Signals & Systems

(Second Edition)

一*Learning Instructions*

(Exercises Answers)

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Contents

Chapter 1 Answers

1.1 Converting from polar to Cartesian coordinates:

$$\frac{1}{2}e^{j\pi} = \frac{1}{2}\cos\pi = -\frac{1}{2} \qquad\qquad \frac{1}{2}e^{-j\pi} = \frac{1}{2}\cos(\pi) = -\frac{1}{2}$$

$$e^{j\frac{\pi}{2}} = \cos(\frac{\pi}{2}) + j\sin(\frac{\pi}{2}) = j \qquad\qquad e^{-j\frac{\pi}{2}} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = -j$$

$$e^{j\frac{5\pi}{2}} = e^{j\frac{\pi}{2}} = j \qquad\qquad \sqrt{2}e^{j\frac{\pi}{4}} = \sqrt{2}(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}) = 1+j$$

$$\sqrt{2}e^{j\frac{9\pi}{4}} = \sqrt{2}e^{j\frac{\pi}{4}} = 1-j \qquad \sqrt{2}e^{-j\frac{9\pi}{4}} = \sqrt{2}e^{-j\frac{\pi}{4}} = 1-j$$

$$\sqrt{2}e^{-j\frac{\pi}{4}} = 1-j$$

1.2 converting from Cartesian to polar coordinates:

$$5 = 5e^{j0}, \qquad\qquad -2 = 2e^{j\pi}, \qquad\qquad -3j = 3e^{-j\frac{\pi}{2}}$$

$$\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{-j\frac{\pi}{2}}, \qquad 1+j = \sqrt{2}e^{j\frac{\pi}{4}}, \qquad\qquad (1-j)^2 = 2e^{-j\frac{\pi}{2}}$$

$$j(1-j) = e^{\frac{\pi}{4}}, \qquad\qquad \frac{1+j}{1-j} = e^{\frac{\pi}{4}} \qquad\qquad \frac{\sqrt{2}+j\sqrt{2}}{1+j\sqrt{3}} = e^{-\frac{\pi}{12}}$$

1.3.
(a) $E_\infty = \int_0^\infty e^{-4t}dt = \frac{1}{4}$, $P_\infty = 0$, because $E_\infty < \infty$

(b) $x_2(t) = e^{j(2t+\frac{\pi}{4})}$, $|x_2(t)| = 1$. Therefore, $E_\infty = \int_{-\infty}^{+\infty}|x_2(t)|^2 dt = \int_{-\infty}^{+\infty}dt = \infty$,

$$P_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x_2(t)|^2 dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt = \lim_{T\to\infty}1 = 1$$

(c) $x_2(t) = \cos(t)$. Therefore, $E_\infty = \int_{-\infty}^{+\infty}|x_3(t)|^2 dt = \int_{-\infty}^{+\infty}\cos(t)^2 dt = \infty$,

$$P_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\cos(t)^2 dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\frac{1+\cos(2t)}{2}dt = \frac{1}{2}$$

(d) $x_1[n] = \left(\frac{1}{2}\right)^n u[n]$, $|x_1[n]|^2 = \left(\frac{1}{4}\right)^n u[n]$. Therefore, $E_\infty = \sum_{-\infty}^{+\infty}|x_1[n]|^2 = \sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n = \frac{4}{3}$

$P_\infty = 0$, because $E_\infty < \infty$.

(e) $x_2[n] = e^{-j(\frac{\pi n}{2}+\frac{\pi}{8})}$, $|x_2[n]|^2 = 1$. therefore, $E_\infty = \sum_{-\infty}^{+\infty}|x_2[n]|^2 = \infty$,

$$P_\infty = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|x_2[n]|^2 = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}1 = 1$$

(f) $x_3[n] = \cos\left(\frac{\pi}{4}\right)^n$. Therefore, $E_\infty = \sum_{-\infty}^{+\infty}|x_3[n]|^2 = \sum_{-\infty}^{+\infty}\cos(\frac{\pi}{4}n)^2 = \sum_{-\infty}^{+\infty}\cos(\frac{\pi}{4}n)^2$,

$$P_\infty = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}\cos\left(\frac{\pi}{4}\right)^n = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}(\frac{1+\cos(\frac{\pi}{2}n)}{2}) = \frac{1}{2}$$

1.4.
(a) The signal x[n] is shifted by 3 to the right. The shifted signal will be zero for n<1, And n>7.

(b) The signal x[n] is shifted by 4 to the left. The shifted signal will be zero for n<-6. And n>0.

(c) The signal x[n] is flipped signal will be zero for n<-1 and n>2.

(d) The signal x[n] is flipped and the flipped signal is shifted by 2 to the right. The new Signal will be zero for n<-2 and n>4.

(e) The signal x[n] is flipped and the flipped and the flipped signal is shifted by 2 to the left. This new signal will be zero for n<-6 and n>0.

1.5.
(a) x(1-t) is obtained by flipping x(t) and shifting the flipped signal by 1 to the right. Therefore, x (1-t) will be zero for t>-2.

(b) From (a), we know that x(1-t) is zero for t>-2. Similarly, x(2-t) is zero for t>-1, Therefore, x (1-t) +x(2-t) will be zero for t>-2.

(c) x(3t) is obtained by linearly compression x(t) by a factor of 3. Therefore, x(3t) will be zero for t<1.

(d) x(t/3) is obtained by linearly compression x(t) by a factor of 3. Therefore, x(3t) will be zero for t<9.

1.6 (a) $x_1(t)$ is not periodic because it is zero for t<0.

(b) $x_2[n]=1$ for all n. Therefore, it is periodic with a fundamental period of 1.

(c) $x_3[n]$ is as shown in the Figure S1.6.
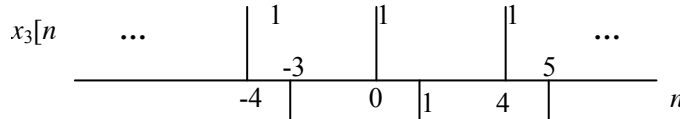
Therefore, it is periodic with a fundamental period of 4.

1.7. (a)

$$\varepsilon_v\left(x_1[n]\right)=\frac{1}{2}\left\{x_1[n]+x_1[-n]\right\}=\frac{1}{2}(u[n]-u[n-4]+u[-n]-u[-n-4])$$

Therefore, $\varepsilon_v\left(x_1[n]\right)$ is zero for $\left|x_1[n]\right|>3$.

(b) Since $x_1(t)$ is an odd signal, $\varepsilon_v\left(x_2[n]\right)$ is zero for all values of t.

(c)

$$\varepsilon_v\left(x_3[n]\right)=\frac{1}{2}\left\{x_1[n]+x_1[-n]\right\}=\frac{1}{2}\left[\left(\frac{1}{2}\right)^n u[n-3]-\left(\frac{1}{2}\right)^{-n} u[-n-3]\right]$$

Therefore, $\varepsilon_v\left(x_3[n]\right)$ is zero when $|n|<3$ and when $|n|\to\infty$.

(d)

$$\varepsilon_v\left(x_1(t)\right)=\frac{1}{2}(x_4(t)+x_4(-t))=\frac{1}{2}\left[e^{-5t}u(t-2)-e^{5t}u(-t+2)\right]$$

Therefore, $\varepsilon_v\left(x_4(t)\right)$ is zero only when $|t|\to\infty$.

1.8. (a) $\left(\Re\ell\{x_1(t)\}\right)=-2=2e^{0t}\cos(0t+\pi)$

(b) $\left(\Re\ell\{x_2(t)\}\right)=\sqrt{2}\cos(\frac{\pi}{4})\cos(3t+2\pi)=\cos(3t)=e^{0t}\cos(3t+0)$

(c) $\left(\Re\ell\{x_3(t)\}\right)=e^{-t}\sin(3\pi+t)=e^{-t}\sin(3t+\frac{\pi}{2})$

(d) $\left(\Re\ell\{x_4(t)\}\right)=-e^{-2t}\sin(100t)=e^{-t}\sin(100t+\pi)=e^{-2t}\cos(100t+\frac{\pi}{2})$

1.9. (a) $x_1(t)$ is a periodic complex exponential.

$$x_1(t)=je^{j10t}=e^{j\left(10t+\frac{\pi}{2}\right)}$$

(b) $x_2(t)$ is a complex exponential multiplied by a decaying exponential. Therefore, $x_2(t)$ is not periodic.

(c) $x_3[n]$ is a periodic signal. $x_3[n]=e^{j7\pi n}=e^{j\pi n}$.

$x_3[n]$ is a complex exponential with a fundamental period of $\frac{2\pi}{2}=\pi$.

(d) $x_4[n]$ is a periodic signal. The fundamental period is given by N=m($\frac{2\pi}{3\pi/5}$)

$=m(\frac{10}{3})$. By choosing m=3. We obtain the fundamental period to be 10.

(e) $x_5[n]$ is not periodic. $x_5[n]$ is a complex exponential with w_0=3/5. We cannot find any integer m such that m($\frac{2\pi}{w_0}$) is also an integer. Therefore, $x_5[n]$ is not periodic.

1.10. x(t)=2cos(10t+1)-sin(4t-1)

Period of first term in the RHS $=\frac{2\pi}{10}=\frac{\pi}{5}$.

Period of first term in the RHS $=\frac{2\pi}{4}=\frac{\pi}{2}$.

Therefore, the overall signal is periodic with a period which the least common multiple of the periods of the first and second terms. This is equal to π.

1.11. $x[n] = 1 + e^{j\frac{7}{4}\pi n} - e^{j\frac{2}{5}\pi n}$

Period of first term in the RHS =1.

Period of second term in the RHS $= \left(\dfrac{2\pi}{4/7}\right) = 7$ （when m=2）

Period of second term in the RHS $= \left(\dfrac{2\pi}{2\pi/5}\right) = 5$ (when m=1)

Therefore, the overall signal x[n] is periodic with a period which is the least common Multiple of the periods of the three terms inn x[n].This is equal to 35.

1.12. The signal x[n] is as shown in figure S1.12. x[n] can be obtained by flipping u[n] and then Shifting the flipped signal by 3 to the right. Therefore, x[n]=u[-n+3]. This implies that M=-1 and no=-3.
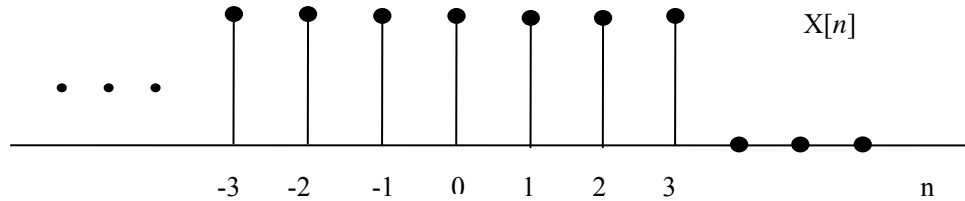

Figure   S 1.12

1.13

$$y(t) = \int_{-\infty}^{t} x(\tau)dt = \int_{-\infty}^{t} (\delta(\tau+2) - \delta(\tau-2))dt = \begin{cases} 0, t < -2 \\ 1, -2 \le t \le 2 \\ 0, t > 2 \end{cases}$$

Therefore $E\infty = \int_{-2}^{2} dt = 4$

1.14 The signal x(t) and its derivative g(t) are shown in Figure S1.14.

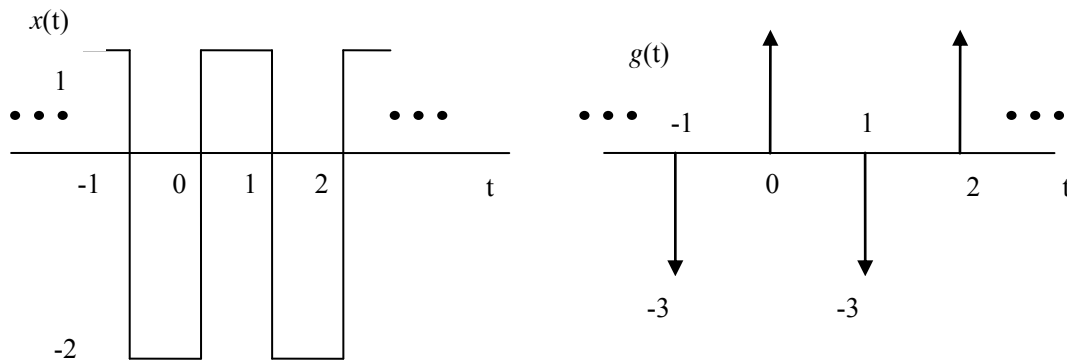Figure S 1.14

Therefore

$$g(t) = 3\sum_{k=-\infty}^{\infty} \delta(t-2k) - 3\sum_{k=-\infty}^{\infty} \delta(t-2k-1)$$

This implies that A_1=3, t_1=0, A_2=-3, and t_2=1.

1.15 (a) The signal x_2 [n], which is the input to S_2, is the same as y_1[n].Therefore ,

$$y_2 [n] = x_2 [n-2] + \frac{1}{2} x_2 [n-3]$$

$$= y_1 [n-2] + \frac{1}{2} y_1 [n-3]$$

$$= 2x_1 [n-2] + 4x_1 [n-3] + \frac{1}{2}(2x_1 [n-3] + 4x_1 [n-4])$$

$$= 2x_1 [n-2] + 5x_1 [n-3] + 2x_1 [n-4]$$

The input-output relationship for S is

$$y[n] = 2x[n-2] + 5x [n-3] + 2x [n-4]$$

(b) The input-output relationship does not change if the order in which S_1 and S_2 are connected series reversed. . We can easily prove this assuming that S_1 follows S_2 . In this case , the signal x_1 [n], which is the input to S_1 is the same as y_2 [n].

Therefore
$$y_1 [n] = 2x_1[n] + 4x_1 [n-1]$$
$$= 2y_2 [n] + 4 y_2 [n-1]$$
$$= 2(x_2 [n-2] + \frac{1}{2} x_2 [n-3]) + 4(x_2 [n-3] + \frac{1}{2} x_2 [n-4])$$
$$= 2 x_2 [n-2] + 5x_2 [n-3] + 2 x_2 [n-4]$$

The input-output relationship for S is once again
$$y[n] = 2x[n-2] + 5x [n-3] + 2x [n-4]$$

1.16 (a)The system is not memory less because y[n] depends on past values of x[n].

   (b)The output of the system will be $y[n] = \delta[n]\delta[n-2] = 0$

   (c)From the result of part (b), we may conclude that the system output is always zero for inputs of the form $\delta[n-k]$, $k \in$ r. Therefore , the system is not invertible .

1.17 (a) The system is not causal because the output y(t) at some time may depend on future values of x(t). For instance , $y(-\pi) = x(0)$.

(b) Consider two arbitrary inputs x_1 (t)and x_2 (t).
$$x_1 (t) \rightarrow y_1 (t) = x_1(\sin(t))$$

$$x_2 (t) \rightarrow y_2 (t) = x_2 (\sin(t))$$

Let x_3 (t) be a linear combination of x_1 (t) and x_2 (t).That is , $\quad x_3 (t) = a x_1 (t) + b x_2 (t)$

Where a and b are arbitrary scalars .If x_3 (t) is the input to the given system ,then the corresponding output y_3 (t) is
$$y_3 (t) = x_3 (\sin(t))$$
$$= a x_1(\sin(t)) + x_2 (\sin(t))$$
$$= a y_1 (t) + b y_2 (t)$$

Therefore , the system is linear.

1.18.(a) Consider two arbitrary inputs x_1[n]and x_2 [n].
$$x_1[n] \rightarrow y_1 [n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$$

$$x_2 [n] \rightarrow y_2 [n] = \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

Let x_3 [n] be a linear combination of x_1[n] and x_2 [n]. That is :
$$x_3 [n] = ax_1[n] + b x_2 [n]$$

where a and b are arbitrary scalars. If x_3 [n] is the input to the given system, then the corresponding output y_3 [n] is
$$y_3 [n] = \sum_{k=n-n_0}^{n+n_0} x_3[k]$$

$$= \sum_{k=n-n_0}^{n+n_0}(ax_1[k] + bx_2[k]) = a \sum_{k=n-n_0}^{n+n_0} x_1[k] + b \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

$$= ay_1[n] + b y_2 [n]$$

Therefore the system is linear.

(b) Consider an arbitrary input x_1[n].Let

$$y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$$

be the corresponding output .Consider a second input $x_2[n]$ obtained by shifting $x_1[n]$ in time:

$$x_2[n] = x_1[n-n_1]$$

The output corresponding to this input is

$$y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k] = \sum_{k=n-n_0}^{n+n_0} x_1[k-n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x_1[k]$$

Also note that $$y_1[n-n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x_1[k].$$

Therefore , $$y_2[n] = y_1[n-n_1]$$

This implies that the system is time-invariant.

(c) If $|x[n]| < B$, then $$y[n] \leq (2n_0+1)B.$$

Therefore ,$C \leq (2n_0+1)B$.

1.19 (a) (i) Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.
$$x_1(t) \rightarrow y_1(t) = t^2 x_1(t-1)$$
$$x_2(t) \rightarrow y_2(t) = t^2 x_2(t-1)$$

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$.That is $\quad x_3(t) = a x_1(t) + b x_2(t)$

where a and b are arbitrary scalars. If $x_3(t)$ is the input to the given system, then the corresponding output

$y_3(t)$ is $\quad\quad y_3(t) = t^2 x_3(t-1)$

$$= t^2(ax_1(t-1) + b x_2(t-1))$$

$$= a y_1(t) + b y_2(t)$$

Therefore , the system is linear.

(ii) Consider an arbitrary inputs $x_1(t)$.Let $\quad\quad y_1(t) = t^2 x_1(t-1)$

be the corresponding output .Consider a second input $x_2(t)$ obtained by shifting $x_1(t)$ in time:

$$x_2(t) = x_1(t-t_0)$$

The output corresponding to this input is $\quad y_2(t) = t^2 x_2(t-1) = t^2 x_1(t-1-t_0)$

Also note that $\quad\quad y_1(t-t_0) = (t-t_0)^2 x_1(t-1-t_0) \neq y_2(t)$

Therefore the system is not time-invariant.

(b) (i) Consider two arbitrary inputs $x_1[n]$ and $x_2[n]$.
$$x_1[n] \rightarrow y_1[n] = x_1^2[n-2]$$
$$x_2[n] \rightarrow y_2[n] = x_2^2[n-2].$$

Let $x_3(t)$ be a linear combination of $x_1[n]$ and $x_2[n]$.That is $\quad x_3[n] = ax_1[n] + b x_2[n]$

where a and b are arbitrary scalars. If $x_3[n]$ is the input to the given system, then the corresponding output

$y_3[n]$ is $\quad\quad y_3[n] = x_3^2[n-2]$

$$= (a x_1[n-2] + b x_2[n-2])^2$$

$$= a^2 x_1^2[n-2] + b^2 x_2^2[n-2] + 2ab x_1[n-2] x_2[n-2]$$

$$\neq a y_1[n] + b y_2[n]$$

Therefore the system is not linear.

(ii) Consider an arbitrary input $x_1[n]$. Let $\quad y_1[n] = x_1^2[n-2]$

be the corresponding output .Consider a second input $x_2[n]$ obtained by shifting $x_1[n]$ in time:

$$x_2[n] = x_1[n-n_1]$$

The output corresponding to this input is

$$y_2[n] = x_2^2[n-2] = x_1^2[n-2-n_1]$$

Also note that $\qquad\qquad y_1[n-n_1]=x_1^2[n-2-n_1]$

Therefore, $\qquad\qquad y_2[n]=y_1[n-n_1]$

This implies that the system is time-invariant.

(c) (i) Consider two arbitrary inputs $x_1[n]$ and $x_2[n]$.

$$x_1[n] \rightarrow y_1[n]=x_1[n+1]-x_1[n-1]$$
$$x_2[n] \rightarrow y_2[n]=x_2[n+1]-x_2[n-1]$$

Let $x_3[n]$ be a linear combination of $x_1[n]$ and $x_2[n]$. That is :

$$x_3[n]=ax_1[n]+bx_2[n]$$

where a and b are arbitrary scalars. If $x_3[n]$ is the input to the given system, then the

corresponding output $y_3[n]$ is $\qquad y_3[n]=x_3[n+1]-x_3[n-1]$

$$=ax_1[n+1]+bx_2[n+1]-ax_1[n-1]-bx_2[n-1]$$
$$=a(x_1[n+1]-x_1[n-1])+b(x_2[n+1]-x_2[n-1])$$
$$=ay_1[n]+by_2[n]$$

Therefore the system is linear.

(ii) Consider an arbitrary input $x_1[n]$. Let $\qquad y_1[n]=x_1[n+1]-x_1[n-1]$

be the corresponding output. Consider a second input $x_2[n]$ obtained by shifting $x_1[n]$ in time: $x_2[n]=x_1[n-n_0]$

The output corresponding to this input is

$$y_2[n]=x_2[n+1]-x_2[n-1]=x_1[n+1-n_0]-x_1[n-1-n_0]$$

Also note that $\qquad\qquad y_1[n-n_0]=x_1[n+1-n_0]-x_1[n-1-n_0]$

Therefore, $\qquad\qquad y_2[n]=y_1[n-n_0]$

This implies that the system is time-invariant.

(d) (i) Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.

$$x_1(t) \rightarrow y_1(t)=Od\{x_1(t)\}$$
$$x_2(t) \rightarrow y_2(t)=Od\{x_2(t)\}$$

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$. That is $\qquad x_3(t)=ax_1(t)+bx_2(t)$

where a and b are arbitrary scalars. If $x_3(t)$ is the input to the given system, then the corresponding output

$y_3(t)$ is $\qquad y_3(t)=Od\{x_3(t)\}$

$$=Od\{ax_1(t)+bx_2(t)\}$$
$$=aOd\{x_1(t)\}+bOd\{x_2(t)\}=ay_1(t)+by_2(t)$$

Therefore the system is linear.

(ii) Consider an arbitrary inputs $x_1(t)$. Let

$$y_1(t)=Od\{x_1(t)\}=\frac{x_1(t)-x_1(-t)}{2}$$

be the corresponding output. Consider a second input $x_2(t)$ obtained by shifting $x_1(t)$ in time:

$$x_2(t)=x_1(t-t_0)$$

The output corresponding to this input is

$$y_2(t)=Od\{x_2(t)\}=\frac{x_2(t)-x_2(-t)}{2}$$
$$=\frac{x_1(t-t_0)-x_1(-t-t_0)}{2}$$

Also note that $\qquad y_1(t-t_0)=\frac{x_1(t-t_0)-x_1(-t-t_0)}{2} \neq y_2(t)$

Therefore the system is not time-invariant.

1.20 (a) Given

$$X(t) = e^{2jt} \longrightarrow y(t) = e^{j3t}$$

$$X(t) = e^{-2jt} \longrightarrow y(t) = e^{-j3t}$$

Since the system liner

$$\mathcal{X}_1(t) = 1/2(e^{j2t} + e^{-2jt}) \longrightarrow y_1(t) = 1/2(e^{j3t} + e^{-j3t})$$

Therefore

$$\mathcal{X}_1(t) = \cos(2t) \longrightarrow y_1(t) = \cos(3t)$$

(b) we know that

$$\mathcal{X}_2(t) = \cos(2(t-1/2)) = (e^{-j} e^{2jt} + e^{j} e^{-2jt})/2$$

Using the linearity property, we may once again write

$$\mathcal{X}_1(t) = \frac{1}{2}(e^{-j} e^{2jt} + e^{j} e^{-2jt}) \longrightarrow y_1(t) = (e^{-j} e^{3jt} + e^{j} e^{-3jt}) = \cos(3t-1)$$

Therefore,

$$\mathcal{X}_1(t) = \cos(2(t-1/2)) \longrightarrow y_1(t) = \cos(3t-1)$$
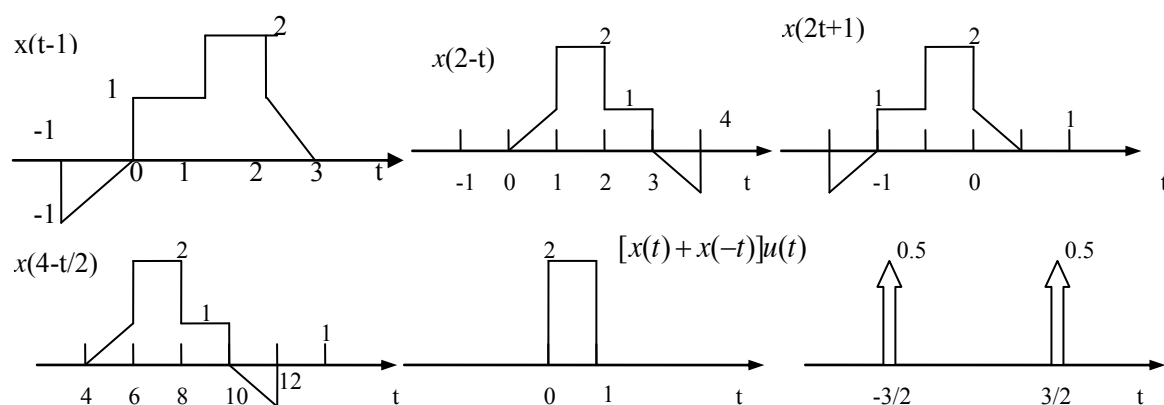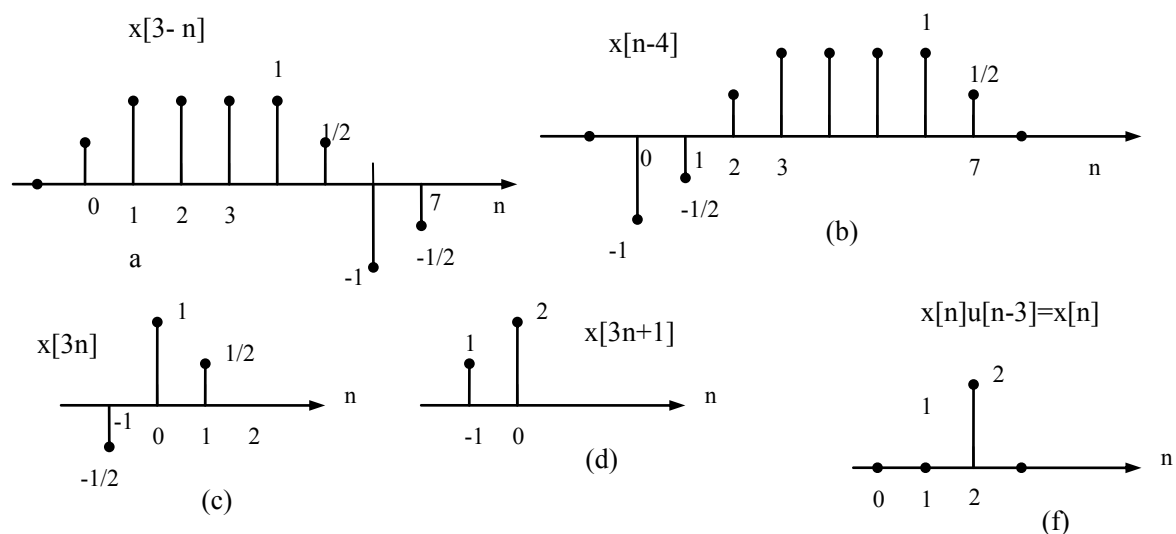
1.21.The signals are sketched in figure S1.21.

Figure S1.21

1.22 The signals are sketched in figure S1.22
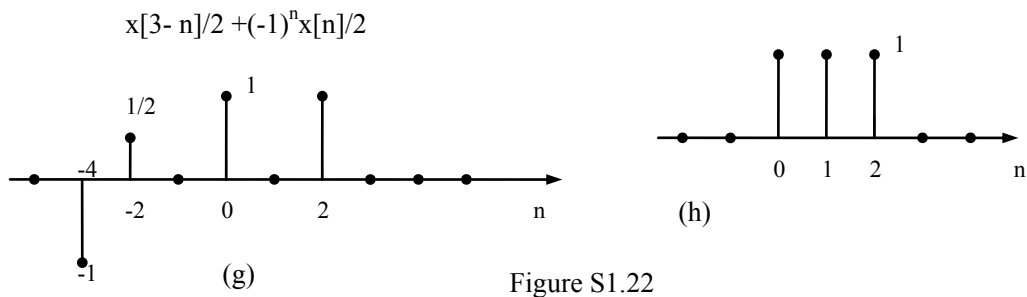**1.23** The even and odd parts are sketched in Figure S1.23

x[3- n]/2 +(-1)^{n}x[n]/2

(g)

(h)

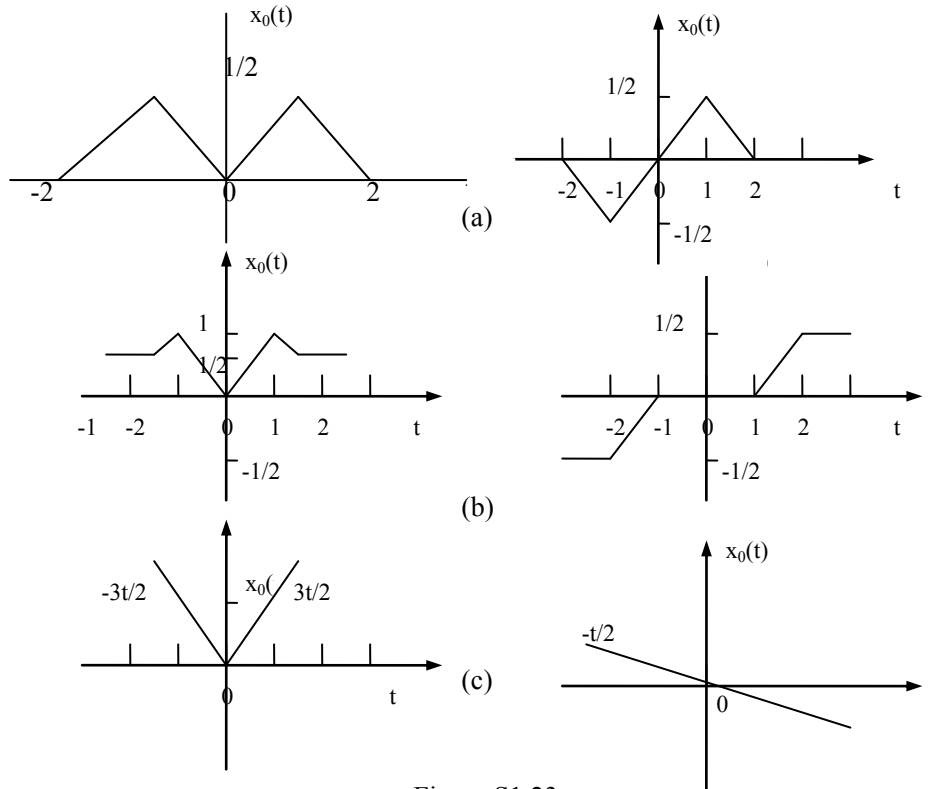Figure S1.22

(a)
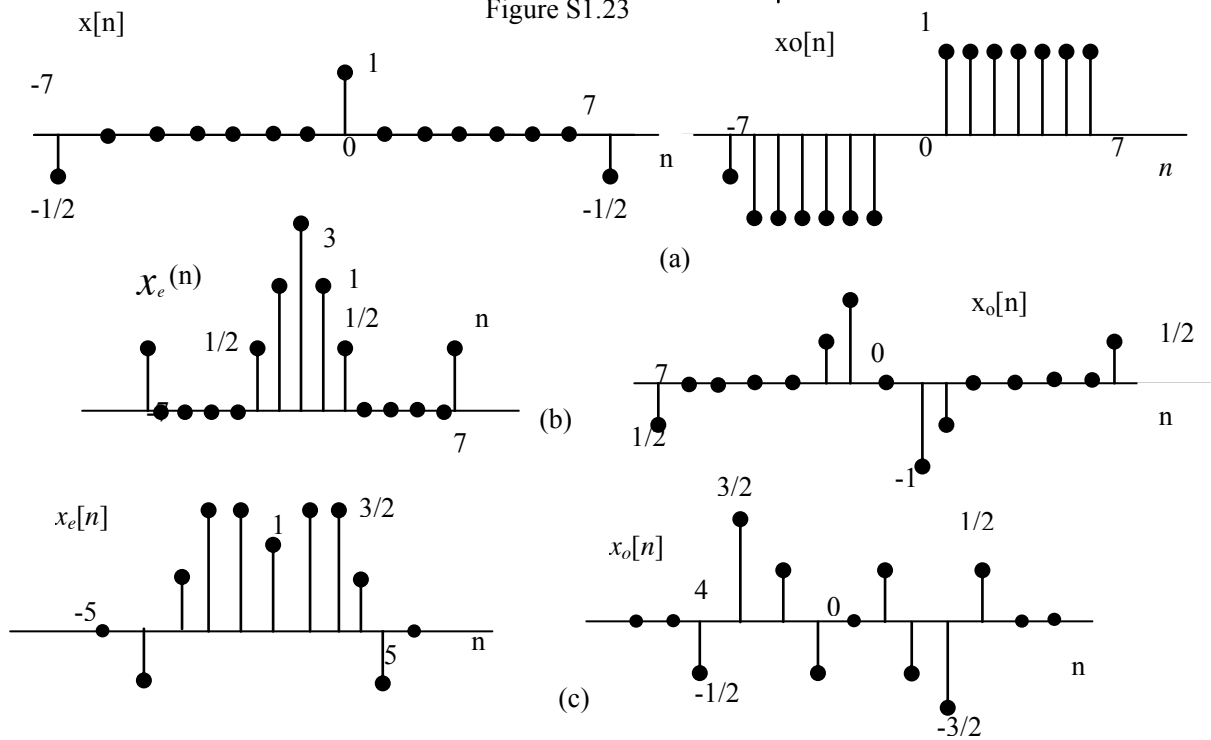

(b)

(c)

Figure S1.23

(a)

(b)

(c)

Figure S1.24

9

1.24 The even and odd parts are sketched in Figure S1.24

1.25 (a) periodic period=$2\pi/(4)=\pi/2$

  (b) periodic period=$2\pi/(4)=2$

  (c) x(t)=[1+cos(4t-2π/3)]/2.   periodic period=$2\pi/(4)=\pi/2$

  (d) x(t)=cos(4πt)/2.   periodic period=$2\pi/(4)=1/2$

  (e) x(t)=[sin(4πt)u(t)-sin(4πt)u(-t)]/2. Not period.

  (f)   Not period.

1.26 (a) periodic, period=7.

   (b) Not period.

   (c) periodic, period=8.

   (d) x[n]=(1/2)[cos(3πn/4+cos(πn/4)). periodic, period=8.

   (e) periodic, period=16.

1.27 (a) Linear, stable

   (b) Not period.

   (c) Linear

   (d) Linear, causal, stable

   (e) Time invariant, linear, causal, stable

   (f) Linear, stable

  (g)   Time invariant, linear, causal

1.28 (a) Linear, stable

   (b) Time invariant, linear, causal, stable

   (c)Memoryless, linear, causal

   (d) Linear, stable

   (e) Linear, stable

   (f) Memoryless, linear, causal, stable

   (g) Linear, stable

1.29 (a) Consider two inputs to the system such that

$$x_1[n]\xrightarrow{\ S\ }y_1[n]=\Re_e\{x_1[n]\}.\ \text{and}\ \ x_2[n]\xrightarrow{\ S\ }y_2[n]=\Re_e\{x_1[n]\}.$$

Now consider a third input $x_3[n]=x_2[n]+x_1[n]$. The corresponding system output

$$y_3[n]=\Re_e\{x_3[n]\}$$

Will be

$$=\Re_e\{x_1[n]+x_2[n]\}$$

$$=\Re_e\{x_1[n]\}+\Re_e\{x_2[n]\}$$

$$=y_1[n]+y_2[n]$$

   therefore, we may conclude that the system is additive

   Let us now assume that inputs to the system such that

$$x_1[n]\xrightarrow{\ S\ }y_1[n]=\Re_e\{e^{j\pi/4}x_1[n]\}.$$

  and

$$x_2[n]\xrightarrow{\ S\ }y_2[n]=\Re_e\{e^{j\pi/4}x_2[n]\}.$$

Now consider a third input $x_3[n]=x_2[n]+x_1[n]$. The corresponding system output
Will be

$$y_3[n]=\Re_e\{e^{j\pi/4}x_3[n]\}$$

$$=\cos(\pi n/4)\Re_e\{x_3[n]\}-\sin(\pi n/4)\mathbf{I}_m\{x_3[n]\}$$

$$+\cos(\pi n/4)\Re_e\{x_1[n]\}-\sin(\pi n/4)\mathbf{I}_m\{x_1[n]\}$$

$$+\cos(\pi n/4)\Re_e\{x_2[n]\}-\sin(\pi n/4)\mathbf{I}_m\{x_2[n]\}$$

$$=\Re_e\{e^{j\pi/4}x_1[n]\}+\Re_e\{e^{j\pi/4}x_2[n]\}$$

$$=y_1[n]+y_2[n]$$

therefore, we may conclude that the system is additive

(b) (i) Consider two inputs to the system such that

$$x_1(t) \xrightarrow{\ S\ } y_1(t) = \frac{1}{x_1(t)}\left[\frac{dx_1(t)}{dt}\right]^2 \qquad \text{and} \qquad x_2(t) \xrightarrow{\ S\ } y_2(t) = \frac{1}{x_1(t)}\left[\frac{dx_2(t)}{dt}\right]^2$$

Now consider a third input $x_3[t] = x_2[t] + x_1[t]$. The corresponding system output
Will be

$$y_3(t) = \frac{1}{x_3(t)}\left[\frac{dx_3(t)}{dt}\right]^2$$

$$= \frac{1}{x_1(t) + x_1(t)}\left[\frac{d[x_1(t) + x_1(t)]}{dt}\right]^2$$

$$\neq y_1(t) + y_2(t)$$

therefore, we may conclude that the system is not additive
Now consider a third input $x_4[t] = a\, x_1[t]$. The corresponding system output
Will be

$$y_4(t) = \frac{1}{x_4(t)}\left[\frac{dx_4(t)}{dt}\right]^2$$

$$= \frac{1}{ax_1(t)}\left[\frac{d[ax_1(t)]}{dt}\right]^2$$

$$= \frac{a}{x_1(t)}\left[\frac{dx_1(t)}{dt}\right]^2$$

$$= ay_1(t)$$

Therefore, the system is homogeneous.

(ii) This system is not additive. Consider the fowling example .Let $\delta[n] = 2\delta[n+2] +$
$2\delta[n+1] + 2\delta[n]$ and $x_2[n] = \delta[n+1] + 2\delta[n+1] + 3\delta[n]$. The corresponding outputs evaluated at n=0 are

$$y_1[0] = 2\, and \quad y_2[0] = 3/2$$

Now consider a third input $x_3[n] = x_2[n] + x_1[n] .= 3\delta[n+2] + 4\delta[n+1] + 5\delta[n]$
The corresponding outputs evaluated at n=0 is $y_3[0] = 15/4$. Gnarly, $y_3[0] \neq y_1[n] + y_2[0]$.This

$$y_4[n] = \begin{cases} \dfrac{x_4[n]x_4[n-2]}{x_4[n-1]}, & x_4[n-1] \neq 0 \\ & \text{otherwise} \\ 0, \end{cases}$$

$$y_5[n] = \begin{cases} a\dfrac{x_4[n]x_4[n-2]}{x_4[n-1]}, & x_4[n-1] \neq 0 \\ & \text{otherwise} \\ 0, \end{cases} = ay_4[n]$$

Therefore, the system is homogenous.
1.30 (a) Invertible. Inverse system y(t)=x(t+4)
  (b)Non invertible. The signals x(t) and $x_1(t) = x(t) + 2\pi$ give the same output
  (c) $\delta[n]$ and $2\delta[n]$ give the same output
  d) Invertible. Inverse system; y(t)=dx(t)/dt
  (e) Invertible. Inverse system y(n)=x(n+1) for n≥0 and y[n]=x[n] for n<0
  (f) Non invertible. $x(n)$ and $-x(n)$ give the same result
 (g)Invertible. Inverse system y(n)=x(1-n)
  (h) Invertible. Inverse system y(t)=dx(t)/dt
  (i) Invertible. Inverse system y(n) = x(n)-(1/2)x[n-1]
  (j) Non invertible. If x(t) is any constant, then y(t)=0
  (k) $\delta[n]$ and $2\delta[n]$ result in y[n]=0
  (l) Invertible. Inverse system: y(t)=x(t/2)
  (m) Non invertible $x_1[n] = \delta[n]_+ \delta[n-1]$and $x_2[n] = \delta[n]$ give y[n]= $\delta[n]$
  (n) Invertible. Inverse system: y[n]=x[2n]
1.31 (a) Note that $x_2[t] = x_1[t] - x_1[t-2]$. Therefore, using linearity we get $y_2(t) =$
  $y_1(t) - y_1(t-2)$.this is shown in Figure S1.31
  (b)Note that x3 (t)= x1 [t]+ x1 [t+1]. .Therefore, using linearity we get Y3 (t)= y1 (t)+ y1 (t+2). this is

shown in Figure S1.31
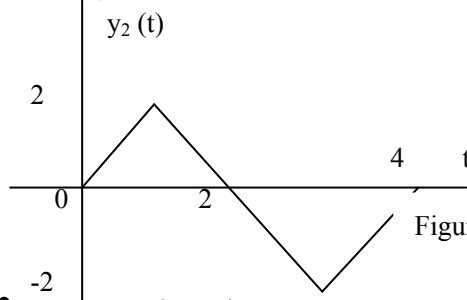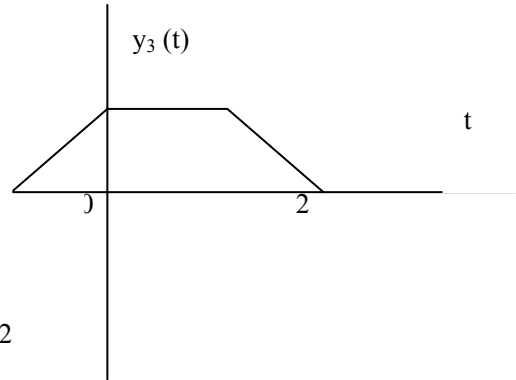

Figure S1.31

1.32 ... statements are true
    (1) $x(t)$ periodic with period T; $y_1(t)$ periodic, period T/2
    (2) $y_1(t)$ periodic, period T; bx(t) periodic, period2T
    (3) $x(t)$ periodic, period T; $y_2(t)$ periodic, period2T
    (4) $y_2(t)$ periodic, period T; x(t) periodic, period T/2;

1.33 (1) True $x[n]=x[n+N]$; $y_1(n)= y_1(n+ N_0)$ i.e. periodic with $N_0=n/2$
      if N is even and with period $N_0=n$ if N is odd.

(2) False. $y_1[n]$ periodic does no imply $x[n]$ is periodic i.e. Let $x[n] = g[n]+h[n]$ where

$$g[n] = \begin{cases} 1, & n\ even \\ 0, & n\ odd \end{cases} and \qquad h[n] = \begin{cases} 0, & n\ even \\ (1/2)^n, & n\ odd \end{cases}$$

Then $y_1[n] = x[2n]$ is periodic but $x[n]$ is clearly not periodic.

(3) True. $x[n+N]=x[n]$; $y_2[n+N_0]=y_2[n]$ where $N_0=2N$

(4) True. $y_2[n+N]=y_2[n]$; $y_2[n+N_0]=y_2[n]$ where $N_0=N/2$

1.34. (a) Consider

$$\sum_{n=-\infty}^{\infty} x[n] = x[0] + \sum_{n=1}^{\infty} \{x[n]+ x[-n]\}$$

If $x[n]$ is odd, $x[n] +x[-n]=0$. Therefore, the given summation evaluates to zero.

(b) Let $y[n] =x_1[n]x_2[n]$. Then

$$y[-n] =x_1[-n]\ x_2[-n] =-x_1[n]x_2[n] =-y[n].$$

This implies that $y[n]$ is odd.

(c) Consider

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n]+ \sum_{n=-\infty}^{\infty} x_o^2[n]$$

$$= \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$$

Using the result of part (b), we know that $x_e[n]x_o[n]$ is an odd signal. Therefore, using the result of part (a) we may conclude that

$$2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n] = 0$$

Therefore,

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n]+ \sum_{n=-\infty}^{\infty} x_o^2[n].$$

(d) Consider

$$\int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} x_e^2(t)dt + \int_{-\infty}^{\infty} x_o^2(t)dt + 2\int_{-\infty}^{\infty} x_e(t)x_0(t)dt.$$

Again, since $x_e(t) x_o(t)$ is odd,

$$\int_{-\infty}^{\infty} x_e(t)x_0(t)dt = 0.$$

Therefore,

$$\int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} x_e^2(t)dt + \int_{-\infty}^{\infty} x_o^2(t)dt.$$

1.35. We want to find the smallest N_0 such that $m(2\pi /N) N_0 =2\pi k$ or $N_0 =kN/m$,

where k is an integer, then N must be a multiple of m/k and m/k must be an integer .this implies that m/k is a divisor of both m and N .Also, if we want the smallest possible N_0, then m/k should be the GCD of m and N. Therefore, N_0=N/gcd(m,N).

1.36. (a)If x[n] is periodic $e^{j\omega_0(n+N)T}$, $where.\omega_o = 2\pi/T_0$. This implies that

$$\frac{2\pi}{T_0}NT = 2\pi k \Rightarrow \frac{T}{T_o} = \frac{k}{N} = \text{a rational number}.$$

(b)T/T_0 =p/q then x[n] = $e^{j2\pi n(p/q)}$,The fundamental period is q/gcd(p,q) and the fundmental frequency is

$$\frac{2\pi}{q}\gcd(p,q) = \frac{2\pi}{p}\frac{p}{q}\gcd(p,q) = \frac{\omega_0}{p}\gcd(p,q) = \frac{\omega_0 T}{p}\gcd(p,q).$$

(c) p/gcd(p,q) periods of x(t) are needed .

1.37.(a) From the definition of $\phi_{xy}(t)$. We have

$$\phi_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)y(\tau)d\tau$$

$$= \int_{-\infty}^{\infty} y(-t+\tau)x(\tau)d\tau$$

(b) Note from $= \phi_{yx}(-t)$. part(a) that $\phi_{xx}(t) = \phi_{xx}(-t)$. This implies that $\phi_{xy}(t)$ is even .Therefore,

the odd part of $\phi_{xx}(t)$. is zero.

(c) Here, $\phi_{xy}(t) = \phi_{xx}(t-T)$. and $\phi_{yy}(t) = \phi_{xx}(t)$.

1.38.(a) We know that $2\delta_\Box(2t) = \delta_{\Box/2}(t)$. Therefore

This implies that

$$\lim_{\Box\to\infty}\delta_\Box(2t) = \lim_{\Box\to\infty}\frac{1}{2}\delta_{\Box/2}(t).$$

$$\delta(2t) = \frac{1}{2}\delta(t).$$

(b)The plot are as shown in Figure s3.18.

1.39 We have

$$\lim_{\Box\to 0}u_\Box(t)\delta(t) = \lim_{\Box\to 0}u_\Box(0)\delta(t) = 0.$$

Also,

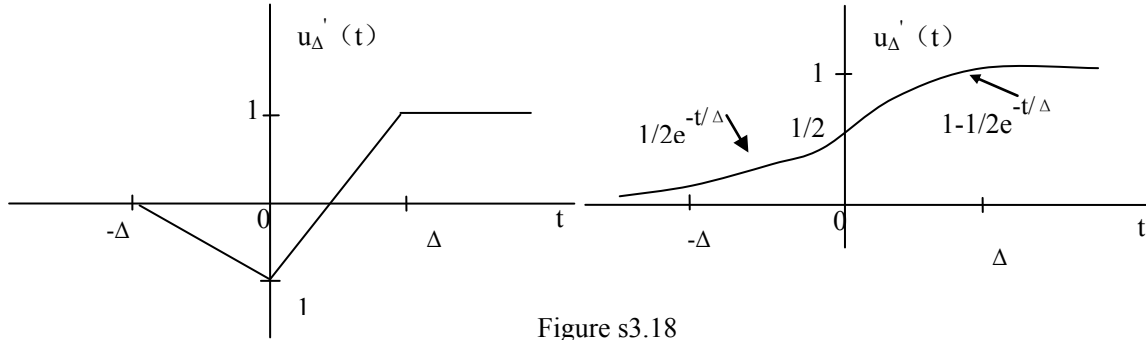$$\lim_{\Box\to 0}u_\Box(t)\delta_\Box(t) = \frac{1}{2}\delta(t).$$
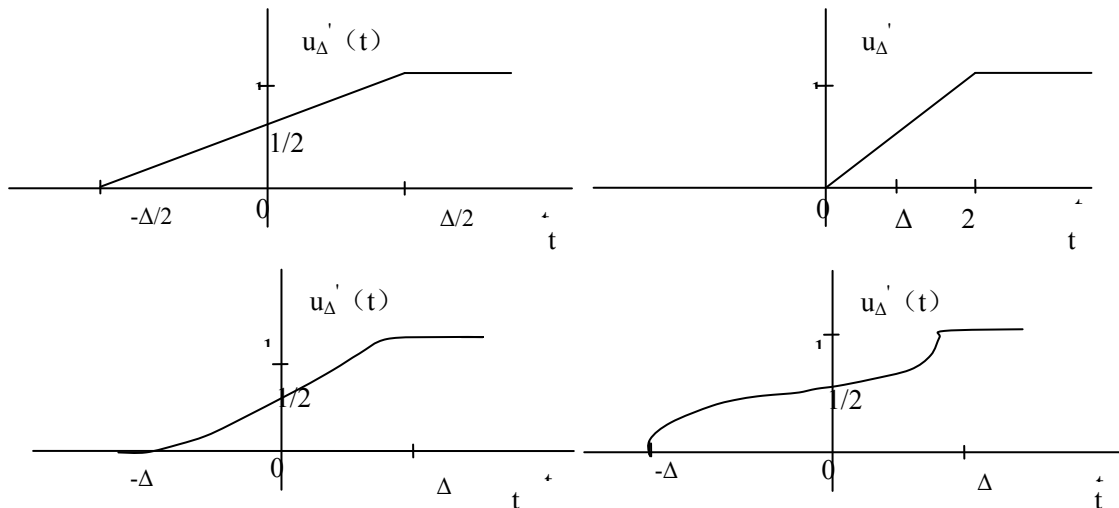
Figure s3.18

We have

$$g(t) = \int_{-\infty}^{\infty} u(\tau)\delta(t-\tau)d\tau = \int_{0}^{\infty} u(\tau)\delta(t-\tau)d\tau$$

Therefore,

$$g(t) = \begin{cases} 0, & t > 0 \\ 1, & t < 0 \\ undefined \; for \; t = 0 \end{cases} \quad \begin{aligned} &\because \delta(t-\tau) = 0 \\ &\because u(\tau)\delta(t-\tau) = \delta(t-\tau) \end{aligned}$$

1.40.(a) If a system is additive ,then
also, if a system is homogeneous,then

$$0 = x(t) - x(t) \rightarrow y(t) - y(t) = 0$$

$$0 = 0.x(t) \rightarrow y(t).0 = 0$$

(b) $y(t) = x^2(t)$ is such a systerm .

(c) No.For example,consider y(t) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$ with $x(t) = u(t) - u(t-1)$. Then x(t)=0

for t>1,but y(t)=1 for t>1.

1.41. (a) y[n]=2x[n].Therefore, the system is time invariant.
   (b) y[n]=(2n-1)x[n].This is not time-invariant because $y[n- N_0] \neq (2n-1)\, 2x[n- N_0]$.
   (c) $y[n]=x[n]\{1+(-1)^n+1+(-1)^{n-1}\}=2x[n]$.Therefore, the system is time invariant .

1.42.(a) Consider two system S_1 and S_2 connected in series .Assume that if $x_1(t)$ and $x_2(t)$ are
   the inputs to S_1.then $y_1(t)$ and $y_2(t)$ are the outputs.respectively .Also,assume that
   if $y_1(t)$ and $y_2(t)$ are the input to S_2 ,then $z_1(t)$ and $z_2(t)$ are the outputs, respectively . Since S_1 is
   linear ,we may write

$$ax_1(t) + bx_2(t) \xrightarrow{s_1} ay_1(t) + by_2(t),$$

where a and b are constants. Since S_2 is also linear ,we may write

$$ay_1(t) + by_2(t) \xrightarrow{s_2} az_1(t) + bz_2(t),$$

We may therefore conclude that

$$a\, x_1(t) + b\, x_2(t) \xrightarrow{\;s_1 s_2\;} a\, z_1(t) + b\, z_2(t)$$

Therefore ,the series combination of S_1 and S_2 is linear.
Since S_1 is time invariant, we may write

$$x_1(t-T_0) \xrightarrow{s_1} y_1(t-T_0)$$

and

$$y_1(t-T_0) \xrightarrow{s_2} z_1(t-T_0)$$

Therefore,

$$x_1(t-T_0) \xrightarrow{s_1 s_2} z_1(t-T_0)$$

Therefore, the series combination of S_1 and S_2 is time invariant.
(b) False, Let y(t)=x(t)+1 and z(t)=y(t)-1.These corresponds to two nonlinear systems. If these systems are
connected in series ,then z(t)=x(t) which is a linear system.

14

(c) Let us name the output of system 1 as w[n] and the output of system 2 as z[n] .Then

$$y[n] = z[2n] = w[2n] + \frac{1}{2}w[2n-1] + \frac{1}{4}w[2n-2]$$

$$= x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2]$$

The overall system is linear and time-invariant.

1.43. (a) We have

$$x(t) \xrightarrow{\ s\ } y(t)$$

Since S is time-invariant.

$$x(t-T) \xrightarrow{\ s\ } y(t-T)$$

Now if x (t) is periodic with period T. x{t}=x(t-T). Therefore, we may conclude that y(t)=y(t-T).This implies that y(t) is also periodic with T .A similar argument may be made in discrete time .

(b)

1.44 (a) Assumption : If x(t)=0 for t<t_0 ,then y(t)=0 for t< t_0. To prove That : The system is causal.

Let us consider an arbitrary signal x_1(t) .Let us consider another signal x_2(t) which is the same as x_1(t)for t< t_0. But for t> t_0 , x_2(t) $\neq x_1$(t),Since the system is linear,

$$x_1(t) - x_2(t) \rightarrow y_1(t) - y_2(t),$$

Since $x_1(t) - x_2(t) = 0$ for t< t_0 ,by our assumption $= y_1(t) - y_2(t) = 0$ for t< t_0 .This implies that $y_1(t) = y_2(t)$ for t< t_0 . In other words, t he output is not affected by input values for $t \geq t_0$. Therefore, the system is causal .

Assumption: the system is causal . To prove that :If x(t)=0 for t< t_0 .then y(t)=0 for t< t_0 .

Let us assume that the signal x(t)=0 for t< t_0 .Then we may express x(t) as $x(t) = x_1(t) - x_2(t)$,

Where $x_1(t) = x_2(t)$ for   t< t_0 .   the   system   is   linear   .the   output   to   x(t)   will   be

$y(t) = y_1(t) - y_2(t)$ .Now ,since the system is causal . $y_1(t) = y_2(t)$ for t< t_0 .implies that

$y_1(t) = y_2(t)$ for t< t_0 .Therefore y(t)=0 for t< t_0 .

(b) Consider y(t)=x(t)x(t+1) .Now , x(t)=0 for t< t_0 implies that y(t)=0 for t< t_0 .Note that the system is nonlinear and non-causal .

(c) Consider y(t)=x(t)+1.   the system is nonlinear and causal .This does not satisfy the condition of part(a).

(d) Assumption: the system is invertible. To prove that :y[n]=0 for all n only if x[n]=0 for all n .
Consider

$$x[n] = 0 \rightarrow y[n].$$

Since the system is linear :

$$2x[n] = 0 \rightarrow 2y[n].$$

Since the input has not changed in the two above equations ,we require that y[n]= 2y[n].This implies that y[n]=0. Since we have assumed that the system is invertible , only one input could have led to this particular output .That input must be x[n]=0 .

Assumption: y[n]=0 for all n if x[n]=0 for all n . To prove that : The system is invertible .
Suppose that

$$x_1[n] \rightarrow y_1[n]$$

and

$$x_2[n] \rightarrow y_1[n]$$

Since the system is linear ,

$$x_1[n] - x_2[n] \Longrightarrow y_1[n] - y_2[n] = 0$$

By the original assumption ,we must conclude that $x_1[n] = x_2[n]$ .That is ,any particular y$_1$[n] can be produced that by only one distinct input x$_1$[n] .Therefore , the system is invertible.

(e) y[n]=x^2[n].

1.45. (a) Consider ,

$$x_1(t) \xrightarrow{s} y_1(t) = \phi_{hx_1}(t)$$

and

$$x_2(t) \xrightarrow{s} y_2(t) = \phi_{hx_2}(t).$$

Now, consider $x_3(t) = ax_1(t) + bx_2(t)$. The corresponding system output will be

$$y_3(t) = \int_{-\infty}^{\infty} x_3(\tau)h(t+\tau)d\tau$$

$$= a\int_{-\infty}^{\infty} x_1(\tau)h(t+\tau)d\tau + b\int_{-\infty}^{\infty} x_2(t)h(t+\tau)d\tau$$

$$= a\phi_{hx_1}(t) + b\phi_{hx_2}(t)$$

$$= ay_1(t) + by_2(t)$$

Therefore, S is linear .

Now ,consider $x_4(t)=x_1(t-T)$.The corresponding system output will be

$$y_4(t) = \int_{-\infty}^{\infty} x_4(\tau)h(t+\tau)d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau-T)h(t+\tau)d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau)h(t+\tau+T)d\tau$$

$$= \phi_{hx_1}(t+T)$$

Clearly, $y_4(t) \neq y_1(t-T)$.Therefore ,the system is not time-invariant.

The system is definitely not causal because the output at any time depends on future values of the input signal x(t).

(b) The system will then be linear ,time invariant and non-causal.

1.46. The plots are in Figure S1.46.

1.47.(a) The overall response of the system of Figure P1.47.(a)=(the response of the system to x[n]+x₁[n])-the response of the system to x₁[n]=(Response of a linear system L to x[n]+x₁[n]+ zero input response of S)- (Response of a linear system L to x₁[n]+zero input response of S)=((Response of a linear system L to x[n]).

Chapter 2 answers

2.1 (a) We have know that $y_1 = x[n]*h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$ (S2.1-1)
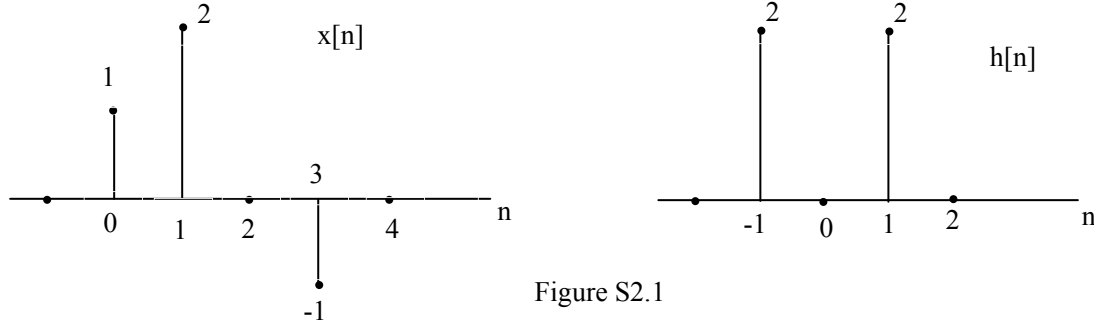
The signals x[n] and h[n] are as how in Figure S2.1

Figure S2.1

From this figure, we can easily see that the above convolution sum reduces to

$$y_1[n] = h[-1]x[n+1] + h[1]x[n-1]$$
$$= 2x[n+1] + 2x[n-1]$$

This gives

$$y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$$

(b) We know that

$$y_2[n] = x[n+2]*h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n+2-k]$$

Comparing with eq.(S2.1-1), we see that

$$y_2[n] = y_1[n+2]$$

(c) We may rewrite eq.(S2.1-1) as

$$y_1[n] = x[n]*h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Similarly, we may write

$$y_3[n] = x[n]*h[n+2] = \sum_{k=-\infty}^{\infty} x[k]h[n+2-k]$$

Comparing this with eq.(S2.1), we see that

$$y_3[n] = y_1[n+2]$$

2.2 Using given definition for the signal h[n], we may write

$$h[k] = \left(\frac{1}{2}\right)^{k-1} \{u[k+3] - u[k-10]\}$$

The signal h[k] is non zero only in the rang $h[n] = h_1[n+2]$. From this we know that the signal h[-k] is non zero only in the rage $-9 \le k \le 3$. If we now shift the signal h[-k] by n to the right, then the resultant signal h[n-k] will be zero in the range $(n-9) \le k \le (n+3)$.

Therefore ,

$$A = n - 9, \qquad B = n + 3$$

2.3 Let us define the signals

$$x_1[n] = \left(\frac{1}{2}\right)^n u[n]$$

and

$$h_1[n] = u[n].$$

We note that

$$x[n] = x_1[n-2] \qquad \text{and} \qquad h[n] = h_1[n+2]$$

Now,

$$y[n] = x[n] * h[n] = x_1[n-2] * h_1[n+2] = \sum_{k=-\infty}^{\infty} x_1[k-2]h_1[n-k+2]$$

By replacing k with m+2 in the abovr summation ,we obtain

$$y[n] = \sum_{m=-\infty}^{\infty} x_1[m]h_1[n-m] = x_1[n]h_1[n]$$

Using the results of Example 2.1 in the text book ,we may write

$$y[n] = 2\left[1-\left(\frac{1}{2}\right)^{n+1}\right]u[n]$$

2.4   We know that

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

The signals x[n] and y[n] are as shown in Figure S2.4.From this figure,we see that the above summation reduces to

$$y[n] = x[3]h[n-3] + x[4]h[n-4] + x[5]h[n-5] + x[6]h[n-6] + x[7]h[n-7] + x[8]h[n-8]$$ This gives

$$y[n] = \begin{cases} n-6, & 7 \le n \le 11 \\ 6, & 12 \le n \le 18 \\ 24-n, & 19 \le n \le 23 \\ 0, & otherwise \end{cases}$$
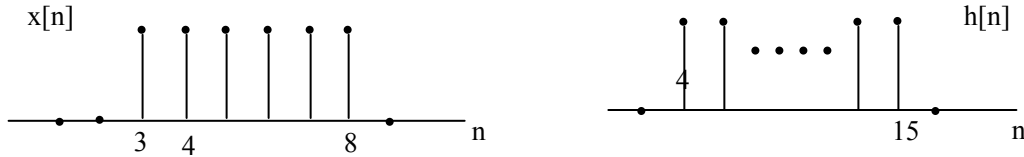

Figure.S2.4

2.5. The signal y[n] is

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k].$$

In this case , this summation reduces to

$$y[n] = \sum_{k=0}^{9} x[k]h[n-k] = \sum_{k=0}^{9} h[n-k].$$

From this it is clear that y[n] is a summation of shifted replicas of h[n]. Since the last replicas will begin at n=9 and h[n] is zero for n>N, y[n] is zero for n>N+9. Using this and the fact y[14]=0,we may conclude that N can at most be 4. Furthermore ,since y[4]=5,we can conclude that h[n] has at least 5 non-zero points . The only value of N which satisfies both these conditions is 4.

2.6. From the given information, we have :

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k].$$

$$= \sum_{k=-\infty}^{\infty} (\frac{1}{3})^{-k} u[-k-1]u[n-k-1].$$

$$= \sum_{k=-\infty}^{-1} (\frac{1}{3})^{-k} u[n-k-1].$$

$$= \sum_{k=1}^{\infty} (\frac{1}{3})^{-k} u[n+k-1].$$

Replacing k by p-1,

$$y[n] = = \sum_{p=0}^{\infty} (\frac{1}{3})^{p+1} u[n+p] \qquad\qquad (S2.6-1)$$

For n≥0 the above equation reduces to,

$$y[n] = \sum_{p=0}^{\infty} \left(\frac{1}{3}\right)^{p+1} = \frac{1}{3}\frac{1}{1-\frac{1}{3}} = \frac{1}{2}$$

For n<0 eq.(S2.6-1) reduces to,

$$y[n] = \sum_{p=-n}^{\infty} \left(\frac{1}{3}\right)^{p+1} = \left(\frac{1}{3}\right)^{-n+1} \sum_{p=0}^{\infty} \left(\frac{1}{3}\right)^{p} = \left(\frac{1}{3}\right)^{-n+1} \frac{1}{1-\frac{1}{3}} = \left(\frac{1}{3}\right)^{-n}\frac{1}{2} = \frac{3^{n}}{2}$$

Therefore,

$$y[n] = \begin{cases} (3^{n}/2), n < 0 \\ (1/2), n \geq 0 \end{cases}$$

2.7.(a) Given that

$$x[n] = \delta[n-1],$$

we see that

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k] = g[n-2] = u[n-2]-u[n-6]$$

(b) Given that $x[n] = \delta[n-2]$,

we see that $y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k] = g[n-4] = u[n-4]-u[n-8]$

(c) The input to system in part(b) is the same as the input in part(a) shifted by 1 to the right . If S is the time invariant then the system output obtained in part(b) has to the be the same as the system output obtained in part (a) shifted by 1 to the right . Clearly , this is not the case .Therefore ,the system is not LTI.
（d）If x[n]=u[n], then

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k]$$

$$= \sum_{k=-\infty}^{\infty} g[n-2k]$$

The signals g[n-2k] is plotted for k=0,1,2 in Figure S2.7. From this figure it is clear that

$$y[n] = \begin{cases} 1, n = 0,1 \\ 2, n > 1 \\ 0, otherwise \end{cases} = 2u[n] - \delta[n] - \delta[n-1]$$
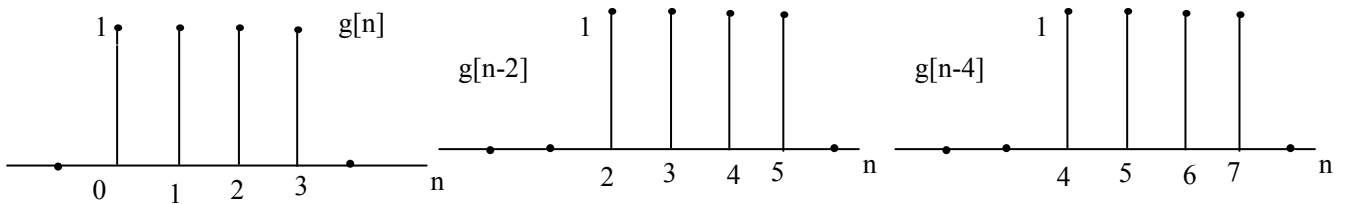

Figure S2.1

2.8 Using the convolution integral.

$$x(t)*h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

Given that $h(t) = \delta(t+2) + 2\delta(t+1)$,the above integral reduces to

$$x(t)*y(t) = x(t+2) + 2x(t+1)$$

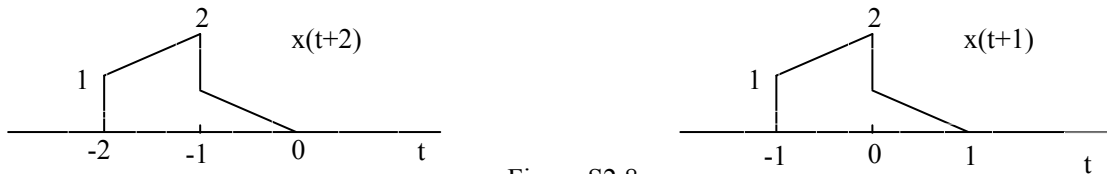This signal x(t+2) and 2x(t+1) are plotted in Figure S2.8.

Figure S2.8

Using these plots, we can easily show that

$$y(t) = \begin{cases} t+3, & -2 < t \le -1 \\ t+4, & -1 < t \le 0 \\ 2-2t, & 0 < t \le 1 \\ 0, & otherwise \end{cases}$$

2.9 Using the give definition for the signal h(t), we may write

$$h(\tau) = e^{2\tau}u(-\tau+4) + e^{-2\tau}u(\tau-5) = \begin{cases} e^{-2\tau}, & \tau > 5 \\ e^{2\tau}, & \tau < 4 \\ 0, & 4 < \tau < 5 \end{cases}$$

Therefore,

$$h(-\tau) = \begin{cases} e^{2\tau}, & \tau < -5 \\ e^{-2\tau}, & \tau > -4 \\ 0, & -5 < \tau < -4 \end{cases}$$

If we now shift the signal h(-τ) by t to the right, then the resultant signal h(t-τ) will be

$$h(t-\tau) = \begin{cases} e^{-2(t-\tau)}, & \tau < t-5 \\ e^{2(t-\tau)}, & \tau < t-4 \\ 0, & (t-5)\tau < (t-4) \end{cases}$$

Therefore

$$A = T - 5, \qquad B = t - 4.$$

2.10 From the given information, we may sketh x(t) and h(t) as show, in Figure S2.10.

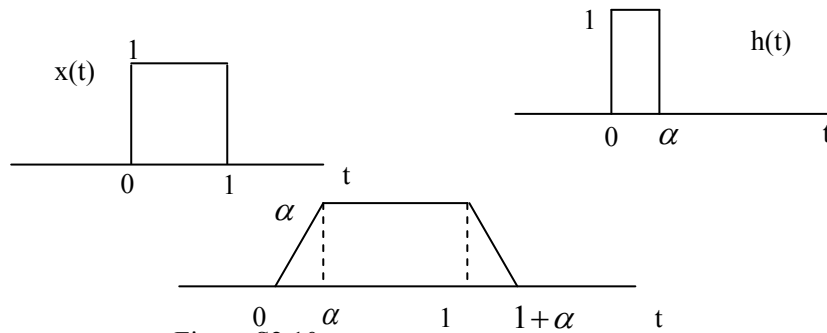(a) With the aid of the plots in Figure S2.10, we can show that $y(t) = x(t) * h(t)$ is as shown in Figure S2.10.

Figure S2.10

Therefore,

$$y(t) = \begin{cases} t, & 0 \le t \le \alpha \\ \alpha, & \alpha \le t \le 1 \\ 1+\alpha - t, & 1 \le t \le (1+\alpha) \\ 0, & otherwise \end{cases}$$

(b) From the plot of y(t), it is clear that $\dfrac{dy(t)}{dt}$ has discontinuities at $0, \alpha, 1$, and $1+\alpha$. If we want $\dfrac{dy(t)}{dt}$ to have only three discontinuities, then we need to ensure that $\alpha = 1$.

2.11(a)From the given information, we see that $h(t)$ is non zero only for $0 \le t \le \infty$ .Therefore,

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} e^{-3\tau}(u(t-\tau-3) - u(t-\tau-5))d\tau$$

We can easily show that $(u(t-\tau-3) - u(t-\tau-5))$ is non zero only in the range $(t-5) < \tau < (t-3)$.

Therefore, for $t \le 3$, the above integral evaluates to zero .For $3 < t \le 5$, the above integral is

$$y(t) = \int_{0}^{t-3} e^{-3\tau}d\tau = \frac{1-e^{-3(t-3)}}{3}$$

For $t > 5$, the integral is

$$y(t) = \int_{t-5}^{t-3} e^{-3\tau} d\tau = \frac{(1-e^{-6})e^{-3(t-5)}}{3} = \frac{1}{3}e^{-3t}\left(e^{15} - e^{9}\right)$$

Therefore, the result of this convolution may be expressed as

$$y(t) = \begin{cases} 0, & -\infty < t \le 3 \\ \dfrac{1-e^{-3(t-3)}}{3}, & 3 < t \le 5 \\ \dfrac{(1-e^{-6})e^{-3(t-3)}}{3}, & 5 < t \le \infty \end{cases}$$

(b) By differentiating x(t) with respect to time we get

$$\frac{dx(t)}{dt} = \delta(t-3) - \delta(t-5)$$

Therefore ,

$$g(t) = \frac{dx(t)}{d(t)} * h(t) = e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5)$$

(c) From the result of part (a),we m ay compute the derivative of y(t) to be

$$\frac{dy(t)}{dt} = \begin{cases} 0, & -\infty < t \le 3 \\ \dfrac{1-e^{-3(t-3)}}{3}, & 3 < t \le 5 \\ \dfrac{(1-e^{-6})e^{-3(t-3)}}{3}, & 5 < t \le \infty \end{cases}$$

This is exactly equal to g(t) .therefore , $g(t) = \dfrac{dy(t)}{dt}$

2.12. The result y(t) may be written as

$$y(t) = \cdots + e^{-(t+6)}u(t+6) + e^{-(t+3)}u(t+3) + e^{-t}u(t) + e^{-(t-3)}u(t-3) + e^{-(t-6)}u(t-6) + \cdots$$

In the range $0 \le t < 3$,we may write y(t) as

$$y(t) = \cdots + e^{-(t+6)}u(t+6) + e^{-(t+3)}u(t+3) + e^{-t}u(t)$$

$$= e^{-t} + e^{-(t+3)} + e^{-(t+6)} + \cdots$$

$$= e^{-t}(1 + e^{-3} + e^{-6} + \cdots)$$

$$= e^{-t}\frac{1}{1-e^{-3}}$$

Therefore , $\quad A = \dfrac{1}{1-e^{-3}}$.

2.13. (a) we require that

$$(\frac{1}{2})^n u[n] - A(\frac{1}{2})^{n-1} u[n-1] = \delta[n]$$

Putting n=1 and solving for A gives $\quad A = \dfrac{1}{5}$

(b) From part (a),we know that

$$h[n] - \frac{1}{5}h[n-1] = \delta[n]$$

$$h[n] * (\delta[n] - \frac{1}{5}\delta[n-1] = \delta[n]$$

21

From the definition of an inverse system ,we may argue that

$$g[n] = \delta[n] - \frac{1}{5}\delta[n-1]$$

2.14. (a) We first determine if $h_1(t)$ is absolutely integrable as follows

$$\int_{-\infty}^{\infty} |h_1(\tau)| \, d\tau = \int_0^{\infty} e^{-t} \, d\tau = 1$$

Therefore , is the impulse response of a stable LTI system .

(b) we determine if $h_2(t)$ is absolutely integrable as follows

$$\int_{-\infty}^{\infty} |h_2(\tau)| \, d\tau = \int_0^{\infty} e^{-t} |\cos(2t)| \, d\tau = 1$$

This integral is clearly finite-valued because $e^{-t}|\cos (2t)|$ is an exponentially decaying function in the range $-\infty < t \leq \infty$. Therefore , $h_2(t)$ is impulse response of a stable LTI system.

2.15. (a) We determine if $h_1[n]$ is absolutely summable as follows

$$\sum_{k=-\infty}^{\infty} |h_1[k]| = \sum_0^{\infty} k |\cos(\frac{\pi}{4}k)|$$

This sum does not have a finite value because the function $k|\cos(\frac{\pi}{4}k)|$ increase as the value of k

increase .Therefore , $h_1[n]$ cannot be the impulse response of a stable LTI system .

(b)We determine if $h_2[n]$ is absolutely summable as follows

$$x(-t) * h(-t)$$

Therefore , $h_2[n]$ is the response of a stable LTI system .

2.16. (a) **True.** This may be easily argued by noting that convolution may be viewed as the process of carrying out the superposition of a number of echos of $h[n]$. The first such echo will occur at the location of the first non zero samples $h[x]$. In this case, the first echo will occur at $n = N_1$ will have its first non zero sample at the time location $N_1 + N_2$. Therefore ,for all values of n which are lesser that $N_1 + N_2$,the output $y[n]$ is zero.

(b) **False** . Consider

$$y[n] = x[n] * h[n]$$
$$= \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

From this ,

$$y[n-1] = \sum_{k=-\infty}^{\infty} x[k]h[n-1-k]$$
$$= x[n] * h[n-1]$$

This shows that the given statement is false.

(c) True. Consider

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

From this ,

$$y(-t) = \int_{-\infty}^{\infty} x(\tau)h(-t-\tau)d\tau$$
$$= \int_{-\infty}^{\infty} x(-\tau)h(-t+\tau)d\tau$$
$$= x(-t) * h(-t)$$

This shows that the given statement is true.

