Quiz 2 (Signals and Systems)

---- C3 /C4/C8 for UoG 20162003&20162004

1 (40 points). Each of the following questions have one or two right answers, justify your answers and write it in the blank.

(1)Let $x_1(t)=u(t+1)-u(t-1)$ **and** $x(t)=x_1(t)*\sum_{k=-\infty}^{+\infty}\delta(t-6k)$ **. The Fourier series coefficients of** $x(t)$ **may be (c).**

(a) $a_{-k}=-a_k$ and $\mathrm{Re}\{a_k\}=0$ **(b)** $a_{-k}=-a_k$ and $\mathrm{Im}\{a_k\}=0$

(c) $a_{-k}=a_k$ and $\mathrm{Im}\{a_k\}=0$ **(d)** $a_{-k}=-a_k$ and $\mathrm{Im}\{a_k\}=0$

(2) Consider two signals $x_1(t)$ **and** $x_2(t)$**, as shown in Figure 1. The Fourier transform of** $x_1(t)$ **is** $X_1(j\omega)$**. Then the Fourier transform of** $x_2(t)$ **should be（ a c ）.**

(a) $X_1(-j\omega)e^{-3j\omega}$

(b) $X_1(j\omega)e^{3j\omega}$

(c) $-X_1(j\omega)e^{-j\omega}$

(d) $X_1(-j\omega)e^{j\omega}$

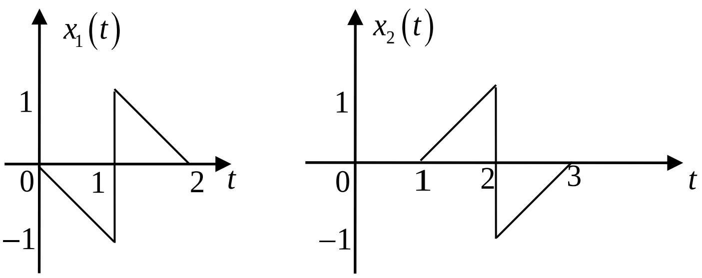


Figure 1

(3) Consider a continuous-time LTI system whose frequency response is $H(j\omega)=\frac{\sin(2\omega)}{\omega}$**. If we know the input signal** $x(t)$ **is an odd periodic signal with fundamental period** $T=4$**, the output** $y(t)$ **may be (a).**

(a) $y(t)=0$ **(b)** $y(t)=1$ **(c)** $y(t)=0.5$ **(d)** $y(t)=-1$

(4) The convolution integral $2e^{2t}*e^{-2t}u(t)=$ **(c).**

(a) 2 (b) $\frac{1}{4}e^{2t}$ **(c)** $\frac{1}{2}e^{2t}$ **(d)** $\frac{1}{2}e^{2t}u(t)$

2. If $y_1(t) = x_1(t) * h(t)$**, where** $x_1(t)$ **and** $h(t)$ **are shown in Figure 3, determine** $\int_{-\infty}^{+\infty} y_1(t)dt = ?$ **（10 points）**

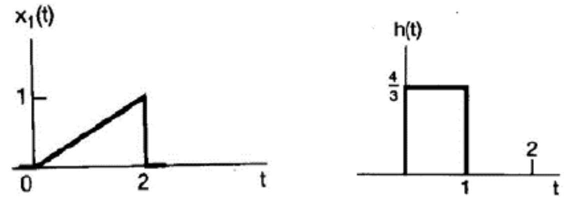


Figure 3

$$\int_{-\infty}^{+\infty} y_1(t)\,dt = Y(0).$$

$$Y(0) = X(0) \cdot H(0)$$

$$X(0) = \int_{-\infty}^{+\infty} x(t)\,dt = 2 \cdot 1 \cdot \frac{1}{2} = 1$$

$$H(0) = \int_{-\infty}^{+\infty} h(t)\,dt = 1 \cdot \frac{4}{3} = \frac{4}{3}$$

$$\therefore Y(0) = X(0) \cdot H(0) = \frac{4}{3}$$

3. Consider a signal x(t) with Fourier transform $X(j\omega) = \begin{cases} 2\cos\omega, & |\omega| \le \pi \\ 0, & |\omega| > \pi \end{cases}$. **Find a close-form expression for x(t).**（10 points）

$$X(j\omega) = \begin{cases} 2\cos\omega & |\omega| \le \pi \\ 0 & |\omega| > \pi \end{cases}$$

Let $X_1(j\omega) = 2\cos\omega$ $\quad X_2(j\omega) = \begin{cases} 1 & |\omega| \le \pi \\ 0 & |\omega| > \pi \end{cases}$

$X(j\omega) = X_1(j\omega)\cdot X_2(j\omega)$

$\therefore x_1(t) = \delta(t-1) + \delta(t+1)$

$x_2(t) = \dfrac{\sin(\pi t)}{\pi t}$

$\therefore x(t) = x_1(t) * x_2(t) = \dfrac{\sin[\pi(t-1)]}{\pi(t-1)} + \dfrac{\sin[\pi(t+1)]}{\pi(t+1)}$

Tips： give the corresponding relationship between the signal in time domain and frequency domain.

