DSP HOMEWORK

杨昊澍
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Chapter 2

2.7 Consider the following causal finite-length sequences with their first samples at $n = 0$:

(a) $\{x_1[n]\} = \{1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1\}$,

(b) $\{x_2[n]\} = \{1,\ 1\}$,

(c) $\{x_3[n]\} = \{1,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 1,\ 1\}$,

(d) $\{x_4[n]\} = \{1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1\}$.

Show that $x_1[n] \circledast x_2[n] = x_3[n] \circledast x_4[n]$.

Solve:

n : 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

$x_1[n]$: 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1

$x_2[n]$: 1, 1

1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1

  1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1

1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

Thus, $x_1[n] * x_2[n] = \{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$
$\uparrow$

n: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

$x_3[n]$: 1, 1, 0, 0, 0, 0, 0, 0, 1, 1

$x_4[n]$: 1, 0, 1, 0, 1, 0, 1

1, 1, 0, 0, 0, 0, 0, 0, 1, 1

0, 0, 0, 0, 0, 0, 0, 0, 0, 0

1, 1, 0, 0, 0, 0, 0, 0, 1, 1

0, 0, 0, 0, 0, 0, 0, 0, 0, 0

1, 1, 0, 0, 0, 0, 0, 0, 1, 1

0, 0, 0, 0, 0, 0, 0, 0, 0, 0

1, 1, 0, 0, 0, 0, 0, 0, 1, 1

1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

Thus, $x_3[n] * x_4[n] = \{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$
$\uparrow$

Therefore, $x_1[n] * x_2[n] = x_3[n] * x_4[n]$

2.8 Evaluate the linear convolution of each of the following sequences with itself:

(a) $x_1[n] = \{1,\ -1,\ 1\}, -1 \le n \le 1$,

(b) $x_2[n] = \{1,\ -1,\ 0,\ 1,\ -1\}, 0 \le n \le 4$,

(c) $x_3[n] = \{-1,\ 2,\ 0,\ -2,\ 1\}, -3 \le n \le 1$.

Solve:

(a) n: 1 2 3 4 5

$x_1[n]$: 1 -1 1

$x_1[n]$: 1 -1 1

1 -1 1

-1 1 -1

1 -1 1

1 -2 3 -2 1

Thus, $x_1[n] * x_1[n] = \{1, -2, 3, -2, 1\}$
$\uparrow$ $\uparrow$

(b) n: 1 2 3 4 5 6 7 8 9

$x_2[n]$: 1 -1 0 1 -1

$x_2[n]$: 1 -1 0 1 -1

1 -1 0 1 -1

-1 1 0 -1 1

0 0 0 0 0

1 -1 0 1 -1

-1 1 0 -1 1

1 -2 1 2 -4 2 1 -2 1

Thus, $x_2[n] * x_2[n] = \{1, -2, 1, 2, -4, 2, 1, -2, 1\}$
$\uparrow$

(c) n: 1 2 3 4 5 6 7 8 9

$x_3[n]$: -1 2 0 -2 1

$x_3[n]$: -1 2 0 -2 1

1 -2 0 2 -1

-2 4 0 -4 2

0 0 0 0 0

2 -4 0 4 -2

-1 2 0 -2 1

1 -4 4 4 -10 4 4 -4 1

Thus, $x_3[n] * x_3[n] = \{1, -4, 4, 4, -10, 4, 4, -4, 1\}$
$\uparrow$

2.13 Let $y[n]$ denote the linear convolution of the two sequences $\{x[n]\} = \{2,\ -3,\ 4,\ 1\}$, $-1 \le n \le 2$, and $\{h[n]\} = \{-3,\ 5,\ -6,\ 4\}$ $\ -2 \le n \le 1$. Determine the value of $y[-1]$ without computing the convolution sum.

Solve:

n:	1	2	3	4	5	6	7
$x[n]$:	2	-3	4	1			
$h[n]$:	-3	5	-6	4			

-3×4

$5\times(-3)$

-6×2

-39

, which indicates that $y[-1] = x[-1]h[0] + x[0]h[-1] + x[1]h[-2]$

$\Rightarrow y[-1] = -39$

2.23 Is an absolutely summable sequence a bounded sequence? Justify your answer.

Solve: if a sequence is absolutely summable, it must satisfy $\sum_{n=-\infty}^{\infty}|x[n]| = S < \infty$,

thus, for any n, $|x[n]| \le S < \infty$, which indicates that it is a bounded sequence.

