpha}} \int_{-\infty}^{\infty} e^{-u^2} \cdot \frac{1}{\sqrt{\alpha}}\, du = \sqrt{\frac{\pi}{\alpha}}\, e^{-\frac{\Omega^2}{4\alpha}}$

3.6

(a) As $X_a(j\Omega) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j\Omega t} dt$

So $X_a(j\Omega) e^{-j\Omega t_0} = \int_{-\infty}^{\infty} x_a(t)\, e^{-j\Omega(t+t_0)} dt$

let $t' = t + t_0$

$dt' = dt$

So: $X_a(j\Omega) e^{-j\Omega t_0} = \int_{-\infty}^{\infty} x_a(t' - t_0)\, e^{-j\Omega t'} dt'$

$\therefore x_a(t - t_0) \xleftrightarrow{CTFT} X_a(j\Omega) e^{-j\Omega t_0}$

(b) As we have: $X_a(j\Omega) = \int_{-\infty}^{\infty} x_a(t) e^{-j\Omega t} dt$

So $X_a(j(\Omega - \Omega_0)) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j(\Omega-\Omega_0)t} dt$

$= \int_{-\infty}^{\infty} x_a(t) e^{j\Omega_0 t} e^{-j\Omega t} dt$

$\therefore x_a(t) e^{j\Omega_0 t} \xleftrightarrow{CTFT} X_a(j(\Omega - \Omega_0))$

(c) As we have:

$x_a(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_a(j\Omega)\, e^{j\Omega t} d\Omega$

$\therefore x_a(-t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_a(j\Omega)\, e^{-j\Omega t} d\Omega$

We change $t \to j\Omega$ and $j\Omega \to t$

$\therefore x_a(-j\Omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_a(t)\, e^{-j\Omega t} dt$

$\therefore 2\pi x_a(-j\Omega) = \int_{-\infty}^{\infty} X_a(t) e^{-j\Omega t} dt$

$\therefore X_a(t) \xleftrightarrow{CTFT} 2\pi x_a(-j\Omega)$

(d) As we have:

$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\Omega)\, e^{j\Omega t} d\Omega$

$x(at) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\Omega)\, e^{j\Omega a t} d\Omega$

let $\Omega = \frac{\Omega'}{a}$, then we can get:

$x(at) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j \cdot \frac{\Omega'}{a}) e^{j\Omega' t} d\Omega' \cdot \frac{1}{|a|}$

$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{|a|} X(j \cdot \frac{\Omega}{a}) e^{j\Omega t} d\Omega$

$\therefore x_a(at) \xleftrightarrow{CTFT} \frac{1}{|a|} X(j \frac{\Omega}{a})$

(e) As $x_a(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(j\Omega) e^{j\Omega t} d\Omega$

So: $\frac{d x_a(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(j\Omega) j\Omega e^{j\Omega t} d\Omega$

$\therefore \frac{d x_a(t)}{dt} \xleftrightarrow{CTFT} j\Omega X_a(j\Omega)$

3.18.

(a) From the definition:

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y[n] e^{-j\omega n} = \sum_{n=-N}^{N} e^{-j\omega n} = e^{j\omega N} + e^{j\omega(N-1)} + \cdots + e^{-j\omega N}$$

$$= e^{j\omega N} \cdot \frac{1-e^{-j\omega(2N+1)}}{1-e^{-j\omega}} = e^{j\omega N} \frac{e^{-j\omega(N+\frac{1}{2})}(e^{j\omega(N+\frac{1}{2})} - e^{-j\omega(N+\frac{1}{2})})}{e^{-\frac{1}{2}j\omega}(e^{\frac{1}{2}j\omega} - e^{-\frac{1}{2}j\omega})}$$

$$= \frac{2j\sin(N+\frac{1}{2})\omega}{2j\sin\frac{\omega}{2}} = \frac{\sin(N+\frac{1}{2})\omega}{\sin\frac{\omega}{2}}$$

(b) $Y(e^{j\omega}) = \sum_{n=0}^{N} e^{-j\omega n}$

$$= 1 \cdot \frac{1-(e^{-j\omega})^{N+1}}{1-e^{-j\omega}} = \frac{e^{-\frac{j\omega(N+1)}{2}}(e^{\frac{j\omega(N+1)}{2}} - e^{-\frac{j\omega(N+1)}{2}})}{e^{-\frac{j\omega}{2}}(e^{\frac{j\omega}{2}} - e^{-\frac{j\omega}{2}})}$$

$$= e^{-\frac{j\omega N}{2}} \frac{\sin\frac{N+1}{2}\omega}{\sin\frac{\omega}{2}}$$

(c) It can be transformed to the convolution of two sequences:

$y_3[n] = \frac{1}{N}(y_0[n] * y_1[n])$

① when N is odd: $y_0[n] = y_1[n] = 1 \quad (-\frac{N-1}{2} \le n \le \frac{N-1}{2})$

② when N is even: we can add 0 to the original sequence: $[0\ 1\ 2\ 3 \cdots\cdots 3\ 2\ 1\ 0]$

then we can let: $y_0[n] = 1 \ (-\frac{N}{2}+1 \le n \le \frac{N}{2})$, $y_1[n] = 1 \ (-\frac{N}{2} \le n \le \frac{N}{2}-1)$

Therefore, when N is odd, $Y_3(e^{j\omega}) = \frac{1}{N} Y_0(e^{j\omega}) Y_0(e^{j\omega}) = \frac{1}{N} \cdot \frac{\sin\frac{\omega N}{2}}{\sin\frac{\omega}{2}} \cdot \frac{\sin\frac{\omega N}{2}}{\sin\frac{\omega}{2}} = \frac{1}{N} \cdot \frac{\sin^2\frac{\omega N}{2}}{\sin^2\frac{\omega}{2}}$

when N is odd, $Y_0(e^{j\omega}) = \sum_{-\frac{N}{2}+1}^{\frac{N}{2}} e^{-j\omega n} = e^{-j\omega(1-\frac{N}{2})} \frac{1-e^{-j\omega N}}{1-e^{-j\omega}}$

$$Y_1(e^{j\omega}) = \sum_{-\frac{N}{2}}^{\frac{N}{2}-1} e^{-j\omega n} = e^{\frac{N}{2}j\omega} \cdot \frac{1-e^{-j\omega N}}{1-e^{-j\omega}} = e^{\frac{N}{2}\omega j} \frac{1-e^{-\omega N j}}{1-e^{-\omega j}}$$