Sol: $\cancel{X(j\omega)=}$ Let $Y_1(j\omega) = \begin{cases} 2, & |\omega| \le \pi \\ 0, & |\omega| > \pi \end{cases} \overset{F}{\longleftrightarrow} y_1(t) = \dfrac{2\sin\pi t}{\pi t}$

Let $Y_2(j\omega) = \cos\omega$, $\overset{F}{\longleftrightarrow} y_2(t) = \frac{1}{2}\delta(t-1) + \frac{1}{2}\delta(t+1)$.

$X(j\omega) = Y_1(j\omega)\,Y_2(j\omega) \overset{F}{\longleftrightarrow} x(t) = y_1(t) * y_2(t)$

$x(t) = \dfrac{\sin\pi(t-1)}{\pi(t-1)} + \dfrac{\sin\pi(t+1)}{\pi(t+1)}$.

$$X(j\omega)=\begin{cases}2\cos\omega, & |\omega|\le\pi\\ 0, & |\omega|>\pi\end{cases}=\begin{cases}e^{j\omega}+e^{-j\omega}, & |\omega|\le\pi\\ 0, & |\omega|>\pi\end{cases}$$

~~$x(t)=[\delta(t+1)+\delta(t-1)]$~~ Assume $H(j\omega)=\begin{cases}1, & |\omega|<\pi\\ 0, & |\omega|>\pi\end{cases}$, $G(j\omega)=e^{j\omega}+e^{-j\omega}$

$\therefore X(j\omega)=$ ~~$e^{j\omega}+e^{-j\omega}$~~ $G(j\omega)H(j\omega) \Rightarrow x(t)=g(t)*h(t)$.

$g(t)=\delta(t+1)+\delta(t-1)$. $h(t)=\frac{\sin\pi t}{\pi t}$

$$x(t)=[\delta(t+1)+\delta(t-1)]*\frac{\sin\pi t}{\pi t}$$

$$=\frac{\sin[\pi(t+1)]}{\pi(t+1)}+\frac{\sin[\pi(t-1)]}{\pi(t-1)}$$

4. Consider a continuous-time LTI system whose frequency response $H(j\omega)$ is illustrated in Figure 4. If the input signal $x(t)=1+\cos 2t-\sin 4t+2\cos 10t$, determine the output $y(t)$ of this system. (20 points)

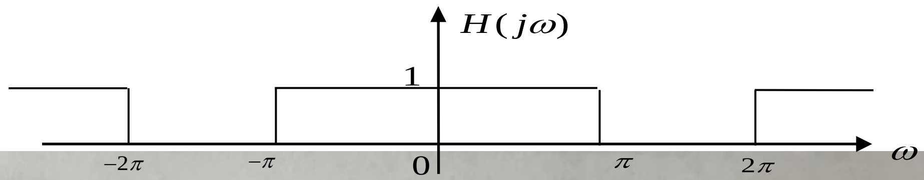


Figure 4

$\cos\omega_0 t \xleftrightarrow{FT} \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$ $\sin\omega_0 t \xleftrightarrow{FT} \frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$

$X(j\omega)=2\pi\delta(\omega)+\pi[\delta(\omega-2)+\delta(\omega+2)]-\frac{\pi}{j}[\delta(\omega-4)-\delta(\omega+4)]+2\pi[\delta(\omega-10)+\delta(\omega+1$

$y(t)=x(t)*h(t) \xleftrightarrow{FT} Y(j\omega)=X(j\omega)H(j\omega)$.

$X(j\omega)H(j\omega)=2\pi\delta(\omega)+\pi[\delta(\omega-2)+\delta(\omega+2)]+2\pi[\delta(\omega-10)+\delta(\omega+10)]$.

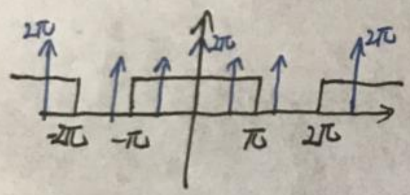
$y(t)=1+\cos 2t+2\cos 10t$.