2.30 Show that the following sequences are absolutely summable:

(a) $x_1[n] = \alpha^n \mu[n-1]$,

(b) $x_2[n] = n\alpha^n \mu[n-1]$,

(c) $x_3[n] = n^2\alpha^n \mu[n-1]$,

where $|\alpha| < 1$.

Solve: (a) $S_1 = \sum_{n=-\infty}^{+\infty}|x_1[n]| = \sum_{n=-\infty}^{+\infty}|\alpha^n\mu[n-1]| = \sum_{n=1}^{+\infty}|\alpha^n| = |\alpha|\cdot\sum_{n=0}^{+\infty}|\alpha^n|$

when $|\alpha|<1$, $S_1 = |\alpha|\cdot\frac{1}{1-|\alpha|} < \infty$. Thus, it is absolutely summable.

(b) $S_2 = \sum_{n=-\infty}^{+\infty}|x_2[n]| = \sum_{n=-\infty}^{+\infty}|n\alpha^n\mu[n-1]| = \sum_{n=1}^{+\infty}|n\alpha^n| = \sum_{n=0}^{+\infty}|(n+1)\alpha^{n+1}| = |\alpha|\left[\sum_{n=0}^{+\infty}|n\alpha^n| + \sum_{n=0}^{+\infty}|\alpha^n|\right]$

when $|\alpha|<1$, $S_2 = |\alpha|\cdot\left[\frac{|\alpha|}{(1-|\alpha|)^2} + \frac{1}{1-|\alpha|}\right] = \frac{|\alpha|}{(1-|\alpha|)^2} < \infty$. Thus, it's absolutely summable.

(c) $S_3 = \sum_{n=-\infty}^{+\infty}|x_3[n]| = \sum_{n=-\infty}^{+\infty}|n^2\alpha^n\mu[n-1]| = \sum_{n=1}^{+\infty}|n^2\alpha^n|$

$= |\alpha| + 2^2|\alpha|^2 + 3^2|\alpha|^3 + \cdots + n^2|\alpha|^n + \cdots$

$= \sum_{n=1}^{+\infty}|\alpha|^n + 3\sum_{n=2}^{+\infty}|\alpha|^n + 5\sum_{n=3}^{+\infty}|\alpha|^n + \cdots$

$= \frac{|\alpha|}{1-|\alpha|} + \frac{3|\alpha|^2}{1-|\alpha|} + \frac{5|\alpha|^3}{1-|\alpha|} + \cdots$

$= \frac{1}{1-|\alpha|}\left(|\alpha| + 3|\alpha|^2 + 5|\alpha|^3 + \cdots\right)$

$= \frac{1}{1-|\alpha|}\sum_{n=1}^{+\infty}\left[(2n-1)|\alpha|^n\right] = \frac{1}{1-|\alpha|}\left[2\sum_{n=1}^{+\infty}n|\alpha|^n - \sum_{n=1}^{+\infty}|\alpha|^n\right]$

when $|\alpha|<1$, $S_3 = \frac{1}{1-|\alpha|}\left[\frac{2|\alpha|}{(1-|\alpha|)^2} - \frac{|\alpha|}{1-|\alpha|}\right] < +\infty$. Thus, it's absolutely summable.

2.31 Show that the following sequences are absolutely summable.

(a) $x_a[n] = \frac{1}{4^n}\mu[n]$, (b) $x_b[n] = \frac{1}{(n+2)(n+3)}\mu[n]$.

Solve: (a) $S_1=\sum_{n=-\infty}^{+\infty} x_a[n] = \sum_{n=-\infty}^{+\infty}\left(\frac{1}{4^n}\mu[n]\right) = \sum_{n=0}^{+\infty}\left(\frac{1}{4}\right)^n = \frac{1}{1-\frac{1}{4}} = \frac{4}{3} < +\infty$. Thus, it's absolutely summable.

(b) $S_2=\sum_{n=-\infty}^{+\infty} x_b[n] = \sum_{n=0}^{+\infty}\frac{1}{(n+2)(n+3)} = \sum_{n=0}^{+\infty}\left(\frac{1}{n+2}-\frac{1}{n+3}\right) = \frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\cdots = \frac{1}{2} < +\infty$. Thus, it's absolutely summable.

2.38 The following sequences represent one period of a sinusoidal sequence of the form $\tilde{x}[n] = A\sin(\omega_o n+\phi)$:

?

