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Quiz 2

1. (20') Each of the following questions may have only one right answers, justify your answers and write it in the blank.

(1) Let $x_1(t) = e^{|t|}$ and $x(t) = x_1(t) * \sum_{k=\infty}^{+\infty} \delta(t-4k)$. The Fourier series coefficients of $x(t)$ may be (a).

(a) $a_{-k} = a_k$ and $Im\{a_k\} = 0$ (b) $a_k = -a_k$ and $Im\{a_k\} = 0$

(c) $a_{-k} = a_k$ and $\mathrm{Re}\{a_k\} = 0$ (d) $a_k = -a_k$ and $\mathrm{Re}\{a_k\} = 0$

(2) Consider two signals $x_1(t)$ and $x_2(t)$, as known in Figure 1. The Fourier transform of $x_1(t)$ is $X_1(jw)$.

Then the Fourier transform of $x_2(t)$ should be (a).

(a) $X_1(-jw)e^{-3jw}$ (b) $X_1(jw)e^{-3jw}$ (c) $X_1(-jw)e^{3jw}$ (d) $X_1(jw)e^{3jw}$

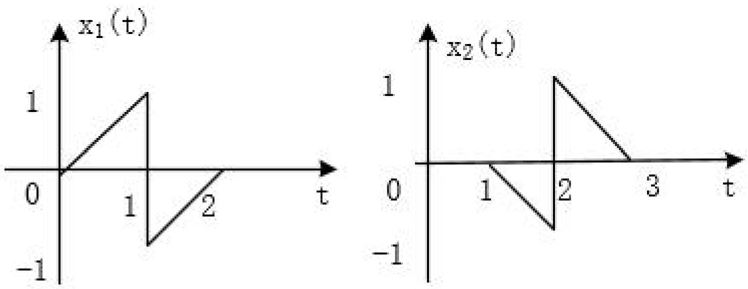


Figure 1

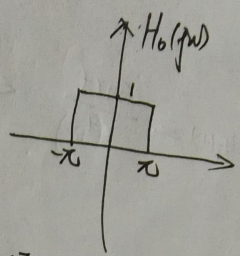
2. (20 points) Consider an LTI system with unit impulse response $h(t) = \frac{\sin \pi t}{\pi t} \cos 4\pi t$, if the input is $x(t) = 1 + \cos 2\pi t + \sin 4\pi t + \frac{\sin 4\pi t}{\pi t}$, determine the output $y(t)$.

Solution: solutions.

2.

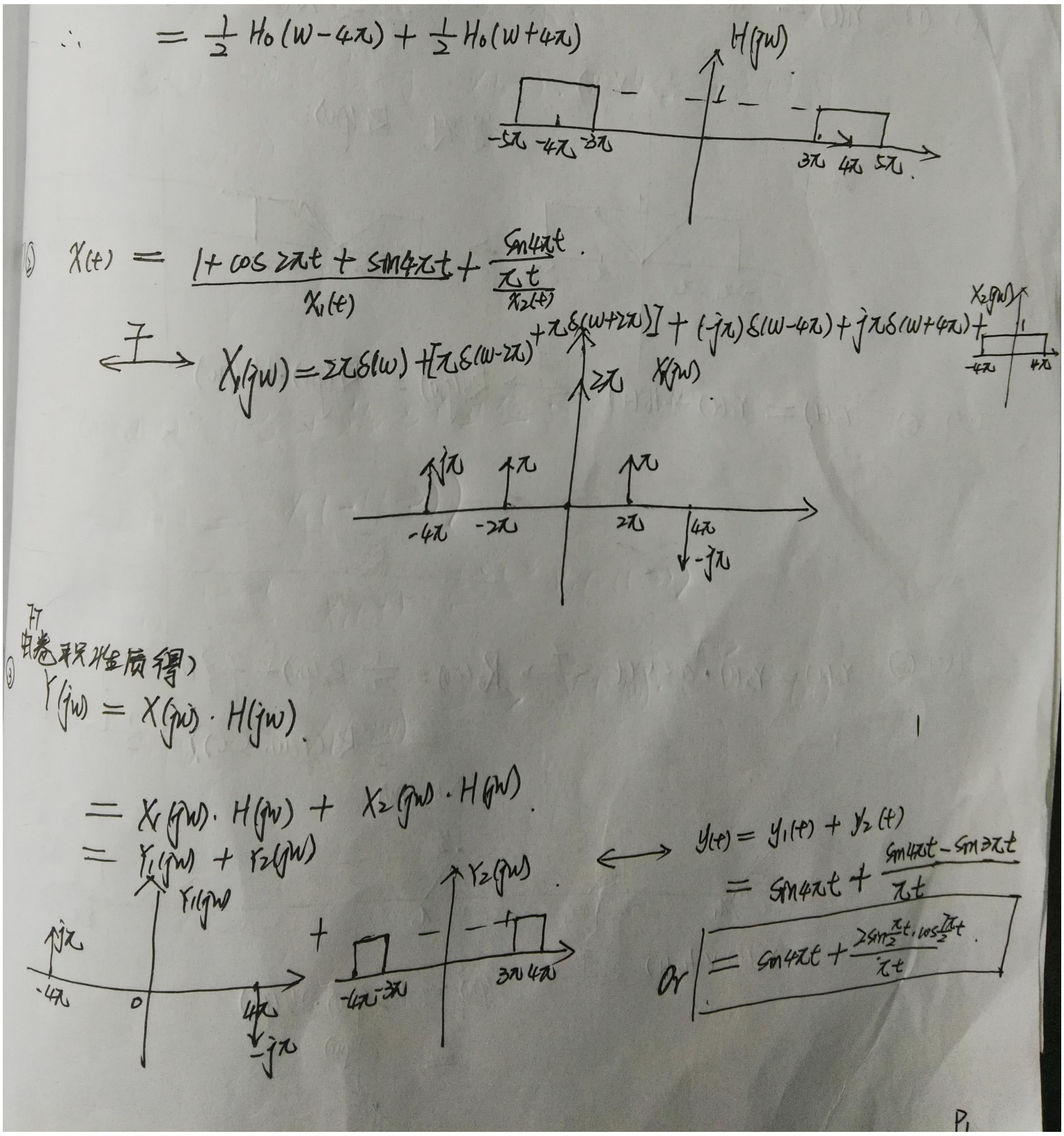
① $h(t) = \frac{\sin \pi t}{\pi t} \cdot \cos 4\pi t$

