电子科技大学
格拉斯哥学院
Glasgow College, UESTC

Assignments for
Digital Signal Processing

Class: EEE Class 4

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Date Performed:

Chapter 7

7.1

Soln: know that:

$H_0(z) = G_L(z)G_H(z^2)$

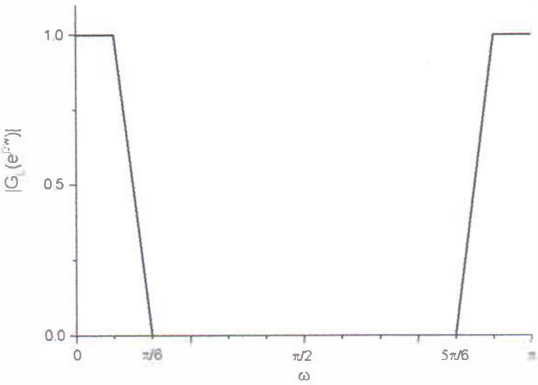
$H_1(z) = G_H(z)G_H(z^2)$

$H_2(z) = G_H(z)G_L(z^2)$

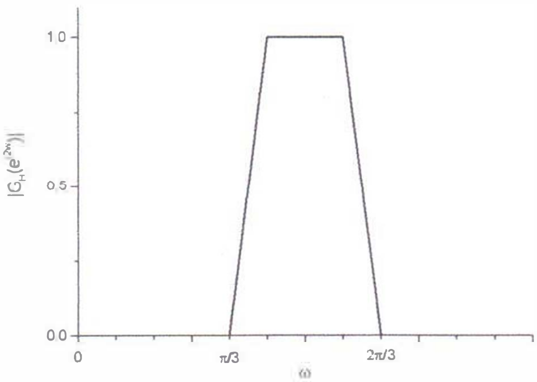
$H_3(z) = G_L(z)G_L(z^2)$

Know that:

The graph of $G_L(z^2)$:

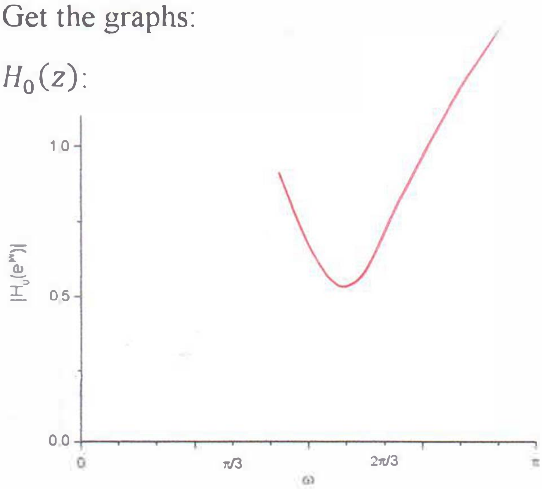


The graph of $G_H(z^2)$:

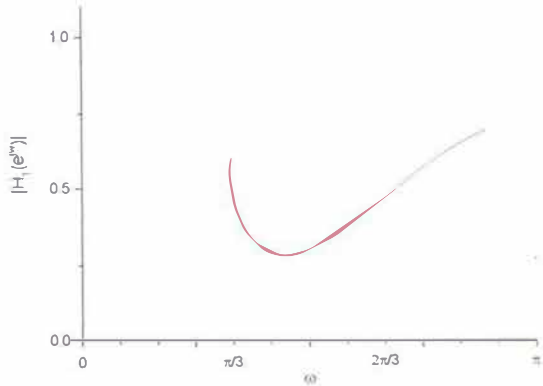


Get the graphs:

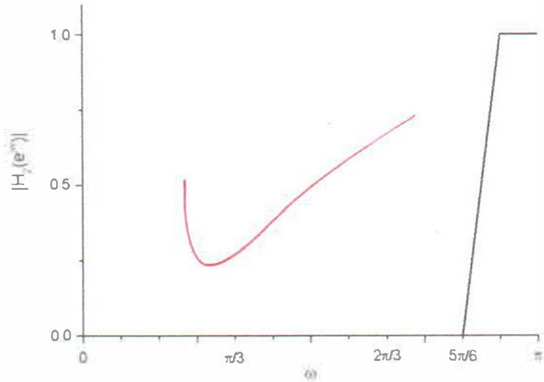
$H_0(z)$:



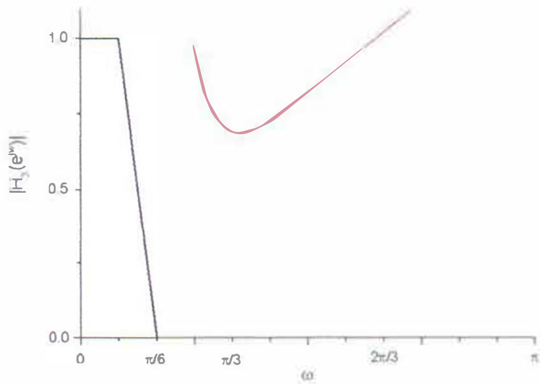
$H_1(z)$:



$H_2(z)$:



$H_3(z)$:



7.6

Soln: know that:

$$M_0(z) = X(ze^{j\omega_0}) + X(ze^{-j\omega_0})$$

$$N_0(z) = \frac{1}{j}X(ze^{j\omega_0}) - \frac{1}{j}X(ze^{-j\omega_0})$$

$$M_1(z) = \frac{1}{2}M_0(ze^{j\omega_0})H_{LP}(ze^{j\omega_0}) + \frac{1}{2}M_0(ze^{-j\omega_0})H_{LP}(ze^{-j\omega_0})$$

$$N_1(z) = -\frac{j}{2}N_0(ze^{j\omega_0})H_{LP}(ze^{j\omega_0}) + \frac{j}{2}M_0(ze^{-j\omega_0})H_{LP}(ze^{-j\omega_0})$$

Get:

$$\begin{aligned} Y(z) &= M_1(z) + N_1(z) \\ &= H_{LP}(ze^{j\omega_0})\left[\frac{1}{2}M_0(ze^{j\omega_0}) - \frac{j}{2}N_0(ze^{j\omega_0})\right] \\ &+H_{LP}(ze^{-j\omega_0})\left[\frac{1}{2}M_0(ze^{-j\omega_0}) + \frac{j}{2}N_0(ze^{-j\omega_0})\right] \\ &= H_{LP}(ze^{j\omega_0})X(z) + H_{LP}(ze^{-j\omega_0})X(z) \end{aligned}$$

Thus:

$$H(z) = \frac{Y(z)}{X(z)} = H_{LP}(ze^{j\omega_0}) + H_{LP}(ze^{-j\omega_0})$$

7.14

Soln: know that:

$$F(e^{j\omega}) = e^{j\omega}\left(\frac{G(e^{j\omega}) + \alpha}{1 + \alpha G(e^{j\omega})}\right)$$

$$= e^{j\omega}\left(\frac{G(e^{j\phi(\omega)}) + \alpha}{1 + \alpha G(e^{j\phi(\omega)})}\right)$$

For $G(z)$ is LBR function, get:

$$\begin{aligned} |F(e^{j\omega})|^2 &= \left|\frac{e^{j\phi(\omega)} + \alpha}{1 + \alpha e^{j\phi(\omega)}}\right|^2 \\ &= \frac{[\cos(\phi(\omega)) + \alpha]^2 + \sin^2(\phi(\omega))}{[1 + \alpha\cos(\phi(\omega))]^2 + [\alpha\sin(\phi(\omega))]^2} \\ &= \frac{1 + 2\alpha\cos(\phi(\omega)) + \alpha^2}{1 + 2\alpha\cos(\phi(\omega)) + \alpha^2} = 1 \end{aligned}$$

Let $z = \lambda$ be a pole of $F(z)$, get:

$$G(z)|_{z=\lambda} = \left.\frac{F(z) - \alpha z}{z - \alpha F(z)}\right|_{z=\lambda} = -\frac{1}{\alpha}$$

$$\Rightarrow |G(\lambda)| = \left|\frac{1}{\alpha}\right|$$

Know that, $|\alpha| < 1 \Rightarrow |G(\lambda)| > 1$, which is satisfied by the LBR function $G(z)$ if $|\lambda| < 1$.

Thus, $F(z)$ is LBR function, and the order of $F(z)$ is the same as $G(z)$.

Know that:

To realize $G(z)$ in terms of $F(z)$, an expression is shown below:

$$G(z) = \frac{-\alpha + z^{-1}F(z)}{1 - \alpha z^{-1}F(z)} = \frac{C + DF(z)}{A + BF(z)}$$

Where A, B, C and D are the chain parameters of the two-pair. Comparing the above two expressions can get:

$A = 1,\ B = -\alpha z^{-1},\ C = -\alpha,\ D = z^{-1}$

The corresponding transfer parameters are given by:

