Glasgow College, University of Electronic Science and Technology of China

Digital Signal Processing **— Spring 2018**

Mid-Term Exam (A)

19:00—21:00 (*Time*), Sunday, May 20, 2018

Notice: Please make sure that both your UESTC and UoG Student IDs are written on the top of every sheet. This examination is open-book and the use of electronic materials or a cell phone is not permitted. All scratch paper must be adequately labeled. Unless indicated otherwise, answers must be derived or explained clearly. Please write within the space given below on the answer sheets.

All questions are compulsory. There are 6 problems and a maximum of 100 marks in total.

The following table is for grader only:

Question	1	2	3	4	5	6	Total	Grader
Score								

Score

PROBLEM 1 (10 POINTS)

Determine the DTFT of the sequence

$$r[n]=\begin{cases}1, & 0\le n\le M,\\ 0, & otherwise.\end{cases}$$

We have

$$r[n]=\begin{cases}1, & \text{for } 0\le n\le M\\ 0, & \text{otherwise}\end{cases}$$

Taking the Fourier transform

$$\begin{aligned}R(e^{j\omega}) &= \sum_{n=0}^{M} e^{-j\omega n} \quad \text{(3 points)}\\ &= \frac{1-e^{-j\omega(M+1)}}{1-e^{-j\omega}} \quad \text{(3 points)}\\ &= e^{-j\frac{M}{2}\omega}\left(\frac{e^{j\frac{M+1}{2}\omega}-e^{-j\frac{M+1}{2}\omega}}{e^{j\omega}-e^{-j\omega}}\right) \quad \text{(2 points)}\\ &= e^{-j\frac{M}{2}\omega}\left(\frac{\sin(\frac{M+1}{2}\omega)}{\sin(\omega/2)}\right) \quad \text{(2 points)}\end{aligned}$$

Score

PROBLEM 2 (18 POINTS)

Determine whether each of the following signals is periodic. If the signal is periodic, state its period.

(a) $x[n] = e^{j(\pi n/6)}$.

(b) $x[n] = e^{j(3\pi n/4)}$.

(c) $x[n] = [\sin(\pi n/5)]/(\pi n)$.

$x[n]$ is periodic with period N if $x[n] = x[n+N]$ for some integer N.

(a) $x[n]$ is periodic with period 12:

$$e^{j(\frac{\pi}{6}n)} = e^{j(\frac{\pi}{6})(n+N)} = e^{j(\frac{\pi}{6}n+2\pi k)}$$

$$\Longrightarrow 2\pi k = \frac{\pi}{6}N, \text{for integers } k, N$$

(3 points)

Making $k = 1$ and $N = 12$ shows that $x[n]$ has period 12. (3 points)

(b) $x[n]$ is periodic with period 8:

$$e^{j(\frac{3\pi}{4}n)} = e^{j(\frac{3\pi}{4})(n+N)} = e^{j(\frac{3\pi}{4}n+2\pi k)}$$

$$\Longrightarrow 2\pi k = \frac{3\pi}{4}N, \text{for integers } k, N$$

$$\Longrightarrow N = \frac{8}{3}k, \text{for integers } k, N$$

(3 points)

The smallest k for which both k and N are integers are is 3, resulting in the period N being 8. (3 points)

(c) $x[n] = [\sin(\pi n/5)]/(\pi n)$ is not periodic because the denominator term is linear in n.

(3 points) (3 points)

Score

PROBLEM 3 (12 POINTS)

The continuous-time signal $x_c(t) = \sin(20\pi t) + \cos(40\pi t)$ **is sampled with a sampling period** T **to obtain the discrete-time signal** $x[\mathrm{n}] = \sin(\frac{\pi n}{5}) + \cos(\frac{2\pi n}{5})$.

(a) Determine a choice for T **consistent with this information.**

(b) Is your choice for T **in part (a) unique? If so, explain why. If not, specify another choice of** T **consistent with the information given.**

(a) Letting $T = 1/100$ gives

$$\begin{aligned} x[n] &= x_c(nT) \quad \text{(3 points)} \\ &= \sin\left(20\pi n \frac{1}{100}\right) + \cos\left(40\pi n \frac{1}{100}\right) \\ &= \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right) \quad \text{(3 points)} \end{aligned}$$