令 $h_0(t) = \frac{\sin \pi t}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow} H_0(j\omega) =$



$\therefore H(j\omega) = \frac{1}{2\pi} \cdot H_0(j\omega) * \mathcal{F}\{\cos 4\pi t\}$

$= \frac{1}{2\pi} \cdot H_0(j\omega) * [\pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)]$



∴ $= \frac{1}{2}H_0(w-4\pi) + \frac{1}{2}H_0(w+4\pi)$

② $x(t) = \underbrace{1+\cos 2\pi t + \sin 4\pi t}_{x_1(t)} + \underbrace{\frac{\sin 4\pi t}{\pi t}}_{x_2(t)}$

$\overset{F}{\longleftrightarrow} X_1(jw) = 2\pi\delta(w) + [\pi\delta(w-2\pi) + \pi\delta(w+2\pi)] + (-j\pi)\delta(w-4\pi) + j\pi\delta(w+4\pi) +$

③ 由卷积性质得：

$Y(jw) = X(jw)\cdot H(jw)$

$= X_1(jw)\cdot H(jw) + X_2(jw)\cdot H(jw)$

$= Y_1(jw) + Y_2(jw)$

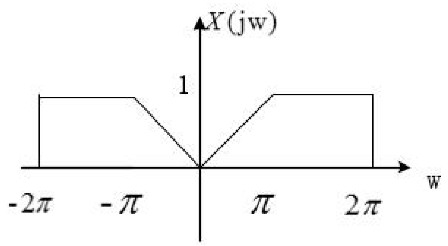
$\longleftrightarrow y(t) = y_1(t) + y_2(t)$

$= \sin 4\pi t + \frac{\sin 4\pi t - \sin 3\pi t}{\pi t}$

Or $\boxed{= \sin 4\pi t + \frac{2\sin\frac{\pi}{2}t\cdot\cos\frac{7\pi}{2}t}{\pi t}}$

P1

3. (20 points) Consider the system illustrated in Figure 2, if we know $h_1(t) = \frac{\sin 5\pi t - \sin 4\pi t}{\pi t}$ and $h_2(t) = \frac{\sin \pi t}{\pi t}$, sketch the spectrum of $r_1(t)$, $r_2(t)$, $r_3(t)$ and $y(t)$.