(d) **True** .This may be argued by considering

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$

In figure S2.16 , we plot $x(\tau)$ and $h(t-\tau)$ under the assumptions that (1) $x(t) = 0$ for $t > T_1$ and (2) $h(t) = 0$ for $t > T_2$.Clearly ,the product

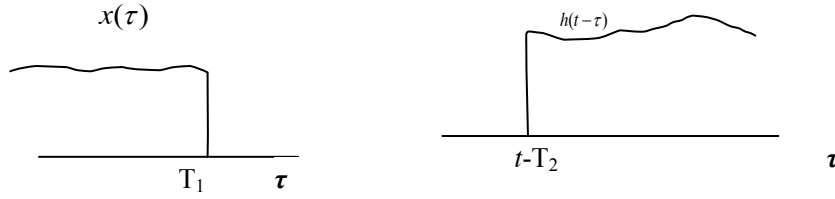$x(\tau)h(t-\tau)$ is zero if $t-T_2 > T_1$. Therefore, $u(t)=0$ for $t > T_1 + T_2$

$x(\tau)$

$h(t-\tau)$

$t\text{-}T_2$

τ

T_1 τ

Figure S2.16

2.17. (a) We know that $y(t)$ is the sum of the particular and homogeneous solutions to the given differential equation .We first determine the particular solution $y_p(t)$ by using the method specified in Example 2.14. Since we are given that the input is $x(t) = e^{(-1+3j)t}u(t)$,for t>0 ,we hypothesize that for t>0

$$y_p(t) = Ke^{(-1+3j)t}.$$

Substituting for $x(t)$ and $y(t)$ in the given differential equation,

$$(-1+3j)Ke^{(-1+3j)t} + 4Ke^{(-1+3j)t} = e^{(-1+3j)t}$$

This gives

$$(-1+3j)K + 4K = 1, \quad \Rightarrow \quad K = \frac{1}{3(1+j)}$$

Therefore ,

$$y_p(t) = \frac{1}{3(1+j)}e^{(-1+3j)t}, \quad t>0$$

In order to determine the homogeneous solution, we hypothesize that

$$y_h(t) = Ae^{st}$$

Since the homogeneous solution has to satisfy the following differential equation

$$\frac{dy_h(t)}{dt} + Ay_h(t) = 0,$$

we obtain

$$Ase^{st} + 4Ae^{st} = Ae^{st}(s+4) = 0,$$

This implies that s=-4 for any A .The overall solution to the differential equation now becomes

$$A + \frac{1}{3(1+j)} = 0, \quad A = \frac{-1}{3(1+j)}$$

Therefore for t > 0,

$$y(t) = \frac{1}{3(1+j)}[-e^{-4t} + e^{(-1+3j)t}], \quad t>0$$

Since the system satisfies the condition of initial rest , $y(t) = 0$ for t < 0. Therefore,

$$y(t) = \frac{1-j}{6}[-e^{-4t} + e]u(t)^{(-1+3j)t}$$

(b) The output will now be the real part of the answer obtained in part (a).

$$y(t) = \frac{1}{6}[e^{-t}\cos 3t + e^{-t}\sin 3t - e^{-4t}]u(t).$$

2.18. Since the system is causal, y[n] =0 for n < 1. Now,

$$y[1] = \frac{1}{4}y[0] + x[1] = 0 + 1 = 1$$

$$y[2] = \frac{1}{4}y[1] + x[2] = \frac{1}{4} + 0 = \frac{1}{4}$$

$$y[3] = \frac{1}{4}y[2] + x[3] = \frac{1}{16} + 0 = \frac{1}{16}$$

$$\vdots$$

$$y[m] = (\frac{1}{4})^{m-1}$$

$$\vdots$$

23

Therefore,

$$y[n] = (\frac{1}{4})^{n-1} u[n-1]$$

2.19. (a) Consider the difference equation relating $y[n]$ and $w[n]$ for s_2:

$$y[n] = \alpha y[n-] + \beta w[n]$$

From this we may write

$$w[n] = \frac{1}{\beta} y[n] + \frac{\alpha}{\beta} y[n-1]$$

and

$$w[n-1] = \frac{1}{\beta} y[n-1] + \frac{\alpha}{\beta} w[n-2]$$

Weighting the previous equation by 1/2 and subtracting from the one before ,we obtain

$$w[n] - \frac{1}{2} w[n-1] = \frac{1}{\beta} y[n] - \frac{\alpha}{\beta} y[n-1] - \frac{1}{2\beta} y[n-1] + \frac{\alpha}{\beta} y[n-2]$$

Substituting this in the difference equation relating $w[n]$ and $x[n]$ for S_1.

$$\frac{1}{\beta} y[n] - \frac{\alpha}{\beta} y[n-1] - \frac{1}{2\beta} y[n-1] + \frac{\alpha}{2\beta} y[n-2] = x[n]$$

Comparing with the given equation relating $y[n]$ and $x[n]$, we obtain

$$\alpha = \frac{1}{4}, \quad \beta = 1$$

(b) The difference equation relating the input and output of the system S_1 and S_2 are

$$w[n] = \frac{1}{2} w[n-1] + x[n] \quad \text{and} \quad y[n] = \frac{1}{4} y[n-1] + w[n]$$

From these, we can use the method specified in Example 2.15 to show that the impulse response of S_1 and S_2 are

$$h_1[n] = (\frac{1}{2})^n u[n]$$

and

$$h_2[n] = (\frac{1}{4})^n u[n] \cdot$$

Respectively. The overall impulse response of the system made up of a cascade of S_1 and S_2 will be

$$h[n] = h_1[n] * h_2[n] = \sum_{k=-\infty}^{\infty} h_1[k] h_2[n-k]$$

$$= \sum_{k=0}^{\infty} (\frac{1}{2})^k (\frac{1}{4})^{n-k} n[n-k]$$

$$= \sum_{k=0}^{n} (\frac{1}{2})^k (\frac{1}{4})^{n-k} n[n-k]$$

$$= \sum_{k=0}^{\infty} (\frac{1}{2})^{2(n-k)} = [2(\frac{1}{2})^n - (\frac{1}{4})^n] u[n]$$

2.20. (a)

$$\int_{-\infty}^{\infty} u_0(t) \cos(t) dt = \int_{-\infty}^{\infty} \delta(t) dt = 1$$

(b)

$$\int_{0}^{5} \sin(2\pi t) \delta(t+3) dt = \sin(6\pi) = 0$$

(c) In order to evaluate the integral

$$\int_{-5}^{5} u_1(1-\tau) \cos(2\pi\tau) d\tau,$$

consider the signal

$$x(t) = \cos(2\pi t)[u[(t+5) - u(t-5)]$$

We know that

24

$$\frac{dx(t)}{dt} = u_1(t) * x(t) = \int_{-\infty}^{\infty} u_1(t-\tau)x(\tau)d\tau$$

$$= \int_{-5}^{5} u_1(t-\tau)\cos(2\pi\tau)d\tau$$

Now

$$\frac{dy}{dx} \Big|_{t=1} = \int_{-5}^{5} u_1(1-\tau)\cos(2\pi\tau)d\tau$$

Which is the desired integral. We now evaluate the value of the integral as

$$\frac{dy}{dx} \Big|_{t=1} = \sin(2\pi t)\big|_{t=1} = 0$$

2.21 (a) the desired convolution is

$$y[n] = x[n]*h[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= \beta^n \sum_{k=0}^{n} (\alpha/\beta)^k \quad \text{for n>=0}$$

$$= \left[(\beta^{n+1} - \alpha^{n+1})/(\beta - \alpha)\right]u[n] \text{ for } \alpha \neq \beta$$

(b)   from (a)

$$y[n] = \alpha^n \left[\sum_{k=0}^{n} 1\right] u[n] = (n+1)\,\alpha^n\, u[n]$$

(c)   for n <=6

$$y[n] = 4^n \left\{ \sum_{k=0}^{\infty}(-1/8)^k - \sum_{k=0}^{3}(-1/8)^k \right\}$$

for n>6

$$y[n] = 4^n \left\{ \sum_{k=0}^{\infty}(-1/8)^k - \sum_{k=0}^{n-1}(-1/8)^k \right\}$$

therefore

$$y[n] = \begin{cases} (8/9)(-1/8)^4 4^n, & n <= 6 \\ (8/9)(-1/2)^n, & n > 6 \end{cases}$$

(d)   the desired convolution is

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= x[0]h[n] + x[1]h[n-1] + x[2]h[n-2] + x[3]h[n-3] + x[4]h[n-4]$$

$$= h[n] + h[n-1] + h[n-2] + h[n-3] + h[n-4]$$

This is shown in figure s2.21

Figure s2.21
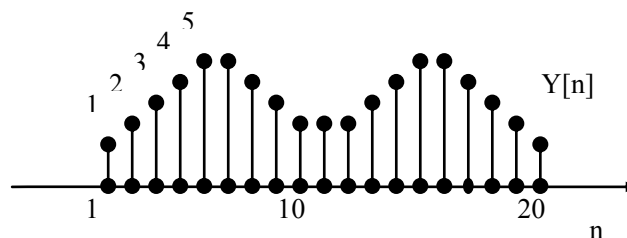
2.22 (a)the desire convolution is

$$y(t)= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$= \int_0^t e^{-\alpha t} e^{-\beta(t-\tau)} d\tau \quad \text{t>=0}$$

Then

$$y(t) = \begin{cases} [(e^{-\alpha t} - e^{-\beta t})/(\beta - \alpha)]u(t) \rightarrow \alpha \neq \beta \\ te^{-\alpha t}u(t) \rightarrow \alpha = \beta \end{cases}$$

(b) the desire convolution is

$$y(t)= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$= \int_0^2 h(t-\tau)d\tau - \int_2^5 h(t-\tau)d\tau$$

This may be written as
y(t)=

$$\begin{cases} \int_0^2 e^{2(t-\tau)}d\tau - \int_2^5 e^{2(t-\tau)}d\tau, t <= 1 \\ \int_{t-1}^2 e^{2(t-\tau)}d\tau - \int_2^5 e^{2(t-\tau)}d\tau, 1 <= t <= 3 \\ -\int_{t-1}^5 e^{2(t-\tau)}d\tau, 3 <= t <= 6 \\ 0, 6 < t \end{cases}$$

Therefore

$$y(t)= \begin{cases} (1/2)e^{2t}[1-2e^{-4}+e^{-10}], t<=1 \\ (1/2)e^{2t}[e^{2-2t}+e^{-10}-2e^{-4}], 1<=t<=3 \\ (1/2)e^{2t}[e^{-10}-e^{2-2t}], 3<=t<=6 \\ 0, 6<t \end{cases}$$

(c) the desire convolution is

$$y(t)= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_0^2 \sin(\pi\tau)h(t-\tau)d\tau,$$

This give us

$$y(t)= \begin{cases} 0, t<1 \\ (2/\pi)[1-\cos\{\pi(t-1)\}], 1<t<3 \\ (2/\pi)[\cos\{\pi(t-3)\}-1], 3<t<5 \\ 0, 5<t \end{cases}$$

(e)   let

$$h(t)= h_1(t)- \frac{1}{3}\delta(t-2)$$

where

$$h_1(t)= \begin{cases} 4/3, 0<=t<=1 \\ 0, otherwise \end{cases}$$

Now $Y(t)=h(t)*x(t)=[h(t)*x(t)]- \frac{1}{3}x(t-2)$

We have

$$h_1(t)* x(t)= \int_{t-1}^t \frac{4}{3}[(a\tau+b)]d\tau = \frac{4}{3}[\frac{1}{2}at^2 \frac{1}{2}a(t-1)^2 +bt-b(t-1)]$$

Therefore

$$y(t)= \frac{4}{3}[\frac{1}{2}at^2 \frac{1}{2}a(t-1)^2 +bt-b(t-1)]- \frac{1}{3}[a(t-2)+b]= at+b = x(t)$$

(f)   x(t) periodic implies y(t) periodic ∴ determine 1 period only . we have

$$y(t)= \begin{cases} \int_{t-1}^{-\frac{1}{2}}(t-\tau-1)d\tau +\int_{-\frac{1}{2}}^t (1-t+\tau)d\tau = \frac{1}{4}+t-t^2, -\frac{1}{2}<t<\frac{1}{2} \\ \int_{t-1}^{\frac{1}{2}}(1-t+\tau)d\tau +\int_{\frac{1}{2}}^t (t-1-\tau)d\tau = t^2-3t+7/4, \frac{1}{2}<t<\frac{3}{2} \end{cases}$$
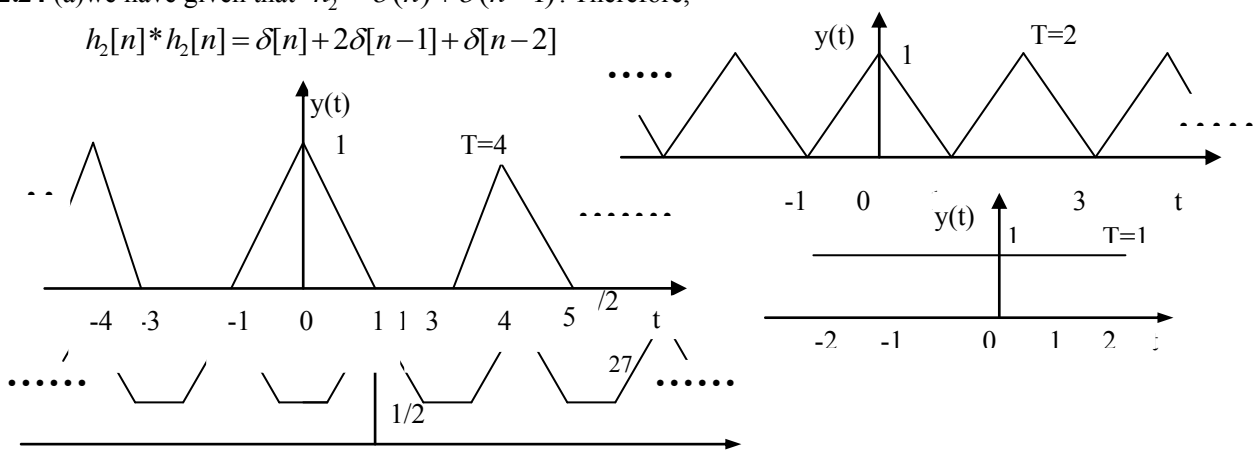
The   periodic of y(t)   is 2.

2.23 Y(t) is sketched   in figure s2.23 for the   different values   of T.
Therefore

2.24 (a)we have given that $h_2 = \delta(n)+\delta(n-1)$. Therefore,

$$h_2[n]*h_2[n]=\delta[n]+2\delta[n-1]+\delta[n-2]$$

since

$$h[n]= h_1[n]*[h_2[n]*h_2[n]]$$

we get

$$h[n]= h_1[n]+2h[n-1]+ h_1[n-2]$$

Therefore

$$h[0]= h_1[0] \Rightarrow h_1[0]=1$$
$$h[1]= h_1[1]+ 2h_1[0] \Rightarrow h_1[1]=3$$
$$h[2]= h_1[2]+ 2h_1[1]+ h_1[0] \Rightarrow h_1[2]=3$$
$$h[3]= h_1[3]+ 2h_1[2]+h_1[1] \Rightarrow h_1[3]=2$$
$$h[4]= h_1[4]+ 2h_1[3]+h_1[2] \Rightarrow h_1[4]=1$$
$$h[5]= h_1[5]+2h_1[4]+h_1[3] \Rightarrow h_1[5]=0.$$

$h_1[n]=0$ for n<0 and n>=5.

(b) in this case

$$Y[n]=x[n]*h[n]=h[n]-h[n-1].$$

2.25 (a)we may write   x[n] as

$$x[n]= (\frac{1}{3})^{|n|}$$

Now   the desire convolution is

$$y[n]= h[n]*x[n]$$
$$= \sum_{k=-\infty}^{-1} (1/3)^{-k}(1/4)^{n-k}u[n-k+3]+\sum_{k=0}^{\infty}(1/3)^{k}(1/4)^{n-k}u[n-k+3]$$

By consider each summation in the above equation separately .we may show that

$$y[n]= \begin{cases} (12^4/11)3^n, n<-4 \\ (1/11)4^4, n=-4 \\ (1/4)^n(1/11)-3(1/4)^n 3(256)(1/3)^n, n>=3 \end{cases}$$

(b) now consider the convolution

$$y_1[n]= [(1 ^n i3 n) \quad [\quad]]*u(n+/4$$

We may show   that

$$y_1[n]= \begin{cases} 0, n<-3 \\ -3(1/4)^n+3(256)(1/3)^n, n\ge-3 \end{cases}$$

Also consider the convolution

$$y_2[n]=[(3)^n u[-n-1]]*[(1/4)^n u[n+3]].$$

We   may show   that

$$y_2[n]= \begin{cases} (12^4/11)3^n, n<-4 \\ (1/4)^n(1/11), n\ge-3 \end{cases}$$

Clearly , $y_1[n]+y_2[n]=$ y[n] obtained in the previous part .

2.26 (a) we have

$$y_1[n]=x_1[n]*x_2[n]=\sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k]$$
$$=\sum_{k=0}^{\infty}(0.5)^k u[n+3-k]$$

This evaluates to

$$y_1[n]=x_1[n]*x_2[n]=\begin{cases}2[1-(1/2)^{n+4}],n\ge-3\\0,otherwise\end{cases}$$

(b) now

$$y[n]=x_3[n]*y_1[n]=y_1[n]-y_1[n-1]$$

Therefore

$$y[n]=\begin{cases}2(1-(1/2)^{n+3})+2(1-(1/2)^{n+4})=(1/2)^{n+3},n\ge-2\\1,n=-3\\0,otherwise\end{cases}$$

Therefore, $y[n]=(1/2)^{n+3}u[n+3]$.

(c) we have

$$y_2[n]=x_2[n]*x_3[n]=u[n+3]-u[n+2]=\delta[n+3]$$

(d) from the result of part (c), we get

$$Y[n]=y_2[n]*x_1[n]=x_1[n+3]=(1/2)^{n+3}u[n+3]$$

2.27 the proof is as follows

$$Ay=\int_{-\infty}^{\infty}y(t)d(t)$$

$$=\int_{\infty}^{\infty}\int_{\infty}^{\infty}x(\tau)h(t-\tau)d\tau dt$$

$$=\int_{-\infty}^{\infty}x(\tau)\int_{-\infty}^{\infty}h(t-\tau)dtd\tau$$

$$=\int_{-\infty}^{\infty}x(\tau)A_hd(\tau)$$

$$=A_zA_h$$

2.28 (a) causal because h[n]=0 for n<0 stable because $\sum_{n=0}^{\infty}(\frac{1}{5})^n=\frac{5}{4}<\infty$

(b)not causal because h[n] $\ne$ 0 for n<0 stable because $\sum_{n=-2}^{\infty}(0.8)^n=5<\infty$

(c)anti- causal because h[n]=0 for n>0 unstable because $\sum_{n=-\infty}^{0}(1/2)^n=\infty$

(d) not causal because h[n] $\ne$ 0 for n<0 stable because $\sum_{n=-\infty}^{3}(5)^n=\frac{625}{4}<\infty$

(e) causal because h[n]=0 for n<0 unstable because the second term becomes infinite as n$\to\infty$.

(f) not causal because h[n] $\ne$ 0 for n<0 stable because $\sum_{n=-\infty}^{\infty}|h[n]|=\frac{305}{3}<\infty$

（g）causal because h[n]=0 for n<0. stable because $\sum_{n=\infty}^{\infty}|h[n]|=1<\infty$

2.29. (a) causal because h(t)=0for t<0. stable because $\int_{\infty}^{\infty}|h(t)|dt=e^{-8}/4<\infty$.

(b) Not causal because h(t)$\ne$0 for t<0. Unstable because $\int_{\infty}^{\infty}|h(t)|=\infty$.

(c) Not causal because h(t)$\ne$0 for t<0. a Stable because $\int_{\infty}^{\infty}|h(t)|dt=e^{100/2}<\infty$

(d) Not causal because h(t)$\ne$0 for t<0. stable because $\int_{\infty}^{\infty}|h(t)|dt=e^{-2}/2<\infty$

(e) Not causal because h(t)$\ne$0 for t<0. stable because $\int_{\infty}^{\infty}|h(t)|dt=1/3<\infty$

(f) Causal because h(t)=0 for t<0. Stable because $\int_{\infty}^{\infty}|h(t)|dt=1<\infty$

(g) Causal because h(t)=0 for t<0. Unstable because $\int_{\infty}^{\infty}|h(t)|dt=\infty$

2.30. We need to find the output of the system when the input is x[n]=δ[n].Since we are asked to assume initial

rest ,we may conclude that y[n]=0. for n<0.now,

$$y[n]=x[n]-2y[n-1],$$

Therefore,

$$y[0]=x[0]-2y[-1]=1, \quad y[1]=x[1]-2\ y[0]=-2, \quad y[2]=x[2]+2y[2]=-4$$

and so on. In closed form,

$$y[n]=(-2)^n u(n).$$

this is the impulse response of the system.

2.31. Initial rest implies that y[n]=0 for n< -2. Now

$$y[n]=x[n]+2x[n-2]-2y[n-1].$$

Therefore,

$$y[-2]=1, y[-1]=0, y[0]=5, y[4]=56, y[0]=-110 \text{ for } n \geq 5.$$

2.32. (a) If ,then we need to verify

$$A\left(\frac{1}{2}\right)^n - \frac{1}{2}A\left(\frac{1}{2}\right)^{n-1} = 0$$

Clearly this is true.

(b) We now require that for n≥0

$$B\left(\frac{1}{3}\right)^n - \frac{1}{2}B\left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^n$$

Therefore, B= -2.

(c) From eq.(P2.32-1), we know that y[0]+(1/2)y[-1]=x[0]=1. Now we also have

$$y[0]= A+B => A=1-B=3.$$

2.33. (a) (i) From Example 2.14,we have know that

$$y_1(t) = \left[\frac{1}{5}e^{3t} - \frac{1}{5}e^{-2t}\right]u(t)$$

(ii) We solve this along the lines of Example 2.14. First assume that $y_p(t)$ is of the

Form Ke^{2t} For t>0. Then using eq.(P2.33-1).we get for t>0

$$2\,Ke^{2t} + 2\,Ke^{2t} = e^{2t} \quad => k = \frac{1}{4}$$

We know that $y_p(t) = \frac{1}{4}e^{2t}$ for t>0. We may hypothesize the homogeneous solution to be of the

form

$$y_h(t) = Ae^{-2t}$$

Therefore,

$$y_2(t) = Ae^{-2t} + \frac{1}{4}e^{2t}, \quad \text{for t>0}$$

Assuming initial rest. we can conclude that =0 for t≤0. Therefore,

$$y_2(t) = 0 = A + \frac{1}{4} \quad => A = \frac{1}{4}$$

Then,

$$y_2(t) = \left[-\frac{1}{4}e^{2t} - \frac{1}{4}e^{-2t}\right]u(t)$$

(iii) Let the input be $x_3(t) = \alpha e^{3t}u(t) + \beta e^{2t}u(t)$ .Assume that the particular solution $y_p(t)$ is of

the form

$$y_p(t) = K_1\,\alpha\,e^{3t}u(t) + K_2\,\beta\,e^{2t}u(t)$$

For t>0. Using eq.(P2.33-1), we get

$$3\,K_1\,\alpha\,e^{3t}u(t) + 2\,K_2\,\beta\,e^{2t}u(t) + 2\,K_1\,\alpha\,e^{3t}u(t) + 2\,K_2\,\beta\,e^{2t}u(t) = \alpha^{3t} + \beta\,e^{2t}.$$

Equating the coefficients of e^{3t} and e^{2t}. On both sides, we get

$$K_1 = 1/5 \quad \text{and} \quad K_2 = 1/4$$

Now hypothesizing that $y_p(t) = \frac{1}{4}e^{-2t}$, we get

$$y_3(t) = \frac{1}{5}\,\alpha\,e^{3t}u(t) + \frac{1}{4}\,\beta\,e^{2t}u(t) + A\,e^{-2t}$$

For t>0. Assuming initial rest,
$$y_0(t) = 0 = A + \alpha/5 + \beta/4 \quad \Rightarrow A = -(\alpha/5 + \beta/4)$$

Therefore,
$$y_3(t) = \{\frac{1}{5}\,\alpha\,e^{3t} + \frac{1}{4}\,\beta\,e^{2t} + A\,e^{-2t}\}\,u(t)$$

Clearly, $y_3(t) = \alpha\,y_1(t) + \beta\,y_2(t)$.

(iv) for the input-output pair $x_1(t)$ and $y_1(t)$, we may use eq.(P2.33-1) and the initial rest Condition to write

$$\frac{dy_1(t)}{dt} + 2y_1(t) = x_1(t), \qquad y_1(t) = 0 \ \text{ for t<}t_1 \quad (S2.33\text{-}1)$$

For the input-output pair $x_2(t)$ and $y_2(t)$, we may use eq.(P2.33-1) and the initial rest condition to write

$$\frac{dy_2(t)}{dt} + 2y_2(t) = x_2(t), \qquad y_2(t) = 0 \ \text{ for t<}t_2 \quad (S2.33\text{-}2)$$

Scaling eq.(S2.33-1) by α and eq.(S2.33-2) by β and summing, we get

$$\frac{d}{dt}\{\alpha y_1(t) + \beta y_2(t)\} + 2\{\alpha y_1(t) + \beta y_2(t)\} = \alpha x_1(t) + \beta x_2(t),$$

And
$$y_1(t) + y_2(t) = 0 \qquad \text{for} \quad \text{t<min}(t_1, t_2)$$

By inspection, it is clear that the output is $y_3(t) = \alpha\,y_1(t) + \beta\,y_2(t)$ when the input is $x_3(t) = \alpha\,x_1(t) + \beta\,x_2(t)$. Furthermore, $y_3(t) = 0$ for t< t_3, where t_3 denotes the time until which $x_3(t)=0$.

(b) (i) Using the result of (a-ii), we may write
$$y_1(t) = \frac{K}{4}\left[e^{2t} - e^{-2t}\right]u(t)$$

(ii) We solve this along the lines of Example 2.14. First assume that $y_p(t)$ is of the form $KY\,e^{2(t-T)}$ for t >T. Then using eq.(P2.33-1), we get for t>T
$$2K\,e^{2(t-T)} + 2K\,e^{2(t-T)} = e^{2t} \quad \Rightarrow \quad K = 1/4.$$

We know that $y_p(t) = \frac{K}{4}\,e^{2(t-T)}$ for t<T. We may hypothesize the homogeneous solution to be of the form
$$y_h(t) = A\,e^{-2t}.$$

Therefore,
$$y_2(t) = A\,e^{-2t} + \frac{K}{4}\,e^{2(t-T)} \qquad \text{for t>T.}$$

Assuming initial rest, we can conclude the $y_2(t)=0$ for t≤T, Therefore,
$$y_2(T) = 0 = A\,e^{-2T} + \frac{K}{4} \quad \Rightarrow \quad A = -\frac{K}{4}\,e^{-2T}$$

Then,
$$y_2(t) = \left[-\frac{K}{4}\,e^{-2(t-T)} + \frac{K}{4}\,e^{2(t-T)}\right]u(t-T)$$

Clearly, $y_2(t) = y_1(t-T)$.

(iii) consider the input-output pair $x_2(t) \to y_1(t)$ where $x_1(t)=0$ for t<t_0. Note that
$$\frac{dy_1(t)}{dt} + 2y_1(t) = x_1(t), \qquad y_1(t) = 0 \ \text{ for t<}t_0$$

Since the derivative is a time-invariant operation, we may now write

31

$$\frac{dy_1(t-T)}{dt} + 2y_1(t-T) = x_1(t-T), \qquad y_1(t) = 0 \text{ for } t<t_0$$

This suggests that if the input is a signal of the form $x_2(t) = x_1(t-T)$, then the output is a signal of the form $y_2 = y_1(t-T)$. Also, note that new output $y_2(t)$ will be zero for $t<t_0+T$. This supports time-invariance since $x_2(t)$ is zero for $t<t_0+T$. Therefore, we may conclude that the system is time-invariant.

2.34. (a) Consider $x_1(t)\xrightarrow{S}y_1(t)$ and $x_2(t)\xrightarrow{S}y_2(t)$. We know that $y_1(t) = y_2(t) =1$. now consider a third input to the system which is $x_3(t) = x_1(t) + x_2(t)$. Let the corresponding output be $y_3(t)$. Now , note that $y_3(1) = 1 \neq y_1(1) + y_2(1)$. Therefore, the system is not linear. A specific example follows.

Consider an input signal $x_1(t) = e^{2t}u(t)$ .Form Problem 2.33(a-ii), we know that the corresponding output for t>0 is

$$y_1(t) = \frac{1}{4}e^{2t} + Ae^{-2t}$$

Using the fact that $y_1(t) =1$, we get for t>0

$$y_1(t) = \frac{1}{4}e^{2t} + (1-\frac{e}{4})e^{-2(t-1)}$$

Now. Consider a second signal $x_2(t) =0$. Then, the corresponding output is

$$y_2(t) = Be^{-2t}$$

For t>0. Using the fact that $y_2(1) = 1$,we get for t>0

$$y_2(t) = e^{-2(t-1)}$$

Now consider a third signal $x_3(t) = x_1(t) + x_2(t) = x_1(t)$ .Note that the output will still be $y_3(t) = y_1(t)$ for t>0. Clearly, $y_3(t) \neq y_1(t) + y_2(t)$ for t>0. Therefore, the system is not linear.

(b) Again consider an input signal $x_2(t) = e^{2t}u(t)$. From part (a), we know that the corresponding output for t>0 with $y_1(1) = 1$ is

$$y_1(t) = \frac{1}{4}e^{2t} + (1-\frac{e}{4})e^{-2(t-1)}$$

Now, consider an input signal of the form $x_2(t) = x_1(t-T) = e^{2(t-T)}u(t-T)$. Then for t>T,

$$y_2(t) = \frac{1}{4}e^{2(t-T)} + Ae^{-2t}$$

Using the fact that $y_2(1) = 1$ and also assuming that T<1, we get for t<T

$$y_2(t) = \frac{1}{4}e^{2(t-T)} + (1-\frac{1}{4}e^{2(1-T)})e^{-2(t-1)}$$

Now note that for t>T. Therefore, the system is not time invariant.

(c) In order to show that the system is incrementally linear with the auxiliary condition specified as $y_1(1) = 1$, we need to first show that the system is linear with the auxiliary Condition specified as $y_1(1) = 0$.

For an input-output pair $x_1(t)$ and $y_1(t)$, we may use eq.(P2.33-1) and the fact that $y_1(1) = 0$ to write

$$\frac{dy_1(t)}{dt} + 2y_1(t) = x_1(t), \qquad y_1(1) = 0 \qquad (S2.34\text{-}1)$$

For an input-output pair $x_2(t)$ and $y_2(t)$, we may use eq.(P2.33-1) and the initial rest condition to write

$$\frac{dy_2(t)}{dt} + 2y_2(t) = x_2(t), \qquad y_2(1) = 0$$

Scaling eq.(S2.34-1) by α and eq.(S2.34-2) by β and summing, we get

$$\frac{d}{dt}\{\alpha y_1(t)+\beta y_2(t)\}+2\{\alpha y_1(t)+\beta y_2(t)\}=\alpha x_1(t)+\beta x_2(t)$$

And

$$y_3(1)=y_1(1)+y_2(1)=0$$

By inspection, it is clear that the output $y_3(t)=\alpha y_1(t)+\beta y_2(t)$ is when the input is $x_3(t)=\alpha x_1(t)+\beta x_2(t)$. Furthermore; $y_3(1)=0=y_1(1)+y_2(1)$ therefore, the system is linear.

Therefore, the overall system may be treated as the cascade of a linear system with an adder which adds the response of the system to the auxiliary condition alone.

(d) In the previous part, we show that the system is linear when $y(1)=0$. In order to show that the system is not time-invariant, consider an input of the form $x_1(t)=e^{2t}u(t)$.

From part(a), we know that the corresponding output will be

$$y_1(t)=\frac{1}{4}e^{2t}+Ae^{-2t}$$

Using the fact that $y_1(1)=0$, we get for t>0

$$y_1(t)=\frac{1}{4}e^{2t}-\frac{1}{4}e^{-2(t-2)}$$

Now consider an input of the form $x_2(t)=x_1(t-1/2)$. Note that $y_2(1)=0$. Clearly,

$$y_2(t)\neq y_1(1-1/2)=(1/4)(e-e^3).$$ Therefore, $y_2(t)\neq y_1(t-1/2)$ for all t. This implies that the system is not time invariant.

(e) A proof which is very similar to the proof for linear used in part(c) may be used here. We may show that the system is not time invariant by using the method outlined in part (d).

2.35 (a) since the system is linear, the respond $y_1(1)=0$ for all t.

(b) Now let us find the output $y_2(t)$ when the input is $x_2(t)$. The particular solution is of the form

$$y_p(t)=Y, \qquad t>-1$$

Substituting in eq.(P2.33-1),we get

$$2Y=1.$$

Now, including the homogeneous solution which is of the form $y_h(t)=Ae^{-2t}$.we get the overall solution

$$y_2(t)=Ae^{-2t}+\frac{1}{2}, \qquad t>-1$$

Since $y(0)=0$, we get

$$y_2(t)=-\frac{1}{2}e^{-2t}+\frac{1}{2}, \qquad t>-1$$

For t< -1, we note that $x_2(t)=0$. Thus the particular solution is zero in this range and

$$y_2(t)=Be^{-2t}, \qquad t>-1$$

Since the tow pieces of the solution for $y_2(t)$ in aqs.(S2.35-1)and (S2.35-2)must match at t= -1, we can determine B from the equation

$$\frac{1}{2}-\frac{1}{2}e^2=Be^2$$

Which yields

$$y_2(t)=\left(\frac{1}{2}-\frac{1}{2}e^2\right)e^{-2(t+1)}, \qquad t<-1$$

Now note that since $x_1(t)=x_2(t)$ for t<-1,it must be true that for a causal system $y_1(t)=y_2(t)$ for t<-1.However the results of parts(a) and (b) show that this is not true. Therefore ,the system is not causal.

2.36. Consider an input $x_1[n]$ such that $x_1[n]=0$ for $n<n_1$.The corresponding output will be

$$y_1[n]=\frac{1}{2}\ y_1[n-1]+x_1[n], \qquad y_1[n]=0\ \text{for}\ n<n_1. \qquad \text{(S2.36-1)}$$

Also, consider another output $x_2[n]$ such that $x_2[n] = 0$ for $n < n_2$ The corresponding output will be

Chapter 3 Answers

3.1 Using the Fourier series synthesis eq. (3.38)

$$x(t) = a_1 e^{j(2\pi/T)t} + a_{-1} e^{-j(2\pi/T)t} + a_3 e^{j3(2\pi/T)t} + a_{-3} e^{-j3(2\pi/T)t}$$

$$= 2e^{j(2\pi/8)t} + 2e^{-j(2\pi/8)t} + 4e^{j3(2\pi/8)t} - 4e^{-j3(2\pi/8)t}$$

$$= 4\cos(\frac{\pi}{4}t) - 8\sin(\frac{6\pi}{8}t)$$

$$= 4\cos(\frac{\pi}{4}t) + 8\cos(\frac{3\pi}{4}t + \frac{\pi}{2})$$

3.2 Using the Fourier series synthesis eq. (3.95)

$$x[n] = a_0 + a_2 e^{j2(2\pi/N)n} + a_{-2} e^{-j2(2\pi/N)n} + a_4 e^{j4(2\pi/N)n} + a_{-4} e^{-j4(2\pi/N)n}$$

$$= 1 + e^{j(\pi/4)} e^{j2(2\pi/5)n} + e^{-j(\pi/4)} e^{-j2(2\pi/5)n}$$

$$= 1 + 2\cos(\frac{4\pi}{5}n + \frac{\pi}{4}) + 4\cos(\frac{8\pi}{5}n + \frac{\pi}{3})$$

$$= 1 + 2\sin(\frac{4\pi}{5}n + \frac{3\pi}{4}) + 4\sin(\frac{8\pi}{5}n + \frac{5\pi}{6})$$

3.3 The given signal is

$$x(t) = 2 + \frac{1}{2}e^{j(2\pi/3)t} + \frac{1}{2}e^{-j(2\pi/3)t} - 2je^{j(5\pi/3)t} + 2je^{-j(5\pi/3)t}$$

$$= 2 + \frac{1}{2}e^{j2(2\pi/6)t} + \frac{1}{2}e^{-j2(2\pi/6)t} - 2je^{j5(2\pi/6)t} + 2je^{-j5(2\pi/6)t}$$

Form this we may conclude that the fundamental frequency of x(t) is $2\pi/6 = \pi/3$. The non-zero Fourier series coefficients of x(t) are

$$a_0 = 2 \quad , \quad a_2 = a_{-2} = \frac{1}{2} \quad , \quad a_5 = a^*_{-5} = -2j$$

3.4 Since $\omega_0 = \pi$, $T = 2\pi/\omega_0 = 2$, Therefore,

$$a_k = \frac{1}{2}\int_0^2 x(t)e^{-jk\pi t}dt$$

Now ,

$$a_0 = \frac{1}{2}\int_0^1 1.5dt - \frac{1}{2}\int_1^2 1.5dt = 0$$

and for $k \neq 0$

$$a_k = \frac{1}{2}\int_0^1 1.5e^{-jk\pi t}dt - \frac{1}{2}\int_1^2 1.5e^{-jk\pi t}dt$$

$$= \frac{3}{2k\pi j}[1 - e^{-jk\pi}]$$

$$= \frac{3}{k\pi}e^{-jk(\pi/2)}\sin(\frac{k\pi}{2})$$

3.5 Both $x_1(1-t)$ and $x_1(t-1)$ are periodic with fundamental period $T_1 = \frac{2\pi}{\omega_1}$, Since y(t) is a linear combination of

$x_1(1-t)$ and $x_1(t-1)$,it is also periodic with fundamental period $T_2 = \frac{2\pi}{\omega_1}$, Therefore $\omega_2 = \omega_1$.

Since $x_1(t) \xleftrightarrow{FS} a_k$ .using the results in Table 3.1 we have

$$x_1(t+1) \xleftrightarrow{FS} a_k e^{jk(2\pi/T_1)}$$

$$x_1(t-1)\xleftrightarrow{FS}a_ke^{-jk(2\pi/T_1)}\Rightarrow x_1(-t+1)\xleftrightarrow{FS}a_{-k}e^{-jk(2\pi/T_1)}$$

Therefore

$$x_1(t-1)+x_1(1-t)\xleftrightarrow{FS}a_ke^{jk(2\pi/T_1)}+a_{-k}e^{-jk(2\pi/T_1)}=e^{-j\omega_1 k}(a_k+a_{-k})$$

3.6 (a) Comparing $x_1(t)$ with the Fourier series synthesis eq. (3.38) , we obtain the Fourier series coefficients $x_1(t)$ to be

$$a_k=\begin{cases}(\dfrac{1}{2})^k & 0\le k\le 100 \\ \\ 0,otherwise \\ \\ \qquad\qquad x_1(t) \end{cases}$$

Form Table 3.1 we know that if $x_1(t)$ is real ,then a_k has to be conjugate-symmetric,
i.e, $a_k=a^*{}_{-k}$ Since this is not true for , the signal is not real valued .

Similarly , the Fourier series coefficients of $x_2(t)$ are

$$a_k=\begin{cases}\cos(k\pi),100\le k\le 100 \\ \\ 0,otherwise \end{cases}$$

Form Table 3.1 we know that if $x_2(t)$ is real ,then a_k has to be conjugate-symmetric,
i.e, $a_k=a^*{}_{-k}$ Since this is not true for $x_2(t)$, the signal is real valued .

Similarly , the Fourier series coefficients of $x_3(t)$ are

$$a_k=\begin{cases}j\sin(k\pi/2),100\le k\le 100 \\ \\ 0,otherwise \end{cases}$$

Form Table 3.1 we know that if $x_3(t)$ is real ,then a_k has to be conjugate-symmetric,

i.e, $a_k=a^*{}_{-k}$ Since this is not true for

$$x_3(t)$$, the signal is real valued .

(b) For a signal to be even , its Fourier series coefficients must be even . This is true only for $x_2(t)$.

3.7 Given that

$$x(t)\xleftrightarrow{FS}a_k$$

we have

$$g(t)=\frac{dx(t)}{dt}\xleftrightarrow{FS}b_k=jk\frac{2\pi}{T}a_k$$

Therefore ,

$$a_k=\frac{b_k}{j(2\pi/T)k}\qquad k\ne 0$$

When $k=0$

$$a_k = \frac{1}{T}\int_{<T>} x(t)dt = \frac{2}{T} \quad \text{using given information}$$

Therefore ,

$$a_k = \begin{cases} \frac{2}{T}, k=0 \\ \\ \frac{b_k}{j(2\pi/T)k}, k \neq 0 \end{cases}$$

3.8 Since x(t) is real and odd(clue 1), its Fourier series coefficients a_k are purely imaginary and odd (See Table 3.1) Therefore , $a_k = -a_{-k}$ and $a_0 = 0$,Also since it is given that $a_k = 0$ for $|k| > 1$, the only unknown Fourier series coefficients are a_1 and a_{-1}.Using Parseval's relation

$$\frac{1}{T}\int_{<T>} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

for the given signal we have

$$\frac{1}{2}\int_0^2 |x(t)|^2 dt = \sum_{k=-1}^{1} |a_k|^2$$

Using the information given in clue (4) along with the above equation ,

$$|a_1|^2 + |a_{-1}|^2 = 1 \quad \Rightarrow \quad 2|a_1|^2 = 1$$

Therefore

$$a_1 = -a_{-1} = \frac{1}{\sqrt{2}j} \quad \text{or} \quad a_1 = -a_{-1} = -\frac{1}{\sqrt{2}j}$$

The two possible signals which satisfy the given information are

$$x_1(t) = \frac{1}{\sqrt{2}j}e^{j(2\pi/2)t} - \frac{1}{\sqrt{2}j}e^{-j(2\pi/2)t} = -\sqrt{2}\sin(\pi t)$$

and

$$x_2(t) = -\frac{1}{\sqrt{2}j}e^{j(2\pi/2)t} + \frac{1}{\sqrt{2}j}e^{-j(2\pi/2)t} = \sqrt{2}\sin(\pi t)$$

3.9 The period of the given signal is 4 .Therefore ,

$$a_k = \frac{1}{4}\sum_{n=0}^{3} x[n]e^{-j\frac{2\pi}{4}kn} = \frac{1}{4}\left[4 + 8e^{-j\frac{\pi}{2}k}\right]$$

This gives

$$a_0 = 3, \qquad a_1 = 1 - 2j \qquad a_2 = -1, \qquad a_3 = 1 + 2j$$

3.10. Since the Fourier series coefficients repeat every N , we have

$$a_1 = a_{15} \ , \quad a_2 = a_{16} \quad \text{and} \quad a_3 = a_{17}$$

Furthermore ,since the signal is real and odd ,the Fourier series coefficients a_k will be purely imaginary and odd . Therefore , $a_0 = 0$ and

$$a_1 = -a_{-1} \qquad a_2 = -a_{-2} \qquad a_3 = -a_{-3}$$

Finally

$$a_{-1} = -j \qquad a_{-2} = -2j \qquad a_{-3} = -3j$$

3.11 Since the Fourier series coefficients repeat every N=10,we have $a_1 = a_{11} = 5$ Furthermore ,since x[n] is real and even , a_k is also real and even .Therefore $a_1 = a_{-1} = 5$ We are also given that

$$\frac{1}{10}\sum_{n=0}^{9} |x[n]|^2 = 50$$

Using Parseval's relation ,

$$\sum_{k=<N>} |a_k|^2 = 50$$

$$\sum_{k=-1}^{8} |a_k|^2 = 50$$

$$|a_{-1}|^2 + |a_1|^2 + a_0^2 + \sum_{k=2}^{8} |a_k|^2 = 50$$

$$a_0^2 + \sum_{k=2}^{8} |a_k|^2 = 0$$

Therefore $a_k = 0$ for k=2 ,....8,   Now   using   the synthesis eq. (3.94) , we have

$$x[n] = \sum_{k=<N>} a_k e^{j\frac{2\pi}{N}kn} = \sum_{k=-1}^{8} a_k e^{j\frac{2\pi}{10}kn}$$

$$= 5e^{j\frac{2\pi}{10}n} + 5e^{-j\frac{2\pi}{10}n} = 10\cos(\frac{\pi}{5}n)$$

3.12. Using   the multiplication   property   (see Table 3.2)   , we   have

$$x_1[n]x_2[n] \xleftrightarrow{FS} \sum_{l=<N>} a_l b_{k-l} = \sum_{k=0}^{3} a_l b_{k-l}$$

$$\xleftrightarrow{FS} \quad a_0 b_k + a_1 b_{k-1} + a_2 b_{k-2} + a_3 b_{k-3}$$

$$\xleftrightarrow{FS} b_k + 2b_{k-1} + 2b_{k-2} + 2b_{k-3}$$

Since b_k is 1 for all values of k, it is clear that $b_k + 2b_{k-1} + 3b_{k-3}$ will be   for all values of k, Therefore,

$$x_1[n]x_2[n] \xleftrightarrow{FS} 6, \quad \text{for all k,}$$

3.13 Let us first evaluate the Fourier series coefficients of $x(t)$.Clearly ,since $x(t)$ is eal and odd, a_k is purely imaginary and odd Therefore, a_0 =0. Now,

$$a_k = \frac{1}{8}\int_0^8 x(t)e^{-j(2\pi/8)kt} dt$$

$$= \frac{1}{8}\int_0^4 e^{-j(2\pi/8)kt} dt - \frac{1}{8}\int_0^4 e^{-j(2\pi/8)kt} dt$$

$$= \frac{1}{j\pi k}\left[1 - e^{-j\pi k}\right]$$

Clearly, the above expression evaluates to zero or all even values of k Therefore.

$$a_k = \begin{cases} 0, k = 0, \pm 2, \pm 4 \cdots \\ \frac{2}{j\pi k}, k = \pm 1, \pm 3, \pm 5 \cdots \end{cases}$$

When $x(t)$is passed through an LTI system with frequency response $H(j\omega)$, the output $y(t)$is given by (see Section 3.8)

$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0)e^{jk\omega_0 t}$$

Where $\omega_0 = \frac{2\pi}{T} = \frac{\pi}{4}$, Since a_k is non zero only or odd values of k, we need to evaluate the above summation only or odd k, Furthermore ,note that

$$H(jk\omega_0) = H(jk(\pi/4)) = \frac{\sin(k\pi)}{k(\pi/4)}$$

is always zero or odd values of k, Therefore,

$$y(t) = 0,$$

3.14 The signal $x[n]$ is periodic with period N=4, Its Fourier series coefficients are

$$a_k = \frac{1}{4}\sum_{n=0}^{3} x[n]e^{-j\frac{2\pi}{4}kn}$$

$$= \frac{1}{4}, \quad \text{for all k}$$

From the results presented in Section 3.8   ,   we know that the output $y[n]$ is given by

$$y[n] = \sum_{k=0}^{3} a_k H(e^{j(2\pi/4)k})e^{jk(2\pi/4)n}$$

$$= \frac{1}{4}H(e^{j0}) + \frac{1}{4}H(e^{j(\pi/2)})e^{j(\pi/2)}$$

$$+ \frac{1}{4}H(e^{j(3\pi/2)})e^{j(3\pi/2)} + \frac{1}{4}H(e^{j(\pi)})e^{j(\pi)}$$

From the given information , we know that $y[n]$ is

$$y[n] = \cos(\frac{5\pi}{2}n + \frac{\pi}{4})$$

$$= \cos(\frac{\pi}{2}n + \frac{\pi}{4})$$

$$= \frac{1}{2}e^{j(\frac{\pi}{2}n+\frac{\pi}{4})} + \frac{1}{2}e^{-j(\frac{\pi}{2}n+\frac{\pi}{4})}$$

$$= \frac{1}{2}e^{j(\frac{\pi}{2}n+\frac{\pi}{4})} + \frac{1}{2}e^{j(3\frac{\pi}{2}n-\frac{\pi}{4})}$$

Comparing this with eq. (S3.14-1), we have

$$H(e^{j0}) = H(e^{j\pi}) = 0$$

And

$$H(e^{j\frac{\pi}{2}}) = 2e^{j\frac{\pi}{4}}, \quad \text{and} \quad H(e^{3j\frac{\pi}{2}}) = 2e^{-j\frac{\pi}{4}},$$

3.15 From the results o Section 3.8,

$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0)e^{jk\omega_0 t}$$

Where $\omega_0 = \frac{2\pi}{T} = 12$, Since $H(j\omega)$ is zero for $|\omega| > 100$, the largest value of |k| or which a_k is nonzero should be such that

$$|k||\omega| \le 100$$

This implies that $|k| \le 8$, Therefore , for $|k| > 8$, a_k is guaranteed to be zero.

3.16 (a) The given signal $x_1[n]$ is

$$x_1[n] = (-1)^n = e^{j\pi n} = e^{j(2\pi/2)n}$$

Therefore, $x_1[n]$ is periodic with period N=2 and it's Fourier series coefficients in the range $0 \le k \le 1$ are

$$a_0 = 0 \quad \text{and} \quad a_0 = 1$$

Using the results derived in Section 3.8 , the output $y_1[n]$ is given by

$$y_1[n] = \sum_{k=0}^{1} a_k H(e^{j2\pi k/2})e^{k(2\pi/2)}$$

$$= 0 + a_1 H(e^{j\pi})e^{j\pi}$$

$$= 0,$$

(b) The signal $x_2[n]$ is periodic with period N= 16 The signal $x_2[n]$ may be written as

$$x_2[n] = e^{j(2\pi/16)(0)n} - (j/2)e^{(\pi/4)}e^{j(2\pi/16)(3)n} + (j/2)e^{-j(\pi/4)}e^{-j(2\pi/16)(3)\pi}$$

$$= e^{j(2\pi/16)(0)n} - (j/2)e^{(\pi/4)}e^{j(2\pi/16)(3)n} + (j/2)e^{-j(\pi/4)}e^{-j(2\pi/16)(13)\pi}$$

Therefore, the non-zero Fourier series coefficients of $x_2[n]$ in the range $0 \le k \le 15$ are

$$a_0 = 1, \quad a_3 = -(j/2)e^{j(\pi/4)},$$

Using the results derived in Section 3.8,the output $y_2[n]$ is given by

$$y_2[n] = \sum_{k=0}^{15} a_k H(e^{j16})e^{k(16)}$$

$$= 0 - (j/2)e^{j(\pi/4)}e^{j(2\pi/16)(3)n} + (j/2)e^{-j(\pi/4)}e^{j(2\pi/16)(3)n}$$

$$= \sin(\frac{3\pi}{8}n + \frac{4\pi}{4})$$

(c) The signal $x_3[n]$ may be written as

$$x_3[n] = \left[(\frac{1}{2})^n u[n]\right] * \sum_{k=-\infty}^{\infty} \delta[n-4k] = g[n]*r[n]$$

where $g[n] = (\frac{1}{2})^n u[n]$ and $r[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$ .Therefore, $y_3[n]$ may be obtained by passing the signal $r[n]$ through the filter with frequency response $H(e^{jw})$, and then convolving the result with $g[n]$.

The signal $r[n]$ is periodic with period 4 and its Fourier series confidents are

$$a_k = \frac{1}{4}, \quad \text{for all k (See Problem 3.14)}$$

The output $q[n]$ obtained by passing $r[n]$ through the filter with frequency response $H(e^{jw})$ is

$$q[n] = \sum_{k=0}^{3} a_k H(e^{j2\pi k/4})e^{k(2\pi/4)}$$

$$= (1/4)H(e^{j0})e^{j0} + H(e^{j(\pi/2)})e^{j(\pi/2)} + H(e^{j\pi})e^{j\pi} + H(e^{j3(\pi/2)})e^{j3(\pi/2)}$$

$$= 0$$

Therefore ,the final output $y_3[n]=q[n]*g[n]=0$

3.17 (a) Since complex exponentials are Eigen functions of LTI systems , the input $x_1(t)=e^{j5t}$ has to produce an output of the form Ae^{j5t} , where A is a complex constant . by clearly , in this case the output is not of this form. Therefore . systems S_1 is definitely not LTI.

(b)This sys tem may be LTI because it satisfies the Eagan function property of LTI systems.

(c)In the case , the output is of the form $y_3(t)=(1/2)e^{j5t}+(1/2)e^{-j5t}$. Clearly, the output contains a complex exponential with frequency −5 which was not present in the input $x_3(t)$.We know that an LTI system can never produce a complex exponential of frequency −5 unless there was complex exponential of same frequency at its input, Since this is not the case in this problem , S_3 is definitely not LTI.

3.18 (a)By using an argument similar to the one used 8in part (a) of the previous problem , we conclude that S_1 is definitely not LTI.

(b)The output in this case is $y_2[n]=e^{j(3\pi/2)n}=e^{-j(\pi/2)n}$. Clearly this violates the eigen function property of LTI systems . therefore, S_2 is definitely not LTI.

(c) the output in this case is $y_3[n]=2e^{j(5\pi/2)n}=2e^{-j(\pi/2)n}$ .This does not violate the eigen function property of LTI systems. Therefore, S_3 could possibly be an LTI system.

3.19 (a)voltage across inductor $=L\dfrac{dy(t)}{dt}$.

Current through resistor $=\dfrac{L}{R}\dfrac{dy(t)}{dt}$.

Input current $x(t)$ = current through resistor + current through inductor

Therefore,
$$x(t)=\frac{L}{R}\frac{dy(t)}{dt}+y(t)$$
Substituting for R and L we obtain
$$\frac{dy(t)}{dt}+y(t)=x(t)$$

(b)Using the approach outlined in Section 3.10.1, we know that the output of this system will be $H(j\omega)e^{j\omega t}$ when the input is $e^{j\omega t}$ .Substituting in the differential equation of part (a),
$$j\omega H(j\omega)e^{j\omega t}+H(j\omega)e^{j\omega t}=e^{j\omega t}$$
Therefore,
$$H(j\omega)=\frac{1}{1+j\omega}$$

(c)The signal $x(t)$ is periodic with period 2π .Since $x(t)$ can be expressed in the form
$$x(t)=\frac{1}{2}e^{j(2\pi/2\pi)t}+\frac{1}{2}e^{-j(2\pi/2\pi)t}$$
the non-zero Fourier series coefficients of $x(t)$ are
$$a_1=a_{-1}=\frac{1}{2}$$
Using the results derived in Section 3.8 (see eq.(3.124)),we have
$$y(t)=a_1 H(j)e^{jt}+a_{-1}H(-j)e^{-jt}$$
$$=(1/2)(\frac{1}{1+j}e^{jt}+\frac{1}{1-j}e^{-jt})$$
$$=(1/2\sqrt{2})(e^{-j\pi/4}e^{jt}+e^{j\pi/4}e^{-jt})$$
$$=(1/\sqrt{2})\cos(t-\frac{\pi}{4})$$

3.20. (a) Current through the capacitor $=C\frac{dy(t)}{dt}$.

Voltage across resistor $=RC\frac{dy(t)}{dt}$.

Voltage across inductor = $LC\dfrac{d^2 y(t)}{dt^2}$.

Input voltage = Voltage across resistor + Voltage across inductor + Voltage across capacitor, Therefore,

$$x(t) = LC\frac{d^2 y(t)}{dt^2} + RC\frac{dy(t)}{dt} + y(t)$$

Substituting for R,L and C, we have

$$\frac{d^2 y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t) = x(t)$$

(b)We will now use an approach similar to the one used in part (b) of the previous problem. If we assume that the input is of the form $e^{j\omega t}$, then the output will be of the form $H(j\omega)e^{j\omega t}$, Substituting in the above differential equation and simplifying, we obtain

$$H(j\omega) = \frac{1}{-\omega^2 + j\omega + 1}$$

(c)The signal $x(t)$ is periodic with period 2π, Since $x(t)$ can be expressed in the form

$$x(t) = \frac{1}{2j}e^{j(2\pi/2\pi)t} - \frac{1}{2j}e^{-j(2\pi/2\pi)t}$$

the non-zero Fourier series coefficients of $x(t)$ are

$$a_1 = a_{-1}^* = \frac{1}{2j}$$

Using the results derived in Section 3.8(see eq.(3.124)), we have

$$y(t) = a_1 H(j)e^{jt} - a_{-1}H(-j)e^{-jt}$$

$$= (1/2j)(\frac{1}{j}e^{jt} - \frac{1}{-j}e^{-jt})$$

$$= (-1/2)(e^{jt} + e^{-jt})$$

$$= -\cos(t)$$

3.21. Using the Fourier series synthesis eq.(3.38),

$$x(t) = a_1 e^{j(2\pi/T)t} + a_{-1}e^{-j(2\pi/T)t} + a_5 e^{j5(2\pi/T)t} + a_{-5}e^{-j5(2\pi/T)t}$$

$$= je^{j(2\pi/8)t} - je^{-j(2\pi/8)t} + 2e^{j5(2\pi/8)t} + 2e^{-j5(2\pi/8)t}$$

$$= -2\sin(\frac{\pi}{4}t) + 4\cos(\frac{5\pi}{4}t)$$

$$= -2\cos(\frac{\pi}{4}t - \pi/2) + 4\cos(\frac{5\pi}{4}t)$$

3.22. (a) (i) $T = 1, a_0 = 0, a_k = \dfrac{j(-1)^k}{k\pi}, k \neq 0$

(ii)Hear,

$$x(t) = \begin{cases} t+2, & -2 < t < -1 \\ 1, & -1 < t < 1 \\ 2-t & 1 < t < 2 \end{cases}$$

T=6, a_0=1/2, and

$$a_k = \begin{cases} 0, & k \quad even \\ \dfrac{6}{\pi^2 k^2}\sin(\dfrac{\pi k}{2})\sin(\dfrac{\pi k}{6}), & k \quad odd \end{cases}$$

(iii) T=3, a_0=1, and

$$a_k = \frac{3j}{2k^2\pi^2}[e^{jk2\pi/3}\sin(k2\pi/3) + 2e^{jk2\pi/3}\sin(k\pi/3)], k \neq 0$$

(iv) T=2, , a_k=$^1/_2$−(-1)^k, k≠0

(v)

$$a_k = \frac{\cos(2k\pi/3) - \cos(k\pi/3)}{jk\pi/3}$$

Note that a_0=0, a_k $_{even}$=0.

(vi) T=4, ω_0=π/2, a_0=3/4 and

$$a_k = \frac{e^{-jk\pi/2}\cos(k\pi/2) - e^{-jk\pi/4}\sin(k\pi/4)}{k\pi}, \forall k.$$

(b) $T = 2, a_k = \dfrac{(-1)^k}{2(1+jk\pi)}[e - e^{-1}], \text{for } all \quad k$

(c) T=3, $\omega_0 = 2\pi/3$, ,$a_0 = 1$, and

$$a_k = \frac{2e^{-jk\pi/3}}{k\pi}\sin(2\pi k/3) + \frac{e^{-jk\pi}}{\pi k}\sin(\pi k)$$

3.23.(a)First let us consider a signal y(t) with FS coefficients
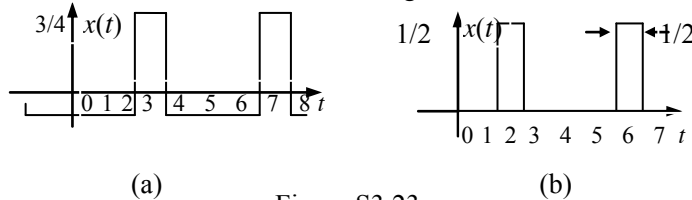
$$b_k = \frac{\sin(k\pi/4)}{k\pi}.$$

Form Example 3.5,we know that * must be a periodic square wave which over one period is

$$y(t) = \begin{cases} 1, & |t| < 1/2 \\ 0, & 1/2 < |t| < 2 \end{cases}$$

Now , note that * , Let us define another signal * whose only nonzero FS coefficient is * , The signal * will have FS coefficients

$$d_k = a_k + c_k = \begin{cases} 0, & k = 0 \\ \frac{\sin(k\pi/4)}{k\pi}, & \text{otherwise.} \end{cases}$$

Now note that * , Therefore , therefore , the signal * which is as shown in Figure S2.23(a).

(a)   Figure S3.23   (b)

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(b) First let us consider a signal y(t) with FS coefficients

$$b_k = \frac{\sin(k\pi/8)}{2k\pi}$$

From Example 3.5,we know that y(t) must be a periodic square wave which over one period is

$$y(t) = \begin{cases} 1/2, |t| < 1/4 \\ 0, 1/4 < |t| < 2 \end{cases}$$

Now note that $a_k = b_k e^{j\pi k}$.Therefore, the signal x(t)=y(t+2) which is as show in Figure S2.23(b).

(c) The only nonzero FS coefficients are $a_1 = a_{-1}^* = j$ and $a_1 = a_{-2}^* = 2j$ .Using the FS synthesis equation ,we got

$$\begin{aligned}
x(t) &= a_1 e^{j(2\pi/T)t} + a_{-1}e^{-j(2\pi/T)t} + a_{2e}e^{2j(2\pi/T)t} + a_{-2}e^{-2j(2\pi/T)t} \\
&= je^{j(2\pi/4)t} - je^{-j(2\pi/4)t} + 2je^{2j(2\pi/4)t} - 2je^{-2j(2\pi/4)t} \\
&= -2\sin(\frac{\pi}{2}t) - 4\sin(\pi t)
\end{aligned}$$

(b)

(d) The FS coefficients a_k may be written as the sum of two sets of coefficients b_k and c_k ,where
b_k=1,for all k
And

$$c_k = \begin{cases} 1, k odd \\ 0, k even \end{cases}$$

The FS coefficients bk correspond to the signal

$$y(t) = \sum_{k=-\infty}^{\infty} \delta(t-4k)$$
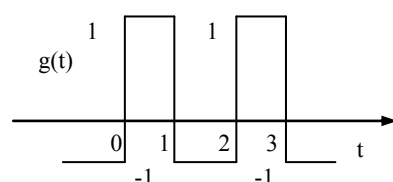
and the FS coefficients ck correspond to the signal

$$z(t) = \sum_{k=-\infty}^{\infty} e^{j(\pi/2)t}\delta(t-2k)$$

Therefore,

$$x(t) = y(t) + p(t) = \sum_{k=-\infty}^{\infty} \delta(t-4k) + \sum_{k=-\infty}^{\infty} e^{j(\pi/2)t}\delta(t-2k)$$

3.24.(a) We have

$$a_0 = \frac{1}{2}\int_0^1 tdt + \frac{1}{2}\int_1^2 (2-t)dt = 1/2$$

(b) The signal g(t)=dx(t)/dt is show in Figure S3.24

The FS coefficients b_k of g(t) may be found as follows:

$$b_0 = \frac{1}{2}\int_0^1 dt - \frac{1}{2}\int_1^2 dt = 0$$

and

$$b_k = \frac{1}{2}\int_0^1 e^{-j\pi kt}dt - \frac{1}{2}\int_1^2 e^{-j\pi kt}dt$$
$$= \frac{1}{j\pi k}\left[1 - e^{-j\pi k}\right]$$

(c) Note that

$$g(t) = \frac{dx(t)}{dt} \xleftarrow{FS} b_k - j\pi k a_k$$

Therefore,

$$a_k = \frac{1}{jk\pi}b_k = -\frac{1}{\pi^2 k^2}\left\{1 - e^{-jk\pi}\right\}$$

3.25.(a) The nonzero FS coefficients of x(t) are a1=a-1=1/2.

(b) The nonzero FS coefficients of x(t) are b1=b_{-1}^*=1/2.

(c)Using the multiplication property, we know that

$$z(t) = x(t)y(t) \xleftarrow{FS} c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$$

Therefore,

$$c_k = a_k * b_k = \frac{1}{4j}\delta[k-2] - \frac{1}{4j}\delta[k+2]$$

This implies that the nonzero Fourier series coefficients of z(t) are c2=c_{-2}^*=(1/4j)

(d) We have z(t)=sin(4t)cos(4t)=sin(8t)/2

Therefore, the nonzero Fourier series coefficients of z(t) are c2=c-2=(1/4j)

3.26.(a) If x(t) is real. Then x(t)=x*(t).This implies that for x(t) real $a_k = a_{-k}^*$.Since this is not true in this case problem, x(t) is not real.

(b) If x(t) is even ,then x(t)=x(-t) and $a_k=a_{-k}$. Since this is true for this case, x(t) is even.

(c) We have

$$g(t) = \frac{dx(t)}{dt} \xleftarrow{FS} b_k = jk\frac{2\pi}{T_0}a_k$$

Therefore,

$$b_k = \begin{cases} 0, k=0 \\ -k(1/2)^{|k|}(2\pi/T_0), otherwise \end{cases}$$

Since b_k is not even.

3.27.Using the Fourier series synthesis eq.(3.38),

$$x[n] = a_0 + a_2 e^{j2(2\pi/N)n} + a_{-2}e^{-j2(2\pi/N)n} + a_4 e^{j4(2\pi/N)n} + a_{-4}e^{-j4(2\pi/N)n}$$
$$= 2 + 2e^{j\pi/6}e^{j(4\pi/5)n} + 2e^{-j\pi/6}e^{-j(4\pi/5)n} + e^{j\pi/3}e^{j(8\pi/5)n} + e^{-j\pi/3}e^{-j(8\pi/5)n}$$
$$= 2 + 4\cos[(4\pi n/5) + \pi/6] + 2\cos[(8\pi n/5) + \pi/3]$$
$$= 2 + 4\sin[(4\pi n/5) + 2\pi/3] + 2\sin[(8\pi n/5) + 5\pi/6]$$

3.28.(a)N=7,

$$a_k = \frac{1}{7}\frac{e^{-j4k\pi/7}\sin(5\pi k/7)}{\sin(\pi k/7)}$$

(b)N=6,ak over period $(0 \leq k \leq 5)$ may be specified as:a0=4/6,

$$a_k = \frac{1}{6}e^{-j\pi k/2}\frac{\sin(\frac{2\pi k}{3})}{\sin(\frac{\pi k}{6})}, 1 \leq k \leq 5$$

(c)N=6,

$$a_k = 1 + 4\cos(\pi k/3) - 2\cos(2\pi k/3)$$

(d)N=12,ak over one period $(0 \le k \le 11)$ may be specified as: $a_1 = \frac{1}{4j} = a_{11}^*$, $a_5 = -\frac{1}{4j} = a_7^*$, $a_k = 0$

Otherwise
(e) N=4,
$$a_k = 1 + 2(-1)^k (1 - \frac{1}{\sqrt{2}}) \cos(\frac{\pi k}{2})$$

(f) N=12
$$a_k = 1 + (1 - \frac{1}{\sqrt{2}}) 2\cos(\frac{\pi k}{6}) + 2(1 - \frac{1}{\sqrt{2}}) 2\cos(\frac{\pi k}{2}) + 2(1 + \frac{1}{\sqrt{2}}) 2\cos(\frac{5\pi k}{6}) + 2(-1)^k + 2\cos(\frac{2\pi k}{3})$$

3.29.(a) N=8,Over one period $(0 \le n \le 7)$
$$x[n] = 4\delta[n-1] + 4\delta[n-7] + 4j\delta[n-3] - 4j\delta[n-5]$$

(b)N=8,Over one period $(0 \le n \le 7)$
$$x[n] = \frac{1}{2j} \left[\frac{-e^{j\frac{3\pi n}{4}} \sin\{\frac{7}{2}(\frac{\pi n}{4} + \frac{\pi}{3})\}}{\sin\{\frac{1}{2}(\frac{\pi n}{4} + \frac{\pi}{3})\}} + \frac{e^{j\frac{3\pi n}{4}} \sin\{\frac{7}{2}(\frac{\pi n}{4} - \frac{\pi}{3})\}}{\sin\{\frac{1}{2}(\frac{\pi n}{4} - \frac{\pi}{3})\}} \right]$$

(c) N=8,Over one period $(0 \le n \le 7)$
$$x[n] = 1 + (-1)^n + 2\cos(\frac{\pi n}{4}) + 2\cos(\frac{3\pi n}{4})$$

(d) N=8,Over one period $(0 \le n \le 7)$
$$x[n] = 2 + 2\cos(\frac{\pi n}{4}) + \cos(\frac{\pi n}{2}) + \frac{1}{2}\cos(\frac{3\pi n}{4})$$

3.30.(a)The nonzero FS coefficients of x(t) are a0=1,a1=a_{-1}=1/2
(b)The nonzero FS coefficient FS coefficient of x(t) are b1= b_{-1}^* = $e^{-j\pi/4}/2$
(c)Using the multiplication property, we know that
$$z[n] = x[n]y[n] \xleftrightarrow{FS} c_k = \sum_{l=-2}^{2} a_l b_{k-l}$$

This implies that the nonzero Fourier series coefficients of z[n] are c0=cos(π/4)/2, $c_1 = c_{-1}^* = e^{-j\pi/4}/2$, $c_2 = c_{-2}^* = ee^{-j\pi/4}/4$
(d) We have
$$z[n] = \sin(\frac{2\pi}{n} + \frac{\pi}{4}) + \sin(\frac{2\pi}{6}n + \frac{\pi}{4})\cos(\frac{2\pi}{6}n)$$
$$= \sin(\frac{2\pi}{6}n + \frac{\pi}{4}) + \frac{1}{2}[\sin(\frac{4\pi}{6}n + \frac{\pi}{4}) + \sin(\frac{\pi}{4})]$$

This implies that the nonzero Fourier series coefficients of z[n] are c0=cos(π/4)/2,
$c_1 = c_{-1}^* = e^{-j\pi/4}/4$

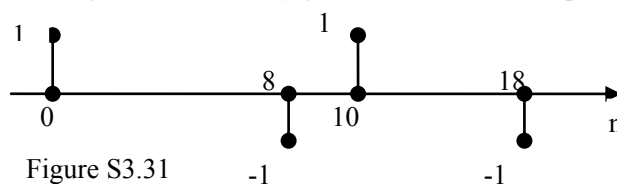**3.31**.(a)g[n] is as show in Figure S3.31.Clearly,g[n] has a fundamental period of 10.

Figure S3.31

(b)The Fourier series coefficients of g[n] are b_k=(1/10)[1- $e^{-j(2\pi/10)8k}$]
(c)Since g[n]=x[n]-x[n-1],the FS coefficients a_k and b_k must be related as
$$b_k = a_k - e^{-j(2\pi/10)k} a_k$$

Therefore,
$$a_k = \frac{b_k}{1 - e^{-j(2\pi/10)k}} = \frac{(1/10)[1 - e^{-j(2\pi/10)8k}]}{1 - e^{-j(2\pi/10)k}}$$

3.32.(a) The four equations are
$$a_0 + a_1 + a_2 + a_3 = 1, a_0 + ja_1 - a_2 - ja_3 = 0$$
$$a_0 - a_1 + a_2 - a_3 = 2, a_0 - ja_1 - a_2 + ja_3 = -1$$

Solving, we got a_0=1/2, $a_1 = -\frac{1+j}{4}, a_2$=-1, $a_3 = -\frac{1-j}{4}$

(b)By direct calculation, $a_k = \frac{1}{4}[1 + 2e^{-j\pi k} - e^{-jk3\pi/2}]$

This is the same as the answer we obtained in part (a) for $0 \le k \le 3$
3.33 We will first evaluate the frequency response of the system. Consider an input x(t) of the form $e^{j\omega t}$ .From

the discussion in Section 3.9.2 we know that response to this input will be $y(t)=H(j\omega)e^{j\omega t}$ .Therefore, substituting these in the given differential equation, we got

$$H(j\omega)j\omega e^{j\omega t}+4e^{j\omega t}=e^{j\omega t}$$

Therefore,

$$H(j\omega)=\frac{1}{j\omega+4}$$

From eq.(3.124),we know that

$$y(t)=\sum_{k=-\infty}^{\infty}a_k H(jk\omega_0)e^{jk\omega_0 t}$$

when the input is x(t).x(t) has the Fourier series coefficients a_k and fundamental frequency ω_0 .Therefore, the Fourier series coefficients of y(t) are $a_k H(jk\omega_0)$.

(a) Here $\omega_0=2\pi$ and the nonzero FS coefficients of x(t) are $a_1=a_{-1}=1/2$ .Therefore, the nonzero FS coefficients of y(t) are

$$b_1=a_1 H(j2\pi)=\frac{1}{2(4+j2\pi)},b_{-1}=a_{-1}H(-j2\pi)=\frac{1}{2(4-j2\pi)}$$

(b) Here, $\omega_0=2\pi$ and the nonzero FS coefficients of x(t) are $a_2=a_{-2}^*=1/2j$ and $a_3=a_{-3}^*=e^{j\pi/4}/2$ .Therefore, the nonzero FS coefficients of y(t) are

$$b_2=a_2 H(j4\pi)=\frac{1}{2j(4+j4\pi)},b_{-2}=a_{-2}H(-j4\pi)=-\frac{1}{2j(4-j4\pi)}$$

$$b_3=a_3 H(j6\pi)=\frac{e^{j\pi/4}}{2(4+j6\pi)},b_{-3}=a_{-3}H(-j6\pi)=\frac{e^{-j\pi/4}}{2(4-j6\pi)}$$

3.34.The frequency response of the system is given by

$$H(j\omega)=\int_{-\infty}^{\infty}e^{-4|t|}e^{-j\omega t}dt=\frac{1}{4+j\omega}+\frac{1}{4-j\omega}$$

(a) Here, T=1 and $\omega_0=2\pi$ and $a_k=1$ .for all k. The FS coefficients of the output are

$$b_k=a_k H(j\omega_0 k)=\frac{1}{4+j2\pi k}+\frac{1}{4-j2\pi k}$$

(b) Here, T=2 and $\omega_0=\pi$ and,

$$a_k=\begin{cases}0,keven\\1,kodd\end{cases}$$

Therefore, the FS coefficients of the outputs are

$$b_k=a_k H(jk\omega_0)=\begin{cases}0,keven\\\dfrac{1}{4+jk\pi}+\dfrac{1}{4-jk\pi},kodd\end{cases}$$

(c) Here, T = 1,$w_0=2\pi$ and

$$a_k=\begin{cases}1/2, & k=0\\0, & k\text{ even},k\neq 0\\\frac{\sin(\pi k/2)}{\pi k}, & k\text{ odd}\end{cases}$$

Therefore, the FS coefficients of the output are

$$b_k=a_k H(jk\omega_0)=\begin{cases}1/4, & k=0\\0, & k\text{ even},k\neq 0\\\frac{\sin(\pi k/2)}{\pi k}\left[\frac{1}{4+j2\pi k}+\frac{1}{4-j2\pi k}\right], & k\text{ odd}\end{cases}$$.

3.35 We know that the Fourier series coefficient of y(t) are b_k=H(j k w_0)a_k, where w_0 is the fundamental of x(t) and a_k, are the FS coefficient of x(t).

If y(t) id identical to x(t),then $b_k=a_k$ for all k. No thing that

H(j w_0)=0 for $|w|\geq 250$.We know that H(j k w_0)=0 for $|k|\geq 18$

(because w_0=14). Therefore a_k must be zero for $|k|\geq 18$.

3.36. We will first evaluate the frequency response of the system. Consider an input x[n] of the form e^{jwn} .From the discussion in Section 3.9 we know that the response to this input will be y[n]=H(e^{jw}) e^{jwn} .Therefore , substituting these in the given difference equation. We get

$$H(e^{jw})e^{jwn}-\frac{1}{4}e^{-jw}e^{jwn}H(e^{jw})=e^{jwn}.$$

Therefore,

$$H(j\omega) = \frac{1}{1 - \frac{1}{4}e^{-j\omega}}.$$

Form eq. (3.131), we know that coefficients

$$y[n] = \sum_{k=<N>} a_k H(e^{j2\pi k/N}) e^{jk(2\pi/N)n}$$

when the input is x[n]. x[n] has the Fourier series coefficients a_k

and fundamental frequency 2/N. therefore, the Fourier series coefficients of y[n] are $a_k \; H(e^{j2\pi k/N})$

(a) Here, N=4 and the nonzero FS coefficients of x[n] are $a_3 = a^*_3 = 1/2j$. Therefore, the nonzero FS coefficients of y[n] are

$$b_3 = a_1 H(e^{3j\pi/4}) = \frac{1}{2j(1-(1/4)e^{-j3\pi/4})}, \qquad b_{-3} = a_{-1}H(e^{-3j\pi/4}) = \frac{-1}{2j(1-(1/4)e^{j3\pi/4})}.$$

(b) Here, N=8 and the nonzero FS coefficients of x[n] are $a_1 = a_{-1}$
=1\2 and $a_2 = a_{-2} = 1$. Therefore, the nonzero FS coefficients of y(t) are

$$b_1 = a_1 H(e^{j\pi/4}) = \frac{1}{2(1-(1/4)e^{-j\pi/4})}, \qquad b_{-1} = a_{-1}H(e^{-j\pi/4}) = \frac{1}{2(1-(1/4)e^{j\pi/4})},$$

$$b_2 = a_2 H(e^{j\pi/2}) = \frac{1}{(1-(1/4)e^{-j\pi/2})}, \qquad b_{-2} = a_{-2}H(e^{-j\pi/2}) = \frac{1}{(1-(1/4)e^{j\pi/2})}.$$

3.37 The frequency response of the system may be easily shown to be

$$H(e^{jw}) = \frac{1}{1-\frac{1}{2}e^{-jw}} - \frac{1}{1-2e^{-jw}}$$

(a) the Fourier series coefficients of x[n] are
$$a_k = 1/4, \text{ for all } k$$
Also, N=4. Therefore, the Fourier series coefficients of y[n] are

$$b_k = a_k H(e^{j2k\pi/N}) = \frac{1}{4}\left[\frac{1}{1-\frac{1}{2}e^{-j\pi k/2}} - \frac{1}{1-2e^{-j\pi k/2}}\right].$$

(b) In this case, the Fourier series coefficients of x[n] are

$$a_k = \frac{1}{6}[1 + 2\cos(k\pi/3)], \qquad \text{for all } k.$$

Also N=6 . Therefore, the Fourier series coefficients of y[n] are

$$b_k = a_k H(e^{j2k\pi/N}) = \frac{1}{6}[1 + 2\cos(k\pi/3)]\left[\frac{1}{1-\frac{1}{2}e^{-j\pi k/3}} - \frac{1}{1-2e^{-j\pi k/3}}\right]$$

3.38 The frequency response of the system may be evaluated as

$$H(e^{j\omega}) = -e^{2j\omega} - e^{j\omega} + 1 + e^{-j\omega} + e^{-2j\omega}.$$

For x[n],N=4 and $w_0 = \pi/2$. the FS coefficients of input x[n] are
$$a_k = 1/4, \text{ for all } k$$
Therefore, the FS coefficients of output are

$$b_k = a_k H(e^{jkw_0}) = \frac{1}{4}[1 - e^{jk\pi/2} + e^{-jk\pi/2}].$$

3.39 Let the FS coefficients of input be a_k. the FS coefficients of output are of the form $b_k = a_k H(e^{jw})$,
where $w_0 = 2\pi/3$ .Note that in the range $0 \le k \le 2$, $H(e^{jw}) = 0$ for k=1,2. Therefore, only b_0 has a nonzero value among b_k in the range $0 \le k \le 2$.

3.40 Let the FS coefficients of x(t) be a_k
(a)x(t-t_0) is also periodic with period T. The FS coefficients b_k of x(t-t_0) are

$$\begin{aligned} b_k &= \frac{1}{T}\int_T x(t-t_0)e^{-jk(2\pi/T)t}dt \\ &= \frac{e^{-jk(2\pi/T)t_0}}{T}\int_T x(\tau)e^{-jk(2\pi/T)\tau}d\tau \\ &= e^{-jk(2\pi/T)t_0}a_k \end{aligned}$$

Similarly, the FS coefficients of x(t-t_0) are

$$c_k = e^{jk(2\pi/T)t_0}a_k.$$

Finally, the FS coefficients of x(t-t_0)+ x(t+t_0) are

$$d_k = b_k + c_k = e^{-jk(2\pi/T)t_0}a_k + e^{jk(2\pi/T)t_0}a_k = 2\cos(k2\pi t_0/T)a_k.$$

(b)Note that $ev\{x(t)\} = [x(t)+x(-t)]/2$. the FS coefficients of x(-t) are

$$b_k = \frac{1}{T}\int_T x(-t)e^{-jk(2\pi/T)t}dt$$
$$= \frac{1}{T}\int_T x(\tau)e^{jk(2\pi/T)\tau}d\tau$$
$$= a_{-k}$$

Therefore, the FS coefficients of $ev\{x(t)\}$ are

$$c_k = \frac{a_k+b_k}{2} = \frac{a_k+a_{-k}}{2}.$$

(c) Note that $Re\{x(t)\} = [x(t)+x^*(t)]/2$. the FS coefficients of $x^*(t)$ are

$$b_k = \frac{1}{T}\int_T x^*(t)e^{-jk(2\pi/T)t}dt.$$

Conjugating both sides ,we get

$$b_k^* = \frac{1}{T}\int_T x(t)e^{jk(2\pi/T)t}dt = a_{-k}.$$

Therefore, the FS coefficients of $Re\{x(t)\}$ are

$$c_k = \frac{a_k+b_k}{2} = \frac{a_k+a_{-k}^*}{2}$$

(d) the FS synthesis equation gives

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/T)kt}.$$

Differentiating both sides wrt t twice ,we get

$$\frac{d^2 x(t)}{dt^2} = \sum_{k=-\infty}^{\infty} -k^2\frac{4\pi^2}{T^2}a_k e^{j(2\pi/T)kt}.$$

By inspection, we know that the FS coefficients of $d^2 x(t)/dt^2$ are $-k^2\frac{4\pi^2}{T^2}a_k$.

(e) The period of $x(3t)$ is a third of The period of $x(t)$. Therefore, the signal $x(3t-1)$ is period with period T/3. the FS coefficients of $x(3t)$ are still a_k .Using the analysis of part (a),we know that the FS coefficients of $x(3t-1)$ is $e^{-jk(6\pi/T)}a_k$

3.41 Since $a_k = a_{-k}$,we require that x(t)=x(-t).Also, note that since $a_k = a_{k+2}$. we require that
$$x(t) = x(t)e^{-jk(4\pi/3)t}$$

This in turn implies that $x(t)$ may have nonzero values only for t=0,$\pm$1.5,$\pm$3,$\pm$4.5,......
Since $\int_{-0.5}^{0.5} x(t)dt = 1$,we may conclude that $x(t) = \delta(t)$ for $-0.5 \le t \le 0.5$ .Also. Since $\int_{0.5}^{1.5} x(t)dt = 2$, we may conclude that $x(t) = 2\delta(t-2/3)$ in the range $0.5 \le t \le 1.5$ .Therefore $x(t)$ may be written as

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-k3) + 2\sum_{k=-\infty}^{\infty} \delta(t-3k-3/2).$$

3.42 (a)From Problem 3.40(and Table 3.1),we know that FS coefficients of $x^*(t)$ are a_{-k}^*. Now, we know $x(t)$ is real, then $x(t) = x^*(t)$. Therefore, $a_k = a_{-k}^*$ .Note that this implies $a_0 = a_0^*$. Therefore, a_0 must be real.

(b)From Problem 3.40(and Table 3.1), we know that FS coefficients of $x(-t)$ are a_{-k} .If $x(t)$ is even, then $x(t) = x(-t)$ .This implies that
$$a_k = a_{-k}$$

This implies that the FS coefficients are even. From the previous part ,we know that if x(t) is real, then

$$a_k = a_k^*$$

Using eqs.(S3.42-1) and (S3.42-2), we know that $a_k = a_k^*$. Therefore, a_k is real for all k. Hence, we may conclude that a_k is real and even.

(c)   Form Problem 3.40(and Table 3.1), we know that FS   coefficients of $x(-t)$ are a_{-k} .If $x(t)$ is odd, then $x(t)=-x(-t)$ .This implies that $$a_k = -a_{-k}$$

This implies that the   FS   coefficients are odd. From previous part ,we know that if $x(t)$ is real ,then
$$a_k = a_k^*$$

Using eqs. (S3.42-3) and (S3.42-4), we know that $a_k = -a_k^*$. Therefore, a_k is imaginary for all k. Hence, we may conclude that a_k is real and even. Noting that eq. (S3.42-3)require that $a_0 = -a_0$,we may also conclude that a_0 =0.

(d) Note that $\varepsilon v\{x(t)\} = [x(t)+x(-t)]/2$ .From the pervious parts, we know that the   FS coefficients of $\varepsilon v\{x(t)\}$,will be $\frac{[a_k + a_{-k}]}{2}$ .Using eq. (S3.42-2) ,we nay write the FS coefficients of $od\{x(t)\} = [x(t)-x(-t)]/2$ .From the previous parts, we known that the FS coefficients of $od\{x(t)\}$ as $\frac{[a_k - a_{-k}]}{2}$ =Re$\{a_k\}$.

(e) Note that $od\{x(t)\} = [x(t)-x(-t)]/2$. From the pervious parts, we know that the   FS coefficients of $\varepsilon v\{x(t)\}$,will be $\frac{[a_k - a_{-k}]}{2}$. Using eq. (S3.42-2) ,we nay write the FS coefficients of $od\{x(t)\}$ as $\frac{[a_k - a_{-k}]}{2}$ =j$Im\{a_k\}$.

3.43 (a) (i)We have
$$x(t) = \sum_{\text{odd } k} a_k e^{jk\frac{2\pi}{T}t}.$$

Therefore,
$$x(t+T/2) = \sum_{\text{odd } k} a_k e^{jk\frac{2\pi}{T}t} e^{jk\pi}.$$

Since $e^{jk\pi}$ =--1 for k odd
$$x(t+T/2) = -x(t).$$

(ii) the Fourier series coefficients of x(t) are
$$a_k = \frac{1}{T}\int_0^{T/2} x(t)e^{-jk\omega_0 t}dt + \frac{1}{T}\int_{T/2}^T x(t)e^{-jk\omega_0 t}dt$$
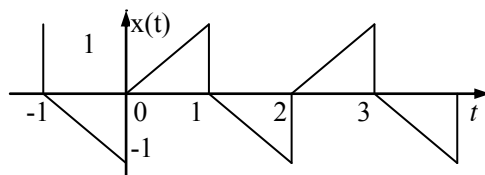$$= \frac{1}{T}\int_0^{T/2} [x(t)+x(t+T/2)e^{-jk\pi}]e^{-jk\omega_0 t}dt$$

Note that the right-hand side of the above equation evaluates to zero for even values of k if $x(t) = -x(t+T/2)$.

(b)The function is as shown in Figure S3.43.
Note that T=2 and $w_0=\pi$ .Therefore,
$$a_k = \begin{cases} 0 & k \text{ even} \\ \frac{1}{jk\pi} + \frac{2}{k^2\pi^2} & k \text{ odd} \end{cases}$$

(c)No. For an even harmonic signal we may follow the reasoning of part (a-i) to show that x(t)= x(t+T/2).

(d)(1) If a_1 or a_{-1} is nonze Figure s3.43
$$x(t) = a_{\pm 1}e^{\pm j2\pi t/T} + \dots..$$
and $x(t+t_0) = a_{\pm 1}e^{\pm j2\pi(t+t_0)/T} + \dots$

The smallest value of $\{t_0\}$ (other than $\{t_0\}$ =0 for which $e^{\pm j2\pi t_0/T}$ =1 is the fundamental period. Only

then is

$$x(t+t_0) = a_{\pm 1}e^{\pm j2\pi t/T} + ... = x(t).$$

Therefore, t_0 has to be the fundamental period.

(2)The period of $x(t)$ is the least common multiple of the periods of $e^{\pm jk2\pi t_0/T_0}$. The period of $e^{jk(2\pi/T)/t}$ is T/k and The period of $e^{jl(2\pi/T)/t}$ and T/l. Since k and l have no common factors, the common multiple of T/k and T/l is T.