So: $Y_3(e^{j\omega}) = \frac{1}{N} Y_0(e^{j\omega}) Y_3(e^{j\omega}) = e^{j\omega(N-1)} \left(\frac{e^{-\frac{\omega N}{2}j}(e^{\frac{\omega N}{2}j} - e^{-\frac{\omega N}{2}j})}{e^{-\frac{\omega}{2}}(e^{\frac{\omega}{2}} - e^{-\frac{\omega}{2}j})} \right)^2 = \frac{1}{N} \cdot \frac{\sin^2\frac{\omega N}{2}}{\sin^2\frac{\omega}{2}}$

$\therefore Y_3(e^{j\omega}) = \frac{1}{N} \cdot \frac{\sin^2\frac{\omega N}{2}}{\sin^2\frac{\omega}{2}}$ for N both odd and even

(d) obviously, $y_4[n] = y_1[n] + N y_3[n]$

By using linearity property, $Y_4(e^{j\omega}) = Y_1(e^{j\omega}) + N Y_3(e^{j\omega})$

$$= \frac{\sin(N+\frac{1}{2})\omega}{\sin\frac{\omega}{2}} + \frac{\sin^2\frac{\omega N}{2}}{\sin^2\frac{\omega}{2}}$$

(e) From the definition, $Y_5(e^{jw}) = \sum_{n=-N}^{N} y_5[n] e^{-jwn} = \sum_{n=-N}^{N} \frac{1}{2}(e^{\frac{n}{2N}\pi j} + e^{-\frac{n}{2N}\pi j}) e^{-jwn}$

$$\sum_{n=-N}^{N} \frac{1}{2} e^{(\frac{\pi}{2N}-w)nj} e^{-jwn} = \frac{1}{2} e^{(\frac{\pi}{2N}-w)j(-N)} \cdot \frac{1-e^{(\frac{\pi}{2N}-w)(2N+1)j}}{1-e^{(\frac{\pi}{2N}-w)j}} = \frac{e^{(N+\frac{1}{2})(\frac{\pi}{2N}-w)j}(e^{-(N+\frac{1}{2})(\frac{\pi}{2N}-w)j} - e^{(\frac{\pi}{2N}-w)(N+\frac{1}{2})j})}{e^{(\frac{\pi}{4N}-\frac{w}{2})j}(e^{-(\frac{\pi}{4N}-\frac{w}{2})j} - e^{(\frac{\pi}{4N}-\frac{w}{2})j})}$$

$$= \frac{1}{2} e^{(-\frac{\pi}{2}+wN)j} \cdot e^{(\frac{\pi}{2}-wN)j} \cdot \frac{\sin(w-\frac{\pi}{2N})(N+\frac{1}{2})}{\sin(\frac{w}{2}-\frac{\pi}{4N})} = \frac{1}{2} \frac{\sin[(w-\frac{\pi}{2N})(N+\frac{1}{2})]}{\sin\frac{1}{2}(w-\frac{\pi}{2N})}$$

$$\sum_{n=-N}^{N} \frac{1}{2} e^{-\frac{n}{2N}\pi j} e^{-jwn} = \frac{1}{2} e^{(-\frac{\pi}{2N}-w)j(-N)} \cdot \frac{1-e^{(-\frac{\pi}{2N}-w)j(2N+1)}}{1-e^{(-\frac{\pi}{2N}-w)j}} = \frac{1}{2} e^{(\frac{\pi}{2}+wN)j} \cdot \frac{e^{(N+\frac{1}{2})(-\frac{\pi}{2N}-w)}(e^{-(N+\frac{1}{2})(\frac{\pi}{2N}+w)} - e \ldots)}{e^{(-\frac{\pi}{4N}-\frac{w}{2})j}(e^{(\frac{\pi}{4N}+\frac{w}{2})j} - e^{(-\frac{\pi}{4N}-\frac{w}{2})} \ldots)}$$

$$= \frac{1}{2} \cdot \frac{\sin[(N+\frac{1}{2})(w+\frac{\pi}{2N})]}{\sin\frac{1}{2}(w+\frac{\pi}{2N})}$$

Therefore, $Y_5(e^{jw}) = \frac{1}{2}\left[\frac{\sin[(w-\frac{\pi}{2N})(N+\frac{1}{2})]}{\sin\frac{1}{2}(w-\frac{\pi}{2N})}\right] + \frac{1}{2} \cdot \frac{\sin[(N+\frac{1}{2})(w+\frac{\pi}{2N})]}{\sin\frac{1}{2}(w+\frac{\pi}{2N})}$

3.23

(a) $H_1(e^{jw}) = -4 + 3\cos w + 4\cos 2w$

$$= -4 + \frac{3}{2}(e^{wj}+e^{-wj}) + 2(e^{2wj}+e^{-2wj})$$

$$= -4e^{0wj} + \frac{3}{2}e^{wj} + \frac{3}{2}e^{-wj} + 2e^{2wj} + 2e^{-2wj}$$

As $H_1(e^{jw}) = \sum_{n=-\infty}^{\infty} h_1[n] e^{-jwn}$

$\therefore h_1[0] = -4$, $h_1[-1] = \frac{3}{2}$, $h_1[1] = \frac{3}{2}$, $h_1[2] = 2$, $h_1[-2] = 2$

$\therefore h_1[n] = [2, \frac{3}{2}, \underset{\uparrow}{-4}, \frac{3}{2}, 2]$, $-2 \le n \le 2$

(b) $H_2(e^{jw}) = H_1(e^{jw}) \cdot \cos\frac{w}{2} e^{-\frac{w}{2}j}$

$$= H_1(e^{jw}) \cdot \frac{1}{2}(e^{\frac{w}{2}j} + e^{-\frac{w}{2}j}) e^{-\frac{w}{2}j}$$

$$= H_1(e^{jw}) \cdot \frac{1}{2}(e^0 + e^{-wj})$$

$$= \frac{1}{2}(e^{-wj}+1)(-4 + \frac{3}{2}e^{wj} + \frac{3}{2}e^{-wj} + 2e^{2wj} + 2e^{-2wj})$$

$$= -2e^{-wj} + \frac{3}{4}e^0 + \frac{3}{4}e^{-2wj} + e^{wj} + e^{-3wj} - 2 + \frac{3}{4}e^{wj} + \frac{3}{4}e^{-wj} + e^{2wj} + e^{-2wj}$$

$$= -\frac{5}{4} + e^{-3wj} + \frac{7}{4}e^{-2wj} - \frac{5}{4}e^{-wj} + \frac{7}{4}e^{wj} + e^{2wj}$$

Same with above, we can get:

$h_2[n] = [1 \quad \frac{7}{4} \quad \underset{\uparrow}{-\frac{5}{4}} \quad -\frac{5}{4} \quad \frac{7}{4} \quad 1]$ $\quad -2 \le n \le 3$