(a) $\{1, \quad 1, \quad -1, \quad -1\}$,

(b) $\{0.5, \quad -0.5, \quad 0.5, \quad -0.5\}$,

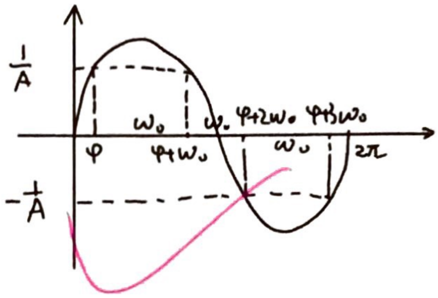
(c) $\{0, \quad 0.5878, \quad -0.9511, \quad 0.9511, \quad -0.5878\}$,

(d) $\{2 \quad 0 \quad -2 \quad 0\}$.

Determine the values of the parameters A, ω_o, and ϕ for each case.

Solve: (a)
$x[0]=A\sin\phi=1$
$x[1]=A\sin(\omega_0+\phi)=1$
$x[2]=A\sin(2\omega_0+\phi)=-1$
$x[3]=A\sin(3\omega_0+\phi)=-1$

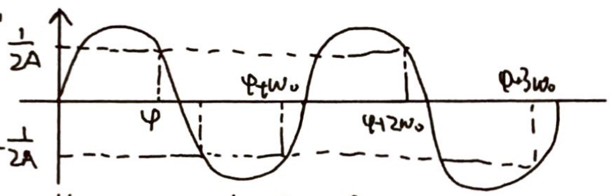
$\Rightarrow$



$\Rightarrow \begin{cases} 2\varphi=\omega_0 \\ 2\varphi+3\omega_0=2\pi \end{cases}$

Thus, $\omega_0=\frac{\pi}{2}$, $\varphi=\frac{\pi}{4}$, $A=\frac{1}{\sqrt{2}}$.

(b) Namely, we have,



$\Rightarrow \omega_0=\frac{4\pi}{4}=\pi$

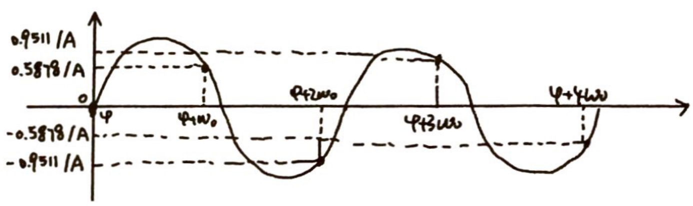
The sequence doesn't represent the fundamental period.

$\begin{cases} A\sin\phi = A\sin(2\pi+\phi)=\frac{1}{2} \\ A\sin(\pi+\phi)=A\sin(3\pi+\phi)=-\frac{1}{2}, \end{cases} \Rightarrow \begin{cases} 0<\frac{1}{2A}\leq 1 \\ \varphi\neq k\pi\ (k\in\mathbb{Z}) \end{cases} \Rightarrow \begin{cases} A\geq\frac{1}{2} \\ \varphi\neq k\pi\ (k\in\mathbb{Z}) \end{cases}$

Thus, $\omega_0=\pi$, $\varphi\neq k\pi\ (k\in\mathbb{Z})$, $A\geq\frac{1}{2}$.

especially, $\omega_0=\pi$, $\varphi=\frac{\pi}{2}$, $A=\frac{1}{2}$.

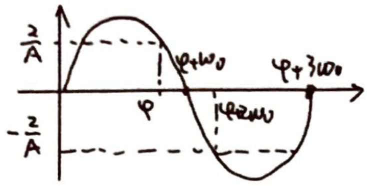
(c) Namely, we have,



$\begin{cases} A\sin\phi=0 \\ A\sin(\varphi+\omega_0)=\sin(\frac{4}{5}\pi) \\ A\sin(\varphi+2\omega_0)=\sin(\frac{8}{5}\pi) \\ A\sin(\varphi+3\omega_0)=\sin(\frac{12}{5}\pi) \\ A\sin(\varphi+4\omega_0)=\sin(\frac{16}{5}\pi) \end{cases}$

$\Rightarrow$ $\phi=0$, especially, $A=1$, $\omega_0=\frac{4}{5}\pi$.