3.44 The only unknown FS coefficients are a_{-1}, a_1, a_2 and a_{-2} .Since $x(t)$ is real . $a_1 = a_{-1}^*$ and $a_2 = a_{-2}^*$ .Since a_{-1} is real, $a_1 = a_{-1}$ .Now, $x(t)$ is of the form

$$x(t) = A_1\cos(w_0 t) + \cos(2w_0 t + \theta)$$

where $w_0 = 2\pi/6$,Form this we get

$$x(t-3) = A_1\cos(w_0 t - 3w_0) + A_2\cos(2w_0 t + \theta - 6w_0)$$

Now , if we need $x(t) = -tx(t-3)$,then $3w_0$ and $6w_0$ should both multiple of π .Clearly, this is impossible, $a_2 = a_{-2} = 0$ and

$$x(t) = A_1\cos(w_0 t)$$

Now using Parseval's relation in Clue 5,we get

$$\sum_{k=-\infty}^{\infty}|a_k|^2 = |a_1|^2 + |a_{-1}|^2 = \frac{1}{2}$$

Therefore, $|a_1|=1/2$ .Since a_1 is positive, we have $a_1 = a_{-1} = \frac{1}{2}$. Therefore, $x(t) = \cos(\pi t/3)$.

3.45 By inspection，we may conclude that the FS coefficients of x(t) are

$$\gamma_k = \begin{cases} a_0, k = 0 \\ B_k + jC, k > 0 \\ B_k - jC_k, k < 0 \end{cases}$$

(a) We know from problem 3.42 that if x(t) is real, the FS coefficients of $\varepsilon v\{x(t)\}$ are $\text{Re}\{\gamma_k\}$ therefore,

$$\alpha_0 = a_0, a_k = B_{|k|}$$

We know from problem 3.42 that if x(t) is real ,the FS coefficients of $Od\{x(t)\}$ are $jI_m\{\gamma_k\}$. therefore.

$$\beta_0 = 0, \beta_k = \begin{cases} jC_k, k > 0 \\ -jC_k, k < 0 \end{cases}$$

(b) $\alpha_k = \alpha_{-k} and \beta_k = -\beta_{-k}$

(c) the signals is

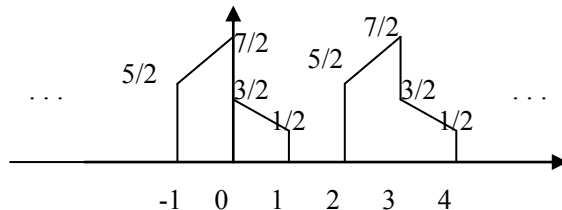$$y(t) = 1 + \xi v\{x(t)\} + \tfrac{1}{2}\xi v\{z(t)\} - Od\{z(t)\}$$

This is as shown in figure S3.45.

Figure S3.45

3.46 (a) The Fourier series coefficients of z(t) are

$$C_k = \tfrac{i}{T}\int_T \sum_n \sum_l a_n b_l e^{j(n+1)w_0 t} e^{-jkw_0 t} dt$$

$$= \tfrac{1}{T}\sum_n \sum_l a_n b_l \delta \,(k\text{-}(n+1))$$

$$= \sum_n a_n b_{k-n}$$

(b) (i) Here ,To=3 and $2\pi/3$ .therefore

$$C_k = \left[\tfrac{1}{2}\delta(k-30) + \tfrac{1}{2}\delta(k+30)\right] * \tfrac{2\sin(k2\pi/3)}{3k2\pi/3}$$

Simplifying

$$C_k = \frac{\sin\{(k-30)2\pi/3\}}{3(k-30)2\pi/3} + \frac{\sin\{(k+30)2\pi/3\}}{3(k+30)2\pi/3}$$

And $C_{\pm 30} = 1/3$.

(ii) We may express $x_2(t)$ as

$x_2(t)$=sum of two shifted square waves x cos(20π t).

Here, To=3, $\omega_0 = 2\pi/3$. therefore,

$$C_k = \tfrac{1}{3} e^{-j(k-30)(2\pi/3)} \frac{\sin\{(k-30)2\pi/3\}}{(k-30)2\pi/3} + \tfrac{1}{3} e^{-j(k+30)(2\pi/3)} \frac{\sin\{(k=30)2\pi/3\}}{(k+30)2\pi/3} +$$

$$\tfrac{1}{3} e^{-j(k-30)(2\pi/3)} \frac{\sin\{(k-30)2\pi/3\}}{(k-30)2\pi/3} + \tfrac{1}{3} e^{-j(k+30)(2\pi/3)} \frac{\sin\{(k+30)2\pi/3\}}{(k+30)2\pi/3}$$

(iii) here, To=4, $\omega_0 = \pi/2$. therefore

$$C_k = \left[\tfrac{1}{2}\delta(k-40) + \tfrac{1}{2}\delta(k+40)\right] * \frac{j\left[kw_0 + e^{-1}\{\sin w_0 t - \cos w_0 t\}\right]}{2\left[1 + (kw_0)^2\right]}$$

Simplifying,

$$C_k = \frac{j\left[(k-40)w_0 + e^{-1}\{\sin(k-40)w_0 - \cos(k-40)w_0\}\right]}{4\left[1 + \{(k-40)w_0\}^2\right]} + \frac{j\left[(k+40)w_0 + e^{-1}\{\sin(k+40)w_0 - \cos(k+40)w_0\}\right]}{4\left[1 + \{(k+40)w_0\}^2\right]}$$

(C) From problom3.42,We know that $b_k = a_k^*$. From part (a),We know that the FS coefficients of $z(t) = x(t)y(t) = x(t)\,x^*(t) = |x(t)|^2$ will be

$$C_k = \sum_{n=-\infty}^{\infty} a_n b_{n-k} = \sum_{n=-\infty}^{\infty} a_n b_{n+k}$$

From the Fourier series analysis equation ,We have

$$C_k = \frac{1}{T_0}\int_0^{T_0} |x(t)|^2 \, e^{-j(2\pi/T_0)kt} dt = \sum_{n=-\infty}^{\infty} a_n a_{n+k}^*$$

Putting k=0 in this equation ,We get

$$\frac{1}{T_0}\int_0^{T_0} |x(t)|^2 \, dt = \sum_{n=-\infty}^{\infty} |a_n|^2$$

3.47 considering x(t) to be periodic with period1,the nonzero FS coefficients of x(t) are $a_1 = a_{-1} = 1/2$. If We now consider x(t) to be periodic with period 3, then nonzero FS coefficients of x(t) are $b_3 = b_{-3} = 1/2$.

3. 48 (a) The FS coefficients of x[n- n_0] are

$$\hat{a}_k = \tfrac{1}{N}\sum_{n=0}^{N-1} x[n-n_0]e^{-j2\pi nk/N}$$

$$= \tfrac{1}{N} e^{-j\frac{2\pi kw_0}{N}} \sum_{n=0}^{N-1} x[n]e^{-j2\pi nk/N}$$

$$= e^{-j2\pi kn_0/N} a_k$$

(b) Using the results of part (a).the FS coefficients of x[n]-x[n-1] are given by

$$\hat{a}_k = a_k - e^{-j2\pi k/n} a_k = \left[1 - e^{-j2\pi/n}\right]a_k.$$

(c) Note the results of part (a). the FS coefficients of x[n]-x[n-N/2] are given by

$$\hat{a}_k = a_k[1 - e^{-j\pi k}]a_k = \begin{cases} 0, even \\ 2a_k, odd \end{cases}$$

(d) Note that x[n]+x[n+N/2] has a period of N/2. The FS coefficients of x[n]+x[n-N/2] are given by

$$\hat{a}_k = \tfrac{2}{N}\sum_{n=0}^{\frac{2}{n}-1}\left[x[n] + x[n+\frac{N}{2}]\right]e^{-j4\pi k/?N} = 2a_{2k}$$

for $0 \le k \le (N/2-1)$.

(e) The FS coefficients of $x^*[-n]$ are

$$\hat{a}_k = \frac{1}{N}\sum_{n=0}^{N-1} x^*[-n]e^{-j2\pi n/N} = a_k^*$$

(f) With N even the FS coefficients of $(-1)^n[n]$ are

$$\hat{a}_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-j(2\pi n/N)(k-\frac{N}{2})} = a_{k-N/2}$$

(g) With N odd, the period of $(-1)^n[n]$ is 2N. therefore ,the FS coefficients are

$$\hat{a}_k = \frac{1}{2N}[\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi n}{N}\left(\frac{k-N}{2}\right)} + \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi n}{N}\left(\frac{k-N}{2}\right)}e^{-j\pi(k-N)}\ ]$$

Note that for k odd $\frac{K-N}{2}$ is an integer and k-N is an even integer. Also, for k even ,K-N is odd

integer r and $e^{-j\pi(K-N)} = -1$. Therefore,

$$\hat{a}_k = \begin{cases} a_{\frac{k-N}{2}}, odd \\ 0, even \end{cases}$$

(h) Here,

$$y[n] = \frac{1}{2}\{x[n] + (-1)^n x[n]\ \}$$

For N even ,

$$\hat{a}_k = \frac{1}{2}\left[a_k + a_{k-\frac{N}{2}}\right]$$

For N odd,

$$\hat{a}_k = \begin{cases} \frac{1}{2}\left[a_k + a_{k-\frac{N}{2}}\right], even \\ \frac{1}{2}a_k, odd \end{cases}$$

3.49 (a) The FS coefficients are given by

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\rho nk}{N}}$$

$$= \frac{1}{N}\sum_{n=0}^{N/2-1} x[n]e^{-j\frac{2\rho nk}{N}} + \frac{1}{N}\sum_{n=N/2}^{N-1} x[n]e^{-j\frac{2\rho nk}{N}}$$

$$= \frac{1}{N}\sum_{n=0}^{N/2-1} x[n]e^{-j\frac{2\rho nk}{N}} + \frac{e^{-j\frac{2\rho nk}{N}}}{N}\sum_{n=0}^{(N/2)-1} x[n]e^{-j\frac{2\rho nk}{N}}$$

$$= 0,$$
    For k even

(b) By adopting an approach similar to part (a),We may show that

$$a_k = \frac{1}{N}\left[\sum_{n=0}^{\frac{N}{4}-1} \{1 - e^{-j\pi k/2} + e^{-j\pi k} + e^{-3j\pi k/2}\}x[n]e^{\frac{-j2nk\pi}{N}}\right]$$

$$= 0$$
    For k 4r, r $\ni \tau$

(c) If N/M is an integer, We may generalize the approach of part (a)to show that

$$a_k = \frac{1}{N}\left[\sum_{K=0}^{B-1} \{1 - e^{-j\pi r} + e^{-j\pi 4r} - e^{-j\pi 2(M-1)}\}x[n]e^{\frac{-j2nk\pi}{N}}\right]$$

Where B=N/M and r=k/m. form the above equation ,it is clear that

$$a_k = 0, \qquad \text{if k=rM, r} \ni \tau$$

3.49   from Table 3.2,we know that if

$$x[n] \xleftarrow{\ FS\ } a_k,$$

then ,

$$(-1)^n x[n] = e^{(2\pi/N)(N/2)n}x[n] \xleftarrow{\ FS\ } a_{k-N/2}$$

In this case, N =8. Therefore,
$$(-1)^n x[n] \overset{FS}{\longleftrightarrow} a_{k-4}$$
This implies that x[0]=x[± 2]x[± 4] $= \cdots = 0$.

We are also given that $x[1] = x[5] = \cdots = and = x[3]x[7] = -1$.therefore , one period of x[n] is as shown in Figure S3.50
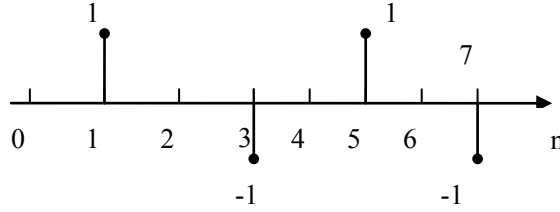

Figure S3.50

3.51.we have
$$e^{j4(2\pi/8)n} x[n] = e^{j\pi n} x[n] = (-1)^n x[n] \overset{FS}{\longleftrightarrow} a_{K-4}$$

And therefore ,
$$(-1)^{n+1} x[n] \overset{FS}{\longleftrightarrow} a_{K-4}$$

If $a_k = a_{k-4}$, then x[0]=x[± 2]x[± 4] $= \cdots = 0$. Now ,note that in the signal p[n]=x[n-1], p$[\pm 1] = p[\pm 3] = \cdots = 0,$ now let us plot the signal z[n]=$(1+(-1)^n)/2$.

This is as shown in Figure S3.51.

Clearly, the signal y[n]=z[n]p[n]=p[n] because p[n] is zero whenever z[n] is zero. therefore , y[n]x[n-1].the FS coefficients of y[n] are $a_{-k} e^{-j(2\pi/8)}$.

3.52 (a)if x[n] is real , x[n]= $x^*[n]$ .therefore,
$$a_{-k} = \sum_n x[n]e^{j2\pi nK/N} = a_k^* .$$

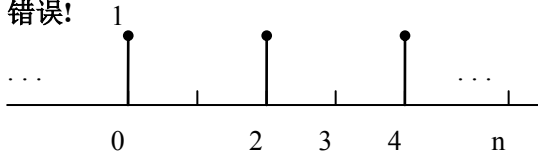From this result , we get $b_{-k} = b_k and c_k = c_{-k}$.

Figure s3.51

(b) if N is even then

$$a_{N/2} = \frac{1}{N}\sum_n x[n]e^{-jn\pi} = \frac{1}{N}\sum_n (-1)^n x[n] = real$$

(C) if N is odd then x[n] is
$$= \sum_{k=0}^{(N-1)/2} a_k e^{j(2\pi/N)kn}$$
$$= a_0 + (-1)^n a_{N/2} + \sum_{k=1}^{(N-2)/2} a_k e^{j(2\pi/N)kn} + a_{N-k}e^{j(2\pi/N)(N-k)n}$$
$$= a_0 + (-1)^n a_{N/2} + \sum_{k=1}^{(N-2)/2} a_k e^{j(2\pi/N)kn} - a_k^* e^{j(2\pi/N)kn}$$
$$= a_0 + (-1)^n a_{N/2} + \sum_{k=1}^{(N-1)/2} b_k \cos(2\pi kn/N) - c_k \sin(2\pi kn/N).$$

if N is even ,then x[n] is

$$= \sum_{k=0}^{(N-1)/2} a_k e^{j(2\pi/N)kn}$$

$$= a_0 + (-1)^n a_{N/2} + \sum_{k=1}^{(N-2)/2} a_k e^{j(2\pi/N)kn} + a_{N-k} e^{j(2\pi/N)(N-k)n}$$

$$= a_0 + (-1)^n a_{N/2} + \sum_{k=1}^{(N-2)/2} a_k e^{j(2\pi/N)kn} - a_k^* e^{j(2\pi/N)kn}$$

$$= a_0 + (-1)^n a_{N/2} + \sum_{k=1}^{(N-1)/2} b_k \cos(2\pi kn/N) - c_k \sin(2\pi kn/N).$$

（d）If $a_k = A_k e^{j\theta_k}$,then $b_k = A\cos(\theta_k)$ and $c_k = A\sin(\theta_k)$ Substituting in the result of the previous part ,we get for N odd:

$$x[n] = a_0 + 2 \sum_{k=1}^{(N-1)/2} A\cos(\theta_k)\cos(2\pi kn/N) - c_k \sin(\theta_k)\sin(2\pi kn/N)$$

$$= a_0 + 2 \sum_{k=1}^{(N-1)/2} A_k \cos\{2\pi kn/N + \theta_k\}$$

Similarly, for N even ,

$$x[n] = a_0 + (-1)^n a_{N/2} + 2 \sum_{k=1}^{(N-1)/2} A\cos(\theta_k)\cos(2\pi kn/N) - c_k \sin(\theta_k)\sin(2\pi kn/N)$$

$$= a_0 + (-1)^n a_{N/2} + 2 \sum_{1}^{(N-2)/2} A_k \cos\{2\pi kn/N + \theta_k\}$$

(e)The signal is :
$$y[n] = d.c\{x[n]\} - d.c\{z[n]\} + \varepsilon v\{z\} + Od\{x\} - 2Od\{z\}$$

This is as shown Figure S3.52.

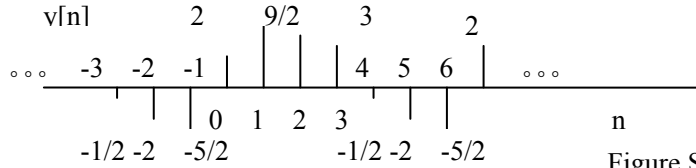Figure S3.52

3.53.We have
$$a_k = \frac{1}{N} \sum_{<N>} x[n] e^{-j(2\pi/N)kn}$$

Note that
$$a_0 = \frac{1}{N} \sum_{<N>} x[n]$$

Which is real if x[n] is real.

(a) If N is even, then
$$a_{N/2} = \frac{1}{N} \sum_{<N>} x[n] e^{-j\pi n} = \frac{1}{N} \sum_{<N>} x[n](-1)^n$$

Clearly, $a_{N/2}$ is also real if x[n] is real.

(b) If N is odd, only a_o is guaranteed to be real.

3.54 (a)Let k =pN, $p \in \tau$ .Then,
$$a[pN] = \sum_{n=0}^{N-1} e^{j(2\pi/N)pNn} = \sum_{n=0}^{N-1} e^{j2\pi pn} = \sum_{n=0}^{N-1} 1 = N$$

(b)Using the finite sum formula ,we have
$$a[k] = \frac{1 - e^{j2\pi k}}{1 - e^{j(2\pi/N)k}} = 0 \qquad \text{if } k \neq pN , p \in \tau$$

(c) Let
$$a[k] = \sum_{n=q}^{q+N-1} e^{j(2\pi/N)kn}$$

Where q is some arbitrary integer .By putting k=pN, we may again easily show that
$$a[pN] = \sum_{n=q}^{q+N-1} e^{j(2\pi/N)pNn} = \sum_{n=q}^{q+N-1} e^{j2\pi pn} = \sum_{n=q}^{q+N-1} 1 = N$$

Now
$$a[k] = e^{j(2\pi/N)kq} \sum_{n=0}^{N-1} e^{j(2\pi/N)kn}$$

Using part (b), we may argue that a[k]=0 for $k \neq pN , p \in \tau$

3.55 (a)Note that

$$x_m[n+mN] = \begin{cases} x[n/m+N] & n=0, \pm m \ldots \\ 0 & \text{otherwise} \end{cases} = \begin{cases} x[n/m], & n=0, \\ 0 & \pm m \ldots \end{cases} = x_m[n]$$

Therefore, $x_m[n]$ is periodic with mN.

(b)The time-scaling operation discussed in this problem is a linear operation. Therefore, if x[n]=v[n]+w[n], then $x_m[n]=v_m[n]+w_m[n]$,

(c) Let us consider
$$y[n] = \frac{1}{m}\sum_{l=0}^{m-1} e^{j(2\pi/mN)(k_0+lN)n} = \frac{1}{m}e^{j(2\pi/mN)k_0 n}\sum_{l=0}^{m-1} e^{j(2\pi/m)ln}$$

This may be written as [From problem 3.54]

$$y[n] = \begin{cases} e^{j(2\pi/mN)k_0 n} & n=0, \pm N, \pm 2N \ldots \\ 0 & \text{otherwise} \end{cases}$$

Now, also note that by applying time-scaling on x[n],we get

$$x_m[n] = \begin{cases} e^{j(2\pi mN\, k_0)n} & n=0, \pm N, \pm 2N \ldots \\ 0 & \text{otherwise} \end{cases}$$

Comparing eps .(s3.55-1)and (s3.55-2),we see that y[n]=

(d) We have
$$b_k = \frac{1}{mN}\sum_{n=0}^{mN-1} x_m[n]e^{-j(2\pi/mN)kn}$$

we know that only every mth value in the above summation is nonzero. Therefore ,

$$b_k = \frac{1}{mN}\sum_{n=0}^{N-1} x_m[nm]e^{-j(2\pi/mN)kmn} = \frac{1}{mN}\sum_{n=0}^{N-1} x_m[nm]e^{-j(2\pi/N)kn}$$

Note that $x_m[nM]=x[n]$ .Therefore , $b_k = \frac{1}{mN}\sum_{n=0}^{N-1} x[n]e^{-j(2\pi/N)kn} = a_k/m$,

3.56 (a)We have $x[n]\xleftrightarrow{FS} a_k$ and $x^*[n]\xleftrightarrow{FS} a^*_{-k}$

Using the multiplication property,
$$x[n]x^*[n] = |x[n]|^2 \xleftrightarrow{FS} \sum_{l=<N>} a_l a^*_{l+k}$$

(b)From above ,it is clear that the answer is yes.

3.57.(a)We have $x[n]y[n] = \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} a_k b_l e^{j(2\pi/N)(k+l)n}$

Putting $l' = k+l$ we get
$$x[n]y[n] = \sum_{k=0}^{(N-1)}\sum_{l'=k}^{(k+N-1)} a_k b_{l'-k} e^{j(2\pi/N)l'n}$$

But since both $b_{l'-k}$ and $e^{j(2\pi/N)l'n}$ are periodic with period N, we may rewrite this as

$$x[n]y[n] = \sum_{k=0}^{(N-1)}\sum_{l'=0}^{N-1} a_k b_{l'-k} e^{j(2\pi/N)l'n} = \sum_{l=0}^{N-1}[\sum_{k=0}^{N-1} a_k b_{l-k}]e^{j(2\pi/N)ln}$$

Therefore , $c_k = \sum_{k=0}^{N-1} a_k b_{l-k} c_k$

By interchanging and ,we may show that
$$c_k = \sum_{k=0}^{N-1} b_k a_{l-k}$$

(b)note that since both a_k and b_k are periodic with period N, we may rewrite the above summation as

$$c_k = \sum_{<N>} a_k b_{l-k} = \sum_{<N>} b_k a_{l-k}$$

(c)(i)Here $c_k = \sum_{l=0}^{N-1}\frac{1}{2}[\delta[l-3]+\delta[l-N+3]]a_{k-l}$

Therefore $c_k = \frac{1}{2}a_{k-3} + \frac{1}{2}a_{k+3-N}$

(ii) period= N. Also
$$b_k = \frac{1}{N} \text{,for all k.}$$

Therefore, $c_k = \frac{1}{N}\sum_{l=0}^{N-1} a_l$

(iii) Here $b_k = \frac{1}{N}[1 + e^{-j2\pi k/3} + e^{-j4\pi k/3}]$

Therefore, $c_k = \frac{1}{N}\sum_{l=0}^{N-1}[1 + e^{-j2\pi l/3} + e^{-j4\pi l/3}]a_{k-l}$

(d)period= 12.Also,

$$x[n]\xleftrightarrow{FS} a_2 = a_{10} = 1/2, \quad \text{All other } a_k = 0, \quad 0 \le k \le 11$$

and $\quad y[n]\xleftrightarrow{FS} b_k = (\frac{1}{12})\frac{\sin 7\pi k/12}{\sin \pi k/12} \quad 0 \le k \le 11$

Therefore one period of c_k is

$$c_k = (\frac{1}{24})[\frac{\sin\{7\pi(k-2)/12\}}{\sin\{\pi(k-2)/12\}} + \frac{\sin\{7\pi(k-10)/12\}}{\sin\{\pi(k-10)/12\}} \quad 0 \le k \le 11$$

(e)Using the FS analysis equation .we have

$$N\sum_{l=<N>} a_l b_{k-l} = \sum_{<N>} x[n]y[n]e^{-j(2\pi/N)kn}$$

Putting k=0 in this , we get

$$N\sum_{l=<N>} a_l b_{-l} = \sum_{<N>} x[n]y[n]$$

Now Let $y[n] = x^*[n]$, then $b_l = a_{-l}^*$ Therefore

$$N\sum_{l=<N>} a_l a_l^* = \sum_{<N>} x[n]x^*[n]$$

Therefore $\quad N\sum_{l=<N>}|a_l|^2 = \sum_{<N>}|x[n]|^2$

3.58 (a) We have

$$z[n+N] = \sum_{<L>} x[r]y[n+N-r]$$

since y[n] is periodic with period N, y[n+N-r]= y[n -r],Therefore

$$z[n+N] = \sum_{<L>} x[r]y[n-r] = z[n]$$

Therefore, z[n] is periodic with period N.
(b) The FS coefficients of z[n] are

$$c_l = \frac{1}{N}\sum_{n=<N>}\sum_{k=<N>} a_k b_{n-k} e^{-j2\pi nl/N}$$

$$= \frac{1}{N}\sum_{k=<N>} a_k e^{-j2\pi kl/N}\sum_{n=<N>} b_{n-k} e^{-j2\pi(n-k)l/N}$$

$$= \frac{1}{N} Na_l Nb_l$$

$$= Na_l b_l$$

(c)Here n=8.the nonzero FS coefficient in the range $0 \le k \le 7$ for x[n]
are $a_3 = a_5^* = 1/2j$. Note that for y[n] ,we need only evaluate b_3 and b_5.

We have $\quad b_3 = b_5^* = \frac{1}{4(1-e^{-j3\pi/4})}$

Therefore ,the only nonzero FS coefficient in the range $0 \le k \le 7$ for the periodic convolution of these signals are $c_3 = 8a_3b_3$ and $c_5 = 8a_5b_5$

(d) Here $x[n]\xleftrightarrow{FS} a_k = \frac{1}{16j}[\frac{1-e^{j(3\pi/7-\pi k/4)4}}{1-e^{-j(3\pi/7-\pi k/4)}} - \frac{1-e^{j(3\pi/7+\pi k/4)4}}{1-e^{-j(3\pi/7+\pi k/4)}}]$

And $\quad y[n]\xleftrightarrow{FS} b_k = \frac{1}{8}[\frac{1-(1/2)^8}{1-(1/2)e^{-jk\pi/4}}]$

Therefore $\quad z[n] = x[n]y[n]\xleftrightarrow{FS} 8a_k b_k$

3.59 (a) Note that the signals x(t) is periodic with period NT. The FS coefficients of x(t) are

$$a_k = \frac{1}{NT}\int_0^{NT}[\sum_{-\infty}^{\infty} x[p]\delta(t-pT)]e^{-j(2\pi/NT)kt} dt$$

Note that the limits of the summation may changed in accordance with limits of the integration so that we get

$$a_k = \tfrac{1}{NT}\int_0^{NT}[\sum_{p=0}^{N-1}x[p]\delta(t-pT)]e^{-j(2\pi/NT)kt}dt$$

Interchanging the summation and the integration and simplifying

$$a_k = \tfrac{1}{NT}\sum_{p=0}^{N-1}x[p]\int_0^{NT}\delta(t-pT)e^{-j(2\pi/NT)kt}dt$$

$$= \tfrac{1}{NT}\sum_{p=0}^{N-1}x[p]e^{-j(2\pi/N)pk}$$

$$= (1/T)[1/N\sum_{p=0}^{N-1}x[p]e^{-j(2\pi/N)pk}].$$

Note that the term within brackets on the RHS of the above equation constitutes the FS coefficients of the signal x[n].Since ,this is periodic with period N, a_k must also be periodic with period N

(b) If the FS coefficients of x(t) are periodic with period N, then

$$a_k = a_{k-N}.$$

This implies that

$$X(t)=x(t)\ e^{j(2\pi/T)Nt}.$$

This is possible only if x(t) is zero for all t other then when $(2\pi/T)N = 2\pi\ k$, where $k \in I$.Therefore ,x(t) is of the form

$$\sum_{k=-\infty}^{\infty}g[k]\delta(t-kT/N)$$

(c) A simple example would be x(t)= $\sum_{k=-\infty}^{\infty}\delta(t-kT)$.

3.60 (a) The system is not LTI. $(1/2)^n$ is an eigen function of LTI systems. Therefore, the output should have been of the form $K(1/2)^n$ .Where K is a complex constant.

(b) It is possible to find an LTI system with this input-output relationship. The frequency response of this system would be $H(e^{j\omega})$=(1- (1/2) $e^{-j\omega}$)/(1-(1/4) $e^{-j\omega}$). The system is unique.

(c) It is possible to find an LTI system with this input-output relationship. The frequency response of this system would be $H(e^{j\omega})$=(1- (1/2) $e^{-j\omega}$)/(1-(1/4) $e^{-j\omega}$). The system is unique.

(d) It is possible to find an LTI system with this input-output relationship. The system is not unique because we only require that $H(e^{j/8})$=2.

(e) It is possible to find an LTI system with this input-output relationship. The frequency response of this system would be $H(e^{j\omega})$=2.The system is unique.

(f) It is possible to find an LTI system with this input-output relationship. The system is not unique because we only require that $H(e^{j\pi/2})$=2(1- $e^{j\pi/2}$).

(g) It is possible to find an LTI system with this input-output relationship. The system is not unique because we only require that $H(e^{j\pi/3})= 1- j\sqrt{3}$.

(h) Note that x[n] and y_1 [n] are periodic with the same fundamental frequency. Therefore, it is possible to find an LTI system with this input-output relationship without violating the Eigen function property. The system is not unique because $H(e^{j\omega})$ needs to be have specific values only for $H(e^{j(2\pi/12)k})$ .The rest of $H(e^{j\omega})$ may be chosen arbitrarily.

(i) Note that x[n] and y_1 [n] are not periodic with the same fundamental frequency. Furthermore, note that $y_2[n]$ has 2/3 the period of x[n].Therefore, y[n] will be made up of complex exponentials which are not present in x[n].This violates the eigen function property of LTI systems. Therefore , the system cannot be LTI.

3.61. (a)   For this system,

$$x(t) \rightarrow \boxed{\delta(t)} \rightarrow x(t).$$

Therefore, all functions are eigenfunctions with an eigenvalue of one.

(b) The following is an eigen function with an eigen value of 1:

$$x(t) = \sum_k \delta(t - kT) \cdot$$

The following is an eigen function with an eigen value of 1/2:

$$x(t) = \sum_k (\frac{1}{2})^k \delta(t - kT) \cdot$$

The following is an eigen function with an eigen value of 2:

$$x(t) = \sum_k (2)^k \delta(t - kT) \cdot$$

(c) If h(t) is real and even then $H(\omega)$ is real and even.

$$e^{j\omega t} \to \boxed{H(j\omega)} \to H(j\omega)e^{j\omega t}$$

And

$$e^{-j\omega t} \to \boxed{H(j\omega)} \to H(-j\omega)e^{-j\omega t} = H(j\omega)e^{-j\omega t}.$$

From these two statements, we may argue that

$$\cos(\omega t) = \frac{1}{2}[e^{j\omega t} + e^{-j\omega t}] \to \boxed{H(j\omega)} \to H(j\omega)\cos(\omega t) \cdot$$

Therefore, $\cos(\omega t)$ is an eigenfunction. We may similarly show hat $\sin(\omega t)$ is an eigenfunction.

(d) We have

$$\phi(t) \to \boxed{u(t)} \to \lambda\phi(t) \cdot$$

Therefore,

$$\lambda\phi(t) = \int_{-\infty}^{t} \phi(\tau)d\tau \cdot$$

Differentiating both sides wrt t, we get

$$\lambda\phi'(t) = \phi(\tau) \cdot$$

Let $\phi(0) = \phi_0$. Then

$$\phi(t) = \phi_0 e^{t/\lambda}.$$

3.62. (a) The fundamental period of the input is $T = 2\pi$. The fundamental period of the input is $T = \pi$ .the signals are as shown in Figure S3.62.

(b) The Fourier series confidents of the output are
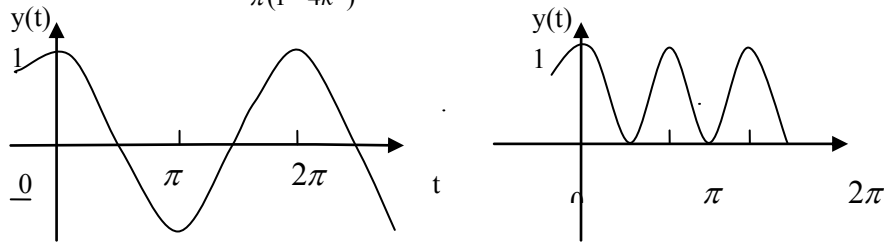
$$b_k = \frac{2(-1)^k}{\pi(1-4k^2)} \cdot$$

Figure S3.62

(c) The dc component of the input is 0. The dc component of the output is $2/\pi$.

3.63. The average energy per period is

$$\frac{1}{T}\int_T |x(t)|^2 \, dt = \sum_k |a_k|^2 = \sum_k |a|^{2|k|} = \frac{1+\alpha^2}{1-\alpha^2}$$

We want N such that

$$\sum_{-N+1}^{N-1} |a_k|^2 = 0.9\frac{1+\alpha^2}{1-\alpha^2}.$$

This implies that

$$\frac{1-2\alpha^{2N}+2\alpha^2}{1-\alpha^2} = \frac{1+\alpha^2}{1-\alpha^2}.$$

Solving

$$N = \frac{\log[1.45\alpha^2 + 0.95]}{2\log\alpha} .$$

And

$$\frac{\pi N}{4} < W < \frac{(N-1)\pi}{4} .$$

3.64 (a) Due to linearity, we have

$$y(t) = \sum_k c_k \lambda_k \phi_k(t) .$$

(b) Let

$$x_1(t) \to y_1(t) \text{ and } x_2(t) \to y_2(t) .$$

Also, let

$$x_3(t) = ax_1(t) + bx_2(t) \to y_3(t) .$$

Then,

$$y_3(t) = t^2[ax_1^n(t) + bx_2^n(t)] + t[ax_1'(t) + bx_2'(t)] = ay_1(t) + by_2(t)$$

Therefore, the system is linear.

Now consider

$$x_4(t) = x(t - t_0) \to y_4(t).$$

We have

$$y_4(t) = t^2 \frac{d^2 x(t - t_0)}{dt^2} + t\frac{dx(t - t_0)}{dt} \neq y(t - t_0) .$$

Therefore, the system is not time invariant.

(c) For inputs of the form $\phi_k(t) = t^k$ .The output is

$$y(t) = k^2 t^k = k^2 \phi_k(t) .$$

The output is

$$y(t) = 10^3 t^{-10} + 3t + 8t^4 .$$

3.65 (a) Pairs (a) and (b) are orthogonal. Pairs (c) and (d) are not orthogonal.

(b) Orthogonal. But not orthogonal. $A_m = 1/\omega_0$.

(c) Orthogonal.

(d) We have

$$\int_{t_0}^{t_0+T} e^{jm\omega_0\tau} e^{-jn\omega_0\tau} d\tau = e^{j(m-n)\omega_0 t_0} \frac{[e^{j(m-n)2\pi} - 1]}{(m-n)\omega_0} .$$

This evaluates to 0when m $\neq$ n and to jT when m=n . Therefore, the functions are orthogonal but not orthogonal.

(e) We have

$$\int_{-T}^{T} x_e(t)x_o(t)dt = \frac{1}{4}\int_{-T}^{T}[x(t) + x(-t)][x(t) - x(-t)]dt = \frac{1}{4}\int_{-T}^{T} x^2(t)dt - \frac{1}{4}\int_{-T}^{T} x^2(-t)dt = 0$$

(f) Consider

$$\int_a^b \frac{1}{\sqrt{A_k}}\phi_k(t)\frac{1}{\sqrt{A_k}}\phi_l^\#(t)dt = \frac{1}{\sqrt{A_k A_l}}\int_a^b\int_a^b \phi_k(t)\phi_l^\#(t)dt .$$

This valuates to zero for k $\neq$ l. For k=l, it evaluates to A_k / A_k =1. Therefore, the functions are orthogonal

(g) We have

$$\int_a^b |x(t)|^2 dt = \int_a^b x(t)x^*(t)dt$$

$$= \int_a^b \sum_i a_i\phi_j^2(t)dt$$

$$= \sum_i \sum_j a_i a_j^* \int_a^b \phi_i(t)\phi_j^*(t)dt$$

$$= \sum_i |a_i|^2 .$$

(h) We have

$$y(T) = \int_{-\infty}^{\infty} h_i(T - \tau)\phi_j(\tau)d\tau$$

$$= \int_{-\infty}^{\infty} \phi_i(\tau)\phi_j(\tau)d\tau$$

$$= \delta_{ij} = 1 \, for \quad i = j \text{ and } 0 \text{ for } i \neq j.$$

3.66. (a) We have

$$E = \int_a^b [x(t) - \sum_{k=-N}^{N} a_k\phi_k(t)][x^*(t) - \sum_{k=-N}^{N} a_k^*\phi_k^*(t)]d$$

Now, let $a_i = b_i + jc_i$. Then

$$\frac{\partial E}{\partial b_i} = 0 = -\int_a^b \phi_i^*(t)x(t)dt + 2b_i - \int_a^b \phi_i(t)x^*(t)dt \cdot$$

And

$$\frac{\partial E}{\partial b_i} = 0 = j\int_a^b \phi_i(t)x^*(t)dt + 2c_i - j\int_a^b \phi_i^*(t)x(t)dt \cdot$$

Multiplying, the equation by j and adding to the before. We get

$$2b_i = 2jc_i + \int_a^b x(t)\phi^*(t)dt \cdot$$

This implies that

$$a_i = \int_a^b x(t)\phi^*(t)dt \cdot$$

(b) In this case, a_i would be

$$a_i = \frac{1}{A_i}\int_a^b x(t)\phi^*(t)dt \cdot$$

(c) Choosing

$$a_k = \frac{1}{T_0}\int_b^{b+T_0} x(t)e^{-j\omega_0 t}dt \,,$$

We have

$$E = \int_{T_0} |x(t) - \sum_{k=-N}^{N} a_k e^{-j\omega_0 kt}|^2 \, dt \cdot$$

Putting $\frac{\partial E}{\partial a_k} = 0$, we get

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt \cdot$$

(d) $a_0 = 2/\pi$, $a_1 = a_3 = 0$, $a_2 = 2(1 - 2\sqrt{2})/\pi$,
   $a_4 = (1/\pi)[2 - 4\cos(\pi/8) + 4\cos(3\pi/8)]$.
(e) We have

$$\int_0^1 \sum_i (a_i\phi_i(t))^*[x(t) - \sum_i a_i\phi_i(t)]dt = \sum_i a_i^* \int_0^1 x(t)\phi_i^*(t)dt - \sum_i \sum_j a_i^* a_j \int_0^1 \phi_i^*(t)\phi_i(t)dt$$

$$= \sum_i a_i^* a_i - \sum_i a_i^* a_i = 0$$

(f) Not orthogonal. Example: $\int_0^1 \phi_0(t)\phi_1(t) = \int_0^1 tdt = 1 \neq 0 \cdot$

(g) Here,

$$a_0 = \int_0^1 e^t \phi_0^*(t)dt = e - 1.$$

(h) Here, x(t)= $a_0 + a_1 t$. Therefore,

$$E = \int_0^1 (e^t - a_0 - a_1 t)(e^t - a_0 - a_1 t)dt \cdot$$

Setting $\partial E/\partial a_0 = 0 = \partial E/\partial a_1$, we get a_0 =c(2e-5) and a_1 =6(3-e).
3.67. (a) From e.g.(P3.67-1) and (P3.67-4), we get

$$\sum_{n=-\infty}^{\infty} j2\pi n b_n(x)e^{j2\pi nt} = \frac{1}{2}k^2 \sum_{n=-\infty}^{\infty} \frac{\partial^2 b_n(x)}{\partial x^2} e^{j2\pi nt}.$$

Equating coefficients of $e^{j2\pi nt}$ on both sides, we get

$$\frac{\partial^2 b_n(x)}{\partial x^2} = \frac{j4\pi n}{k^2} b_n(x).$$

$sin\,ce$ $s^2 = 4\pi jn/k^2$,

$$s = \pm \frac{2\sqrt{\pi n}\ell^{j\pi/4}}{k},$$

for n>0,

$$s = \frac{\sqrt{2\pi n}(1+j)}{k},$$

is a stable solution. for n<0,

$$s = -\frac{\sqrt{2\pi|n|}(1-j)}{k}$$

is a stable solution. also, $b_n(0) = a_n$ and

$$b_n(x) = \begin{cases} a_n e^{-\sqrt{2\pi n}(1+j)x/k}, & n>0 \\ a_n e^{-\sqrt{2\pi n}(1-j)x/k}, & n<0 \end{cases}$$

(c)$b_0 = 2$, $b_1 = (1/2j)e^{-(1+j)\pi}$, $b_{-1} = -(1/2j)e^{-(1+j)\pi}$,

$T(k\sqrt{\pi/2}, t) = 2 + e^{-\pi}\sin(2\pi t - \pi)$,

Phase reversed.

3.68. (a)$x(\theta) = r(\theta)\cos(\theta) = \frac{1}{2}r(\theta)e^{j\theta} + \frac{1}{2}r(\theta)e^{-j\theta}$ .if

$$x(\theta) = \sum_{k=-\infty}^{\infty} b_k e^{jk\theta},$$

then $b_k = (1/2)a_{k+1} + (1/2)a_{k-1}$.

(b) $x(\theta) \xleftarrow{Fs} b_k$ .then $x(\theta) = r(\theta + \frac{\pi}{4})$ .the sketch is as show in Figure s3.68

(c)$b_0 = a_0$ .rest of b_k is all zero. Therefore, the sketch will be a circle of radius a_0 as shown in figure s3.68.

(d)(i) $r(\theta) = r(-\theta)$,even. Sketch as shown in figure s3.68.

(ii)$r(\theta + k\pi) = r(\theta)$ .sketch as shown in figure s3.68.

(iii)$r(\theta + k\pi/2) = r(\theta)$ .sketch as shown in figure s3.68.

3.69.(a) $\sum_{n=-N}^{N} \phi_k[n]\phi^*_k[m] = \sum_{n=-N}^{N}\delta[n-k]\delta[n-m]$ .this is 1 for k =m and 0 for $k \neq m$. therefore, orthogonal.

(b)we have

$$\sum_{n=r}^{r+N-1} \phi_k[n]\phi_m^*[n] = e^{j(2\pi/N)r(k-m)}\left[\frac{1-e^{j2\pi(k-m)}}{1-e^{j(2\pi/N)(k-m)}}\right] = \begin{cases} 0, k \neq m \\ N, k = m. \end{cases}$$
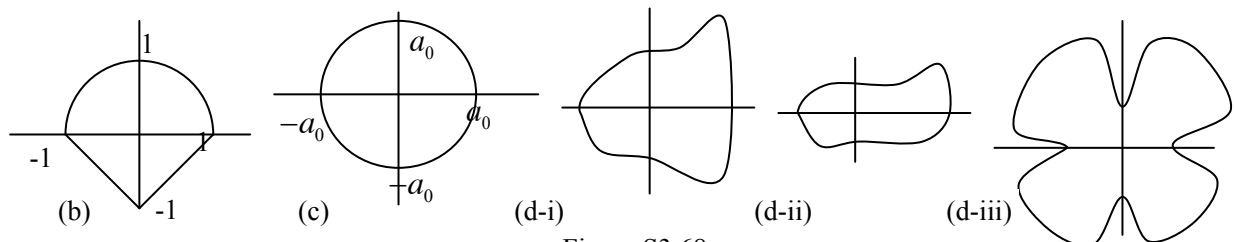
therefore, orthogonal.

(b)   (c)   (d-i)   (d-ii)   (d-iii)

Figure S3.68

(c)we have

$$\sum_{n=N_1}^{N_2}|x[n]|^2 = \sum_{n=N_1}^{N_2}\sum_{i=1}^{M} a_i\phi_i[n]\sum_{k=1}^{M} a_k^*\phi_k^*[n] = \sum_{k=1}^{M}\sum_{i=1}^{M} a_i a_k^* \sum_{n=N_1}^{N_2}\phi_k^*[n]\phi_i[n] = \sum_{k=1}^{M}\sum_{i=1}^{M} a_i a_k^* A_i \delta[i-k] = \sum_{i=1}^{M}|a_i|^2 A_i$$

(d) let $-a_0$ then

$$E = \sum_{n=N_1}^{N_2}|x[n]|^2 + \sum_{i=1}^{M}(b_i^2 + c_i^2)A_i - \sum_{n=N_1}^{N_2} x[n]\sum_{i=1}^{M}(b_i - jc_i)\phi_i^*[n] - \sum_{n=N_1}^{N_2} x^*[n]\sum_{i=1}^{M}(b_i + jc_i)\phi_i[n]$$

Set $\partial E/\partial b_i = 0.$ then

$$b_i = [2A_i]^{-1}\left[\sum_{n=N_1}^{N_2}\{x[n]\phi_i^*[n] + x^*[n]\phi_i[n]\}\right] = \frac{1}{A_i}\text{Re}\left\{\sum_{n=N_1}^{N_2} x[n]\phi_i^*[n]\right\}$$

Similarly,

$$c_i = \frac{1}{A_i}\text{Im}\left\{\sum_{n=N_1}^{N_2} x[n]\phi_i^*[n]\right\}.$$

therefore,

$$a_i = b_i + jc_i = \frac{1}{A_i}\sum_{n=N_1}^{N_2} x[n]\phi_i^*[n].$$

(e) $\phi_i[n] = \delta[n-i].$ then,

$$a_i = \sum_{n=N_1}^{N_2} x[n]\delta[n-i]x[i].$$

3.70 (a) we get

$$a_{mn} = \frac{1}{T_1 T_2}\int_0^{T_1}\int_0^{T_2} x(t_1,t_2)\ell^{-jm\omega_1 t_1}\ell^{-jn\omega_2 t_2} dt_1 dt_2.$$

(b) (i) $T_1 = 1, T_2 = \pi, a_{11} = 1/2, a_{-1,-1} = 1/2.$ Rest of the coefficients are all zero.

(ii) here,

$$a_{mn} = \begin{cases} 1/(\pi^2 mn), m, n \, odd \\ 0, otherwise \end{cases}.$$

3.71 (a) the differential equation $f_s(t)$ and $f(t)$ is

$$\frac{B}{K}\frac{df_s(t)}{dt} + f_s(t) = f(t).$$

The frequency response of this system may be easily shown to be

$$H(j\omega) = \frac{1}{1+(B/K)j\omega}.$$

Note that for $\omega = 0, H(j\omega) = 1$ and for $\omega \to \infty, H(jw) = 0.$ Therefore, the system approxima as a lowpass filter.

(b) The differential equation $f_d(t)$ and $f(t)$ is

$$\frac{df_d(t)}{dt} + \frac{K}{B}f_d(t) = \frac{df(t)}{dt}.$$

The frequency response of this system may be easily shown to be

$$H(j\omega) = \frac{j\omega}{j\omega + (K/B)}.$$

Note that for $\omega = 0, H(j\omega) = 0$ and for $\omega \to \infty, H(j\omega) = 1.$ Therefore, the system Approximates a highpass filter

Chapter 4 Answers

4.1 (a)Let $x(t)=\ell^{-2(t-1)}u(t-1)$, then the Fourier transform $x(j\omega)$ of $x(t)$ is :

$$X(j\omega)=\int_{-\infty}^{\infty}\ell^{-2(t-1)}u(t-1)\ell^{-j\omega t}dt$$

$$=\int_{1}^{\infty}\ell^{-2(t-1)}\ell^{-j\omega t}dt=\ell^{-j\omega}/(2+j\omega)$$

$|X(j\omega)|$ is as shown in figure s4.1.

(b) Let $x(t)=\ell^{-2|t-1|}$, then the Fourier transform $X(j\omega)$ of $x(t)$ is :

$$X(j\omega)=\int_{-\infty}^{\infty}\ell^{-2|t-1|}\ell^{-j\omega t}dt=\int_{1}^{\infty}\ell^{-2(t-1)}\ell^{-j\omega t}dt+\int_{-\infty}^{1}\ell^{2(t-1)}\ell^{-j\omega t}dt$$

$$=\ell^{-j\omega}/(2+j\omega)+\ell^{-j\omega}/(2-j\omega)=4\ell^{-j\omega}/(4+\omega^2)$$
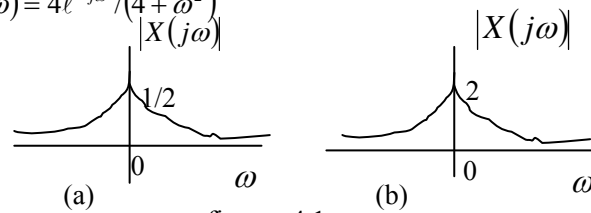
$|X(j\omega)|$ is as shown in figure s4.1

figure s4.1

4.2 (a) Let $x_1(t)=\delta(t+1)+\delta(t-1)$, then the Fourier transform $X_1(j\omega)$ of $x(t)$ is :

$$X_1(j\omega)=\int_{-\infty}^{\infty}[\delta(t+1)+\delta(t-1)]\ell^{-j\omega t}dt=\ell^{j\omega}+\ell^{-j\omega}=2\cos\omega$$

$|X_1(j\omega)|$ is as sketched in figure s4.2.

(b)the signal $x_2(t)=u(-2-t)+u(t-2)$ is as shown in the figure below .Clearly,

$$\frac{d}{dt}\{u(-2-t)+u(t-2)\}=\delta(t-2)-\delta(t+2)$$

Therefore

$$x_2(j\omega)=\int_{-\infty}^{\infty}[\delta(t-2)-\delta(t+2)]e^{-j\omega t}dt$$

$$=e^{-2j\omega}-e^{2j\omega}$$

$$=-2j\sin(2\omega)$$

$|x_1(j\omega)|$ is as sketched in figure s4.2.
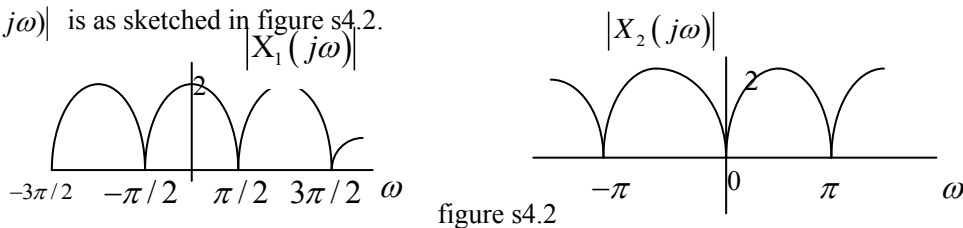

figure s4.2

4.3 (a) the signal $x_1(t)=\sin(2\pi t+\pi/4)$ is periodic with a fundamental periodic of T=1.

This translations to a fundamental frequency of $\omega_0=2\pi$ .the nonzero Fourier series coefficients of this signals of this signal may be found by writing it in the form

$$x_1(t)=\frac{1}{2j}(e^{j(2\pi t+\pi/4)}-e^{-j(2\pi t+\pi/4)})$$

$$=\frac{1}{2j}e^{j\pi/4}e^{j2\pi t}-\frac{1}{2j}e^{-j\pi/4}e^{-j2\pi t}$$

therefore, the nonzero Fourier series coefficients of $x_1(t)$ are

$$a_1=\frac{1}{2j}e^{j\pi/4}e^{j2\pi t},a_2=\frac{1}{2j}e^{-j\pi/4}e^{-j2\pi t}$$

Form Section 4.2 we know that for periodic signals, the Fourier transforms consists of train of impulse occurring at $k\omega_0$ .Furthermore, the area under each impulse is 2π times the Fourier series coefficients

a_k .Therefore, for $x_1(t)$ the corresponding Fourier transforms $x_1(j\omega)$ is given by

$$x_1(j\omega) = 2\pi a_1\delta(\omega-\omega_0)+2\pi a_{-1}\delta(\omega+\omega_0)$$

$$= (\pi/j)e^{j\pi/4}\delta(\omega-2\pi)-(\pi/j)e^{-j\pi/4}\delta(\omega+2\pi)$$

(b) The signal $x_2(t)=1+\cos(6\pi t+\pi/8)$ is periodic with a fundamental period of $T=1/3$ .This translates to a fundamental frequency of $\omega_0=6\pi$ .The nonzero Fourier series coefficients of this signal may be found by writing it in the form

$$x_2(t)=1+\frac{1}{2}(e^{j(6\pi t+\pi/8)}+e^{-j(6\pi t+\pi/8)})$$

$$=1+\frac{1}{2}e^{j\pi/8}e^{j6\pi t}+\frac{1}{2}e^{-j\pi/8}e^{-j6\pi t}$$

Therefore ,The nonzero Fourier series coefficients of $x_2(t)$ are

$$a_0=1, a_1=\frac{1}{2}e^{j\pi/8}e^{j6\pi t}, a_{-1}=\frac{1}{2}e^{-j\pi/8}e^{-j6\pi t}.$$

Form Section 4.2 we know that for periodic signals, the Fourier transforms consists of train of impulse occurring at $k\omega_0$ .Furthermore, the area under each impulse is 2π times the Fourier series coefficients a_k .Therefore, for $x_2(t)$ the corresponding Fourier transforms $x_2(j\omega)$ is given by

$$x_2(j\omega)=2\pi a_0\delta(\omega)+2\pi a_1\delta(\omega-\omega_0)+2\pi a_{-1}\delta(\omega+\omega_0)$$

$$=2\pi\delta(\omega)+\pi e^{j\pi/8}\delta(\omega-6\pi)+\pi e^{-j\pi/8}\delta(\omega+6\pi)$$

4.4 (a) The inverse Fourier transforms is

$$x_1(t)=(1/2\pi)\int_{-\infty}^{\infty}[2\pi\delta(\omega)+\pi\delta(\omega-4\pi)+\pi\delta(\omega+4\pi)]e^{j\omega t}d\omega$$

$$=(1/2\pi)[2\pi e^{j0t}+\pi e^{j4\pi t}+\pi e^{-j4\pi t}]$$

$$=1+(1/2)e^{j4\pi t}+(1/2)e^{-j4\pi t}$$

$$=1+\cos(4\pi t)$$

(b) The inverse Fourier transforms is

$$x_2(t)=(1/2\pi)\int_{-\infty}^{\infty}X_2(jw)e^{jwt}dw$$

$$=(1/2\pi)\int_{0}^{2}2e^{jwt}dw+\int_{-2}^{0}-2e^{jwt}dw$$

$$=(e^{j2t}-1)/(\pi jt)-(1-e^{-j2t})/(\pi jt)$$

$$=-(4j\sin^2 t)/(\pi t)$$

4.5 Form the given information

$$x(t)=(1/2\pi)\int_{-\infty}^{\infty}X(jw)e^{jwt}dw$$

$$=(1/2\pi)\int_{-\infty}^{\infty}|X(jw)|e^{j\leq\{X(e^{jw})\}wt}e^{jw}dw$$

$$=(1/2\pi)\int_{-3}^{3}2e^{-2/3w+\pi}e^{jw}dw$$

$$=\frac{2}{\pi(t-2/3)}\sin[3(t-2/3)]$$

The signal $x(t)$ is zero when $3(t-2/3)$ is a nonzero integer multiple of π this gives

$$t=\frac{k\pi}{2}+2/3, for k\in I and k\neq 0$$

4.6. Throughout this problem ,we assume that

$$x(t)\xleftarrow{FT}X_1(j\omega)$$

(a) Using the time reversal property (Sec. 4.3.5), we have

$$x(-t)\xleftarrow{FT}X_1(-j\omega)$$

Using the time shifting property (Sec. 4.3.2) on this, we have

$$x(-t+1)\xleftarrow{FT}e^{-j\omega t}X_1(-j\omega) \quad and \ x(-t-1)\xleftarrow{FT}e^{j\omega t}X_1(-j\omega)$$

therefore

$x_1(t)= x(-t+1)+ x(-t-1)$ $\xleftrightarrow{\ FT\ }$ $e^{-j\omega t} X_1(-j\omega)+ e^{j\omega t} X_1(-j\omega)$ $\xleftrightarrow{\ FT\ }$ $2 X(-j\omega)\cos\omega$

(b) Using the time scaling property (Sec. 4.3.5), we have

$\qquad x(3t)$ $\xleftrightarrow{\ FT\ }$ $1/3 X(j\omega/3)$

Using the time shifting property on this, we have $\quad x_2(3(t-2))$ $\xleftrightarrow{\ FT\ }$ $e^{-2j\omega} 1/3 X(j\omega/3)$

(c) Using the differentiation in time property (Sec. 4.3.4), we have $\quad \dfrac{dx(t)}{dt}$ $\xleftrightarrow{\ FT\ }$ $j\omega X(j\omega)$

Applying this property again, we have $\dfrac{d^2x(t)}{dt^2}$ $\xleftrightarrow{\ FT\ }$ $-\omega^2 X(j\omega)$

Using the time shifting property, we have $x_3=\dfrac{d^2x(t-1)}{dt^2}$ $\xleftrightarrow{\ FT\ }$ $-\omega^2 X(j\omega)\, e^{-j\omega t}$

4.7 (a)Since $X_1(-j\omega)$ is not conjugate symmetric ,the corresponding signal $x_1(t)$ is not real

Since $X_1(-j\omega)$ is neither even nor odd ,the corresponding signal $x_1(t)$ is neither even nor odd

(b) the FT of a real and odd signal is purely imaginary and odd. therefore ,we may conclude that the FT of a purely imaginary and odd signal is real and odd . since $X_2(j\omega)$ is real and odd we may therefore conclude that the corresponding signal $x_2(t)$ is purely imaginary and odd.

(c) Consider a signal $y_3(t)$ whose magnitude of the FT is $|\ Y_3(j\omega)|=A(\omega)$,and whose phase of the FT is $\sphericalangle\{Y_3(j\omega)\} = 2\omega$. Since $|Y_3(j\omega)|$ and $\sphericalangle\{Y_3(j\omega)\}=-\sphericalangle\{Y_3(-j\omega)\}$,we may conclude that the signal $y_3(t)$ is real

(d) Since $X_4(j\omega)$ is both real and even , corresponding signal $x_4(t)$ is real and even

4.8 (a)The signal x(t) is as shown in Figure S4.8.

We may express this signal as

$$x(t) = \int_{-\infty}^{t} y(t)dt$$
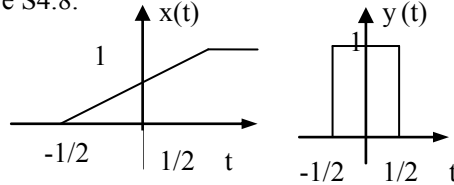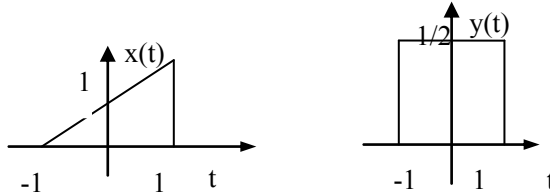

Figure S4.8

Where y (t) is the rectangular pulse shown in S4.8 Using the integration property of FT we have

$\qquad x(t)$ $\xleftrightarrow{\ FT\ }$ $X(j\omega) = \dfrac{1}{j\omega}Y(j\omega)+\pi Y(j0)\sigma(\omega)$

we know from 4.2 that

$$Y(j\omega)=\frac{2\sin(w/2)}{w}$$

Therefore $X(j\omega)= \dfrac{2\sin(w/2)}{jw^2}+\pi\sigma(\omega)$

(b) if g(t)=x(t)-(1/2) $\pi\sigma(\omega)= \dfrac{2\sin(w/2)}{jw^2}$

4.9 (a) the signal x(t) is plotted in figure S4.9

$x(t)= \displaystyle\int_{-\infty}^{t} y(t)dt - u(t-1/2)$

using the result obtain ed in part (a) of the previous problem ,the FT $X(j\omega)$ of x(t) is

$X(j\omega)=\dfrac{2\sin(w/2)}{jw^2}+\pi\sigma(\omega)$ -FT{u(t-1/2)}= $\dfrac{\sin\omega}{j\omega^2}-\dfrac{e^{-j\omega}}{j\omega}$

(b) the even part of x(t) is given by

$\varepsilon v\{x(t)\}=(x(t)+x(-t))/2$

This is as shown in the 4.9

Therefore

$FT\{\varepsilon v\{x(t)\}\} = \dfrac{\sin\omega}{\omega}$

Now the real part of answer to part (a) is

$\mathrm{Re}\{-\dfrac{e^{j\omega}}{j\omega}\} = \dfrac{1}{\omega}\mathrm{Re}\{j(\cos\omega - j\sin j\omega)\} = \dfrac{\sin\omega}{\omega}$

(c) the FT of the odd part of x(t) is same as j times imaginary part of the answer to part (a),we have

$\mathrm{Im}\{\dfrac{\sin\omega}{j\omega^2}-\dfrac{e^{-j\omega}}{j\omega}\} =-\dfrac{\sin\omega}{\omega^2}+\dfrac{\cos\omega}{\omega}$

Therefore ,the desired result is

FT{Odd part of x(t)}$= \dfrac{\sin\omega}{j\omega^2} = \dfrac{\cos\omega}{j\omega}$

4.10（a） we know from table 4.2 that

$$\dfrac{\sin t}{\pi t} \xleftrightarrow{\ FT\ } \text{Rectangular} \quad \text{function y(}\,j\omega\,) [\quad \text{see figure s4.10}]$$

Therefore

$$(\dfrac{\sin t}{\pi t})^2 \xleftrightarrow{\ FT\ } (1/2\,\pi\,)[\ \text{Rectangular} \quad \text{function y(}\,j\omega\,)*\text{Rectangular} \quad \text{function y(}\,j\omega\,)]$$

This is a triangular function $y_1(\,j\omega)$ as show in the figure s4.10
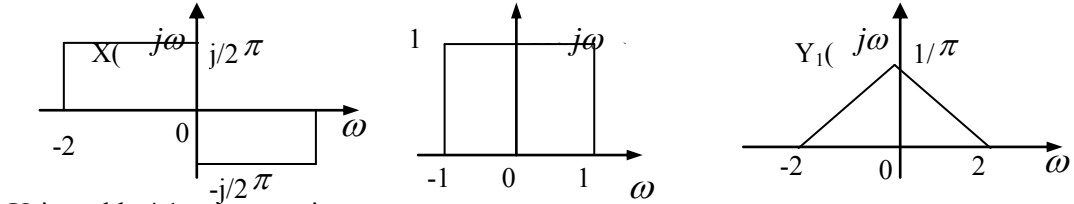
Using table 4.1 w　　ite

$$\text{t } (\dfrac{\sin t}{\pi t}) \xleftrightarrow{\ FT\ } \text{x(}\,j\omega\,)=\text{j}\ \dfrac{dy_1(\,j\omega)}{d\omega}$$

this is as show in the figure above .x($j\omega$) may be expressed mathematically as

$$\text{x(}\,j\omega\,)=\begin{cases} j/2\pi, -2 \le \omega < 0 \\ -j/2\pi, 0 \le \omega < 2 \\ 0, otherwise \end{cases}$$

(b) using Pareseval's relation

$$\int_{-\infty}^{\infty} t^2 (\dfrac{\sin t}{\pi t})^4 dt = \dfrac{1}{2\pi} \int_{-\infty}^{\infty} |\,x(j\omega)\,|^2\ d\omega = \dfrac{1}{2\pi^3}$$

4.11 We know that

$$x(3t) \xleftrightarrow{\ FT\ } \dfrac{1}{3} X(j\dfrac{\omega}{3}), h(3t) \xleftrightarrow{\ FT\ } \dfrac{1}{3} H(j\dfrac{\omega}{3})$$

Therefore,

$$G(\,j\omega) = \mathcal{F}T\{x(3t)*h(3t)\} = \dfrac{1}{9} X(j\dfrac{\omega}{3})H(j\dfrac{\omega}{3})$$

Now note that

$$Y(\,j\omega) = \mathcal{F}T\{x(t)*h(t)\} = X(\,j\omega)H(\,j\omega)$$

From this we may write

$$Y(j\dfrac{\omega}{3}) = X(j\dfrac{\omega}{3})H(j\dfrac{\omega}{3})$$

Using this in eq.(**),we have

$$G(\,j\omega) = \dfrac{1}{9} Y(j\dfrac{\omega}{3})$$

and

$$g(t) = \dfrac{1}{3} y(3t)$$

Therefore, A=1/3 and B=3.

4.12 (a) From Example 4.2 we know that

$$e^{-|t|} \xleftrightarrow{\ FT\ } \dfrac{2}{1+\omega^2}$$

Using the differentiation in frequency property, we have

$$te^{-|t|} \xleftrightarrow{\ FT\ } j\dfrac{d}{d\omega}\dfrac{2}{1+\omega^2} = -\dfrac{4\,j\omega}{(1+\omega^2)}$$

(b) The duality property states that if

$$g(t) \xleftrightarrow{\ FT\ } G(\,j\omega)$$

then

$$G(t) \xleftrightarrow{\ FT\ } 2\pi g(\,j\omega)$$

Now since

$$te^{-|t|} \xleftrightarrow{\ FT\ } -\dfrac{4\,j\omega}{(1+\omega^2)}$$

we may use duality to write

$$-\frac{4jt}{(1+t^2)^2} \xleftrightarrow{FT} 2\pi\omega e^{-|\omega|}$$

Multiplying both sides by j, we obtain

$$\frac{4t}{(1+t^2)^2} \xleftrightarrow{FT} j2\pi\omega e^{-|\omega|}$$

4.13 (a)Taking the inverse Fourier transform of X(j ω),we obtain

$$x(t) = \frac{1}{2\pi} + \frac{1}{2\pi}e^{j\pi t} + \frac{1}{2\pi}e^{j5t}$$

The signal x(t) is therefore a constant summed with two complex exponentials whose fundamental frequencies are 2 π /5rad/sec and 2 rad/sec, These two complex exponentials are not harmonically related. That is ,the fundamental frequencies of these complex exponentials can never be integral multiples of a common fundamental frequency . Therefore, the signal is not periodic.

(b)Consider the signal y(t)=x(t)*h(t).From the convolution property, we know that Y(j ω)=X(j ω)H(j ω).Also ,from h(t),we know that

$$H(j\omega) = e^{-j\omega}\frac{2\sin\omega}{\omega}$$

This gives $\quad y(t) = \frac{1}{2\pi} + \frac{1}{2\pi}e^{j5t}$

Therefore, y(t) is a complex exponential summed with a constant. We know that a complex exponential is periodic. Adding a constant to a complex exponential does not affect its periodic. Adding a constant to a complex exponential does not affect its periodicity. Therefore, y(t) will be a signal with a fundamental frequency of 2 π /5.

(c) From the Fourier transform of parts (a) and (b),we see that the answer is yes.

4.14 Taking the Fourier transform of both sides of the equation

$$\mathcal{F}^{-1}\{(1+j\omega)X(j\omega)\} = A2^{-2t}u(t)$$

we obtain

$$X(j\omega) = \frac{A}{(1+j\omega)(2+j\omega)} = A\{\frac{1}{1+j\omega} - \frac{1}{2+j\omega}\}$$

Taking the inverse Fourier transform of the above equation

$$x(t) = Ae^{-t}u(t) - Ae^{-2t}u(t)$$

Using Parseval's relation, we have

$$\int_{-\infty}^{\infty}|X(j\omega)|^2\,d\omega = 2\pi\int_{-\infty}^{\infty}|X(t)|^2\,dt$$

Using the fact that $\int_{-\infty}^{\infty}|X(j\omega)|^2\,d\omega = 2\pi$,we have

$$\int_{-\infty}^{\infty}|X(t)|^2\,dt = 1$$

Substituting the previously obtained expression for x(t) in the above equation, we have

$$\int_0^\infty[A^2e^{-2t} + A^2e^{-4t} - 2A^2e^{-3t}]dt = 1$$

$$A^2/12 = 1$$

$$\Rightarrow A = \sqrt{12}$$

We choose A to be $\sqrt{12}$ instead of $-\sqrt{12}$ because we know that x(t) is none negative.

4.15. since x(t) is real.

$$Ev\{x(t)\} = \frac{x(t)+x(-t)}{2} \xleftrightarrow{FT} Re\{X(jw)\}$$

We are given that

$$IFT\{Re\ \{X(jw)\}\} = |t|e^{-|t|}$$

Therefore

$$Ev\{x(t)\} = \frac{x(t)+x(-t)}{2} = |t|e^{-|t|}$$

We also know that x(t)=0 for t≤ 0 .this implies that x(-t) is zero for t>0. we may conclude that

x(t)=2|t|e$^{-|t|}$ for t≥ 0

therefore x(t)=2te^{-t} u(t)

4.16 (a) we may write

$$x(t) = \sum_{-\infty}^{\infty}\frac{\sin(k\pi/4)}{k\pi/4}\delta(t-k\pi/4)$$

$$= \frac{\sin(t)}{\pi t}\sum_{-\infty}^{\infty}\pi\delta(t-k\pi/4)$$

Therefore, g(t) = $\sum_{-\infty}^{\infty} \pi\delta(t - k\pi/4)$

(b) Since g(t) is an impulse train , its Fourier transform G(jw) is also an impulse train
From Table 4.2,

$$G(j\omega) = \pi \frac{2\pi}{\pi/4} \sum_{-\infty}^{\infty} \delta(\omega - \frac{2\pi k}{\pi/4})$$

$$= 8\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 8k)$$

We see that G(jw) is periodic with a period of 8.Using the multiplication property,
We know that

$$X(j\omega) = \frac{1}{2\pi}[FT\{\frac{\sin t}{\pi t}\} * G(j\omega)]$$

If we denote $FT\{\frac{\sin t}{\pi t}\}$ by A(jw),then

$$X(j\omega) = 1/2\pi[A(j\omega) * 8\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 8k)]$$

$$= 4 \sum_{k=-\infty}^{\infty} A(j\omega - 8k)$$

$X(j\omega)$ may thus be viewed as a replication of 4A(jw) every 8 rad/sec. this is obviously
Periodic.
Using Table 4.2 , we obtain

$$A(j\omega) = \begin{cases} 1. & |w| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Therefore, we may specify X(jw) over one period as

$$X(jw) = \begin{cases} 4, & |w| \leq 1 \\ 0, & 1 < |w| \leq 4 \end{cases}$$

4.17. (a) From Table 4.1 ,we know that a real and odd signal signal x(t) has a purely imaginary and odd Fourier transform X(jw). Let us now consider the purely imaginary and odd signal jx(t) ,using linearity ,we obtain the Fourier transform of this signal to be jX(jw) . The function jX(jw) will clearly be real and odd. Therefore, the given statement is **false.**

(b) An odd Fourier transform corresponds to an odd signal, while an even Fourier transform
Corresponds to an even signal. .The convolution of an even Fourier transform with an
odd Fourier may be viewed in the time domain as a multiplication of an even and odd signal Such a multiplication will always result in a an odd time signal .The Fourier transform of this odd signal will always be odd ,Therefore ,the given statement is true.

4.18. Using Table 4.2, we see that the rectangular pulse $x_1(t)$ shown in Figure S4.18 has a Fourier transform $X_1(jw) = \sin(3w)/w$. Using the convolution property of the Fourier transform,
We may write

$$x_2(t) = x_1(t) * x_1(t) \xleftarrow{FT} x_2(jw) = x_1(jw)x_1(jw) = (\frac{\sin(3w)}{w})^2$$

The signal $x_2(t)$ is shown in Figure S4.18. Using the shifting property ,we also note that

$$\frac{1}{2}x_2(t+1) \xleftarrow{FT} \frac{1}{2}e^{j\omega}(\frac{\sin w 3}{w})^2$$

And $\frac{1}{2}x_2(t-1) \xleftarrow{FT} \frac{1}{2}e^{-j\omega}(\frac{\sin(3\omega)}{\omega})^2$

Adding the two above equation ,we obtain

$$h(t) = \frac{1}{2}x_2(t+1) + \frac{1}{2}x_2(t-1) \xleftarrow{FT} \cos(\omega)(\frac{\sin(3\omega)}{\omega})^2$$

The signal h(t) is as shown in Figure S4.18 .we note that h(t) has the given Fourier transform H(jw)

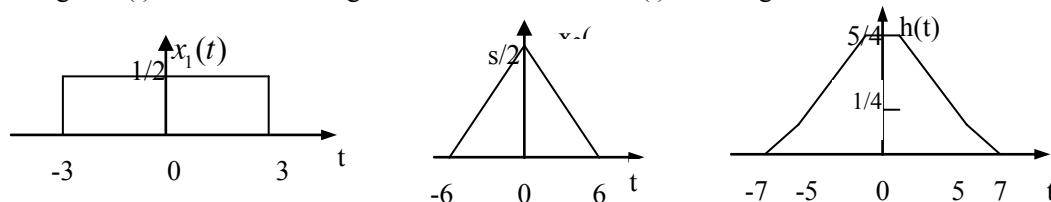
Figure S4.18

Mathematically h(t) may be expressed as

$$h(t) = \begin{cases} \frac{5}{4}, & |t| < 1 \\ -\frac{|t|}{4} + \frac{3}{2}, & 1 \leq |t| \leq 5 \\ -\frac{|t|}{8} + \frac{7}{8} & 5 < |t| \leq 7 \end{cases}$$

$$0 \qquad\qquad \text{otherwise}$$

4.19 we know that

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}.$$

Since it is given that $y(t) = e^{-3t}u(t) - e^{-4t}u(t)$, we can compute Y(jw) to be

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}$$

Since H(jw) = 1/(3+jw), we have

$$X(\text{jw}) = \frac{Y(j\omega)}{X(j\omega)} = 1/(4+j\omega)$$

Taking the inverse Fourier transform of X(jw), we have

$$x(t) = e^{-4t}u(t)$$

4.20 From the answer to Problem 3.20 we know that the frequency response of the circuit is

$$\text{H(jw)} = \frac{1}{-w^2 + j\omega + 1}$$

Breaking this up into partial fractions, we may write

$$H(j\omega) = -\frac{1}{j\sqrt{3}}\left[\frac{-1}{\frac{1}{2}-\frac{\sqrt{3}}{2}j+j\omega} + \frac{-1}{\frac{1}{2}+\frac{\sqrt{3}}{2}j+j\omega}\right]$$

Using the Fourier transform pairs provided in Table 4.2, we obtain the Fourier transform of H(jw) to be

$$h(t) = -\frac{1}{j\sqrt{3}}[-e^{(-\frac{1}{2}+\frac{\sqrt{3}}{2}j)t} + e^{(-\frac{1}{2}-\frac{\sqrt{3}}{2}j)t}]u(t).$$

Simplifying ,

$$h(t) = \frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \sin(\frac{\sqrt{3}}{2}t)u(t).$$

4.21. (a)   The given signal is

$$e^{-at}\cos(\omega_0 t)u(t) = \tfrac{1}{2} e^{-at} e^{j\omega_0 t}u(t) + \tfrac{1}{2} e^{-at} e^{-j\omega_0 t}u(t)$$

Therefore .

X(jw) $=$

$$\frac{1}{2(\alpha - j\omega_0 + j\omega)} + \frac{1}{2(\alpha + j\omega_0 + j\omega)} = \frac{\alpha + j\omega}{(\alpha + j\omega)^2 + \omega_0^2}$$

(b). the given signal is

$$\underset{x(t)}{e^{-3t}} \sin(2t)u(t) + e^{3t}\sin(2t)u(-t).$$

we have

$$x_1(t) = e^{-3t}\sin(2t)u(t) \xleftarrow{FT} X_1(j\omega) = \frac{1/2\,j}{3-j2+j\omega} - \frac{1/2\,j}{3+j2+j\omega} = \frac{2}{(3+j\omega)^2+4}$$

Also

$$x_2(t) = e^{3t}\sin(2t)u(-t) = -x_1(-t) \xleftarrow{FT} X_2(j\omega) = -X_1(-jw) = \frac{1/2\,j}{3-j2-j\omega} - \frac{1/2\,j}{3+j2-j\omega}$$

Therefore

$$X(\text{jw})= X_1(j\omega) + X_2(j\omega) = \frac{3\,j}{9+(\omega+2)^2} - \frac{3\,j}{9+(\omega-2)^2}.$$

(c) Using the Fourier transform analysis equation (4.9) we have

$$X(j\omega) = \frac{2\sin\omega}{\omega} + \frac{\sin(\omega)}{\pi-\omega} - \frac{\sin(\omega)}{\pi+\omega} = \frac{2\pi^2\sin\omega}{\omega(\pi^2-\omega^2)}$$

(d) Using the Fourier transform analysis equation (4.9) we have

$$X(j\omega) = \frac{1}{1-\alpha e^{-j\omega T}}$$

(e) we have

$$x(t) = (1/2j)te^{-2t}e^{j4t}u(t) - (1/2j)te^{-2t}e^{-j4t}u(t)$$

Therefore

$$X(j\omega) = \frac{1/2j}{(2-j4+j\omega)^2} - \frac{1/2j}{(2+j4-j\omega)^2}$$

(f). We have $\qquad x_1(t) = \frac{\sin\pi t}{\pi t} \xleftarrow{\quad FT \quad} X_1(j\omega) = \begin{cases} 1 & |\omega| < \pi \\ 0 & otherwise \end{cases}$

Also $\qquad x_2(t) = \frac{\sin 2\pi(t-1)}{\pi(t-1)} \xleftarrow{\quad FT \quad} X_2(j\omega) = \begin{cases} e^{-2\omega} & |\omega| < 2\pi \\ 0 & otherwise \end{cases}$

$$x(t) = x_1(t)x_2(t) \xleftarrow{\quad FT \quad} X(j\omega) = \frac{1}{2\pi}\{X_1(j\omega) * X_2(j\omega)\}$$

Therefore ,

$$X(jw) = \begin{cases} e^{-j\omega} & |\omega| < \pi \\ (1/2\pi)(3\pi+\omega)e^{-j\omega}, & -3\pi < w < -\pi \\ (1/2\pi)(3\pi+\omega)e^{-j\omega}, & \pi < \omega < 3\pi \\ 0 & otherwise \end{cases}$$

(g)   Using the Fourier transform analysis eq. (4.9) we obtain

$$X(j\omega) = \frac{2j}{\omega}\left[\cos 2\omega - \frac{\sin\omega}{w}\right]$$

if

$$x_1(t) = \sum_{k=-\infty}^{\infty}\delta(t-2k),$$

Then

$$X(t) = 2x_1(t) + x_1(t-1).$$

Therefore

$$X(j\omega) = X_1(j\omega)[2+e^{-\omega}] = \pi\sum_{-\infty}^{\infty}\delta(\omega-k\pi)[2+(-1)^k].$$

Using the Fourier transform analysis eq.(4.9) we obtain

$$X(j\omega) = \frac{1}{j\omega} + \frac{2e^{-j\omega}}{-\omega^2} - \frac{2e^{-j\omega}-2}{j\omega^2}$$

x(t) is periodic with period 2 therefore .

$$X(j\omega) = \pi\sum_{k=-\infty}^{\infty}X(jk\pi)\delta(\omega-k\pi)$$

Where X(jw) is the fourier transform of one period of x(t) .that is

$$X(j\omega) = \frac{1}{1-e^{-2}}\left[\frac{1-e^{-2(1+j\omega)}}{1+j\omega} - \frac{e^{-2}[1-e^{-2(1+j\omega)}]}{1-j\omega}\right]$$

4.22. (a) $x(t) = \begin{cases} e^{j2\pi t} & |t| < 3 \\ 0 & otherwise \end{cases}$

   (b) x(t)$= \frac{1}{2}e^{-j\pi/3}\delta(t-4) + \frac{1}{2}e^{j\pi/3}\delta(t+4)$

the Fourier transform synthesis eq.(4.8) may be written as

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(j\omega)|e^{j\angle X(j\omega)}e^{j\omega t}d\omega$$

   From the given figure we have

$$x(t) = \frac{1}{\pi}\left[\frac{\sin(t-3)}{t-3} + \frac{\cos(t-3)-1}{(t-3)^2}\right]$$

x(t) $= \frac{2j}{\pi}\sin t + \frac{3}{\pi}\cos(2\pi t)$

Using the Fourier transform synthesis equation (4.8) ,

$$x(t) = \frac{\cos 3t}{j\pi t} + \frac{\sin t - \sin 2t}{j\pi t^2}$$

4.23. For the given signal $x_0(t)$, we use the Fourier transform analysis eq.(4.8) to evaluate the corresponding Fourier transform

$$X_0(j\omega) = \frac{1-e^{-(1+j\omega)}}{1+j\omega}$$

we know that

$$x_1(t) = x_0(t) + x_0(-t)$$

Using the linearity and time reversal properties of the Fourier transform we have

$$X_1(j\omega) = X_0(j\omega) + X_0(-j\omega) = \frac{2-2e^{-1}\cos\omega - 2\omega e^{-1}\sin\omega}{1+\omega^2}$$

（ii）we know that

$$x_2(t) = x_0(t) - x_0(-t)$$

Using the linearity and time reversal properties of Fourier transform we have

$$X_2(j\omega) = X_0(j\omega) - X_0(-j\omega) = \frac{-2\omega + 2e^{-1}\sin\omega + 2\omega e^{-1}\cos\omega}{1+\omega^2}$$

（iii）we know that

$$x_3(t) = x_0(t) + x_0(t+1)$$

Using the linearity and time shifting properties of Fourier transform we have

$$X_3(j\omega) = X_0(j\omega) + e^{j\omega}X_0(-j\omega) = \frac{-2\omega + 2e^{-1}\sin\omega + 2\omega e^{-1}\cos\omega}{1+\omega^2}$$

（iv）we know that

$$x_3(t) = x_0(t) + x_0(t+1)$$

Using the differentiation frequency property $X_4(j\omega) = j\dfrac{d}{d\omega}X_0(j\omega)$

Therefore,

$$X_4(j\omega) = \frac{1+j\omega e^{-1-j\omega}}{(1+j\omega)^2}$$

4.24 (a) (ii) For $\mathrm{Re}\{X(j\omega)\}$ to be 0, the signal x(t) must be real or odd. Therefore , signals in figures (a) and (c) have this property.

(ii) For $\mathrm{Im}\{X(j\omega)\}$ to be 0, the signal x(t) must be real or even. Therefore , signals in figures (e) and (f) have this property.

(iii)For these exist a real α such that $e^{-j\alpha\omega}X(j\omega)$ is real ,we require that x(t+α) be a real and even signal. Therefore , signals in figures (a), (b), (c), (d), and (f) have this property.

(iv) For the condition to be true , x(0)=0,. Therefore ,signals in figures (a), (b), (c), (d), and (f)have this property.