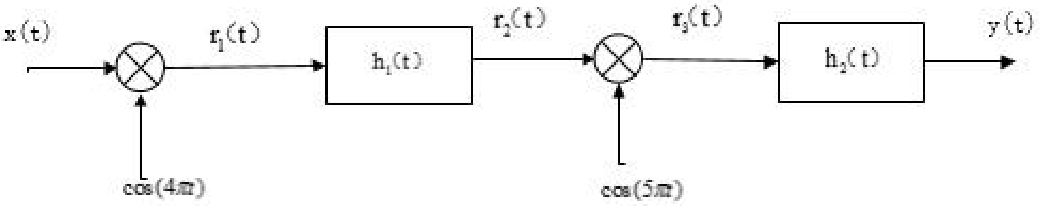
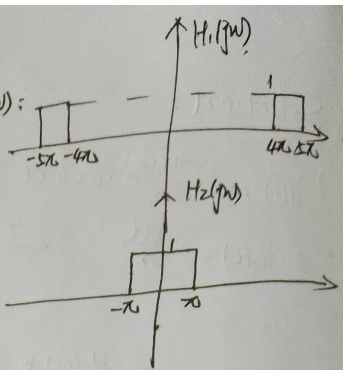


Figure 2.

3. Solution:

$h_1(t) = \frac{\sin 5\pi t - \sin 4\pi t}{\pi t} \xleftrightarrow{\mathcal{F}} H_1(j\omega)$:

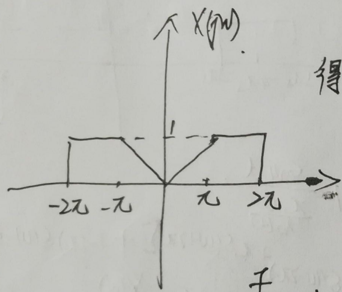
$h_2(t) = \frac{\sin \pi t}{\pi t} \xleftrightarrow{\mathcal{F}} H_2(j\omega)$:



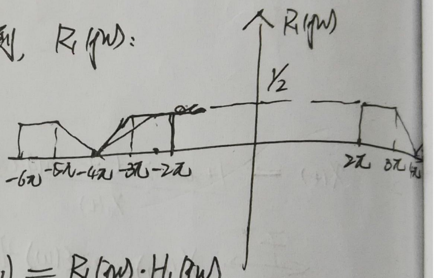
(5') ① $r_1(t) = x(t)\cdot \cos 4\pi t \xleftrightarrow{\mathcal{F}} R_1(j\omega) = \frac{1}{2\pi} X(j\omega) * [\pi\delta(\omega-4\pi) + \pi\delta(\omega+4\pi)]$

$= \frac{1}{2} X(j(\omega-4\pi)) + \frac{1}{2} X(j(\omega+4\pi))$

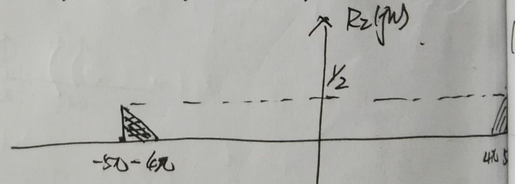
由



得到, $R_1(j\omega)$:

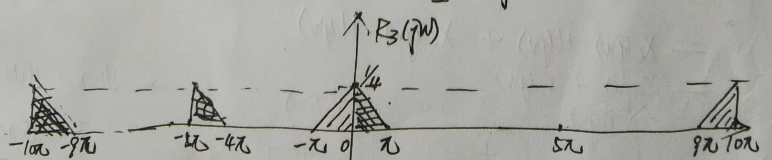


(5') ② $r_2(t) = r_1(t) * h_1(t) \xleftrightarrow{\mathcal{F}} R_2(j\omega) = R_1(j\omega)\cdot H_1(j\omega)$

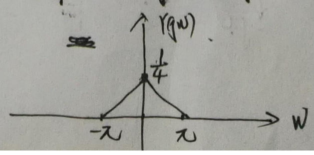


(5') ③ $r_3(t) = r_2(t)\cdot \cos 5\pi t \xleftrightarrow{\mathcal{F}} R_3(j\omega) = \frac{1}{2\pi} R_2(j\omega) * \mathcal{F}\{\cos(5\pi t)\}$

$= \frac{1}{2} R_2(j(\omega-5\pi)) + \frac{1}{2} R_2(j(\omega+5\pi))$



(5') ④ $y(t) = r_3(t) * h_2(t) \xleftrightarrow{\mathcal{F}} Y(j\omega) = R_3(j\omega)\cdot H_2(j\omega)$



4. (20 points) It is known that $x(t) \overset{FT}{\leftrightarrow} X(jw) = \mathrm{Re}\{X(jw)\} + j\,\mathrm{Im}\{X(jw)\}$, and $x(t)$ is shown as Figure 3.