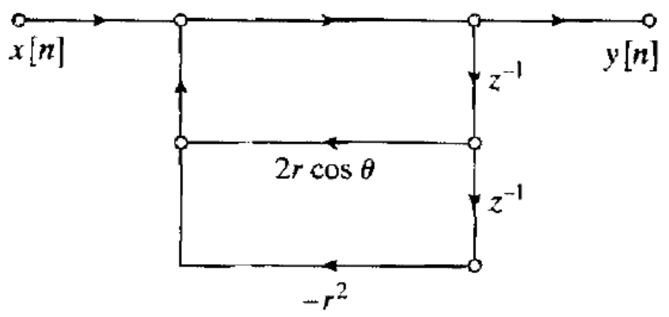
(b) No, another choice is $T = 11/100$:

$$\begin{aligned} x[n] &= x_c(nT) \quad \text{(3 points)} \\ &= \sin\left(20\pi n \frac{11}{100}\right) + \cos\left(40\pi n \frac{11}{100}\right) \\ &= \sin\left(\frac{11\pi n}{5}\right) + \cos\left(\frac{22\pi n}{5}\right) \\ &= \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right) \quad \text{(3 points)} \end{aligned}$$

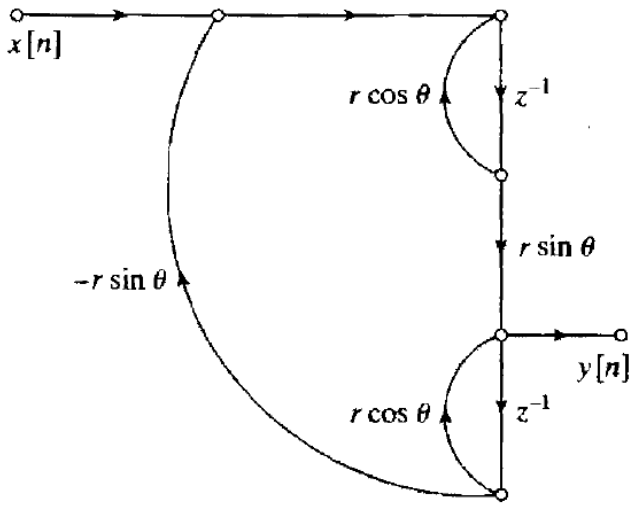
Score

PROBLEM 4 (12 POINTS)

Determine the system function of the following flow graphs.



(a)



(b)

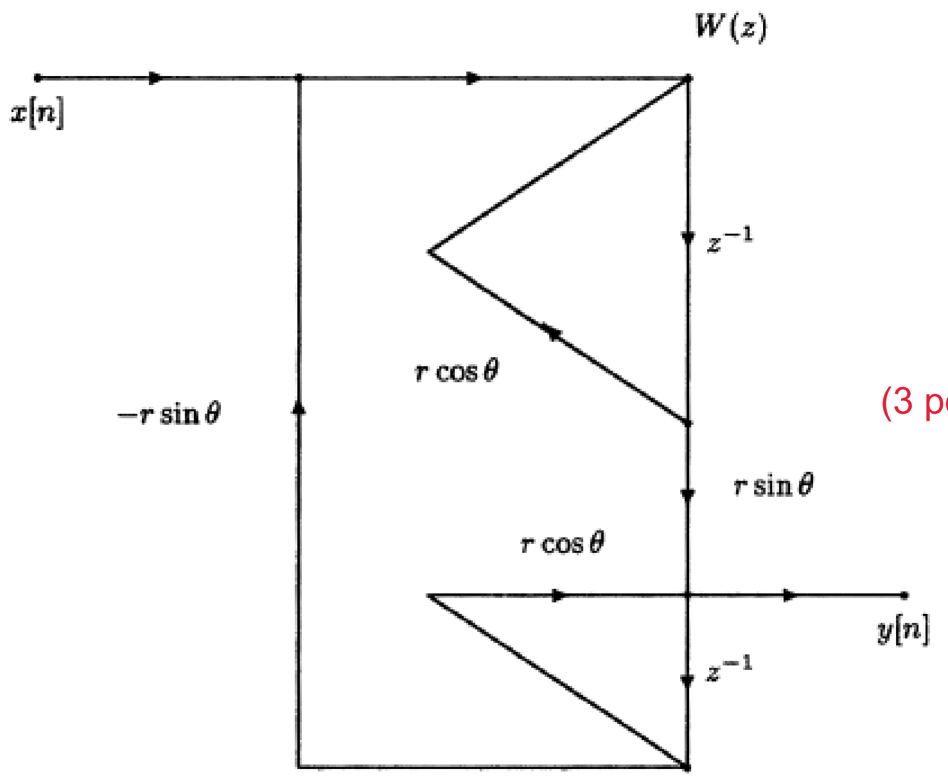
We proceed by obtaining the transfer functions for each of the networks. For network 1,

$$Y(z) = 2r\cos\theta z^{-1}Y(z) - r^2 z^{-2}Y(z) + X(z) \quad \text{(3 points)}$$

or

$$H_1(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 2r\cos\theta z^{-1} + r^2 z^{-2}}$$