(d) Namely, we have,



$\Rightarrow (\varphi+3\omega_0)-(\varphi+\omega_0)=\pi$

$\Rightarrow \omega_0=\frac{\pi}{2}.$

$$\begin{cases} A\sin\varphi = 2 \\ A\sin(\varphi+\frac{\pi}{2})=0 \\ A\sin(\varphi+\pi)=-2 \\ A\sin(\varphi+\frac{3}{2}\pi)=0 \end{cases} \Rightarrow \varphi+\frac{\pi}{2}=\pi \Rightarrow \varphi=\frac{\pi}{2},$$

$A=2$

Thus, $\omega_0=\frac{\pi}{2}$, $\varphi=\frac{\pi}{2}$, $A=2$

Ch 2: Matlab

M 2.2 (a) Using Program 2_2, generate the sequences shown in Figures 2.23 and 2.24.
(b) Generate and plot the complex exponential sequence $-2.7e^{(-0.4+j\pi/6)n}$ for $0 \le n \le 82$ using Program 2_2.

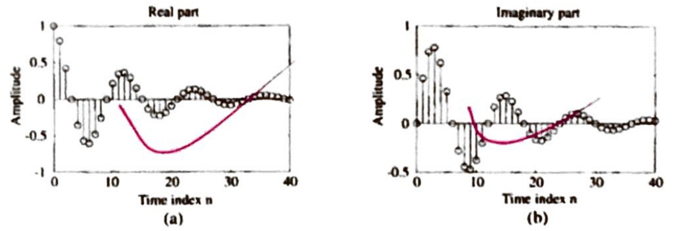


Figure 2.23: A complex exponential sequence $x[n]=e^{(-1/12+j\pi/6)n}$. (a) Real part and (b) imaginary part.

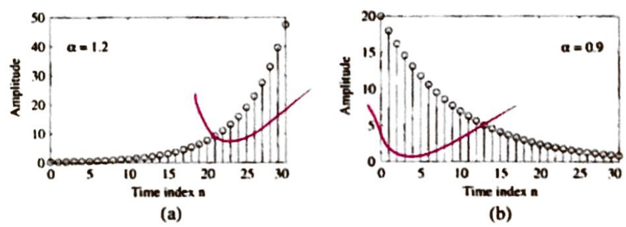
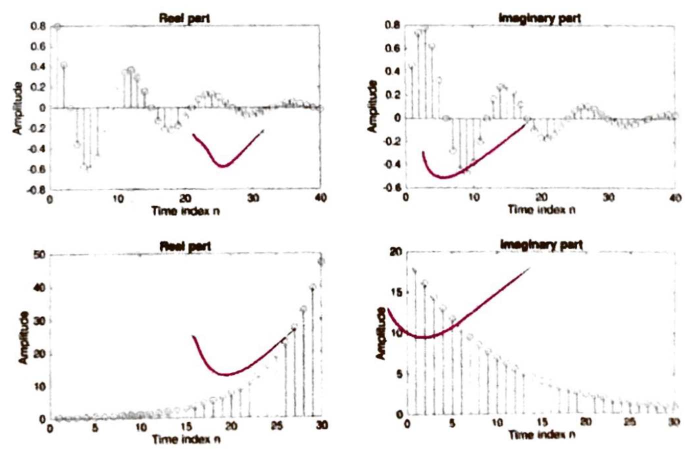


Figure 2.24: Examples of real exponential sequences: (a) $x[n]=0.2(1.2)^n$, (b) $x[n]=20(0.9)^n$.

(a) code:

```
%M2.2(a)
a = -1/12;
b = pi/6;
c = a + b*i;
K = 1;
N = 41;
n = 1:N;
x = K*exp(c*n);

subplot(2,2,1);
stem(n,real(x));
xlabel('Time index n');ylabel('Amplitude');
title('Real part');

subplot(2,2,2);
stem(n,imag(x));
xlabel('Time index n');ylabel('Amplitude');
title('Imaginary part');

a1 = log(1.2);
a2 = log(0.9);
b1 = 0;
b2 = 0;
c1 = a1 + b1*i;
c2 = a2 + b2*i;
K1 = 0.2;
K2 = 20;
N = 30;
n = 1:N;
x1 = K1*exp(c1*n);
x2 = K2*exp(c2*n);

subplot(2,2,3);
stem(n,x1);
xlabel('Time index n');ylabel('Amplitude');
title('Real part');

subplot(2,2,4);
stem(n,x2);
xlabel('Time index n');ylabel('Amplitude');
title('Imaginary part');
```