(v) For the condition to be true the derivative of x(t) has to be zero at t=0 . Therefore , signals in figures (b), (c), (e), and (f)have this property.

(vi) For this to be true , the signals x(t) has to be periodic .Only the signals in figures (a) has this property.

(b) For signal to satisfy only properties (i), (iv), and (v), it must be real and odd , and
x(t)=0 , $x'(0) = 0$
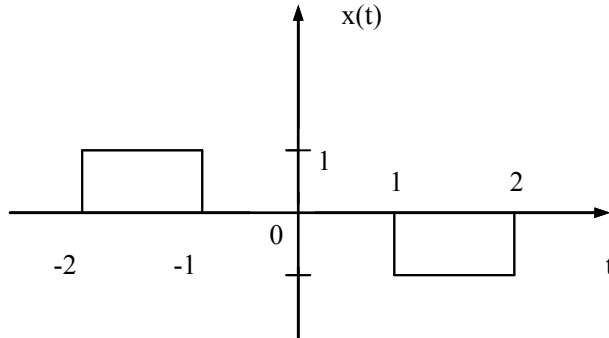
the signal shown below is example of that.

Figure 4.24

4.25 (a) Note that y(t)=x(t+1) is a real and even signal. Therefore , $Y(j\omega)$ is also real and even .this implies

that $\triangleleft Y(j\omega)$=0.Also ,since $Y(j\omega) = e^{j\omega}X(j\omega)$,we know that $\triangleleft Y(j\omega) = -\omega$

(b) we have

$$X(j0) = \int_{-\infty}^{\infty} x(t)dt = 7$$

(c)we have

$$\int_{-\infty}^{\infty} x(j\omega)d\omega = 2\pi x(0) = 4\pi$$

(d) Let $Y(j\omega)\dfrac{2\sin\omega}{\omega}e^{2j\omega}$. The corresponding signal y(t) is

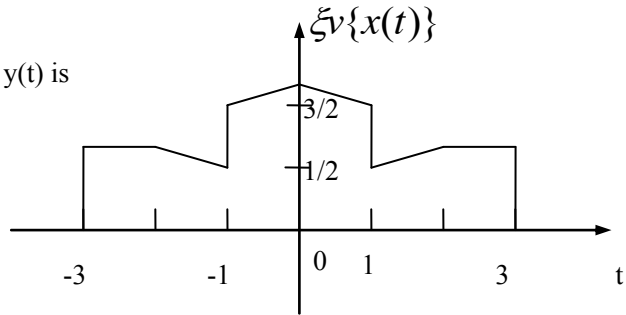$$y(t) = \begin{cases} 1, -3 < t < -1 \\ 0, otherwize \end{cases}$$

Then the given integral is

$$\int_{-\infty}^{\infty} X(j\omega)Y(j\omega)d\omega = 2\pi\{x(t) * y(t)\}_{t=0} = 7\pi$$

(e) we have

$$\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi\int_{-\infty}^{\infty} |x(t)|^2 dt = 26\pi$$

(f)The inverse Fourier transform of $\Re e\{X(j\omega)\}$ is the $\xi v\{x(t)\}$ which is [x(t)+x(-t)]/2.this is as shown is the figure below.

4.26 (a) (i) We have

$$Y(j\omega) = X(j\omega)H(j\omega) = [\frac{1}{(2+j\omega)^2}][\frac{1}{4+j\omega}] = \frac{(1/4)}{4+j\omega} - \frac{(1/4)}{(2+j\omega)^2}$$

Taking the inverse Fourier transform we obtain

$$y(t) = \frac{1}{4}e^{-4t}u(t) - \frac{1}{4}e^{-2t}u(t) + \frac{1}{2}te^{-2t}u(t)$$

(ii) We have

$$Y(j\omega) = X(j\omega)H(j\omega) = [\frac{1}{(2+j\omega)^2}][\frac{1}{(4+j\omega)^2}] = \frac{-(1/4)}{2+j\omega} + \frac{(1/4)}{(2+j\omega)^2} + \frac{(1/4)}{4+j\omega} + \frac{(1/4)}{(4+j\omega)^2}$$

Taking the inverse Fourier transform we obtain

$$y(t) = \frac{1}{4}e^{-2t}u(t) + \frac{1}{4}te^{-2t}u(t) + \frac{1}{4}e^{-4t}u(t) + \frac{1}{4}te^{-4t}u(t)$$

(iii) We have

$$Y(j\omega) = X(j\omega)H(j\omega) = [\frac{1}{1+j\omega}][\frac{1}{1-j\omega}] = \frac{1/2}{1+j\omega} + \frac{1/2}{1-j\omega}$$

Taking the inverse Fourier transform we obtain

$$y(t) = \frac{1}{2}e^{|-t|}$$

(b) By direct convolution of x(t) with h(t) we obtain

$$y(t) = \begin{cases} 0, t < 1 \\ 1 - e^{-(t-1)}, 1 < t \le 5 \\ e^{-(t-5)} - e^{-(t-1)}, t > 5 \end{cases}$$

Taking the Fourier transform of y(t),

$$Y(j\omega) = \frac{2e^{-j3\omega}\sin(2\omega)}{\omega(1+j\omega)} = [\frac{e^{-2j\omega}}{1+j\omega}]\frac{e^{-j\omega}2\sin(2\omega)}{\omega} = X(j\omega)H(j\omega)$$

4.27 (a) The Fourier transform X($j\omega$) is

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_1^2 e^{-j\omega t}dt - \int_2^3 e^{-j\omega t}dt = 2\frac{\sin(\omega/2)}{\omega}\{1 - e^{-j\omega}\}e^{-j3\omega/2}$$

(b) The Fourier series coefficients a_k are

$$a_k = \frac{1}{T}\int_{\langle T \rangle} \overline{x}(t)e^{-j\frac{2\pi}{T}kt} = \frac{1}{2}\{\int_1^2 e^{-j\frac{2\pi}{T}kt}dt - \int_2^3 e^{-j\frac{2\pi}{T}kt}dt\} = \frac{\sin(k\pi/2)}{k\pi}\{1 - e^{-jk\pi}\}e^{-j3k\pi/2}$$

Comparing the answer to parts (a) and (b) ,it is clear that

$$a_k = \frac{1}{T}X(j\frac{2\pi k}{T})$$

Where T=2.

4.28

(a) Form Table 4.2 we know that

$$p(t)\sum_{n=\infty}^{\infty} a_n e^{jn\omega_0 t} \xleftrightarrow{FT} p(j\omega) = 2\pi\sum_{n=\infty}^{\infty} a_k \delta(\omega - k\omega_0)$$

From this ,

$$Y(j\omega) = \frac{1}{2\pi}\{X(j\omega)*H(j\omega)\} = \sum_{n=\infty}^{\infty} a_k X(j(\omega - k\omega_0))$$

(b)The spectra are sketched in Figure S4.28

4.29 (i)   We have

$$X_a(j\omega) = |X(j\omega)|e^{j\angle X(j\omega)-ja\omega} = X(j\omega)e^{-ja\omega}$$

From the shifting property we know that

$$x_a(t) = x(t-a)$$

(ii)   We have

$$X_b(j\omega) = |X(j\omega)|e^{j\angle X(j\omega)+jb\omega} = X(j\omega)e^{-jb\omega}$$

From the shifting property we know that

$$x_b(t) = x(t+b)$$

(iii)   We have

$$X_c(j\omega) = |X(j\omega)|e^{j\angle X(j\omega)} = X^*(j\omega)$$

From the conjugation property we know that

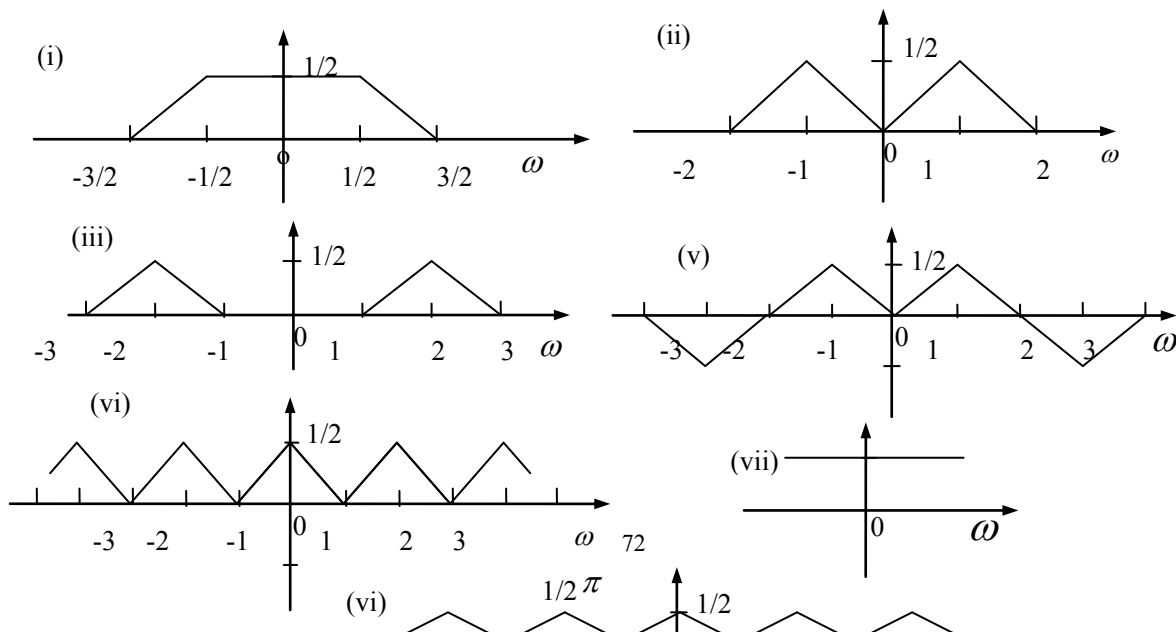$$x_c(t) = x^*(-t)$$

Since x(t) is real , $x_c(t) = x(-t)$

(i)

(ii)

(iii)

(v)

(vi)

(vii)

(vi)

$1/2\pi$ $1/2$

(iv)   We have

$$X_d(j\omega) = |X(j\omega)|e^{j\sphericalangle X(j\omega)+jd\omega} = X^*(j\omega)e^{jd\omega}$$

From the conjugation , time reversal and time shifting properties we know that

$$x_d(t) = x^*(-t-d)$$

Since x(t) is real , $x_d(t) = x(-t-d)$

4.30 (a) We have that

$$w(t) = \cos t \xleftarrow{\ FT\ } W(j\omega) = \pi[\delta(\omega-1)+\delta(\omega+1)]$$

   And

$$g(t) = x(t)\cos t \xleftarrow{\ FT\ } G(j\omega) = \frac{1}{2\pi}[X(j\omega)*W(j\omega)]$$

Therefore,

$$G(j\omega) = \frac{1}{2}X(j(\omega-1))+\frac{1}{2}X(j(\omega+1))$$

Since $G(j\omega)$ is as shown in Figure s4.30 , it is clear from the above equation that $X(j\omega)$ is as shown in the Figure s4.30.
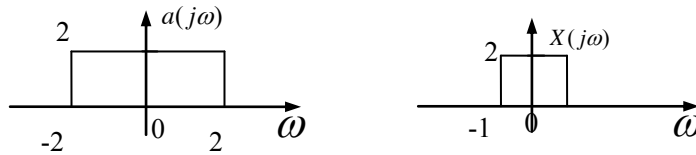

Figure s4.30

   (b) $X_1(j\omega)$ is as shown in the Figure s4.30

4.31 (a) We have

$$x(t) = \cos t \xleftarrow{\ FT\ } X(j\omega) = \pi[\delta(\omega+1)+\delta(\omega-1)]$$

(i) We have

$$h_1(t) = u(t) \xleftarrow{\ FT\ } H_1(j\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$$

Therefore,

$$Y(j\omega) = X(j\omega)H_1(j\omega) = \frac{\pi}{j}[\delta(\omega+1)-\delta(\omega-1)]$$

Taking the inverse Fourier transform ,we obtain

$$y(t) = \sin(t)$$

(ii)We have

$$h_2(t) = -2\delta(t)+5e^{-2t}u(t) \xleftarrow{\ FT\ } H_2(j\omega) = -2+\frac{5}{2+j\omega}$$

Therefore,

$$Y(j\omega) = X(j\omega)H_1(j\omega) = \frac{\pi}{j}[\delta(\omega+1) - \delta(\omega-1)]$$

Taking the inverse Fourier transform ,we obtain

$$y(t) = \sin(t)$$

(iii)we have

$$h_3(t) = 2t\,e^{-t}\,u(t) \longleftrightarrow H_2(j\omega) = \frac{2}{(1+j\omega)^2}.$$

Therefore,

$$Y(j\omega) = X(j\omega)H_1(j\omega) = \frac{\pi}{j}[\delta(j\omega+1) - \delta(j\omega-1)].$$

Taking the inverse Fourier transform ,we obtain

$$y(t) = \sin(t).$$

(b)An LTI system with impulse response

$$h_4(t) = \frac{1}{2}[h_1(t) + h_2(t)]$$

will have the same response to x(t) =cos(t),we can find other such impulse responses by suitably scaling and linearly combining h1(t),h2(t),and h3(t).

4.32 Note that h(t)=h(t-1); where

$$h1(t) = \frac{\sin 4t}{\pi t}$$

The Fourier transform H1(jw) of h1(t) is as shown in figure s4.32.

From the above figure it is clear that h1(t) is the impulse response of an ideal lowpass filter whose passband is in the rage |w|<4.therefore, h(t) is the impulse response of an ideal lowpass filter shifted by one to the right .using the shift property,

$$H(j\omega) = \begin{cases} e^{-t}, & |\omega| < 4 \\ 0, & otherwise \end{cases}$$

(a) we have

$$X_1(j\omega) = \pi e^{j\frac{\pi}{12}}\delta(\omega-6) + \pi e^{j\frac{\pi}{12}}\delta(\omega+6)$$

It is clear that

$$Y_1(j\omega) = X_1(j\omega)\,H(j\omega) = 0 \Longrightarrow y_1(t) = 0$$

this result is equivalent to saying that x1(jw) is zero in the passband of H(jw).

(b) We have

$$X_2(j\omega) = \frac{\pi}{j}\left[\sum_{k=0}^{\infty}(\frac{1}{2})^k\{\delta(\omega-3k) - \delta(\omega+3k)\}\right].$$

Therefore,

$$Y_2(j\omega) = X_2(j\omega)\,H(j\omega) = \frac{\pi}{j}\left[\sum_{k=0}^{\infty}(\frac{1}{2})\{\delta(\omega-3) - \delta(\omega+3)\}e^{-j\omega}\right].$$

this implies that

$$y_2(t) = {}^1\!/_2\,\sin(3t-1).$$

we may have obtained the same result by noting that only the sinusoid with frequency 3 in x2(jw) lies in the passband of H(jw).

(c) We have

$$X_2(j\omega) = \begin{cases} e^{-j\omega}, & |\omega| < 4 \\ 0, & otherwise \end{cases}$$

$$Y_3(j\omega) = X_3(j\omega)\,H(j\omega) = X_3(j\omega)\,e^{-j\omega}$$

this implies that

$$y_3(t) = x_3(t-1) = \frac{\sin(4t)}{\pi t}.$$

we may have obtained the same result by noting that x2(jw) lies entirely in the
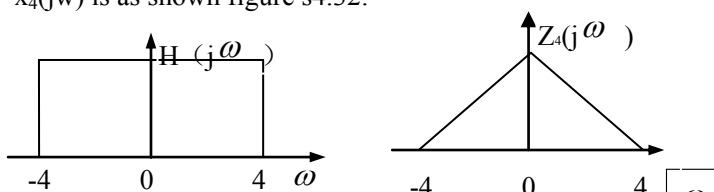
(d) x4(jw) is as shown figure s4.32.

Figure S4.32

Therefore,
$$Y_4(j\omega) = X_4(j\omega)\,H(j\omega) = X_4(j\omega)\,e^{-j\omega}$$

This implies that
$$Y_4(t) = x_4(t-1) = \left(\frac{\sin(2(t-1))}{\pi(t-1)}\right)^2.$$

We may have obtained the same result by noting that x4(jw) lies entirely in the passband of H(jw) .

4.33 (a)Taking the Fourier transform of both sides of the given differential equation .we obtain
$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2}{-\omega^2 + 2j\omega + 8}.$$

Using partial fraction expansion, we obtain
$$H(j\omega) = \frac{2}{j\omega + 2} - \frac{1}{j\omega + 4}$$

Taking the inverse Fourier transform
$$h(t) = e^{-2t}u(t) - e^{-4t}u(t)$$

(b) for the given signal x(t) ,we have
$$X(j\omega) = \frac{1}{(2+j\omega)^2}$$

Therefore,
$$Y(j\omega) = X(j\omega)H(j\omega) = \frac{2}{(-\omega^2 + 2j\omega + 8)}\frac{1}{(2+j\omega)^2}$$

Using partial fraction expansion, we obtain
$$Y(j\omega) = \frac{1/4}{j\omega + 2} - \frac{1/2}{(j\omega+2)^2} + \frac{1}{(j\omega+2)^3} - \frac{1/4}{j\omega+4}$$

Taking the inverse Fourier transform
$$h(t) = \frac{1}{4}e^{-2t}u(t) - \frac{1}{2}e^{-2t}u(t) + t^2 e^{-2t}u(t) - \frac{1}{4}e^{-4t}u(t)$$

(c) Taking the Fourier transform of both sides of the given differential equation ,we obtain
$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2(-\omega^2 - 1)}{-\omega^2 + \sqrt{2}j\omega + 1}.$$

Using partial fraction expansion ,we obtain
$$H(j\omega) = 2 + \frac{-\sqrt{2} - 2\sqrt{2}j}{j\omega - \frac{-\sqrt{2}+j\sqrt{2}}{2}} + \frac{-\sqrt{2} + 2\sqrt{2}j}{j\omega - \frac{-\sqrt{2}-j\sqrt{2}}{2}}$$

Taking the inverse Fourier transform
$$h(t) = 2\delta(t) - \sqrt{2}(1+2j)e^{-(1+j)t/\sqrt{2}}u(t) - \sqrt{2}(1-2j)e^{(-1-j)t/\sqrt{2}}u(t).$$

4.34.(a)We have
$$\frac{Y(j\omega)}{X(j\omega)} = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}.$$

Cross-multiplying and taking the inverse Fourier transform, we obtain
$$\frac{d^2 y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t).$$

(b)we have
$$H(j\omega) = \frac{2}{2+j\omega} - \frac{1}{3+j\omega}.$$

Taking the inverse Fourier transform we obtain,
$$h(t) = 2e^{-2t}u(t) - e^{-3t}u(t).$$

(c) we have
$$X(j\omega) = \frac{1}{4+j\omega} - \frac{1}{(4+j\omega)^2}$$

Therefore,
$$Y(j\omega) = X(j\omega)H(j\omega) = \frac{1}{(4+j\omega)(2+j\omega)}$$

Finding the partial fraction expansion of Y(jω) and taking the inverse Fourier transform.
$$h(t) = \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}e^{-4t}u(t).$$

4.35 (a)from the given information

$$|H(j\omega)| = \frac{\sqrt{a^2 + \omega^2}}{\sqrt{a^2 + \omega^2}} = 1.$$

Also.

$$\angle H(j\omega) = -\tan^{-1}\frac{\omega}{\alpha} - -\tan^{-1}\frac{\omega}{\alpha} = -2\tan^{-1}\frac{\omega}{\alpha}$$

Also.

$$H(j\omega) = -1 + \frac{2\alpha}{\alpha + j\omega} \quad \Rightarrow h(t) = -\delta(t) + 2ae^{-a}u(t)$$

(b)if a=1,we have

$$|H(j\omega)| = 1. \qquad \angle H(j\omega) = -2\tan^{-1}\omega$$

Therefore,

$$y(t) = \cos(\frac{t}{\sqrt{3}} - \frac{\pi}{3}) - \cos(t - \frac{\pi}{2}) - \cos(\sqrt{3}t - \frac{2\pi}{3}).$$

4.36. (a)the frequency response is

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{3(3 + j\omega)}{(4 + j\omega)(2 + j\omega)}.$$

(b)finding the partial fraction expansion of answer in par (a) and taking its inverse Fourier Transform, we obtain

$$h(t) = \frac{3}{2}[e^{-4t} + e^{-2t}]u(t).$$

(c) we have

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{(9 + 3j\omega)}{8 + 6j\omega - \omega^2}.$$

Cross-multiplying and taking the inverse Fourier transform. we obtain

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 3\frac{dx(t)}{dt} + 9x(t).$$

4.37.(a)Note that

$$x(t) = x_1(t) \quad x_1(t)$$

where

$$x_1(t) = \begin{cases} 1, & |\omega| < \frac{1}{2} \\ 0, & otherwise \end{cases}$$

Also, the Fourier transform $X_1(j\omega)$ of $x_1(t)$ is

$$X_1(j\omega) = 2\frac{\sin(\omega/2)}{\omega}$$

Using the convolution property we have

$$X(j\omega) = X_1(j\omega)X_1(j\omega) = \left[2\frac{\sin(\omega/2)}{\omega}\right]^2$$
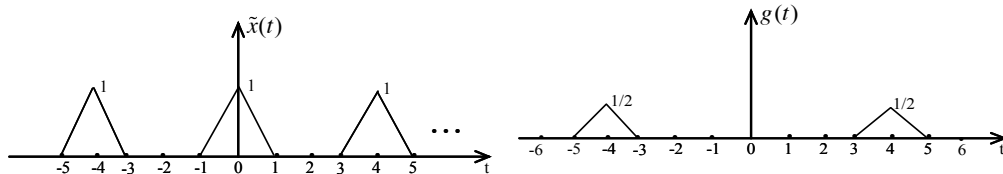
(b) The signal $\tilde{x}(t)$ is shown in Figure S4.37

Figure S4.37

(c)Note that

$$\tilde{X}(j\omega) = X(j\omega)\frac{\pi}{2}\sum_{k=-\infty}^{\infty}\delta(j(\omega - k\frac{\pi}{2})) = G(j\omega)\frac{\pi}{2}\sum_{k=-\infty}^{\infty}\delta(j(\omega - k\frac{\pi}{2}))$$

This may also be written as

$$\tilde{X}(j\omega) = \frac{\pi}{2}\sum_{k=-\infty}^{\infty}X(j\pi k/2)\delta(j(\omega - k\frac{\pi}{2})) = \frac{\pi}{2}\sum_{k=-\infty}^{\infty}G(j\pi k/2)\delta(j(\omega - k\frac{\pi}{2}))$$

Clearly, this is possible only if

$$G(j\pi k/2) = X(j\pi k/2)$$

4.38.(a)Apply a frequency shift to the analysis equation ,we have

(b) We have

$$\omega(t) = e^{j\omega_0 t} \xleftarrow{\quad FS \quad} W(j\omega) = 2\pi\delta(\omega - \omega_0)$$

Also

$$x(t)\omega(t) \xleftrightarrow{FT} \frac{1}{2\pi}[X(j\omega)*W(j\omega)]$$
$$= X(j\omega)*\delta(\omega-\omega_0)$$
$$= X(j(\omega-\omega_0))$$

4.39. (a) From the Fourier analyses equation. We have

$$G(j\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} X(jt)e^{-j\omega t}dt \qquad \text{(S4.39-1)}$$

Also from the Fourier transform equation, we have

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

Switching the variables t and ω, we have

$$x(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jt)e^{j\omega t}dt$$

We may also write this equation as

$$2\pi x(-\omega) = \int_{-\infty}^{\infty} X(jt)e^{-j\omega t}dt$$

Substituting this equation in eq. (S4.39-1), we obtain

$$G(j\omega) = 2\pi x(-\omega)$$

(b) If in part (a) we have $x(t) = \delta(t+B)$, then we would have $g(t) = X(jt) = e^{jBt}$ and

$$G(j\omega) = 2\pi x(-\omega) = 2\pi\delta(-\omega+B) = 2\pi\delta(\omega-B)$$

4.40. When n=1, $x_1(t) = e^{-at}u(t)$ and $X_1(j\omega) = 1/(a+j\omega)$

When n=2, $x_2(t) = e^{-at}u(t)$ and $X_2(j\omega) = 1/(a+j\omega)^2$

Now, let us assume that the given statement is true when n=m, that is,

$$x_m(t) = \frac{t^{m-1}}{(m-1)!}e^{-at}u(t) \xleftrightarrow{FS} X_m(jw) = \frac{1}{(1+j\omega)^m}$$

For n=m+1 we may use the differentiation in frequency property to write,

$$x_{m+1}(t) = \frac{t}{m}x_m(t) \xleftrightarrow{FS} X_{m+1}(j\omega) = \frac{1}{m}j\frac{dX_m(j\omega)}{d\omega} = \frac{1}{(1+j\omega)^{m+1}}$$

This shows that if we assume that the given statement is true for n=m, then it is true for n=m+1 .Since we also shown that the given statement is true for n=2, we may argue that it is true for n=2+1=3, n=3+1=4, and so on. Therefore, the given statement is true for any n.

4.41. (a) We have

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{2\pi}[X(j\omega)*Y(j\omega)]e^{j\omega t}d\omega$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{2\pi}\left[\int_{-\infty}^{\infty} X(j\theta)Y(j(\omega-\theta))d\theta\right]e^{j\omega t}d\omega$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j(\omega-\theta))e^{j\omega t}d\omega\right]d\theta$$

(b) Using the frequency shift property if the Fourier transform we have

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j(\omega-\theta))e^{j\omega t}d\omega = e^{j\theta t}y(t)$$

(c) Combining the results of parts (a) and (b)

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)e^{j\theta t}y(t)d\theta$$
$$= y(t)\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)e^{j\theta t}d\theta$$
$$= y(t)x(t)$$

4.42. x(t) is periodic signal with Fourier series coefficients a_k. The fundamental frequency of x(t) is ω_f =100 rad/sec. From Section 4.2 we know that the Fourier transform $X(j\omega)$ of x(t) is

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k\delta(\omega-100k)$$

(a) Since

$$y_1(t) = x(t)\cos(\omega_0 t) \xleftrightarrow{FT} Y_1(j\omega) = \frac{1}{2}\{X(j(\omega-\omega_0))+X(j(\omega+\omega_0))\}$$

We have

$$Y_1(j\omega) = \pi\sum_{k=-\infty}^{\infty}[a_k\delta(\omega-100k-\omega_0)+a_k\delta(\omega-100k+\omega_0)]$$

$$= \pi \sum_{k=-\infty}^{\infty} \left[a_{-k}\delta(\omega+100k-\omega_0) + a_k\delta(\omega-100k+\omega_0) \right] \qquad \text{(S4.42-1)}$$

If $\omega_0 = 500$. then the term in the above summation with k=5 becomes

$$\pi a_{-5}\delta(\omega) + \pi a_5\delta(\omega)$$

Since x(t) is real, $a_k = a^*_{-k}$. Therefore, the above expression becomes $2\pi \Re e\{a_5\}\delta(\omega)$,

Which is an impulse at $\omega = 0$. Note that the inverse Fourier transform of $2\pi \Re e\{a_5\}\delta(\omega)$

is $g_1(t) = \text{Im}\{a_5\}$. Therefore ,we now need to find a $H(j\omega)$ such that

$$Y_1(j\omega)H(j\omega) = G_1(j\omega) = 2\pi \Re e\{a_5\}\delta(\omega)$$

We may easily obtain such a $H(j\omega)$ by noting that the other terms (other than that for k=5) in the summation of eq.(S4.42-1) result in impulses at $\omega = 100m, m \neq 0$. Therefore, we may choose any $H(j\omega)$ which is zero for $\omega = 100m$, where $m = \pm 1, \pm 2, \cdots$

Similarly since

$$y_2(t) = x(t)\sin(\omega_0 t) \xleftrightarrow{\ FT\ } Y_2(j\omega) = \frac{1}{2j}\{X(j(\omega-\omega_0)) - X(j(\omega+\omega_0))\}$$

we have

$$Y_2(j\omega) = \frac{\pi}{j} \sum_{k=-\infty}^{\infty} [a_k\delta(\omega-100k-\omega_0) - a_k\delta(\omega-100k+\omega_0)]$$

$$= \frac{\pi}{j} \sum_{k=-\infty}^{\infty} [a_{-k}\delta(\omega+100k-\omega_0) - a_k\delta(\omega-100k+\omega_0)] \qquad \text{(S4.42-2)}$$

If $\omega_0 = 500$, then the term in the above summation with k=5 becomes

$$\frac{\pi}{j}a_{-5}\delta(\omega) - \frac{\pi}{j}a_5\delta(\omega)$$

Since x(t) is real, 2π ,Therefore, the above expression becomes $2\pi \Re e\{a_5\}\delta(\omega)$,

Which is an impulse at $\omega = 0$. Note that the inverse Fourier transform of $2\pi \Re e\{a_5\}\delta(\omega)$

Is $g_2(t) = \text{Im}\{a_k\}$. Therefore ,we now need to find a $H(j\omega)$ such that

$$Y_2(j\omega)H(j\omega) = G_2(j\omega) = 2\pi \Re e\{a_5\}\delta(\omega)$$

(b) An example of a valid $H(j\omega)$ which be the frequency response of an ideal lowpass filter with passband gain of unity and cutoff frequency of 50 rad/sec. In this case,

$$h(t) = \frac{\sin(50t)}{\pi t}$$

4.43. Since

$$y_1(t) = \cos^2 t = \frac{1+\cos(2t)}{2}$$

we obtain

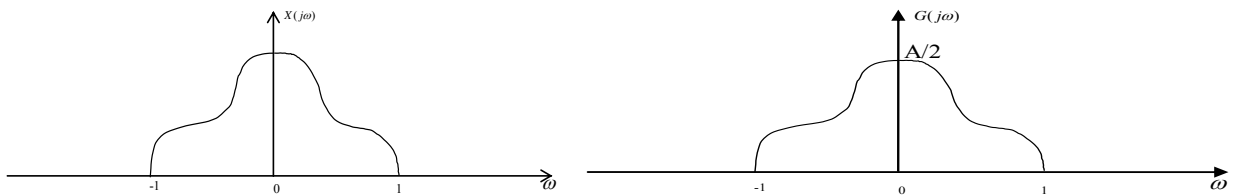$$Y_1(j\omega) = \pi\delta(\omega) + \frac{\pi}{2}\delta(\omega-2) + \frac{\pi}{2}\delta(\omega+2)$$

Therefore,

$$y_2(t) = x(t)y_1(t) = x(t)\cos^2(t) \xleftrightarrow{\ FT\ } Y_2(j\omega) = \frac{1}{2\pi}\{X(j\omega) * Y_1(j\omega)\}$$

This gives

$$Y_2(j\omega) = \frac{1}{2}X(j\omega) + \frac{1}{4}X(j(\omega-2)) + \frac{1}{4}X(j(\omega+2))$$

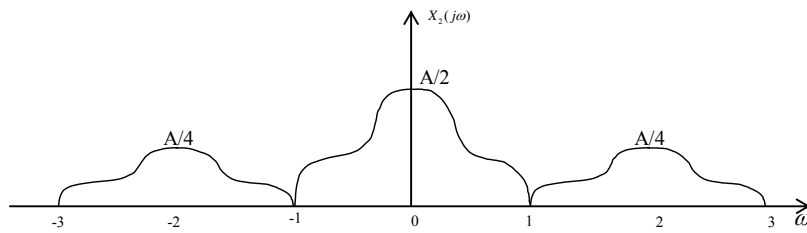$X(j\omega)$ and $Y_2(j\omega)$ are as shown in Figure S4.43.

78

Figure S4.43

Now,

$$y_3(t) = \frac{\sin t}{\pi t} \xleftarrow{\quad FT \quad} Y_3(j\omega) = \begin{cases} 1, & |\omega| < 1 \\ 0, & otherwise \end{cases}$$

Also,

$$g(t) = y_2(t) * y_3(t) \xleftarrow{\quad FT \quad} G(j\omega) = Y_2(j\omega)Y_3(j\omega)$$

From Figure S.4.43 it is clear that

$$G(j\omega) = \frac{1}{2} X(j\omega)$$

Therefore, an LTI system with impulse response $h(t) = \frac{1}{2}\delta(t)$ may be used to obtain $g(t)$ from $x(t)$

4.44. (a) Taking the Fourier transform of both sides of the given differential equation, we have

$$Y(j\omega)[10 + j\omega] = X(j\omega)[Z(j\omega) - 1]$$

Since, $Z(j\omega) = \dfrac{1}{1 + j\omega} + 3$, we obtain from the above equation

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{3 + 2j\omega}{(1 + j\omega)(10 + j\omega)}$$

(b) Finding the partial fraction expansion of $H(j\omega)$ and then taking its inverse Fourier Transform we obtain

$$h(t) = \frac{1}{9} e^{-t} u(t) + \frac{17}{9} e^{-10t} u(t)$$

4.45. We have

$$y(t) = x(t) * h(t) \Rightarrow Y(j\omega)H(j\omega)$$

From Parseval's relation the total energy in y(t) is

$$E = \int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |Y(j\omega)|^2 d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 |H(j\omega)|^2 d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_0 - \Delta/2}^{-\omega_0 + \Delta/2} |X(j\omega)|^2 d\omega + \frac{1}{2\pi} \int_{\omega_0 - \Delta/2}^{-\omega_0 + \Delta/2} |X(j\omega)|^2 d\omega$$

$$E = \frac{1}{\pi} |X(j\omega_0)|^2 \Delta$$

For real x(t) , $|X(-j\omega_0)|^2 = |X(j\omega_0)|^2$. Therefore,

$$E = \frac{1}{\pi} |X(j\omega_0)|^2 \Delta$$

4.46. Let $g_1(t)$ be the response of $H_1(j\omega)$ to $x(t)\cos\omega_c t$ .Let $g_2(t)$ be the response of $H_2(j\omega)$ to $x(t)\sin\omega_c t$ .Then, with reference to Figure 4.30,

$$y(t) = x(t)e^{j\omega_c t} = x(t)\cos\omega_c t + x(t)\sin\omega_c t$$

and

$$\omega(t) = g_1(t) + jg_2(t)$$

Also,

$$f(t) = e^{-j\omega_c t} \omega(t) = [\cos\omega_c t - j\sin\omega_c t][g_1(t) + jg_2(t)]$$

Therefore,

$$\Re e\{f(t)\} = g_1(t)\cos\omega_c t + g_2(t)\sin\omega_c t$$

This is exactly what Figure P4.46 implements.

4.47. (a)We have

79

$$h_e(t) = \frac{h(t) + h(-t)}{2}.$$

Since h(t) is causal, the non-zero portions of h(t) and h(-t) overlap only at t=0.Therefore,

$$h(t) = \begin{cases} 0, & t < 0 \\ h_e(t), & t = 0 \\ 2h_e(t), & t > 0 \end{cases} \qquad \text{(s4.47-1)}$$

Also, from Table 4.1 we have

$$h_e(t) \xleftrightarrow{\ FT\ } \text{Re}\{H(jw)\}.$$

Given Re$\{H(jw)\}$,we can obtain $h_e(t)$ .From $h_e(t)$,we can recover $h(t)$ (and consequently $H(jw)$) by using eq.(s4.47-1).

Therefore, $H(jw)$ is completely specified by Re$\{H(jw)\}$.

(b)If

$$\text{Re}\{H(jw)\} = \cos t = \frac{1}{2}e^{jwt} + \frac{1}{2}e^{-jwt}$$

then,

$$h_e(t) = \frac{1}{2}\delta(t+1) + \frac{1}{2}\delta(t-1).$$

Therefore from eq.(s.4.47-1),

$$h(t) = \delta(t-1).$$

(c)We have

$$h_o(t) = \frac{h(t) + h(-t)}{2}.$$

Since $h(t)$ is causal, the non-zero portions of $h(t)$ and $h(-t)$ overlap only at t=0 and $h_o(t)$ will be zero at t=0,Therefore,

$$h(t) = \begin{cases} 0, & t < 0 \\ unknow, & t = 0 \\ 2h_o(t), & t > 0 \end{cases} \qquad \text{(s4.47-2)}$$

Also, from Table4.1 we have

$$h_o(t) \xleftrightarrow{\ FT\ } \text{Im}\{H(jw)\}.$$

Given Im$\{H(jw)\}$,we can obtain $h_o(t)$ .From $h_o(t)$,we can recover $h(t)$ except for t=0 by using eq.(s4.47-1).If there

Are no singularities in $h(t)$ at t=0,then $H(jw)$ can be recovered from $h(t)$ even if $h(0)$ is unknown. Therefore $H(jw)$

Is completely specified by IM$\{H(jw)\}$ in this case.

4.48. (a)Using the multiplication property we have

$$h(t) = h(t)u(t) \xleftrightarrow{\ FT\ } H(jw) = \frac{1}{2\pi}\left\{H(jw) * [\frac{1}{jw} + \pi\delta(w)]\right\}.$$

The right-hand side may be written as

$$H(jw) = \frac{1}{2}H(jw) + \frac{1}{2\pi j}[H(jw) * \frac{1}{w}].$$

That is,

$$H(jw) = \frac{1}{\pi j}\int_{-\infty}^{\infty}\frac{H(j\eta)}{w-\eta}d\eta.$$

Breaking up $H(jw)$ into real and imaginary parts,

$$H_R(jw) + jH_I(jw) = \frac{1}{\pi j}\int_{-\infty}^{\infty}\frac{H_R(j\eta) + jH_I(j\eta)}{w-\eta}d\eta = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{H_I(j\eta) - jH_R(j\eta)}{w-\eta}d\eta.$$

Comparing real and imaginary parts on both sides, we obtain

$$H_R(jw) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{H_I(j\eta)}{w-\eta}d\eta \quad \text{and} \quad H_I(jw) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{H_R(j\eta)}{w-\eta}d\eta.$$

(b)From eq.(P4.48-3),we may write

$$y(t) = x(t) * \frac{1}{\pi t} \Rightarrow Y(jw) = X(jw)FT\{1/(\pi t)\}. \qquad \text{(S.4.48-1)}$$

Also, from Table 4.2

$$u(t) \xleftrightarrow{\ FT\ } \frac{1}{jw} + \pi\delta(w).$$

Therefore,

$$2u(t) - 1 \xleftrightarrow{\ FT\ } 2\frac{1}{jw}.$$

Using the duality property, we have

$$\frac{2}{jt} \xleftrightarrow{\ FT\ } 2\pi[2u(-w) - 1].$$

or

$$\frac{1}{\pi t} \xleftrightarrow{\ FT\ } j[2u(-w) - 1].$$

Therefore, from eq.(S4.48-1),we have

$$Y(jw) = X(jw)H(jw).$$

where

$$H(jw) = j[2u(-w) - 1] = \begin{cases} -j, w > 0 \\ j, w < 0 \end{cases}.$$

(c)Let *y(t)* be the Hilbert transform of *x(t)=cos(3t)*.Then,

$$Y(jw) = X(jw)H(jw) = \pi[\delta(w-3) + \delta(w+3)]H(jw) = -j\pi\delta(w-3) + j\pi\delta(w+3).$$

Therefore,

y(t)=sin(3t).

4.49.(a) (i)Since *H(jw)* is real and even, *h(t)* is also real and even.

(ii) $|h(t)| = \left| \frac{1}{2\pi} \int_{-\infty}^{\infty} H(jw)e^{jwt}dw \right| \le \frac{1}{2\pi} \int_{-\infty}^{\infty} |H(jw)| e^{jwt}dw$

Since *H(jw)* is real and positive,

$|h(t)| \le \frac{1}{2\pi} \int_{-\infty}^{\infty} H(jw)e^{jwt}dw = h(0).$

Therefore,

max[|h(t)|]=h[0].

(b)The bandwidth of this system is *2w*.

(c)We have

$$B_w H(j0) = \text{Area under } H(jw).$$

Therefore,

$$B_w = \frac{1}{H(j0)} \int_{-\infty}^{\infty} H(jw)dw.$$

(d)We have

$$t_r = \frac{s(\infty)}{h(0)} = \frac{\int_{-\infty}^{\infty} h(t)dt}{\frac{1}{2\pi} \int_{-\infty}^{\infty} H(jw)dw} = \frac{H(j0)}{\frac{1}{2\pi} \int_{-\infty}^{\infty} H(jw)dw} = \frac{2\pi}{B_w}.$$

(e)Therefore,

$$B_w t_r = B_w \frac{2\pi}{B_w} = 2\pi.$$

4.50.(a)We know from problems 1.45 and 2.67 that

$$\phi_{xy}(t) = \phi_{yx}(-t).$$

Therefore,

$$\Phi_{xy}(jw) = \Phi_{yx}(-jw).$$

Since $\phi_{yx}(t)$ is real,

$$\Phi_{xy}(jw) = \Phi^*_{yx}(jw).$$

(b)We may write

$$\phi_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)y(\tau)d\tau = x(t) * y(-t).$$

Therefore,

$$\Phi_{xy}(jw) = X(jw)Y(-jw).$$

Since y(t) is real, we may write this as

$$\Phi_{xy}(jw) = X(jw)Y^*(jw).$$

(c)Using the results of part(b) with y(t)=x(t),

$$\Phi_{xx}(jw) = X(jw)X^*(jw) = |X(jw)|^2 \ge 0.$$

(d)From part(b) we have

$$\Phi_{xy}(jw) = X(jw)Y^*(jw)$$
$$= X(jw)[H(jw)X(jw)]^*$$
$$= \Phi_{xx}(jw)H^*(jw).$$

Also,
$$\Phi_{yy}(jw) = Y(jw)Y^*(jw)$$
$$= [H(jw)X(jw)][H(jw)X(jw)]^*$$
$$= \Phi_{xx}(jw)|H(jw)|^2.$$

(e)From the given information, we have
$$X(jw) = \frac{e^{-jw}-1}{w^2} - j\frac{e^{-jw}}{w}$$

and
$$H(jw) = \frac{1}{a+jw}.$$

Therefore,
$$\Phi_{xx}(jw) = |X(jw)|^2 = \frac{2-2\cos w}{w^4} - \frac{2\sin w}{w^2} + \frac{1}{w^2},$$
$$\Phi_{xy}(jw) = \Phi_{xx}(jw)H^*(jw) = \left[\frac{2-2\cos w}{w^4} - \frac{2\sin w}{w^2} + \frac{1}{w^2}\right]\left[\frac{1}{a-jw}\right],$$

and
$$\Phi_{yy}(jw) = \Phi_{xx}(jw)|H(jw)|^2 = \left[\frac{2-2\cos w}{w^4} - \frac{2\sin w}{w^2} + \frac{1}{w^2}\right]\left[\frac{1}{a^2+w^2}\right].$$

(f) We require that
$$|H(jw)|^2 = \frac{w^2+100}{w^2+25}.$$

The possible causal and stable choices for $H(jw)$ are
$$H_1(jw) = \frac{1+jw}{5+jw} \quad \text{and} \quad H_2(jw) = \frac{10-jw}{5+jw}.$$

The corresponding impulse responses are
$$h_1(t) = \delta(t) + 5e^{-5t}u(t) \quad \text{and} \quad h_2(t) = -\delta(t) + 15e^{-5t}u(t).$$

Only the system whit impulse response $h_1(t)$ has a causal and stable inverse.

4.51.(a)$H(jw)=1/G(jw)$.

(b)(i)If we denote the output by $y(t)$,then we have
$$Y(j0) = \frac{1}{2}.$$

Since $H(j0)=0$,it impossible for us to have $Y(j0)=X(j0)H(j0)$.Therefore, we cannot find an $x(t)$ which produces an output which looks like Figure P4.50.

(ii)This system is not invertible because $1/H(jw)$ is not defined for all w.

(c)We have
$$H(jw) = \sum_{k=0}^{\infty} e^{-kT}e^{-jwkT} = \frac{1}{1-e^{-(1+jw)T}}.$$

We now need to find a $G(jw)$ such that
$$H(jw)G(jw)=1.$$

Thus $G(jw)$ is the inverse system of $H(jw)$, and is given by
$$G(jw) = 1-e^{-(1+jw)T}.$$

(d)Since $H(jw)=2+jw$,
$$G(jw) = \frac{Y(jw)}{X(jw)} = \frac{1}{2+jw}.$$

Cross-multiplying and taking the inverse Fourier transform, we obtain
$$\frac{dy(t)}{dt} + 2y(t) = x(t).$$

(e)We have
$$H(jw) = \frac{-w^2+3jw+2}{-w^2+6jw+9}.$$

Therefore, the frequency response of the inverse is
$$G(jw) = \frac{1}{H(jw)} = \frac{-w^2+6jw+9}{-w^2+3jw+2}.$$

The differential equation describing the inverse system is

$$\frac{d^2 y(t)}{dt} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{d^2 x(t)}{dt} + 6\frac{dx(t)}{dt} + 9x(t).$$

Using partial fraction expansion followed by application of the inverse Fourier transform. We find the impulse responses to be

$$h(t) = \delta(t) - 3e^{-3t}u(t) + 2te^{-3t}u(t)$$

and

$$g(t) = \delta(t) - e^{-2t}u(t) + 4e^{-t}u(t).$$

4.52.(a)Since the step response is $s(t) = (1 - e^{-t/2})u(t)$,the impulse response has to be

$$h(t) = \frac{1}{2}e^{-t/2}u(t).$$

The frequency response of the system is

$$H(jw) = \frac{1/2}{\frac{1}{2} + jw}.$$

We now desire to build an inverse for the above system. Therefore, the frequency response of the inverse system has to be

$$G(jw) = \frac{1}{H(jw)} = 2\left[\frac{1}{2} + jw\right].$$

Taking the inverse Fourier transform we obtain

$$g(t) = \delta(t) + 2u_1(t).$$

(b)When sin(wt) passes through the inverse system, the output will be

$$t(t) = \sin(wt) + 2w\cos(wt).$$

We see that the output is directly proportional to *w*. Therefore, as *w* increases, the contribution to the output due to the noise also increases.

(c)In this case we require that $|H(jw)| \le \frac{1}{4}$ when *w=6*.Since

$$|H(jw)|^2 \le \frac{1}{a^2 + w^2},$$

we require that

$$\frac{1}{a^2 + 36} \le \frac{1}{16}.$$

Therefore, $a \le \frac{6}{\sqrt{15}}.$

Chapter 5 Answers

5.1 (a) let x[n]= $(1/2)^{n-1}$u[n-1].Using the Fourier transform analysis equation (5.9).the Fourier transform $X(e^{jw})$ of this signal is

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jwn}$$

$$= \sum_{n=1}^{\infty} (1/2)^{n-1} e^{-jwn}$$

$$= \sum_{n=0}^{\infty} (1/2)^{n} e^{-jw(n+1)}$$

$$= e^{-jw} \frac{1}{(1-(1/2)e^{-jw})}$$

(b) Let x[n]=$(1/2)^{|n-1|}$.Using the Fourier transform analysis equation (5.9).The Fourier transform $x(e^{jw})$ of signal is

$$x(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jwn} = \sum_{n=-\infty}^{0} (1/2)^{-(n-1)} e^{-jwn} + \sum_{n=1}^{\infty} (1/2)^{n-1} e^{-jwn}$$

The second summation in the right—hand side of the above equation is exactly the same as result of part (a).Now ,

$$\sum_{n=-\infty}^{0} (1/2)^{-(n-1)} e^{-jwn} = \sum_{n=0}^{\infty} (1/2)^{n+1} e^{jwn} = (1/2) \frac{1}{(1-(1/2)e^{jw})}$$

Therefore

$$x(e^{jw}) = (1/2) \frac{1}{(1-(1/2)e^{jw})} + e^{-jw} \frac{1}{(1-(1/2)e^{-jw})} = \frac{0.75e^{-jw}}{(1.25 - \cos w)}$$

5.2 (a) let $$x[n] = \delta[n-1] + \delta[n+1]$$. Using the Fourier transform analysis equation (5.9).the Fourier transform $x(e^{jw})$ of this signal is

$$x(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jwn}$$

$$= e^{-jw} + e^{jw} = 2\cos w$$

(b) Let $x[n] = \delta[n+2]+\delta[n-2]$ .using the Fourier transform analysis equation (5.9). the Fourier transform $x(e^{jw})$ of this signal is

$$x(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jwn}$$

$$= e^{2jw} - e^{-2jw} = 2j\sin(2w)$$

5.3 We note from section 5.2 that a periodic signal with Fourier series representation

$$x[n]= \sum_{k=<N>} a_k e^{jk(2\pi/N)n}$$

has a Fourier transform

$$X(e^{jw}) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(w - \frac{2\pi k}{N})$$

(a) Consider the signal $x_1[n] = \sin(\frac{\pi}{3}n + \frac{\pi}{4})$ .We note that the fundamental period of the signal $x_1[n]$ is N=6.

The signal may be written as

$$x_1[n] = \frac{1}{2j}e^{j(\frac{\pi}{3}n+\frac{\pi}{4})} - \frac{1}{2j}e^{-j(\frac{\pi}{3}n+\frac{\pi}{4})} = \frac{1}{2j}e^{j\frac{\pi}{4}}e^{j\frac{2\pi}{6}n} - \frac{1}{2j}e^{-j\frac{\pi}{4}}e^{-j\frac{2\pi}{6}n}$$

Form this , we obtain the non-zero Fourier series coefficients a_k of $x_1[n]$ the range

$-2 \le \pi \le 3$ as

$$a_1 = (1/2j)e^{j\frac{\pi}{4}} \qquad\qquad a_{-1} = -(1/2j)e^{-j\frac{\pi}{4}}$$

Therefore , in the range $-\pi \le w \le \pi$,we obtain

$$X(e^{jw}) = 2\pi a_1\delta(w-\frac{2\pi}{6}) + 2\pi a_{-1}\delta(w+\frac{2\pi}{6})$$

$$= (\pi/j)\{e^{j\pi/4}\delta(w-2\pi/6) - e^{-j\pi/4}\delta(w+2\pi/6)\}$$

(b) consider the signal $x_2[n] = 2 + \cos(\frac{\pi}{6}n+\frac{\pi}{8})$ .we note that the fundamental period of the signal $x_1[n]$ is

N=12.the signal maybe written as

$$x_1[n] = 2 + (1/2)e^{j(\frac{\pi}{6}n+\frac{\pi}{8})} + (1/2)e^{-j(\frac{\pi}{6}n+\frac{\pi}{8})}$$

$$= 2 + (1/2)e^{j\frac{2\pi}{12}n}e^{j\frac{\pi}{8}} + (1/2)e^{-j\frac{2\pi}{12}n}e^{-j\frac{\pi}{8}}$$

Form this ,we obtain the non-zero Fourier series coefficients a_k of $x_2[n]$ in the range $-5 \le k \le 6$ as

$$a_0 = 2 \qquad a_1 = (1/2)e^{j\frac{\pi}{8}} \qquad a_{-1} = (1/2)e^{-j\frac{\pi}{8}}$$

Therefore ,in the range ,we obtain

$$X(e^{jw}) = 2\pi a_0\delta(w) + 2\pi a_1\delta(w-\frac{2\pi}{12}) + 2\pi a_{-1}\delta(w+\frac{2\pi}{12})$$

$$= 4\pi\delta(w) + \pi\{e^{j\pi/8}\delta(w-\frac{\pi}{6}) + e^{-j\pi/8}\delta(w+\frac{\pi}{6})\}$$

5.4 (a)Using the Fourier transform synthesis equation (5.8)

$$x_1[n] = (1/2\pi)\int_{-\pi}^{\pi} X_1(e^{jw})e^{jwn}dw$$

$$= (1/2\pi)\int_{-\pi}^{\pi}[2\pi\delta(w) + \pi\delta(w-\pi/2) + \pi\delta(w+\pi/2)]e^{jwn}dw$$

$$= e^{j0} + (1/2)e^{j(\pi/2)n} + (1/2)e^{-j(\pi/2)n}$$

$$= 1 + \cos(\pi n/2)$$

(b)Using the transform synthesis equation (5.8)

$$x_2[n] = (1/2\pi)\int_{-\pi}^{\pi} X_2(e^{jw})e^{jwn}dw$$

$$= -(1/2\pi)\int_{-\pi}^{0} 2je^{jwn}dw + (1/2\pi)\int_{0}^{\pi} 2je^{jwn}dw$$

$$= (j/\pi)[-\frac{1-e^{-j\pi n}}{jn} + \frac{e^{j\pi n}-1}{jn}]$$

$$= -(4/(n\pi))\sin^2(n\pi/2)$$

5.5 From the given information

$$x[n] = (1/2\pi)\int_{-\pi}^{\pi} x(e^{jw})e^{jwn}dw$$

$$= (1/2\pi)\int_{-\pi}^{\pi} |X(e^{jw})e^{j\sphericalangle\{X(e^{jw})\}}e^{jwn}|)dw$$

$$= (1/2\pi)\int_{-\pi/4}^{\pi/4} e^{-\frac{3}{2}w}e^{jwn}dw$$

$$= \frac{\sin(\frac{\pi}{4}(n-3/2))}{\pi(n-3/2)}$$

The signal x[n]is zero when $\frac{\pi}{4}(n-3/2)$ is a nonzero integer multiple of π or when $|n| \to \infty$ .the value of

$\frac{\pi}{4}(n-3/2)$ can never be such that it is a nonzero integer multiple of π .Therefore .x[n]=0 only for n=$\pm\infty$

5.6 Throughout this problem, we assume that

$$X[n] \xleftarrow{\quad FT \quad} x_1(e^{jw})$$

(a) Using the time reversal property (Sec.5.3.6),we have

$$\text{x[-n]} \xleftrightarrow{\text{FT}} X(e^{-jw})$$

Using the time shift property (Sec.5.3.3) on this .we have

$$\text{x[-n+1]} \xleftrightarrow{\text{FT}} e^{-jwn} x(e^{-jw}) \quad \text{and} \quad \text{x[-n-1]} \xleftrightarrow{\text{FT}} e^{jwn} x(e^{-jw})$$

Therefore

$$x_1[n] = \text{x[-n+1]} + \text{x[-n-1]} \xleftrightarrow{\text{FT}} e^{-jwn} X(e^{-jw}) + e^{jwn} X(e^{-jw})$$

$$\xleftrightarrow{\text{FT}} 2X(e^{-jw})\cos w$$

(b) Using the time reversal property (Sec.5.3.6) ,we have

$$\text{x[-n]} \xleftrightarrow{\text{FT}} X(e^{-jw})$$

Using the same conjugation property on this ,we have

$$x^*[\text{-n}] \xleftrightarrow{\text{FT}} X^*(e^{-jw})$$

Therefore

$$x_2[n] = (1/2)(x^*[\text{-n}] + x[n]) \xleftrightarrow{\text{FT}} (1/2) X(e^{jw}) + X^*(e^{jw})$$

$$\xleftrightarrow{\text{FT}} \text{Re}\{X(e^{jw})\}$$

(c) Using the differentiation frequency property (Sec.5.3.8),we have

$$nx[n] \xleftrightarrow{\text{FT}} j\frac{dX(e^{jw})}{dw}$$

Using the same property second time ,

$$n^2 x[n] \xleftrightarrow{\text{FT}} -\frac{d^2 X(e^{jw})}{dw^2}$$

Therefore

$$x_3[n] = n^2 x[n] - 2nx[n] + 1 \xleftrightarrow{\text{FT}} -\frac{d^2 X(e^{jw})}{dw^2} - 2j\frac{d^2 X(e^{jw})}{dw^2} + X(e^{jw})$$

5.7 (a) Consider the signal $y_1[n]$ with Fourier transform

$$Y_1(e^{jw}) = \sum_{k=1}^{10} \sin(kw)$$

We see that $Y_1(e^{jw})$ is real and odd .From Table 5.1 , we know that the Fourier transform of a real and odd signal is purely imaginary and odd. Therefore ,we may say that the Fourier transform of a purely imaginary and odd signal is real and odd. Using this observation, we conclude that $y_1[n]$ is purely imaginary and odd

Note now that

$$X_1(e^{jw}) = e^{-jw} Y_1(e^{jw})$$

Therefore , $x_1[n] = y_1[n-1]$ .therefore , is also purely imaginary .but $x_1[n]$ is neither even nor odd

(b)We note that $X_2(e^{jw})$ is purely imaginary and odd. Therefore, $x_2[n]$ has to be real and odd.

(d) ©Consider a signal $y_3[n]$ whose magnitude of the Fourier transform is $|Y_3(e^{jw})| = A(w)$ and whose phase of the Fourier transform is $\sphericalangle\{Y_3(e^{jw})\} = -(3/2)w$ .since $|Y_3(e^{jw})| = |Y_3(e^{-jw})|$ and ,we may conclude that the signal $y_3[n]$ is real (see Table 5.1,property5.3.4).

Now, consider the signal $x_3[n]$ with Fourier transform $X_3(e^{jw}) = Y_3(e^{jw})e^{j\pi} = Y_3(jw)$ .Using the result from previous paragraph and the linearity property of the Fourier transform .we may conclude that has to real .since the Fourier transform ,we may conclude that has to real . since the Fourier transform $X_3(e^{jw})$ is neither purely imaginary nor purely real .the signal $x_3[n]$ is neither even nor odd

5.8 Consider the signal

$$x_1[n] = \begin{cases} 1, & |n| \leq 1 \\ 0, & |n| > 1 \end{cases}$$

From the table 5.2, we know that

$$x_1[n] \xleftrightarrow{\text{FT}} X_1(e^{jw}) = \frac{\sin(3w/2)}{\sin(w/2)}$$

Using the accumulation property (Table 5.1，Property 5.3.5),we have

$$\sum_{k=-\infty}^{n} x_1[k] \xleftrightarrow{\text{FT}} \frac{1}{1-e^{-jw}} X_1(e^{jw}) + \pi X_1(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(w - 2\pi k)$$

Therefore , in the range $-\pi \leq w \leq \pi$,

$$\sum_{k=-\infty}^{n} x_1[k] \xleftrightarrow{\ FT\ } \frac{1}{1-e^{-jw}} X_1(e^{jw}) + 3\pi\delta(w)$$

Also, in the range $-\pi \le w \le \pi$,

$$1 \xleftrightarrow{\ FT\ } 2\pi\delta(w)$$

Therefore , in the range $-\pi \le w \le \pi$,

$$x[n] = 1 + \sum_{k=-\infty}^{n} x_1[k] \xleftrightarrow{\ FT\ } \frac{1}{1-e^{-jw}} X_1(e^{jw}) + 5\pi\delta(w)$$

The signal x[n] has the desired Fourier transform .We may express x[n] mathematically as

$$x[n] = 1 + \sum_{k=-\infty}^{n} x_1[k] = \begin{cases} 1 & n \le -2 \\ n+3 & -1 \le n \le 1 \\ 4 & n \ge 2 \end{cases}$$

5.9 From property 5.3.4 in Table 5.1 , we know that for a real signal x[n],

$$Od\{x[n]\} \xleftrightarrow{\ FT\ } j\,\text{Im}\{X_1(e^{jw})\}$$

From the given information

$$j\,\text{Im}\,X_1(e^{jw}) = j\sin w - j\sin 2w$$
$$= (1/2)(e^{jw} - e^{-jw} - e^{2jw} + e^{-2jw})$$

Therefore,

$$Od\{x[n]\} = IFT\{j\,\text{Im}\,X_1(e^{jw})\} = (1/2)(\delta[n+1] - \delta[n-1] - \delta[n+2] + \delta[n-2])$$

We also know that

$$Od\{x[n]\} = \frac{x[n] - x[-n]}{2}$$

And that x[n]=0 for n>0. therefore

$$x[n] = 2Od\{x[n]\} = \delta[n+1] - \delta[n+2] \qquad \text{for n<0}$$

Now we only have to find x[0] .Using Parseval's relation ,we have

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} |X(e^{jw})|^2 \, dw = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Form the given information, we can write

$$3 = (x[n])^2 + \sum_{n=-\infty}^{-1} |x[n]|^2 = (x[n])^2 + 2$$

This gives x[0]=1.but since we are given that x[0]>0.we conclude that x[0]=1
Therefore

$$x[n] = \delta[n] + \delta[n+1] - \delta[n+2]$$

5.10 From table 5.2 we know that

$$(\frac{1}{2})^n u[n] \xleftrightarrow{\ FT\ } \frac{1}{1 - \frac{1}{2}e^{-jw}}$$

Using property 5.3.8 in table 5.1,

$$x[n] = n(\frac{1}{2})^n u[n] \xleftrightarrow{\ FT\ } X(e^{jw}) = j\frac{d}{dw}\{\frac{1}{1-\frac{1}{2}e^{-jw}}\} = \frac{\frac{1}{2}e^{-jw}}{(1-\frac{1}{2}e^{-jw})^2}$$

Therefore , $x[n] = \sum_{n=0}^{\infty} n(\frac{1}{2})^n = \sum_{-\infty}^{\infty} x[n] = X(e^{j0}) = 2$

5.11 We know from the time expansion property (Table 5.1, Property 5.3.7) that

$$g[n] = x_{(2)}[n] \xleftrightarrow{\ FT\ } G(e^{j\omega}) = X(e^{j2\omega})$$

Therefore, $G(e^{j\omega})$ is obtained by compressing $X(e^{j\omega})$ by a factor of 2. Since we know that $X(e^{j\omega})$ is periodic with a periodic of 2π, we my conclude that $G(e^{j\omega})$ has a periodic which is $(1/2)2\pi = \pi$. Therefore,

$$G(e^{j\omega}) = G(e^{j9\omega - \pi}) \quad and \quad a = \pi.$$

5.12 Consider the signal

$$x_1[n] = \left(\frac{\sin\frac{\pi}{4}n}{\pi n}\right)$$

For Table 5.2, we obtain the Fourier transform of x₁[n] to be

$$X_1(e^{j\omega}) = \begin{cases} 1, & 0 \le |\omega| \le \pi/4 \\ 0, & \pi/4 \le |\omega| \ge \pi \end{cases}$$

The plot of $X_1(e^{j\omega})$ is as shown in Figure S5.12. Now consider the signal $x_1[n]=(x_2[n])^2$. Using the multiplication property (Table 5.1, Property 5.5), we obtain the Fourier transform of $x_2[n]$ to be

$$X_2(e^{j\omega}) = (1/2\pi)[X_1(e^{j\omega}) * X_1(e^{j\omega})]$$

This is plotted in Figure S5.12.

From Figure S5.12. It is clear that $X_2(e^{j\omega})$ is zero for $\omega > \pi/2$. By using the convolution property (Table 5.1, Property 5.4), we know that

$$Y(e^{j\omega}) = X_1(e^{j\omega})FT\left\{\frac{\sin()\omega_c n}{\pi n}\right\}$$
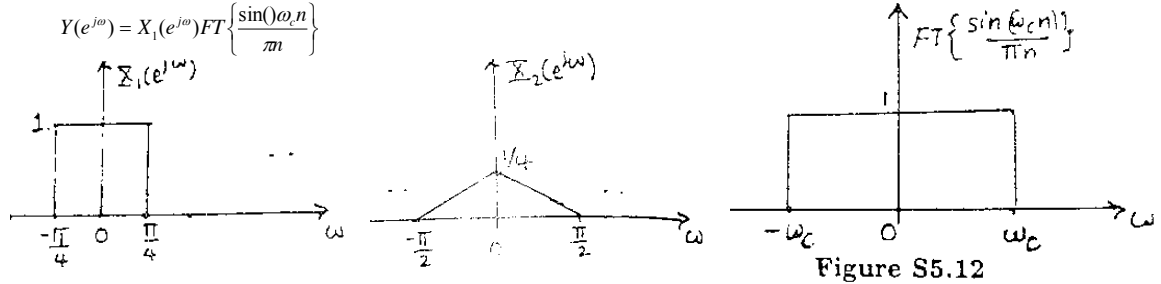
Figure S5.12

The plot of $FT\left(\frac{\sin \omega_c n}{\pi n}\right)$ is shown in Figure S5.12. It is clear that of then $\pi/2 \le \omega \le \pi$.

5.13 When two LTI systems connected in parallel, the impulse response of the overall system is the sum of the impulse response of the individual. Therefore,

$$h[n]= h_1[n]+ h_2[n]$$

using the linearity property (Table 5.1, Property 5.3.2)

$$H(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega})$$

Given that $h_1[n]=(1/2)^n u[n]$, we obtain

$$H_1(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}}$$

Therefore,

$$H_2(e^{j\omega}) = \frac{-12 + 5^{-j\omega}}{12 - 7e^{-j\omega} + e^{-2j\omega}} - \frac{1}{1 - \frac{1}{2}e^{-j\omega}} = \frac{-2}{1 - \frac{1}{4}e^{-j\omega}}$$

Taking the inverse Fourier transform, $h_2[n]=-2(1/4)^n u[n]$.

5.14 From the given information, we have the Fourier transform $G(e^{j\omega})$ of $g[n]$ to be

$$G(e^{j\omega}) = g[0] + g[1]e^{-j\omega}.$$

Also, when the input to the system is $x[n]=(1/4)^n u[n]$, the output is $g[n]$. Therefore,

$$H(e^{j\omega}) = \frac{G(e^{j\omega})}{X(e^{j\omega})}.$$

For Table 5.2, we obtain

$$X(e^{j\omega}) = \frac{1}{1 - \frac{1}{4}e^{-j\omega}}$$

Therefore,

$$H(e^{j\omega}) = \{g[0] + g[1]e^{-j\omega}\}\{1 - \frac{1}{4}e^{-j\omega}\} = g[0] + \{g[1] - \frac{1}{4}g[0]\}e^{-j\omega} - g[1]e^{-2j\omega}$$

Clearly, $h[n]$ is a three point sequence.

We have

$$H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-2j\omega}$$

and

$$H(e^{j(\omega-\pi)}) = h[0] + h[1]e^{-j(\omega-\pi)} + h[2]e^{-2j(\omega-\pi)}$$
$$= h[0] - h[1]e^{-j\omega} + h[2]e^{-2j\omega}$$

We see that $H(e^{j\omega}) = H(e^{j(\omega-\pi)})$ only if $h[1]=0$.

We also have

$$H(e^{j\pi/2}) = h[0] + h[1]e^{-j\pi/2} + h[2]e^{-2j\pi/2}$$
$$= h[0] - h[2]$$

Since we are also given that $H(e^{j\pi/2}) = 1$, we have $h[0]-h[2]=1$ (S5.14-1)

Now not that

$$g[n] = h[n] * \{(1/4)^n u[n]\}$$
$$= \sum_{k=0}^{2} h[k](1/4)^{n-k} u[n-k]$$

88

Evaluating this equation at n=2, we have

$$g[2] = 0 = \frac{1}{16}h[0] + \frac{1}{4}h[1] + h[2]$$

Since h[1]=0,

$$\frac{1}{16}h[0] + h[2] = 0 \qquad (S5.14-2)$$

Solving equation (S5.14-1) and (S5.14-2), we obtain

$$h[0] = \frac{16}{17}, \qquad \text{and} \qquad h[2] = -\frac{1}{17}.$$

Therefore,

$$h[n] = \frac{16}{17}\delta[n] - \frac{1}{17}\delta[n-2].$$

5.15 Consider $x[n]=\sin(w_c n)/(\pi n)$, the Fourier transform $X(e^{j\omega})$ of x[n] is also shown in Figure S5.15. We note that the given signal y[n]=x[n]x[n]. Therefore, the Fourier transform $Y(e^{j\omega})$ of y[n] is

$$Y(e^{j\omega}) = \frac{1}{2\pi}\int_{<2\pi>} X(e^{j\theta})X(e^{j(\omega-\theta)})d\theta.$$

Employing the ____ 5, we can convert the above periodic convolution into an aperiodic signal by defining

$$\hat{X}(e^{j\omega}) = \begin{cases} X(e^{j\omega}), & -\pi < \omega \leq \pi \\ 0, & \text{otherwise} \end{cases}$$

Then we may write

$$Y(e^{j\omega}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{X}(e^{j\theta})X(e^{j(\omega-\theta)})d\theta.$$

This is the aperiodic convolution of the rectangular pulse $\hat{X}(e^{j\omega})$ shown in Figure S5.15 with the periodic square wave $X(e^{j\omega})$. The result of this convolution is also shown in Figure S5.15.

Figure S5.15
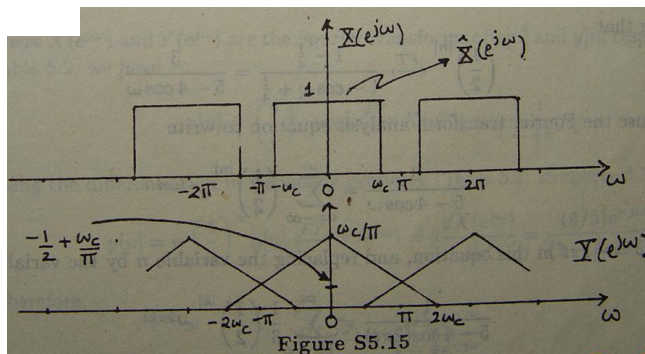
From the figure, it is clear that we require $-1+(2w_c/\pi)$ to be 1/2. therefore, $w_c=3\pi/4$.

5.16 We may write

$$X(e^{j\omega}) = \frac{1}{2\pi}\left\{ \frac{1}{1-\frac{1}{4}e^{-j\omega}} * \left[2\pi\sum_{k=0}^{3}\delta(\omega - \frac{\pi k}{2}) \right] \right\}$$

where * denotes aperiodic convolution. We may also rewrite this as a periodic convolution

where

$$X(e^{j\omega}) = \frac{1}{2\pi}\int_{0}^{2\pi} G(e^{j\theta})Q(e^{j(\omega-\theta)})d\theta$$

and

$$G(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

$$Q(e^{j\omega}) = 2\pi\sum_{k=0}^{3}\delta(\omega - \frac{\pi k}{2}) \qquad \text{for } 0 \leq \omega < 2\pi.$$

(a) Taking the inverse Fourier transform of $G(e^{j\omega})$, we get g[n]=(1/4)^{n}u[n]. therefore, a=1/4.

(b) Taking the inverse Fourier transform of $Q(e^{j\omega})$, we get

$$q[n] = 1 + \frac{1}{2}e^{j(\pi/2)n} + \frac{1}{4}e^{j\pi n} + \frac{1}{8}e^{j(3\pi/2)n}.$$

This signal is periodic with a fundamental periodic N=4.

(c) We can easily show that $X(e^{j\omega})$ is not conjugate symmetric. Therefore, x[n] is not real.

5.17 Using the duality property, we have

$$(-1)^n \overset{FS}{\longleftrightarrow} a_k \Rightarrow a_n \overset{FS}{\longleftrightarrow} \frac{1}{N}(-1)^{-k} = \frac{1}{2}(-1)^k.$$

5.18 Knowing that

$$\left(\frac{1}{2}\right)^{|n|} \overset{FT}{\longleftrightarrow} \frac{1 - \frac{1}{4}}{1 - \cos\omega + \frac{1}{4}} = \frac{3}{5 - 4\cos\omega},$$

we may use the Fourier transform analysis equation to write

$$\frac{3}{5 - 4\cos\omega} = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|n|} e^{-j\omega n}$$

Putting ω=-2πt in this equation, and replacing the variable n by the variable k

$$\frac{1}{5 - 4\cos(2\pi t)} = \sum_{k=-\infty}^{\infty} \frac{1}{3}\left(\frac{1}{2}\right)^{|k|} e^{j2\pi kt}.$$

By comparing this with the continuous-time Fourier series synthesis equation, it is immediately apparent that $a_k = (1/3)(1/2)^{|k|}$ are the Fourier series coefficients of the signal 1/(5-4cos(2πt)).

5.19 (a) Taking the Fourier transform of both sides of the difference equation, we have

$$Y(e^{j\omega})\left[1 - \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-2j\omega}\right] = X(e^{j\omega}).$$

Therefore,

$$H(ej\omega) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1 - \frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-2j\omega}} = \frac{1}{(1 - \frac{1}{2}e^{-j\omega})(1 + \frac{1}{3}e^{-j\omega})}.$$

(b) Using Partial faction expansion,

$$H(ej\omega) = \frac{3/5}{1 - \frac{1}{2}e^{-j\omega}} + \frac{2/5}{1 + \frac{1}{3}e^{-j\omega}}.$$

Using Table 5.2, and taking the inverse Fourier transform, we obtain

$$h[n] = \frac{3}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{2}{5}\left(-\frac{1}{3}\right)^n u[n].$$

5.20 (a) Since the LTI system is causal and stable, a signal input-output pair is sufficient to determine the frequency response of the system. In this case, the input is x[n]=(4/5)^{n}u[n] and output is y[n]=n(4/5)^{n}u[n]. The frequency response is given by

$$H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega}),$$

Where $X(e^{j\omega})$ and $Y(e^{j\omega})$ are the Fourier transforms of x[n] and y[n] respectively. Using Table 5.2, we have

$$x[n] = \left(\frac{4}{5}\right)^n u[n] \overset{FT}{\longleftrightarrow} X(e^{j\omega}) = \frac{1}{1 - \frac{4}{5}e^{-j\omega}}.$$

Using the differentiation in frequency property (Table 5.1, Property 5.3.8), we have

Therefore,
$$y[n] = n\left(\frac{4}{5}\right)^n u[n] \overset{FT}{\longleftrightarrow} Y(e^{j\omega}) = j\frac{dX(e^{j\omega})}{d\omega} = \frac{(4/5)e^{-j\omega}}{(1 - \frac{4}{5}e^{-j\omega})^2}$$

$$H(e^{j\omega}) = \frac{(4/5)e^{-j\omega}}{1 - \frac{4}{5}e^{-j\omega}}.$$

(b) Since $H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega})$, we may write $Y(e^{j\omega})\left[1 - \frac{4}{5}e^{-j\omega}\right] = X(e^{j\omega})\left[(4/5)e^{-j\omega}\right]$.

Taking the inverse Fourier transform of both sides
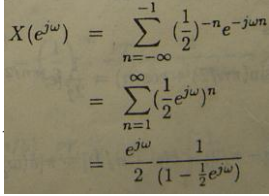
$$y[n] - \frac{4}{5}y[n-1] = \frac{4}{5}x[n].$$

5.21 (a) The given signal is

$x[n]=u[n-2]- u[n-6]=\delta[n-2]+\delta[n-3]+ \delta[n-4]+\delta[n-5]$

Using the Fourier transform analysis eq. (5.9), we obtain

$$X(e^{j\omega}) = e^{-2j\omega} + e^{-3j\omega} + e^{-4j\omega} + e^{-5j\omega}.$$

(b) Using the Fourier transform analysis eq. (5.9), we obtain

$$
\begin{aligned}
X(e^{j\omega}) &= \sum_{n=-\infty}^{-1} (\tfrac{1}{2})^{-n} e^{-j\omega n} \\
&= \sum_{n=1}^{\infty} (\tfrac{1}{2} e^{j\omega})^{n} \\
&= \frac{e^{j\omega}}{2} \frac{1}{(1 - \tfrac{1}{2}e^{j\omega})}
\end{aligned}
$$

（c）Using the Fourier transform analysis (5.9),we obtain

$$
\begin{aligned}
X(e^{jw}) &= \sum_{n=-\infty}^{-2} (\tfrac{1}{3})^{-n} e^{-jwn} \\
&= \sum_{n=2}^{\infty} (\tfrac{1}{3} e^{jw})^{n} \\
&= \frac{e^{2jw}}{9} \frac{1}{(1-1/3e^{jw})} e^{2jw}
\end{aligned}
$$

(d) using the Fourier transform analysis eq.(5.9),we obtain

$$
\begin{aligned}
X(e^{jw}) &= \sum_{n=-\infty}^{0} 2^{n} \sin(\pi n/4) e^{-jwn} \\
&= -\sum_{n=-\infty}^{0} 2^{-n} \sin(\pi n/4) e^{jwn} \\
&= -\frac{1}{j}\sum_{n=0}^{\infty}[(1/2)^{n} e^{j\pi n/4} e^{jwn} -(1/2)^{n} e^{-j\pi n/4} e^{jwn}] \\
&= -\frac{1}{2j}[\frac{1}{1-(1/2)e^{j\pi/4}e^{jw}} - \frac{1}{1-(1/2)e^{-j\pi/4}e^{jw}}]
\end{aligned}
$$

(e) using the Fourier transform analysis eq(5.9),we obtain

$$
\begin{aligned}
X(e^{jw}) &= \sum_{n=-\infty}^{0} (1/2)^{|n|} \cos[\pi(n-1)/8] e^{-jwn} \\
&= \frac{1}{2}[\frac{e^{-j\pi/8}}{1-(1/2)e^{j\pi/8}e^{-jw}} - \frac{e^{j\pi/8}}{1-(1/2)e^{-j\pi/8}e^{-jw}}] \\
&\quad + \frac{1}{4}[\frac{e^{j\pi/4}e^{jw}}{1-(1/2)e^{j\pi/8}e^{jw}} - \frac{e^{-j\pi/4}e^{jw}}{1-(1/2)e^{-j\pi/8}e^{jw}}]
\end{aligned}
$$

（f）the given signal is

$$x[n]=-3 - 3\delta[n+3] - 2\delta[n+2] - \delta[n+1] + \delta[n-1] + 2\delta[n-2] + 3\delta[n-3]$$

Using the Fourier transform analysis eq(5.9),we obtain

$$x(e^{jw}) = -3e^{3jw} - 2e^{2jw} - e^{jw} + e^{jw} + 2e^{-2jw} + 3e^{-3jw}$$

(g) the given signal is

$$x[n] = \sin(\pi n/2) + \cos(n) = \frac{1}{2j}[e^{j\pi n/2} - e^{-j\pi n/2}] + \frac{1}{2}[e^{jn/2} + e^{-jn/2}]$$

therefore

$$x(e^{jw}) = \frac{\pi}{j}[\delta(w-\pi/2) - \delta(n+\pi/2)] + \pi[\delta(w-1)+\delta(w+1)] \quad 0\le|w|<\pi$$

(h) the given signal is

$$
\begin{aligned}
X[n] &= \sin(5\pi n/3) + \cos(7\pi n/3) \\
&= -\sin(\pi n/3) + \cos(\pi n/3) \\
&= -\frac{1}{2j}[e^{j\pi n/3} - e^{-j\pi n/3}] + \frac{1}{2}[e^{jn/3} + e^{-jn/3}]
\end{aligned}
$$

$$x(e^{jw}) = -\frac{\pi}{j}[\delta(w-\pi/3) - \delta(w+\pi/3)] + \pi[\delta(w-\pi/3)+\delta(w+\pi/3)] \quad 0\le|w|<\pi$$

(i)   x[n] is periodic with periodic 6. the Fourier series coefficients of x[n] are give by

$$a_k = \frac{1}{6}\sum_{n=0}^{5} x[n] e^{-j(2\pi/6)kn}$$

$$= \frac{1}{6} \sum_{n=0}^{4} e^{-j(2\pi/6)kn}$$

$$= \frac{1}{6} \left[\frac{1 - e^{-j5\pi k/3}}{1 - e^{-j(2\pi/6)k}} \right]$$

Therefore, form the result of section 5.2

$$x(e^{jw}) = \sum_{l=-\infty}^{\infty} 2\pi \frac{1}{6} \left[\frac{1 - e^{-j5\pi k/3}}{1 - e^{-j(2\pi/6)k}} \right] \delta(w - \frac{2\pi}{6} - 2\pi l))$$

(j) using the Fourier transform analysis eq.(5.9) we obtain

$$(\frac{1}{3})^{|n|} \xleftrightarrow{FT} \frac{4}{5 - 3\cos w}$$

Using the differentiation in frequency property of the Fourier transform

$$n(\frac{1}{3})^{|n|} \xleftrightarrow{FT} -j \frac{12\sin w}{(5 - 3\cos w)^2}$$

Therefore

$$X[n] = n(\frac{1}{3})^{|n|} - (\frac{1}{3})^{|n|} \xleftrightarrow{FT} \frac{4}{5 - 3\cos w} - j\frac{12\sin w}{(5 - 3\cos w)^2}$$

(k)we have

$$x_1[n] = \frac{\sin(\pi n/5)}{\pi n} \xleftrightarrow{FT} x_1(e^{jw}) = \begin{cases} 1, |w| < \frac{\pi}{5} \\ 0, \frac{\pi}{5} \leq |w| < \pi \end{cases}$$

$$x_2[n] = \cos(7\pi n/2) = \cos(\pi n/2) \xleftrightarrow{FT} x_2(e^{jw}) = \pi\{\delta(w - \pi/2) + \delta(w + \pi/2)\}$$

In the range $0 \leq |w| < \pi$, therefore, if $x[n] = x_1[n]x_2[n]$, then

$$x(e^{jw}) = \text{periodic convolution of } x_1(e^{jw}) \text{ and } x_2(e^{jw})$$

Using the mechanics of periodic convolution demonstrated in example 5.15 ,we obtain
In the range $0 \leq |w| < \pi$

$$x(e^{jw}) = \begin{cases} 1, \frac{3\pi}{10} < |w| < \frac{7\pi}{10} \\ 0, otherwise \end{cases}$$

5.22 (a) Using the Fourier transform analysis eq(5.8),we obtain

$$x[n] = \frac{1}{2\pi} \int_{-3\pi/4}^{-\pi/4} e^{jwn}dw + \frac{1}{2\pi} \int_{\pi/4}^{3\pi/4} e^{jwn}dw$$

$$= \frac{1}{\pi n}[\sin(3\pi n/4) - \sin(\pi n/4)]$$

（b）comparing the given Fourier transform analysis eq(5.8),we obtain

$$x[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-2] - 4\delta[n-3] + \delta[n-10]$$

(c) Using the Fourier transform analysis eq(5.8),we obtain

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-jw/2} e^{jwn}dw$$

$$= \frac{(-1)^{n+1}}{\pi(n - 1/2)}$$

(d) the given Fourier transform is

$$x(e^{jw}) = \cos^2 w + \sin^2(3w)$$

$$= \frac{1 + \cos(2w)}{2} + \frac{1 - \cos(3w)}{2}$$

$$= 1 + \frac{1}{4}e^{2jw} + \frac{1}{4}e^{-2jw} + -\frac{1}{4}e^{3jw} - \frac{1}{4}e^{-3jw}$$

Comparing the given Fourier transform with the analysis eq(5.8),we obtain

$$x[n] = \delta[n] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n+2] - \frac{1}{4}\delta[n-3] - \frac{1}{4}\delta[n+3]$$

（e）this is the Fourier transform of a periodic signal with fundamental frequency $\frac{\pi}{2}$

Therefore its fundamental periodic is 4. also, the Fourier series coefficient of this
Signal are $a_k = (-1)^k$. Therefore, the signal is given by

$$x[n] = \sum_{k=0}^{3} (-1)^k e^{jk(\pi/2)n} = 1 - e^{j\pi n/2} + e^{j\pi n} - e^{3j\pi n/2}$$

(f) the given Fourier transform may be written as

$$x(e^{jw}) = e^{-jw}\sum_{n=0}^{\infty}(1/5)^n e^{-jwn} - (1/5)\sum_{n=0}^{\infty}(1/5)^n e^{-jwn}$$

$$= 5\sum_{n=0}^{\infty}(1/5)^n e^{-jwn} - (1/5)\sum_{n=0}^{\infty}(1/5)^n e^{-jwn}$$

Compare each of two terms in the right-hand side of the above equation with the Fourier transform analysis eq. (5.9) we obtain

$$x[n] = (\frac{1}{5})^{n-1}u[n-1] - (\frac{1}{5})^{n+1}u[n]$$

(g) the given Fourier transform may be written as

$$x(e^{jw}) = \frac{2/9}{1-1/2e^{-jw}} + \frac{7/9}{1+1/4e^{-jw}}$$

Therefore

$$x[n] = \frac{2}{9}(\frac{1}{2})^n u[n] + \frac{7}{9}(-\frac{1}{4})^n u[n]$$

(h) the given Fourier transform may be written as

$$x(e^{jw}) = 1 + \frac{1}{3}e^{-jw} + \frac{1}{3^2}e^{-2jw} + \frac{1}{3^3}e^{-3jw} + \frac{1}{3^4}e^{-4jw} + \frac{1}{3^5}e^{-5jw}$$

Compare the given Fourier transform with the analysis eq. (5.8), we obtain

$$x[n] = \delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{9}\delta[n-2] + \frac{1}{27}\delta[n-3] + \frac{1}{81}\delta[n-4] + \frac{1}{243}\delta[n-5]$$

5.23 (a) we have form eq.(5.9)

$$x(e^{j0}) = \sum_{n=-\infty}^{\infty}x[n] = 6$$

(b) note that y[n]=x[n+2] is an even signal. Therefore , $Y(e^{jw})$ is real and even . This Implies that $\forall Y(e^{jw}) = 0$ .furthermore , form the time shifting property of the Fourier Transform we have $\forall Y(e^{jw}) = e^{j2w}X(e^{jw})$ .therefore, $\forall X(e^{jw}) = e^{-j2w}$

 (c) we have form eq. (5.8)

$$2\pi x\,|\,0\,| = \int_{-\pi}^{\pi}X(ej^w)dw$$

Therefore
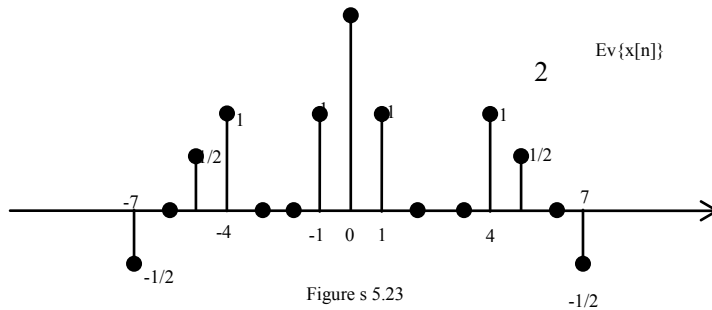
$$\int_{-\pi}^{\pi}X(e^{jw})dw = 4\pi$$

(d) we have form eq.(5.9)

$$X(e^{j\pi}) = \sum_{n=-\infty}^{\infty}x[n](-1)^n = 2$$

(e) from table 5.1, we have

$$\varepsilon v\{x[n]\} \xleftarrow{\ FT\ } \mathrm{Re}\{X(e^{jw})\}$$

Therefore, the desired signal is $\varepsilon v\{x[n]\} = \{x[n] + x[-n]\}/2$ .this is as shown in figure

Figure s 5.23

(f) (i) from Parseval's theorem we have

$$\int_{-\infty}^{\infty}|X(e^{jw})|^2\,dw = 2\pi\sum_{-\infty}^{\infty}|x[n]|^2 = 28\pi$$

(ii) using the differentiation in frequency property of the Fourier transform we obtain

$$nx[n] \xleftarrow{\ FT\ } j\frac{dX(e^{jw})}{dw}$$

Again using Parseval's theorem, we obtain

$$\int_{-\infty}^{\infty} |\frac{dX(e^{jw})}{dw}|^2 \, dw = 2\pi \sum_{-\infty}^{\infty} |n|^2 |x[n]|^2 = 316\pi$$

5.24 (1) for $\text{Re}\{X(e^{jw})\}$ to be zero, the signal must be real and odd. Only signal (b) and (i)

Are real and odd.