For network 2, define $W(z)$ as in the figure below:



(3 points)

then

$$W(z) = X(z) - r\sin\theta z^{-1}Y(z) + r\cos\theta z^{-1}W(z)$$

and

$$Y(z) = r\sin\theta z^{-1}W(z) + r\cos\theta z^{-1}Y(z) \quad \text{(3 points)}$$

Eliminate $W(z)$ to get

$$H_2(z) = \frac{Y(z)}{X(z)} = \frac{r\sin\theta z^{-1}}{1 - 2r\cos\theta z^{-1} + r^2 z^{-2}}$$

Hence the two networks have the same poles. (3 points)

Score

PROBLEM 5 (24 POINTS)

Compute the DFT of each of the following finite-length sequences considered to be of length N (where N is even):

(a) $x[n] = \delta[n]$,

(b) $x[n] = \delta[n - n_0]$, $\quad 0 \le n_0 \le N-1$,

(c) $x[n] = \begin{cases} 1, & 0 \le n \le N/2-1, \\ 0, & N/2 \le n \le N-1, \end{cases}$

(d) $x[n] = \begin{cases} a^n, & 0 \le n \le N-1, \\ 0, & otherwise. \end{cases}$

(a)

$$\begin{aligned} x[n] &= \delta[n] \\ X[k] &= \sum_{n=0}^{N-1} \delta[n] W_N^{kn}, \quad 0 \le k \le (N-1) \quad \text{(3 points)} \\ &= 1 \quad \text{(3 points)} \end{aligned}$$

(b)

$$\begin{aligned} x[n] &= \delta[n-n_0], \quad 0 \le n_0 \le (N-1) \\ X[k] &= \sum_{n=0}^{N-1} \delta[n-n_0] W_N^{kn}, \quad 0 \le k \le (N-1) \quad \text{(3 points)} \\ &= W_N^{kn_0} \quad \text{(3 points)} \end{aligned}$$

(c)

$$\begin{aligned} x[n] &= \begin{cases} 1, & n \text{ even} \\ 0, & n \text{ odd} \end{cases} \\ X[k] &= \sum_{n=0}^{N-1} x[n] W_N^{kn}, \quad 0 \le k \le (N-1) \quad \text{(3 points)} \\ &= \sum_{n=0}^{(N/2)-1} W_N^{2kn} \\ &= \frac{1 - e^{-j2\pi k}}{1 - e^{-j(\pi k/N)}} \\ X[k] &= \begin{cases} N/2, & k = 0, N/2 \\ 0, & \text{otherwise} \end{cases} \quad \text{(3 points)} \end{aligned}$$

(d)

$$\begin{aligned} x[n] &= \begin{cases} a^n, & 0 \le n \le (N-1) \\ 0, & \text{otherwise} \end{cases} \\ X[k] &= \sum_{n=0}^{N-1} a^n W_N^{kn}, \quad 0 \le k \le (N-1) \quad \text{(3 points)} \\ &= \frac{1 - a^N e^{-j2\pi k}}{1 - a e^{-j(2\pi k)/N}} \\ X[k] &= \frac{1 - a^N}{1 - a e^{-j(2\pi k)/N}} \quad \text{(3 points)} \end{aligned}$$

Score

PROBLEM 6 (24 POINTS)

Suppose we have two four-point sequences $x[n]$ **and** $h[n]$ **as follows:**

$$x[n] = \cos\left(\frac{\pi n}{2}\right), \qquad n = 0,1,2,3,$$

$$h[n] = 2^n, \qquad n = 0,1,2,3,$$

(a) Calculate the four-point DFT $X[\mathrm{k}]$

(b) Calculate the four-point DFT $H[\mathrm{k}]$

(c) Calculate $y[n] = x[\mathrm{n}] \otimes h[n]$ **by doing the four-point circular convolution directly**

(d) Calculate $y[n]$ **of part (c) by multiplying the DFTs of** $x[n]$ **and** $h[n]$ **and performing an inverse DFT.**

(a)

$$x[n] = \cos(\frac{\pi n}{2}), \qquad 0 \le n \le 3$$

transforms to

$$X[k] = \sum_{n=0}^{3} \cos(\frac{\pi n}{2}) W_4^{kn}, \qquad 0 \le k \le 3$$ (3 points)