(c) $H_3(e^{jw}) = H_1(e^{jw}) j \cdot \frac{1}{2j}(e^{jw} - e^{-jw})$

$$= \frac{1}{2} H_1(e^{jw})(e^{jw} - e^{-jw}) = 0 + \frac{3}{4}e^{2wj} - 3e^{wj} + e^{3wj} + 3e^{-wj} - \frac{3}{4}e^{-2wj} - e^{-3wj}$$

$\therefore h_3[n] = [1 \quad \frac{3}{4} \quad -3 \quad \underset{\uparrow}{0} \quad 3 \quad -\frac{3}{4} \quad -1]$, $-3 \le n \le 3$

(d) $H_4(e^{jw}) = H_1(jw) j \cdot \frac{1}{2j}(e^{\frac{w}{2}j} - e^{-\frac{w}{2}j}) e^{\frac{w}{2}j}$

$= H_1(jw) \frac{1}{2}(e^{wj} - 1)$

$= -2e^{wj} + \frac{3}{4}e^{2wj} + \frac{3}{4} + e^{3wj} + e^{-wj} + 2 - \frac{3}{4}e^{wj} - \frac{3}{4}e^{-wj} - e^{2wj} - e^{-2wj}$

$\therefore h_4[n] = [1 \quad -\frac{1}{4} \quad -\frac{11}{4} \quad \underset{\uparrow}{\frac{11}{4}} \quad \frac{1}{4} \quad -1], \ -3 \le n \le 2$

3.43

(a) To solve this problem, we firstly need to configure a DTFT:

The DTFT of $\{-\alpha^n u[-n-1]\}_{-\infty}^{\infty}$ is: $X(e^{jw}) = \sum_{n=-\infty}^{\infty}(-\alpha^n u[-n-1]) e^{-jwn} = \sum_{-\infty}^{n=-1} -\alpha^n e^{-jwn} = \sum_{n=-\infty}^{n=-1} -(\alpha e^{-jw})^n$

$= -\frac{1}{(\alpha e^{-jw})^{-\infty}} \cdot \frac{1-(\alpha e^{-jw})^{\infty}}{1-\alpha e^{-jw}}$ when $|\alpha| > 1$, $X(e^{jw}) = \frac{1}{1-\alpha e^{-jw}}$

$|X(e^{jw})|^2 = \left|\frac{1}{1-\alpha e^{-jw}}\right|^2 = \left|\frac{1}{(1-\alpha\cos w + \alpha \sin w j)}\right|^2 = \frac{1}{\alpha^2 - 2\alpha\cos w + 1}$

$\int_0^{\pi} \frac{4}{5+4\cos w} dw = \frac{1}{2}\int_{-\pi}^{\pi} \frac{4}{5+4\cos w} dw = \frac{1}{2} 4 \int_{-\pi}^{\pi} |X(e^{jw})|^2 dw$

Simpicitily, we let $\alpha = -2$, $X(e^{jw}) = \frac{1}{1+2e^{-jw}}$, $x[n] = -(-2)^n u[-n-1]$

By parseval's relation:

$4 \times \frac{1}{2}\int_{-\pi}^{\pi} |X(e^{jw})|^2 dw = 4\pi \sum_{n=-\infty}^{\infty} |x[n]|^2 = 4\pi \sum_{n=-\infty}^{\infty} |(-2)^n|^2 u[-n-1] = 4\pi \sum_{n=1}^{\infty} 4^{-n} = \frac{4}{3}\pi$

(b) $\int_0^{\pi} \frac{1}{3.25 - 3\cos w} dw = \frac{1}{2}\int_{-\pi}^{\pi} \frac{1}{3.25 - 3\cos w} dw$

let $\alpha = 1.5$, $|X(e^{jw})|^2 = \frac{1}{3.25 - 3\cos w}$, $x[n] = -1.5^n u[-n-1]$

So: $\frac{1}{2}\int_{-\pi}^{\pi} \frac{1}{3.25 - 3\cos w} dw = \pi \sum_{n=-\infty}^{\infty} |x[n]|^2 = \pi \sum_{n=-\infty}^{n=-1} (1.5^n)^2 = \frac{4}{5}\pi$

(c) In order to configure the form in (c), we can firstly take a derivative:

$j \frac{d(X(e^{jw}))}{dw} = j \cdot \frac{j - \alpha(-1)e^{-jw}}{(1-\alpha e^{-jw})^2} = \frac{\alpha e^{-jw}}{(1-\alpha e^{-jw})^2} \longleftrightarrow -n\alpha^n u[-n-1]$

By linearity, the DTFT of $x[n] = -(n+1)\alpha^n u[-n-1]$ is: $\frac{\alpha e^{-jw}}{(1-\alpha e^{-jw})^2} + \frac{1}{1-\alpha e^{-jw}} = \frac{1}{(1-\alpha e^{-jw})^2}$

let $\alpha = 2$, $X(e^{jw}) = \frac{1}{(1-2e^{-jw})^2}$, $|X(e^{jw})|^2 = \frac{1}{(5-4\cos w)^2}$

$\therefore \int_0^{\pi} \frac{4}{(5-4\cos w)^2} dw = 2\int_{-\pi}^{\pi} \frac{1}{(5-4\cos w)^2} dw = 4\pi \sum_{n=-\infty}^{\infty} |x[n]|^2 = 4\pi \sum_{n=-\infty}^{-1} (n+1)^2 \cdot 4^n$

let $n' = -n - 1$, So: $4\pi \sum_{n=-\infty}^{-1} (n+1)^2 4^n = 4\pi \sum_{n=0}^{\infty} n^2 4^{-n-1} = \pi \sum_{n=0}^{\infty} n^2 4^{-n}$

As $S = \pi \sum_{n=0}^{\infty} n^2 4^{-n} = \pi(1^2 \cdot 4^{-1} + 2^2 \cdot 4^{-2} + 3^2 \cdot 4^{-3} + 4^2 \cdot 4^{-4} + \dots)$

$\frac{1}{4}S = (1^2 4^{-2} + 2^2 4^{-3} + 3^2 4^{-4} + \dots)\pi$, So $\frac{3}{4}S = (\frac{1}{4} + 3(\frac{1}{4})^2 + 5(\frac{1}{4})^3 + 7(\frac{1}{4})^4 + \dots)\pi$ $\quad \frac{5\pi}{12} \times \frac{16}{9}$ [illegible]

$\frac{3}{16}S = [(\frac{1}{4})^2 + 3(\frac{1}{4})^3 + 5(\frac{1}{4})^4 + \dots]\pi$, So [illegible] $\Rightarrow S = \frac{5}{12}\pi \Rightarrow \int_0^{\pi} \frac{4dw}{(5-4\cos w)^2} =$ [illegible]