4. $x(t) = 1 + \cos 2t - \sin 4t + 2\cos 10t$

$\cancel{x(t) = 1 + \frac{1}{2}(e^{j2t} + e^{-j2t})}$

$x(t) \xleftrightarrow{FT} 2\pi\delta(\omega) + \pi[\delta(\omega-2)+\delta(\omega+2)] - \frac{\pi}{j}[\delta(\omega-4)-\delta(\omega+4)] + 2\pi[\delta(\omega-10)+\delta(\omega+10)]$

$\therefore X(j\omega) = 2\pi\delta(\omega) + \pi[\delta(\omega-2)+\delta(\omega+2)] - \frac{\pi}{j}[\delta(\omega-4)-\delta(\omega+4)] + 2\pi[\delta(\omega-10)+\delta(\omega+10)]$



$Y(j\omega) = H(j\omega)X(j\omega)$

$= 2\pi\delta(\omega)\times 1 + \pi[\delta(\omega-2)+\delta(\omega+2)]\times 1 + 2\pi[\delta(\omega-10)+\delta(\omega+10)]\times 1$

$= 2\pi\delta(\omega) + \pi[\delta(\omega-2)+\delta(\omega+2)] + 2\pi[\delta(\omega-10)+\delta(\omega+10)]$

$\therefore Y(j\omega) \xleftarrow{FT} = 1 + \cos 2t + 2\cos 10t$

$x(t) = 1 + \frac{1}{2}(e^{-j2t} + e^{j2t}) - \frac{1}{2j}(e^{j4t} - e^{-j4t}) + (e^{j10t} + e^{-j10t})$

$= \sum_{k=-\infty}^{\infty} a_k e^{jkt} \quad (\omega_0 = 1)$

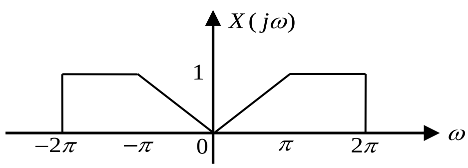
where $a_0 = 1$, $a_2 = a_{-2} = \frac{1}{2}$, $a_4 = -\frac{1}{2j}$, $a_{-4} = \frac{1}{2j}$, $a_{10} = a_{-10} = 1$.

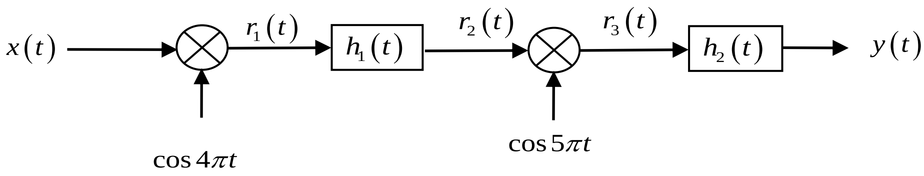
$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k)$

Given $H(j\omega)$, only $k = 0, \pm 2, \pm 10$ remain

so, $y(t) = 1 + \cos 2t + 2\cos 10t$

5.(20 points) Consider the system illustrated in Figure 5, if we know $h_1(t) = \frac{\sin 5\pi t - \sin 4\pi t}{\pi t}$ **and** $h_2(t) = \frac{\sin \pi t}{\pi t}$, **sketch the spectrum of** $r_1(t)$, $r_2(t)$, $r_3(t)$ **and** $y(t)$.



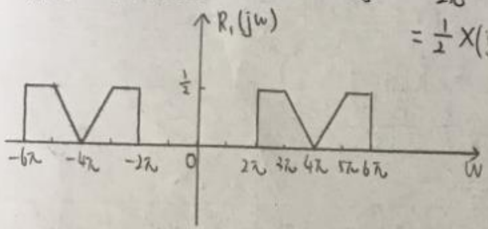


5. Sol: $h_1(t) = \frac{\sin 5\pi t}{\pi t} - \frac{\sin 4\pi t}{\pi t} \xleftrightarrow{F} \begin{cases} 1, & -5\pi < \omega < -4\pi,\ 4\pi < \omega < 5\pi \\ 0, & \text{otherwise} \end{cases} = H_1(j\omega)$

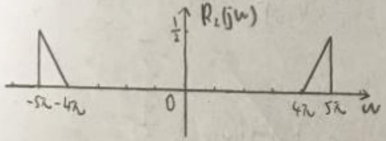
$h_2(t) = \frac{\sin \pi t}{\pi t} \xleftrightarrow{F} H_2(j\omega) = \begin{cases} 1, & |\omega| < \pi \\ 0, & |\omega| > \pi \end{cases}$

$r_1(t) = x(t)\cos 4\pi t \xleftrightarrow{F} R_1(j\omega) = \frac{1}{2\pi} X(j\omega) * [\pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)]$

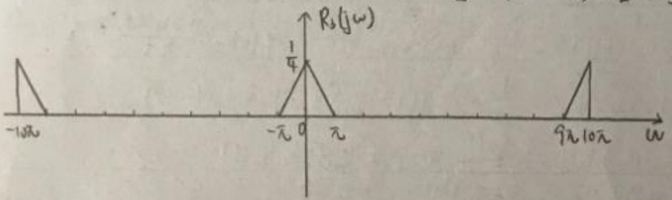
$= \frac{1}{2} X(j(\omega - 4\pi)) + \frac{1}{2} X(j(\omega + 4\pi))$



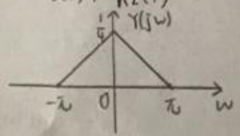
$r_2(t) = r_1(t) * h_1(t) \xleftrightarrow{F} R_2(j\omega) = R_1(j\omega) H_1(j\omega)$.

