Quiz 3.

一、Session 1:

(10') (1.1) Solution: (B)

$$f(t) = t\,u(t-1) \overset{\mathcal{L}}{\longleftrightarrow} F(s) = ?$$

令 $f_1(t) = (t-1)\,u(t-1)$, $f_0(t) = t\,u(t) \overset{\mathcal{L}}{\longleftrightarrow} F_0(s) = \frac{1}{s^2}$, $Re\{s\} > 0$

$\because f_1(t) = f_0(t-1) \overset{\mathcal{L}}{\longleftrightarrow} F_1(s) = e^{-s} \cdot F_0(s) = \frac{e^{-s}}{s^2}$, $Re\{s\} > 0$

$\therefore f(t) = t\,u(t-1) = f_1(t) + u(t-1) \overset{\mathcal{L}}{\longleftrightarrow} F_1(s) + \frac{e^{-s}}{s} = \frac{e^{-s}}{s^2} + \frac{e^{-s}}{s} = \frac{s+1}{s^2} \cdot e^{-s}$

$Re\{s\} > 0$.

(10') (1.2). Solution: (B)

$$\omega_m = 10^4 \pi \Rightarrow \underset{=2\pi f_s}{\omega_s} \geq 2\omega_m = 2\times 10^4 \pi \Rightarrow f_s \geq 1\times 10^4\ Hz.$$

$T_s \leq 1\times 10^{-4}\ s$.

$\omega_c = 2\pi f_c = \omega_m \Rightarrow f_c = 5\times 10^3\ Hz$.

~~(1.3) Solution: ( )~~

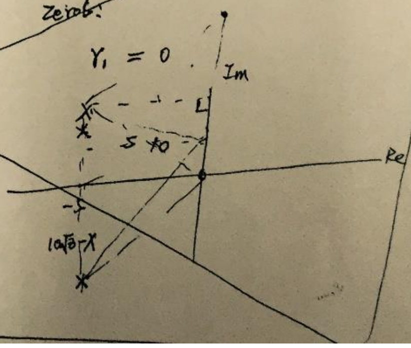
$$H(s) = \frac{s}{s^2+10s+100} = \frac{s}{(s-\lambda_1)(s-\lambda_2)}.$$

Poles: $\lambda_1, \lambda_2 = \frac{-10\pm\sqrt{10^2-400}}{2} = -5 \pm 5\sqrt{3}\, i$

Zeros: $r_1 = 0$.

$l_1^2 = x^2 + 5^2$; $l_2^2 = (10\sqrt{3} - x)^2 + 5^2$.

$l_1^2 + l_2^2 - 2 l_1 \cdot l_2 = (l_1 - l_2)^2 \geq 0$.



二、Session 2.

(10')(2.1) 由已知可得：满足初值定理，

$$x(0^+) = \lim_{s\to\infty} sX(s) = \lim_{s\to\infty} \frac{s(2s+1)}{s(s^2+3s+2)} = 0.$$

满足终值定理

$$x(\infty) = \lim_{s\to 0} sX(s) = \lim_{s\to 0} \frac{s(2s+1)}{s(s^2+3s+2)} = \frac{1}{2} = 0.5$$

(10')(2.2) $x(t) = e^{2t}$ ~~是LTI~~系统特征函数。

$$x(t) = e^{2t} \longrightarrow y(t) = H(s)|_{s=2} \cdot e^{2t}$$

$$H(s)|_{s=2} = \frac{1}{s^2+5s+6}\Big|_{s=2} = \frac{1}{20}.$$

$$\therefore y(t) = \frac{1}{20} e^{2t}.$$

H(s) Rational，ROC：$Re\{s\} > -2$ 在最右极点右边，则系统 stable.（Yes）

(10')(2.3) 两边 LT：

$$sY(s) + 5Y(s) = X(s)\cdot Z(s) - X(s).$$

$$Z(s) = 3\cdot\frac{1}{s+1} + 2 = \frac{3+2(s+1)}{s+1} = \frac{2s+5}{s+1}.$$

$$\therefore sY(s) + 5Y(s) = X(s)\cdot\frac{2s+5}{s+1} - X(s) = X(s)\cdot\frac{s+4}{s+1}$$

$$\therefore H(s) = \frac{Y(s)}{X(s)} = \frac{s+4}{(s+5)(s+1)}$$，Causal，ROC：$Re\{s\} > -1$.

$$\therefore H(j\omega) = \frac{j\omega+4}{(j\omega+5)(j\omega+1)}$$

$$\therefore h(t) = \mathcal{L}^{-1}\{H(s)\}$$

H(s) 部分分式展开 $\frac{A}{s+5} + \frac{B}{s+1}$

$A = H(s)\cdot(s+5)|_{s=-5} = \frac{s+4}{s+1}\Big|_{s=-5} = \frac{1}{4}.$

$B = H(s)\cdot(s+1)|_{s=-1} = \frac{s+4}{s+5}\Big|_{s=-1} = \frac{3}{4}.$

$$H(s) = \frac{1}{4}\frac{1}{s+5} + \frac{3}{4}\frac{1}{s+1}.$$

$$\therefore h(t) = \frac{1}{4}e^{-5t}u(t) + \frac{3}{4}e^{-t}u(t)$$

Quiz 3

Section 3:

3.1. Solution:

$H(s) = \frac{N(s)}{D(s)}$, Rational, causal, stable.