The cosine term contributes only two non-zero values to the summation, giving:

$$\begin{aligned} X[k] &= 1 - e^{-j\pi k}, \qquad 0 \le k \le 3 \\ &= 1 - W_4^{2k} \end{aligned}$$ (3 points)

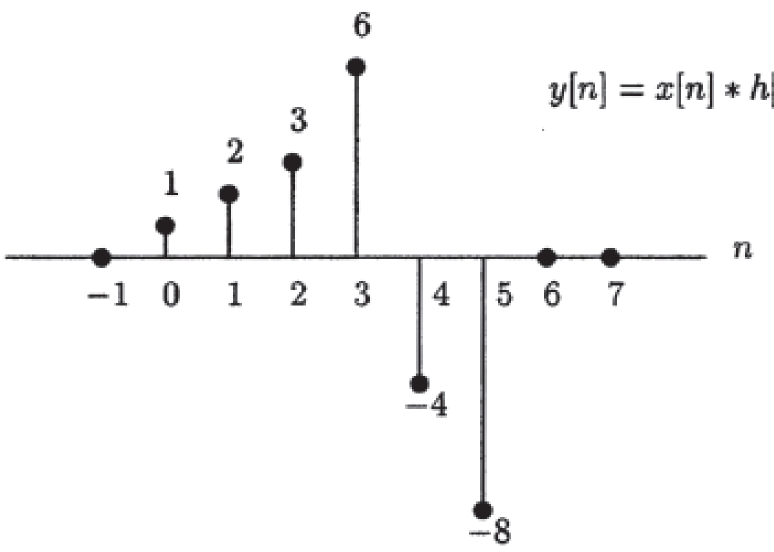
(b)

$$h[n] = 2^n, \qquad 0 \le n \le 3$$

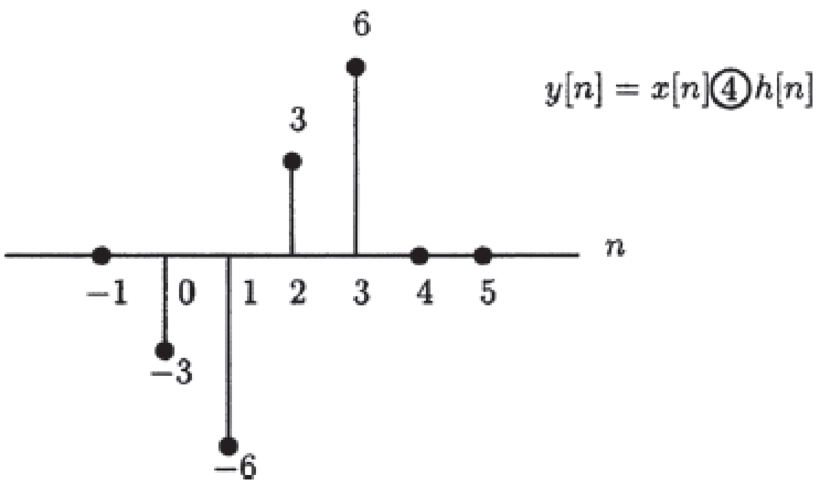
$$H[k] = \sum_{n=0}^{3} 2^n W_4^{kn}, \qquad 0 \le k \le 3$$ (3 points)

$$= 1 + 2W_4^k + 4W_4^{2k} + 8W_4^{3k}$$ (3 points)

(c) Remember, circular convolution equals linear convolution plus aliasing. We need $N \geq 3+4-1=6$ to avoid aliasing. Since $N = 4$,we expect to get aliasing here. First, find $y[n] = x[n] * h[n]$:



For this problem, aliasing means the last three points ($n = 4, 5, 6$) will wrap-around on top of the first three points, giving $y[n] = x[n] ④ h[n]$: (3 points)



(3 points)

(d) Using the DFT values we calculated in parts (a) and (b):

$$\begin{aligned} Y[k] &= X[k]H[k] \quad \text{(3 points)} \\ &= 1 + 2W_4^k + 4W_4^{2k} + 8W_4^{3k} - W_4^{2k} - 2W_4^{3k} - 4W_4^{4k} - 8W_4^{5k} \end{aligned}$$

Since $W_4^{4k} = W_4^{0k}$ and $W_4^{5k} = W_4^k$

$$Y[k] = -3 - 6W_4^k + 3W_4^{2k} + 6W_4^{3k}, \quad 0 \leq k \leq 3$$

Taking the inverse DFT:

$$y[n] = -3\delta[n] - 6\delta[n-1] + 3\delta[n-2] + 6\delta[n-3], \quad 0 \leq n \leq 3$$ (3 points)