(2) $\text{Im}\{X(e^{jw})\}$ to be zero , the signal must be real and even Only signal (b) and (h)

Are real and even

(3) Assume $Y(e^{jw}) = e^{jaw} X(e^{jw})$. Using the time shifting property of the Fourier transform

We have y[n]=x[n+a] , if $Y(e^{jw})$ is real, then y[n] is real and even (assuming that x[n] is real).

Therefore, x[n] has to be symmetric about a/ this is true only for signal (a) , (b), (d) , (e) , (f) , and (h).

(4) since $\int_{-\pi}^{\pi} X(e^{jw}) dw = 2\pi x[0]$, the given condition is satisfied only is x[n]=0. this is

True for signal (b), (e) , (f) , (h) , and (f).

(5) $X(e^{jw})$ is always periodic with period 2π. Therefore ,all signal satisfy this condition

(6) since $x(e^{j0}) = \sum_{n=-\infty}^{\infty} x[n]$, the given condition is satisfied only the samples of the signal

Add up to zero. This is true for signal (b), (g) , and (i).

5.25. if the inverse Fourier transform of $X(e^{jw})$ is x[n], then

$$x_e[n] = \varepsilon v\{x[n]\} = \frac{x[n] + x[-n]}{2} \xleftarrow{FT} A(w)$$

And

$$x_o[n] = od\{x[n]\} = \frac{x[n] + x[-n]}{2} \xleftarrow{FT} jB(w)$$

Therefore , the inverse Fourier transform of B(w) is $-jx_0[n]$. Also, the inverse Fourier transform of

$A(w)e^{jw}$ is. Therefore, the time function corresponding to the inverse

Fourier transform of $B(w) + A(w)e^{jw}$ will be $x_e[n+1] - jx_0[n]$. this is as shown in the

figure s 5.25

Figure s 5.25
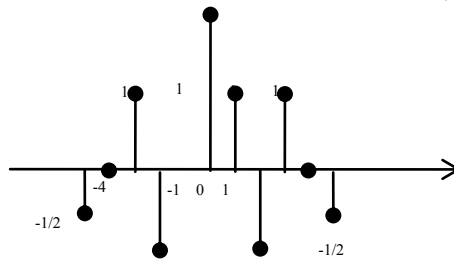
5.26 (a) we may express $X_2(e^{jw})$ as

$$X_2(e^{jw}) = \text{Re}\{X_1(e^{jw})\} + \text{Re}\{X_1(e^{j(w-2\pi/3)})\} + \text{Re}\{X_1(e^{j(w+2\pi/3)})\}$$

Therefore

$$x_2[n] = \varepsilon v\{x_1[n]\}[1 + e^{j2\pi/3} + e^{-j2\pi/3}]$$

(b) We may express $X_3(e^{j\omega})$ as

$$X_3(e^{j\omega}) = \text{Im}\{X_1(e^{j(\omega-\pi)})\} + \text{Im}\{X_1(e^{j(\omega+\pi)})\}.$$

Therefore,

$$x_3[n] = Od\{x_1[n]\}\{e^{j\pi n} + e^{-j\pi n}\} = 2(-1)^n Od\{x_1[n]\}.$$

(c) We may express α as

$$\alpha = \frac{j \frac{dX_1(e^{j\omega})}{d\omega}\Big|_{\omega=0}}{X_1(e^{j\omega})\Big|_{\omega=0}} = \frac{j(-6j/\pi)}{1} = \frac{6}{\pi}.$$

(d) Using the fact $H(e^{j\omega})$ is the frequency of an idea lowpass filter with cutoff frequency $\pi/6$, we may draw $X_4(e^{j\omega})$ as show in figure s5.26
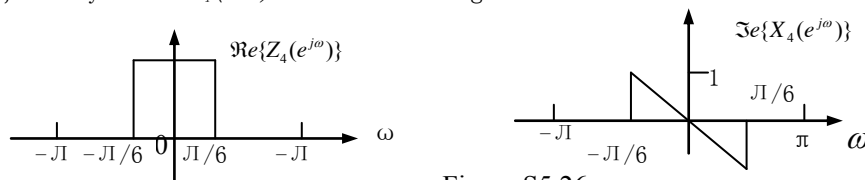

Figure S5.26

5.27. (a) $W(e^{j\omega})$ will be the periodic convolution of $X(e^{j\omega})$ with $P(e^{j\omega})$. The Fourier transforms are sketched in figure S5.27.

 (b) The Fourier transform of $Y(e^{j\omega})$ of y[n] is $Y(e^{j\omega})=P(e^{j\omega})H(e^{j\omega})$. The LTI system with unit sample response h[n] is an idea lowpass filter with cutoff Frequency $\pi/2$. Therefore, $Y(e^{j\omega})$ for each choice of p[n] are as shown in Figure S5.27. Therefore, y[n] in each case is :

 (i) y[n]=0

 (ii) $y[n]= \dfrac{\sin(\pi n/2)}{2\pi n} - \dfrac{1-\cos(\pi n/2)}{\pi^2 n^2}$

 (iii) $y[n]= \dfrac{\sin(\pi n/2)}{\pi^2 n^2} - \dfrac{\cos(\pi n/2)}{2\pi n}$

 (iv) $y[n]= 2\left[\dfrac{\sin(\pi n/4)}{\pi n}\right]^2$

 (v) $y[n]= \dfrac{1}{4}\left[\dfrac{\sin(\pi n/2)}{\pi n}\right]$

5.28. Let

$$\frac{1}{2\pi}\int X\left(e^{j\theta}\right)G\left(e^{j(\omega-\theta)}\right)d\theta = 1+e^{-j\omega} = Y\left(e^{j\omega}\right).$$

Taking the inverse Fourier transform of the above equation ,we obtain

   g[n]x[n]= δ[n]+ δ[n-1]=y[n].
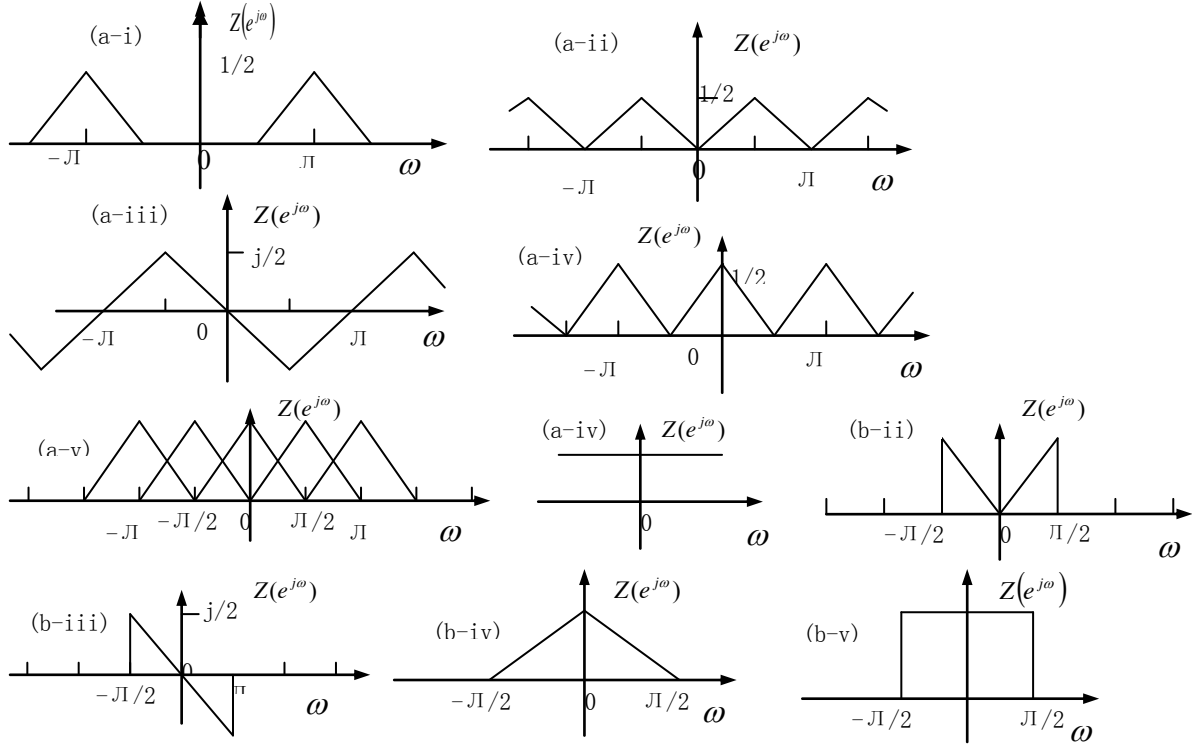

Figure S5.27

(a) If x[n]=(-1)^n,

    g[n]= δ[n]- δ[n-1].

(b)If x[n]=(1/2)^{n}u[n],g[n] has to be chosen such that

$$g[n]=\begin{cases} 1, & n=0 \\ 2, & n=1 \\ 0, & n>1 \\ anyvalue, & otherwise \end{cases}$$

 Therefore, there are many possible chosen for g[n].

5.29.(a)Let the output of the system be y[n].We known that

   $X(e^{j\omega})=X(e^{j\omega})H(e^{j\omega})$.

 In this part of the problem

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}} \quad .$$

（i）we have

$$X(e^{jw}) = \frac{1}{1 - \frac{3}{4}e^{-jw}}$$

Therefore

$$Y(e^{jw}) = [\frac{1}{1 - \frac{3}{4}e^{-jw}}][\frac{1}{1 - \frac{1}{2}e^{-jw}}]$$

$$= \frac{-2}{1 - \frac{1}{2}e^{-jw}} + \frac{3}{1 - \frac{3}{4}e^{-jw}}$$

Taking the inverse Fourier transform, we obtain

$$y[n] = 3(\frac{3}{4})^n u[n] - 2(\frac{1}{2})^n u[n]$$

(ii) we have

$$Y(e^{jw}) = [\frac{1}{(1 - \frac{3}{4}e^{-jw})^2}][\frac{1}{1 - \frac{1}{2}e^{-jw}}]$$

$$= \frac{4}{1 - \frac{1}{2}e^{-jw}} + \frac{2}{1 - \frac{1}{4}e^{-jw}} - \frac{3}{(1 - \frac{1}{4}e^{-jw})^2}$$

Taking the inverse Fourier transform, we obtain

$$y[n] = 4(\frac{1}{2})^n u[n] - 2(\frac{1}{4})^n u[n] - 3(n+1)(\frac{1}{4})^n u[n]$$

(iii) We have

$$X(e^{jw}) = 2\pi \sum_{-\infty}^{\infty} \delta(w - (2k+1)\pi)$$

Therefore

$$Y(e^{jw}) = [2\pi \sum_{-\infty}^{\infty} \delta(w - (2k+1)\pi)][\frac{1}{1 - \frac{1}{2}e^{-jw}}]$$

$$= \frac{4\pi}{3} \sum_{-\infty}^{\infty} \delta(w - (2k+1)\pi)$$

Taking the inverse Fourier transform, we obtain

$$x[n] = \frac{2}{3}(-1)^n$$

(b) Given

$$h[n] = \frac{1}{2}(\frac{1}{2}e^{j\pi/2})^n u[n] + \frac{1}{2}(\frac{1}{2}e^{-j\pi/2})^n u[n]$$

we obtain

$$H[e^{jw}] = \frac{1/2}{1 - \frac{1}{2}e^{j\pi/2}e^{-jw}} + \frac{1/2}{1 - \frac{1}{2}e^{-j\pi/2}e^{-jw}}$$

(i)We have

$$X(e^{jw}) = \frac{1}{1 - \frac{1}{2}e^{-jw}}$$

Therefore

$$Y[e^{jw}] = [\frac{1/2}{1 - \frac{1}{2}e^{j\pi/2}e^{-jw}} + \frac{1/2}{1 - \frac{1}{2}e^{-j\pi/2}e^{-jw}}][\frac{1}{1 - \frac{1}{2}e^{-jw}}]$$

$$= \frac{A}{1 - \frac{1}{2}e^{j\pi/2}e^{-jw}} + \frac{B}{1 - \frac{1}{2}e^{-jw}} + \frac{C}{1 - \frac{1}{2}e^{-j\pi/2}e^{-jw}}$$

Where A=-j/[2(1-j)], B=1/2, and C= 1/[2(1+j)], therefore

$$y[n] = \frac{-j}{2(1-j)}(\frac{j}{2})^n u[n] + \frac{1}{2(1+j)}(-\frac{j}{2})^n u[n] + \frac{1}{2}(\frac{1}{2})^n u[n]$$

(ii)In this case

$$y[n] = \frac{\cos(\pi n/2)}{3}[4 - (\frac{1}{2})^n]u[n]$$

(c) Here

$$Y[e^{jw}] = X(e^{jw})H(e^{jw}) = -3e^{-2jw} - e^{jw} + 1 - 2e^{-2jw}$$
$$+ 6e^{-jw} + 2e^{-jw} - 2e^{-2jw} + 4e^{-j5w}$$
$$+ 3e^{5jw} + 2e^{4jw} - e^{3jw} + 2e^{jw}$$

Therefore，

$$y[n] = 3\delta[n+5] + \delta[n+4] - \delta[n+3] - 3\delta[n+2]$$
$$+ \delta[n+1] + \delta[n] + 6\delta[n-1] - 2\delta[n-3] + 4\delta[n-5]$$

5.30 (a) the frequency response of the system is as shown in figure s5.30

(b) the Fourier transform in figure s5.30 $X(e^{jw})$ of x[n] is as shown in figure s5.30

(i) the frequency response $H(e^{jw})$ is as shown in figure s5.30. therefore , y[n]=sin(π n/8)

(ii)the frequency response $H(e^{jw})$ is as shown in figure s5.30. therefore , y[n]=2sin(π n/8)-cos(π n/4)

(iii) the frequency response $H(e^{jw})$ is as shown in figure s5.30. therefore , y[n]= 1/6sin(π n/8)-1/4 cos(π n/4)
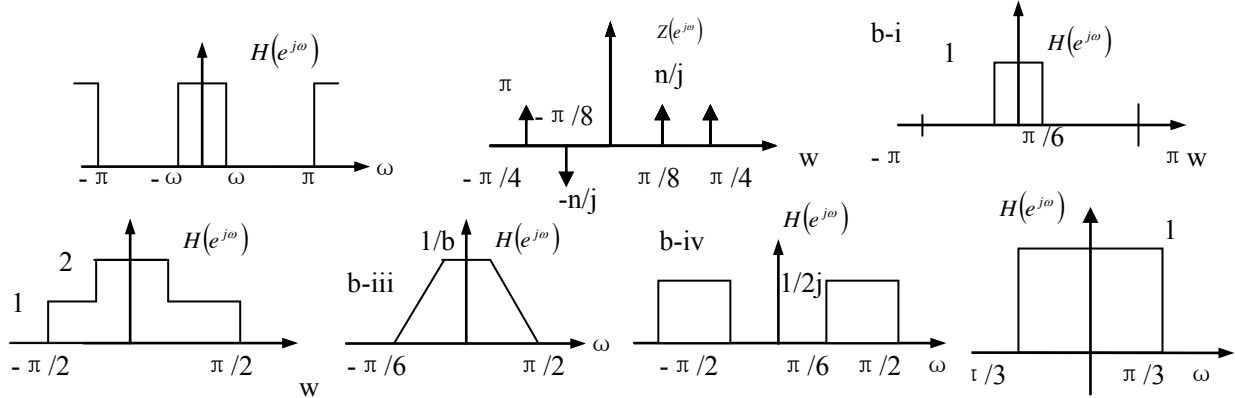

Figure S5.30

(iv)The frequency response $H(e^{j\omega})$ is as show in Figure S5.30. Therefore, y[n]=-sin(π n/4).

(c) The frequency response $H(e^{j\omega})$ is as show in Figure S5.30.

(i)   The signal x[n] is periodic with period 8.The Fourier series coefficients of the signal are

$$a_k = \tfrac{1}{8}\sum_{n=0}^{7}x[n]e^{-j(2\pi/8)kn}$$

The Fourier transform of this signal is

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty}2\pi a_k\delta(\omega - 2\pi k/8) \cdot$$

The Fourier transform $Y(e^{j\omega})$ of the output is $Y(e^{j\omega})=X(e^{j\omega})H(e^{j\omega})$.Therefore,

$$Y(e^{j\omega}) = 2\pi[a_0\delta(\omega) + a_1\delta(\omega - \pi/4) + a_{-1}\delta(\omega + \pi/4)]$$

In the range $0 \le |\omega| \le \pi$. Therefore,

$$y[n] = a_0 + a_1 e^{j\pi n/4} + a_{-1}e^{-j\pi/4} = \tfrac{5}{8} + [(1/4) + (1/2)(1/\sqrt{2})]\cos(\pi n/4).$$

(ii)   The signal *x[n]* is periodic with periodic 8.The Fourier series coefficients of the signal are

$$a_k = \tfrac{1}{8}\sum_{n=0}^{7}x[n]e^{-j(2\pi/8)k\pi}$$

The Fourier transform of this signal is

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty}2\pi a_k\delta(\omega - 2\pi k/8)$$

The Fourier transform $Y(e^{j\omega})$ of the output is $Y(e^{j\omega}) =X(e^{j\omega})H(e^{j\omega})$.Therefore,

$$Y(e^{j\omega}) = 2\pi[a_0\delta(\omega) + a_1\delta(\omega - \pi/4) + a_{-1}\delta(\omega + \pi/4)]$$

In the range $0 \le |\omega| \le \pi$.Therefore,

$$y[n] = a_0 + a_1 e^{j\pi n/4} + a_{-1}e^{-j\pi/4} = \tfrac{1}{8} + [(1/4) - (1/2)(1/\sqrt{2})]\cos(\pi n/4).$$

(iii)   Again in this case, the Fourier transform $X(e^{j\omega})$ of the signal x[n] is of the form show in part(i).Therefore,

$$y[n] = a_0 + a_1 e^{j\pi n/4} + a_{-1} e^{-j\pi/4} = \tfrac{1}{8} + \left[(1/4) - (1/2)(1/\sqrt{2})\right]\cos(\pi n/4).$$

(iv)   in this case, the output is

$$y[n] = h[n] * x[n] = \frac{\sin[\pi/3(n-1)]}{\pi(n-1)} + \frac{\sin[\pi/3(n+1)]}{\pi(n+1)}$$

5.31.(a) Form the given information, it is clear that when the input to the system is a complex exponential of frequency ω_0, the output is a complex exponential of the same frequency but scaled by the $|\omega_0|$. Therefore, the frequency response of the system is

$$H(e^{j\omega}) = |\omega|, \text{ for } 0 \le |\omega_0| \le \pi.$$

(b) Taking the inverse Fourier transform of the frequency response, we obtain

$$h[n] = \tfrac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \tfrac{1}{2\pi} \int_{-\pi}^{0} -\omega e^{j\omega} d\omega + \tfrac{1}{2\pi} \int_{0}^{\pi} \omega e^{j\omega n} d\omega$$

$$= \tfrac{1}{\pi} \int_{0}^{\pi} \omega \cos(\omega n) d\omega$$

$$= \tfrac{1}{\pi} \left[\frac{\cos(n\pi) - 1}{n^2} \right]$$

5.32   From the synthesis equation (5.8) we have

$$\left[\frac{1}{2\pi} \int_{-\pi}^{\pi} H_1(e^{j\omega}) d\omega \right] \quad \left[\frac{1}{2\pi} \int_{-\pi}^{\pi} H_2(e^{j\omega}) d\omega \right] = h_1[0] h_2[0]$$

Also, since

$$h_1[n] * h_2[n] \xleftarrow{\ FT\ } H_1(e^{j\omega}) H_2(e^{j\omega})$$

we have

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} H_1(e^{j\omega}) H_2(e^{j\omega}) d\omega = [h_1[n] * h_2[n]]_{n=0}$$

therefore, the question here amounts to asking whether it is true that

$$h_1[0] h_2[0] = [h_1[n] * h_2[n]]_{n=0}$$

since $h_1[n]$ and $h_2[n]$ are causal, this is indeed true.

5.33   (a) Taking the Fourier transform of the given difference equation we have

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1 + \tfrac{1}{2} e^{-j\omega}}$$

(b) The Fourier transform of the output will be $Y(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega})$.

(i) In this case

$$X(e^{j\omega}) = \frac{1}{1 - \tfrac{1}{2} e^{-j\omega}}$$

Therefore

$$Y(e^{j\omega}) = \left[\frac{1}{1 - \tfrac{1}{2} e^{-j\omega}} \right] \left[\frac{1}{1 + \tfrac{1}{2} e^{-j\omega}} \right]$$

$$= \frac{1/2}{1 - \tfrac{1}{2} e^{-j\omega}} + \frac{1/2}{1 + \tfrac{1}{2} e^{-j\omega}}$$

Taking the inverse transform, we obtain

$$y[n] = \frac{1}{2} \left(\frac{1}{2} \right)^n u[n] + \frac{1}{2} \left(-\frac{1}{2} \right)^n u[n].$$

(ii) In this case

$$X(e^{j\omega}) = \frac{1}{1 + \tfrac{1}{2} e^{-j\omega}}$$

Therefore,

$$Y(e^{j\omega}) = \left[\frac{1}{1 - \tfrac{1}{2} e^{-j\omega}} \right]^2$$

Taking the inverse Fourier transform, we obtain

$$y[n] = (n+1) \left(-\frac{1}{2} \right)^n u[n],$$

(iii) In this case $X(e^{j\omega}) = 1 + \dfrac{1}{2} e^{-j\omega}$.

Therefore $Y(e^{j\omega}) = 1$.

Taking the inverse Fourier transform , we have

$$y[n] = \delta[n] \ .$$

(iv) In this case

$$X(e^{j\omega}) = 1 - \dfrac{1}{2} e^{-j\omega} \ .$$

Therefore

$$Y(e^{j\omega}) = \left[1 - \dfrac{1}{2} e^{-j\omega}\right]\left[\dfrac{1}{1 + \dfrac{1}{2} e^{-j\omega}}\right]$$

$$= -1 + \dfrac{2}{1 + \dfrac{1}{2} e^{-j\omega}}$$

Taking the inverse Fourier transform , we obtain

$$y[n] = -\delta[n] + 2\left(-\dfrac{1}{2}\right)^n u[n] \ .$$

(c) (i) We have

$$Y(e^{j\omega}) = \left[\dfrac{1 - \dfrac{1}{4} e^{-j\omega}}{1 + \dfrac{1}{2} e^{-j\omega}}\right]\left[\dfrac{1}{1 + \dfrac{1}{2} e^{-j\omega}}\right]$$

$$= \dfrac{1}{(1 + \dfrac{1}{2} e^{-j\omega})^2} + \dfrac{\dfrac{1}{4} e^{-j\omega}}{(1 + \dfrac{1}{2} e^{-j\omega})^2}$$

Taking the inverse Fourier transform . we obtain

$$Y(e^{j\omega}) = (n+1)\left(-\dfrac{1}{2}\right)^n u[n] - \dfrac{1}{4} n\left(-\dfrac{1}{2}\right)^{n-1} u[n-1] \ .$$

(ii) We have

$$Y(e^{j\omega}) = \left[\dfrac{1 + \dfrac{1}{2} e^{-j\omega}}{1 - \dfrac{1}{4} e^{-j\omega}}\right]\left[\dfrac{1}{1 + \dfrac{1}{2} e^{-j\omega}}\right]$$

$$= \dfrac{1}{1 - \dfrac{1}{4} e^{-j\omega}} \ .$$

Taking the inverse Fourier transform , we obtain

$$y[n] = \left(\dfrac{1}{4}\right)^n u[n]$$

.(iii) We have

$$Y(e^{j\omega}) = \left[\dfrac{1}{\left(1 + \dfrac{1}{2} e^{-j\omega}\right)\left(1 - \dfrac{1}{4} e^{-j\omega}\right)}\right]\left[\dfrac{1}{1 + \dfrac{1}{2} e^{-j\omega}}\right]$$

$$= \dfrac{2/3}{\left(1 + \dfrac{1}{2} e^{-j\omega}\right)^2} + \dfrac{2/9}{1 + \dfrac{1}{2} e^{-j\omega}} + \dfrac{1/9}{1 - \dfrac{1}{4} e^{-j\omega}}$$

Taking the inverse Fourier transform , we obtain

$$y[n] = \dfrac{2}{3}(n+1)\left(-\dfrac{1}{2}\right)^n u[n] + \dfrac{2}{9}\left(-\dfrac{1}{2}\right)^n u[n] + \dfrac{1}{9}\left(\dfrac{1}{4}\right)^n u[n] \ .$$

(iv) We have

$$Y(e^{j\omega}) = \left[1 - \dfrac{1}{2} e^{-3j\omega}\right]\left[\dfrac{1}{1 + \dfrac{1}{2} e^{-j\omega}}\right]$$

$$= \dfrac{1}{1 + \dfrac{1}{2} e^{-j\omega}} + \dfrac{2e^{-3j\omega}}{1 + \dfrac{1}{2} e^{-j\omega}}$$

Taking the inverse Fourier transform , we obtain

$$y[n] = \left(-\frac{1}{2}\right)^n u[n] + 2\left(-\frac{1}{2}\right)^{n-3} u[n-3]$$

5.34 (a) Since the two systems are cascaded , the frequency response of the overall system is

$$H(e^{j\omega}) = H_1(e^{j\omega})H_2(e^{j\omega})$$

$$= \frac{2 - e^{-j\omega}}{1 + \frac{1}{8}e^{-j3\omega}}$$

Therefore , the Fourier transform of the input and output of the overall system are related by

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2 - e^{-j\omega}}{1 + \frac{1}{8}e^{-3j\omega}}$$

Cross-multiplying and taking the inverse Fourier transform , we get

$$y[n] + \frac{1}{8}y[n-3] = 2x[n] - x[n-1] \quad \cdot$$

(b) We may rewriter the overall frequency response as

$$H(e^{j\omega}) = \frac{4/3}{\left(1 + \frac{1}{2}e^{-j\omega}\right)} + \left(\frac{(1 + j\sqrt{3})/3}{1 + \frac{1}{2}e^{j120}e^{-j\omega}}\right) + \frac{(1 - j\sqrt{3})/3}{1 - \frac{1}{2}e^{-j120}e^{-j\omega}}$$

Taking the inverse Fourier transform , we get

$$h[n] = \frac{4}{3}\left(-\frac{1}{2}\right)^n u[n] + \frac{1 + j\sqrt{3}}{3}\left(\frac{1}{2}e^{j120}\right)^n u[n] + \frac{1 - j\sqrt{3}}{3}\left(\frac{1}{2}e^{-j120}\right)^n u[n] \quad \cdot$$

5.35 (a) Taking the Fourier transform of both sides of the given difference equation we obtain

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{b + e^{-j\omega}}{1 - ae^{-j\omega}}$$

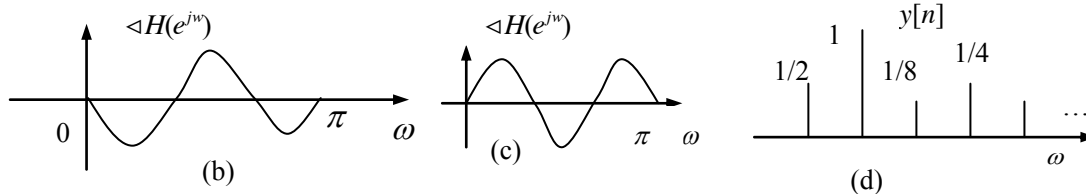In order for $|H(e^{j\omega})|$ to be one . we must ensure that

$$|b + e^{-j\omega}| = |1 - ae^{-j\omega}|$$

$$1 + b^2 + 2b\cos\omega = 1 + a^2 - 2a\cos\omega$$

this is possible only if $b = -a$.

(b) The plot is as shown Figure S5.35

(c) The plot is as shown Figure S5.35

(b) (c) (d)

(d) When $a = -\frac{1}{2}$.

$$H(e^{jw}) = \frac{\frac{1}{2} + e^{-j\omega}}{1 + \frac{1}{2}e^{-jw}}$$

Also

$$X(e^{jw}) = \frac{1}{1 - \frac{1}{2}e^{-jw}}$$

Therefore

$$Y(e^{jw}) = \frac{\frac{1}{2} + e^{-j\omega}}{\left(1 + \frac{1}{2}e^{-jw}\right)\left(1 - \frac{1}{2}e^{-j\omega}\right)}$$

$$= \frac{5/4}{1 - \frac{1}{2}e^{-j\omega}} - \frac{3/4}{1 + \frac{1}{2}e^{-j\omega}}$$

Taking the inverse Fourier transform we obtain

$$y[n] = \frac{5}{4}\left(\frac{1}{2}\right)^n u[n] - \frac{3}{4}\left(-\frac{1}{2}\right)^n u[n] \quad \cdot$$

This is as sketched in Figure S5.35

5.36 (a) The frequency responses are related by the following expression

$$G(e^{j\omega}) = \frac{1}{H(e^{j\omega})}$$

(b) (i) Here $H(e^{j\omega})=1-\frac{1}{4}e^{-j\omega}$. Therefore , $G(e^{j\omega})=1/\left(1-\frac{1}{4}e^{-j\omega}\right)$ and $g[n]=\left(\frac{1}{4}\right)^{n}u[n]$ since

$$G(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

the difference equation relating the input $x[n]$ and output $y[n]$ is

$$y[n]-\frac{1}{4}y[n-1]=x[n]$$

(ii) Here $H(e^{j\omega})=1/\left(1+\frac{1}{2}e^{-j\omega}\right)$. Therefore , $G(e^{j\omega})=1+\frac{1}{2}e^{-j\omega}$ and $g[n]=\delta[n]+\frac{1}{2}\delta[n-1]$. since

$$G(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=1+\frac{1}{2}e^{-j\omega}$$

the difference equation relating the input $x[n]$ and output $y[n]$ is

$$y[n]=x[n]+\frac{1}{2}x[n-1]$$

(iii) Here $H(e^{j\omega})=\left(1-\frac{1}{4}e^{-j\omega}\right)/\left(1+\frac{1}{2}e^{-j\omega}\right)$. therefore , $G(e^{j\omega})=\left(1+\frac{1}{2}e^{-j\omega}\right)/\left(1-\frac{1}{4}e^{-j\omega}\right)$

and $g[n]=\left(\frac{1}{4}\right)^{n}u[n]+\frac{1}{2}\left(\frac{1}{4}\right)^{n-1}u[n-1]$ since

$$G(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1+\frac{1}{2}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}}$$

the difference equation relating the input $x[n]$ and output $y[n]$ is

$$y[n]-\frac{1}{4}y[n-1]=x[n]+\frac{1}{2}x[n-1]$$

(iv) Here $H(e^{j\omega})=\left(1-\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}\right)/\left(1+\frac{5}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}\right)$ .therefore

$$G(e^{j\omega})=\left(1+\frac{5}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}\right)/\left(1-\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}\right) \text{ therefore}$$

$$G(e^{j\omega})=1+\frac{2}{1-(1/2)e^{-j\omega}}-\frac{2}{1+(1/4)e^{-j\omega}}$$

and

$$g[n]=\delta[n]+2\left(\frac{1}{2}\right)^{n}u[n]-2\left(-\frac{1}{4}\right)^{n}u[n]$$

since

$$G(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{\left(1+\frac{5}{4}e^{-jw}-\frac{1}{8}e^{-2j\omega}\right)}{\left(1-\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}\right)}$$

the difference equation relating the input $x[n]$ and output $y[n]$ is

$$y[n]-\frac{1}{4}y[n-1]-\frac{1}{8}y[n-1]=x[n]+\frac{5}{4}x[n-1]-\frac{1}{8}x[n-2]$$

(v) Here $H(e^{j\omega})=\left(1-\frac{1}{2}e^{-j\omega}\right)/\left(1+\frac{5}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}\right)$ .therefore

$$G(e^{j\omega})=\left(1+\frac{5}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}\right)/\left(1-\frac{1}{2}e^{-j\omega}\right) \text{ since}$$

$$G(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{\left(1+\frac{5}{4}e^{-jw}-\frac{1}{8}e^{-2j\omega}\right)}{\left(1-\frac{1}{2}e^{-j\omega}\right)}$$

the difference equation relating the input $x[n]$ and output $y[n]$ is

$$y[n]-\frac{1}{2}y[n-1]=x[n]+\frac{5}{4}x[n-1]-\frac{1}{8}x[n-2]$$

(v) Here $H(e^{j\omega})=1/\left(1+\frac{5}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}\right)$ .therefore

$$G(e^{j\omega})=\left(1+\frac{5}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}\right) \text{ since}$$

$$G(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\left(1+\frac{5}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}\right)$$

We have

$$y[n] = \delta[n] + \frac{5}{4}\delta[n-1] - \frac{1}{8}\delta[n-2]$$

and the difference equation relating the input $x[n]$ and output $y[n]$ is

$$y[n] = x[n] + \frac{5}{4}x[n-1] - \frac{1}{8}x[n-2]$$

(c) The frequency response of the given system is

$$H(e^{j\omega}) = \frac{e^{-j\omega} - \frac{1}{2}e^{-2j\omega}}{1 + e^{-j\omega} + \frac{1}{4}e^{-2j\omega}}$$

The frequency response of the given system is

$$G(e^{j\omega}) = \frac{1}{H(e^{j\omega})} = \frac{e^{j\omega} + 1 + \frac{1}{4}e^{-j\omega}}{1 + \frac{1}{2}e^{-j\omega}}$$

Therefore

$$g[n] = \left(\frac{1}{2}\right)^{n+1} u[n+1] + \left(\frac{1}{2}\right)^{n} u[n] + \frac{1}{4}\left(\frac{1}{2}\right)^{n-1} x[n-1]$$

Clearly, $g[n]$ is not a causal impulse response

If we delay this impulse response by 1 sample , then it become causal . Furthermore the output of the inverse system will then be $x[n-1]$ .the impulse response of this causal system is

$$g_1[n] = g[n-1] = \left(\frac{1}{2}\right)^{n} u[n] + \left(\frac{1}{2}\right)^{n-1} x[n-1] + \frac{1}{4}\left(\frac{1}{2}\right)^{n-2} u[n-2]$$

5.37 Give that

$$x[\ ] \xleftarrow{FT} X(e^{j\omega})$$

(i)Since

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jwn}$$

We may write

$$X^*(e^{-jw}) = \sum_{n=-\infty}^{\infty} x^*[n]e^{-jwn}$$

Comparing with the analysis eq 5.9),we conclude that

$$x^*[n] \quad \xleftarrow{FT} X^*(e^{-jw})$$

Therefore

$$\Re e\{x[n]\} = \frac{x[n] + x^*[n]}{2} \xleftarrow{FT} \frac{X(e^{jw}) + X^*(e^{jw})}{2}$$

(ii)Since

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jwn}$$

We may write

$$X(e^{-jw}) = \sum_{n=-\infty}^{\infty} x[-n]e^{-jwn}$$

Therefore,

$$x[-n] \quad \xleftarrow{FT} X(e^{-jw})$$

Form the previous part we know that

$$x^*[n] \quad \xleftarrow{FT} X^*(e^{-jw})$$

Therefore, putting these two statements together we get

$$x^*[-n] \quad \xleftarrow{FT} X^*(e^{jw})$$

(iii) Form our previous part we know that

$$\mathscr{E}\nu \{x[n]\} = \frac{x[n] + x[-n]}{2} \xleftarrow{FT} \frac{X(e^{jw}) + X(e^{-jw})}{2}$$

5.38. From the synthesis equation (5.8) we obtain

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{jw})e^{jwn} d_w$$

$$= \frac{1}{2\pi} \int_{0}^{\pi} X(e^{jw})e^{jwn} d_w + \frac{1}{2\pi} \int_{0}^{\pi} X(e^{-jw})e^{-jwn} d_w$$

Since x[n] is real. $X(e^{-jw}) = X*(e^{jw})$. Therefore

$$x[n] = \frac{1}{\pi}\int_0^\pi \Re e\{X(e^{jw})\} \{e^{wn} + e^{-jwn}\} d_w + \frac{j}{2\pi}\int_0^\pi \text{Im}\{X(e^{jw})\}\{e^{wn} - e^{-jwn}\} d_w$$

$$= \frac{1}{\pi}\int_0^\pi \Re e\{X(e^{jw})\} 2\cos(wn) d_w - \frac{1}{\pi}\int_0^\pi \text{Im}\{X(e^{jw})\}\sin(wn) d_w$$

Therefore,

$$B(w) = \frac{1}{\pi}\Re e\{X(e^{jw})\}\cos(wn), \text{ and } -\frac{1}{\pi}\text{Im}\{X(e^{jw})\}\sin(wn)$$

5.39. Let y[n]=x[n]*h[n]. Then

$$Y(e^{jw}) = \sum_{n=-\infty}^{\infty}\{x[n]*h[n]\}e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}x[n]*h[n-k]e^{-jwn}$$

$$= \sum_{k=-\infty}^{\infty}x[k]\sum_{n=-\infty}^{\infty}h[n-k]e^{-jwn}$$

$$= H(e^{jw})\sum_{k=-\infty}^{\infty}x[n]e^{-jwk}$$

$$H(e^{jw})X(e^{jw})$$

5.40. Let y[n]=x[n]*[n]. Then using the convolution sum

$$y[n] = \sum_{k=-\infty}^{\infty}x[k]h[-k] \qquad (S5.40\text{-}1)$$

Using the convolution property of the Fourier transform,

$$y[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw})H(e^{jw})d_w \qquad (S5.40\text{-}2)$$

Now let h[n]=x*[-n]. Then $H(e^{jw}) = X*(e^{jw})$ Substituting in the right-hand sides of equations(S5.40-2)and equating them,

$$\sum_{k=-\infty}^{\infty}x[k]x*[k] = \frac{1}{2\pi}\int_{-\infty}^{\infty}X(e^{jw})X*(e^{jw})d_w$$

Therefore，

$$\sum_{n=-\infty}^{\infty}|x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}|X(e^{jw})|^2 d_w$$

Now let h[n]=z*[-n]. Then $H(e^{jw}) = Z*(e^{jw})$ .Substituting in the right-hand sides of equation (S5.40-1)and(S5.40-2) and equation then.

$$\sum_{n=-\infty}^{\infty}x[k]z*(k) = \frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{jw})Z*(e^{jw})d_w$$

5.41. (a)The Fourier transform $X(e^{jw})$ of the signal x[n] is

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty}x[n]e^{-jwn} = \sum_{n_0}^{n_0+N-1}x[n]e^{-jwn}$$

Therefore,

$$X(e^{j2\pi k/N}) = \sum_{n=n_0}^{n_0+N-1}x[n]e^{-j(2\pi/N)kn}$$

Now, we may write the expression for the FS coefficients of x[n] $\tilde{x}[n]$ as

$$a_k = \frac{1}{N}\sum_{<N>}x[n]e^{-j(2\pi/N)kn} = \frac{1}{N}\sum_{n=n_0}^{n_0+N-1}x[n]e^{-j(2\pi/N)kn}$$

(Because x[n]= $x[n]$ in the range $n_0 \le n \le n_0 + N-1$) . Comparing the above equation with eq. (S5.41-1), we get

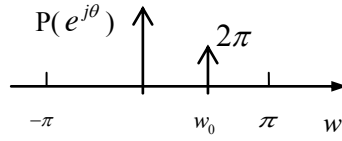$$a_k = \frac{1}{N}X(e^{j2\pi k/N})$$

(b) (i) From the given information,

$$X(e^{jw}) = 1 + e^{-jw} + e^{-2jw} + e^{-3jw}$$

$$= e^{-j(3/2)w}\{e^{j(3/2)w} + e^{-j(3/2)w}\} + e^{-j(3/2)w}\{e^{j(1/2)w} + e^{-j(1/2)w}\}$$

$$= 2e^{-j(3/2)w}\{\cos(3w/2) + \cos(w/2)\}$$

(ii)From part (a),

$$a_k = \frac{1}{N}X(e^{j2\pi k/N}) = \frac{1}{N}2e^{-j(3/2)2\pi k/N}\{\cos(6\pi k/(2N)) + \cos(\pi k/N)\}$$

5.42.(a) $P(e^{jw}) = 2\pi\delta(w - w_0)$ for $|w| < \pi$. This is as shown in Figure S5.42

(b) From the multiplication property of the Fourier transform we have

$$G(e^{jw}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw})P(e^{j(w-\theta)})d_\theta$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw})2\pi\delta(w - \theta - w_0)d_\theta$$

$$= X(e^{j(w-w_0)})$$

5.31. (a) Using frequency shift and linearity properties

$$V(e^{jw}) = \frac{X(e^{j(w-\pi)}) + X(e^{jw})}{2}$$

(b) Let y[n]=v[2n]. Then

$$Y(e^{jw}) = \sum_{n=-\infty}^{\infty} v[2n]e^{-jwn}$$

Since the odd-indexed samples of v[n] are zero, we may put m=2n in the above equation to get

$$Y(e^{jw}) = \sum_{m=-\infty}^{\infty} v[m]e^{-jwm/2} = V(e^{jw/2})$$

(Note that the substitution of n by 2m is balid only if the odd-indexed samples in the summation are zero.)
(c) x[2n] is a new sequence which consists of only the even indexed samples of x[n].v[n] is a sequence whose even-indexed samples are equal to x[n].The odd-indexed samples of v[n] are zero .v[2n] is a new sequence which consists of only the even indexed samples of v[n].This implies that v[2n] is a sequence which consists of only the even indexed samples of x[n].This idea is illustrated in Figure S5.43.

From part (a).

$$G(e^{jw}) = \frac{X(e^{j(w/2-\pi)}) + X(e^{jw/2})}{2}$$

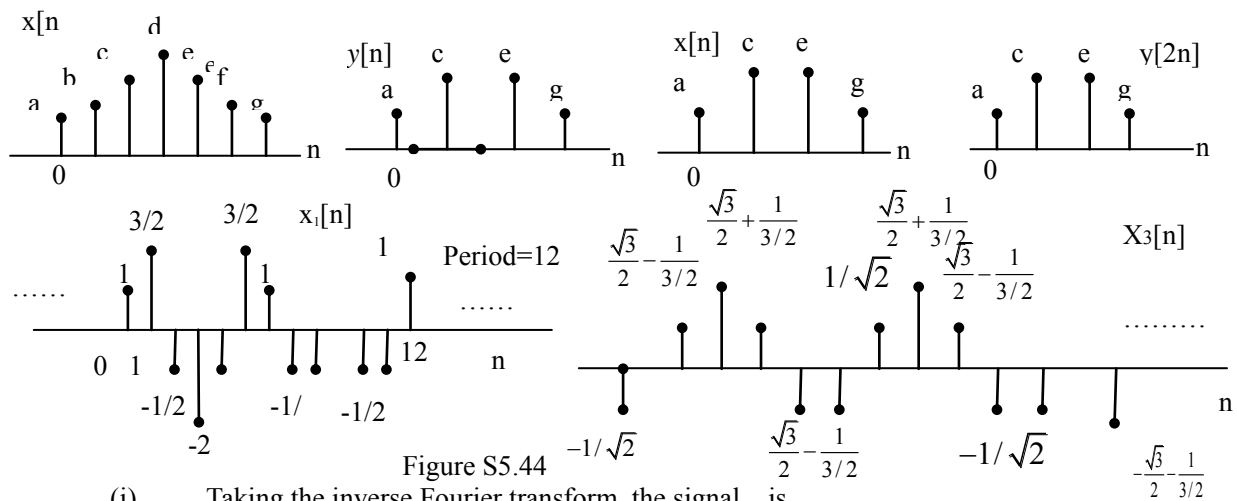**5.44**. (a). The signal is show in Figure S5.44.

Figure S5.44

(i)   Taking the inverse Fourier transform, the signal is

$$x_2[n] = x_1[n+1]$$

(ii)   Taking the inverse Fourier transform ,the signal is

$$x_2[n] = x_1[n-3/2] = \sin(\pi n/3) + \sin(\pi n/2)\cos(3\pi/4) - \cos(\pi n/2)\sin(3\pi/4)$$

This is as shown in Figure S5.44.

(b). Form part (a),

$$x_2[n] = x_1[n+1] = w(nT+T)$$

Also,

$$x_3[n] = x_1[n-3/2] = w(nT-3T/2)$$

Therefore, $\alpha = -1$ and $\beta = 3/2$.

5.45. From the Fourier transform analysis equation

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jwn}$$

(a). Comparing the equation for $x_1(t)$ with the above equation, we obtain

$$x_1(t) = X(e^{-j(2\pi/10)t})$$

Therefore is as $x_1(t)$ is as shown in Figure S5.45.

(b). Comparing the equation for $x_2(t)$ with the equation $X(e^{jw})$, we obtain

$$x_2(t) = X(e^{j(2\pi/10)t}) = x_1(-t)$$

Therefore is as $x_2(t)$ is as shown in Figure S5.45.

(C). We know that $od\{x[n]\}=(x[n]-x[-n])/2$. Therefore.

$$\frac{X(e^{jw}) - X(e^{-jw})}{2} = \sum_{n=-\infty}^{\infty} od\{x[n]\}e^{-jwn}$$

Comparing this with the given equation for $x_3(t)$ .we obtain

$$x_3(t) = \frac{X(e^{-j(2\pi/8)t}) - X(e^{j(2\pi/8)t})}{2}$$

Therefore $x_3(t)$ is as shown in Figure S5.45.

(d). We know that $Re\{x[n]\}=(x[n]+x^*[n])/2$. Therefore,

$$\frac{X(e^{jw}) - X^*(e^{-jw})}{2} = \sum_{n=-\infty}^{\infty} \Re e\{x[n]\}e^{-jwn}$$

Comparing this with the given equation for   .we obtain

$$x_4(t) = \frac{X(e^{-j(2\pi/6)t}) + X^*(e^{j(2\pi/6)t})}{2}$$

Therefore   is as shown in the Figure S5.45.

5.46. (a). Let $x[n] = \alpha^n u[n]$ .Then $X(e^{jw}) = \dfrac{1}{1 - \alpha e^{-jw}}$ .Using the differentiation in frequency property,

$$n\alpha^n u[n] \xleftrightarrow{FT} j\frac{d_{X(e^{jw})}}{d_w} = \frac{\alpha e^{-jw}}{(1 - \alpha e^{-jw})^2}$$

Therefore,

$$(n+1)n\alpha^n u[n] \xleftrightarrow{FT} j\frac{d_{X(e^{jw})}}{d_w} + X(e^{jw}) = \frac{1}{(1 - \alpha e^{-jw})^2}$$

(b). From part (a),it is clear that the result is true for r=1 and r=2. Let us assume that it is also true for k=r-1. We will now attempt to prove that result is true for k=r. We have

$$x_{r-1}[n] = \frac{(n+r-2)}{n!(r-2)!}\alpha^n u[n] \xleftrightarrow{FT} X_{r-1}(e^{jw}) = \frac{1}{(1 - \alpha e^{-jw})^{r-1}}$$
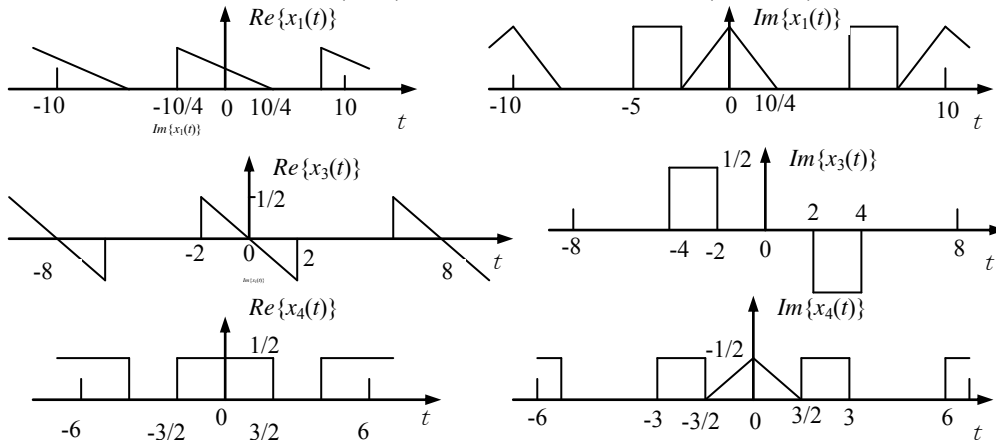

Figure S5.45

From the differentiation in frequency property,

$$nx_{r-1}[n] \xleftrightarrow{FT} \frac{\alpha(r-1)e^{-j\omega}}{(1 - \alpha e^{-j\omega})r - 1} \quad .$$

Therefore,

$$\frac{(n+1)x_{r-1}[n+1]}{\alpha(r-1)} \xleftrightarrow{\ FT\ } \frac{1}{(1-\alpha e^{-jw})^r} \quad .$$

The left hand side of the above expression is

$$\frac{(n+1)x_{r-1}[n+1]}{\alpha(r-1)} = \frac{(n+r-1)!}{n!(r-1)!}\alpha^n u[n] = x_r[n] \quad .$$

Therefore, we have shown that the result is valid for r if it is valid for $r-1$. Since,
We know that the result is valid for $r=2$, we may conclude that it is valid for $r=3$,
$r=4$, and so on.

5.47. (a) If $X(e^{j\omega}) = X(e^{j(\omega-1)})$ then $X(e^{j\omega})$ is periodic with a period of 1. But we already know that $X(e^{j\omega})$ is periodic with a period of 2π. This is only possible if $X(e^{j\omega})$ is a constant for all ω. This implies that $x[n]$ is of the form $k\delta[n]$ where k is a constant. Therefore, the given statement is true.

 (b) If $X(e^{j\omega}) = X(e^{j(\omega-\pi)})$ then $X(e^{j\omega})$ is periodic with a period of π. We also know that $X(e^{j\omega})$ is periodic with a period of 2π. Both these conditions can be satisfied even if $X(e^{j\omega})$ has some arbitrary shape in the region $0 \le |\omega| \le \pi/2$. Therefore, $X(e^{j\omega})$ need not necessarily be a constant. Consequently, $x[n]$ need not be just an impulse. Therefore, the given statement is false.

 (c) We know from Problem 5.43 that the inverse Fourier transform of $X(e^{j\omega/2})$ is the sequence $v[n] = (x[n] + e^{j\pi n}x[n])/2$. The even-indexed samples of $v[n]$ are identical to the even-indexed samples of $x[n]$. The odd-indexed samples of $v[n]$ are zero. If $X(e^{j\omega}) = X(e^{j\omega/2})$, then $x[n] = v[n]$. This implies that the even-indexed samples of $x[n]$ are zero. Consequently, $x[n]$ does not necessarily have to be an impulse. Therefore, the given statement is false.

 (d) From Table 5.1 we know that the inverse Fourier transform of $X(e^{j2\omega})$ is the time –expanded signal

$$x_{(2)}[n] = \begin{cases} x[n/2], n = 0, \pm 2, \pm 4, \cdots \\ 0, otherwise \end{cases}$$

If $X(e^{j\omega}) = X(e^{j2\omega})$, then $x[n] = x_{(2)}[n]$. This is possible only if $x[n]$ is an impulse. Therefore, the given statement is true.

5.48. (a) Taking the Fourier transform of both equations and eliminating $W(e^{j\omega})$, we obtain

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{3 - \frac{1}{2}e^{-j\omega}}{(1 - \frac{1}{2}e^{-j\omega})(1 - \frac{1}{4}e^{-j\omega})} \quad .$$

Taking the inverse Fourier transform of the partial fraction expansion of the above expression, we obtain

$$h[n] = 4(\frac{1}{2})^n u[n] - (\frac{1}{4})^n u[n] \quad .$$

 (b) We know that

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{3 - \frac{1}{2}e^{-j\omega}}{(1 - \frac{1}{2}e^{-j\omega})(1 - \frac{1}{4}e^{-j\omega})}$$

Cross-multiplying and taking the inverse Fourier transform, we obtain

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 3x[n] - \frac{1}{2}x[n-1] \quad .$$

5.49. (a) (i) Consider the signal $x[n] = ax_1[n] + bx_2[n]$, where a and b are constants. Then ,
$X(e^{j\omega}) = aX_1(e^{j\omega}) + bX_2(e^{j\omega})$. Also let the responses of the system to $x_1[n]$ and $x_2[n]$ by $y_1[n]$ and $y_2[n]$, respectively. Substituting for $X(e^{j\omega})$ in the equation given in the problem and simplifying we obtain $Y(e^{j\omega}) = aY_1(e^{j\omega}) + bY_2(e^{j\omega})$. Therefore, the system is linear.

 (ii) Consider the signal $x_1[n] = x[n-1]$. Then $X_1(e^{j\omega}) = e^{-j\omega}X(e^{j\omega})$ Let the response of the system to this signal be $y_1[n]$. From the given equation,

$$Y_1[e^{j\omega}] = 2X_1(e^{j\omega}) + e^{-j\omega}X_1(e^{j\omega}) - \frac{dX_1(e^{j\omega})}{d\omega}$$

$$= e^{-j\omega}[2X(e^{j\omega}) + e^{-j\omega}X(e^{j\omega}) - \frac{dX(e^{j\omega})}{d\omega} + je^{-j\omega}X(e^{j\omega})$$

$$\ne e^{-j\omega}Y(e^{j\omega})$$

106
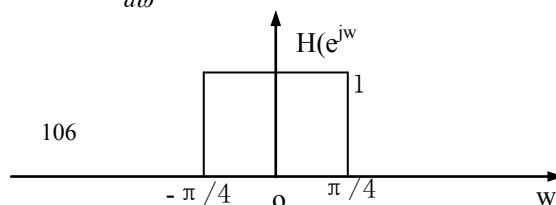
Therefore, the system is not invariant.

(iii) If $x[n] = \delta[n]$, $X(e^{j\omega}) = 1$ Then,

$$Y(e^{j\omega}) = 2 + e^{-j\omega}$$

Therefore, $y[n] = 2\delta[n] + \delta[n-1]$

(b) We may write

$$Y(e^{j\omega}) = \frac{1}{2\pi}\int_{\omega-\pi/4}^{\omega+\pi/4} X(e^{j\theta})H(e^{j(\omega-\theta)})d\theta \qquad \text{Figure S5.49}$$

Where $H(e^{j\omega})$ is as shown in the Figure S5.49.

Using the multiplication property of the Fourier transform and Table 5.2, we obtain

$$y[n] = 2x[n]\frac{\sin(\pi n/4)}{n}$$

5.50. (a) (i) From the given information,

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1-\frac{1}{2}e^{-j\omega}}{(1-\frac{1}{3}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})}$$

Taking the inverse Fourier transform, we obtain

$$h[n] = 3(\frac{1}{4})^n u[n] - 2(\frac{1}{3})^n u[n]$$

(ii) From part (a), we know that

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1-\frac{1}{2}e^{-j\omega}}{(1-\frac{1}{3}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})}$$

Cross-multiplying and taking inverse Fourier transform

$$y[n] - \frac{7}{12}y[n-1] + \frac{1}{12}y[n-2] = x[n] - \frac{1}{2}x[n-1]$$

(b) Form the given information,

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{(1-\frac{1}{2}e^{-j\omega})^2}{2(1-\frac{1}{4}e^{-j\omega})^2}$$

We now want to find $X(e^{j\omega})$ when $Y(e^{j\omega}) = (1/2)e^{-j\omega}/(1+\frac{1}{2}e^{-j\omega})$. From the above equation we

obtain

$$X(e^{j\omega}) = \frac{e^{-j\omega}(1-\frac{1}{4}e^{-j\omega})^2}{(1-\frac{1}{2}e^{-j\omega})^2(1+\frac{1}{2}e^{-j\omega})}$$

Taking the inverse Fourier transform of the partial fraction expansion of the above expression, we obtain

$$x[n] = \frac{3}{8}(-\frac{1}{2})^{n-1}u[n-1] + \frac{3}{8}(\frac{1}{2})^{n-1}u[n-1] + \frac{1}{8}n(\frac{1}{2})^{n-1}u[n-1]$$

5.51. (a) Taking the Fourier transform of $h[n]$ we obtain

$$H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega}) = \frac{\frac{3}{2} - \frac{1}{2}e^{-j\omega}}{1-\frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}}$$

Cross-multiplying and taking the inverse Fourier transform we obtain

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = \frac{3}{2}x[n] - \frac{1}{2}x[n-1]$$

(b) (i) Let us name the intermediate output $\omega[n]$ (See Figure S5.51).
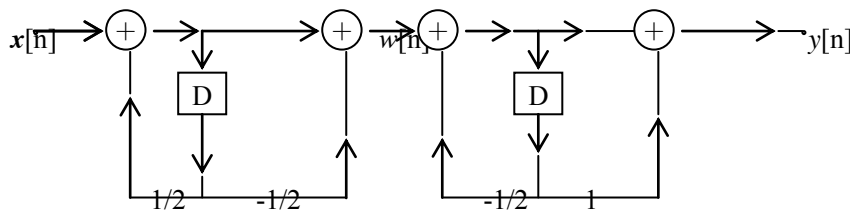

Figure S5.51

We may then write the following difference equations:

$$y[n] + \frac{1}{2} y[n-1] = \frac{1}{4} \omega[n] + \omega[n-1]$$

and

$$\omega[n] - \frac{1}{3} \omega[n-1] = x[n] - \frac{1}{2} x[n-1]$$

Taking the Fourier transform of both these equations and eliminating $W(e^{j\omega})$, we obtain

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\frac{1}{4} + \frac{7}{8} e^{-j\omega} - \frac{1}{2} e^{-2j\omega}}{1 - \frac{1}{4} e^{-2j\omega}}$$

Cross-multiplying and taking the inverse Fourier transform we obtain

$$y[n] - \frac{1}{4} y[n-2] = \frac{1}{4} x[n] + \frac{7}{8} x[n-1] - \frac{1}{2} x[n-2]$$

(ii) From (i)

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\frac{1}{4} + \frac{7}{8} e^{-j\omega} - \frac{1}{2} e^{-2j\omega}}{1 - \frac{1}{4} e^{-2j\omega}}$$

(iii) Taking the inverse Fourier transform of the partial fraction expansion of $H(e^{j\omega})$ we obtain

$$h[n] = 2\delta[n] - \frac{21}{16} (-\frac{1}{2})^n u[n] + \frac{7}{16} (\frac{1}{2})^n u[n]$$

5.52. (a) Since $h[n]$ is causal, the nonzero sample values of $h[n]$ and $h[-n]$ overlap only at $n = 0$
Therefore,

$$\mathcal{E}v\{h[n]\} = \frac{h[n] + h[-n]}{2} = \begin{cases} h[n]/2, & n > 0 \\ h[0], & n = 0 \\ h[-n]/2, & n < 0 \end{cases}$$

In other words,

$$h[n] = \begin{cases} 2\mathcal{E}v\{h[n]\}, & n > 0 \\ \mathcal{E}v\{h[0]\}, & n = 0 \\ 0, & n < 0 \end{cases} \qquad \text{(S5.52-1)}$$

Now note that if

$$h[n] \xleftarrow{FT} H(e^{j\omega})$$

Then

$$\mathcal{E}v\{h[n]\} = \frac{h[n] + h[-n]}{2} \xleftarrow{FT} \Re e\{H(e^{j\omega})\}$$

Clearly, we can recover $\mathcal{E}v\{h[n]\}$ from $\Re e\{H(e^{j\omega})\}$ From $\mathcal{E}v\{h[n]\}$ we can use eq.(S5.52-1) to recover h[n]. Obviously, from $h[n]$ we can again obtain $H(e^{j\omega})$. Therefore, the system is completely specified by $\Re e\{H(e^{j\omega})\}$.

(b) Taking the inverse Fourier transform of $\Re e\{H(e^{j\omega})\}$, we obtain

$$\mathcal{E}v\{h[n]\} = \delta[n] + \frac{\alpha}{2} \delta[n-2] + \frac{\alpha}{2} \delta[n+2]$$

Therefore, In other words

$$h[n] = \delta[n] + \alpha\delta[n-2]$$

and

$$H(e^{j\omega}) = 1 + \alpha e^{-j2\omega}$$

(c) Since $h[n]$ is causal, the nonzero sample values of $h[n]$ and $h[-n]$ overlap only at $n = 0$. Therefore

$$od\{h[n]\} = \frac{h[n] - h[-n]}{2} = \begin{cases} h[n]/2, & n > 0 \\ 0, & n = 0 \\ -h[-n]/2, & n < 0 \end{cases}$$

In other words,

$$h[n] = \begin{cases} 2od\{h[n]\}, & n > 0 \\ some \quad value, & n = 0 \\ 0, & n < 0 \end{cases}$$

Now note that if

$$h[n] \xleftarrow{FT} H(e^{j\omega})$$

Then

$$od\{h[n]\} = \frac{h[n] - h[-n]}{2} \xleftrightarrow{\ FT\ } j\,\mathrm{Im}\{H(e^{j\omega})\}$$

Clearly, we can recover $od\{h[n]\}$ from $\mathrm{Im}\{H(e^{j\omega})\}$. From $od\{h[n]\}$ we can use eq.(S5.52-1) to recover $h[n]$ (provided $h[0]$ is given).Obviously, from $h[n]$ we can once again obtain $H(e^{j\omega})$.Therefore, the system is completely specified by $\mathrm{Im}\{H(e^{j\omega})\}$ and h[0].

(d) Let $\mathrm{Im}\{H(e^{j\omega})\} = \sin\omega$. Then

$$od\{x[n]\} = \frac{1}{2}\delta[n-1] - \frac{1}{2}\delta[n+1]$$

Therefore,

$$h[n] = h[0]\delta[n] + \delta[n+1]$$

We may choose two different values for $h[0]$ (say 1 and 2) to obtain two different systems whose frequency responses have imaginary parts equal to $\sin\omega$.

5.53. (a) The analysis equation of the Fourier transform is

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Comparing with eq. (P5.53-2) , we have

$$\widetilde{X}[k] = \frac{1}{N} X(e^{j(2\pi k/N)})$$

(b) From the figures we obtain

$$X_1(e^{j\omega}) = 1 - e^{-j\omega} + 2e^{-3j\omega}$$

and

$$X_2(e^{j\omega}) = -e^{2j\omega} - e^{j\omega} - 1 + e^{-2j\omega} + e^{-3j\omega} + 2e^{-j4\omega} - e^{-j5\omega} + 2e^{-j7\omega}$$

Now,

$$X_1(e^{j(2\pi k/4)}) = 1 - e^{-j\pi k/2} + 2e^{-3j\pi k/2}$$

and

$$X_2(e^{j(2\pi k/4)}) = 1 - e^{-j\pi k/2} + 2e^{-3j\pi k/2} = X_1(e^{j(2\pi k/4)})$$

Chapter 6 Answers

6.6 (b) the impulse response h1[n] is as shown in figure s6.6,as was increase ,it is clear that the significant central lobe of h1[n] becomes more concentrated around the origin. consequently. h[n]=h1[n](-1)^n also becomes more concentrated about the origin.

6.7 the frequency response magnitude |H(jw)| is as shown in figure s6.7.the frequency response of the bandpass filter G(jw) will be given by

$$G(j\omega) = FT\{2h(t)\cos(4000\pi t)\}$$
$$= H(j(\omega - 4000\pi)) + H(j(\omega + 4000\pi))$$
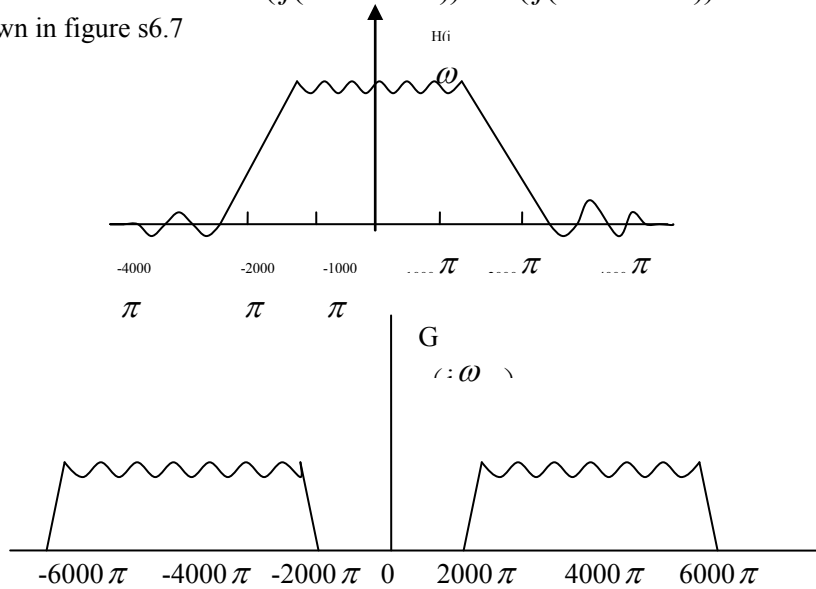
This is as shown in figure s6.7

Figure S6.7

(a) from the figure ,it is obvious that the passband edges are at 2000Πrad/sec and 6000Πrad/sec. this translates to 1000HZ and 3000Hz,respectively.

(b) (b)from the figure ,it is obvious that the stopband edges are at 1600Π rad/sec.this translates to 800Hz and 3200 Hz, respectively.

6.8 taking the Fourier transform of both sides of the first difference equation and simplifying, we obtain the frequency response H(e^jw)of the first filter.

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\sum_{k=0}^{M} b_k e^{-j\omega k}}{1 - \sum_{k=1}^{N} a_k e^{-j\omega k}}.$$

Taking the Fourier transform of both sides of the second difference equation and simplifying ,we obtain the frequency response H1(e^jw) of the second filter.

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\sum_{k=0}^{M} (-1)^k b_k e^{-j\omega k}}{1 - \sum_{k=1}^{N} (-1)^k a_k e^{-j\omega k}}.$$

This may also be written as

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\sum_{k=0}^{M} b_k e^{-j(\omega - \pi)k}}{1 - \sum_{k=1}^{N} a_k e^{-j(\omega - \pi)k}} = H(e^{j(\omega - \pi)}).$$

Therefore .the frequency response of the second filter is obtained bu shifting the frequency response of the first filter by Π.although the first fitter has its passband between-wp and wp. Therefore, the second filter will have its passband between Π-wp and Π+wp.

6.9 taking the Fourier transform of the given differential equation and simplifying .we obtain the frequency of the LTI system to be

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2}{5 + j\omega}$$

Taking the inverse Fourier transform, we obtain the impulse response to be

$$h(t) = 2e^{-5t}u(t).$$

Using the result derived in section 6.5.1,we have the step response of the system

$$s(t) = h(t) * u(t) = \frac{2}{5}[1 - e^{-5t}]u(t).$$

The final value of the step response is

$$s(\infty) = \frac{2}{5}.$$

We also have

$$s(\infty) = \frac{2}{5}[1 - e^{-5t_0}].$$

Substituting s(t0)=(2/5)[1-1/e^2],in the above equation ,we obtain t0=2/5 sec

(a) we may rewrite H1(jw)to be

$$H(j\omega) = (\frac{1}{j\omega + 40})(j\omega + 0.1).$$

we may then treat each of the two factors as individual first order systems and draw their bode magnitude plots .the final bode will then be asum of these two bode plots .this is shown in the figures6.10 mathematically. the straight-line approximation of the bode magnitude plot is

$$20\log_{10}|H(j\omega)| = \begin{cases} -20, \omega \square \ 0.1 \\ 20\log_{10}(\omega), 0,1 \square \ \omega \square \ 40 \\ 32, \omega \square \ 40 \end{cases}$$
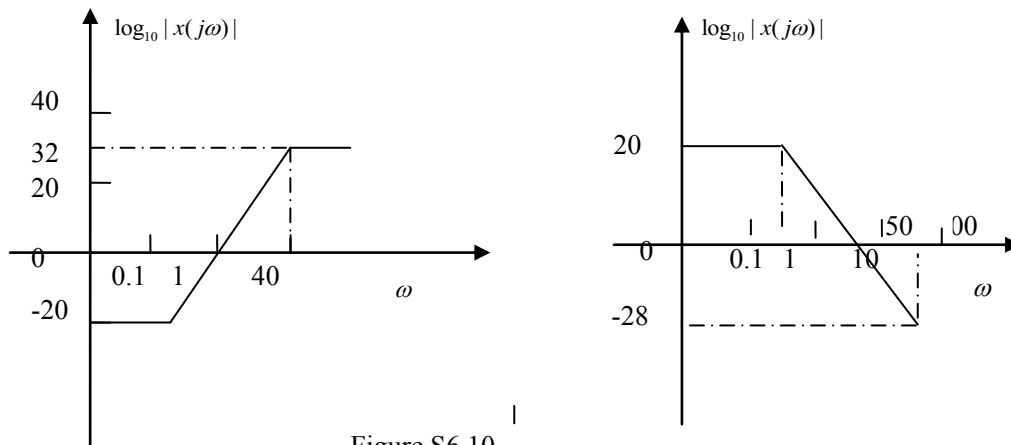

Figure S6.10

(b) Using a similar approach as in part (a),we obtain the Bode plot to be as shown in Figure S6.10.

Mathematically, the straight-line approximation of the BODE magnitude plot is

$$20\log_{10}|H(j\omega)| \approx \begin{cases} 20, & \omega << 0.2 \\ -20\log_{10}(\omega), 0.2 << \omega << 50 \\ -28, & \omega >> 50 \end{cases}$$

6.10. (a) We may rewrite the given frequency response $H_1(j\omega)$ as

$$H_1(j\omega) = \frac{250}{(j\omega)^2 + 50.5 j\omega + 25} = \frac{250}{(j\omega + 0.5)(j\omega + 50)}.$$

We may then use an approach similar to the one used in Example 6.5 and in Problem **6.11** to obtain the Bode magnitude plot(with straight line approximations) shown in Figure S6.11.
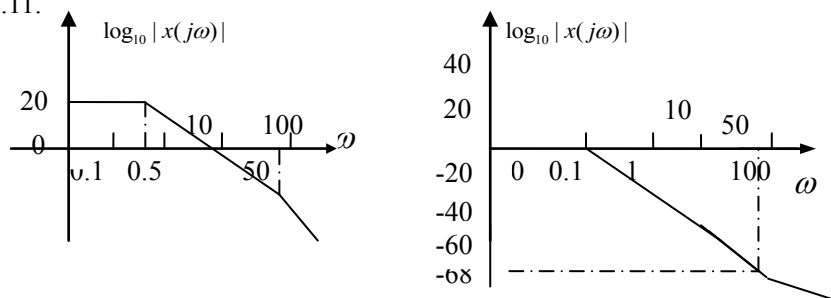
Mathematically, the straight-line approximation of the Bode magnitude plot is

$$20\log_{10}|H(j\omega)| \approx \begin{cases} 20, & \omega << 0.5 \\ -20\log_{10}(\omega)+14, & 0.5 << \omega << 50 \\ -40\log_{10}(\omega)+48, & \omega >> 50 \end{cases}$$

(b)We may rewrite the frequency response $H_2(j\omega)$ as

$$H_2(j\omega) = {}_{(j\omega+50)}\left(\frac{0.02}{(j\omega)^2+0.2\,j\omega+1}\right).$$

Again using an approach similar to the one used in Example 6.5,we may draw the Bode magnitude plot by treating the first and second order factors separately. This Givens us a Bode magnitude plot (using straight line) approximations as shown below:
Mathematically , the straight-line approximation of the Bode magnitude plot is

$$20\log_{10}|H(j\omega)| \approx \begin{cases} 0, & \omega << 1 \\ -40\log_{10}\omega, & 1 << \omega << 50 \\ -20\log_{10}(\omega)-34, & \omega >> 50 \end{cases}$$

6.12. Using the Bode magnitude plot, specified in Figure P6.12(a). we may obtain an expression
For $H_1(j\omega)$. The figure shows that $H_1(j\omega)$ has the break frequencies $\omega_1=1$, $\omega_2=8$,And $\omega_3=40$. The frequency response rises as 20dB/decade after ω_1. At ω_2,this rise is canceled by a -20 dB/decade contribution. Finally, at ω_3 ,an additional -20 dB/decade. Contribution results in the subsequent decay at the rate of -20 dB/decade, therefore, we may conclude that

$$H_1(j\omega) = \frac{A(j\omega+\omega_1)}{(j\omega+\omega_2)(j\omega+\omega_3)}.$$

We now need to find A. Note that when $\omega=0$, $20\log_{10}|H_1(j0)|=2$.Therefore. $H_1(j0)=0.05$. From eq . (S6.12-1),we know that

$$H_1(j0) = \frac{A}{320}.$$

Therefore, A =640. This gives us

$$H_1(j\omega) = \frac{640(j\omega+1)}{(j\omega+8)(j\omega+40)}.$$

Using a similar approach on Figure P6.12(b), we obtain

$$H(j\omega) = \frac{6.4}{(j\omega+8)^2}.$$

Since the overall system (with frequency response H(jω)) is constructed by cascading Systems with frequency responses $H_1(j\omega)$ and $H_2(j\omega)$,

$$H(j\omega) = H_1(j\omega)\,H_2(j\omega).$$

Using the previously obtained expressions for H(jω) and $H_1(j\omega)$,

$$H_2(j\omega) = H(j\omega)/H_1(j\omega) = \frac{0.01(j\omega+40)}{(j\omega+1)(j\omega+8)}.$$

6.13. Using an approach similar to the one used in the previous problem, we obtain

$$H(j\omega) = \frac{320}{(j\omega+2)(j\omega+8)}.$$

(a) Let us assume that we desire to construct this system by cascading two systems with frequency responses $H_1(j\omega)$ and $H_2(j\omega)$, respectively. We require that

$$H(j\omega) = H_1(j\omega)\,H_2(j\omega).$$

We see that $H_1(j\omega) = \frac{40}{(j\omega+2)}$ and $H_2(j\omega) = \frac{8}{(j\omega+80)}$

And

$$H_1(j\omega) = \frac{32}{(j\omega+2)} \quad \text{and} \quad H_2(j\omega) = \frac{10}{(j\omega+80)}$$

are both valid combinations.
(b) Let us assume that we desire to construct this system by connecting two systems with frequency responses $H_1(j\omega)$ and $H_2(j\omega)$ in parallel. We require that

$$H(j\omega) = H_1(j\omega) + H_2(j\omega)$$

Using partial fraction expansion on $H(j\omega)$, we obtain

$$H(j\omega) = \frac{160/39}{(j\omega+2)} - \frac{160/39}{(j\omega+80)}$$

From the above expression it is clear that we can define $H_1(j\omega)$ and $H_2(j\omega)$ in only one way

6.14. Using an approach similar to the one used in Problem 6.12 ,we have

$$H(j\omega) = \frac{5000(j\omega+0.2)^2}{(j\omega+50)(j\omega+10)}.$$

The inverse to this system has a frequency response

$$H_1(j\omega) = \frac{1}{H(j\omega)} = \frac{0.2 \times 10^{-4}(j\omega+50)(j\omega+10)}{(j\omega+0.2)^2}.$$

6.15. We will use the results from Section 6.5 in this problem.

(a) We may write the frequency response of the system described by the given differential Equation as

$$H_1(j\omega) = \frac{1}{(j\omega)^2 + 4j\omega + 4}.$$

This may be rewritten as

$$H_1(j\omega) = \frac{1/4}{(j\omega/2)^2 + 2j(\omega/2) + 1}.$$

From this we obtain the damping ratio to be $\varsigma = 1$.Therefore , the system is critically damped

(b) We may write the frequency response of the system described by the given differential equation as

$$H_2(j\omega) = \frac{7}{5(j\omega)^2 + 4j\omega + 5}.$$

This may be rewritten as

$$H_2(j\omega) = \frac{7/5}{(j\omega)^2 + 2(2/5)j(\omega) + 1}.$$

From this we obtain the damping ratio to be $\varsigma = 2/5$. Therefore , the system is under-damped.

(c) We may write the frequency response of the system described by the given differential equation as

$$H_3(j\omega) = \frac{1}{(j\omega)^2 + 20j\omega + 1}.$$

This may be rewritten as

$$H_3(j\omega) = \frac{1}{(j\omega)^2 + 2(10)j(\omega) + 1}.$$

From this we obtain the damping ratio to be $\varsigma = 10$. Therefore , the system is under-damped.

(d) We may write the frequency response of the system described by the given differential equation as

$$H_3(j\omega) = \frac{7 + (1/3)j\omega}{5(j\omega)^2 + 4j\omega + 5}.$$

The terms in the numerator do not affect the ringing behavior of the impulse response of this system .Therefore , we need to only consider the denominator in order to determine if the system. is critically damped, under-damped ,or over-damped .We see that this frequency response has the same denominator as the one obtained in part (b).Therefore . this system is still under-damped.