① $H(s)|_{s=1} = 1$

② $x(t) = u(t) \xleftrightarrow{\mathcal{L}} X(s) = \frac{1}{s}$.

$Y(s) = X(s) \cdot H(s) = \frac{1}{s} \cdot H(s)$, $y(t)$绝对可积，则 $Y(s)$的ROC包含 $j\omega$轴。 $H(s)$在$s=0$处有零0.

$N(s) = s \cdot P(s)$. (5')

③ $x_2(t) = t u(t) \xleftrightarrow{\mathcal{L}} X_2(s) = \frac{1}{s^2}$.

$Y(s) = X_2(s) \cdot H(s) = \frac{1}{s^2} \cdot \frac{s P(s)}{D(s)}$, $y(t)$不绝对可积，则 $Y(s)$ ROC不包含 $j\omega$轴. $s=0$是其一极点.

③ $f(t) = h''(t) + 5h'(t) + 6h(t) \xleftrightarrow{\mathcal{L}} F(s) = s^2 H(s) + 5sH(s) + 6H(s)$.

$= (s^2 + 5s + 6) \cdot H(s)$

ROC: 整个 s-plane.

∵ $H(s)$ 包含 $(s+2)(s+3)$ 两个因子 (5')

④ $H(s)$在无穷远处有一零点.

$O_D - O_N = 1$

∴ $N(s)$ 含 s 因子， $D(s)$ 含 $(s+2)$、$(s+3)$ 两个因子.

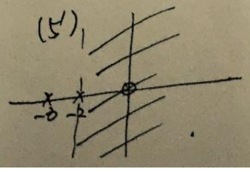
∴ $H(s) = \frac{N(s)}{D(s)} = \frac{s \cdot P(s)}{(s+2)(s+3) \cdot q(s)}$. $\frac{P(s)}{q(s)} = k$.

∴ $H(s) = k \cdot \frac{s}{(s+2)(s+3)}$.

由①可得， $H(s)|_{s=1} = k \cdot \frac{1}{(1+2)(1+3)} = 1 \Rightarrow k = 12$. (5')

∴ $H(s) = \frac{12s}{(s+2)(s+3)}$ (5')

由于 causal, stable. ROC: $Re\{s\} > -2$ (5')



Quiz 3

(3.2) Solution:

(a) Zeros: $\gamma_1 = 1$

Poles: $\lambda_1 = -2$, $\lambda_2 = -1$.

$\therefore \quad H(s) = K\dfrac{(s-1)}{(s+1)(s+2)}$; $\quad h(0^+) = 3 = \lim\limits_{s\to\infty} s\cdot H(s) = K = 3$.

for Rational H(s)

Causal, ROC: $Re\{s\} > -1$. $\quad \therefore K = 3$.

(b) $h(t) = \mathcal{L}^{-1}\{H(s)\}$.

$H(s) \overset{\text{部分分式展开}}{=\!=} 3\left[\dfrac{A}{s+1} + \dfrac{B}{s+2}\right]$

$A = H(s)(s+1)\big|_{s=-1} = \dfrac{s-1}{s+2}\Big|_{s=-1} = -2$

$B = H(s)(s+2)\big|_{s=-2} = \dfrac{s-1}{s+1}\Big|_{s=-2} = 3$.

$\therefore H(s) = 3\left[\dfrac{-2}{s+1} + \dfrac{3}{s+2}\right]$, $Re\{s\} > -1$.

$\therefore h(t) = -6\cdot e^{-t}u(t) + 9e^{-2t}u(t)$.

ROC 包含 $j\omega$-轴，故 stable.

(c) $x(t) = e^{t} \longrightarrow y(t) = H(s)\big|_{s=1}\cdot e^{t}$

$= 0\cdot e^{t} = 0$.

(d) $\dfrac{Y(s)}{X(s)} = H(s) = \dfrac{3(s-1)}{(s+1)(s+2)} = \dfrac{3(s-1)}{s^2+3s+2} \Rightarrow s^2Y(s) + 3sY(s) + 2Y(s) = 3[sX(s) - X(s)$.

$\therefore$ 两边 $\mathcal{L}^{-1}$，得 LCCDE: $\dfrac{d^2y(t)}{dt^2} + 3\dfrac{dy(t)}{dt} + 2y(t) = 3\dfrac{dx(t)}{dt} - 3x(t)$

(e) $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{3(s-1)}{s^2+3s+2} = \dfrac{1}{s^2+3s+2}\cdot 3(s-1)$ ，令 $H_1(s) = \dfrac{1}{s^2+3s+2}$

$H_2(s) = s-1$.

$\therefore Y_1(s) = H_1(s)\cdot X(s)$

$\therefore s^2Y_1(s) = X(s) - 3sY_1(s) - 2Y_1(s)$

$Y(s) = 3(s-1)Y_1(s)$

$= 3sY_1(s) - 3Y_1(s)$