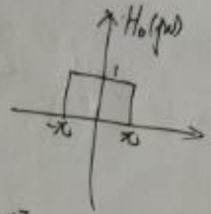
Solution :

2.

① $h(t) = \frac{\sin\pi t}{\pi t}\cdot\cos 4\pi t$

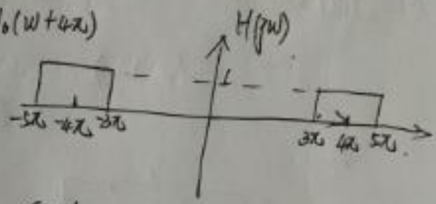
令 $h_0(t) = \frac{\sin\pi t}{\pi t} \xleftrightarrow{\mathcal{F}} H_0(j\omega) =$



$\therefore H(j\omega) = \frac{1}{2\pi}\cdot H_0(j\omega) * \mathcal{F}\{\cos 4\pi t\}$

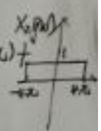
$= \frac{1}{2\pi}\cdot H_0(j\omega) * [\pi\delta(\omega-4\pi) + \pi\delta(\omega+4\pi)]$

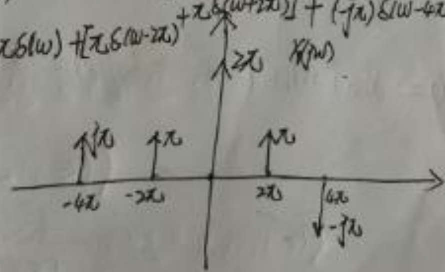
$\therefore \quad = \frac{1}{2}H_0(\omega-4\pi) + \frac{1}{2}H_0(\omega+4\pi)$



② $x(t) = \underbrace{1+\cos 2\pi t + \sin 4\pi t}_{x_1(t)} + \underbrace{\frac{\sin 4\pi t}{\pi t}}_{x_2(t)}$

$\xleftrightarrow{\mathcal{F}} X_1(j\omega) = 2\pi\delta(\omega) + [\pi\delta(\omega-2\pi) + \pi\delta(\omega+2\pi)] + (-j\pi)\delta(\omega-4\pi) + j\pi\delta(\omega+4\pi) +$





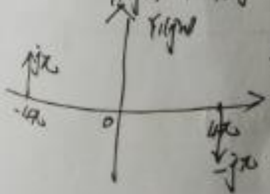
③ 由卷积性质得)

$Y(j\omega) = X(j\omega)\cdot H(j\omega)$

$= X_1(j\omega)\cdot H(j\omega) + X_2(j\omega)\cdot H(j\omega)$

$= Y_1(j\omega) + Y_2(j\omega) \longleftrightarrow y(t) = y_1(t) + y_2(t)$

$= \sin 4\pi t + \frac{\sin 4\pi t - \sin 3\pi t}{\pi t}$

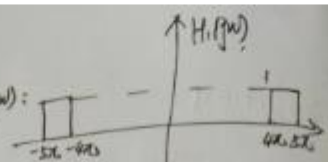


+

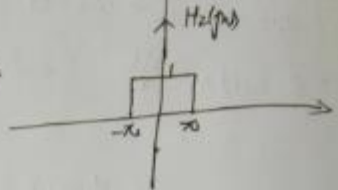
or $\boxed{= \sin 4\pi t + \frac{2\sin\frac{\pi}{2}t\cdot\cos\frac{7\pi}{2}t}{\pi t}}$

3. Solution:

$h_1(t) = \frac{\sin 5\pi t - \sin 4\pi t}{\pi t} \xleftrightarrow{\mathcal{F}} H_1(j\omega):$



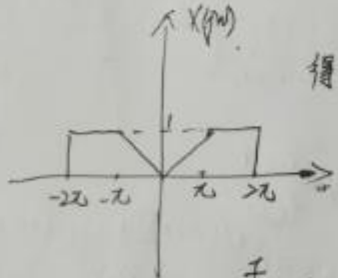
$h_2(t) = \frac{\sin \pi t}{\pi t} \xleftrightarrow{\mathcal{F}} H_2(j\omega):$



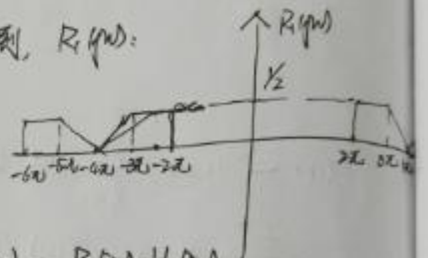
(5') ① $r_1(t) = x(t)\cdot \cos 4\pi t \xleftrightarrow{\mathcal{F}} R_1(j\omega) = \frac{1}{2\pi} X(j\omega) * [\pi\delta(\omega-4\pi) + \pi\delta(\omega$

$= \frac{1}{2} X(j(\omega-4\pi)) + \frac{1}{2} X(j(\omega+$

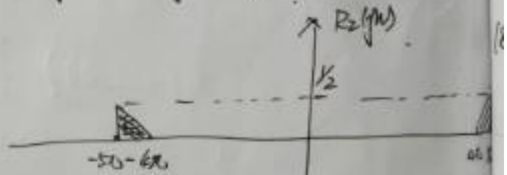
由



得到 $R_1(j\omega)$:

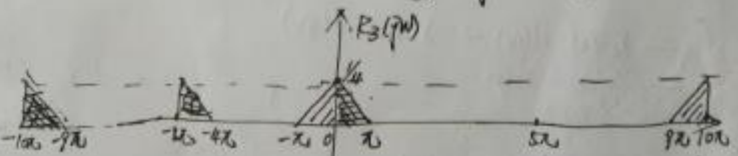


(5') ② $r_2(t) = r_1(t) * h_1(t) \xleftrightarrow{\mathcal{F}} R_2(j\omega) = R_1(j\omega)\cdot H_1(j\omega)$

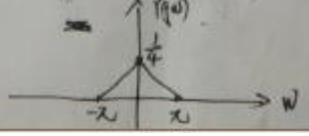


(5') ③ $r_3(t) = r_2(t)\cdot \cos 5\pi t \xleftrightarrow{\mathcal{F}} R_3(j\omega) = \frac{1}{2\pi} R_2(j\omega) * \mathcal{F}\{\cos(5\pi t)\}$

$= \frac{1}{2} R_2(j(\omega-5\pi)) + \frac{1}{2} R_2(j(\omega+5\pi))$



(5') ④ $y(t) = r_3(t) * h_2(t) \xleftrightarrow{\mathcal{F}} Y(j\omega) = R_3(j\omega)\cdot H_2(j\omega)$

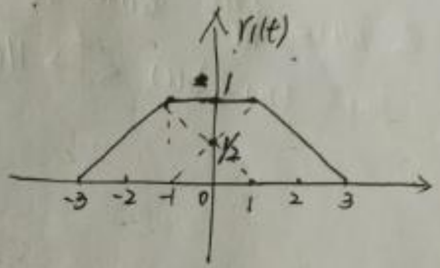


Quiz 2 Solutions.

Solution:

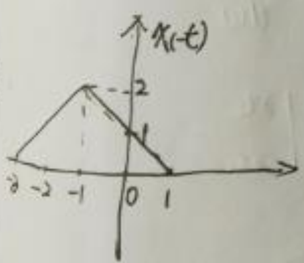
(a) ∵ $x(t)$ is real, $\mathcal{F}\{x_e(t)\} = Re\{X(j\omega)\}$

∴ $r(t) = x_e(t) = \frac{x(t)+x(-t)}{2}$



(b) 由 $x(t) \xleftrightarrow{\mathcal{F}} X(j\omega)$ 可得

$x(-t) \xleftrightarrow{\mathcal{F}} X(-j\omega)$.



$\frac{1}{2\pi}\int_{-\infty}^{\infty} X(-j\omega)\cdot d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(-j\omega)\cdot e^{j\omega t} d\omega \Big|_{t=0}$

$= x(-t)|_{t=0}$

由图x(-t)可得

$= 1$

$\int_{-\infty}^{\infty} X(-j\omega)\cdot d\omega = 2\pi$

(c) 令 $Y_1(j\omega) = \frac{1}{3} X(-\frac{j\omega}{3}) \leftrightarrow y_1(t) = x(-3t)$

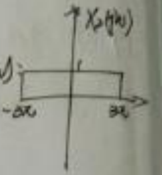
$Y(j\omega) = 3Y_1(j\omega)\cdot e^{-j\omega} \xleftrightarrow{\mathcal{F}} y(t) = 3\cdot x(-3(t-1))$.

~~$= 3x(-3t+3)$~~

5. Solution:

(7)(a) $h(t) = \left[\frac{d\delta(t)}{dt}\right] * \frac{\sin 3\pi t}{\pi t}$

令 $x_1(t) = \frac{d\delta(t)}{dt}$, $x_2(t) = \frac{\sin 3\pi t}{\pi t} \longleftrightarrow X_1(j\omega) = j\omega$, $X_2(j\omega)$:



$\therefore h(t) = x_1(t) * x_2(t) \xleftrightarrow{\mathcal{F}} H(j\omega) = X_1(j\omega) \cdot X_2(j\omega)$

$$= \begin{cases} j\omega, & |\omega| < 3\pi \\ 0, & |\omega| > 3\pi \end{cases}$$

$\because h(t)$ is real

(7)(b) $\int_{-\infty}^{\infty} h^2(t)\, dt = \int_{-\infty}^{\infty} |h(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(j\omega)|^2 d\omega$.

$$= \frac{1}{2\pi}\int_{-3\pi}^{3\pi} \omega^2 \, d\omega$$

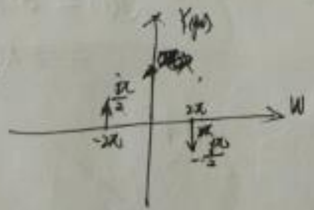
$$= \frac{1}{2\pi} \cdot \frac{\omega^3}{3} \Big|_{-3\pi}^{3\pi}$$

$$= 9\pi^2$$

(7)(c). 令 $x(t) = \left[\sum_{k=0}^{\infty} \left(\frac{1}{k+1}\right) \sin(2k\pi t)\right]$

则 $y(t) = x(t) * h(t)$

$\therefore Y(j\omega) = X(j\omega) \cdot H(j\omega)$



$\therefore y(t) = \frac{1}{2}\sin 2\pi t$.