6.16. The system of interest will have a difference equation of the form

$$y[n] - ay[n-1] = b\, x\,[n].$$

Making slight modifications to the results obtained in Section 6.6.1,we determine the step response of this system to be

$$b\left(\frac{1-a^{n+1}}{1-a}\right)u[n]\ .$$

The final value of the step response will be b/(1-a). The step response exhibits oscillatory behavior only if $|a| < 1$.Using this fact, we may easily show that the maximum overshoot in the step response occurs when n=0. Therefore , the maximum value of the step response is

$$\frac{b}{1-a}(1-a) = b.$$

Since we are given that the maximum, overshoot is 1.5 times the final value, we have

$$1.5 \frac{b}{1-a} = b \quad \Rightarrow \quad a = -\frac{1}{2}$$

Also ,since we are given that the final value us 1,

$$\frac{b}{1-a} = 1 \quad \Rightarrow \quad b = \frac{3}{2}$$

Therefore, the difference equation relating the input and output will be

$$y[n] + \frac{1}{2} y[n-1] = \frac{3}{2} x[n].$$

6.17. We will use the results derived in Section 6.6.2 to solve this problem.
(a)Comparing the given difference equation with eq. (6.56),we obtain

$$T = \frac{1}{2}, \text{ and } \quad \cos\theta = -1.$$

Therefore, $\theta = \pi$,and the system has an oscillatory step response.
(b) Comparing the given difference equation with eq. (6.56),we obtain

$$T = \frac{1}{2}, \text{ and } \quad \cos\theta = -1.$$

Therefore, $\theta = 0$,and the system has non- oscillatory step response.
6.18. Let us first find the differential equation governing the input and output of this circuit.
Current through capacitor $= C \frac{dy(t)}{dx}$.

Voltage across resistor $= RC \frac{dy(t)}{dx}$.

Total input voltage =Voltage across resistor + Voltage across capacitor
Therefore ,

$$x(t) = RC \frac{dy(t)}{dx} + y(t).$$

The frequency response of this circuit is therefore

$$H(j\omega) = \frac{1}{RCj\omega + 1} .$$

Since this is a first order system , the step response has to be non oscillatory.
6.19. Let us first find the differential equation governing the input and output of this circuit .

Current through resistor and inductor = Current through capacitor $= C \frac{dy(t)}{dx}$.

Voltage across resistor $= RC \frac{dy(t)}{dx}$.

Voltage across inductor $= LC \frac{d^2 y(t)}{dt^2}$.

Total input voltage = Voltage across inductor + Voltage across resistor + Voltage across capacitor
Therefore ,

$$x(t) = LC \frac{d^2 y(t)}{dt^2} + RC \frac{dy(t)}{dx} + y(t).$$

The frequency response of this circuit is therefore

$$H(j\omega) = \frac{1}{LC(j\omega)^2 + RCj\omega + 1} .$$

We may rewrite this to be

$$H(j\omega) = \frac{1}{(\frac{j\omega}{1/\sqrt{LC}})^2 + 2(R/2)\sqrt{C/L} \frac{j\omega}{1/\sqrt{LC}} + 1} .$$

Therefore, the damping constant $\varsigma = (R/2)\sqrt{C/L}$. In order for the step response to have no oscillations, we must have $\varsigma \geq 1$ .Therefore, we require

$$R \geq 2 \sqrt{\frac{L}{C}} .$$

6.20. Let us call the given impulse response h[n]. It is easily observed that the signal h_1 [n]=h[n+2] is real and even . Therefore ,(using properties of the Fourier transform) we know that the Fourier transform $H_1(e^{j\omega})$ of h_1 [n] is real and even . Therefore $H_1(e^{j\omega})$ has zero phase, we have

$$\angle H(e^{j\omega}) = -2\omega .$$

Therefore , the group delay is

$$\tau(\omega) = \frac{d}{d\omega} \Box \ H(e^{j\omega}) = 2.$$

6.21. Note that in all parts of this problem $Y(j\omega) = H(j\omega)X(j\omega) = -2 j\omega X(j\omega)$. Therefore ,y(t)= $-2dx(t)/dt$.

(a) Here, x(t)= e^{jt} .Therefore, y(t)=$-2dx(t)/dt = -2 je^{jt}$ .This part could also have been solved by nothing that complex exponentials are Eigen functions of LTI systems. Then when x(t)= e^{jt} ,y(t) should be y(t)=H(j1) e^{jt} =-2j e^{jt} .

(b)Here ,x(t)=sin($\omega_0 t$)u(t).Then , $\frac{dx(t)}{dt} = \omega_0$ cos (ω_0 t)u(t)+ sin (ω_0 t) δ (t)= ω_0 cos (ω_0 t)u(t).

Therefore ,y(t)= $-2\frac{dx(t)}{dt} = -2\omega_0$ cos (ω_0 t)u(t).

(c) Here , $Y(j\omega) = X(j\omega)H(j\omega) = -2/(6+j\omega)$ .Taking the inverse Fourier transform we obtain y(t)= $-2e^{-6t}u(t)$.

(d)Here, x(jw)=1/(2+jw). From this we obtain x(t)= e^{-2t} u(t). Therefore, y(t)= -2dx(t)/dt=4 e^{-2t} u(t) - 2 δ (t).

6.22 Note that $\quad$ H(jw)= $\begin{cases} jw/3\pi, -3\pi \le w \le 3\pi \\ 0, otherwise \end{cases}$

(a)Since x(t)=cos($2\pi t+\theta$),X(jw)= $e^{i\theta}$ π δ (w-2 π)+ $e^{-i\theta}$ π δ (w+2 π). This is zero outside the region -3 π < w<3 π .Thus, Y(jw)=H(jw)X(jw)=(jw/3 π)X(jw). This implies that y(t)=(1/3 π)dx(t)/dt=(-2/3)sin(2π t+ θ).

(b)Since x(t)= cos($4\pi t+\theta$),X(jw)= $e^{i\theta}$ π δ (w-4 π)+ $e^{-i\theta}$ π δ (w+4 π). Therefore, the nonzero portions of X(jw) lie outside the range -3 π < w<3 π . This implies that Y(jw)=H(jw)X(jw)=0.Therefore,y(t)=0.

(c)The Fourier series coefficients of the signal x(t) are given by

$$a_k = \frac{1}{T_0} \int_{<T_0>} x(t) \ e^{-jkw_0 t}$$

Where T_0 =1 and w_0 =2 π /T_0 =2 π , Also,

$$X(jw) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(w - kw_0)$$

The only impulses of X(jw) which lie in the region -3 π < w<3 π are at w=0,2 π ,and 2 π .Defining the signal x_{lp} (t)= a_0 =1/ π , $a_1 = a_{-1}^*$ =-1/(4j).Putting these into the expression for x_{lp} (t) we obtain x_{lp} (t)=(1/ π)+(1/2)sin(2 π t). Finally, y(t)=(1/3 π)d x_{lp} (t)/dt=(1/3)cos(2 π t).

6.23. (a) From the given information, we have

$$H_a (jw) = \begin{cases} 1, |w| \le w_c \\ 0, otherwise \end{cases}$$

Using Table 4.2, we get

$$h_a (t) = \frac{\sin(w_c t)}{\pi t}.$$

(b)Here,

$$H_b(jw) = H_a (jw) \ e^{jwT}$$

Using Table 4.1, we get

$$h_b(t) = h_a (t+T)$$

Therefore,

$$h_b (t) = \frac{\sin[w_c(t+T)]}{\pi(t+T)}$$

(c)Let us consider a frequency response H_0 (jw) given by

$$H_0 \text{(jw)}= \begin{cases} 1, |w| \le w_c/2 \\ 0, otherwise \end{cases}$$

Clearly,

$$H_c \text{(jw)}= \frac{1}{2\pi}[H_0 \text{(jw)}*W\text{(jw)}],$$

Where

$$W\text{(jw)}=j2\pi\delta(w-w_c/2)-j2\pi\delta(w-w_c/2)$$

Therefore , from Table 4.1
$$h_c(t)= h_0(t)w(t)=[\frac{\sin(w_c/2)}{\pi t}][-2\sin(w_c t/2)].$$

6.24. If $\tau(w)= k_1$, where k_1 is a constant, then

$$\triangleleft H\text{(jw)}=- k_1 w + k_2$$

Where k_2 is another constant.

(a) Note that if h(t) is real, the phase of the Fourier transform $\triangleleft H\text{(jw)}$ has to be an odd function. Therefore, the value of k_2 in eq. (S6.24-1) will be zero.

Also, let us define $H_0\text{(jw)}=|H\text{(jw)}|$. Then

$$h_0(t)= \frac{\sin(200\pi t)}{\pi t}$$

(i)   Here $k_1=5$. Hence, $\triangleleft H\text{(jw)}=-5w$. Then

$$H\text{(jw)}=|H\text{(jw)}|e^{j\triangleleft H(jw)} = H_0\text{(jw)} e^{-j5w}$$

Therefore,

$$H(t)= h_0(t-5)= \frac{\sin[200\pi(t-5)]}{\pi(t-5)}$$

(ii)   Here $k_1=5/2$. Hence, $\triangleleft H\text{(jw)}=-(5/2)w$. Then,

$$H\text{(jw)}=|H\text{(jw)}| e^{j\triangleleft H(jw)} = H_0\text{(jw)} e^{-j(5/2)w}$$

Therefore,

$$h(t)= h_0(t-5/2)= \frac{\sin[200\pi(t-5/2)]}{\pi(t-5/2)}$$

(iii)   Here $k_1=-5/2$. Hence, $\triangleleft H\text{(jw)}=(5/2)w$. Then,

$$H\text{(jw)}=|H\text{(jw)}| e^{j\triangleleft H(jw)} = H_0\text{(jw)} e^{j(5/2)w}$$

Therefore,

$$h(t)= h_0(t+5/2)= \frac{\sin[200\pi(t+5/2)]}{\pi(t+5/2)}$$

(b) If h(t) is not specified to be real, then $\triangleleft H\text{(jw)}$ does not have to be an odd function. Therefore, the value of k_2 in eq. (S6.24-1) does not have to be zero. Given only $|H\text{(jw)}|$ and $\tau(w)$, k_2 cannot be determined uniquely. Therefore, h(t) cannot be determined uniquely.

6.25 (a) We may write $H_a\text{(jw)}$ as

$$H_a\text{(jw)}= \frac{(1-jw)}{(1+jw)(1-jw)} = \frac{1-jw}{2}$$

Therefore,

$$\triangleleft H_a\text{(jw)}= \tan^{-1}[-w]$$

And

$$\tau_a(w) = -\frac{d\triangleleft H_a(jw)}{dw} = \frac{1}{1+w^2}$$

Since $\tau_a(0)=1 \ne 2 = \tau_a(1)$, $\tau_a(w)$ is not a constant for w. Therefore, the frequency respons has nonlinear phase.

(b) In the case, $H_b(jw)$ is the frequency response of a system which is a cascade combination of two system, each of which has a frequency response $H_a\text{(jw)}$. Therefore.

$$\sphericalangle H_b(jw) = \sphericalangle H_a(jw) + \sphericalangle H_a(jw)$$

And

$$\tau_b(w) = -2\frac{d\sphericalangle H_a(jw)}{dw} = \frac{2}{1+w^2}$$

Since $\tau_b(0) = 2 \ne 4 = \tau_b(1)$, $\tau_b(w)$ is not a constant for all w. Therefore, the frequency response has nonlinear phase.

(c)IN this case, $H_c(jw)$ is again the frequency response of a system which is a cascade combination of two systems. The first system has a frequency response $H_a(jw)$. While the second system has a frequency response $H_0(jw) = 1/(2+jw)$. Therefore,

$$\sphericalangle H_b = \sphericalangle H_a(jw) + H_0(jw)$$

And

$$\tau_c(w) = -\frac{d\sphericalangle H_a(jw)}{dw} - \frac{d\sphericalangle H_0(jw)}{dw} = \frac{1}{1+w^2} + \frac{2}{4+w^2}$$

Since $\tau_c(0) = (3/2) \ne (3/5) = \tau_c(0)$, $\tau_b(w)$ is not a constant for all w . Therefore, the frequency response has nonlinear phase.

6.26. (a) Note that $H(jw) = 1 - H_0(jw)$, where $H_0(jw)$ is

$$H_0(jw) = \begin{cases} 1, 0 \le |w| \le w_c \\ 0, otherwise \end{cases}$$

Therefore,

h(t)= δ (t)- $h_0(t)$
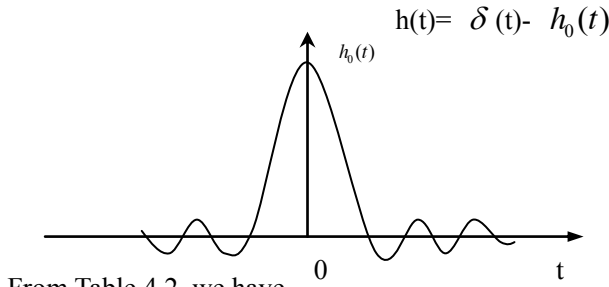
From Table 4.2 .we have

$$h_0(t) = \frac{\sin(w_c t)}{\pi t}$$

Therefore,

h(t)= δ (t)- $\frac{\sin(w_c t)}{\pi t}$

(b)A sketch of $h_0(t)$ is Figure S6.26. Clearly, as w_c increase. h(t) becomes more concentrated about the origin.

(c) Note that the step response is given by

S(t)=h(t)*u(t)=u(t)-u(t)* $h_0(t)$

Also, note that $h_0(t)$ is the impulse response of an ideal lowpass filter. If $s_0(t) = u(t)* h_0(t)$ denotes the step response of the lowpass filter, we know from Figure 6.14 that $s_0(0) = 0$ and $s_\infty = 1$. Therefore,

S(0+)=u(0+)- $s_0(0+)$ =1-(1/2)=1/2

And

S(∞)=u(∞)- $s_0(\infty) = 0$

6.27. (a) Taking the Fourier transform of both sides of the given differential equation, we obtain
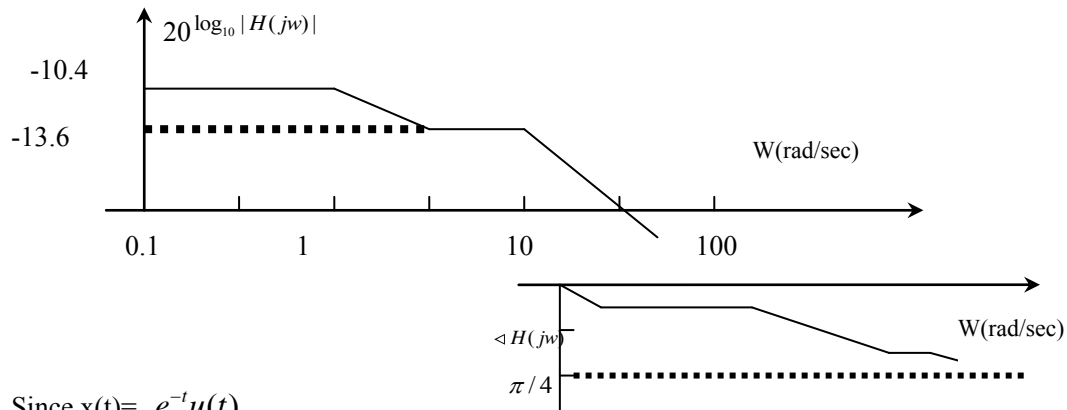
H(jw)= $\frac{Y(jw)}{X(jw)} = \frac{1}{2+jw}$

The Bode plot is as shown in Figure S6.27

(b)From the expression for H(jw) we obtain

$$\sphericalangle H(jw) = -\tan^{-1}(w/2)$$

Therefore,

$$\tau(w) = -\frac{d\sphericalangle H(jw)}{dw} = \frac{2}{4+w^2}$$

(c) Since x(t)= $e^{-t}u(t)$,

$$X(jw)= \frac{1}{1+jw}$$

Therefore,

$$Y(jw)=X(jw)H(jw)= \frac{1}{(1+jw)(2+jw)}$$

(d) Taking the inverse Fourier transform of the partial fraction expansion of Y(jw), we obtain

$$y(t)= e^{-t}u(t)-te^{-2t}u(t)$$

(e) (i) Here,

$$Y(jw)= \frac{1+jw}{(2+jw)^2}$$

Taking the inverse Fourier transform of the partial expansion of Y(jw), we obtain

$$y(t)= e^{-2t}u(t)-te^{-2t}u(t)$$

(ii) Here,

$$Y(jw)= \frac{1}{(1+jw)}$$

Taking the inverse Fourier transform of Y(jw), we obtain

$$y(t)= e^{-t}u(t)$$

(iii) Here,

$$Y(jw)= \frac{1}{(1+jw)(2+jw)^2}$$

Taking the inverse Fourier transform of the partial expansion of Y(jw), we obtain

$$y(t)= e^{-t}u(t)+\frac{1}{2}e^{-2t}u(t)-te^{-2t}u(t)$$

6.28. (a)   The Bode plots are as shown below
   (b)   We may write the frequency response of (iv) as

$$H(jw)= \frac{11/10}{1+jw}-\frac{1}{10}$$

Therefore.

$$h(t)= \frac{11}{10}e^{-t}u(t)-\frac{1}{10}\delta(t)$$

and

$$s(t)=h(t)*u(t)= \frac{11}{10}(1-e^{-t})u(t)-\frac{1}{10}u(t)$$

Both h(t) and s(t) are as shown in Figure S6.28.
   We may write the frequency response of (vi) as

$$H(jw)= \frac{9/10}{1+jw}+\frac{1}{10}$$
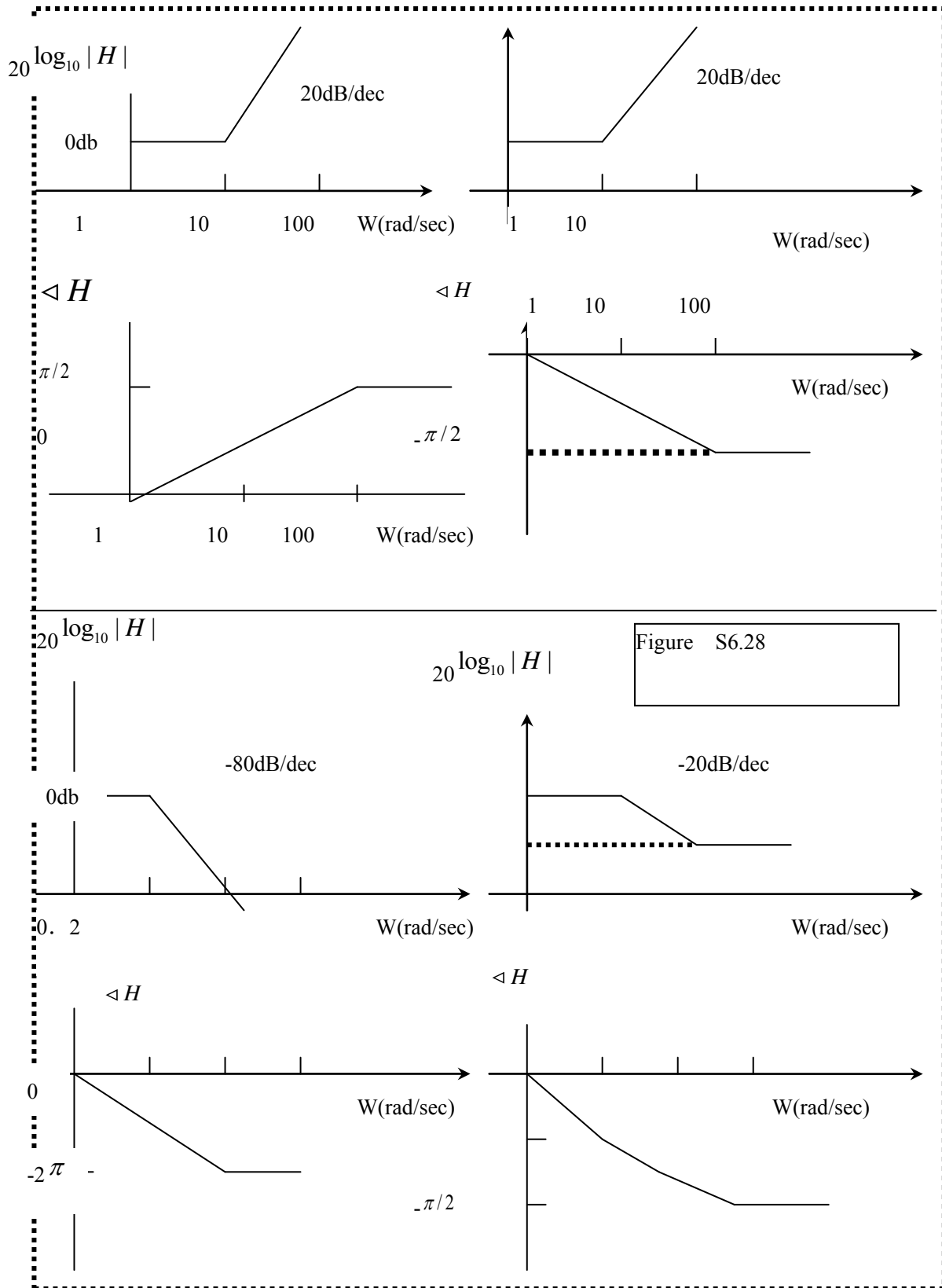
Therefore,

$$h(t)= \frac{9}{10}e^{-t}u(t)+\frac{1}{10}\delta(t)$$

and

$$s(t)=h(t)*u(t)= \frac{9}{10}(1-e^{-t})u(t)+\frac{1}{10}u(t)$$

Both h(t) and s(t) are as shown in Figure S6.28

Figure   S6.28

Chapter 7 Answers

7.1 From the Nyquist sampling theorem , we know that only if X (j w)=0 for |w| > $w_s/2$ will be signal be recoverable from its samples. Therefore, X(jw)>5000 п .

7.2 From the Nyquist theorem ,we know that the sampling frequency in this case must be at least w_s=2000 п .In other words ,the sampling period should be at most T=2 п / (w_s)=1 ∗ 10^{-3} .Clearly ,only (a) and (e) satisfy this condition.

7.3 (a) We can easily show that X(j w)=0 for |w| >4000 п .Therefore, the Nyquist rate for this signal is w_N=2(4000 п)=8000 п .

 (b)From the Tables 4.1 and 4.2 we know that X(j w) is a rectangular pulse for which
X(j w)=0 for |w| > 4000 п .Therefore, the Nyquist rate for this signal is w_N =2(4000 п)=800 п .

 (c) From the Tables 4.1 and 4.2 we know that X(j w) is the convolution of two rectangular pulses each of which is zero for |w| > 4000 п .Therefore ,X(j w)=0 for |w| >8000 п and the Nyquist rate for this signal is w_N=2(8000 п)=16000 п .

7.4 If the signal x(t) has a Nyquist rate of w_o ,then its Fourier transform X (j w)=0 for |w| > $w_o/2$.

(a) From chapter 4,
 $$y(t) = x (t) + x (t-1) \xleftarrow{\quad FT \quad} Y (jw) = X (jw) + e^{-jwt} X (jw).$$
 Clearly, we can only guarantee that Y (jw) =0 for |w| > $w_o/2$. Therefore, the Nyquist rate for y(t) is also w_o.

(b) From chapter 4,
 $$y(t) = \frac{dx(t)}{dt} \xleftarrow{\quad FT \quad} Y (jw)= jw X(jw).$$
 Clearly, we can only guarantee that Y (jw) =0 for |w| > $w_o/2$. Therefore, the Nyquist rate for y(t) is also w_o.

(c) From chapter 4,
 $$y(t) =x^2(t) \xleftarrow{\quad FT \quad} Y (jw)= (1/2 \text{ п })[X(jw)*X(jw)]$$
 Clearly, we can only guarantee that Y (jw) =0 for |w| > w_o. Therefore, the Nyquist rate for y(t) is also $2w_o$.

(d) From chapter 4,
 $$y(t)=x(t)\cos (w_o t) \xleftarrow{\quad FT \quad} Y (jw)= (1/2)X(j(w-w_o)) +(1/2)X(j(w+w_o)).$$
 Clearly, we can guarantee that Y (jw) =0 for |w| > w_o+ $w_o/2$. Therefore, the Nyquist rate for y(t) is $3w_o$.

7.5 Using Table 4.2,
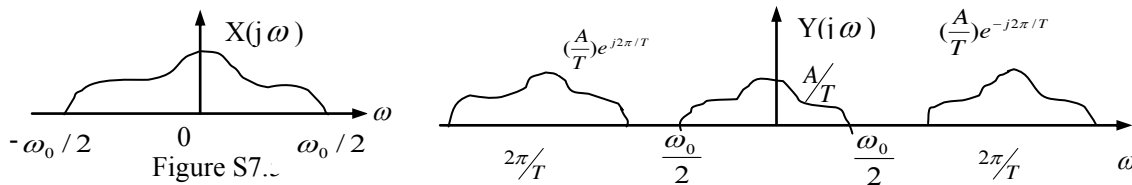 $$p(t) \xleftarrow{\quad FT \quad} \frac{2\pi}{T} \sum_{K=-\infty}^{\infty}\delta(\omega - K2\pi /T)$$

From Table 4.1
 $$p(t-1) \xleftarrow{\quad FT \quad} \frac{2\pi}{T} \ e^{-jw} \sum_{k=-\infty}^{\infty} \delta(\omega - k\frac{2\pi}{T})e^{-jk\frac{2\pi}{T}} .$$

Since y(t)=x(t)p(t-1),we have
 $$Y (jw)= (1/2 \text{ п })[X(jw)*FT\{P(t-1)\}]$$
 $$=(1/T) \sum_{K=-\infty}^{\infty} X (j(\omega - k\frac{2\pi}{T}))e^{-jk\frac{2\pi}{T}}$$

Therefore, Y(jω) consists of replicates of X(jω) shifted by k2 π /T and added to earth other (see Figure S7.5).In order to recover x(t) from y(t).we need to be able to isolate one replica of X(jω) from Y(jω).

Figure S7.5

From the figure ,it is clear that this is possible if we multiply Y(jω) with
$$H(j\omega) = \begin{cases} T, & |\omega| \le \omega_c \\ 0, & otherwise \end{cases}$$

Where ($\omega_0 / 2$)< ω_c <(2 π /T) - ($\omega_0 / 2$).

7.6 Consider the signal w(t)=x_1(t)x_2(t).The Fourier transform W(jω) of w(t) is given by
$$W(j\omega) = \frac{1}{2\pi} [X_1(j\omega) * X_2(j\omega)].$$

Since $X_1(j\omega) = 0$ for $|\omega| \ge \omega_1$ and $X_2(j\omega)$=0 for $|\omega| \ge \omega_2$, we may conclude that W(jω)=0 for $|\omega| \ge \omega_1 + \omega_2$ .Consequently，the Nyquist rate for w(t) is ω_s =2($\omega_1 + \omega_2$).Therefore ,the maximum sampling

period which would still allow w(t) to be recovered is $T=2\pi/(\omega_s)=\pi/(\omega_1+\omega_2)$.

7.7   We note that

$$x_1(t)=h_1(t)*\{\sum_{n=-\infty}^{\infty}x(nT)\delta(t-nT)\}$$

Form Figure 7.7 in the book ,we know that the output of the zero-order hold may be written as

$$x_0(t)=h_0(t)*\{\sum_{n=-\infty}^{\infty}x(nT)\delta(t-nT)\}$$

where $h_0(t)$ is as shown in Figure S7.7 By taking the Fourier transform of the two above equations, we have

$$X_1(j\omega)=H_1(j\omega)X_p(j\omega)$$
$$X_0(j\omega)=H_0(j\omega)X_p(j\omega)$$

We now need to determine a frequency   response $H_d(j\omega)$ for a filter which produces $x_1(t)$ at its output when $x_0(t)$ is its input. Therefore, we need

$$X_0(j\omega)H_d(j\omega)=X_1(j\omega)$$

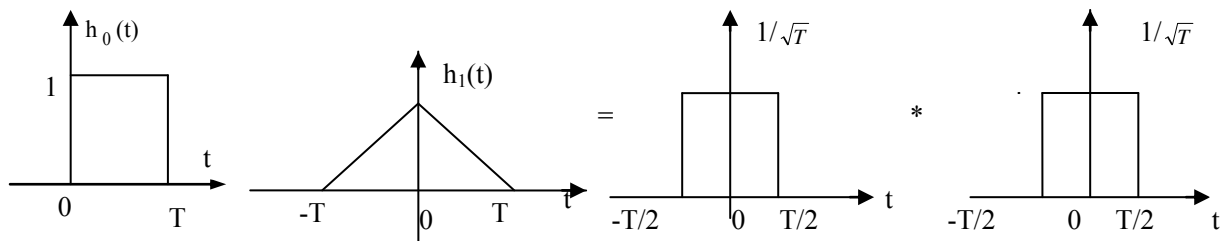The triangular function $h_1(t)$ may be obtained by convolving two rectangular pulses as shown in Figure S7.7

Figure S7.7

Therefore,

$$h_1(t)=\{(1/\sqrt{T})h_0(t+T/2)\}*\{(1/\sqrt{T})h_0(t+T/2)\}$$

Taking the Fourier transform of both sides of the above equation,

$$H_1(j\omega)=\frac{1}{T}e^{j\omega T}H_0(j\omega)H_0(j\omega)$$

Therefore

$$X_1(j\omega)=H_1(j\omega)X_p(j\omega)$$

$$=\frac{1}{T}e^{j\omega T}H_0(j\omega)H_0(j\omega)X_p(j\omega)$$

$$=\frac{1}{T}e^{j\omega T}H_0(j\omega)X_0(j\omega)$$

Therefore

$$H_d(j\omega)=\frac{1}{T}e^{j\omega T}H_0(j\omega)=e^{jwT/2}\frac{2\sin(\omega T/2)}{\omega T}$$

7.8 (a) Yes, aliasing does occur in this case .This may be easily shown by considering the sinusoidal term of x(t) for k=5. This term is a signal of the form $y(t)=(1/2)^5\sin(5\pi t)$.If x(t) is sampled as T=0.2, then we will always be sampling y(t) at exactly its zero-crossings (This is similar to the idea presented in Figure 7.17 of your textbook). Therefore ,the signal y(t) appears to be identical to the signal $(1/2)^5\sin(0\pi t)$ for frequency 5π is a liased into a sinusoid of frequency 0 in the sampled signal.

(b) The lowpass filter performs band limited interpolation on the signal $\hat{x}(t)$ .But since aliasing has already resulted in the loss of the sinusoid $(1/2)^5$ $\sin(5\pi t)$,the output will be of the form

$$x_\gamma(t)=\sum_{k=0}^{4}(\frac{1}{2})^k\sin(k\pi t)$$

The Fourier series representation of this signal is of the form

$$x_\gamma(t) = \sum_{k=-4}^{4} a_k\, e - j(k\pi/t)$$

Where $\quad a_k = \begin{cases} 0 \\ \text{-j}(1/2)^{k+1} \\ \text{j}(1/2)^{-k+1} \end{cases}$

7.9   The Fourier transform $X(j\omega)$ of x(t) is as shown in Figure S7.9
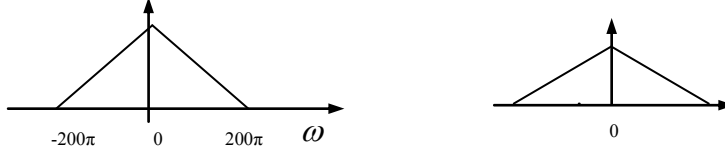

Figure S7.9

We know from the results on impulse-train sampling that

$$G(jw) = \frac{1}{T} \sum_{-\infty}^{\infty} X(j(\omega - k\omega s)),$$

Where $T = 2\pi/\omega_s = 1/75$.therefore,G(jw) is as shown in Figure S7.9 .Clearly, $G(jw) = (1/T)X(j\omega) = 75$ $X(j\omega)$ for $|\omega| \le 50\pi$.

7.10   (a)   We know that x(t) is not a band-limited signal. Therefore, it cannot undergo impulse-train sampling without aliasing.

(b) Form the given $X(j\omega)$ it is clear that the signal x(t) which is bandlimited. That is, $X(j\omega) = 0$ for $|\omega| > \omega_0$.Therefore, it must be possible to perform impulse-train sampling on this signal without experiencing aliasing. The minimum sampling rate required would   be $\omega_s = 2\omega_0$,This implies that the sampling period can at most be $T = 2\pi/\omega_s = \pi/\omega_0$

(c) When x(t) undergoes impulse train sampling with $T = 2\pi/\omega_0$,we would obtain the signal g(t) with Fourier transform

$$G(jw) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(\omega - k2\pi/T))$$
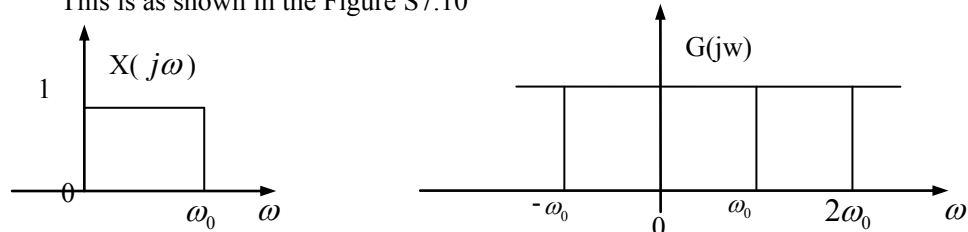
This is as shown in the Figure S7.10

Figure S7.10

It is clear from the figure that no aliasing occurs, and that X(jw) can be recovered by using a filter with frequency response

$H(jw) = \begin{cases} T & 0 \le \omega \le \omega_0 \\ 0 & \text{otherwise} \end{cases}$

Therefore, the given statement is true.

7.11 We know from Section 7.4 that

$$X_d(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\omega - 2\pi k/T))$$

(a)   Since $X_d(e^{j\omega})$ is just formed by shifting and summing replicas of X(jw),we may argue that if $X_d(e^{j\omega})$ is real , then X(jw) must also be real

(b) $X_d(e^{j\omega})$ consists of replicas of X(jw) which are scaled by 1/T,Therefore,if $X_d(e^{j\omega})$ has a maximum of 1, then X(jw) must also be real.

(c)   The region $3\pi/4 \le |\omega| \le \pi$ in   the   discrete-time   domain   corresponds   to   the   region $3\pi/(4T) \le |\omega| \le \pi/T$ in   the   discrete-time   domain. Therefore ,if $X_d(e^{j\omega}) = 0$ for $3\pi/4 \le |\omega| \le \pi$,then X(jw)=0 for $1500\pi \le |\omega| \le 2000\pi$,But since we already have X(jw)=0 for

$|\omega| \geq 2000\pi$,we have X(jw)=0 for $|\omega| \geq 1500\pi$

(d) In this case, since π in discrete-time frequency domain corresponds to 2000π in the continuous-time frequency domain, this condition translates to X(jw)=(j(ω -2000 π))

7.12 Form Section 7.4 ,we know that the discrete and continuous-time frequencies Ω and ω are related by $\Omega = \omega$ .Therefore, in this case for $\Omega = \frac{3\pi}{4}$,we find the corresponding value of ω to

$\omega = \frac{3\pi}{4} \frac{1}{T} = 3000\pi/4 = 7500\pi$

7.13 For this problem ,we use an approach similar to the one used in Example 7.2 .we assume that

$x_c(t) = \frac{\sin(\pi t/T)}{\pi t}$

The overall output is

$y_c(t) = x_c(t-2T) = \frac{\sin[(\pi/T)(t-2T)]}{\pi(t-2T)}$

Form $x_c(t)$. We obtain the corresponding discrete-time signal $x_d[n]$ to be

$x_d[n] = x_c(nT) = \frac{1}{T}\delta[n]$

also, we obtain from $y_c(t)$,the corresponding discrete-time signal $y_d[n]$ to be

$y_d[n] = y_c(nT) = \frac{\sin[(\pi(n-2)]}{\pi T(n-2)}$

We note that the right-hand side of the above equation is always zero when $n \neq 2$.When n=2 ,we may evaluate the value of ratio using L'Hospital's rule to be 1/T ,Therefore

$y_d[n] = \frac{1}{T}\delta[n-2]$

We conclude that the impulse response of the filter is

$h_d[n] = \delta[n-2]$

7.14 For this problem ,we use an approach similar to the one used in Example 7.2.We assume that

$x_c(t) = \frac{\sin[(\pi t/T)]}{\pi t}$

The overall output is

$y_c(t) = \frac{d}{dt}x_c\left(t-\frac{T}{2}\right) = \frac{(\pi/T)COS[(\pi/T)-(t-T/2)]}{\pi(t-T/2)} - \frac{\pi\sin[(\pi/T)(t-T/2)]}{(\pi(t-T/2))^2}$

Form $x_c(t)$, we obtain the corresponding discrete-time signal $x_d[n]$ to be

$x_d[n] = x_c(nT) = \frac{1}{T}\delta[n]$

Also, we obtain from yc(t),the corresponding discrete-time signal $y_d[n]$ to be

$Y_d[n] = y_c(nT) = \frac{(\pi/T)\cos[\pi(n-1/2)]}{\pi T(n-1/2)} - \frac{\sin[\pi(n-1/2)]}{\pi T(n-1/2)}$

The first term in rig π ht-hang side of the above equation is always zero because cos[π (n-1/2)]=0, therefore,

$y_d[n] = \frac{\sin[\pi(n-1/2)]}{\pi T(n-1/2)}$

We conclude that the impulse response of the filter is

$h_d[n] = \frac{\sin[\pi(n-1/2)]}{\pi T(n-1/2)}$

7.15. in this problem we are interested in the lowest rate which x[n] may be sampled without the possibility of aliasing, we use the approach used in Example 7.4 to solve this problem. To find the lowest rate at which x[n] may be sampled while avoiding the possibility of aliasing, we must find an N such that

$\frac{2\pi}{N} \geq 2\left(\frac{3\pi}{7}\right)$ $\Longrightarrow$ N≤7/3

therefore, N can at most be2.

7.16 Although the signal $x_1[n]=2\sin(\pi n/2)/(\pi n)$ satisfies the first tow conditions, it does not satisfy the third condition . This is because the Flurries transform $X_1(e^{j\omega})$ of this signal is rectangular pulse which is zero for $\pi/2 < |\omega| < \pi/2$ We also note that the signal $x[n]=4[\sin(\pi n/2)/(\pi n)]^2$ satisfies the first tow conditions. From our numerous encounters with this signal, we know that its Fourier transform $X(e^{j\omega})$ is given by the periodic

convolution of $X_1(e^{j\omega})$ with itself. Therefore, $X(e^{j\omega})$ will be a triangular function in the range $0 \leq |\omega| \leq \pi$. This obviously satisfies the third condition as well. T therefore, the desired signal is $x[n]=4[\sin(\pi n/2)/(\pi n)]^2$.

7.17 In this problem .we wish to determine the effect of decimating the impulse response of the given filter by a factor of 2. As explained in Section 7.5.2 ,the process of decimation may be broken up into two steps. In the first step we perform impulse train sampling on h[n] to obtain

$$H_p[n] \sum_{k=-\infty}^{\infty} h[2k] \, \delta[n-2k]$$

The decimated sequence is then obtained using

$$h_1[n]=h[2n]=h_p[2n]$$

Using eq (7.37), we obtain the Fourier transform $H_p(e^{j\omega})$ of $h_p[n]$ to be

$$H_p(e^{j\omega})=(1/2) \, H(e^{j\omega})+(1/2)H(e^{j(\omega-\pi)})$$
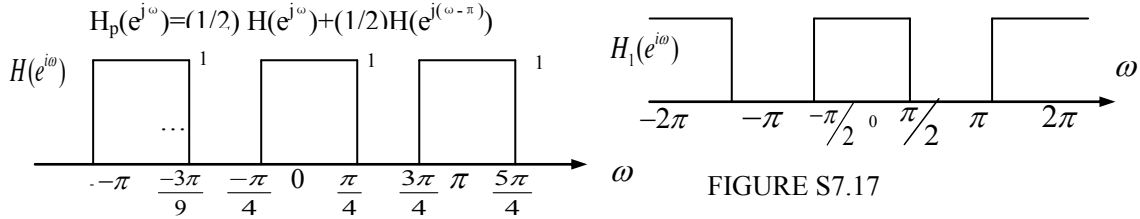
This is as shown that the Fourier transform of decimated impulse response is

$$H_1(e^{j\omega})=H_p(e^{j\omega/2})$$

In other words , $H_1(e^{j\omega})$ is $H_p(e^{j\omega/2})$ expanded by a factor of 2. This is as shown in the figure above. Therefore, $h_1[n]=h[2n]$ is the impulse response of an ideal lowpass filter with a passband gain of unity and a cutoff frequency of $\pi/2$

7.18 From Figure 7.37,it is clear interpolation by a factor of 2 results in the frequency response getting compressed by a factor of 2. Interpolation also results in a magnitude sealing by a factor of 2. Therefore, in this problem, the interpolated impulse response will correspond to an ideal lowpass filter with cutoff frequency $\pi/$ and a passband gain of 2.
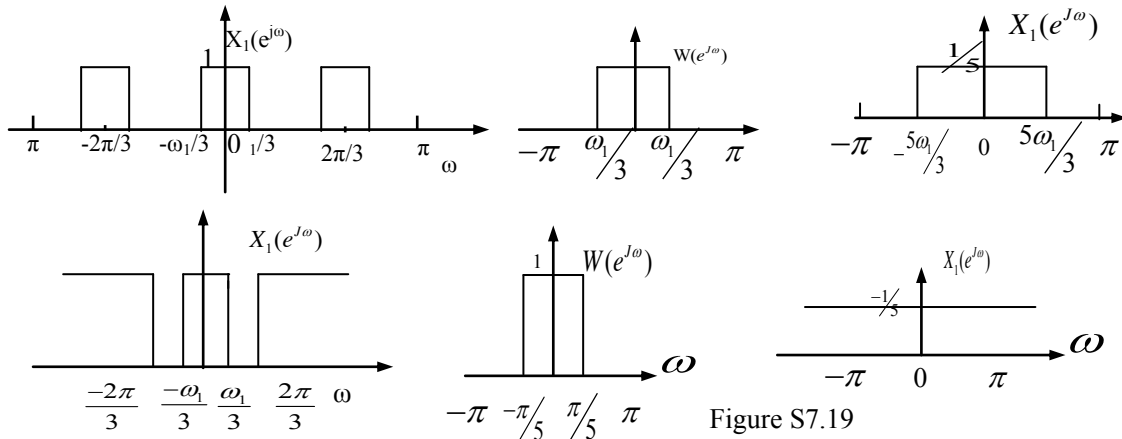
7.19 The Fourier transform of x[n] is given by

$$X(e^{j\omega})= \begin{cases} 1 & |\omega| \leq \omega_1 \\ 0 & \text{otherwise} \end{cases}$$

This is as shown in Figure 7.19.

(a) when $\omega_1 \leq 3\pi/5$, the Fourier transform $X_1(e^{j\omega})$ of the output of the zero-insertion system is shown in Figure 7.19. The output $w(e^{j\omega})$ of the lowpass filter is as shown in Figure 7.19. The Fourier transform of the output of the decimation system $Y(e^{j\omega})$ is an expanded or stretched out version of $W(e^{j\omega})$. This is as shown in Figure 7.19.

therefore, $\quad y[n]=\dfrac{1}{5} \dfrac{\sin(5\omega_1 n/3)}{\pi n}$

(b) When $\omega_1 > 3\pi/5$, the Fourier's transform $X_1(e^{j\omega})$ of the output of the zero-insertion system is as shown in Figure 7.19 The output $W(e^{j\omega})$ of the lowpass filter is as shown in Figure 7.19

Figure S7.19

124

The Fourier transform of the output of the decimation system $Y(e^{j\omega})$ bis an expanded or stretched out version of $W(e^{j\omega})$ .This is as shown in Figure S7.19. Therefore,

$$y[n]=\frac{1}{5}\delta[n]$$

7.20 Suppose that $X(e^{j\omega})$ is as shown in Figure S7.20, then the Fourier transform $X_A(e^{j\omega})$ of the output of the output of S_A, the Fourier transform $X_1(e^{j\omega})$ of the output of the lowpass filter , and the Fourier transform $X_B(e^{j\omega})$ of the output of S_B are all shown in the figures below. Clearly this system accomplishes the filtering task .
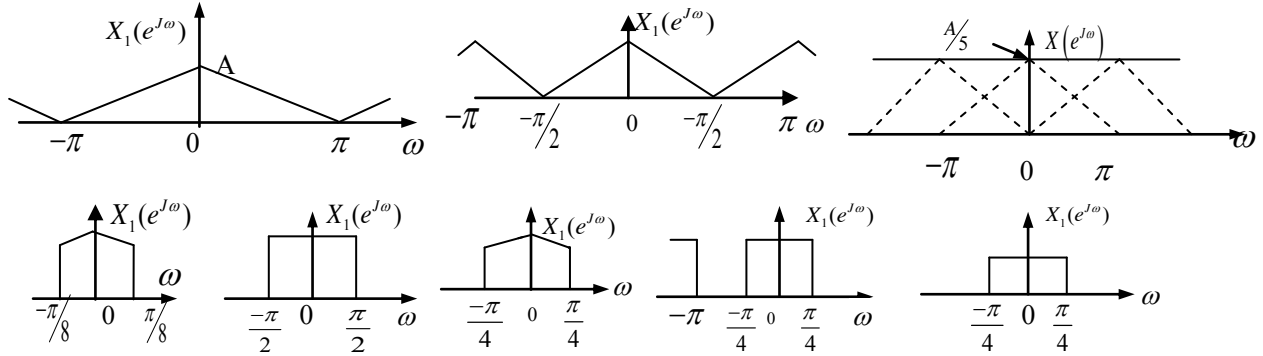

Figure S7.20

(b) Suppose that $X(e^{j\omega})$ is as shown in Figure S7.20 ,then the Fourier transform $X_B(e^{j\omega})$ of the output of S_B ,the Fourier transform $X_1(e^{j\omega})$of the output of the first lowpass filter ,the Fourier transfore $X_A(e^{j\omega})$ of the output of S_A ,the Fourier transform $X_2(e^{j\omega})$ of the output of the first lowpass filter are all shown in the figure below .Clearly this system does not accomplish the filtering task.

7.21

(a) The Nyquist rate for the given signal is $2\times5000\pi=10000\pi$. Therefore in order to be able to recover x(t) from $x_p(t)$,the sampling period must at most be $T_{max}=2\pi/10000\pi=2\times10^{-4}$ sec .Since the sampling period used is $T=10^{-4}<T_{max}$,x(t) can be recovered from $x_p(t)$.

(b) The Nyquist rate for the given signal is $2\times15000\pi=30000\pi$. Therefore in order to be able to recover x(t) from $x_p(t)$,the sampling period must at most be $T_{max}=2\pi/30000\pi=0.66\times10^{-4}$ sec .Since the sampling period used is $T=10^{-4}>T_{max}$, x(t) can not be recovered from $x_p(t)$.

(c) Here,$I_m\{X(j\omega)\}$ is not specified. Therefore, the Nyquist rate for the signal x(t) is indeterminate. This implies that one cannot guarantee that x(t) would be recoverable from $x_p(t)$.

(d) Since x(t) is real,we may conclude that $X(j\omega)=0$ for $|\omega|>5000$. Therefore the answer to this part is identical to that of part (a)

(e) Since x(t) is real, $X(j\omega)=0$ for $|\omega|>15000\pi$. Therefore the answer to this part is identical to that of part (b)

(f) If $X(j\omega)=0$ for $|\omega|>\omega_1$,then $X(j\omega)*X(j\omega)=0$ for $|\omega|>2\omega_1$,Therefore in this part $X(j\omega)=0$ for $|\omega|>7500$. The Nyquist rate for this signal is $2\times7500\pi=15000\pi$. Therefore in order to be able to recover x(t) from $x_p(t)$,the sampling period must at most be $T_{max}=2\pi/15000\pi=1.33\times10^{-4}$ sec .Since the sampling period used is $T=10^{-4}<T_{max}$, x(t) can be recovered from $x_p(t)$.

(g)

If $|X(j\omega)|=0$ for $\omega>5000\pi$,then $X(j\omega)=0$ for $|\omega|>5000\pi$. Therefore the answer to this part is identical to that of part (a).

7.22 Using the properties of the Fourier transform, we obtain

$$Y(j\omega)=X_1(j\omega)X_2(j\omega).$$

Therefore, $Y(j\omega)=0$ for $|\omega|>1000\pi$ .This implies that the Nyquist rate for y(t) is$2\times1000\pi=2000\pi$ .Therefore, the sampling period T can at most be $2\pi/(2000\pi)=10^{-3}$sec. Therefore we have to use $T<10^{-3}$sec in order to be able to recover y(t) from $y_p(t)$.

7.23

(a) We may express p(t) as

$$P(t)=p_1(t)-p_1(t-\triangle);$$

Where $p_1(t)=\sum_{k=-\infty}^{\infty}\delta(t-k2\Delta)$ \qquad now,

$$P_1(j\omega) = \frac{\pi}{\Delta} \sum_{k=-\infty}^{\infty} \delta(\omega - \pi/\Delta)$$

Therefore,

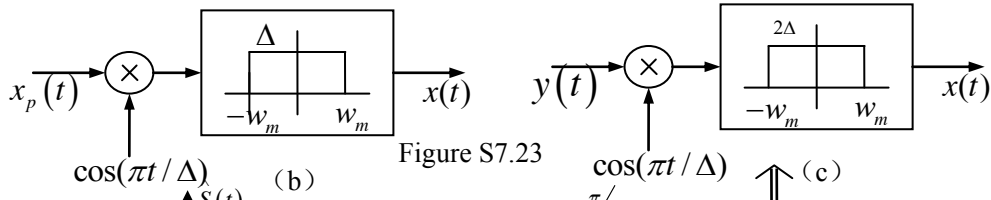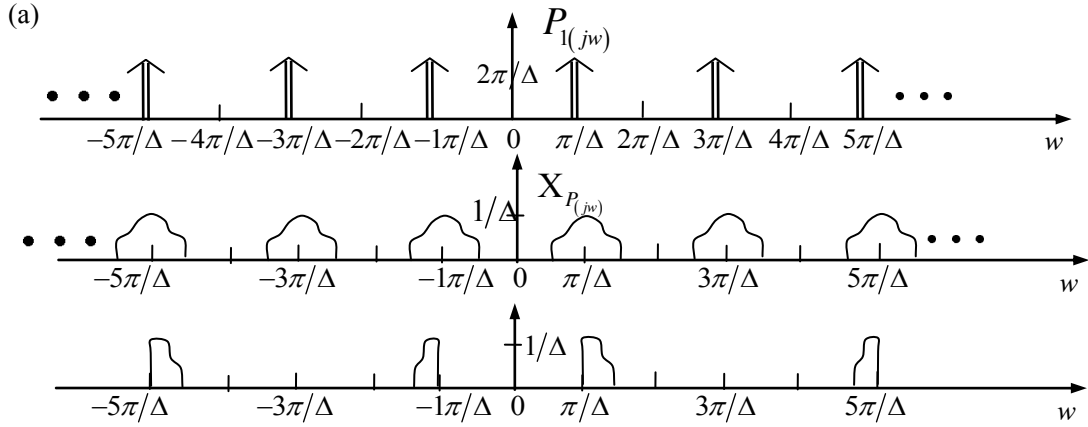$$P(j\omega) = P_1(j\omega) \cdot e^{-j\omega\Delta} P_1(j\omega)$$
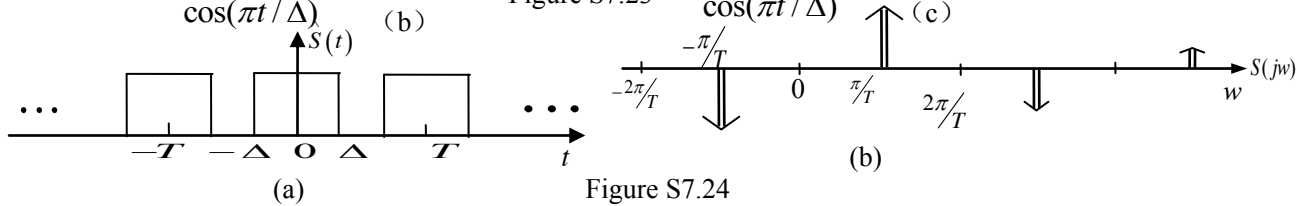
(a)

Figure S7.23

Figure S7.24

Is as shown in figure S7.23.

Now,

$$X_p(j\omega) = \frac{1}{2\pi}[X(j\omega) * P(j\omega)]$$

Therefore, $X_p(j\omega)$ is as sketched below for $\Delta < \pi/(2\omega_M)$, The corresponding $Y(j\omega)$ is also sketched in figure S7.23.

(b) The system which can be used to recover x(t) from $x_p(t)$ is as shown in FigureS7.23.

(c) The system which can be used to recover x(t) from x(t) is as shown in FigureS7.23.

(d) We see from the figures sketched in part (a) that aliasing is avoided when $\omega_M \leq \pi/\Delta$. therefore, $\Delta_{max} = \pi/\omega_M$.

7.24 we may impress s(t) as s(t)=s(t)-1, where s(t) is as shown in Figure S7.24 we may easily show that

$$\widehat{S}(j\omega) = \sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k\Delta/T)}{k} \delta(\omega - k2\pi/T)$$

From this, we obtain

$$S(j\omega) = \widehat{S}(j\omega) - 2\pi\delta(\omega) = \sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k\Delta/T)}{k} \delta(\omega - k2\pi/T) - 2\pi\delta(\omega)$$

Clearly, $S(j\omega)$ consists of impulses spaced every $2\pi/T$.

(a) If $\Delta = T/3$, then

$$S(j\omega) = \sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k/3)}{k} \delta(\omega - k2\pi/T) - 2\pi\delta(\omega)$$

Now, since w(t)=s(t)x(t),

$$W(j\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k/3)}{k} X(j(\omega - k2\pi/T)) - 2\pi X(\omega)$$

Therefore, $W(j\omega)$ consists of replicas of $X(j\omega)$ which are spaced $2\pi/T$ apart. Tn order to avoid aliasing, ω_W should be less that π/T. Therefore, $T_{max} = 2\pi/\omega_W$.

(b) If $\Delta = T/3$, then

$$S(j\omega) = \sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k/4)}{k} \delta(\omega - k2\pi/T) - 2\pi\delta(\omega)$$

we note that $S(j\omega)=0$ for k=0,$\pm$2, $\pm$4,.....This is as sketched in Figure S7.24.

Therefore, the replicas of $X(j\omega)$ in $W(j\omega)$ are now spaced 4π/T apart. Tn order to avoid aliasing, ω_W should be less that 2π/T. Therefore, T_{max}=2π/ω_W.

7.25 Here, $x_T(kT)$ can be written as

$$X_T(kT) = \sum_{k=-\infty}^{\infty} \frac{\sin[\pi(k-n)]}{\pi(k-n)} x(nT)$$

Note that when n$\neq$k,

$$\frac{\sin[\pi(k-n)]}{\pi(k-n)} = 0$$

And when n=k,

$$\frac{\sin[\pi(k-n)]}{\pi(k-n)} = 1$$

Therefore,

$$x_T(kT) = x(kT)$$

7.26. We note that

$$p(j\omega) = \frac{2\pi}{T} \delta(\omega - k2\pi/T)$$

Also, since $x_p(t)=x(t)p(t)$.

$$X_p(j\omega) = \frac{1}{2\pi} \{ x(j\omega) * P(j\omega)\}$$

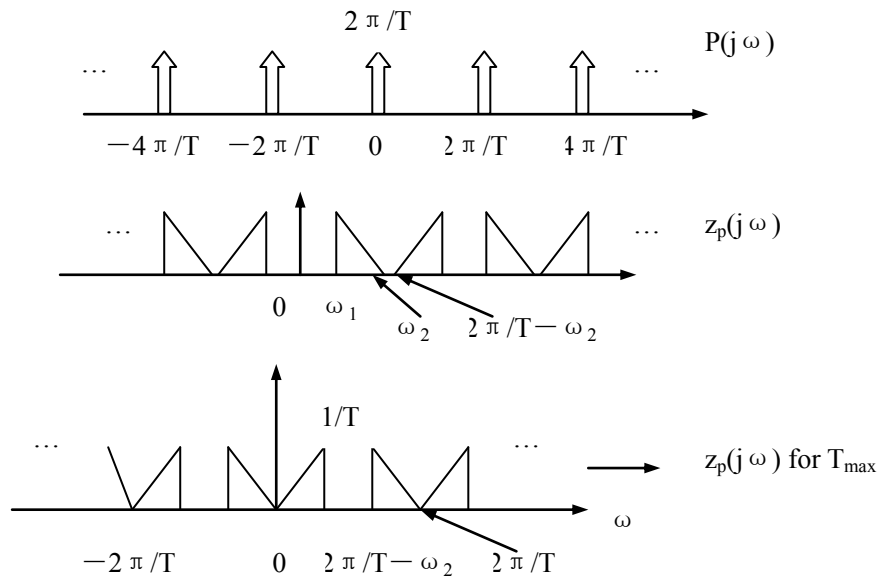$$= \frac{1}{T} x(j(\omega - k2\pi/T))$$

This is sketched in Figure S7.26.

Figure S7.26

Note that as T increase, $\frac{2\pi}{T} - \omega_2$ approaches zero. Also, we note that there is aliasing

When

$$2\omega_1 - \omega_2 < \frac{2\pi}{T} - \omega_2 < \omega_2$$

If $2\omega_1 - \omega_2 \geq 0$(as given) then it is easy to see that aliasing does not occur when

$$0 \leq \frac{2\pi}{T} - \omega_2 \leq 2\omega_1 - \omega_2$$

For maximum T, we must choose the minimum allowable value for $\frac{2\pi}{T} - \omega_2$ (which is zero).

This implies that T_{max}=2π/ω_2. We plot $x_p(j\omega)$ for this case in Figure S7.26. Therefore,

A=T, ω_b=2π/T, and $\omega_a = \omega_b - \omega_1$

7.27.(a) Let $x_1(j\omega)$ denote the Fourier transform of the signal $x_1(t)$ obtained by multiplying $x(t)$ with $e^{-j\omega_0 t}$ Let $x_2(j\omega)$ be the Fourier transform of the signal $x_2(t)$ obtained at the output of the lowpass filter. Then, $x_1(j\omega)$, $x_2(j\omega)$,and $x_p(j\omega)$,are as shown in Figure S7.27

   (b) The Nyquist rate for the signal $x_2(t)$ is $2\times(\omega_2-\omega_1)/2=\omega_2-\omega_1$.Therefore, the sampling period T must be at most $2\pi/(\omega_2-\omega_1)$ in order to avoid aliasing.
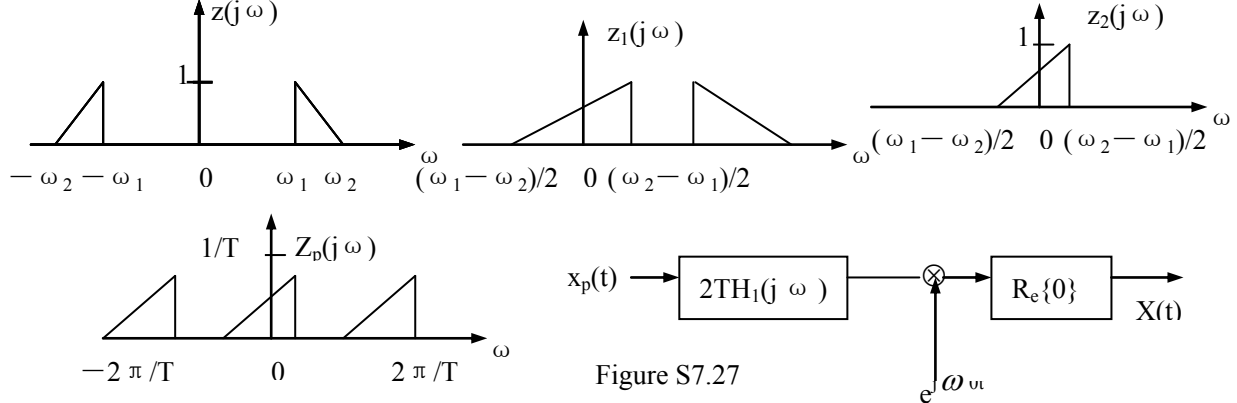
Figure S7.27

   (c) A system that can be used to recover $x(t)$ form $x_p(t)$ is shown in Figure S7.27.

7.28.   (a) The fundamental frequency of $x(t)$ is 20π rad/sec.From Chapter 4 we know that the Fourier transform of $x(t)$ is given by

$$X(j\omega)=2\pi\sum_{k=-\infty}^{\infty} a_k\,\delta(\omega-20\pi k).$$

This is as sketched below. The Fourier transform $x_c(j\omega)$ of the signal $x_c(t)$ is also Sketched in Figure S7.28.

Note that

$$P(j\omega)=\frac{2\pi}{5\times10^{-3}}\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k/(5\times10^{-3}))$$

And

$$x_p(j\omega)=\frac{1}{2\pi}[\,x_c(j\omega)*p(j\omega)\,]$$

Therefore, $x_p(j\omega)$ is as shown in the Figure S7.28.Note that the impulses from adjacent Replicas of $x_c(j\omega)$ add up at 200π.Now the Fourier transform $x(e^{j\Omega})$ of the sequence $x[n]$ is given by

$$x(e^{j\Omega})=x_p(j\omega)|_{\omega=\Omega T.}$$

This is as shown in the Figure S7.28.

   Since the impulses in $x(e^{j\omega})$ are located at multiples of a 0.1π ,the signal $x[n]$ is Periodic. The fundamental period is $2\pi/(0.1\pi)=20$.

$X_p(jw)$:峰 at -400π, -200π, 0, 200π, 400π, w

$X(e^{jw})$: -2π, $-\pi$, 0, π, 2π, w
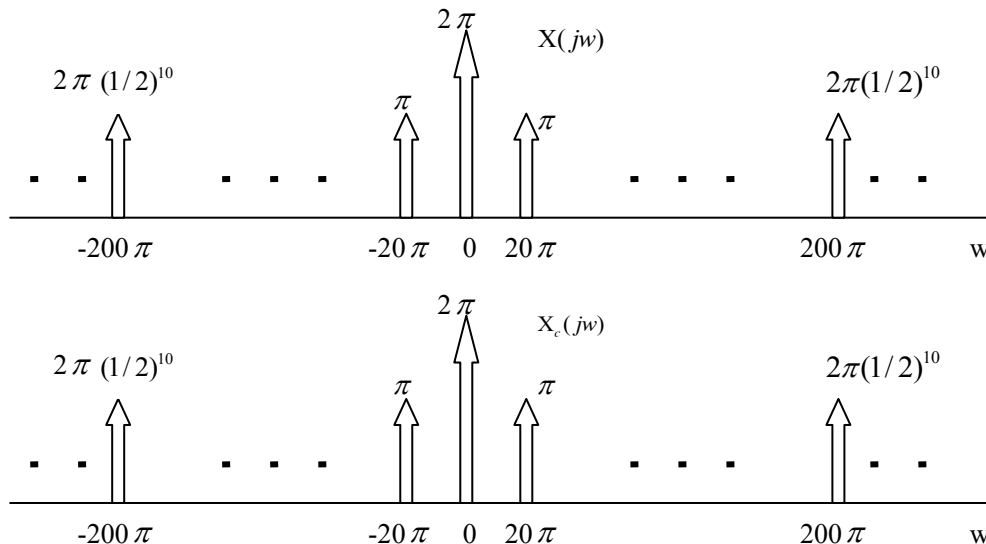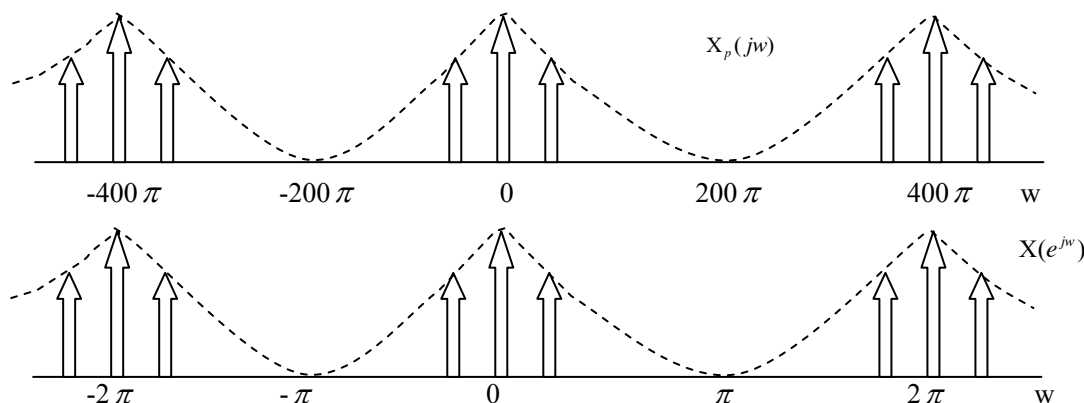
(b) The Fourier series coefficients of X[n] a

$$a_k= \begin{cases} \dfrac{2\pi}{T}(\dfrac{1}{2})^k, & k=0,\pm 1,\pm 2,\ldots,\pm 9 \\[2mm] \dfrac{4\pi}{T}(\dfrac{1}{2})^{10}, & k=10 \end{cases}$$

7.29. From Section 7.1.1 we know that

$$x_p(j\omega)=\frac{1}{T}\sum_{k=-\infty}^{\infty} x(j(\omega-k2\pi/T))$$

$x(e^{jw})$, $Y(e^{jw})$, $Y_p(j\omega)$, and $Y_c(j\omega)$ are as shown in Figure S7.29.

7.30. (a) Since $x_c(t)=\delta(t)$, we have

$$\frac{dy_c(t)}{dt}+y_c(t)=\delta(t)$$

Taking the Fourier transform we obtain

$$j\omega Y(j\omega)+Y(j\omega)=1$$

Therefore ,

$$Y_c(j\omega)=\frac{1}{j\omega+1}, \quad \text{and } y_c(t)=e^{-t}u(t).$$

(b) Since $y_c(t)=e^{-t}u(t)$,

$$y[n]=y_c(nT)=e^{-nT}u[n].$$

Therefore,

$$Y(e^{j\omega})=\frac{1}{1-e^{-T}e^{-j\omega}}$$
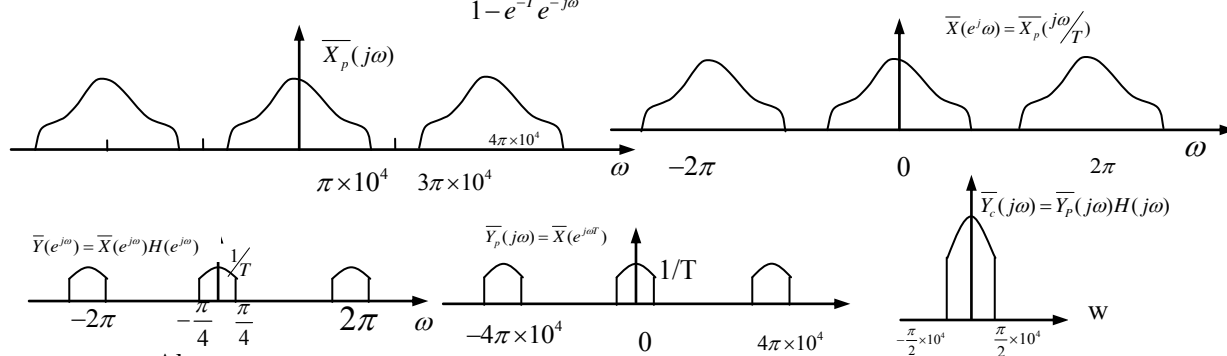
Also,

$$H(e^{j\omega})=\frac{W(e^{j\omega})}{Y(e^{j\omega})}=\frac{1}{1/(1-e^{-T}e^{-j\omega})}=1-e^{-T}e^{-j\omega}$$

Therefore,

$$h[n]=\delta[n]-e^{-T}\delta[n-1]$$

7.31. In this problem for the sake of clarity we will use the variable Ω to denote discrete frequency. Taking the Fourier transform of both sides of the given difference equation we obtain

$$H(e^{j\Omega})=\frac{Y(e^{j\Omega})}{X(e^{j\Omega})}=\frac{1}{1-\dfrac{1}{2}e^{-j\Omega}}$$

Given that the sampling rate is greater than the Nyquist rate, we have

$$x(e^{j\Omega})=\frac{1}{T}x_c(j\,\Omega/T),\quad \text{for } -\pi\leqslant\Omega\leqslant\pi$$

Therefore,

$$Y(e^{j\Omega})=\frac{\frac{1}{T}x_c(j\Omega/T)}{1-\frac{1}{2}e^{-j\Omega}}$$

For $-\pi\leqslant\Omega\leqslant\pi$ .From this we get

$$Y(j\omega)=Y(e^{j\omega T})==\frac{\frac{1}{T}x_c(j\omega)}{1-\frac{1}{2}e^{-j\omega T}}$$

For $-\pi/T\leqslant\omega\leqslant\pi/T$. in this range, $Y(j\omega)=Y_c(j\omega)$.Therefore,

$$H_c(j\omega)=\frac{Y_c(j\omega)}{X_c(j\omega)}=\frac{1/T}{1-\frac{1}{2}e^{-j\omega T}}$$

7.32. Let $p[n]=\sum_{k=-\infty}^{\infty}\delta[n-1-4k]$ .Then from Chapter 5,

$$p(e^{j\omega})=e^{-j\omega}\frac{2\pi}{4}\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k/4)=\frac{\pi}{2}\sum_{k=-\infty}^{\infty}e^{-j2\pi k/4\delta(\omega-2\pi k/4)}$$

Therefore,

$$G(e^{j\omega})=\frac{1}{2\pi}\int_{-\pi}^{\pi}p(e^{j\theta})x(e^{j(\omega-\theta)})d\theta$$
$$=\frac{1}{4}\sum_{k=0}^{3}e^{-j2\pi k/4}x(e^{j(\omega-2\pi k/4)})$$

Since $x(e^{j\omega})=0$ for $\pi/4\leqslant|\omega|\leqslant\pi$, $G(e^{j\omega})$ is as shown in Figure S7.32.
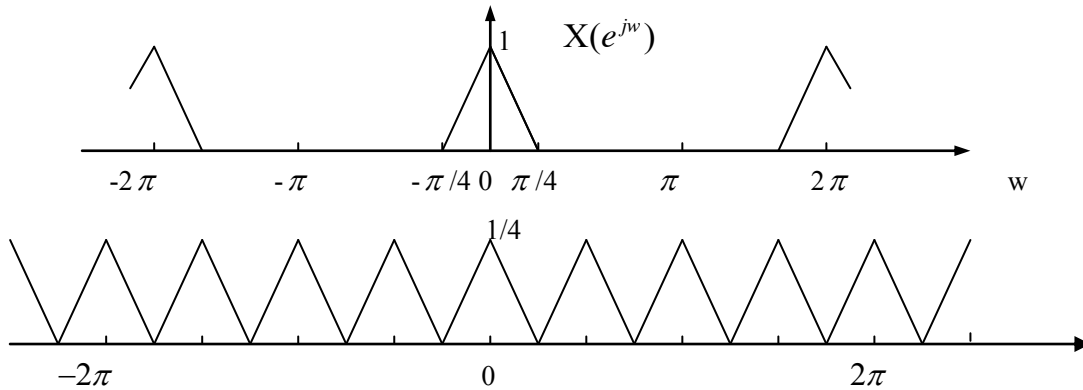
Figure S7.32

Clearly, in order to isolate just $x(e^{j\omega})$ we need to use an ideal lowpass filter with Cutoff frequency $\pi/4$ and passband gain of 4. Therefore, in the range $|\omega|<\pi$,

$$H(e^{j\omega})=\begin{cases}4, & |\omega|<\pi/4\\[2mm]0, & \pi/4\leqslant|\omega|\leqslant\pi\end{cases}$$

7.33. Let $y[n]=x[n]\sum_{k=-\infty}^{\infty}\delta[n-3k]$ .Then

$$Y(e^{j\omega})=\frac{1}{3}\sum_{k=0}^{3}x(e^{j(\omega-2\pi k/3)})$$

Note that $\sin(\pi n/3)/(\pi n/3)$ is the impulse response of an ideal lowpass filter with cutoff frequency π /3 and passband gain of 3.Therefore,we now require that y[n] when passed through this filter should yield x[n].Therefore, the replicas of $x(e^{j\omega})$ contained in $Y(e^{j\omega})$ should not overlap with one another. This is possible only if $x(e^{j\omega})=0$ for $\pi/3\leqslant|\omega|\leqslant\pi$.

7.34. In order to make $x(e^{j\omega})$ occupy the entire region from $-\pi$ to π ,the signal x[n]

must be downsampled by a factor of 14/3. Since it is not possible to directly downsample by a noninteger   factor, we first upsample the signal by a factor of 3. Therefore, after the upsampling we will need to reduce the sampling rate by $14/3 \times 3=14$. Therefore, the overall system for performing the sampling rate conversion is shown in Figure S7.34.

y[n]

$$\omega[n]= \begin{cases} x[\dfrac{n}{2}], & n=0, \pm 3, \pm 6, \dots \\ 0, & \text{otherwise} \end{cases}$$

y[n]=p[14n]
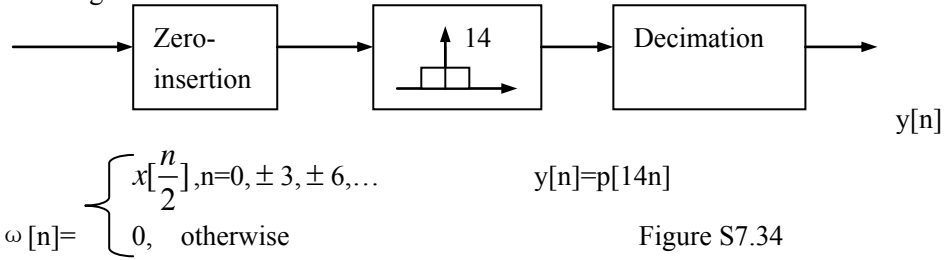
Figure S7.34

7.35 (a) The signals $x_p[n]$ and $x_d[n]$ are sketched in Figure S7.35.
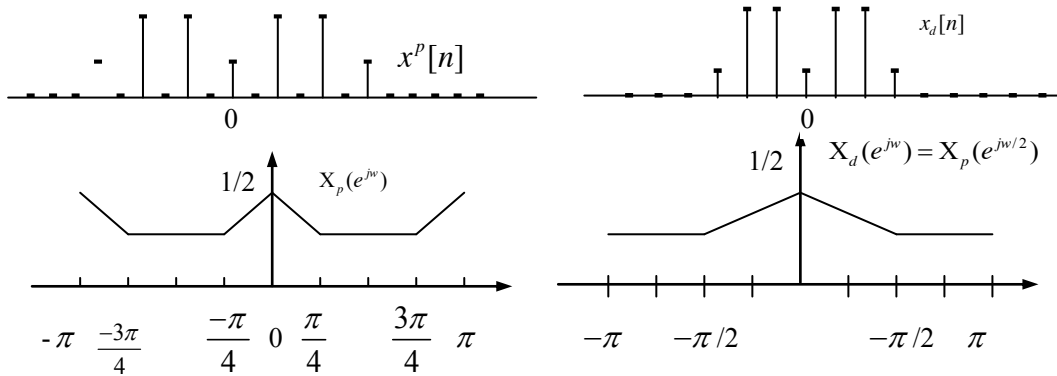
(b) $x_p(e^{j\omega})$ and $x_d(e^{j\omega})$ are sketched in Figure s7.35

7.36. (a) Let us decnote the sampled signaled signal by $x_p(t)$. We have

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

Since the Nyquist rate for the signal x(t) is $2\pi/T$, we can reconstruct the signal from $x_p(t)$. From Section 7.2, we know that

$$x(t) = x_p(t) * h(t)$$

where

$$h(t) = \frac{\sin(\pi t/T)}{\pi t/T}$$

Therefore

$$\frac{dx(t)}{dt} = x_p(t) * \frac{dh(t)}{dt}$$

Denoting $\dfrac{dh(t)}{dt}$ by g(t), we have

$$\frac{dx(t)}{dt} = x_p(t) * g(t) = \sum_{n=-\infty}^{\infty} x(nT)g(t - nT)$$

Therefore,

$$g(t) = \frac{dh(t)}{dt} = \frac{\cos(\pi t/T)}{t} - \frac{T\sin(\pi t/T)}{\pi t^2}$$

(b) No.

7.37.   We may write p(t) as

$$p(t)=p_1(t)+p_1(t-\Delta),$$

where

$$p_1(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2\pi k/W)$$

Therefore,

$$p(j\omega) = (1+e^{-j\Delta\omega})p_1(j\omega)$$

where

$$p_1(j\omega) = w\sum_{k=-\infty}^{\infty} \delta(\omega - kW)$$

131

Let us denote the product p(t)f(t) by g(t).Then,

$$g(t) = p(t)f(t) = p_1(t)f(t) + p_1(t-\Delta)f(t)$$

This may be written as

$$g(t) = ap_1(t) + bp_1(t-\Delta)$$

Therefore,

$$G(j\omega) = (a + be^{-j\omega\Delta})p_1(j\omega)$$

with $p_1(j\omega)$ is specified in eq.(s7.37-1). Therefore

$$G(j\omega) = w\sum_{k=-\infty}^{\infty}\left[a + be^{-jk\Delta w}\right]\delta(\omega - kw)$$

We now have

$$y_1(t) = x(t)p(t)f(t)$$

Therefore,

$$Y_1(j\omega) = \frac{1}{2\pi}\left[G(j\omega)*x(j\omega)\right]$$

This give us

$$Y_1(j\omega) = \frac{W}{2\pi}\sum\left[a + be^{-jk\Delta w}\right]x(j(\omega - kW))$$

In the range $0 < \omega < W$, we may specify $Y_1(j\omega)$ as

$$Y_1(j\omega) = \frac{w}{2\pi}\left[(a+b)x(j\omega) + (a + be^{-jk\Delta w})x(j(\omega - W))\right]$$

since $Y_2(j\omega) = Y_1(j\omega)H_1(j\omega)$, in the range $0 < \omega < W$ we may specify $Y_2(j\omega)$ as

$$Y_2(j\omega) = \frac{jW}{2\pi}\left[(a+b)x(j\omega) + (a + be^{-j\Delta W})x(j(\omega - W))\right]$$

Since $y_3(t) = x(t)p(t)$, in the range $0 < \omega < W$ we may specify $Y_3(j\omega)$ as

$$Y_3(j\omega) = \frac{W}{2\pi}\left[2x(j\omega) + (1 + e^{-j\Delta W})x(j(\omega - W))\right]$$

Give that $0 < W\triangle < \pi$,we require that $Y_2(j\omega) + Y_3(j\omega) = kx(j\omega)$ for $0 < \omega < W$.

That is

$$\frac{W}{2\pi}\left[(a + ja + jb)x(j\omega)\right] + \frac{W}{2\pi}\left[(1 + e^{-j\Delta w})x(j(\omega - W))\right] = kx(j\omega)$$

This implies that

$$1 + e^{-j\Delta W} + ja + jbe^{-j\Delta W} = 0$$

Solving this we obtain

$$A = 1, \quad b = -1,$$

When $W\triangle = \pi/2$. More generally, we also get

$$a = \sin(W\triangle) + \frac{(1 + \cos(W\Delta))}{\tan(W\Delta)} \quad \text{and} \quad b = -\frac{1 + \cos(W\Delta)}{\sin(W\Delta)}$$

except when $W\Delta = \pi/2$ Finally, we also get $k = \frac{2\pi}{W}\left[1/(2 + ja + jb)\right]$

Chapter 8 Answers

8.1 Using Table 4.1, take the inverse Fourier transform of $Y(j(\omega-\omega_c))$. This gives $y(t)=2x(t)e^{jw_ct}$. Therefore,

$$m(t)=2e^{jw_ct}.$$

8.2 (a) The Fourier transform Y(jw) of y(t) is given by

$$Y(jw)=Xt(j(\omega-\omega_c)).$$

Clearly, $Y(jw)$ is just a shifted version of $X(jw)$. Therefore, x(t) may be recovered from y(t) simply by multiplying y(t) by e^{-jw_ct}. There is no constraint that needs to be placed on ω_c to ensure that $x(t)$ is recoverable from $y(t)$.

(b) We know that

$$y_1(t)=\mathrm{Re}\{y(t)\}=x(t)\cos(\omega_ct).$$

The Fourier transform $Y_1(jw)$ of $y_1(t)$ is as shown in Figure S8.2

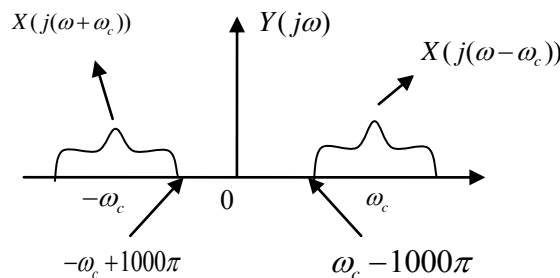$$Y_1(jw)=\frac{1}{2}X(j(\omega-\omega_c))+\frac{1}{2}X(j(\omega+\omega_c))$$

Figure S8.2

If we want to prevent the two shifted replicas of $Y(jw)$ from multiplied by $\cos(2000\pi t)$, the output will be

$$x_1(t)=g(t)\cos(2000\pi t)=x(t)\sin(2000\pi t)\cos(2000\pi t)=\frac{1}{2}x(t)\sin(4000\pi t)$$

The Fourier transform of this signal is

$$X_1(jw)=\frac{1}{4j}X(j(\omega-4000\pi))-\frac{1}{4j}X(j(\omega+4000\pi)).$$

This implies that $X_1(jw)$ is zero for $|\omega|\le 2000\pi$. When $y(t)$ is passed through a lowpass fiter with cutoff frequency 2000π ,the output will clearly be zero .Therefore $y(t)=0$.