(a) $r(t) \overset{FT}{\leftrightarrow} \mathrm{Re}\{X(jw)\}$, where $\mathrm{Re}\{X(jw)\}$ is the real part of $X(\mathrm{jw})$. Please sketch $\mathrm{r}(t)$.

(b) Find the value of $\int_{-\infty}^{+\infty} X(\text{-jw})dw$.

(c) Let $Y(\mathrm{jw}) = X(-jw/3)e^{-jw}$, and $\mathrm{y}(t) \overset{FT}{\leftrightarrow} Y(jw)$. Sketch $\mathrm{y}(t)$.

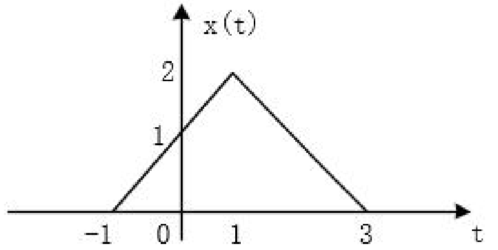
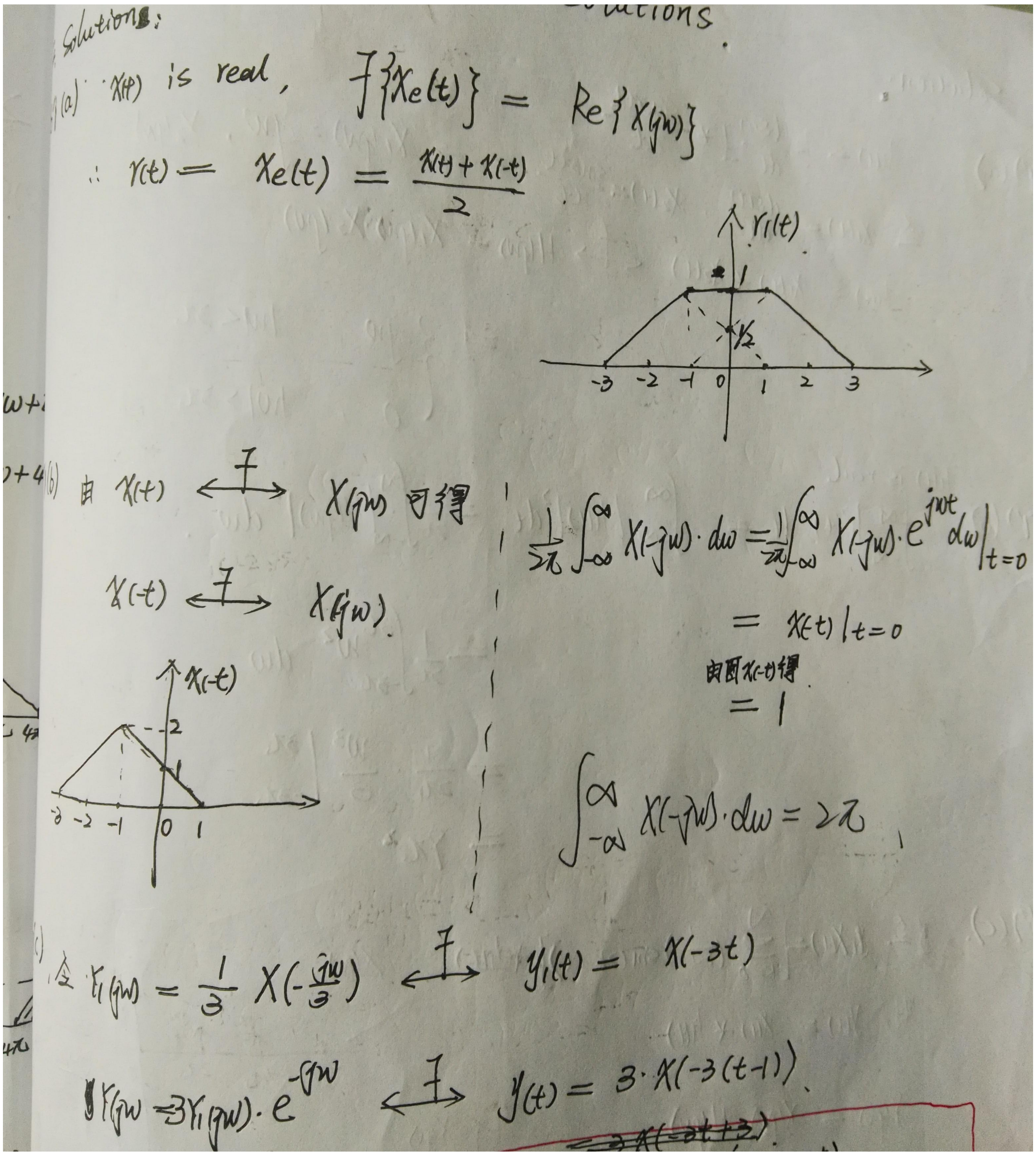


Figure 3.