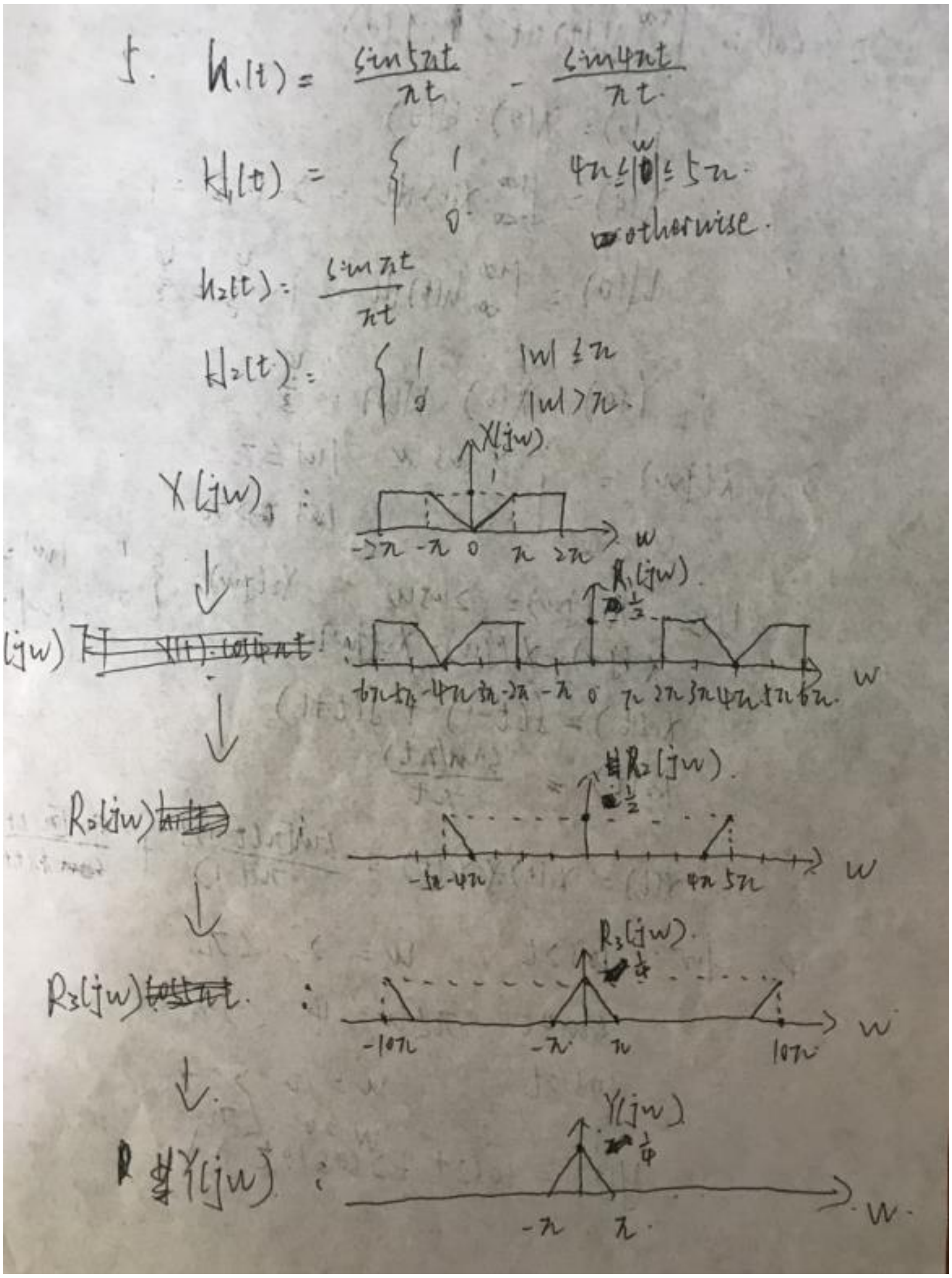


$r_3(t) = r_2(t)\cos 5\pi t \xleftrightarrow{F} R_3(j\omega) = \frac{1}{2} R_2(j(\omega - 5\pi)) + \frac{1}{2} R_2(j(\omega + 5\pi))$



$y(t) = r_3(t) * h_2(t) \xleftrightarrow{F} Y(j\omega) = R_3(j\omega) H_2(j\omega)$





Tips：Pay attention to the amplitude of signal and their relative notation.