3.53

we can firstly find the relationship between those functions:

obviously, $g_2[n]=g_1[n]+g_1[n-4]$

So: $G_2(e^{jw}) = G_1(e^{jw})+e^{-4jw}G_1(e^{jw})$

Similarly, $g_3[n]=g_1[n]+g_1[-(n-7)]$

So: $G_3(e^{jw})=G_1(e^{jw})+e^{-7jw}G_1(e^{-jw})$

Similarly, $g_4[n]=g_1[n-4]+g_1[-(n-3)]$

So: $G_4(e^{jw})=G_1(e^{jw})e^{-4jw}+e^{-3jw}G_1(e^{-jw})$

3.61

As signal $x_a(t)$ is Sampled at a 3.0 kHz rate,

So the frequency of different component will be $(f\pm 3000n)$ Hz, $-\infty<n<\infty$, $n\in N$

for 300 HZ component, it will generate: ... -2700 HZ, 300 HZ, 3300 HZ ...

for 500 HZ component, it will generate: ... -2500 HZ, 500 HZ, 3500 HZ ...

for 1200 HZ component, it will generate: ... -1800 HZ, 1200 HZ, 4200 HZ ...

for 2150 HZ component, it will generate: ... -850 HZ, 2150 HZ, 5150 HZ ...

for 3500 HZ component, it will generate: ... 500 HZ, 3500 HZ, 6500 HZ ...

When the sampled sequence pass through the low pass filter:

There are 300 HZ, 500 HZ, 850 HZ

M 3.1

In this problem, we choose four combinations of θ and r, the generated figures are as following:

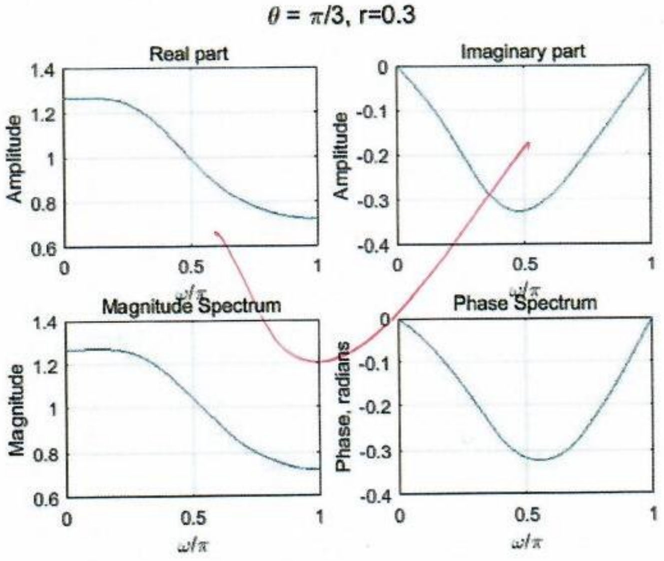

Figure 1: Graph of $G(e^{jw})$ when $\theta = \frac{\pi}{3}, r = 0.3$

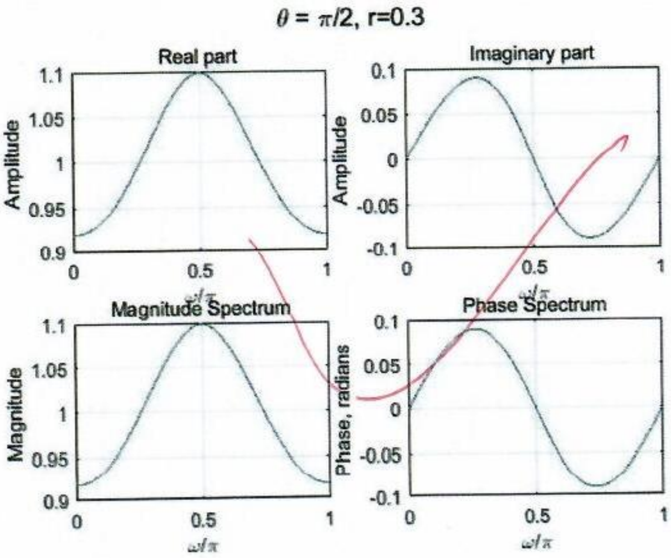

Figure 2: Graph of $G(e^{jw})$ when $\theta = \frac{\pi}{2}, r = 0.3$

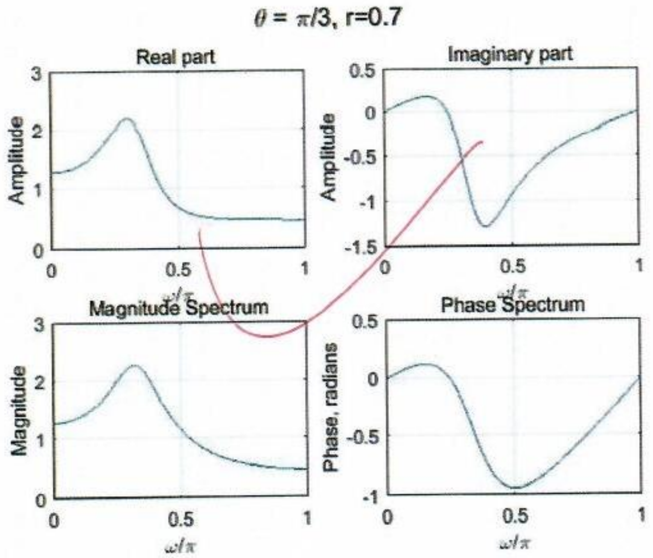

Figure 3: Graph of $G(e^{jw})$ when $\theta = \frac{\pi}{3}, r = 0.7$

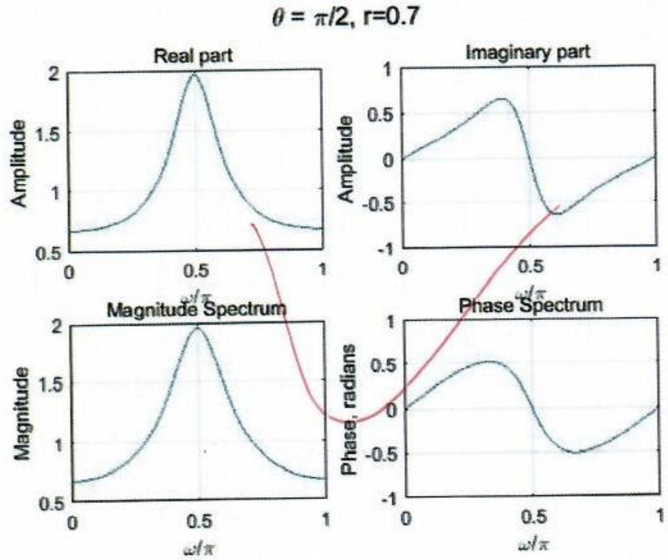

Figure 4: Graph of $G(e^{jw})$ when $\theta = \frac{\pi}{2}, r = 0.7$

M 3.4

- First property: $X(e^{jw}) = X^*(e^{-jw})$

The figures of two DTFT are as following:

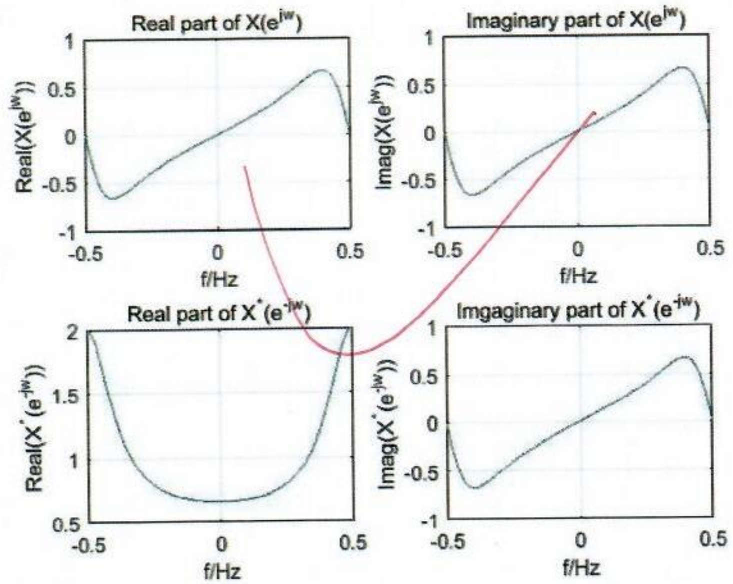

Figure 5: Graph of Two DTFT

Matlab code for plotting figure 5:

```
%M3.4
Fs = 1;                    % Sampling frequency
T = 1/Fs;                     % Sampling period
L = 1000;                     % Length of signal
n = (0:L-1)*T;                % Time vector
x=(1/2).^n;                    %Original signal
X = fft(x);                     %DTFT
f = Fs*(-L/2+1:L/2)/L;    %frequency


%Verify first property
Y_conj=conj(X);
figure(1);
subplot(2,2,1);
plot(f,imag(X));grid
title('Real part of X(e^{jw})')
xlabel('f/Hz'); ylabel('Real(X(e^{jw}))');
subplot(2,2,2)
plot(f,imag(X));grid
title('Imaginary part of X(e^{jw})')
xlabel('f/Hz'); ylabel('Imag(X(e^{jw}))')
subplot(2,2,3)
plot(-f,real(Y_conj));grid
```

```
title('Real part of X^*(e^{-jw})')
xlabel('f/Hz'); ylabel('Real(X^*(e^{-jw}))')
subplot(2,2,4);
plot(-f,imag(Y_conj));grid
title('Imgaginary part of X^*(e^{-jw})')
xlabel('f/Hz'); ylabel('Imag(X^*(e^{-jw}))')
```

- Second property: $X_{re}(e^{jw}) = X_{re}(e^{-jw})$

The figures of two DTFT are as following:

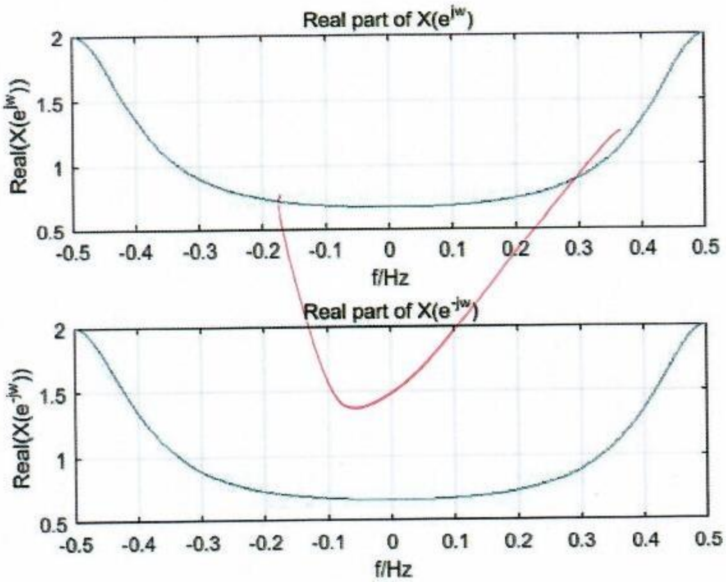

Figure 6: Graph of Two DTFT

Matlab code for plotting figure 6:

```
%Verify Second property
figure(2);
subplot(2,1,1);
plot(f,real(X));grid
title('Real part of X(e^{jw})')
xlabel('f/Hz'); ylabel('Real(X(e^{jw}))')
subplot(2,1,2);
plot(-f,real(X));grid
title('Real part of X(e^{-jw})')
xlabel('f/Hz'); ylabel('Real(X(e^{-jw}))')
```

- Third property: $X_{im}(e^{jw}) = -X_{im}(e^{-jw})$

The figures of two DTFT are as following:

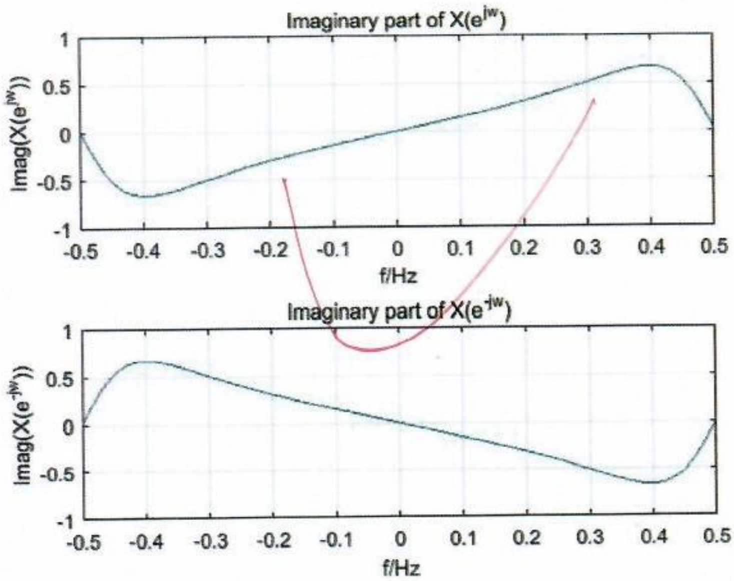

Figure 7: Graph of Two DTFT

Matlab code for plotting figure 7:

```
%Verify the third property
figure(3);
subplot(2,1,1);
plot(f,imag(X));grid
title('Imaginary part of X(e^{jw})')
xlabel('f/Hz'); ylabel('Imag(X(e^{jw}))')
subplot(2,1,2);
plot(-f,imag(X));grid
title('Imaginary part of X(e^{-jw})')
xlabel('f/Hz'); ylabel('Imag(X(e^{-jw}))')
```