Solutions:

(a) ∵ $x(t)$ is real, $\mathcal{F}\{x_e(t)\} = \mathrm{Re}\{X(jw)\}$

∴ $r(t) = x_e(t) = \frac{x(t)+x(-t)}{2}$

$r_1(t)$: 1, 1/2, -3, -2, -1, 0, 1, 2, 3

(b) 由 $x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(jw)$ 可得

$x(-t) \overset{\mathcal{F}}{\longleftrightarrow} X(-jw)$.

$x(-t)$: 2, 1, -3, -2, -1, 0, 1

$\frac{1}{2\pi}\int_{-\infty}^{\infty} X(-jw)\cdot dw = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(-jw)\cdot e^{jwt} dw\Big|_{t=0}$

$= x(-t)\big|_{t=0}$

由图$x(-t)$得

$= 1$

$\int_{-\infty}^{\infty} X(-jw)\cdot dw = 2\pi$

(c) 令 $Y_1(jw) = \frac{1}{3} X(-\frac{jw}{3}) \overset{\mathcal{F}}{\longleftrightarrow} y_1(t) = x(-3t)$

$Y(jw) = 3Y_1(jw)\cdot e^{-jw} \overset{\mathcal{F}}{\longleftrightarrow} y(t) = 3\cdot x(-3(t-1))$.

$\cancel{= 3x(-3t+3)}$

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5. (20 points) Suppose the unit impulse response of a LTI system is $h(t)=\left[\frac{d}{dt}\delta(t)\right]*\frac{\sin 3\pi t}{\pi t}$.

(a) Determine the expression of the frequency response $H(jw)$.

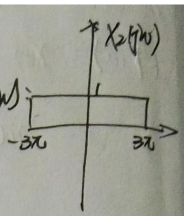
(b) Determine the value of $\int_{-\infty}^{\infty} h^2(t)dt$.

(c) Determine the convolution integral $y(t)=\left[\sum_{k=0}^{\infty}\left(\frac{1}{k+1}\right)\sin(2k\pi t)\right]*h(t)$.

5. Solution:

(7') (a) $h(t)=\left[\frac{d\delta(t)}{dt}\right]*\frac{\sin 3\pi t}{\pi t}$

令 $x_1(t)=\frac{d\delta(t)}{dt}$, $x_2(t)=\frac{\sin 3\pi t}{\pi t}$ $\longleftrightarrow$ $X_1(jw)=jw$, $X_2(jw)$:



$\therefore$ $h(t)=x_1(t)*x_2(t) \xleftrightarrow{F} H(jw)=X_1(jw)\cdot X_2(jw)$

$$=\begin{cases} jw, & |w|<3\pi \\ 0, & |w|>3\pi \end{cases}$$

$\because$ h(t) is real

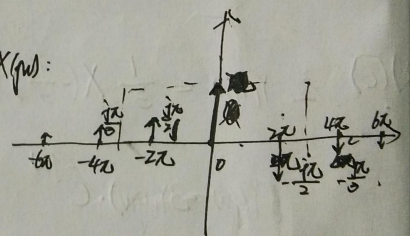
(7') (b) $\int_{-\infty}^{\infty} h^2(t)\,dt=\int_{-\infty}^{\infty}|h(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}|H(jw)|^2dw$.

$$=\frac{1}{2\pi}\int_{-3\pi}^{3\pi}w^2\,dw$$

$$=\frac{1}{2\pi}\cdot\frac{w^3}{3}\Big|_{-3\pi}^{3\pi}$$

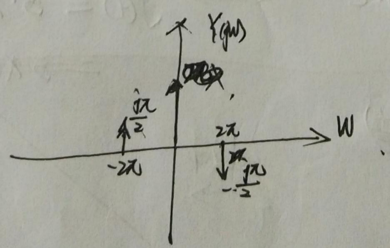
$$=9\pi^2$$

(6') (c) 令 $x(t)=\left[\sum_{k=0}^{\infty}\left(\frac{1}{k+1}\right)\sin(2k\pi t)\right]$



则 $y(t)=x(t)*h(t)$

$\therefore$ $Y(jw)=X(jw)\cdot H(jw)$.



$\therefore$ $y(t)=\frac{1}{2}\sin 2\pi t$.