$t_{11} = -\alpha,\ t_{21} = 1,\ t_{12} = (1-\alpha^2)z^{-1},$

$t_{22} = \alpha z^{-1}$

7.20

Soln: know that:

$$\begin{aligned} A_2(z) &= \frac{d_2 + d_1z^{-1} + z^{-2}}{1 + d_1z^{-1} + d_2z^{-2}} = \frac{d_2z + d_1 + z^{-1}}{z + d_1 + d_2z^{-1}} \\ &= \frac{(d_1 + d_2\cos\omega + \cos\omega) + j(\sin\omega + d_2\sin\omega)}{(d_1 + d_2\cos\omega + \cos\omega) + j(\sin\omega - d_2\sin\omega)} \\ &= \frac{[d_1 + (d_2+1)\cos\omega] + j(d_2-1)\sin\omega}{[d_1 + (d_2+1)\cos\omega] - j(d_2-1)\sin\omega} \end{aligned}$$

So that:

$$\theta(\omega) = 2\arctan\left(\frac{(d_2-1)\sin\omega}{d_1 + (d_2+1)\cos\omega}\right)$$

$$\begin{aligned} \Rightarrow \tau_p(\omega) &= -\frac{\theta(\omega)}{\omega} \\ &= -\frac{2}{\omega}\arctan\left(\frac{(d_2-1)\sin\omega}{d_1 + (d_2+1)\cos\omega}\right) \end{aligned}$$

For $\omega \to 0$, get:

$\sin\omega = \omega,\ \cos\omega = 1$

For $x \to 0$, get $\arctan x \to x$,

Get:

$$\tau_p(\omega) = -\frac{2}{\omega}\arctan\left(\frac{(d_2-1)\omega}{d_1 + (d_2+1)}\right)$$

$$= -\frac{2}{\omega} \cdot \frac{(d_2 - 1)\omega}{d_1 + (d_2 + 1)}$$
$$= \frac{2(1 - d_2)}{d_1 + d_2 + 1}$$

For $d_1 = 2\left(\frac{2-\delta}{1+\delta}\right)$, $d_2 = \frac{(2-\delta)(1-\delta)}{(2+\delta)(1+\delta)}$, get:

$$\tau_p(\omega) = \frac{2\left[1 - \frac{(2-\delta)(1-\delta)}{(2+\delta)(1+\delta)}\right]}{2\left(\frac{2-\delta}{1+\delta}\right) + \frac{(2-\delta)(1-\delta)}{(2+\delta)(1+\delta)} + 1}$$
$$= \frac{2[(2+\delta)(1+\delta) - (2-\delta)(1-\delta)]}{2(2-\delta)(2+\delta) + (2-\delta)(1-\delta) + (2+\delta)(1+\delta)}$$
$$= \frac{12\delta}{12} = \delta$$

7.27

Soln:

The first-order factor: $1 + az^{-1}$

The square-magnitude function:

$(1 + az^{-1})(1 + az)|_{z=e^{j\omega}} = (1 + a^2) + 2a\cos\omega$

Know that:

$$\left|H(e^{j\omega})\right|^2 = H(z)H(z^{-1})$$
$$= \left.\frac{4[1.25 + 0.5(z + z^{-1})][1.36 - 0.6(z + z^{-1})]}{[1.36 + 0.6(z + z^{-1})][1.64 + 0.8(z + z^{-1})]}\right|_{z=e^{j\omega}}$$
$$= \frac{4(1 + 0.5z)(1 + 0.5z^{-1})(1 - 0.6z)(1 - 0.6z^{-1})}{(1 + 0.6z)(1 + 0.6z^{-1})(1 + 0.8z)(1 + 0.8z^{-1})}$$

Know that, there are four possible causal stable transfer functions:

$$H_1(z) = \frac{2(1 + 0.5z)(1 - 0.6z)}{(1 + 0.6z^{-1})(1 + 0.8z^{-1})}$$
$$H_2(z) = \frac{2(1 + 0.5z)(1 - 0.6z^{-1})}{(1 + 0.6z^{-1})(1 + 0.8z^{-1})}$$
$$H_3(z) = \frac{2(1 + 0.5z^{-1})(1 - 0.6z)}{(1 + 0.6z^{-1})(1 + 0.8z^{-1})}$$
$$H_4(z) = \frac{2(1 + 0.5z^{-1})(1 - 0.6z^{-1})}{(1 + 0.6z^{-1})(1 + 0.8z^{-1})}$$

Chapter 8

8.1

Soln: know that:

$$Y(z) = [X(z) - Y(z)C(z)]G(z)$$

$$\Rightarrow Y(z) = \frac{X(z)G(z)}{1 - G(z)C(z)}$$

Get:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{G(z)}{1 + G(z)C(z)}$$

$$= \frac{\frac{z^{-2}}{1 + 1.5z^{-1} + 0.5z^{-2}}}{1 + \frac{Kz^{-2}}{1 + 1.5z^{-1} + 0.5z^{-2}}}$$

$$= \frac{z^{-2}}{1 + 1.5z^{-1} + (0.5 + K)z^{-2}}$$

$\because$ The system is stable.