8.4 Consider the signal

$$\begin{aligned}y(t)&=g(t)\sin(400\pi t)+2\sin^3(400\pi t)\\&=\sin(200\pi t)\sin^2(400\pi t)+2\sin^3(400\pi t)\\&=\sin(200\pi t)\big[(1-\cos(800\pi t))/2\big]+2\sin(400\pi t)\big[(1-\cos(800\pi t))/2\big]\\&=(1/2)\sin(200\pi t)-(1/4)\{\sin(1000\pi t)-\sin(600\pi t)\}\\&\quad+\sin(400\pi t)-(1/2)\{\sin(1200\pi t)-\sin(400\pi t)\}\end{aligned}$$

If this signal is passed through a lowpass filter with cutoff frequency 400π ,then the output will be

$$y_1=\sin(200\pi t).$$
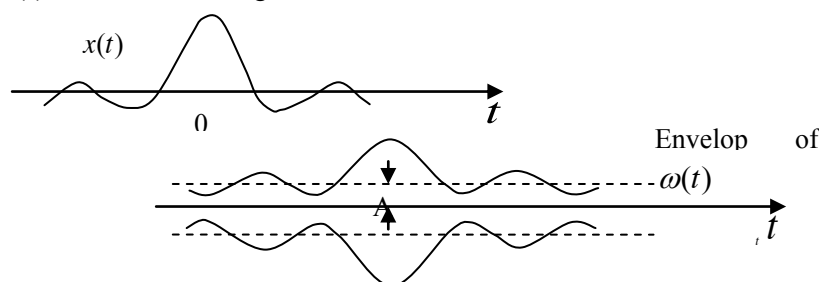
8.5 The signal $x(t)$ is as shown in Figure S8.5

Figure S8.5

The envelope of the signal $\omega(t)$ is as shown in Figure S8.5.Clearly,is we want to use asynchronous demodulation to recover the signal $x(t)$,we need to ensure that A is greater than the height h of the highest sidelobe (see Figure S8.5).Let us now determine the height of the highest sidelobe.The first zero-crossing of the signal $x(t)$ occurs at time t_o such that

$$1000\pi t_0 = \pi, \Rightarrow t_0 = 1/1000.$$

Similarly , the second zero –crossing happens at time t_1 such that

$$1000\pi t_1 = 2\pi, \Rightarrow t_1 = 2/1000.$$

The highest sidelobe occurs at time $(t_0 + t_1)/2$.that is ,at time $t_2 = 3/2000$. At this time, the amplitude of $x(t)$ is

$$x(t_2) = \frac{\sin(3\pi/2)}{\pi 3/2000} = -\frac{2000}{3\pi}$$

Therefore , A should at least be $\dfrac{2000}{3\pi}$ .The modulation index corresponding to the smallest permissible value of A is

$$m = \frac{Max.value}{Min.possible}\frac{of}{value}\frac{x(t)}{of}\frac{1}{A} = \frac{1000}{2000/3\pi} = \frac{3\pi}{2}$$

8.6 Let us denote the Fourier transform of $\sin(\omega_c)/(\pi t)$ by $H(jw)$. This will be rectangular pulse which is nonzero only in the range $|\omega| \le \omega_c$. Taking The Fourier transform of the first equation given in the problem ,we have

$$G(j\omega) = FT\{x(t)\cos(\omega_c t)\} - FT\{x(t)\cos(\omega_c t)\}H(jw)$$
$$= FT\{x(t)\cos(\omega_c t)\}\{1 - H(jw)\}$$
$$= (1/2)[X(j(\omega - \omega_c)) + X(j(\omega + \omega_c))]\{1 - H(jw)\}$$

$G(j\omega)$ is as shown in Figure S8.6
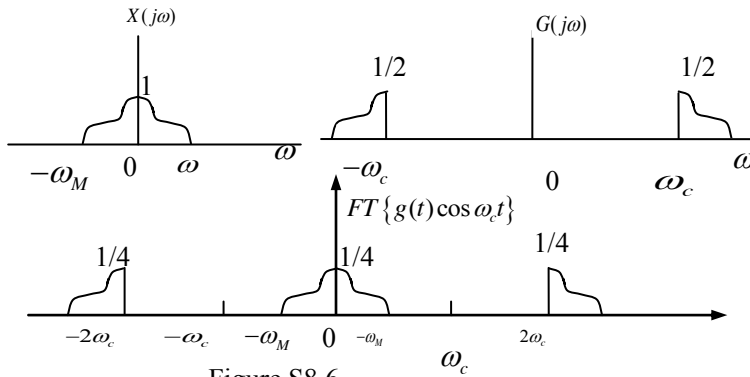
Figure S8.6

The Fourier transform of $g(t)\cos(\omega_c t)$ is also shown in Figure S8.6.Clearly,if we want to recover $x(t)$ from $g(t)\cos(\omega_c t)$,then we have to pass $g(t)\cos(\omega_c t)$ through an ideal lowpass filter with gain 4 and cutoff frequency ω_M .Therefore ,A=4.

8.7 In Figure S8.7 ,we show $X(j\omega), G(j\omega)$,and $Q(j\omega)$.We also show a polt of The Fourier transform of $g(t)\cos(\omega_c t)$,then we need to ensure that (1) $\omega_0 = 2\omega_c$, and (2) an ideal lowpass filter with passband gain of 2 and a cutoff frequency of ω_c is used to filter $g(t)\cos(\omega_0 t)$.

$$FT\{g(t)\cos\omega_0 t\}$$

$$-\omega_0-\omega_c \qquad -\omega_0 \qquad -\omega_0+\omega_c \qquad 0 \qquad \omega_0-\omega_c \qquad \omega_0 \qquad \omega_0+\omega_c$$
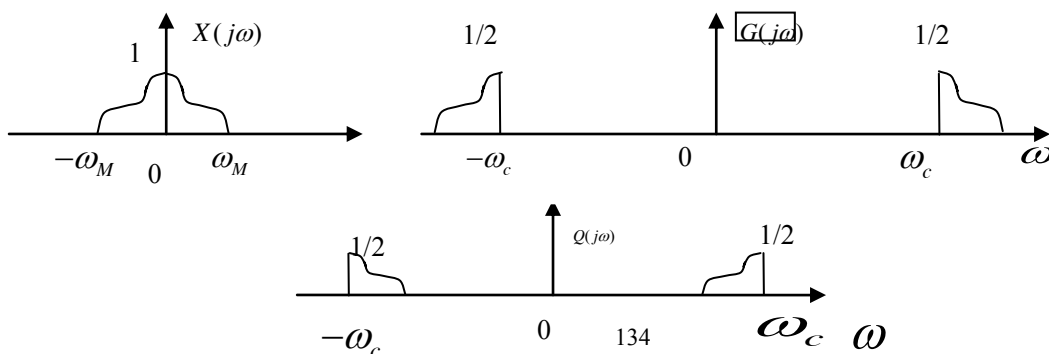
Figure S8.7

8.8 (a) From Figure S8.8,it is clear that $Y(j\omega)$ is conjugate-symmetric .Therefore, $y(t)$ is real.

(b) This part of the problem explores the demodulation of SSB signals through synchronous demodulation. This idea is explored in more detail in problem 8.29.

Let us assume that we use the synchronous demodulation system shown in the Figure S8.8. The Fourier Transform $Y_1(j\omega)$ of the signal $y_1(t)$ is shown in the Figure S8.8. Clearly, if we use an ideal lowpass filter with cutoff frequency ω_c and passhand gain of 2,we should recover the original signal $x(t)$. Therefore,
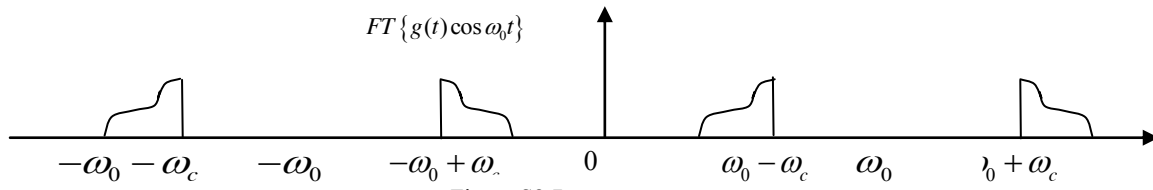
$$x(t) = \left[y(t)\sin(\omega_c t) \right] * \left\{ \frac{2\sin\omega_c t}{\pi t} \right\}.$$
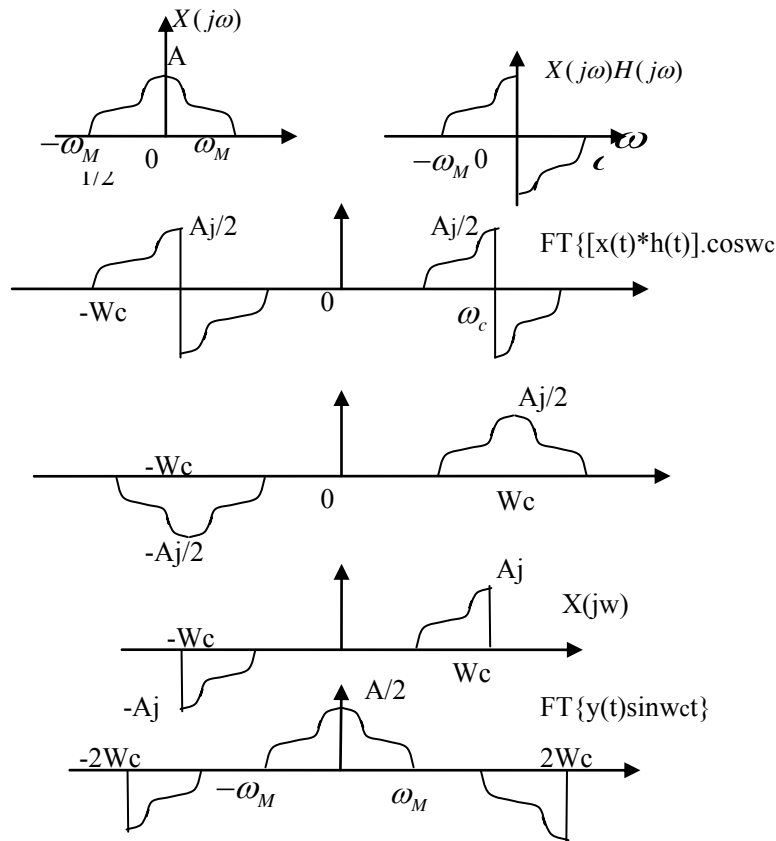
Figure S8.8

8.9 Let the signal $x_1(t)$ and $x_2(t)$ have Fourier Transform $X_1(j\omega)$ and $X_2(j\omega)$ as shown in the Figure S8.9. When SSB modulation is performed on the signals $x_1(t)$ and $x_2(t)$,we would obtain the signal $y_1(t)$ and $y_2(t)$, respectively .The Fourier Transform $Y_1(j\omega)$ and $Y_2(j\omega)$ of these signals would be as shown in the Figure S8.9(see Section 8.4 for details).

(a) From the figure ,it is clear that the signal $y(t) = y_1(t) + y_2(t)$ would have a Fourier Transform $Y(j\omega)$ which is as shown in the Figure S8.9.

135

From this figure ,it is obvious that $Y(j\omega)$ is zero for $|\omega| \geq 2\omega_c$.

（b）In order to obtain $x_1(t)$ from $y(t)$,we have to first remove any contribution in $y(t)$ from $x_2(t)$ .From the previously draw figures , it is clear that we can remove all contribution to $y(t)$ from $x_2(t)$ by first lowpass filtering $y(t)$ using a lowpass filter with cutoff frequency ω_c . We may then follow this by a synchronous demodulation system. This idea is illustrated in the Figure S8.9 . Therefore.

$$x_1(t) = \left[\left\{y(t) * \frac{\sin\omega_c t}{\pi t}\right\}\cos\omega_0 t\right] * \frac{A\sin\omega_c t}{2}$$

In order to determine the value of the gain A ,we first plot the Fourier Transform of $x_1(t) = \left[\left\{y(t) * \frac{\sin\omega_c t}{\pi t}\right\}\cos\omega_0 t\right]$ .From this it is clear that A=4.
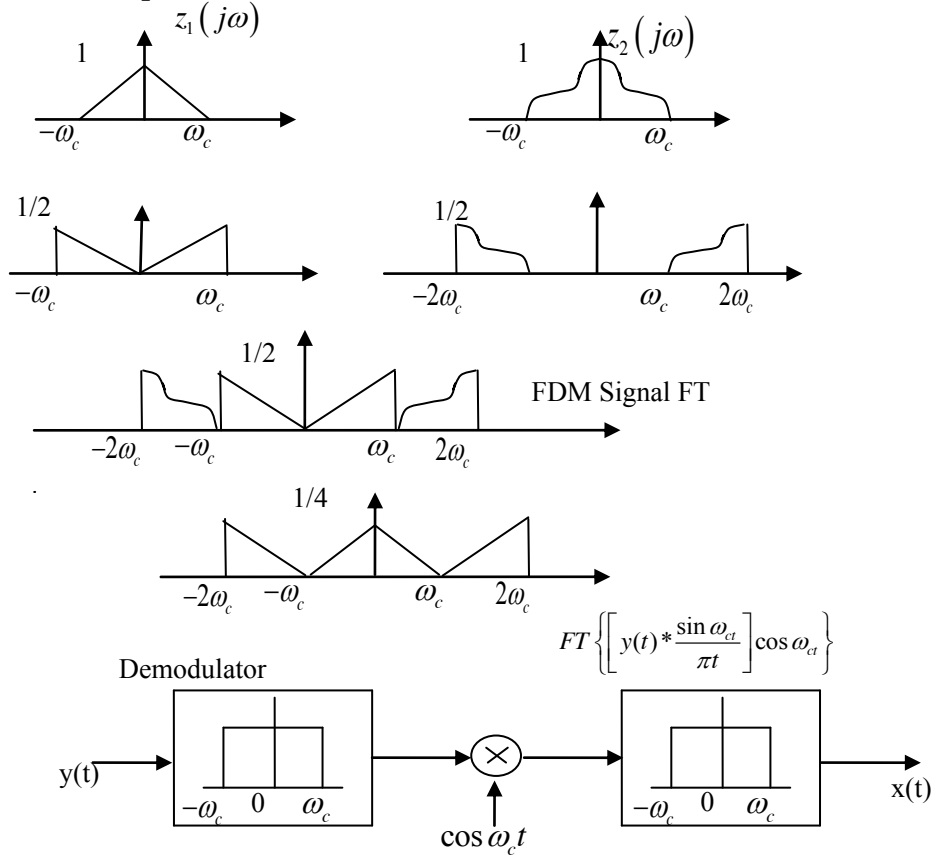
8.10 (a) From Section 8.5 ,we know that in order to avoid aliasing , $2\pi/T > 2\omega_M$, where ω_M is the Maximum frequency in the original signal and T is the period of $c(t)$ .In this case , $T = 10^{-3}$. Therefore $\omega_M \leq 1000\pi$ .Therefore , $X(j\omega) = 0$ for $\omega \geq 1000\pi$.

(b)For Figure 8.24, we know that the Fourier Transform $Y(j\omega)$ of the signal $y(t)$ consists of shifted replicas of $X(j\omega)$. The replica of $X(j\omega)$ centered around $\omega = 0$ is scaled by Δ/T , where Δ is the width of each pulse of $c(t)$ .By using a lowpass filter , we may recover $X(j\omega)$ from $Y(j\omega)$. The lowpass filter needs to have a passband gain of T/Δ .In this case ,this evaluates to $10^{-3}/(0.25\times10^{-3}) = 4$.

8.11 This signal $c(t)$ is

$$c(t) = a_1 e^{jw_c t} + a_{-1} e^{-jw_c t} + a_2 e^{j2w_c t} + a_{-2} e^{-j2w_c t} + \cdots$$

Since $c(t)$ is real , $a_k = a^*_{-k}$. The Fourier Transform $Y(j\omega)$ of the signal $y(t) = x(t)c(t)$ is

$$Y(j\omega) = a_1 X(j(\omega-\omega_c)) + a^*_1 X(j(\omega+\omega_c)) + a_2 X(j(\omega-2\omega_c)) + a_2^* X(j(\omega+2\omega_c))\cdots$$
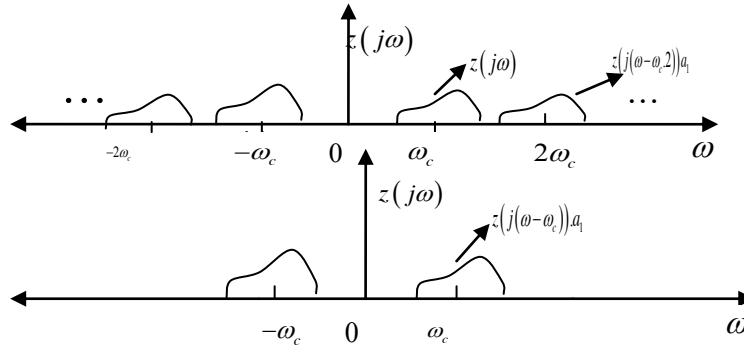
This is plotted in Figure S8.11.

Figure s8.11

(a)The Fourier transform G(jw) of g(t) is

$$G(J\omega) = a_1 X(J(\omega - \omega_c)) + a_1 *(j(\omega + \omega_c))$$

This is as shown in Figure s8.11   clearly, by comparing $G(J\omega)$ and Y(JW).we know
That g(t) may be obtained from y(t) by passing Y(t) through an ideal bandpass filter
Which has a passband gain of unity in the range $(\omega_c/2) <= |\omega| <= (3\omega/2)$.

(b)if $a_1 = |a_1| e^{j<a}$ then $a^*_1 = |a_1| e^{-j<a}$ Also,

$$g(t) = (a_1 e^{j\omega t} + a_1 e^{-j\omega t})\omega_c t + \triangleleft a$$

$$= |a_1| e^{j(\omega_c t + \triangleleft a_1)} x(t) + |a_1| e^{-j(\omega_c t + \triangleleft a_1)}$$

$$= 2|a_1| \cos(\omega_c t + \triangleleft a)\omega_c t + \triangleleft a$$

$$therefore, A = 2|a_1|, and \phi = \triangleleft a_1.$$

8.11 we need to first determine the maximum allowable period t .from section 8.5.1,we know
That t should be chosen such that $\frac{2\pi}{T} > 2\omega_m$ in this case $\omega_{m.} = 2000\pi$. Therefore,

$T \le 0.5 \times 10^{-3}$ sec.we need to how 10 different pulses within a duration of T

Therefore ,each pulled can be at most $\Delta = 0.5 \times 10^{-4}$ sec wide

8.12 (a) we know that

$$p(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} p(j\omega)d\omega$$

Therefore,

$$p(0) = \frac{1}{2\pi} \int_{\frac{-2\pi}{T_1}}^{\frac{2\pi}{T_1}} (\frac{1}{2} + \cos(\frac{\omega T_1}{2}))d\omega$$

(b)since p(jω)statistics eq. (8.28),we know that it must have zero –crossings every T_1

Therefore,,

$p(kT_1) = 0, for k = \pm 1, \pm 2, \square\square\square$

8.13 given

$$y(t) = \cos(\omega_c t + m\cos\omega_m t)$$

$$= \cos(\omega_c t)\cos(m\cos(\omega_m t)) - \sin(\omega_c t)\sin(m\cos(\omega_m t))$$

But since $\omega_c >> \omega_m$ and $m << \frac{\pi}{2}$, we may make the following approximations

$$\cos(m\cos(\omega_m t)) \approx 1$$

And $\sin(m\cos(\omega_m t)) \approx m\cos(\omega_m t)$

Therefore ,

Y(t)= $\cos(\omega_c t) - \sin(\omega_c t)m\cos(\omega_m t)$

$$= \cos(\omega_c t) - \frac{m}{2}\{\sin[(\omega_c + \omega_m)t] + \sin[(\omega_c - \omega_m)t]\}$$

Therefore for w>0,

$$y(j\omega) = \pi\partial(\omega - \omega_c) - \frac{m\pi}{2j}\partial(w - (\omega_c + \omega_m)) - \frac{m\pi}{2j}\partial(\omega - (\omega_c - \omega_m))$$

8.14 when a signal x(t) is amplitude modulated with $e^{j\omega_0 n}$,then the Fourier transform OF the result is

$$y_1(e^{j\omega}) = x(e^{j(\omega-\omega_0)}).$$

8.15 when a signal x(t) is amplitude modulated with cos($\omega_0 n$),then the Fourier transform OF the result is

$$y_1(e^{j\omega}) = (\frac{1}{2})x(e^{j(\omega-\omega_0)}) + (\frac{1}{2})x(e^{j(\omega+\omega_0)}).$$

$y_1(e^{j\omega}) = y_2(e^{j\omega})$ only when ω_0 is either 0 or π.

8.16 we know that c[n]= $\sin(\frac{5\pi n}{2}) = \sin(\frac{\pi n}{2})$

$$y(j\omega) = (\frac{1}{2j})x(e^{j(\omega-\frac{\pi}{2})}) - (\frac{1}{2j})x(e^{j(\omega+\frac{\pi}{2})})$$

This is as shown in the figure s8.16.
From the figure ,it is obvious that

$$y(j\omega) = 0, fo, r0 \le \omega \le \frac{3\pi}{8} and \frac{5\pi}{8} \le \omega \le \pi$$

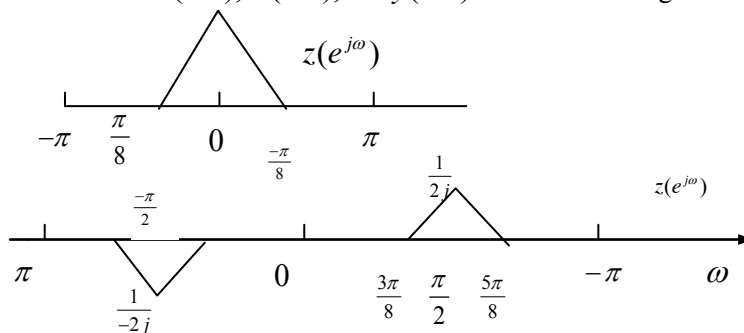**8.17** the Fourier transforms $x(e^{j\omega}), G(e^{j\omega}), and y(e^{j\omega}) are$ shown in figure s8.17.

Figure s8.16

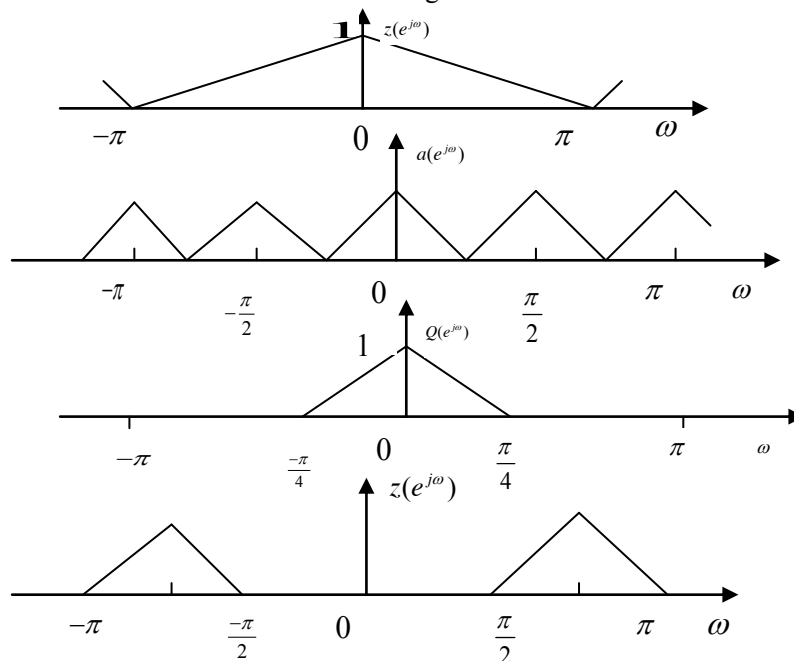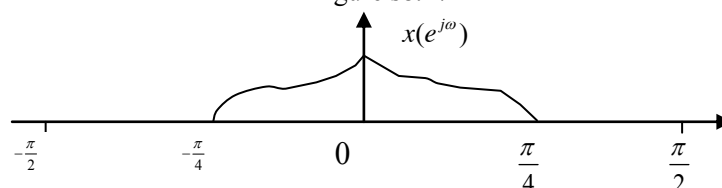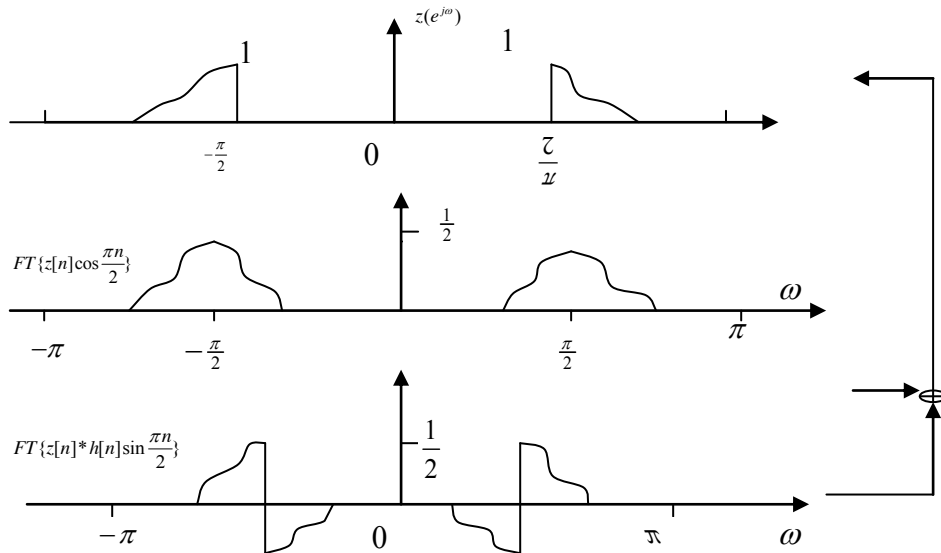Figure s8.17

$z(e^{j\omega})$

1   1

$-\dfrac{\pi}{2}$ 0 $\dfrac{\pi}{2}$

$FT\{z[n]\cos\dfrac{\pi n}{2}\}$

$\dfrac{1}{2}$

$-\pi$ $-\dfrac{\pi}{2}$ $\dfrac{\pi}{2}$ π ω

$FT\{z[n]*h[n]\sin\dfrac{\pi n}{2}\}$

$\dfrac{1}{2}$ $\dfrac{1}{2}$

$-\pi$ 0 π ω

8.18.the Fourier transforms $x(e^{j\omega})$ and $y(e^{j\omega})$ are as shown in figure s8.18.from this figures it is clear that we wish to accomplish single sideband modulation using the system .in particular, we are interest in retaining the upper sidebands of the signal. note that in figure 8.21 of section8.4,is shown a continues-time signal sideband system for retaining the lower sidebands. In this section, it was also mentioned (see eq.(8.21))that in order to retain the upper sidebands ,the frequency response of the filter used in the system had to be charge to

$$H(j\omega) = \begin{cases} j, & \omega>0 \\ -j, & \omega<0 \end{cases}$$

in this problem ,we extend this same idea to discrete-time system, we assume that the frequency response $H(e^{j\omega})$ of the unknown systems is

$$H(e^{j\omega}) = \begin{cases} j, & \omega>0 \\ -j, & \omega<0 \end{cases}$$

let us now show that this does indeed give us the desired output .we redraw the system give in the problem with appropriate labels for the intermediate outputs. The fourier transforms of this intermediate outputs are shown in figure s8.18
from figure s8.18,it is clear that the choice of $H(e^{j\omega})$ was appropriate.

8.19 since 10 different signals have to be squeezed in within a bandwidth of 2π each signal is allowed to occupy a bandwidth of $\dfrac{2\pi}{10}=\dfrac{\pi}{5}$ after sinusoidal modulation .therefore, before from the figure , it is obvious that $\quad y(j\omega)=0, for 0 \le |\omega| \le \dfrac{\pi}{2}$.

sinusoidal modulation each signal can occupy only a bandwidth of $\dfrac{\pi}{10}$. the Fourier transform $y_i(e^{j\omega})$ of the signal obtained by upsampling $x_i[n]$ by a factor of N can be nonzero (in the range$|\omega|<\pi$)only for $|\omega|<\dfrac{\pi}{20}$ .therefore ,n has to be at least 20.

8.20 note that by choosing p[n]= $\displaystyle\sum_{k=-\infty}^{\infty}\partial[n-2k]$, we would be eble to get $\hat{v}_1[n]$ and $\hat{v}_2[n-1]$ at the output of the multipliers .furthermore ,note that $\hat{v}_1(e^{j\omega})=v_2(e^{2j\omega})$ and $\hat{v}_2(e^{j\omega})=v_2(e^{2j\omega})e^{-j\omega}$

this is illustrated in figure s8.20. therefore ,the output of the two branches will be as shown in figure s 8.20. from thus figures ,it is clears ,that the sum of the two outputs will be a FDM signal containing both $v_1[n]$ and $v_2[n]$.

We are given that $W<2\omega+\Delta\omega - \omega_M$, lowpass filtering will result in the output

$$\dfrac{1}{2}x(t)\cos(\Delta\omega t) .$$

(b) We sketch the spectrum of the output for $\Delta\omega = \omega_M/2$ in figure s8.23

8.24 (a)Since s(t)= $\displaystyle\sum_{k=-\infty}^{\infty}\delta(\omega-kT)$. We have

$$S(j\omega)=\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k/T)=\omega_c\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_c)$$

Let us denote x(t)s(t) by u(t).Then the Fourier transform of the signal w(t) is

$$W(j\omega)=\frac{1}{2\pi}[X(j\omega)*S(j\omega)]$$

$$=\frac{\omega_c}{2\pi}\sum_{k=-\infty}^{\infty}X(j(\omega-k\omega_c))$$

This is as shown in FigureS8.24. Therefore , the Fourier transform $Y(j\omega)$ of the output of the bandpass filter in

$$Y(j\omega)=\frac{(A\omega_c)}{2\pi}X(j(\omega-\omega_c))+\frac{(A\omega_c)}{2\pi}X(j(\omega+\omega_c))$$

Therefore.

$$y(t)=\frac{(A\omega_c)}{\pi}X(t)\cos(\omega_c t)$$

(b) If $\Delta\neq0$,then
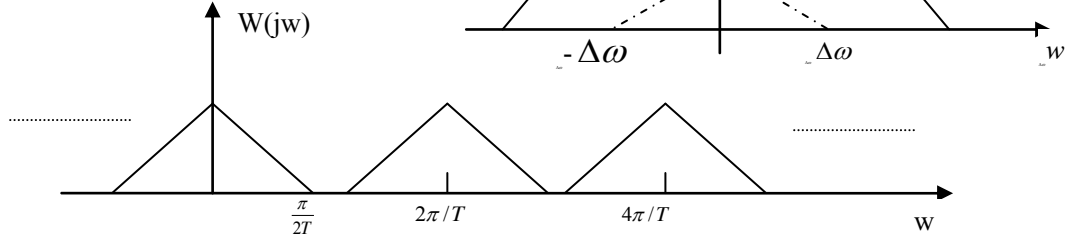
Figure S8.24

$$S(j\omega)=\frac{2\pi}{T}e^{-\Delta\omega}\sum_{k=-\infty}^{\infty}\delta(\omega-2k\pi/T)=\frac{2\pi}{T}e^{-j\Delta k2\pi/T}\sum_{k=-\infty}^{\infty}\delta(\omega-2k\pi/T)$$

Let us denote x(t)s(t) by $\omega(t)$.Then the Fourier transform of the signal $\omega(t)$is

$$W(j\omega)=\frac{1}{2\pi}[X(j\omega)*S(j\omega)]$$

$$=\frac{1}{T}\sum_{k=-\infty}^{\infty}e^{-j\Delta k2\pi/T}X(j(\omega-k2\pi/T))$$

Therefore, the Fourier transform $Y(j\omega)$ of the output of the bandpass filter is

$$Y(j\omega)=\frac{A\omega_c}{2\pi}e^{-j2\pi\Delta/T}X(j(\omega-2\pi/T))+\frac{A\omega_c}{2\pi}e^{-j2\pi\Delta/T}X(j(\omega+2\pi/T))$$

Therefore,

$$y(t)=\frac{(A\omega_c)}{2\pi}X(t)\cos(\frac{2\pi}{T}t-\frac{2\pi}{T}\Delta)$$

(c) Form the analysis in part (b).it is clear that the maximum allowable value for ω_M is π/T.

Chapter 9 Answers

9.1 （a）The given integral may be written as

$$\int_0^\infty e^{-(5+\sigma)t} e^{j\omega t} dt$$

If $\sigma <-5$,then the function $e^{-(5+\sigma)t}$ grows towards ∞ with increasing t and the given integral does not converge .but if >-5,then the integral does converge

(b) The given integral may be written as

$$\int_{-\infty}^0 e^{-(5+\sigma)t} e^{j\omega t} dt$$

If $\sigma >-5$,then the function $e^{-(5+\sigma)t}$ grows towards ∞ as t decreases towards $-\infty$ and the given integral does not converge .but if $\sigma <-5$,then the integral does converge

(c) The given integral may be written as

$$\int_{-5}^5 e^{-(5+\sigma)t} e^{j\omega t} dt$$

Clearly this integral has a finite value for all finite values of σ .

(d) The given integral may be written as

$$\int_{-\infty}^\infty e^{-(5+\sigma)t} e^{j\omega t} dt$$

If $\sigma >-5$,then the function $e^{-(5+\sigma)t}$ grows towards ∞ as t decreases towards $-\infty$ and the given integral does not converge If $\sigma <-5$, ,then function $e^{-(5+\sigma)t}$ grows towards ∞ with increasing t and the given integral does not converge If $\sigma =5$, then the integral stilldoes not have a finite value. therefore, the integral does not converge for any value of σ .

(e) The given integral may be written as

$$\int_{-\infty}^0 e^{-(5+\sigma)t} e^{j\omega t} dt + \int_0^\infty e^{-(5+\sigma)t} e^{j\omega t} dt$$

The first integral converges for $\sigma <-5$, the second internal converges if $\sigma >-5$,therefore, the given internal converges when $|\sigma| <5$.

(f) The given integral may be written as

$$\int_{-\infty}^0 e^{-(5+\sigma)t} e^{j\omega t} dt$$

If $\sigma >5$,then the function $e^{-(-5+\sigma)t}$ grows towards ∞ as t decrease towards $-\infty$ and the given integral does not converge .but if $\sigma <5$,then the integral does converge.

9.2 (a)

$$X(s) = \int_{-\infty}^\infty e^{-5t} u(t-1) e^{-dt} dt$$

$$= \int_0^\infty e^{-(5+s)t} dt$$

$$= \frac{e^{-(5+s)}}{s+5}$$

As shown in Example 9.1 the ROC will be $Re\{s\} >-5$.

(b) By using eg.(9.3), we can easily show that g(t)=A e^{-5t} u(-t-t_0) has the Laplace transform

$$G(s) = \frac{Ae^{(s+5)t_0}}{s+5}$$

The ROC is specified as $Re\{s\} <-5$. Therefore ,A=1 and t_0 =-1

9.3 Using an analysis similar to that used in Example 9.3 we known that given signal has a Laplace transform of the form

$$X(s) \frac{1}{s+5} + \frac{1}{s+\beta}$$

The corresponding ROC is $Re\{s\} >$max(-5,Re$\{\beta\}$). Since we are given that the ROC is Re{s}>-3, we know that Re{β}=3 . there are no constraints on the imaginary part of β .

9.4 We know form Table 9.2 that

$$x_1(t) = -e^{-t} \sin(2t) u(t) \xleftrightarrow{L} X_1(s) = X_1(-s), Re\{s\} >-1$$

We also know form Table 9.1 that

$$x(t) = x_1(-t) \xleftrightarrow{L} X(s) = X_1(-s)$$

The ROC of X(s) is such that if s_0 was in the ROC of $X_1(s)$, then - s_0 will be in the ROC of X(s). Putting

the two above equations together ,we have

x(t)= x_1 (-t) $=e^{-t}\sin(2t)u(-t) \xleftrightarrow{L}$ X(s)= $X_1(-s)$=- $\dfrac{2}{(s-1)^2+2^2}$, $Re\{s\}$ <1

the denominator of the form s^2 -2s+5. Therefore, the poles of X(s) are 1+2j and 1-2j.

9.5 (a)   the given Laplace transform may be written as

$X(s)=\dfrac{2s+4}{(s+1)(s+3)}$.

Clearly ,X(s) has a zero at s=-2 .since in X(s) the order of the denominator polynomial exceeds the order of the numerator polynomial by 1 ,X(s) has a zero at ∞. Therefore ,X(s) has one zero in finite s-plane and one zero at infinity.

(b) The given Laplance transform may be written as

X(s)= $\dfrac{s+1}{(s-1)(s+1)}$ = $\dfrac{1}{s-1}$

    Clearly ,X(s) has no zero in the finite s-plane .Since in X(s) the of the denominator polynomial exceeds the order the numerator polynomial by 1,X(s) has a zero at ∞ .therefore X(s) has no zero in the finite s-plane and one zero at infinity.

(c) The given Laplace transform may be written as

$$X(s)=\dfrac{(s-1)(s^2+s+1)}{(s^2+s+1)}=s-1$$

    Clearly ，X（s）has a zero at s=1.since in X(s) the order of the numerator polynomial exceeds the order of the denominator polynomial by 1,X(s) has zeros at ∞ .therefore , X(s) has one zero in the s-plane and no zero at infinity .

9.6 (a)   No. From property 3 in Section 9.2 we know that for a finite-length signal .the ROC is the entire s-plane .therefore .there can be no poles in the finite s-plane for a finite length signal . Clearly in this problem this not the case.

(b) Yes. Since the signal is absolutely integrable, The ROC must include, the $j\omega$ -axis . Furthermore ,X(s) has a pole at s=2 .therefore, one valid ROC for the signal would be Re{s}<2. From property 5 in section 9.2 we know that this would correspond to a left-sided signal

(C)   No . Since the signal is absolutely integrable, The ROC must include , the $j\omega$ -axis . Furthermore ,X(s) has a pole at s=2. therefore ,we can never have an ROC of the form Re{s}> α . From property 5 in section 9.2 we knew that x(t) can not be a right-side signal

(d) Yes . Since the signal is absolutely integrable, The ROC must include , the $j\omega$ -axis . Furthermore ,X(s) has a pole at s=2 .therefore, one valid ROC for the signal could be α <Re{s}<2 such that α <0 .From property 6 in section 9.2 ,we know that   this would correspond to a two side signal

9.7   We may find different signal with the given Laplace transform by choosing different regions of convergence , the poles of the given Laplace transform are

$s_0=-2$ $s_1=-3$ $s_2=-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}j$ $s_3=-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}j$

Based on the locations of the locations of these poles , we my choose form the following regions of convergence:

(i) Re{s}>- $\dfrac{1}{2}$

(ii)-2< Re{s}<- $\dfrac{1}{2}$

(iii)-3<Re{s}<-2

(iv)Re{s}<-3

Therefore ,we may find four different signals the given Laplace transform.

9.8 From Table 9.1,we know that

G(t)= $e^{2t}x(t) \xleftrightarrow{L} G(s)=X(s-2)$.

The ROC of G(s) is the ROC of X(s) shifted to the right by 2

We are also given that X(s) has exactly 2 poles at s=-1 and s=-3. since G(s)=X(s-2), G(s)also has exactly two poles ,located at s=-1+2=1 and s=-3+2=-1 since we are given G($j\omega$) exists , we may infer that $j\omega$ -axis lies in the ROC of G(s). Given this fact and the locations of the poles ,we may conclude that g(t) is a two side sequence .Obviously x(t)= e^{2t} g(t) will also be two sided
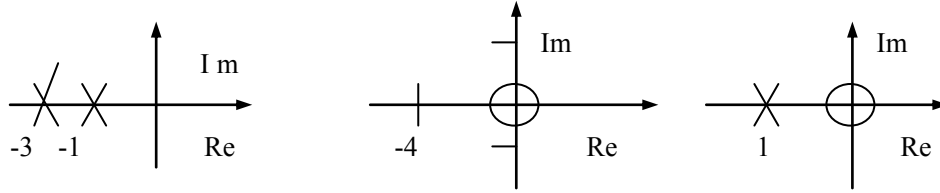
9.9 Using partial fraction expansion

$$X(s)= \frac{4}{s+4} - \frac{2}{s+3}$$

Taking the inverse Laplace transform,

X(t)=$4e^{-4t}u(t) - 2e^{-3t}u(t)$

9.10 The pole-zero plots for each of the three Laplace transforms is as shown in Figure S9.10

(a) form Section 9.4 we knew that the magnitude of the Fourier transform may be expressed as

$$\frac{1}{(\text{Length of vector form } \omega \text{ to -1})(\text{Length of vector form } \omega \text{ to})}$$

we se that the right-hand side of the above expression is maximum for ω=0 and decreases as ω becomes increasing more positive or more negative . Therefore $|H_1(j\omega)|$ is approximately lowpass

(b) From Section 9.4 we know that the magnitude of the Fourier transform may be express as

$$\frac{(\text{length of vector from } \omega \text{ to 0})}{(\text{length of vector from } \omega \text{ to } -\frac{1}{2}+j\frac{\sqrt{3}}{2})(\text{Length of vector from } \omega \text{ to } -\frac{1}{2}-j\frac{\sqrt{3}}{2})}$$

we see that the right-hand side of the above expression is zero for ω=0.It then increams with increasing $|\omega|$ until $|\omega|$ reach 1/2. Then it starts decreasing as $|\omega|$ increase even further.

Therefore $|H_2(j\omega)|$ is approximately bandpass.

(c) From Section 9.4 we know that the magnitude of the Fourier transform may be express as

$$\frac{(\text{Length of vector from } \omega \text{ to 0})^2}{(\text{length of vector from } \omega \text{ to } -\frac{1}{2}+j\frac{\sqrt{3}}{2})(\text{Length of vector from } \omega \text{ to } -\frac{1}{2}-j\frac{\sqrt{3}}{2})}$$

We see that the right-hand side of the above expression is zero for ω=0. It then increases with increasing $|\omega|$ until $|\omega|$ reaches $\frac{1}{2}$. Then $|\omega|$ increases,$|H_3(j\omega)|$ decreases towards a value of 1(because all the vector lengths became almost identical and the ratio become 1) .Therefore $|H_3(j\omega)|$ is approximately highpass.

9.11 X(s) has poles at s=$-\frac{1}{2}+j\frac{\sqrt{3}}{2}$ and $-\frac{1}{2}-j\frac{\sqrt{3}}{2}$ .X(s) has zeros at s=$\frac{1}{2}+j\frac{\sqrt{3}}{2}$ and $\frac{1}{2}-j\frac{\sqrt{3}}{2}$ .From Section 9.4 we know that $|X(j\omega)|$ is

$$\frac{(\text{Length of vector from } \omega \text{ to } \frac{1}{2}+j\frac{\sqrt{3}}{2})\ (\text{Length of vector from } \omega \text{ to } \frac{1}{2}-j\frac{\sqrt{3}}{2})}{(\text{length of vector from } \omega \text{ to } -\frac{1}{2}+j\frac{\sqrt{3}}{2})(\text{Length of vector from } \omega \text{ to } -\frac{1}{2}-j\frac{\sqrt{3}}{2})}$$

The terms in the numerator and denominator of the right-band side of above expression cancel our giving us $|X(j\omega)|$=1.

9.12 (a) If X(s) has only one pole, then x(t) would be of the form Ae^{-at} .Clearly such a signal violates condition 2. Therefore , this statement is inconsistent with the given information.

(b) If X(s) has only two poles, then x(t) would be of the form $Ae^{-at}\sin(\omega_0 t)$ .Clearly such a signal could be made to satisfy all three conditions(Example: ω_0=80π ,α=19200). Therefore, this statement is consistent with the given information.

(c) If X(s) has more than two poles (say 4 poles), then x(t) could be assumed to be of the form $Ae^{-at}\sin(\omega_0 t) + Be^{-bt}\sin(\omega_0 t)$. Clearly such a signal could still be made to satisfy all three conditions. Therefore, this statement is consistent with the given information.

9.13 We have

$$X(s) = \frac{\beta}{s+1}, \text{Re}\{s\} > -1 \cdot$$

Also,

$$G(s) = X(s) + \alpha X(-s), -1 < \text{Re}\{s\} < 1$$

Therefore,
$$G(s) = \beta[\frac{1 - s + \alpha s + \alpha}{1 - s^2}].$$

Comparing with the given equation for G(s),
$$\alpha = -1, \qquad \beta = \frac{1}{2}.$$

9.14. Since X(s) has 4 poles and no zero in the finite s-plane, we many assume that X(s) is of the form
$$X(s) = \frac{A}{(s-a)(s-b)(s-c)(s-d)}.$$

Since x(t) is real ,the poles of X(s) must occur in conjugate reciprocal pairs. Therefore, we may assume that b=a^* and d=c^*. This result in
$$X(s) = \frac{A}{(s-a)(s-a^*)(s-c)(s-c^*)}.$$

Since the signal x (t) is also even , the Laplace transform X(s) must also be even . This implies that the poles have to be symmetric about the jω -axis. Therefore, we may assume that c=$-a^*$. This results in
$$X(s) = \frac{A}{(s-a)(s-a^*)(s+a)(S+a^*)}.$$

We are given that the location of one of the poles is $(1/2)_e{}^{j\pi/4}$. If we assume that this pole is a, we have
$$X(s) = \frac{A}{(s-\frac{1}{2}e^{j\frac{\pi}{4}})(s-\frac{1}{2}e^{-j\frac{\pi}{4}})(s+\frac{1}{2}e^{-j\frac{\pi}{4}})(s+\frac{1}{2}e^{j\frac{\pi}{4}})}.$$

This gives us
$$X(s) = \frac{A}{(s^2 - \frac{s}{\sqrt{2}} + \frac{1}{4})(s^2 + \frac{s}{\sqrt{2}} + \frac{1}{4})}.$$

Also ,we are give that $\int_{-\infty}^{\infty} x(t)dt = X(0) = 4$

Substituting in the above expression for X(s), we have A=1/4. Therefore,
$$X(s) = \frac{1/4}{(s^2 - \frac{s}{\sqrt{2}} + \frac{1}{4})(s^2 + \frac{s}{\sqrt{2}} + \frac{1}{4})}.$$

9.15. Taking the Laplace transform of both sides of the two differential equations, we have
$$s X(s) = -2Y(s) + 1 \quad \text{and} \quad s Y(s) = 2X(s).$$

Solving for X(s) and Y(s), we obtain
$$X(s) = \frac{s}{s^2 + 4} \quad \text{and} \quad Y(s) = \frac{2}{s^2 + 4}.$$

The region of convergence for both X(s) and Y(s) is Re{s}>0 because both are right-hand signals.

9.16. Taking the Laplace transform of both sides of the given differential equations ,we obtain
$$Y(s)[s^3 + (1+\alpha)s^2 + \alpha(1+\alpha)s + \alpha^2] = X(s).$$

therefore,
$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^3 + (1+\alpha)s^2 + \alpha(1+\alpha)s + \alpha^2}.$$

(a) Taking the Laplace transform of both sides of the given equation, we have
$$G(s) = s H(s) + H(s).$$

Substituting for H(s) from above,
$$G(s) = \frac{(s+1)}{s^3 + (1+\alpha)s^2 + \alpha(1+\alpha)s + \alpha^2} = \frac{1}{s^2 + \alpha s + \alpha^2}.$$

Therefore, G(s) has 2 poles.

(b) we know that
$$H(s) = \frac{1}{(s+1)(s^2 + \alpha s + \alpha^2)}.$$

Therefore, H(s) has poles at $-1, \alpha(-\frac{1}{2} + j\frac{\sqrt{3}}{2}), and \quad \alpha(-\frac{1}{2} - j\frac{\sqrt{3}}{2})$. If the system has to be stable, then the real part of the poles has to be less than zero. For this to be true, we require that $-\alpha/2 < 0$, i.e., $\alpha > 0$.

9.17 The overall system show in Figure 9.17 may be treated as two feedback system of the form shown in figure 9.31 connected in parallel. By carrying out an analysis similar to that described in Section 9.8.1, we find the system function of the upper feedback system to be
$$H_1(s) = \frac{2/s}{1 + 4(2/s)} = \frac{2}{s+8}.$$

Similarly, the system function of the lower feedback system is

$$H_2(s) = \frac{1/s}{1+2(1/2)} = \frac{1}{s+2}.$$

The system function of the overall system is now

$$H(s) = H_1(s) + H_2(s) = \frac{3s+12}{s^2+10s+16}.$$

Since H(s)=Y(s)/X(s), we may write

$$Y(s)[s^2+10s+16] = X(s)[3s+12].$$

Taking the inverse Laplace transform, we obtain

$$\frac{d^2 y(t)}{dt} + 10\frac{dy(t)}{dt} + 16y(t) = 12x(t) + 3\frac{dx(t)}{dt}$$

9.18. (a)   From problem 3.20, we know that differential equation relating the input and output of the RLC circuit is

$$\frac{d^2 y(t)}{dt} + \frac{dy(t)}{dt} + y(t) = x(t).$$

Taking the Laplace transform of this (while nothing that the system is causal and stable), we obtain

$$Y(s)[s^2+s+1] = X(s).$$

Therefore ,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2+s+1}, \qquad \Re e\{s\} > -\frac{1}{2}.$$

(b) We note that H(s) has two poles at $s = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$ and $s = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$. It has no zeros in the finite s-plane.

From Section 9.4 we know that the magnitude of the Fourier transform may be expressed as

$$\frac{1}{(\text{Length of vector from } \omega \text{ to } -\frac{1}{2}+j\frac{\sqrt{3}}{2})(\text{Length of vector from } \omega \text{ to } -\frac{1}{2}-j\frac{\sqrt{3}}{2})}$$

We see that the right hand side of the above expression Increases with increasing $|\omega|$ until $|\omega|$ reaches $\frac{1}{2}$. Then it starts decreasing as $|\omega|$ increasing even further. It finally reaches 0 for $|\omega| = \infty$. Therefore $|H_2(j\omega)|$ is approximately lowpass.

(c) By repeating the analysis carried out in Problem 3.20 and part (a) of this problem with R $= 10^{-3}\Omega$, we can show that

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2+s+1}, \qquad \Re e\{s\} > -0.0005.$$

(d) We have

$$\frac{1}{(\text{Vect.Len.from } \omega \text{ to } -0.0005+j\frac{\sqrt{3}}{2})(\text{Vect.Len.from } \omega \text{ to } -0.0005-j\frac{\sqrt{3}}{2})}$$

We see that when $|\omega|$ is in he vicinity 0.0005, the right-hand side of the above equation takes on extremely large value. On either side of this value of $|\omega|$ the value of $|H(j\omega)|$ rolls off rapidly. Therefore, H(s) may be considered to be approximately bandpass.

9.19. (a) The unilateral Laplace transform is

$$X(s) = \int_{0^-}^{\infty} e^{-2t} u(t+1) e^{-st} dt$$

$$= \int_{0^-}^{\infty} e^{-2t} e^{-st} dt$$

$$= \frac{1}{s+2} \qquad \Re e\{s\} > -2.$$

(b) The unilateral Laplace transform is

$$X(s) = \int_{0^-}^{\infty} [\delta(t+1) + \delta(t) + e^{-2(t+3)} u(t+1)] e^{-st} dt$$

$$= \int_{0^-}^{\infty} [\delta(t) + e^{-2(t+3)}] e^{-st} dt$$

$$= 1 + \frac{e^{-6}}{s+2} \qquad \Re e\{s\} > -2.$$

(c) The unilateral Laplace transform is

$$X(s) = \int_{0^-}^{\infty} [e^{-2t} u(t) e^{-4t} u(t)] e^{-st} dt$$

$$= \int_{0^-}^{\infty} [e^{-2t} + e^{-4t}] e^{-st} dt$$

$$= \frac{1}{s+2} + \frac{1}{s+4} \qquad \Re e\{s\} > -2.$$

9.20. In Problem 3.29, we know that the input of the RL circuit are related by

$$\frac{dy(t)}{dt} + y(t) = x(t).$$

Applying the unilateral Laplace transform to this equation, we have

$$sy(s) - y(0^-) + y(s) = x(s).$$

(a) For the zero-state response, set $y(0^-) = 0$. Also we have

$$x(s) = u \, L\{e^{-2t}u(t)\} = \frac{1}{s+2} \ .$$

Therefore,

$$y(s)(s+1) = \frac{1}{s+2}.$$

Computing the partial fraction expansion of the right-hand side of the above equation and then taking its inverse unilateral Laplace transform, we have

$$y(t) = e^{-t}u(t) - e^{-2t}u(t).$$

(b) For the zero-state response, assume that x(t) = 0.Since we are given that $y(0^-) = 1$,

$$sy(s) - 1 + y(s) = 0 \Rightarrow y(s) = \frac{1}{s+1}.$$

Taking the inverse unilateral Laplace transform, we have
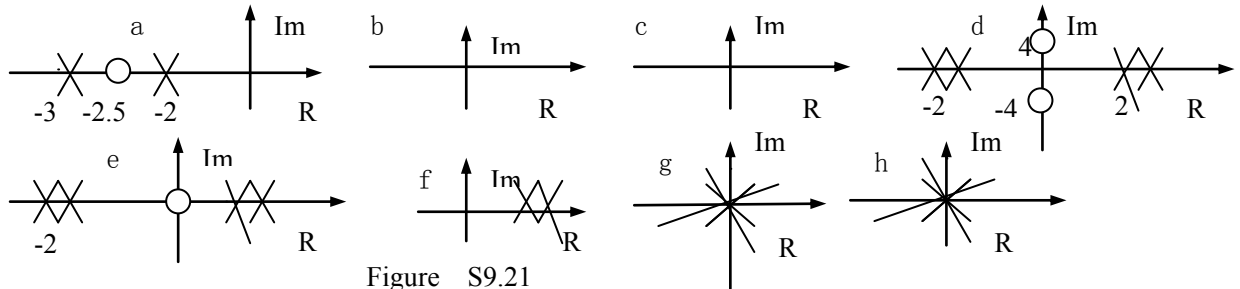
$$y(t) = e^{-t}u(t).$$

Figure   S9.21

(c) The total response is the sum of the zero-state and zero-input response. This is

$$y(t) = 2e^{-t}u(t) - e^{-2t}u(t).$$

9.21. The pole zero plots for all the subparts are shown in figure S9.21.

(a) The Laplace transform of x(t) is

$$X(s) = \int_0^\infty (e^{-2t} + e^{-3t})e^{-st}dt$$
$$= [-e^{-(s+2)t}/(s+2)]|_0^\infty + [-e^{-(s+3)t}/(s+3)]|_0^\infty$$
$$= \frac{1}{s+2} + \frac{1}{s+3} = \frac{2s+5}{s^2+5s+6}$$

(b) Using an approach similar to that show in part (a), we have

$$e^{-4t}u(t) \xleftrightarrow{\ L\ } \frac{1}{s+4}, \qquad \Re e\{s\} > -4$$

Also,

$$e^{-5t}e^{j5t}u(t) \xleftrightarrow{\ L\ } \frac{1}{s+5-j5}, \qquad \Re e\{s\} > -5.$$

and

$$e^{-5t}e^{-j5t}u(t) \xleftrightarrow{\ LT\ } \frac{1}{s+5+j5}, \Re e\{s\} > -5 \cdot$$

From this we obtain

$$e^{-5t}\sin(5t)u(t) = \frac{1}{2j}\left[e^{-5t}e^{j5t} - e^{-5t}e^{-j5t}\right]u(t) \xleftrightarrow{\ LT\ } \frac{5}{(s+5)^2 + 25} \quad ,$$

where $\Re e\{s\} > -5$ .Therefore,

$$e^{-4t}u(t) + e^{-5t}\sin(5t)u(t) \xleftrightarrow{\ LT\ } \frac{s^2 + 15s + 70}{s^3 + 14s^2 + 90s + 100}, \Re e\{s\} > -5 \cdot$$

(c)The Laplace transform of $x(t)$ is

$$X(s) = \int_{-\infty}^{0} \left(e^{2t} + e^{3t}\right)e^{-st}dt$$

$$= \left[-e^{(s-2)t}/(s-2)\right]\big|_{-\infty}^{0} + \left[-e^{-(s-3)t}/(s-3)\right]\big|_{-\infty}^{0}$$

$$= \frac{1}{s-2} + \frac{1}{s-3} = \frac{2s-5}{s^2-5s+6}.$$

The region of convergence (ROC) is $\Re e\{s\} < 2$.

(d)Using an approach along the lines of part (a),we obtain

$$e^{-2t}u(t) \xleftrightarrow{LT} \frac{1}{s+2}, \Re e\{s\} > -2. \qquad \text{(S9.21-1)}$$

Using an approach along the lines of part (c) ,we obtain

$$e^{2t}u(-t) \xleftrightarrow{LT} \frac{1}{s-2}, \Re e\{s\} < 2. \qquad \text{(S9.21-2)}$$

From these we obtain

$$e^{-2|t|} = e^{-2t}u(t) + e^{2t}u(-t) \xleftrightarrow{LT} \frac{2s}{s^2-4}, \quad -2 < \Re e\{s\} < 2.$$

Using the differentiation in the s-domain property , we obtain

$$te^{-2|t|} \xleftrightarrow{LT} -\frac{d}{ds}\left[\frac{2s}{s^2-4}\right] = -\frac{2s^2+8}{\left(s^2-4\right)^2}, -2 < \Re e\{s\} < 2.$$

(e)Using the differentiation in the s-domain property on eq.(S9.21-1),we get

$$te^{-2t}u(t) \xleftrightarrow{LT} -\frac{d}{ds}\left[\frac{1}{s+2}\right] = \frac{1}{\left(s+2\right)^2}, \Re e\{s\} > -2.$$

Using the differentiation in the s-domain property on eq (S9.21-2),we get

$$-te^{2t}u(-t) \xleftrightarrow{LT} \frac{d}{ds}\left[\frac{1}{s-2}\right] = -\frac{1}{\left(s-2\right)^2}, \Re e\{s\} < 2.$$

Therefore,

$$|t|e^{-2|t|} = te^{-2t}u(t) - te^{2t}u(-t) \xleftrightarrow{LT} \frac{-4s}{\left(s+2\right)^2\left(s-2\right)^2}, -2 < \Re e\{s\} < 2.$$

(f)From the previous part ,we have

$$|t|e^{2t}u(-t) = -te^{2t}u(-t) \xleftrightarrow{LT} -\frac{1}{\left(s-2\right)^2}, \Re e\{s\} < 2.$$

(g)Note that the given signal may be written as $x(t) = u(t) - u(t-1)$ .Note that

$$u(t) \xleftrightarrow{LT} \frac{1}{s}, \Re e\{s\} > 0.$$

Using the time shifting property ,we get

$$u(t-1) \xleftrightarrow{LT} \frac{e^{-s}}{s}, \Re e\{s\} > 0.$$

Therefore , $x_1(t)$

$$u(t) - u(t-1) \xleftrightarrow{LT} \frac{1-e^{-s}}{s}, \quad \text{All } s.$$

Note that in this case ,since the signal is finite duration ,the ROC is the entire s-plane.

(h)Consider the signal $x_1(t) = t\left[u(t) - u(t-1)\right]$.Note that the signal $x(t)$ may be

expressed as $x(t) = x_1(t) + x_1(-t+2)$. We have from the previous part

$$u(t) - u(t-1) \xleftrightarrow{LT} \frac{1-e^{-s}}{s}, \qquad \text{All } s.$$

Using the differentiation in s-domain property ,we have

$$x_1(t) = t\left[u(t) - u(t-1)\right] \xleftrightarrow{LT} \frac{d}{ds}\left[\frac{1-e^{-s}}{s}\right] = \frac{se^{-s} - 1 + e^{-s}}{s^2}, \quad \text{All } s.$$

Using the time-scaling property ,we obtain

$$x_1(-t) \xleftrightarrow{LT} \frac{-se^{s} - 1 + e^{s}}{s^2}, \qquad \text{All } s.$$

Then ,using the shift property ,we have

$$x_1(-t+2) \xleftarrow{\ LT\ } e^{-2s}\frac{-se^s-1+e^s}{s^2} \quad ,\text{All } s.$$

Therefore ,

$$x(t)=x_1(t)+x_1(-t+2) \xleftarrow{\ LT\ } \frac{se^{-s}-1+e^{-s}}{s^2}+e^{-2s}\frac{-se^s-1+e^s}{s^2}, \text{ All s.}$$

(i) The Laplace transform of $x(t)=\delta(t)+u(t)$ is $X(s)=1+1/s, \Re\{s\}>0$.

(j) Note that $\delta(3t)+u(3t)=\delta(t)+u(t)$.Therefore ,the Laplace transform is the same as the result of the previous part.

9.22 (a)From Table 9.2,we have

$$x(t)=\frac{1}{3}\sin(3t)u(t).$$

(b)From Table 9.2 we know that

$$\cos(3t)u(t) \xleftarrow{\ LT\ } \frac{s}{s^2+9}, \Re\{s\}>0.$$

Using the time scaling property ,we obtain

$$\cos(3t)u(-t) \xleftarrow{\ LT\ } -\frac{s}{s^2+9}, \Re\{s\}<0$$

Therefore ,the inverse Laplace transform of $X(s)$ is

$$x(t)=-\cos(3t)u(-t).$$

(c)From Table 9.2 we know that

$$e^t\cos(3t)u(t) \xleftarrow{\ LT\ } \frac{s-1}{(s-1)^2+9}, \Re\{s\}>1.$$

Using the time scaling property ,we obtain

$$e^{-t}\cos(3t)u(-t) \xleftarrow{\ LT\ } -\frac{s+1}{(s+1)^2+9}, \Re\{s\}<-1.$$

Therefore ,the inverse Laplace transform of $X(s)$ is

$$x(t)=-e^{-t}\cos(3t)u(-t).$$

(d)Using partial fraction expansion on $X(s)$,we obtain

$$X(s)=\frac{2}{s+4}-\frac{1}{s+3}.$$

From the given ROC ,we know that $x(t)$ must be a two-sided signal .Therefore

$$x(t)=2e^{-4t}u(t)+e^{-3t}u(-t).$$

(e)Using partial fraction expansion on $X(s)$,we obtain

$$X(s)=\frac{2}{s+3}-\frac{1}{s+2}.$$

From the given ROC ,we know that $x(t)$ must be a two-sided signal ,Therefore,

$$x(t)=2e^{-3t}u(t)+e^{-3t}u(-t).$$

(f)We may rewrite $X(s)$ as

$$X(s)=1+\frac{3s}{s^2-s+1}$$

$$=1+\frac{3s}{(s-1/2)^2+(\sqrt{3}/2)^2}$$

$$=1+3\frac{s-1/2}{(s-1/2)^2+(\sqrt{3}/2)^2}+\frac{3/2}{(s-1/2)^2+(\sqrt{3}/2)^2}$$

Using Table 9.2 ,we obtain

$$x(t)=\delta(t)+3e^{-t/2}\cos(\sqrt{3}t/2)u(t)+\sqrt{3}e^{-t/2}\sin(\sqrt{3}t/2)u(t).$$

(g)We may rewrite $X(s)$ as

$$X(s)=1-\frac{3s}{(s+1)^2}.$$

From Table 9.2,we know that

$$tu(t) \xleftrightarrow{\ LT\ } \frac{1}{s^2}, \Re\{s\} > 0.$$

Using the shifting property ,we obtain

$$e^{-t}tu(t) \xleftrightarrow{\ LT\ } \frac{1}{(s+1)^2}, \Re\{s\} > -1.$$

Using the differentiation property ,

$$\frac{d}{dt}\left[e^{-t}tu(t)\right] = e^{-t}u(t) - te^{-t}u(t) \xleftrightarrow{\ LT\ } \frac{s}{(s+1)^2}, \Re\{s\} > -1.$$

Therefore,

$$x(t) = \delta(t) - 3e^{-t}u(t) - 3te^{-t}u(t).$$

9.23.The four pole-zero plots shown may have the following possible ROCs:

- Plot (a): $\Re\{s\} < -2$ or $-2 < \Re\{s\} < 2$ or $\Re\{s\} > 2$.

- Plot (b): $\Re\{s\} < -2$ or $\Re\{s\} > -2$.

- Plot (c): $\Re\{s\} < 2$ or $\Re\{s\} > 2$.

- Plot (d): Entire s-plane.

Also, suppose that the signal $x(t)$ has a Laplace transform $X(s)$ with ROC R.

(1).We know from Table 9.1 that

$$e^{-3t}x(t) \xleftrightarrow{\ LT\ } X(s+3).$$

The ROC R_1 of this new Laplace transform is R shifted by 3 to the left .If $x(t)e^{-3t}$ is absolutely

integrable, then R_1 must include the jw-axis.

- For plot (a), this is possible only if R was $\Re\{s\} > 2$.

- For plot (b), this is possible only if R was $\Re\{s\} > -2$.

- For plot (c), this is possible only if R was $\Re\{s\} > 2$.

- For plot (d), R is the entire s-plane.

(2)We know from Table 9.2 that

$$e^{-t}u(t) \xleftrightarrow{\ LT\ } \frac{1}{s+1}, \Re\{s\} > -1.$$

Also ,from Table 9.1 we obtain

$$x(t)*\left[e^{-t}u(t)\right] \xleftrightarrow{\ LT\ } \frac{X(s)}{s+1}, R_2 = R\cap\left[\Re\{s\} > -1\right]$$

If $e^{-t}u(t)*x(t)$ is absolutely integrable, then R_2 must include the jw-axis.

- For plot (a), this is possible only if R was $-2 < \Re\{s\} < 2$.

- For plot (b), this is possible only if R was $\Re\{s\} > -2$.

- For plot (c), this is possible only if R was $\Re\{s\} < 2$.

- For plot (d), R is the entire s-plane.

(3)If $x(t) = 0$ for $t > 1$,then the signal is a left-sided signal or a finite-duration signal .

- For plot (a), this is possible only if R was $\Re\{s\} < -2$.

- For plot (b), this is possible only if R was $\Re\{s\} < -2$.

- For plot (c), this is possible only if R was $\Re\{s\} < 2$.

- For plot (d), R is the entire s-plane.

(4)If $x(t) = 0$ for $t < -1$,then the signal is a right-sided signal or a finite-duration signal

- For plot (a), this is possible only if R was $\Re\{s\} > 2$.

- For plot (b), this is possible only if R was $\Re\{s\} > -2$.

- For plot (c), this is possible only if R was $\Re\{s\}>2$.
- For plot (d), R is the entire s-plane.

9.24.(a)The pole-zero diagram with the appropriate markings is shown Figure S9.24.

(b)By inspecting the pole-zero diagram of part (a), it is clear that the pole-zero diagram shown in Figure S9.24 will also result in the same $|X(jw)|$ .This would correspond to the Laplace transform

$$X_1(s)=s-\frac{1}{2}, \quad \Re\{s\}<\frac{1}{2}.$$

(c)$\measuredangle X(jw)=\pi-\measuredangle X_1(jw)$.

(d) $X_2(s)$ with the pole-zero diagram shown below in Figure S9.24 would have the property that $\measuredangle X_2(jw)=\measuredangle X(jw)$.Here , $X_2(s)=\dfrac{-1}{s-1/2}$.

(e) $|X_2(jw)|=1/|X(jw)|$.

(f)From the result of part (b),it is clear that $X_1(s)$ may be obtained by reflecting the poles and zeros in the right-half of the s-plane to the left-half of the s-plane .Therefore,

$$X_1(s)=\frac{s+1/2}{s+2}.$$

From part (d),it is clear that $X_2(s)$ may be obtained by reflecting the poles (zeros) in the right-half of the s-plane to the left-half and simultaneously changing them to zeros (poles).Therefore,

$$X_2(s)=\frac{(s+1)^2}{(s+1/2)(s+2)}$$

9.25.The plots are as shown in Figure S9.25.

9.26.From Table 9.2 we have

$$x_1(t)=e^{-2t}u(t)\xleftrightarrow{LT}X_1(s)=\frac{1}{s+2},\Re\{s\}>-2$$

and

$$x_1(t)=e^{-3t}u(t)\xleftrightarrow{LT}X_1(s)=\frac{1}{s+3},\Re\{s\}>-3.$$

Using the time-shifting time-scaling properties from Table 9.1,we obtain

$$x_1(t-2)\xleftrightarrow{LT}e^{-2s}X_1(s)=\frac{e^{-2s}}{s+2},\Re\{s\}>-2$$

and

$$x_2(-t+3)\xleftrightarrow{LT}e^{-3s}X_2(-s)=\frac{e^{-3s}}{3-s},\Re\{s\}>-3.$$

Therefore, using the convolution property we obtain

$$y(t)=x_1(t-2)*x_2(-t+3)\xleftrightarrow{LT}Y(s)=\left[\frac{e^{-2s}}{s+2}\right]\left[\frac{e^{-3s}}{3-s}\right].$$

9.27.From clues 1 and 2,we know that $X(s)$ is of the form

$$X(s)=\frac{A}{(s+a)(s+b)}.$$

Furthermore , we are given that one of the poles of $X(s)$ is $-1+j$ .Since $x(t)$ is real, the poles of $X(s)$ must occur in conjugate reciprocal pairs .Therefore, $a=1-j$ and $b=1+j$ and

$$H(s)=\frac{A}{(s+1-j)(s+1+j)}.$$

From clue 5,we know that $X(0)=8$ .Therefore, we may deduce that $A=16$ and

$$H(s)=\frac{16}{s^2+2s+2}.$$

Let R denote the ROC of $X(s)$ .From the pole locations we know that there are two possible choices of R . R may either be $\Re\{s\}<-1$ or $\Re\{s\}>-1$ .We will now use clue 4 to pick one .Note that

$$y(t) = e^{2t} x(t) \xleftarrow{\quad LT \quad} Y(s) = X(s-2).$$

The ROC of $Y(s)$ is R shifted by 2 to the right .Since it is given that $y(t)$ is not absolutely integrable ,the ROC of $Y(s)$ should not include the $jw - axis$ .This is possible only of R is $\Re e\{s\} > -1$.

9.28.(a) The possible ROCs are

(i) $\Re e\{s\} < -2$.

(ii) $-2 < \Re e\{s\} < -1$.

(iii) $-1 < \Re e\{s\} < 1$.

(iv) $\Re e\{s\} > 1$.

(b)(i)Unstable and anticausal.

(ii) Unstable and non causal.

(iii) Stable and non causal.

(iv) Unstable and causal.

9.29.(a)Using Table 9.2,we obtain

$$X(s) = \frac{1}{s+1}, \Re e\{s\} > -1$$

and

$$H(s) = \frac{1}{s+2}, \Re e\{s\} > -2.$$

(b) Since $y(t) = x(t) * h(t)$,we may use the convolution property to obtain

$$Y(s) = X(s)H(s) = \frac{1}{(s+1)(s+2)}.$$

The ROC of $Y(s)$ is $\Re e\{s\} > -1$.

(c) Performing partial fraction expansion on $Y(s)$,we obtain

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+2}.$$

Taking the inverse Laplace transform, we get

$$y(t) = e^{-t} u(t) - e^{-2t} u(t).$$

(d)Explicit convolution of $x(t)$ and $h(t)$ gives us

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau \\ &= \int_{0}^{\infty} e^{-2\tau} e^{-(t-\tau)} u(t-\tau) d\tau \\ &= e^{-t} \int_{0}^{t} e^{-\tau} d\tau \quad \text{for } t > 0 \\ &= \left[e^{-t} - e^{-2t} \right] u(t). \end{aligned}$$

9.30.For the input $x(t) = u(t)$, the Laplace transform is

$$X(s) = \frac{1}{s}, \Re e\{s\} > 0.$$

The corresponding output $y(t) = \left[1 - e^{-t} - te^{-t} \right] u(t)$ has the Laplace transform

$$Y(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2} = \frac{1}{s(s+1)^2}, \Re e\{s\} > 0.$$

Therefore,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s+1)^2}, \Re e\{s\} > 0.$$

Now ,the output $y_1(t) = \left[2 - 3e^{-t} + e^{-3t} \right] u(t)$ has the Laplace transform

$$Y_1(s) = \frac{2}{s} - \frac{3}{s+1} + \frac{1}{s+3} = \frac{6}{s(s+1)(s+3)}, \Re e\{s\} > 0.$$

Therefore , the Laplace transform of the corresponding input will be

$$X_1(s) = \frac{Y_1(s)}{H(s)} = \frac{6(s+1)}{s(s+3)}, \Re e\{s\} > 0.$$

Taking the inverse Laplace transform of the partial fraction expansion of $X_1(s)$, we obtain

$x_1(t) = 2u(t) + 4e^{-3t}u(t)$.

9.31.(a).Taking the Laplace transform of both sides of the given differential equation and simplifying, we obtain

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2}.$$

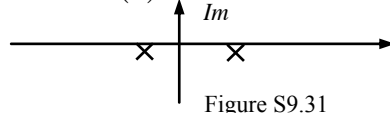The pole-zero plot for $H(s)$ is as shown in figure S9.31.

Figure S9.31

b).The partial fraction expansion of $H(s)$ is

$$H(s) = \frac{1/3}{s-2} - \frac{1/3}{s+1}.$$

(i).If the system is stable ,the ROC for $H(s)$ has to be $-1 < \Re e\{s\} < 2$. Therefore

$$h(t) = -\frac{1}{3}e^{2t}u(-t) - \frac{1}{3}e^{-t}u(t).$$

(ii).If the system is causal, the ROC for $H(s)$ has to be $\Re e\{s\} > 2$ .Therefore

$$h(t) = \frac{1}{3}e^{2t}u(t) - \frac{1}{3}e^{-t}u(t).$$

(iii)If the system is neither stable nor causal ,the ROC for $H(s)$ has to be $\Re e\{s\} < -1$.

Therefore ,

$$h(t) = -\frac{1}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(-t)$$

9.32. If $x(t) = e^{2t}$ produces $y(t) = (1/6)e^{2t}$,then $H(2) = (1/6)$. Also, by taking the Laplace transform of both sides of the given differential equation we get

$$H(s) = \frac{s + b(s+4)}{s(s+4)(s+2)}.$$

Since $H(2) = 1/6$,we may deduce that $b = 1$ .Therefore

$$H(s) = \frac{2(s+2)}{s(s+4)(s+2)} = \frac{2}{s(s+4)}.$$

9.33.Since $x(t) = e^{-|t|} = e^{-t}u(t) + e^{t}u(-t)$,

$$X(s) = \frac{1}{s+1} - \frac{1}{s-1} = \frac{-2}{(s+1)(s-1)}, \ -1 < \Re e\{s\} < 1.$$

We are also given that

$$H(s) = \frac{s+1}{s^2 + 2s + 2}.$$

Since the poles of $H(s)$ are at $-1 \pm j$, and since $h(t)$ is causal ,we may conclude that the ROC of $H(s)$ is $\Re e\{s\} > -1$.Now

$$Y(s) = H(s)X(s) = \frac{-2}{(s^2 + 2s + 2)(s-1)}.$$

The ROC of $Y(s)$ will be the intersection of the ROCs of $X(s)$ and $H(s)$.This is $-1 < \Re e\{s\} < 1$.

We may obtain the following partial fraction expansion for $Y(s)$:

$$Y(s) = -\frac{2/5}{s-1} + \frac{2s/5 + 6/5}{s^2 + 2s + 2}.$$

We may rewrite this as

$$Y(s) = -\frac{2/5}{s-1} + \frac{2}{5}\left[\frac{s+1}{(s+1)^2 + 1}\right] + \frac{4}{5}\left[\frac{1}{(s+1)^2 + 1}\right].$$

Nothing that the ROC of $Y(s)$ is $-1 < \Re e\{s\} < 1$ and using Table9.2,we obtain

$$y(t) = \frac{2}{5}e^t u(-t) + \frac{2}{5}e^{-t}\cos tu(t) + \frac{4}{5}e^{-t}\sin tu(t)$$

9.34. We know that

$$x_1(t) = u(t) \xleftarrow{LT} X_1(s) = \frac{1}{s}, \Re e\{s\} > 0$$

Therefore, $X_1(s)$ has a pole at $s = 0$. Now, the Laplace transform of the output $y_1(t)$ of the system with $x_1(t)$ as the input is

$$Y_1(s) = H(s)X_1(s)$$

Since in clue 2, $Y_1(s)$ is given to be absolutely integrable, $H(s)$ must have a zero at $s = 0$ which cancels out the pole of $X_1(s)$ at $s = 0$.

We also know that

$$x_2(t) = tu(t) \xleftarrow{LT} X_2(s) = \frac{1}{s^2}, \Re e\{s\} > 0$$

Therefore, $X_2(s)$ has two poles at $s = 0$. Now, the Laplace transform of the output $y_2(t)$ of the system with $x_2(t)$ as the input is

$$Y_2(s) = H(s)X_2(s)$$

Since in clue 3, $Y_2(s)$ is given to be not absolutely integrable, $H(s)$ does not have two zeros at $s = 0$. Therefore, we conclude that $H(s)$ has exactly one zero at $s = 0$.

From clue 4 we know that the signal

$$p(t) = \frac{d^2h(t)}{dt^2} + 2\frac{dh(t)}{dt} + 2h(t)$$

is finite duration. Taking the Laplace transform of both sides of the above equation, we get

$$P(s) = s^2 H(s) + 2sH(s) + 2H(s).$$

Therefore,

$$H(s) = \frac{P(s)}{s^2 + 2s + 2}.$$

Since $p(t)$ is of finite duration, we know that $P(s)$ will have no poles in the finite s-plane. Therefore, $H(s)$ is of the form

$$H(s) = \frac{A\prod_{i=1}^{N}(s - z_i)}{s^2 + 2s + 2},$$

where $z_i, i = 1, 2, ..., N$ represent the zeros of $P(s)$. Here, A is some constant.

From clue 5 we know that the denominator polynomial of $H(s)$ has to have a degree which is exactly one greater than the degree of the numerator polynomial. Therefore,

$$H(s) = \frac{A(s - s_1)}{s^2 + 2s + 2}.$$

Since we already know that $H(s)$ has a zero at $s = 0$, we may rewrite this as $H(s) = \frac{As}{s^2 + 2s + 2}$

From clue 1 we know that $H(1)$ is 0.2. From this, we may easily show that $A = 1$. Therefore,

$$H(s) = \frac{s}{s^2 + 2s + 2}.$$

Since the poles of $H(s)$ are at $-1 \pm j$ and since $h(t)$ is causal and stable, the ROC of $H(s)$ is $\Re e\{s\} > -1$.

9.35. (a) We may redraw the given block diagram as shown in Figure S9.35.

From the figure, it is clear that

$$\frac{F(s)}{s} = Y_1(s).$$

Therefore, $f(t) = dy_1(t)/dt$. Similarly, $e(t) = df(t)/dt$. Therefore, $e(t) = d^2 y_1(t)/dt^2$.

From the block diagram it is clear that

$$y(t) = e(t) - f(t) - 6y_1(t) = \frac{d^2 y_1(t)}{dt^2} - \frac{dy_1(t)}{dt} - 6y_1(t).$$

Therefore

$$Y(s) = s^2 Y_1(s) - s Y_1(s) - 6 Y_1(s).$$

Now ,let us determine the relationship between $y_1(t)$ and $x(t)$.This may be done by concentrating on the lower half of the above figure .We redraw this in Figure S9.35.

From Example 9.30,it is clear that $y_1(t)$ and $x(t)$ must be related by the following differential equation :

$$\frac{d^2 y_1(t)}{dt^2} + 2\frac{dy_1(t)}{dt} + y_1(t) = x(t).$$

Therefore,

$$Y_1(s) = \frac{X(s)}{s^2 + 2s + 1}.$$

Using this in conjunction with eq (S9.35-1), we get

$$Y(s) = \frac{s^2 - s - 6}{s^2 + 2s + 1} X(s).$$

Taking the inverse Laplace transform ,we obtain

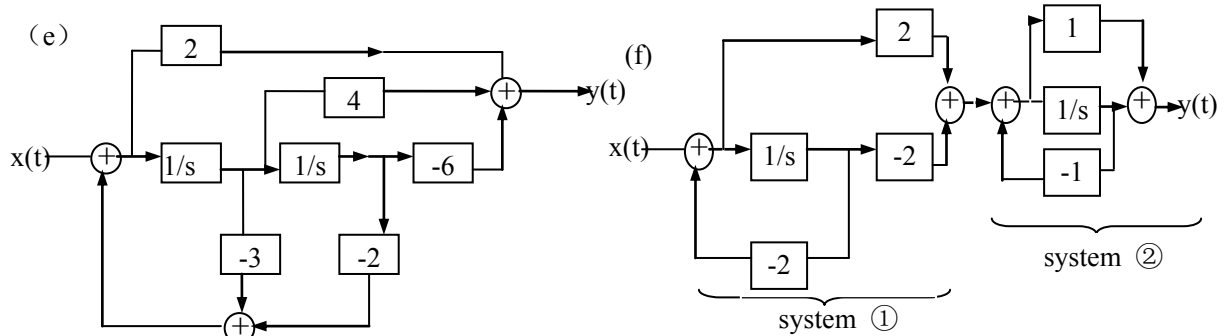$$\frac{d^2 y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \frac{d^2 x(t)}{dt^2} - \frac{dx(t)}{dt} - 6x(t).$$
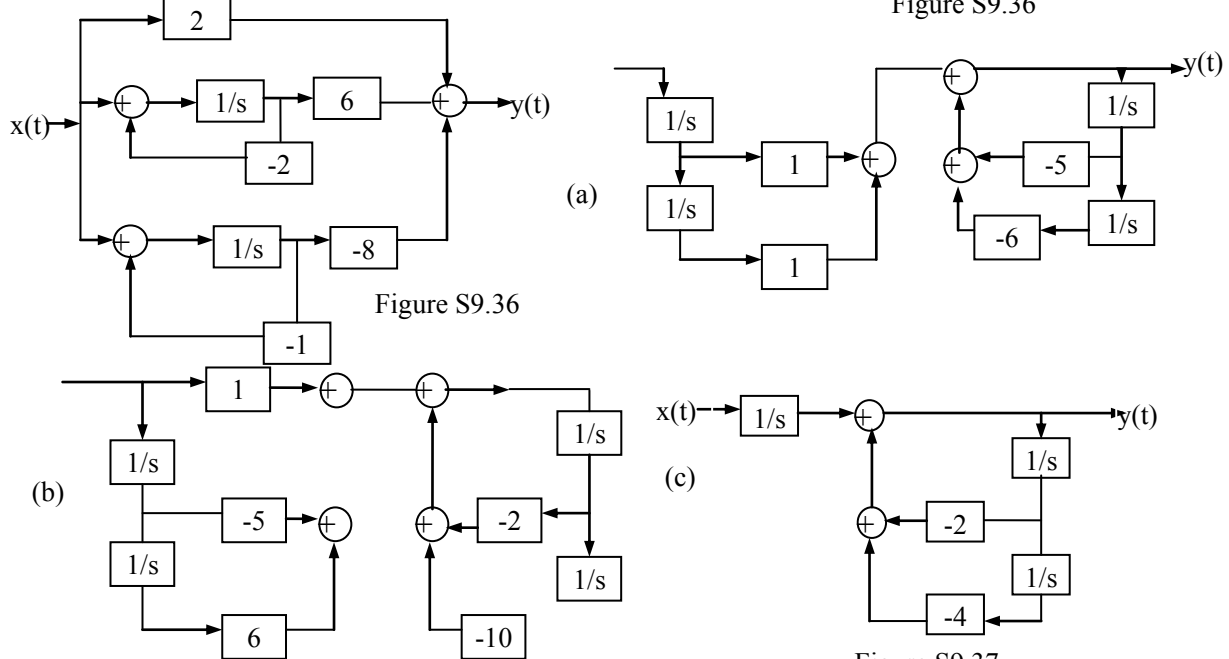

Figure S9.36

Figure S9.36

Figure S9.37

（b）The two poles of the system are at -1.Since the system is causal; the ROC must be to the right of s= - 1.Therefore, the ROC must include the $j\omega$ -axis. Hence, the system is stable.