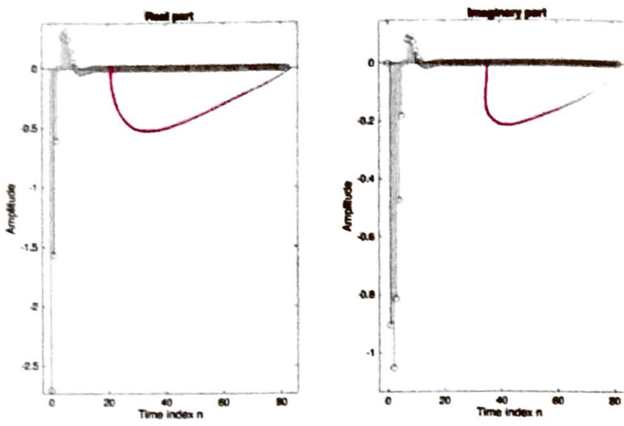
result:



(b) code:

```
a = -0.4;
b = pi/6;
c = a + b*i;
K = -2.7;
N = 82;
n = 0:N;

subplot(1,2,1);
x = K*exp(c*n);%Generate the sequence
stem(n,real(x));%Plot the real part
xlabel('Time index n');ylabel('Amplitude');
title('Real part');

subplot(1,2,2);
stem(n,imag(x));%Plot the imaginary part
xlabel('Time index n');ylabel('Amplitude');
title('Imaginary part');
```

result:



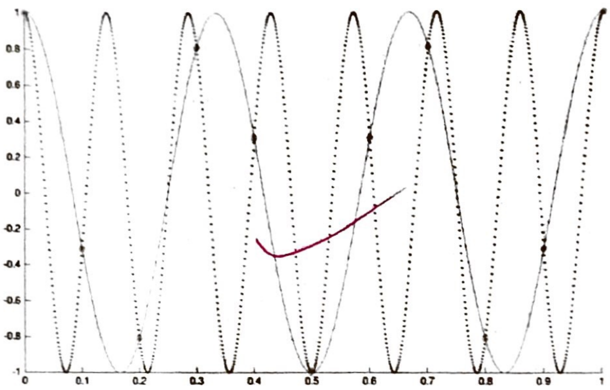
M 2.7 Using the program developed in the previous problem, verify experimentally that the family of continuous-time sinusoids given by Eq. (2.65) lead to identical sampled signals.

$$x_{a,k}(t) = A\cos(\pm(\Omega_o t + \phi) + k\Omega_T t), \qquad k = 0, \pm 1, \pm 2, \ldots \tag{2.65}$$

code:

```
t=linspace(0,1,1000);
x1=cos(6*pi.*t);
x2=cos(14*pi.*t);
x3=cos(26*pi.*t);

n=0:10;
xs1=cos(0.6*pi.*n);
xs2=cos(1.4*pi.*n);
xs3=cos(2.6*pi.*n);

figure;
hold on;
plot(t,x1,'-',t,x2,'.',t,x3,'-.')
plot(n/10,xs1,'x',n/10,xs2,'o',n/10,xs3,'+');
hold off;
```

result:

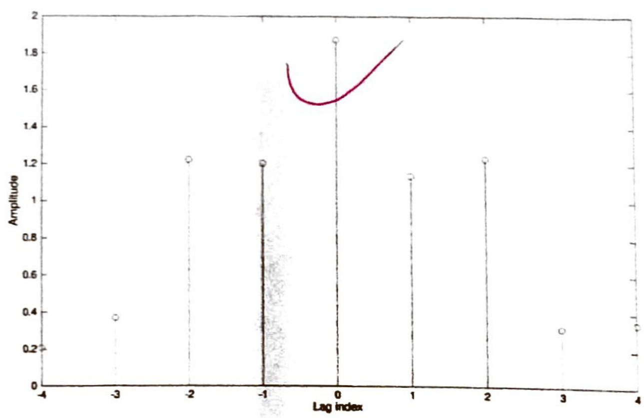


M 2.9 Modify Program 2_5(new) to determine the autocorrelation sequence of a sequence corrupted with a uniformly distributed random signal generated using the M-function `rand`. Using the modified program, demonstrate that the autocorrelation sequence of a noise-corrupted signal exhibits a peak at zero lag.

code:

```
x = rand(1,5);
y = rand(1,5);

n1 = length(y)-1; n2 = length(x)-1;
r = conv(x,fliplr(y));
k = (-n1):n2';
stem(k,r);
xlabel('Lag index'); ylabel('Amplitude');
v = axis;
axis([-n1 n2 v(3:end)]);
```

result:



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