- Fourth property: $|X(e^{jw})| = |X(e^{-jw})|$

The figures of two DTFT are as following:

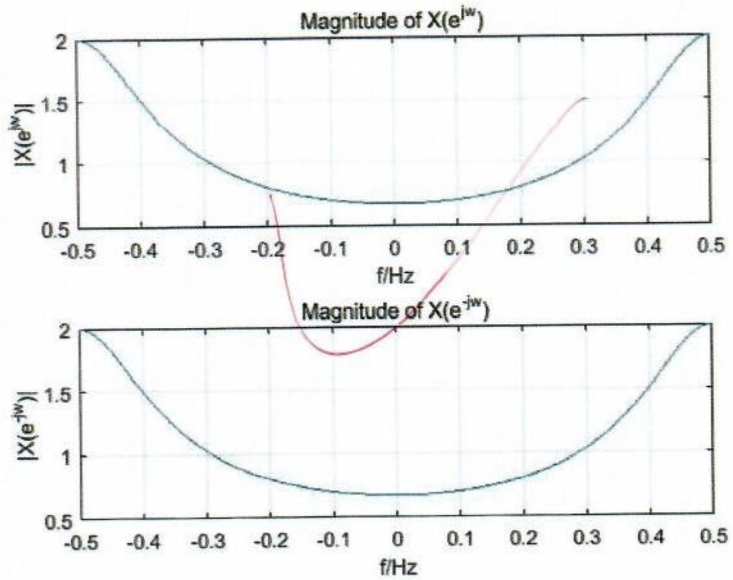

Figure 8: Graph of Two DTFT

Matlab code for plotting figure 8:

```
%Verify the fourth property
figure(4);
subplot(2,1,1);
plot(f,abs(X));grid
title('Magnitude of X(e^{jw})')
xlabel('f/Hz'); ylabel('|X(e^{jw})|')
subplot(2,1,2);
plot(-f,abs(X));grid
title('Magnitude of X(e^{-jw})')
xlabel('f/Hz'); ylabel('|X(e^{-jw})|')
```

- Fifth Property: $arg\{X(e^{jw})\} = -arg\{X(e^{-jw})\}$

The figures of two DTFT are as following:

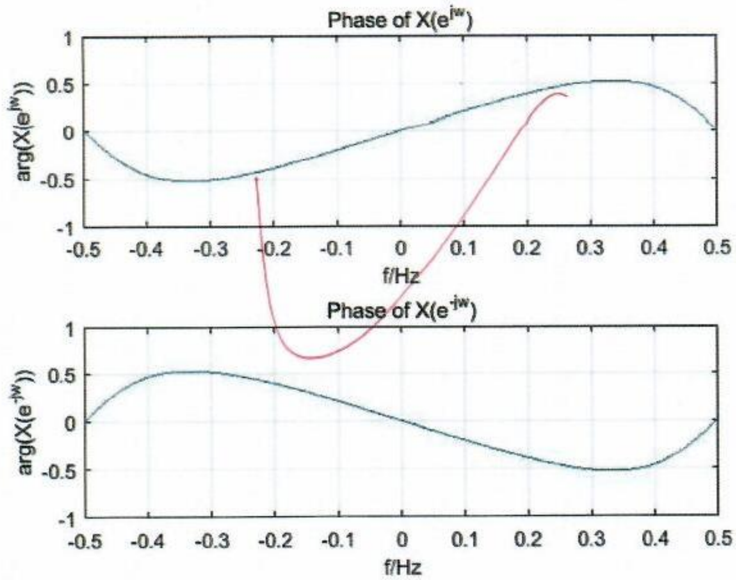

Figure 9: Graph of Two DTFT

- Matlab code for plotting figure 9:

```
%Verify the fifth property
figure(5);
subplot(2,1,1);
plot(f,angle(X));grid
title('Phase of X(e^{jw})')
xlabel('f/Hz'); ylabel('arg(X(e^{jw}))')
subplot(2,1,2);
plot(-f,angle(X));grid
title('Phase of X(e^{-jw})')
xlabel('f/Hz'); ylabel('arg(X(e^{-jw}))')
```

M 3.5

- First property: $x[-n] \Leftrightarrow X(e^{-jw})$

The figure for verifying that $x[-n]$ and $X(e^{-jw})$ are DTFT pairs is:

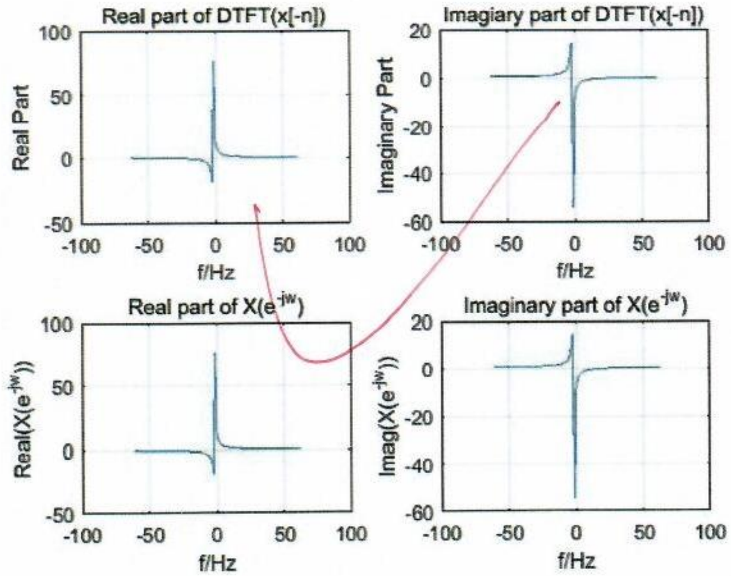

Figure 10: Graph of Two DTFT

Matlab code for plotting figure 10:

```
%M3.5
fs = 125;          % Sampling frequency
N = 100;            %sampling frequency and mumber
n = 0:N-1;
t = n/fs;          %time sequence
f = n*fs/N-fs/2;

x = exp(1i*3*pi*t);            %x[n]
X = fftshift(fft(x,N));        % DTFT of x[n]

%Verify first property
x1=exp(-1i*3*pi*t);            % x[-n]
X1 = fftshift(fft(x1,N));      % DTFT of x[-n]

figure(1);
subplot(221);
plot(f,real(X1)); grid;
title('Real part of DTFT(x[-n])');
xlabel('f/Hz'); ylabel('Real Part');

subplot(222);
plot(f,imag(X1)); grid;
title('Imagiary part of DTFT(x[-n])');
xlabel('f/Hz'); ylabel('Imaginary Part');