$\therefore\ |0.5 + K| < 1,\ \ 1.5 < 1 + |0.5 + K|$

Get:

$K \in (0, 0.5)$

8.2

Soln: know that:

According to the calculation in 8.1, know that:

$$H(z) = \frac{z^{-2}}{1 + (1.5 + K)z^{-1} + 0.5z^{-2}}$$

$\because$ The system is stable.

$\therefore\ |1.5 + K| < 1 + 0.5$

Get:

$K \in (-3, 0)$

8.7

Soln: know that:

$$H_1(z) = \frac{\beta_1}{z - \alpha_1} = \frac{\beta_1 z^{-1}}{1 - \alpha_1 z^{-1}}$$

For the same reason:

$$H_2(z) = \frac{\beta_2 z^{-1}}{1 - \alpha_2 z^{-1}}$$

$$H_3(z) = \frac{\beta_3 z^{-1}}{1 - \alpha_3 z^{-1}}$$

Know that:

$$B(z) = H_1(z)[X(z) + A(z)]$$

$$A(z) = H_2(z)[B(z) + C(z)]$$

$$C(z) = H_3(z)A(z)$$

Get:

$$B(z) = \frac{H_1(z)X(z)}{1 - \frac{H_1(z)H_2(z)}{1 - H_2(z)H_3(z)}}$$

$$= \frac{[H_1(z) - H_1(z)H_2(z)H_3(z)]X(z)}{1 - H_2(z)H_3(z) - H_1(z)H_2(z)}$$

Get:

$$Y(z) = \alpha_0 X(z) + B(z)$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)}$$

$$= \alpha_0 + \frac{H_1(z) - H_1(z)H_2(z)H_3(z)}{1 - H_2(z)H_3(z) - H_1(z)H_2(z)}$$

Where $H_1(z) = \frac{\beta_1 z^{-1}}{1-\alpha_1 z^{-1}},\ H_2(z) = \frac{\beta_2 z^{-1}}{1-\alpha_2 z^{-1}},$

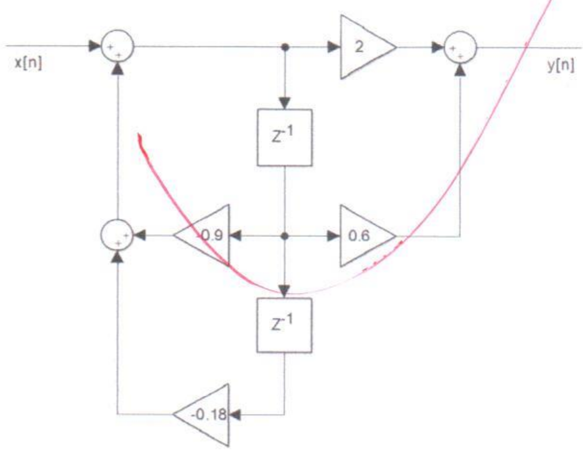
$H_3(z) = \frac{\beta_3 z^{-1}}{1-\alpha_3 z^{-1}}$

8.24

Soln:

a. Know that:

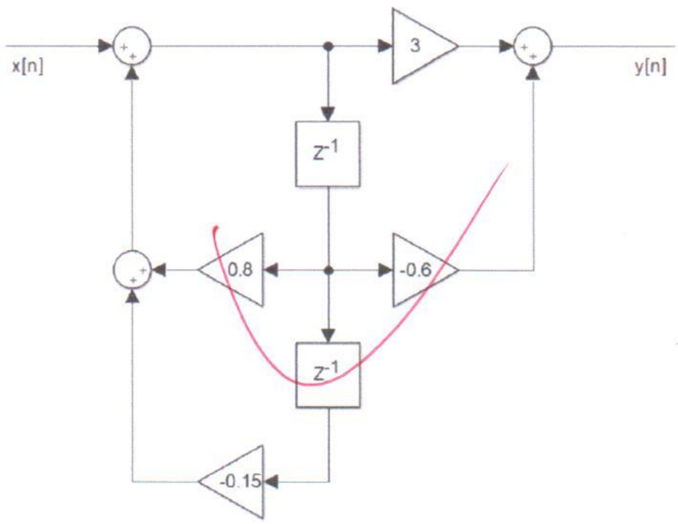
The canonical direct-form II realization is:



b. Know that:

$$H_2(z) = \frac{3 - 0.6z^{-1}}{1 - 0.8z^{-1} + 0.15z^{-2}}$$

The canonical direct-form II realization is:



8.61

Soln:

a. Know that:

$$\begin{aligned} &d_0 y[2l] + d_1 y[2l-1] + d_2 y[2l-2] \\ &\quad + d_3 y[2l-3] + d_4 y[2l-4] \\ &= p_0 x[2l] + p_1 x[2l-1] + p_2 x[2l-2] \\ &\quad + p_3 x[2l-3] + p_4 x[2l-4] \end{aligned}$$

Get:

$$\begin{aligned} &d_0 y[2l+1] + d_1 y[2l] + d_2 y[2l-1] \\ &\quad + d_3 y[2l-2] + d_4 y[2l-3] \\ &= p_0 x[2l] + p_1 x[2l-1] + p_2 x[2l-2] \\ &\quad + p_3 x[2l-3] + p_4 [2l-4] \end{aligned}$$

Know that:

$$\begin{aligned} &\begin{bmatrix} d_0 & 0 \\ d_1 & d_0 \end{bmatrix} \begin{bmatrix} y[2l] \\ y[2l+1] \end{bmatrix} + \begin{bmatrix} d_2 & d_1 \\ d_3 & d_2 \end{bmatrix} \begin{bmatrix} y[2l-2] \\ y[2l-1] \end{bmatrix} \\ &\quad + \begin{bmatrix} d_4 & d_3 \\ 0 & d_4 \end{bmatrix} \begin{bmatrix} y[2l-4] \\ y[2l-3] \end{bmatrix} \\ &= \begin{bmatrix} p_0 & 0 \\ p_1 & p_0 \end{bmatrix} \begin{bmatrix} x[2l] \\ x[2l+1] \end{bmatrix} + \begin{bmatrix} p_2 & p_1 \\ p_3 & p_2 \end{bmatrix} \begin{bmatrix} x[2l-2] \\ x[2l-1] \end{bmatrix} \\ &\quad + \begin{bmatrix} p_4 & p_3 \\ 0 & p_4 \end{bmatrix} \begin{bmatrix} x[2l-4] \\ x[2l-3] \end{bmatrix} \end{aligned}$$

$$\therefore D_0 = \begin{bmatrix} d_0 & 0 \\ d_1 & d_0 \end{bmatrix}, D_1 = \begin{bmatrix} d_2 & d_1 \\ d_3 & d_2 \end{bmatrix}, D_2 = \begin{bmatrix} d_2 & d_1 \\ d_3 & d_2 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} p_0 & 0 \\ p_1 & p_0 \end{bmatrix}, P_1 = \begin{bmatrix} p_2 & p_1 \\ p_3 & p_2 \end{bmatrix}, D_2 = \begin{bmatrix} p_2 & p_1 \\ p_3 & p_2 \end{bmatrix}$$

b. Know that:

For the same principle in part 1:

$$D_0 = \begin{bmatrix} d_0 & 0 & 0 \\ d_1 & d_0 & 0 \\ d_2 & d_1 & d_0 \end{bmatrix}, D_1 = \begin{bmatrix} d_3 & d_2 & d_1 \\ d_4 & d_3 & d_2 \\ 0 & d_4 & d_3 \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 0 & 0 & d_4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, P_0 = \begin{bmatrix} p_0 & 0 & 0 \\ p_1 & p_0 & 0 \\ p_2 & p_1 & p_0 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} p_3 & p_2 & p_1 \\ p_4 & p_3 & p_2 \\ 0 & p_4 & p_3 \end{bmatrix}, P_2 = \begin{bmatrix} 0 & 0 & p_4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

c. Know that:

For the same principle in part 1:

$$D_0 = \begin{bmatrix} d_0 & 0 & 0 & 0 \\ d_1 & d_0 & 0 & 0 \\ d_2 & d_1 & d_0 & 0 \\ d_3 & d_2 & d_1 & d_0 \end{bmatrix} D_1 = \begin{bmatrix} d_4 & d_3 & d_2 & d_1 \\ 0 & d_4 & d_3 & d_2 \\ 0 & 0 & d_4 & d_3 \\ 0 & 0 & 0 & d_4 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} p_0 & 0 & 0 & 0 \\ p_1 & p_0 & 0 & 0 \\ p_2 & p_1 & p_0 & 0 \\ p_3 & p_2 & p_1 & p_0 \end{bmatrix}, P_1 = \begin{bmatrix} p_4 & p_3 & p_2 & p_1 \\ 0 & p_4 & p_3 & p_2 \\ 0 & 0 & p_4 & p_3 \\ 0 & 0 & 0 & p_4 \end{bmatrix}$$

A+