```

```
subplot(223);
plot(-f,real(X)); grid;
title('Real part of X(e^{-jw})');
xlabel('f/Hz'); ylabel('Real(X(e^{-jw}))');

subplot(224);
plot(-f,-imag(X)); grid;
title('Imaginary part of X(e^{-jw})');
xlabel('f/Hz'); ylabel('Imag(X(e^{-jw}))');
```

- Second property: $x^*[-n] \Leftrightarrow X^*(e^{jw})$

The figure for verifying that $x^*[-n]$ and $X^*(e^{jw})$ are DTFT pairs is:

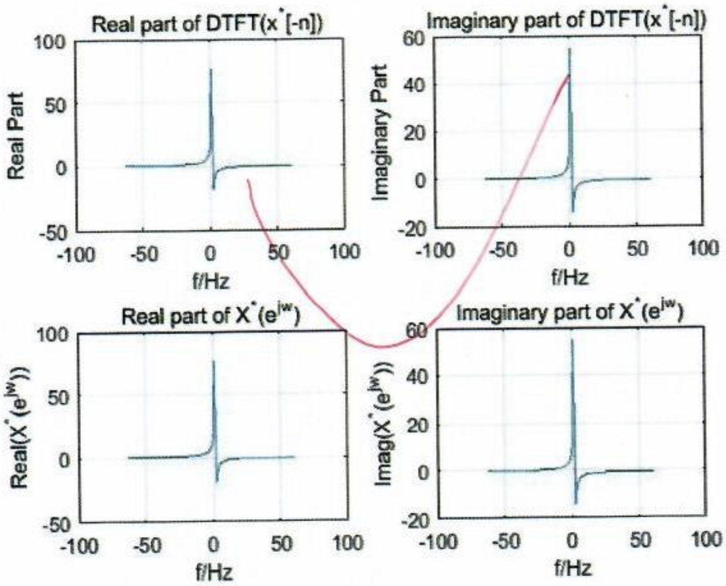

Figure 11: Graph of Two DTFT

Matlab code for plotting figure 12:

```
%Verify Second property
x2 = conj(x1);                 % x*[-n]
X2 = fftshift(fft(x2,N));      % DTFT of x[-n]
X_conj = conj(X);

% DTFT of x*[-n]
figure(2);
subplot(221);
plot(f,real(X2)); grid;
```

```
title('Real part of DTFT(x^{*}[-n])');
xlabel('f/Hz'); ylabel('Real Part');

subplot(222);
plot(f,imag(X2)); grid;
title('Imaginary part of DTFT(x^{*}[-n])');
xlabel('f/Hz'); ylabel('Imaginary Part');

subplot(223); plot(f,real(X_conj)); grid;
title('Real part of X^{*}(e^{jw})');
xlabel('f/Hz'); ylabel('Real(X^{*}(e^{jw}))');

subplot(224); plot(f,-imag(X_conj)); grid;
title('Imaginary part of X^{*}(e^{jw})');
xlabel('f/Hz'); ylabel('Imag(X^{*}(e^{jw}))');
```

- Third property: $x_{re}[n] \Leftrightarrow X_{cs}(e^{jw})$

The figure for verifying that $x_{re}[n]$ and $X_{cs}(e^{jw})$ are DTFT pairs is:

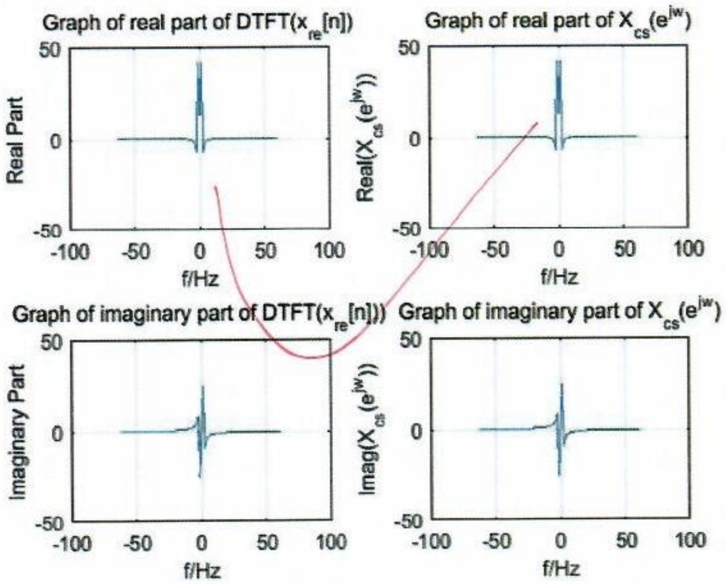

Figure 12: Graph of Two DTFT

Matlab code for plotting figure 12:

```
%Verify property 3
Xcs = 0.5 * (X + conj(X1));
x3 = real(x);
```

```
X3 = fftshift(fft(x3));

figure(3);
subplot(221);
plot(f,real(X3)); grid;
title('Graph of real part of DTFT(x_{re}[n])');
xlabel('f/Hz'); ylabel('Real Part');

subplot(222);
plot(f,real(Xcs)); grid;
title('Graph of real part of X_{cs}(e^{jw})');
xlabel('f/Hz'); ylabel('Real(X_{cs}(e^{jw}))');

subplot(223);
plot(f,imag(X3)); grid;
title('Graph of imaginary part of DTFT(x_{re}[n]))');
xlabel('f/Hz'); ylabel('Imaginary Part');

subplot(224);
plot(f,imag(Xcs)); grid;
title('Graph of imaginary part of X_{cs}(e^{jw})');
xlabel('f/Hz'); ylabel('Imag(X_{cs}(e^{jw}))');
```

- Fourth property: $jx_{im}[n] \Leftrightarrow X_{ca}(e^{jw})$

The figure for verifying that $jx_{im}[n]$ and $X_{ca}(e^{jw})$ are DTFT pairs is:

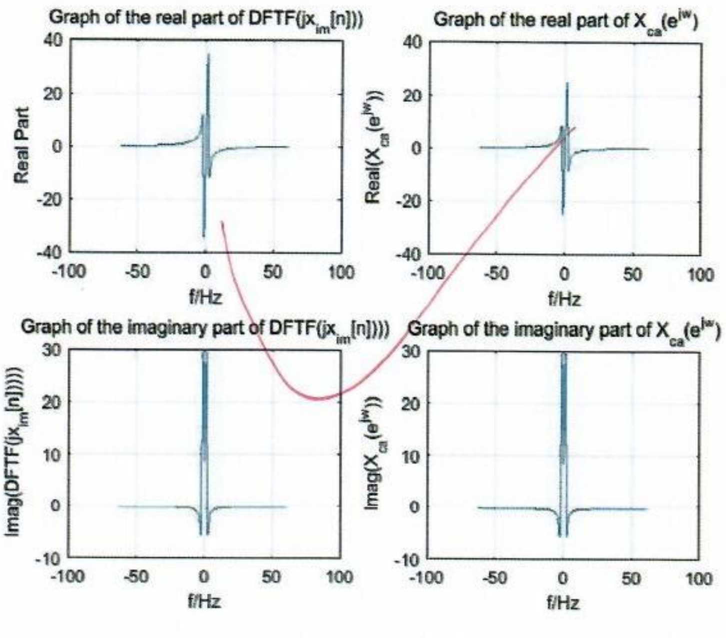

Figure 13: Graph of Two DTFT

Matlab code for plotting figure 13:

```
% Property 4
Xca = 0.5 * (X - conj(X1))/1i;
x4 = 1i * imag(x);
X4 = fftshift(fft(x4));

figure(4);
subplot(221);
plot(f,real(X4)); grid;
title('Graph of the real part of DFTF(jx_{im}[n]))');
xlabel('f/Hz'); ylabel('Real Part');

subplot(222);
plot(f,real(Xca)); grid;
title('Graph of the real part of X_{ca}(e^{jw})');
xlabel('f/Hz'); ylabel('Real(X_{ca}(e^{jw}))');

subplot(223);
plot(f,imag(X4)); grid;
title('Graph of the imaginary part of DFTF(jx_{im}[n])))');
xlabel('f/Hz'); ylabel('Imag(DFTF(jx_{im}[n]))))');

subplot(224);
plot(f,imag(Xca)); grid;
title('Graph of the imaginary part of X_{ca}(e^{jw})');
xlabel('f/Hz'); ylabel('Imag(X_{ca}(e^{jw}))');
```

- Fifth property: $x_{cs}[n] \Leftrightarrow X_{re}(e^{jw})$

The figure for verifying that $x_{cs}[n]$ and $X_{re}(e^{jw})$ are DTFT pairs is:

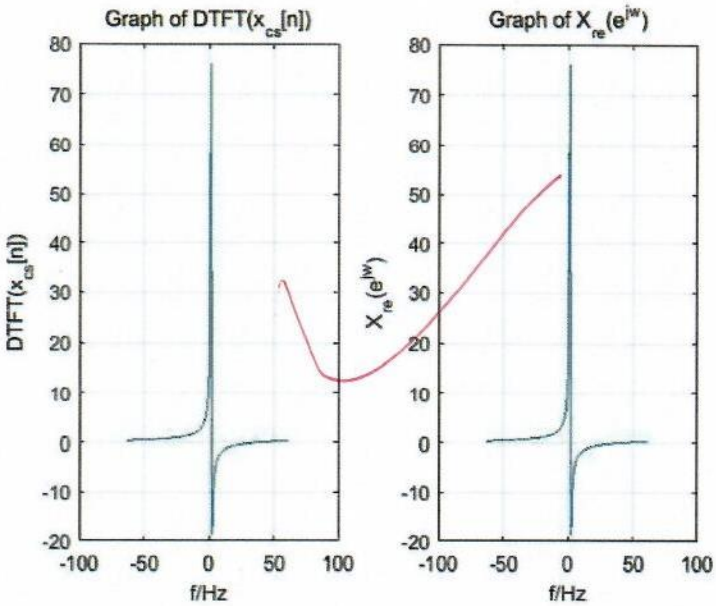

Figure 14: Graph of Two DTFT

Matlab code for plotting figure 14:

```
% Property 5
i=0:length(x)-1;
j=mod(-i,length(x))+1;
xcc=conj(x(j));

xcs=1/2*(x+xcc);
X5 = fftshift(fft(xcs));

figure(5);
subplot(121); plot(f,X5); grid;
title('Graph of DTFT(x_{cs}[n])');
xlabel('f/Hz'); ylabel('DTFT(x_{cs}[n])');

subplot(122); plot(f,real(X)); grid;
title('Graph of X_{re}(e^{jw})');
xlabel('f/Hz'); ylabel('X_{re}(e^{jw})');
```

- Sixth property: $x_{ca}[n] \Leftrightarrow jX_{im}(e^{jw})$

The figure for verifying that $x_{ca}[n]$ and $jX_{im}(e^{jw})$ are DTFT pairs is:

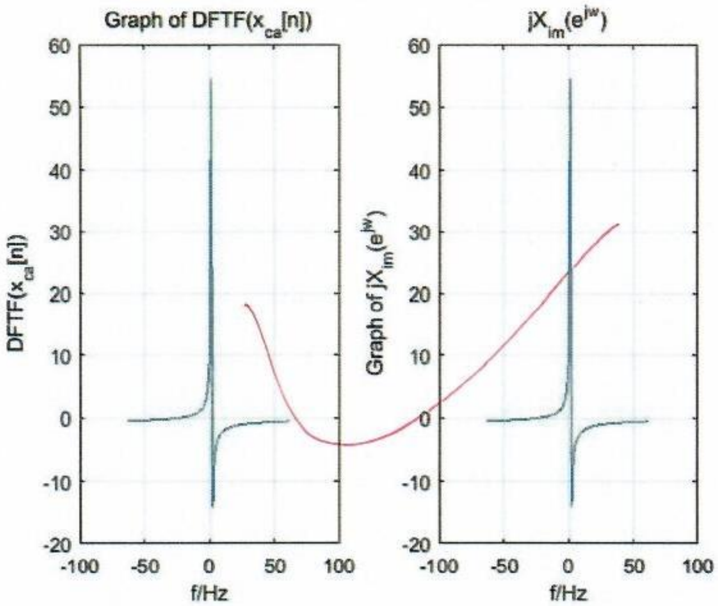

Figure 15: Graph of Two DTFT

Matlab code for plotting figure 15:

```
% Property 6
xca=1/2*(x-xcc);
X6 = fftshift(fft(xca));

figure(6);
subplot(121);
plot(f,imag(X6)); grid;
title('Graph of DFTF(x_{ca}[n])');
xlabel('f/Hz'); ylabel('DFTF(x_{ca}[n])');

subplot(122);
plot(f,imag(X)); grid;
title('jX_{im}(e^{jw})');
xlabel('f/Hz'); ylabel('Graph of jX_{im}(e^{jw})');
```

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