9.36. (a) We know that $Y_1(s)$ and $Y(s)$ are related by

$$Y(s) = (2s^2 + 4s - 6) Y_1(s).$$

taking the inverse Laplace transform, we get

$$y(t) = 2\frac{d^2 y_1(t)}{dt^2} + 4\frac{dy_1(t)}{dt} - 6 y_1(t).$$

(b) Since $Y_1(s) = F(s)/s, f(t) = \frac{dy_1(t)}{dt}.$

(c) Since $F(s)=_{E(s)/s,e(t)=\frac{df(t)}{dt}=\frac{d^2y_1(t)}{dt^2}}$.

(d) From part (a), $y(t)=2e(t)+4f(t)-6y_1(t)$.

(e) The extended block diagram is as shown in Figure S9.36.

(f) The block diagram is as shown in Figure S9.36.

(g) The block diagram is as shown in Figure S9.36.

     The three subsystems may be connected in parallel as show in the figure above to obtain the overall system.

9.37. The block diagrams are shown in Figure S9.37.

9.38 (a) We may rewrite $H(s)$ as

$$H(s)=[\frac{1}{s+1}][\frac{1}{s+1}][\frac{1}{s-\frac{1}{2}+\frac{j\sqrt{3}}{2}}][\frac{1}{s-\frac{1}{2}-\frac{j\sqrt{3}}{2}}].$$

$H(s)$ clearly may be treated as the cascade combination of four first order subsystems. consider one of there subsystems with the system function

$$H_1(s)=[\frac{1}{s-\frac{1}{2}-\frac{j\sqrt{3}}{2}}]$$

the block diagram for this is as show in Figure S9.38.Clearly, it contains multiplications with coefficients that are not real.

(b) We may write $H(s)$ as

$$H(s)=[\frac{1}{s^2+2s+1}][\frac{1}{s^2-s+1}]=H_1(s)H_2(s).$$

the block diagram for $H(s)$ may be constructed as a cascade of the block diagrams of $H_1(s)$ and $H_2(s)$ as show in Figure S9.38.
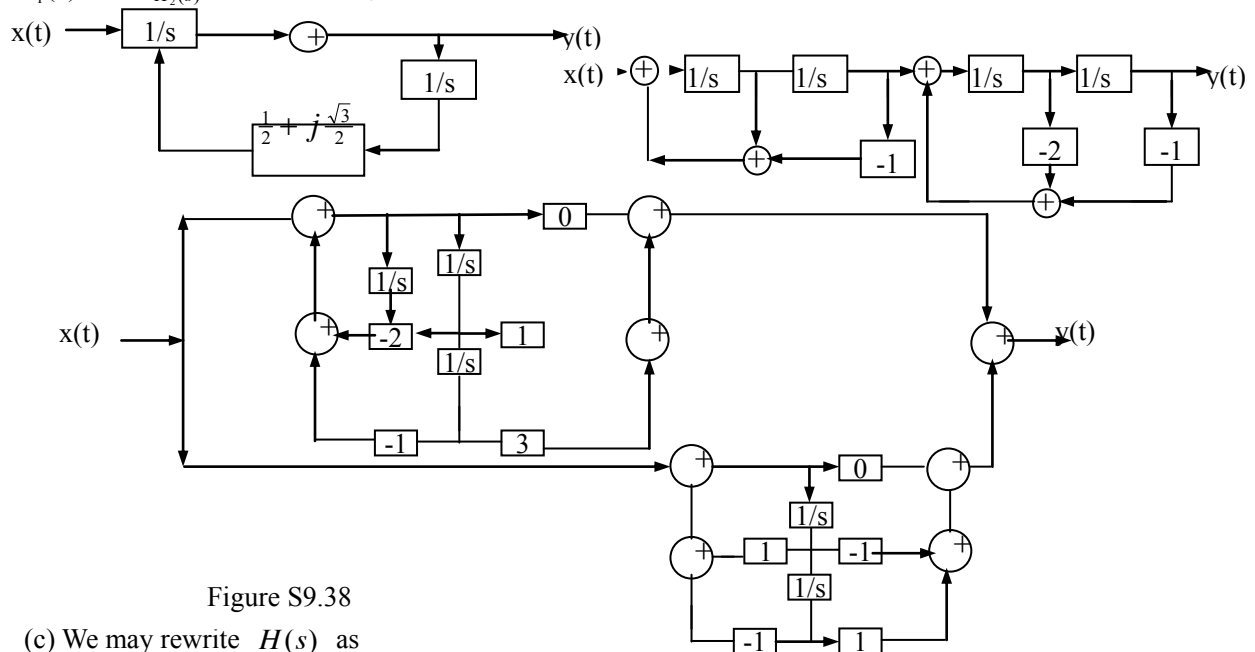

Figure S9.38

(c) We may rewrite $H(s)$ as

$$H(s)=\frac{1}{3}[\frac{s+3}{s^2+2s+1}]+\frac{1}{3}[\frac{1-s}{s^2-s+1}]=H_3(s)+H_4(s).$$

    The block diagram for $H(s)$ may be constructed as a parallel combination of the block Diagrams of $H_3(s)$ and $H_4(s)$.as show in Figure S9.38.

9.39. (a) For $x_1(t)$, the unilateral and bilateral Laplace transforms are identical

$$X_1(s)=x_1(s)=\frac{1}{s+2},\Re e\{s\}>-2.$$

   (b) Here, using Table9.2 and time shifting property we get

$$X_2(s)=\frac{e^s}{s+2},\Re e\{s\}>-3.$$

    The unilateral Laplace transform is

$$X_2(s) = e^{-3} \frac{1}{s+3}, \Re\{s\} > -3 \cdot$$

(c) We have

$$G(s)=$$
$$X_1(s)X_2(s) = \frac{e^s}{(s+2)(s+3)} = e^s[\frac{1}{s+2} - \frac{1}{s+3}]$$

Taking the inverse unilateral transform, we obtain

$$r(t) = e^{-2t-3}u(t) - e^{-3(t+1)}u(t).$$

clearly, $r(t) \neq g(t)$ for t>0^-.

9.40. Taking the inverse unilateral transform of both sides of the given differential equation, we get

$$s^3 y(s) - s^2 y(0^-) - sy'(0^-) - y''(0^-) + 6s^2 y(s) - 6sy(0^-)$$
$$- 6y(0^-) + 11sy(s) - 11y(0^-) + 6y(s) = x(s)$$

(S9.40-1)

(a) For the zero state response, assume that all the initial conditions are zero. Furthermore, Form the given x(t) we may determine

$$x(s) = \frac{1}{s+4}, \qquad \Re\{s\} > -4.$$

Form eq.(S9.40-1), we get

$$y(s)[s^3 + 6s^2 + 11s + 6] = \frac{1}{s+4} \cdot$$

Therefore,

$$y(s) = \frac{1}{(s+4)(s^3 + 6s^2 + 11s + 6)}$$

taking the inverse unilateral Laplace transform of the partial fraction expansion of the above equation, we get

$$y(t) = \frac{1}{6}e^{-t}u(t) - \frac{1}{6}e^{-4t}u(t) + \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}e^{-3t}u(t)$$

(b) For the zero-input response, we assume that X(s)=0 .Assuming that the initial conditions are as given, we obtain from (s9.40-1)

$$y(s) = \frac{s^2 + 5s + 6}{s^3 + 6s^2 + 11s + 6} = \frac{1}{s+1}$$

Taking the inverse unilateral Laplace transform of the above equation, we get

$$y(t) = e^{-t}u(t)$$

(c) the total response is the sum of the zero-state and aero-input responses.

$$y(t) = \frac{7}{6}e^{-t}u(t) - \frac{1}{6}e^{-4t}u(t) + \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}e^{-3t}u(t)$$

9.41. let us first find the Laplace transform of the signal $y(t) = x(-t)$ .We have

$$Y(s) = \int_{-\infty}^{+\infty} x(-t)e^{-st}dt$$
$$= \int_{-\infty}^{+\infty} x(t)e^{st}dt$$
$$= X(-s)$$

(a) Since $x(t) = x(-t)$ for an even signal, we can conclude that LT{x(t)}=LT{x(-t)} therefore, X(s)=X(-s).

(b) Since $x(t) = -x(-t)$ for an odd signal , we can conclude that $LT\{x(t)\} = -LT\{x(-t)\}$ there,

(b) First of all note that for a signal to be even, it must be either two-sided or finite duration. therefore, if has poles, the ROC must be a strip in the s-plane.

(c) Form plot (a),we get

$$x(s) = \frac{As}{(s+1)(s-1)} \cdot$$

Therefore,

$$x(-s) = \frac{-As}{(s+1)(s-1)} = -x(s).$$

Therefore, x(t) is not even (in fact it is odd).

For plot (b), we note that the ROC cannot be chosen to correspond to a two-sided function x(t).therefore, this signal is not even.

Form plot (c), we get

$$x(s) = \frac{A(s-j)(s+j)}{(s+1)(s-1)} = \frac{A(s^2+1)}{s^2-1}$$

Therefore,

$$x(-s) = \frac{A(s^2+1)}{s^2-1} = x(s)$$

Therefore, x(t) is even provided the ROC is chosen to be –1<

For plot (d), we note that the ROC cannot be chosen to correspond to a two-sided function x(t). Therefore, this signal is not even.

9.42. (a) form table 9.2 we know that Laplace transform of $t^2 u(t)$ is $1/s^2$ with the ROC $\Re e\{s\} > 0$ .therefore, the given statement is false.

(b) We know that the Laplace transform of a signal x(t) is the same as the Fourier transform of the signal $x(t)e^{-\sigma t}$ .the ROC is given by the range of σ for which this Fourier transform exists.

Now, if x(t)= $e^{t^2}u(t)$, the we note that as $t \to \infty$,the signal x(t) becomes unbounded. Therefore, for the Fourier transform of of $e^{-\sigma t}x(t)$ to exist, we need to find a range of σ which ensure that $e^{-\sigma t}x(t)$ is bounded as $t \to \infty$. Clearly , this is not possible. Therefore , the given statement is true.

(c) this statement is true. Consider the signal $x(t) = e^{j\omega_0 t}$. Then

$$X(s) = \int_{-\infty}^{+\infty} e^{j\omega_0 t} e^{-st} dt = \frac{e^{t(j\omega_0 - s)}}{j\omega_0 - s} \Big|_{-\infty}^{+\infty}$$

this integral dose not converge for any vale of s.

(d) this statement is false. Consider the signal $x(t) = e^{j\omega_0 t} u(t)$. Then

$$X(s) = \int_0^{\infty} e^{j\omega_0 t} e^{-st} dt = \frac{e^{t(j\omega_0 - s)}}{j\omega_0 - s} \Big|_0^{+\infty}$$

this integral converges for any value of s>0.

(e) this statement is false. Consider the signal $x(t) = |t|$ .Then

$$X(s) = \int_0^{\infty} te^{-st} dt + \int_{-\infty}^0 -te^{-st} dt$$

both integrals on the right-hand side converge for any value of s>0.

9.43. we are given that h(t) is causal and stable .therefore ,all poles are in the left half of the s-plane .

(a) note that

$$g(t) = \frac{dh(t)}{dt} \xleftrightarrow{L} G(s) = sH(s)$$

now, G(s) has the same poles as H(s) and hence the ROC for G(s) remains the same therefore, g(t) is also guaranteed to be causal and stable.

(b) note that

$$r(t) = \int_{-\infty}^t h(\tau) d\tau \xleftrightarrow{L} R(s) = \frac{H(s)}{s}$$

note that R(s) dose not have a pole at s=0 only if H(s) has a zero at s=0. therefore, we cannot guarantee that r(t) is always causal and unstable.

9.44. (a) Note that

$$\delta(t - nT) \xleftrightarrow{c} e^{-snT} , \quad All \ s.$$

Therefore,

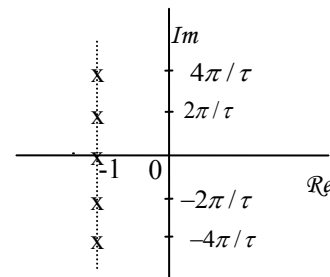$$X(s) = \sum_{n=0}^{\infty} e^{-nT} e^{-snT} = \frac{1}{1 - e^{-T(1+s)}} .$$

Figure S9.44

In order to determine the ROC, let us first find the poles of X(s) . Clearly , the poles occur when

This implies that the poles satisfy the following equation :

$$e^{-T(1+s_k)} = e^{jk2\pi}, k = 0, \pm 1, \pm 2, \cdots.$$

Taking the logarithm of both side of the above equation and simplifying , we get

$$s_k = -1 + \frac{jk2\pi}{T}, \qquad k = 0, \pm 1, \pm 2, \cdots.$$

Therefore , the poles all lie on a vertical line (parallel to the jw-axis) passing though s = -1 . Since the signal is right-sided , the ROC is Re{s}>-1.

(c) The pole-zero poles is as shown in Figure S9.44 .

(c) The magnitude of the Fourier transform $X(jw)$ is given by the product of the reciprocal of the

lengths of the vectors from the poles to the point jw .The phase of $X(jw)$ is given by the negative of the sum of the angles of these vectors . Clearly from the pole-zero plot above it is clear that both the magnitude and phase have to vary periodically with a period of $2\pi/T$.

9.45 . (a) Taking the Laplace transform of the signal x(t) , we get

$$Y(s) = \frac{2/3}{s-2} + \frac{1/3}{s+1} = \frac{s}{(s-2)(s-1)}.$$

The ROC $-1 < \Re e\{s\} < 2$ Also , note that since x(t) is a left-sided signal , the ROC

for *X(s)* is $\Re e\{s\} < 2$

Now ,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s}{(s+2)(s+1)}$$

We know that the ROC of *Y(s)* has to be the intersection of the ROCs of *X(s)* and

H(s) . This leads us to conclude that the ROC of *H(s)* is $\Re e\{s\} > -1$.

(b) The partial fraction expansion of *H(s)* is

$$H(S) = \frac{2}{S+2} - \frac{1}{S+1}$$

Therefore,

$$h(t) = 2e^{-2t}u(t) - e^{-t}u(t).$$

(c) e^{3t} is an Eigen function of the LTI system . Therefore ,

$$y(t) = H(3)e^{3t} = \frac{3}{20}e^{3t}$$

9.46 . Since *y(t)* is real , the third input must be of the form $e^{s_0 t}$ Since *x(t)* is of the form

$\delta(t) + e^{s_0 t} + e^{s_0^* t}$ and the output is $y(t) = -6e^{-t}u(t) + \frac{4}{34}e^{4t}\cos(3t) + \frac{18}{34}e^{4t}\sin(3t)$, we may conclude that $H(4 \pm 3j) = \frac{4}{34} \pm j\frac{18}{34}$

Let us try $h(t) = \delta(t) - 6e^{-t}u(t)$ then

$$H(s) = \frac{s-5}{s+1}$$

We may easily show that $H(4 \pm 3j) = \frac{4}{34} \pm j\frac{18}{34}$ Therefore , *H(s)* as given above is consistent with the given information .

9.47 . (a) Taking the Laplace transform of *y(t)* , we obtain

$$Y(s) = \frac{1}{s+2} \qquad \Re e\{s\} > -2$$

Therefore ,

$$X(s) = \frac{Y(s)}{H(s)} = \frac{s+1}{(s-1)(s+2)}$$

The pole-zero diagram for *X(s)* is as shown in Figure S9.47 . Now , the ROC of *H(s)* is $\Re e\{s\} > -1$.

We know that ROC of *Y(s)* is at lest the intersection of the ROCs of *X(s)* and *H(s)* . Note that the ROC can be larger if some poles are canceled out by zeros at the same location . In this case , we can choose the ROC of *X(s)* to be either -2< $\Re e\{s\} < 1$ or $\Re e\{s\} > 1$. In both cases , we get the same ROC of *Y(s)* because the poles at *s* = -1 and *s* =1 in *H(s)* and *X(s)* , respective are canceled out by zeros. The partial fraction expansion of *X(s)* is

$$X(s) = \frac{2/3}{(s-1)} + \frac{1/3}{(s-2)}$$

Taking the ROC of *X(s)* to be $-2 < \Re e\{s\} < 1$, we get

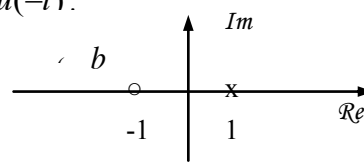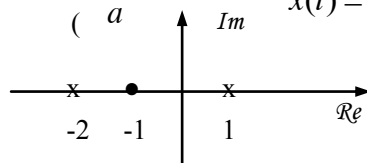$$x(t) = -\frac{2}{3}e^{t}u(-t) + \frac{1}{3}e^{-2t}u(-t).$$

Figure S.9.47

Taking the ROC of *X(s)* to be $\Re e\{s\} > 1$, we get

$$x(t) = \frac{2}{3}e^{t}u(-t) + \frac{1}{3}e^{-2t}u(t)$$

(b) Since it is given that *x(t)* is absolutely integrable , we can conclude that the ROC of *X(s)* must include the *jw*-axis. Therefore, the first choice of *x(t)* given above is the one we want.

(c) We need to first find a *H(s)* such that *H(s)Y(s)=X(s)*. Clearly,

$$H(s) = \frac{X(s)}{Y(s)} = \frac{s+1}{s-1}.$$

The pole-zero plot for *H(s)* is as shown in Figure S9.47. Since *h(t)* is given to be stable, the ROC of *H(s)* has to be . The partial fraction expansion of *H(s)* is

$$H(s) = 1 + \frac{2}{s-1}.$$

Therefore,

$$h(t) = \delta(t) - 2e^{-t}u(-t).$$

Also, *Y(s)* has the ROC . Therefore, *X(s)* must have the ROC $-2 < \Re e\{s\} < 1$ (the intersection of the ROCs of *Y(s)* and *H(s)* . From this we get (as shown in part (a))

$$x(t) = -\tfrac{2}{3}e^{t}u(-t) + \tfrac{1}{3}e^{-2t}u(t).$$

Verification: Now,

$$h(t) * y(t) = \left[\delta(t) - 2e^{-t}u(t)\right] * \left[e^{-2t}u(t)\right]$$

$$= e^{-2t}u(t) - 2\int_0^\infty e^{-2\tau}e^{t-\tau}u(\tau - t)d\tau$$

For t>0, the integral in the above equation is

$$e^t \int_t^\infty e^{-3\tau}d\tau = \tfrac{1}{3}e^{-2t}.$$

For t<0, the integral in the above equation is

$$e^t \int_0^\infty e^{-3\tau}d\tau = \tfrac{1}{3}e^t.$$

Therefore,

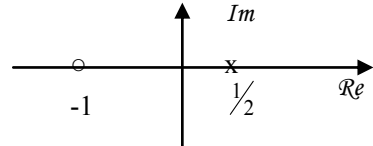$$h(t) * y(t) = -\tfrac{2}{3}e^{t}u(-t) + \tfrac{1}{3}e^{-2t}u(t) = x(t).$$

9.48. (a) $H_1(s) = 1/H(s)$

(b) Form the above relationship it is clear that the poles of the inverse system will be the zeros of original system. Also, the zeros of the inverse system will be the poles of the original system.

Therefore, the pole-zero plot for $H_1(s)$ is as sketched in Figure S9.48.

Figure S.9.48

9.49. If a system is causal and stable, then the poles of its transfer function must all be in the left half of the s-plane. This is because the ROC of a causal system is to the right of the right-most pole. For the ROC to contain the jw-axis, the right-more pole must be in the left-half of the s-plane.

Now, if the inverse system is also causal and stable, then its poles must also all lie in the left half of the s-plane. But we know that the poles of the inverse system are the zeros of the original system. Therefore, the zeros of the original system must also lie in the left-half of the s-plane.

9.50. (a) False. Counter-example:

(b) True. If the system function has more poles than zeros, then h(t) does not have an impulse at t=0. Since we know that h(t) is the derivative of the step response, we may conclude that the step response has no discontinuities at t=0.

(c) False . Causality plays no part in the argument of part (b)

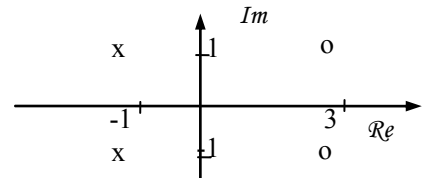(d) False . Counter-example : *H(s)= (s -1)/(s+2)* , *R(e) > -2* .

Figure S9.51

9.51 . Since h(t) is real , its poles and zeros must occur in complex conjugate pairs . Therefore, the known poles and zeros of *H(s)* are as shown in Figure S9.51 . Since *H(S)* has exactly 2 zeros at infinity , *H(s)* has *at least* two more unknown finite poles. In case *H(s)* has more than 4 poles , then it will have a zero at some location for every additional pole . Furthermore , since *h(t)* is causal and stable, all poles of *H(s)* must lie in the left half of the s-plane and the ROC must include the jw-axis .

(a) True. Consider

$$g(t) = h(t)\,e^{-3t} \xleftrightarrow{c} G(t) = H(s+3).$$

The ROC of G(s) will be. the ROC of H(s) shifted by 3 to the left . Clearly this ROC will still include the jw-axis. Therefore , *g(t)* has to be stable .

(b) Insufficient information. As mentioned earlier, H(s) has some unknown poles . So we do not know which the rightmost pole is in H(s) . Therefore, we cannot determine what its exact ROC is .

(c) True, Since H(s) is rational, H(s) may be expressed as a ratio of two polynomials in s. Furthermore,

since h(t) is real, the coefficients of these polynomials will ba real . Now, $H(s) = \frac{Y(s)}{X(s)} = \frac{P(s)}{Q(s)}$.

Here, *P(s)* and *Q(s)* are polynomial in s. The differential equation relating *x(t)* and y(t) is obtained by taking the inverse Laplace transform of *Y(s)Q(s)=X(s)P(s)*.Clearly, this differential equation has to have only real coefficients.

(d) False. We are given that H(s) has 2 zeros at $s = \infty$, Therefore, $\lim_{s \to \infty} H(s) = 0$.

(e) True. See the reasoning at the beginning of the problem.

(f) Insufficient information. H(s) may have other zeros. See reasoning at the beginning of the problem.

(g) False. We know that $e^{3t} \sin(t) = (1/2j)e^{(3+j)t} - (1/2j)e^{(3-j)t}$ .Both $e^{(3+j)t}$ and $e^{(3-j)t}$

are Eigen functions of the LTI system. Therefore, the response of the system to these exponentials is $H(s+j)e^{(3+j)t}$ and $H(s-j)e^{(3-j)t}$, respectively . Since H(s) has

zeros at $3 \pm j$, we know that the output of the system to the two exponentials has to be zero. Hence,

the response of the system to $e^{3t} \sin(t)$ has to be zero.

9.52. (a) Consider the signal y(t) = x(t-t_0). Now,

$$Y(s) = \int_{-\infty}^{\infty} x(t - t_0)e^{-st}$$

Replacing t-t_0 by τ , we get

$$Y(s) = \int_{-\infty}^{\infty} x(\tau)e^{-s(\tau+t_0)}d\tau$$

$$= e^{-st_0}\int_{-\infty}^{\infty} x(\tau)e^{-s\tau}d\tau$$

$$= e^{-st_0} X(s)$$

This obvious converges when *X(s)* converges because e^{-st_0} has no poles. Therefore, the ROC of *Y(s)* is the same as the ROC of X(s).

(b) Consider the signal $y(t) = e^{st_0} x(t)$. Now,

$$Y(s) = \int_{-\infty}^{\infty} x(\tau)e^{s_0 t}e^{-st}dt$$

$$= \int_{-\infty}^{\infty} x(\tau)e^{-(s-s_0)t}dt$$

$$= X(s - s_0)$$

If X(s) converges in the range α<Re{s}<b, then $X(s-s_0)$ converges in the range $\alpha + s_0 < s < b + s_0$. This is the ROC of *Y(s)*.

(c) Consider the signal *y(t)=x(a t)*. Now,

$$Y(s) = \int_{-\infty}^{\infty} x(\alpha t)e^{-st}dt$$

Replacing at by τ and assuming that a >1, we get

$$Y(s) = (1/a)\int_{-\infty}^{\infty} x(at)e^{-s(\tau/a)}d\tau$$

$$= (1/a)X(s/a)$$

If a<0, then

$$Y(s) = -(1/a)\int_{-\infty}^{\infty} x(at)e^{-s(\tau/a)}d\tau$$

$$= -(1/a)X(s/a)$$

Therefore,

$$Y(s) = \tfrac{1}{|a|} X(\tfrac{s}{a})$$

Chapter 10 Answers

10.1 (a)The given summation may be written as

$$\sum_{n=-1}^{\infty} \frac{1}{2}(\frac{1}{2}r^{-1})ne^{-jwn}$$,by replacing z with re^{jw}. If r$<\frac{1}{2}$,then $\frac{1}{2}r^{-1} > 1$

And the function within the summation grows towards infinity with increasing n.. Also , the summation dose not converge.

But if r$>\frac{1}{2}$,then the summation converges.

(b) The given summation may be written as

$$\sum_{n=-1}^{\infty} \frac{1}{2}(\frac{1}{2}r^{-1})ne^{-jwn}$$, by replacing z with re^{jw}. If r$>\frac{1}{2}$, then $2r > 1$

And the function within the summation grows towards infinity with increasing n ,. Also , the summation dose not converge.

But if r$< \frac{1}{2}$, then the summation converges.

(c)The summation may be written as $$\sum_{n=0}^{\infty} \frac{r^{-n} + (-r)^{-n}}{2}e^{-jwn}$$. by replacing z with re^{jw}. If r>1, then the

function within the summation grows towards infinity with increasing n ,. Also , the summation dose not converge.

But if r< 1, then the summation converges.

(a) The summation may be written as $$\sum_{n=-\infty}^{\infty}(\frac{1}{2}r^{-1})^n \cos(\pi n/4)e^{-jwn} + \sum_{n=-\infty}^{0}(\frac{1}{2}r)^{-n}\cos(\pi n/4)e^{-jwn}$$

by replacing z with re^{jw}. The first summation converges for r$>\frac{1}{2}$. The second summation

converge for r$>$2. Therefore, the sum of these two summations converges for 1/2$<$r$<$2 .

10.2 Using eq.910.3).

$$X(z) = \sum_{n=-\infty}^{\infty}(\frac{1}{5})^n u[n-3]z^{-n} = \sum_{n=3}^{\infty}(\frac{1}{5})^n z^{-n}$$

$$= [\frac{z^{-3}}{125}]\sum_{n=0}^{\infty}(\frac{1}{5})^n z^{-n} = [\frac{z^{-3}}{125}]\frac{1}{1-\frac{1}{5}z^{-1}} \quad , \quad |z| > \frac{1}{5}$$

10.18. (a) using the analysis of example10.18,we may show that

$$H(Z) = \frac{1-6Z^{-1}+8Z^{-2}}{1-\frac{2}{3}Z^{-2}+\frac{1}{9Z^{-2}}}$$

Since h(z)=y(z)/x(z), we may write

$$Y(Z)[1-\frac{2}{3}Z^{-1}+\frac{1}{9}Z^{-2}] = X(Z)[1-6Z^{-1}8Z^{-2}]$$

Taking the inverse z-transform we obtain

$$y[n]-\frac{2}{3}y[n-1]+\frac{1}{9}[n-2] = x[n]-6x[n-1]+8x[n-2]$$

(b) H (Z) has only two poles, these are both at z=1/3. Since the system is causal, the ROC of H (Z) will be the form $|z|$>1/3. Since the ROC includes the unit circle, the system is Stable.

10.19. (a) The unilateral z-transform is

$$x(Z) = \sum_{N=0}^{\infty}(\frac{1}{4})^n u[n+5]z^{-n}$$

$$= \sum_{n=0}^{\infty}(\frac{1}{4})^n z^{-n}$$

$$\frac{1}{1-(\frac{1}{4})z^{-1}}, |z| > \frac{1}{4}$$

(b) The unilateral z-transform is

$$x(z) = \sum_{n=0}^{\infty} (\sigma[n+3] + \sigma[n] + 2^n[-n])z^{-n}$$

$$= \sum_{n=0}^{\infty} (0 + \sigma[n] + \sigma[n])z^{-n}$$

$$= 2,$$

All z

The unilateral z-transform is

$$x(z) = (\frac{1}{2})^{|n|} z^{-n}$$

$$= \sum_{n=0}^{\infty} (\frac{1}{2})^n z^{-n}$$

$$= \frac{1}{1 - \left(\frac{1}{2}\right)z^{-1}}, |z| > \frac{1}{2}$$

10.20. Applying the unilateral z-transform to given difference equation, we have

$$z^{-1} y(z) + y[-1] + 2 y(z) = y(z).$$

(a) For the zero-input response, assume that x[n]=0. Since we are given that y[-1]=2,

$$z^{-1} y(z) + y(-1) + 2 y(z) = 0 \Rightarrow y(z) = \frac{-1}{1 + (\frac{1}{2})z^{-1}}.$$

Taking the inverse unilateral z-transform,

$$Y[n] = -(-\frac{1}{2})^n \mu[n].$$

(b) For the zero-state response set y[-1]=0. Also, we have

$$x(z) = \mu z\{(\frac{1}{2})^n \mu[n] = \frac{1}{1 - \frac{1}{2} z^{-1}}, |z| > \frac{1}{2}.$$

Therefore,

$$y(z) = (\frac{1}{1 - \frac{1}{4} z^{-1}})(\frac{2}{2 + z^{-1}}).$$

We use partial fraction expansion followed by the inverse unilateral z-transform to obtain

$$y[n] = \frac{1}{3}(-\frac{1}{2})^n \mu[n] + \frac{1}{6}(\frac{1}{4})^n \mu[n].$$

(c) The total response is the sum of the zero-input response. This is

$$y[n] = -\frac{2}{3}(-\frac{1}{2})^n \mu[n] + \frac{1}{6}(\frac{1}{4})^n \mu[n].$$

10.21. the pole −zero plots are all shown in figure S10.21.

(a) For $x[n] = \sigma[n+5]$, $x(z) = z^5$, all z.

The Fourier transform exists because the ROC includes the unit circle.

(b) For x [n] $= \sigma[n-5]$,

X (z)= z^{-5}, all z expect 0.

The Fourier transform exists because the ROC includes the unit circle.

(c) For x [n]$= (-1)^n \mu[n]$,

$$X(Z) = \sum_{n=\infty}^{\infty} x[n]z^{-n}$$

$$= \sum_{n=0}^{\infty} (-1)^n z^{-n}$$

$$= 1/(1 + z^{-1}), |z| > 1$$

The Fourier transform does not exist because the ROC does not include the unit circle

(d) For x [n]$= (\frac{1}{2})^{n+1} \mu[n+3],$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$= \sum_{n=-3}^{\infty} (\frac{1}{2})^{n+1} z^{-n}$$

$$= \sum_{n=0}^{\infty} (\frac{1}{2})^{n-2} z^{-n+3}$$

$$= \frac{4z^3}{(1-(\frac{1}{2})z^{-1})}, |z| > \frac{1}{2}$$

(e) For $x[n] = (\frac{-1}{3})^n \mu[-n-2]$,

$$x(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{-2} (-1/3)^n z^{-n}$$

$$= \sum_{n=2}^{\infty} (\frac{-1}{3})^{-n} z^n$$

$$= \sum_{n=0}^{\infty} (\frac{-1}{3})^{-n-2} z^{n+2}$$

$$= 9z^2 / (1+3z), |z| < 1/3$$

$$= \frac{3z}{(1+(1/3)^{-1})}, |z| < 1/3$$

The Fourier transform does not exist because the ROC does not include the unit circle.

(f) For $x[n] = (1/4)^n \mu[-n+3]$,

$$x(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{3} (1/4)^n z^{-n}$$

$$= \sum_{n=-3}^{\infty} (1/4)^{-n} z^n$$

$$= \sum_{n=0}^{\infty} (1/4)^{-n+3} z^{n-3}$$

$$= (1/64)z^{-3} / (1-4z), |z| < 1/4$$

$$= (1/16)z^{-4} / (1-(1/4)z^{-1}), |z| < 1/4$$

The Fourier transform does not exist because the ROC includes the unit circle.

(g) Consider $x_1(z) = 2^n \mu[-n]$.

$$x_1(z) = \sum_{n=-\infty}^{\infty} x_1[n] z^{-n}$$

$$= \sum_{n=-\infty}^{0} (2)^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (2)^{-n} z^n$$

$$= 1/(1-(1/2)z), |z| < 2$$

$$= -2z^{-1} / (1-2z^{-1}), |z| < 2$$

Consider $x_2[n] = (1/4)^n \mu[n-1]$.

$$x_2(z) = \sum_{n=-\infty}^{\infty} x_2[n] z^{-n}$$

$$= \sum_{n=1}^{\infty} (1/4)^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (1/4)^{n+1} z^{-n-1}$$

$$= (z^{-1}/4)[1/(1-(1/4)z^{-1})], |z| > 1/4$$

The z-transform of the overall sequence $x[n] = x_1[n] + x_2[n]$ is

$$x(z) = -\frac{2z^{-1}}{(1-2z^{-1})} + \frac{z^{-1}/4}{1-(1/4)z^{-1}}, (1/4) < |z| < 2$$

The Fourier transform exist because the ROC include the unit circle.

(g) Consider x $[n]=(1/3)^{n-2}\mu[n-2]$.

$$x(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$= \sum_{n=2}^{\infty} (1/3)^{n-2} z^{-n}$$

$$= \sum_{n=0}^{\infty} (1/3)^n z^{-n-2}$$

$$= z^{-2}[1/(1-(1/3)z^{-1})], |z| > 1/3$$

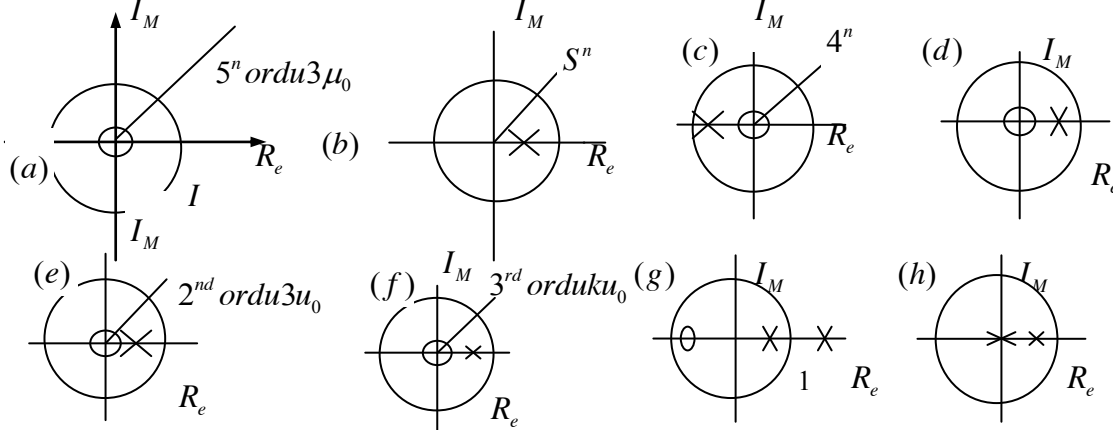The Fourier transform exists because the ROC includes the unit circle.

Figure   S10.21

10.22. (a) Using the z-transform analysis equation,

$$X(z) = (1/2)^{-4} z^4 + (1/2)^{-3} z^3 + (1/2)^{-2} z^2 + (1/2)^{-1} z^1 + (1/2)^0 z^0 + (1/2)^1 z^{-1}$$

$$+ (1/2)^2 z^{-2} + (1/2)^3 z^{-3} + (1/2)^4 z^{-4}$$

This may be express as

$$X(z) = (1/2)^{-4} z^4 [\frac{1-(1/2)^9 z^{-9}}{1-(1/2)z^{-1}}].$$

This has four zero at z=0 and 8 more zero distributed on a circle of radius 1/2. The ROC is the entire z plane. (Although form an inspection of expression for X(z) it seems like these is a pole at 1/2 which cancels with this pole.) Since the ROC includes the unit circle ,the Fourier transform exists.

(b) Consider the sequence

$$x_1[n] = (1/2)^n u[n] + 2^n u[-n-1].$$

Now,

$$(1/2)^n u[n] \quad \xrightarrow{z} \quad \frac{1}{1-(1/2)z^{-1}}, |z| > (1/2)$$

And

$$(2)^n u[-n-1] \quad \xleftrightarrow{z} \quad -\frac{1}{1-2z^{-1}}, |z| < 2.$$

Therefore,

$$x_1(z) = \frac{1}{1-(1/2)z^{-1}} - \frac{1}{1-2z^{-1}}, (1/2) < |z| < 2.$$

Note that x $[n]=n\, x_1[n]$. Therefore,

$$X(z) = -z\frac{d}{dz} X_1(Z) = -\frac{(1/2)Z^{-1}}{(1-(1/2)Z^{-1})^2} + \frac{2Z^{-1}}{(1-2Z^{-1})^2}.$$

The ROC is (1/2)< $|z|$ <2.Therefore, the Fourier transform exists.

 (c) Write x [n] as

$$n(1/2)^n u[n] - n2^n u[-n-1] = nx_1[n] - nx_2[n]$$

Where

$$x_1[n] = (1/2)^n u[n] \quad \xleftrightarrow{\quad z \quad} \quad X_1(z) = \frac{1}{1-(1/2)z^{-1}}, |z| > 1/2$$

And

$$x_2[n] = (2)^n u[-n-1] \quad \xleftrightarrow{\quad z \quad} \quad X_2(Z) = -\frac{1}{1-2Z^{-1}}, |Z| < 2$$

Using the differentiation property, we get

$$X(Z) = -z\frac{d}{dz}X_1(z) + z\frac{d}{dz}X_2(z) = -\frac{(1/2)z^{-1}}{(1-(1/2)z^{-1})^2} - \frac{2z^{-1}}{(1-2z^{-1})^2}.$$

The ROC is $(1/2) < |z| < 2$. Therefore, the Fourier transform exists.

(d) The sequence may be written as

$$x[n] = 4^n \{ \frac{e^{j[(2\pi n/6)+(\pi/4)]}}{2} \} u[-n-1].$$

Now $\dfrac{e^{-j\pi/4}}{2} \dfrac{1}{1-4e^{-j2\pi/6}z^{-1}}, |z| < 4$

$$4^n e^{j[(2\pi n/6)+(\pi/4)]} u[-n-1] \xleftrightarrow{\quad z \quad} \frac{e^{j\pi/4}}{2} \frac{1}{1-4e^{j2\pi/6}z^{-1}}, |z| < 4$$

And

$$4^n e^{-j[(2\pi n/6)+(\pi/4)]} u[-n-1] \xleftrightarrow{\quad z \quad} \frac{e^{-j\pi/4}}{2} \frac{1}{1-4e^{-j2\pi/6}z^{-1}}, |z| < 4$$

Therefore,

$$X(z) = \frac{e^{-j\pi/4}}{2} \frac{1}{1-4e^{-j2\pi/6}z^{-1}} + \frac{e^{j\pi/4}}{2} \frac{1}{1-4e^{j2\pi/6}z^{-1}}, |z| < 4$$

The ROC is $|z| < 4$. Therefore, the Fourier transform exists.

10.23 （i）The partial fraction of the given $x(z)$ is

$$X(z) = \frac{-\dfrac{1}{2}}{1-\dfrac{1}{2}z^{-1}} + \frac{\dfrac{3}{2}}{1+\dfrac{1}{2}z^{-1}}$$

Since the ROC is $|z| > \dfrac{1}{2}$.

$$x[n] = -\frac{1}{2}\left(\frac{1}{2}\right)^n u[n] + \frac{3}{2}\left(\frac{-1}{2}\right)^n u[n].$$

Performing long-division in order to get a right-sided sequence. we obtain

$$X(z) = 1 - z^{-1} + \frac{1}{4}z^{-2} - \frac{1}{4}z^{-3} + \frac{1}{16}z^{-4} - \frac{1}{16}z^{-5} + ...$$

This may be rewritten as

$$X(z) = \frac{3}{2}[1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} - \frac{1}{8}z^{-3} + ...] - \frac{1}{2}[1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} + \frac{1}{8}z^{-3} + ...]$$

Therefore.

$$x[n] = -\frac{1}{2}\left(\frac{1}{2}\right)^n u[n] + \frac{3}{2}\left(\frac{-1}{2}\right)^n u[n].$$

(ii) The partial fraction of the given $X(z)$ is

$$X(z) = \frac{-\dfrac{1}{2}}{1-\dfrac{1}{2}z^{-1}} + \frac{\dfrac{3}{2}}{1+\dfrac{1}{2}z^{-1}}$$

Since the ROC is $|z| < \dfrac{1}{2}$.

$$x[n] = \frac{1}{2}\left(\frac{1}{2}\right)^n u[-n-1] - \frac{3}{2}\left(\frac{-1}{2}\right)^n u[-n-1].$$

Performing long-division in order to get a left-sided sequence. we obtain
$$X(z) = 4z - 4z^2 + 16z^3 - 16z^4 + 64z^5 + ...$$

This may be rewritten as
$$X(z) = \frac{3}{2}[2z - 4z^2 + 8z^3 - 16z^4 + ...] + \frac{1}{2}[2z + 4z^2 + 8z^3 + 16z^4 + ...]$$

Therefore.
$$x[n] = \frac{1}{2}\left(\frac{1}{2}\right)^n u[-n-1] - \frac{3}{2}\left(\frac{-1}{2}\right)^n u[-n-1]$$

(iii) . The partial fraction of   the given $x(z)$ is
$$X(z) = -2 + \frac{\frac{3}{2}}{1 - \left(\frac{1}{2}\right)z^{-1}}$$

Since the ROC is $|z| > \frac{1}{2}$.
$$x[n] = -2\delta[n] + \frac{3}{2}\left(\frac{1}{2}\right)^n u[n]$$

Performing long-division in order to get a right-sided sequence. we obtain
$$X(z) = -\frac{1}{2} + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2} + \frac{3}{8}z^{-3} + ...$$

This may be rewritten as
$$X(z) = -2 + \frac{3}{2}[1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}....].$$

Therefore.
$$x[n] = -2\delta[n] + \frac{3}{2}\left(\frac{-1}{2}\right)^n u[n]$$

(iv) The partial fraction of   the given $x(z)$ is
$$X(z) = -2 + \frac{\frac{3}{2}}{1 - \left(\frac{1}{2}\right)z^{-1}}$$

Since the ROC is $|z| < \frac{1}{2}$
$$x[n] = -2\delta[n] - \frac{3}{2}\left(\frac{1}{2}\right)^n u[-n-1]$$

Performing long-division in order to get a left-sided sequence. we obtain
$$X(z) = -2 - 3z - 6z^2 - 12z^3 - 24z^4 - ...$$

This may be rewritten as
$$X(z) = -2 - \frac{3}{2}[2z + 4z^2 + 8z^3 + 16z^4 + ...].$$

Therefore.
$$x[n] = -2\delta[n] - \frac{3}{2}\left(\frac{1}{2}\right)^n u[-n-1].$$

(v) The partial fraction of   the given $x(z)$ is
$$x[n] = 2n\left(\frac{1}{2}\right)^n u[n] - n\left(\frac{1}{2}\right)^{n+1} u[n+1].$$

(vi) We may similarly show that in this case,
$$x[n] = -2n\left(\frac{1}{2}\right)^n u[-n-1] + n\left(\frac{1}{2}\right)^{n+1} u[-n-2].$$

10.24   (a) We may write $X[z]$ as
$$X(z) = \frac{1 - z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)(1 - 2z^{-1})}.$$

Therefore

$$X(z) = \frac{1 - z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)}.$$

If $x[n]$ is absolutely summable, then the ROC of $X(z)$ has to include the unit circle

Therefore, the ROC is $|z| > \frac{1}{2}$. It follow that

$$x[n] = (\frac{1}{2})^2 u[n].$$

(b) Carrying out long division on $X(z)$, we get

$$X(z) = 1 - z^{-1} + \frac{1}{2}z^{-2} - \frac{1}{4}z^{-2} + \dots.$$

Using the analysis equation (10.3), we get

$$x[n] = \delta[n] - (-\frac{1}{2})^{n-1} u[n-1].$$

(c) We may write $X(z)$ as

$$X[z] = \frac{3z^{-1}}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}} = \frac{3z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{4}z^{-1})}.$$

The partial fraction of $X(z)$ is

$$X[z] = \frac{4}{1 - \frac{1}{2}z^{-1}} - \frac{4}{1 + \frac{1}{4}z^{-1}}.$$

Since $x[n]$ is absolute summable, the ROC must be $|z| > \frac{1}{2}$ in order to include the unit

circle. It follow that

$$X[z] = 4(\frac{1}{2})^2 u[n] - 4(-\frac{1}{4})^2 u[n]$$

10.25. (a) The partial fraction equation of $X(z)$ is

$$X[z] = \frac{1}{1 - \frac{1}{2}z^{-1}} - \frac{2}{1 - z^{-1}}.$$

Since $x[n]$ is absolute summable, the ROC must be $|z| > 1$ in order to include the unit circle. It follow that

$$x[n] = -(\frac{1}{2})^2 u[n] + 2u[n].$$

(b) $X(z)$ may be rewritten as

$$X[z] = \frac{Z^2}{(Z - \frac{1}{2})(Z - 1)}.$$

Using partialfraction expansion, we may rewrite this as

$$X[z] = 2z^2 [\frac{1}{z - \frac{1}{2}} - \frac{2}{z-1}] = 2z^2 [-\frac{z}{z - \frac{1}{2}} + \frac{z}{z-1}].$$

If $x[n]$ is right-side, then the ROC for this signal is $|z| > 1$. Using this fact, we may

Find the inverse z-transform of the term within square bracket above to be $y[n] = -(\frac{1}{2})^n u[n] + u[n].$

Note that $X[z] = 2zX[z]$. Therefore, $x[n] = 2y[n-1]$. This gives

$$x[n] = -2(\frac{1}{2})^{n+1} u[n+1] + 2u[n+1]$$

Noting that $x[-1] = 0$. we may rewrite this as

$$x[n] = -(\frac{1}{2})^n u[n] + 2u[n]$$

This is the answer that we obtainly in part (a).

10.26 (a) Form part (b) of the previous problem,

$$X[z] = \frac{z^2}{(Z-\frac{1}{2})(Z-1)} \cdot$$

(b) Form part (b) of the previous problem ,

$$X[z] = 2z[-\frac{z}{z-\frac{1}{2}} + \frac{z}{z-1}] \cdot$$

(c) If $x[n]$ is left-side, then the ROC for this signal is $|z| < \frac{1}{2}$.Using this fact, we may

.Find the inverse z-transform of the term within square bracket above to be
$y[n] = (\frac{1}{2})^n u[-n-1] - u[-n-1] \cdot$ Note that $X[z] = 2zX[z]$ .Therefore , $x[n] = 2y[n-1]$. This gives

$$x[n] = 2(\frac{1}{2})^{n+1} u[-n-2] - 2u[-n-2] \cdot$$

10.27 We perform long-division on $X(z)$ so as to obtain a right-seder sequence. This gives us

$$X(z) = z^3 + 4z^2 + 5z + \ldots$$
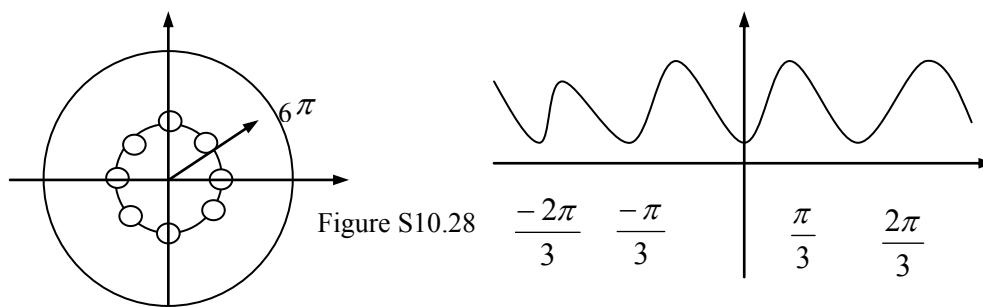
Therefore ,comparing this with eq.(10.3) we get

$x(-3) = 1$ $x(-2) = 4$ $x(-1) = 5$

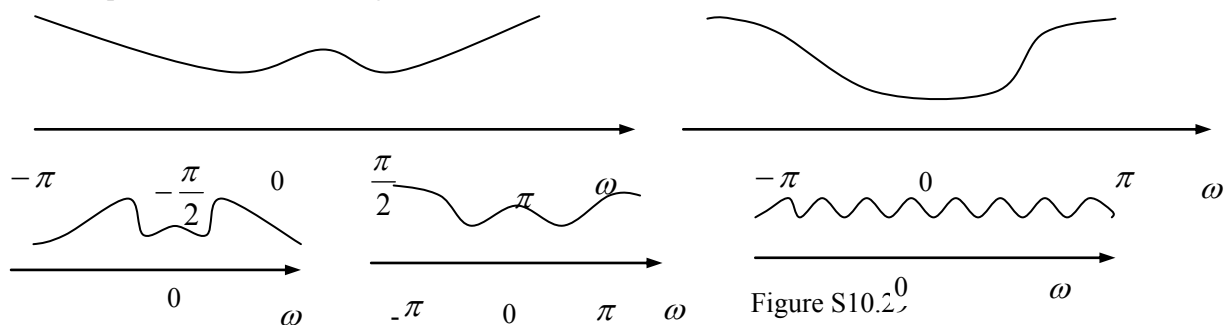And $x(n) = 0$ for n<-3

10.28 (a) Using eq (10.3),we get

$$X(z) = 1 - 0.95z^6 = \frac{z^6 - 0.95}{z^6}$$

(b) Therefore X(z) has zeros lying on a circle of radius 0.95(as shown is Figure S10.28)and 6 poles at z=0

Figure S10.28

(c) The magnitude Fourier transform is as shown in Figure S10.28

10.29 The plots are as shown in Figure S10.29

Figure S10.29

10.30 From the given information, we have

$$x_1 \xleftrightarrow{z} x_1(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} \qquad |z| > \frac{1}{2}$$

And

$$x_2[n] \xleftrightarrow{z} x_2(z) = \frac{1}{1 - \frac{1}{3}z^{-1}} \qquad |z| > \frac{1}{3}$$

Using the time shifting property, we get

$$x_1[n+3] \xleftrightarrow{z} z^3 x_1(z) \qquad |z| > \frac{1}{2}$$

Using the time reversal and properties, we get

$$x_1[-n+1] \xleftrightarrow{z} z^{-1} x_1(z^{-1}) \qquad |z| < 3$$

Now, using the convolution property, we get

$$y[n] = x_1[n+3] * x_2[-n+1] \xleftrightarrow{z} Y(z) = z^2 X_1(z) X_2(z^{-1})$$

$$\frac{1}{2} < |z| < 3$$

Therefore

$$Y(z) = \frac{z^2}{(1 - \frac{1}{2} z^{-1})(1 - \frac{1}{3} z)}$$

10.31 From Clue 1, we know that $x[n]$ is real. Therefore the poles and zeros of $X(z)$ have to occur in conjugate pairs, Since Clue 4 tells us that $X(z)$ has a pole at $z = (\frac{1}{2}) e^{\frac{j\pi}{3}}$ .we can conclude that $X(z)$ must have anther pole at $z = (\frac{1}{2}) e^{-\frac{j\pi}{3}}$ .Now since $X(z)$ has

No more poles, we have to assume that $X(z)$ has 2or less zeros. If $X(z)$ has than 2 zeros, then $X(z)$ must have poles at infinity, Since Clue 3 tell us that $X(z)$ has 2 zeros at the origin, we know that $X(z)$ must be of the from

$$X(z) = \frac{A z^2}{(z - (\frac{1}{2}) e^{\frac{j\pi}{3}})(z - (\frac{1}{2}) e^{-\frac{j\pi}{3}})}$$

Since $x[n]$ is right-sided, the ROC must be $|z| > \frac{1}{3}$.

10.32. (a) We are given that $h[n] = a^n u[n]$ and $x[n] = u[n] - u[n-N]$ therefore

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[n-k] x[k] = \sum_{k=0}^{N-1} a^{n-k} u[n-k]$$

Now. y[n] may be evaluated to be

$$y[n] = \begin{cases} 0, & n<0 \\ \sum_{k=0}^{n} a^n a^{-k}, & 0 \le n \le N-1 \\ \sum_{k=0}^{N-1} a^n a^{-k}, & n > N-1 \end{cases}$$

Simplifying

$$y[n] = \begin{cases} 0, & n<0 \\ (a^n - a^{-1})/1 - a^{-1}, & 0 \le n \le N-1 \\ a^n(1 - a^{-n})/1 - a^{-1}, & n > N-1 \end{cases}$$

(b) Using Table 10.2 we get

$$H(z) = \frac{1}{1 - a z^{-1}}, \qquad |z| > |a|$$

And

$$X(z) = \frac{1 - z^{-N}}{1 - z^{-1}}, \qquad \text{All } z,$$

Therefore

$$Y(z) = X(z)H(z) = \frac{1}{(1 - z^{-1})(1 - a z^{-1})} - \frac{z^{-N}}{(1 - z^{-1})(1 - a z^{-1})},$$

The ROC is |z|>|a|, Consider

$$P(z) = \frac{1}{(1 - z^{-1})(1 - a z^{-1})}$$

With ROC |z|>|a|, The partial fraction expansion of P(z) is

$$P(z) = \frac{1/(1-a)}{1 - z^{-1}} + \frac{1/(1 - a^{-1})}{1 - a z^{-1}},$$

Therefore,

$$P[n] = \frac{1}{1-a}u[n] + \frac{1}{1-a^{-1}}a^n u[n],$$

Now, note that

$$Y(z) = P(z)_{[1-z^{-N}]},$$

Therefore,

$$y[n] = p[n] - p[n-N] = \frac{1}{1-a}\{u[n] - u[n-N]\} + \frac{1}{1-a^{-1}}\{a^n u[n] - a^{n-N}u[n-N]\}, \quad \text{This may be written as}$$

$$y[n] = \begin{cases} 0, & n<0 \\ (a^n - a^{-1})/1 - a^{-1}, & 0 \le n \le N-1 \\ a^n(1 - a^{-n})/1 - a^{-1}, & n > N-1 \end{cases}$$

This is the same as the result of part(a).

10.33. (a) Taking the z-transform of both sides of the give difference equation and simplifying,
We get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}.$$

The poles of H(z) are at $(1/4) \pm j(\sqrt{3}/4)$. Since h[n] is causal, the ROC has to be $|z| > |(1/4) + j(\sqrt{3}/4)| = (1/2)$.

(b) We have

$$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}, \qquad |z| > \frac{1}{2}.$$

Therefore,

$$Y(z) = H(z)X(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2})}.$$

The ROC of Y(z) will be the intersection of the ROCs of X(z) and H(z). This implies that the ROC of Y(z) is |z|>1/2. The partial fraction expansion of Y(z) is

$$Y(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{z^{-1}/2}{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}.$$

Using Table 10.2 we get

$$y[n] = \left(\frac{1}{2}\right)^n u[n] + \frac{2}{\sqrt{3}}\left(\frac{1}{2}\right)^n \sin\left(\frac{\pi n}{2}\right)u[n].$$

10.34.(a) Taking the z-transform lf both sides of the give difference equation and simplifying, we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - z^{-1} - z^{-2}}.$$

The poles of H(z) are at $z = (1/2) \pm \left(\sqrt{5}/2\right)$. H(z) has a zero at z=0. The pole-zero plot for H(z) is as shown in Figure S10.34. since h[n] is causal, ROC for H(z) has to be $|z| > (1/2) + \left(\sqrt{5}/2\right)$.

(b) The partial fraction expansion of H(z) is

$$H(z) = -\frac{1/\sqrt{5}}{1 - \left(\frac{1+\sqrt{5}}{2}\right)z^{-1}} + \frac{1/\sqrt{5}}{1 - \left(\frac{1-\sqrt{5}}{2}\right)z^{-1}}.$$

Therefore,

$$h[n] = -\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n u[n] + \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n u[n].$$

Figure S10.34

(c). Now assuming that the ROC is $\left(\sqrt{5}/2\right) - (1/2) < |z| < (1/2) + \left(\sqrt{5}/2\right)$, we get

$$h[n] = -\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n u[-n-1] + \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n u[n].$$

10.35. Taking the z-transform of both sides of the given difference equation and simplifying. We get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{5}{2} + z^{-1}} = \frac{z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}}.$$
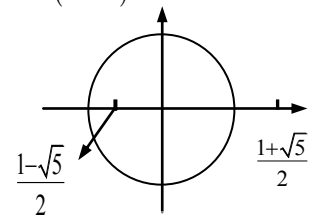
The partial fraction expansion of H(z) is

$$H(z) = \frac{-2/3}{1 - \frac{1}{2}z^{-1}} + \frac{2/3}{1 - 2z^{-1}}.$$

If the ROC is $|z|>2$,then

$$h_1[n] = -\frac{2}{3}\left(\frac{1}{2}\right)^n u[n] + \frac{2}{3}(2)^n u[n].$$

If the ROC is $1/2<|z|<2$,then

$$h_2[n] = -\frac{2}{3}\left(\frac{1}{2}\right)^n u[n] - \frac{2}{3}(2)^n u[-n-1].$$

If the ROC is $|z|<1/2$,then

$$h_3[n] = \frac{2}{3}\left(\frac{1}{2}\right)^n u[-n-1] - \frac{2}{3}(2)^n u[-n-1].$$

For each $h_i[n]$,we now need to show that if y[n]= $h_i[n]$ in the difference equation, then x[n]= $\delta[n]$.Consider substituting $h_1[n]$ into the difference equation .This yields

$$\frac{2}{3}\left(\frac{1}{2}\right)^{n-1} u[n-1] - \frac{2}{3}(2)^{n-1} u[n-1] - \frac{5}{3}\left(\frac{1}{2}\right)^n u[n] + \frac{5}{3}(2)^n u[n] + \frac{2}{3}\left(\frac{1}{2}\right)^{n+1} u[n+1] - \frac{2}{3}(2)^{n+1} u[n+1] = x[n]$$

Then,

$$x[n] = 0, \qquad \text{for n<-1,}$$
$$x[-1] = 2/3\text{-}2/3\text{=}0,$$
$$x[n] = 0, \qquad \text{for n>0.}$$

It follows that $x[n] = \delta[n]$.it can similarly be $h_2[n]$ and $h_3[n]$ satisfy the difference equation.

10.36. Taking the z-transform of both side of the given difference equation and simplifying, we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z^{-1} - \frac{10}{3} + z} = \frac{z^{-1}}{1 - \frac{10}{3}z^{-1} + z^{-2}}.$$

The partial fraction expansion of H(z) is

$$H(z) = \frac{3/8}{1 - \frac{1}{3}z^{-1}} + \frac{3/8}{1 - 3z^{-1}}.$$

Since H(z) corresponds to a stable system, the ROC has ro be $(1/3)<|z|<3$.Therefore,

$$h[n] = -\frac{3}{8}\left(\frac{1}{3}\right)^n u[n] - \frac{3}{8}(3)^n u[-n-1].$$

10.37. (a) The block-diagram may be redrawn as show in part (a) of the figure below . This may be treated as a cascade of the two systems shown within the dotted lines in Figure S10.37. These tow systems may be interchanged as shown in part (b) of the figure Figure S10.37 without changing the system function of the overall system. From the figure below, it is clear that

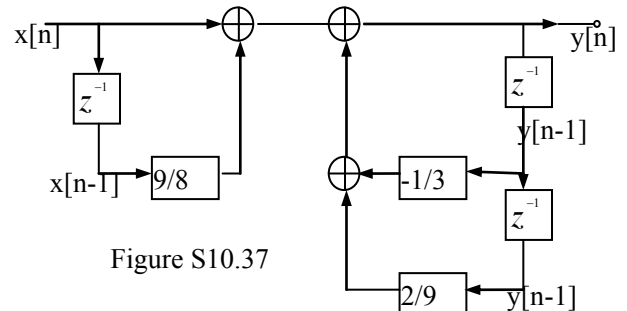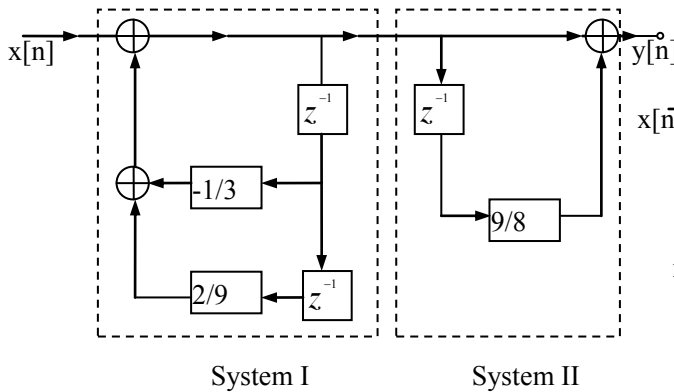$$y[n] = x[n] + \frac{9}{8}x[n-1] - \frac{1}{3}y[n-1] + \frac{2}{9}y[n-2].$$

Figure S10.37

(b) Taking the z-transform of the above difference equation and simplifying , we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{9}{8}z^{-1}}{1 + \frac{1}{3}z^{-1} - \frac{2}{9}z^{-2}} = \frac{1 + \frac{9}{8}z^{-1}}{(1 + \frac{2}{3}z^{-1})(1 - \frac{1}{3}z^{-1})}.$$

H(z) has poles at z=1/3 and z=-2/3. Since the system is causal. The ROC has to be |z|>2/3. The ROC includes the unit circle and hence the system is stable.

10.38.(a) $e_1[n] = f_1[n]$,

(b) $e_2[n] = f_2[n]$,

(c) Using the results of part (a) and (d), we may redraw the block-diagram as shown in Figure S10.38.
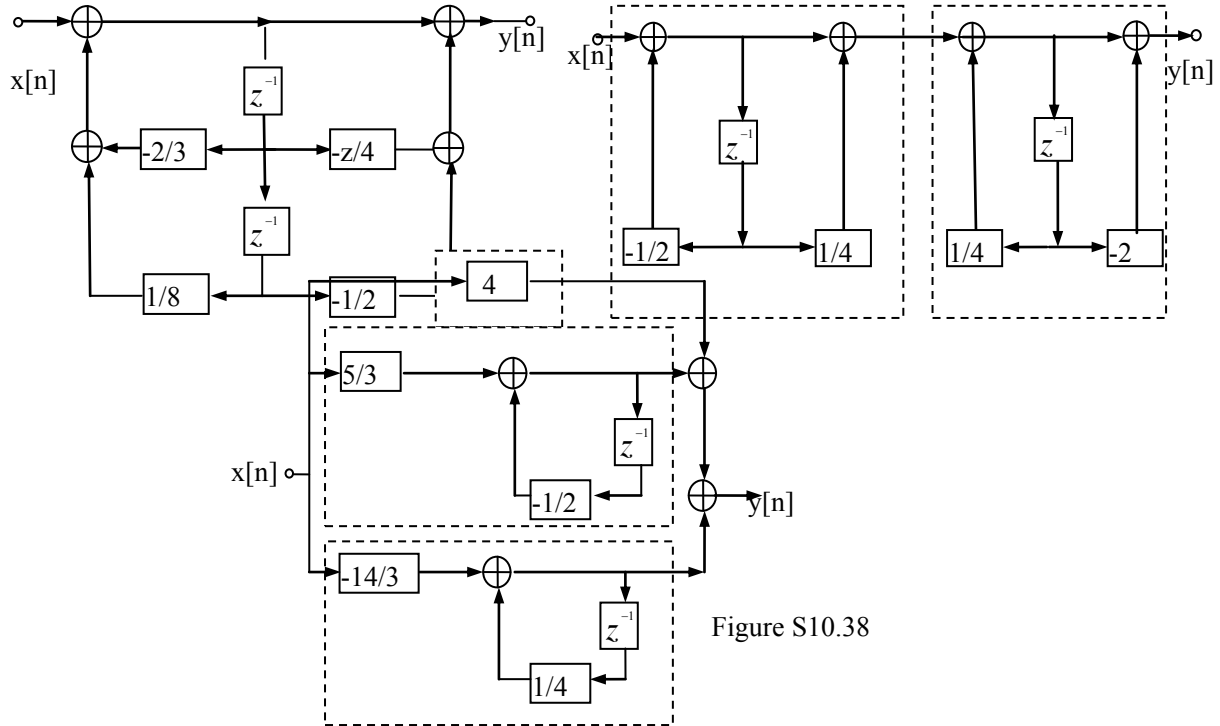

Figure S10.38

(d) Using the approach shown in the examples in the textbook we may draw the block-diagram of $H_1(z) = [1 + (1/4) z^{-1}]/[1 + (1/2) z^{-1}]$ and $H_2(z) = [1 - 2z^{-1}]/[1 - (1/4) z^{-1}]$ as shown in the dotted boxes in the figure below. H(z) is the cascade of these two systems.

(e) Using the approach show in the examples shown in the textbook, we may draw the block-diagram of $H_1(z) = 4$, $H_2(z) = [5/3]/[1 + (1/2) z^{-1}]$ and $H_3(z) = [-14/3]/[1 - (1/4) z^{-1}]$ as shown in the dotted boxes in the figure below. H(z) is the parallel combination of $H_1(z)$, $H_2(z)$, $H_3(z)$.

10.39. (a) The direct form block diagram may be drawn as shown in part(a-i) of Figure S10.39 by noting that

$$H_1(z) = \frac{1}{1 - \frac{5}{3} z^{-1} - \frac{16}{36} z^{-2} - \frac{5}{18} z^{-3} + \frac{1}{36} z^{-4}}.$$

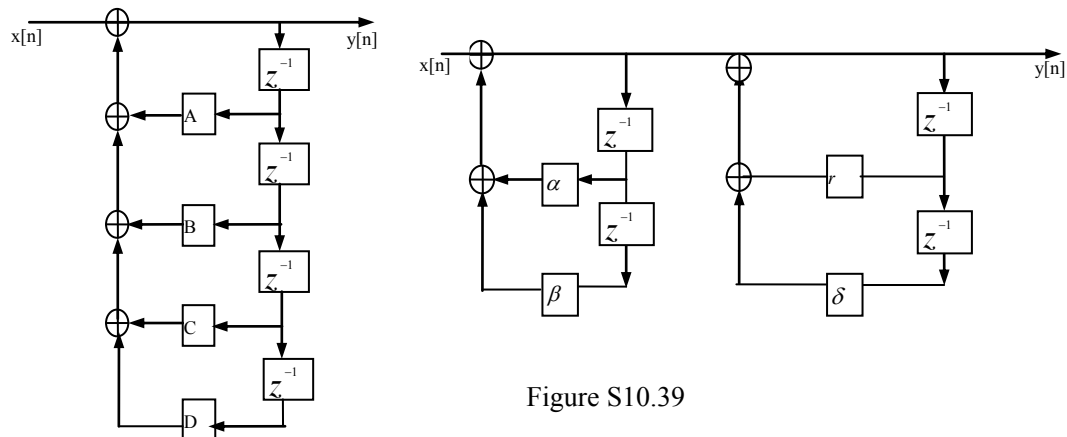The cascade block-diagram is as shown in part (a-ii) of Figure S10.39.

Figure S10.39

Part(a): A=5/3,B=11/36.C=15/54,D=1/36
Part(b): A=3/2,B=-1.C=5/4,D=-1/2
Part(b): A=2,B=-z/4.C=3/4,D=-1/8
Part(a): α =1, β =-1/4,r=2/3, δ =-1/9
Part(a): α =1, β =-1/2,r=1/2 δ =-1
Part(a): α =1, β =-1/2,r=1, δ =-1/4

Note that

$$H_1(z) = \left[\frac{1}{1-\frac{1}{2}z^{-1}}\right]\left[\frac{1}{1-\frac{1}{2}z^{-1}}\right]\left[\frac{1}{1-\frac{2}{3}z^{-1}}\right]\left[\frac{1}{1-\frac{2}{3}z^{-1}}\right].$$

Therefore, $H_1(z)$ may be drawn as a cascade of four systems for which the coefficient multipliers are all real.

(b) The direct form block diagram may be drawn as shown in part (b-i) of Figure S10.39 by noting that

$$H_2(z) = \frac{1}{1-\frac{3}{2}z^{-1}+2z^{-2}-\frac{5}{4}z^{-3}+\frac{1}{2}z^{-4}}.$$

The cascade block-diagram is as shown in part(b-ii) of Figure S10.39.
Now that

$$H_2(z) = \left[\frac{1}{1-\frac{(1+j)}{2}z^{-1}}\right]\left[\frac{1}{1-\frac{1-j}{2}z^{-1}}\right]\left[\frac{1}{1-\frac{1+j\sqrt{15}}{4}z^{-1}}\right]\left[\frac{1}{1-\frac{1-j\sqrt{15}}{4}z^{-1}}\right].$$

Therefore, $H_1(z)$ cannot be drawn as cascade of four systems for which the coefficient multipliers are all real.

(c) The direct form block diagram may be drawn as shown in part(c-i) of the Figure S10.39 by noting that

$$H_3(z) = \frac{1}{1-2z^{-1}+\frac{7}{4}z^{-2}-\frac{3}{4}z^{-3}+\frac{1}{8}z^{-4}}.$$

The cascade block-diagram is as shown in part(c-ii) of the Figure S10.39.

Note that

$$h_3(z) = \left[\frac{1}{1-\frac{(1+j)}{2}z^{-1}}\right]\left[\frac{1}{1-\frac{1-i}{2}z^{-1}}\right]\left[\frac{1}{1-\frac{1}{2}z^{-1}}\right]\left[\frac{1}{1-\frac{1}{2}z^{-1}}\right]$$

Therefore, $H_1(z)$ cannot be drawn as a cascade of four systems for which the coefficient
Multipliers are all real.
10.40. The definition of the unilateral z-transform is

$$x(z) = \sum_{n=0}^{\infty} x[n] z^{-n}$$

(a)   since x[n]= δ [n-5] is zero in the range $0\le$ n $\le \infty$,x(z)=0.
(b)   The unilateral of Laplace transform of x[n]= δ [n-5]
Is

$$x(z) = \sum_{n=0}^{\infty} \delta [n - 5^n] = e^{-5\omega}$$

(c)   The unilateral of Laplace transform of x[n]=$(-1)^n u[n]$ is

$$x(z) = \sum_{n=0}^{\infty}(-1)^n u[n]z^{-n} = \frac{1}{1+z^{-1}}, |z|>1$$

(d)The unilateral of Laplace transform of $x[n]=(1/2)^n u[n+3]$ is

$$x(z) = \sum_{n=0}^{\infty}(1/2)^n u[n+3]z^{-n}, |z|>1/2.$$

$$= \sum_{n=0}^{\infty}(1/2)^n z^{-n}$$

$$= \frac{1}{1-(1/2)z^{-1}}$$

(e)Since x[n]=(-1/3)u[-n-2] is zero in the range $0 \le n \le \infty$, x(z)=0.

(f) The unilateral of Laplace transform of $x[n] = (1/4)^n u[-n+3]$ is

$$x(z) = \sum_{n=0}^{\infty} (1/4)^n u[-n+3]z^{-n}$$

$$= \sum_{n=0}^{\infty} (1/4)^n z^{-n} \qquad \text{,All z.}$$

$$= 1 + \frac{1}{4}z^{-1} + \frac{1}{16}z^{-2} + \frac{1}{64}z^{-3}$$

(g) The unilateral of Laplace transform of $x[n] = 2^n u[-n] + (1/4)^n u[n-1]$ is

$$x(z) = \sum_{n=0}^{\infty} 2^n u[-n] + (1/4)^n u[n-1]z^{-n}$$

$$\qquad \text{,All z.}$$

$$= \sum_{n=0}^{\infty} (1/4)^n z^{-n}$$

$$= \frac{1}{1 - \frac{1}{4}z^{-1}}$$

(h) The unilateral of Laplace transform of $x[n] = (1/3)^{n-2} u[n-2]$ is

$$x(z) = \sum_{n=0}^{\infty} (1/3)^{n-2} u[n-2]z^{-n} \qquad |z|>1/2.$$

$$= z^{-2} \sum_{n=0}^{\infty} (1/3)^n z^{-n}$$

$$= \frac{z^{-2}}{1 - \frac{1}{2}z^{-1}}$$

10.41.from the given information,

$$x_1(z) = \sum_{n=0}^{\infty} (1/2)^{n+1} u[n+1]z^{-n}$$

$$\qquad |z|>1/2.$$

$$= (1/2)\sum_{n=0}^{\infty} (1/2)^n z^{-n}$$

$$+ \frac{1/2}{1-(1/2)z^{-1}}$$

And

$$x_2(z) = \sum_{n=0}^{\infty} (1/4)^n u[n]z^{-n}$$

$$\qquad |z|>1/4.$$

$$= \sum_{n=0}^{\infty} (1/4)^n z^{-n}$$

$$= \frac{1}{1-(1/4)z^{-1}}$$

Using Table 10.2 and the time shift property we get

$$X_1(z) = \frac{z}{1 - \frac{1}{2}z^{-1}} \qquad ,|z|>1/2.$$

And

$$X_2(z) = \frac{1}{1-\frac{1}{4}z^{-1}}, |z|>1/4.$$

(a) We have

$$G(z) = X_1(z)X_2(z) = \frac{z}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}.$$

The ROC is |z|>(1/2).The partial fraction expansion of G(z) is

$$G(z) = z\left[\frac{2}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}}\right].$$

Using Table 10.2 and the time shift property ,we get

$$g[n] = 2(\frac{1}{2})^{n+1} u[n+1] - (\frac{1}{4})^{n+1} u[n+1].$$

(b) We have

$$Q(z) = X_1(z) X_2(z) = \frac{1/2}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}.$$

The ROC of Q(z) is |z|>(1/2).The partial fraction expansion of y(z) is

$$Q(z) = \frac{1}{2}\left[\frac{2}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}} \right].$$

Therefore,

$$q[n] = (\frac{1}{2})^n u[n] - \frac{1}{2}(\frac{1}{4})^n u[n].$$

Clearly, q[n] $\neq$ g[n] for n>0.

10.42. (a)Taking the unilateral z-transform of both sides of the given difference equation .we get

$$Y(z) + 3z^{-1}Y(z) + 3Y[-1] = X[z].$$

Setting x(z) =0,we get

$$Y(z) = \frac{-3}{1+3z^{-1}}.$$

The inverse unilateral z-transform gives the zero-input response

$$y_{zi}[n] = -3(-3)^n u[n] = (-3)^{n+1} u[n].$$

Now, since it is given that $x[n] = (1/2)^n u[n]$, we have

$$|z|>1/2.$$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}},$$

Setting y[-1] to be zero ,we get

$$Y(z) + 3z^{-1}Y(z) = \frac{1}{1-\frac{1}{2}z^{-1}}.$$

Therefore,

$$Y(z) = \frac{1}{(1-\frac{1}{2}z^{-1})(1+3z^{-1})}.$$

The partial fraction expansion of $Y(z)$ is

$$Y(z) = \frac{1/7}{1-\frac{1}{2}z^{-1}} + \frac{6/7}{1+3z^{-1}}.$$

The inverse unilateral z-transform of both sides of the given difference equation ,we get

$$Y(z) = -\frac{1}{2}z^{-1}Y(z) - \frac{1}{2}y[-1] = X(z) - \frac{1}{2}z^{-1}X(z).$$

Setting $X(z) = 0$, we get

$$Y(z)=0.$$

The inverse unilateral z-transform gives the zero-input response

$$y_{zi}[n] = 0.$$

Now , since it is given that $x[n] = u[n]$, we have

$$X(z) = \frac{1}{1-z^{-1}}, \qquad |z|>1.$$

Setting y[-1] to be zero, we get

$$Y(z) - \frac{1}{2}z^{-1}Y(z) = \frac{1}{1-z^{-1}} - \frac{(1/2)z^{-1}}{1-z^{-1}}.$$

Therefore,

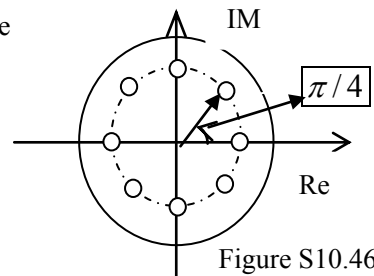$$Y(z) = \frac{1}{1-z^{-1}}.$$

The inverse unilateral z-transform gives the zero-state response

$$y_{zs}[n] = u[n].$$

(c) Taking the unilateral z-transform of both sides of the given difference equation ,we get

$$Y(z) - \frac{1}{2}z^{-1}Y(z) - \frac{1}{2}y[-1] = X(z) - \frac{1}{2}z^{-1}X(z).$$

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Figure S10.46

Setting $X(z) = 0,$, we get

$$Y(z) = \frac{1/2}{1 - \frac{1}{2}z^{-1}}.$$

10.46. Taking the z-transform of both sides of the difference equation relating x[n] and s[n] and simplifying, we get

$$H_1(z) = \frac{X(z)}{s(z)} = 1 - z^8 e^{-8\alpha} = \frac{z^8 - e^{-8\alpha}}{z^8}$$

The system has an 8th order pole at z=0 and 8 zeros distributed around a circle of radius $e^{-\alpha}$. This is show in Figure S10.46. The ROC is everywhere on the z-plane except at z=0.

(b) We have

$$H_2(z) = \frac{Y(z)}{X(z)} = \frac{S(z)}{X(z)} = \frac{1}{H_1(z)}$$

Therefore,

$$H_2(z) = \frac{1}{1 - z^{-8}e^{-8\alpha}} = \frac{z^8}{z^8 - e^{-8\alpha}}$$

There are two possible ROCs for $H_2(z)$: $|z| \prec e^{-\alpha}$ or $|z| \succ e^{-\alpha}$. If the ROC is $|z| \prec e^{-\alpha}$, then the ROC dose not include the unit circle. This in turn implies that the system would be unstable and anti-causal. If the ROC is $|z| \succ e^{-\alpha}$, then the ROC includes the unit circle. This in turn implies that the system would be stable and causal.

(d) We have

$$H_2(z) = \frac{1}{1 - z^{-8}e^{-8\alpha}}.$$

We need to choose the ROC to be $|z| \succ e^{-\alpha}$ in order to get s stable system. Now consider

$$P(z) = \frac{1}{1 - z^{-1}e^{-8\alpha}}$$

With ROC $|z| \succ e^{-\alpha}$. Taking the inverse z-transform, we get

$$p[n] = e^{-8\alpha n}u[n].$$

Now, note that

$$H_2(z) = P(z^8).$$

From Table 10.1 we know that

$$h_2[n] = \begin{cases} p[n/8] = e^{-\alpha n}, n = 0, \pm 8, \pm 10, \ldots \\ \\ 0, \quad \text{otherwise} \end{cases}$$

10.47. (a) From Clue 1,we have H(-2)=0. From Clue 2, we know that when

$$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}, |z| \succ \frac{1}{2}$$

We have

$$Y(z) = 1 + \frac{\alpha}{1 - \frac{1}{4}z^{-1}}, |z| \succ \frac{1}{4}.$$

Therefore,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\left(1 + \alpha - \frac{1}{4}z^{-1}\right)\left(1 - \frac{1}{2}z^{-1}\right)}{1 - \frac{1}{4}z^{-1}}, |z| \succ \frac{1}{4}.$$

Substituting z=-2 in the above equation and nothing that N(-2)=0,we get

$$\alpha = -\frac{9}{8}.$$

(b) The response to the signal x[n]=1=1^n. Therefore ,

$$y[n] = H(1) = \frac{1}{4}.$$

10.48. from the pole-zero diagram, we many write

$$H_2(z) = B\frac{\left(z - \frac{1}{2}e^{j\pi/4}\right)\left(z - \frac{1}{2}e^{-j\pi/4}\right)}{\left(z - \frac{1}{2}e^{j3\pi/4}\right)\left(z - \frac{1}{2}e^{-j3\pi/4}\right)}$$

And

$$H_2(z) = B\frac{\left(z - \frac{1}{2}e^{j3\pi/4}\right)\left(z - \frac{1}{2}e^{-j3\pi/4}\right)}{\left(z - \frac{1}{2}e^{j\pi/4}\right)\left(z - \frac{1}{2}e^{-j\pi/4}\right)}$$

Where A and B are constants. now note that

$$H_2(z) = \frac{B}{A}H_1\left(\frac{3}{2}ze^{j\pi}\right) = \frac{B}{A}H_1\left(-\frac{3}{2}z\right)$$

Using the property 10.53 of the z-transform(see Table 10.1),we get

$$h_2[n] = \frac{B}{A}\left(-\frac{2}{3}\right)^n h_1[n].$$

We may rewrite this as

$$h_2[n] = g[n]h_1[n,]$$

Where $g[n] = (B/a)(-2/3)^n$.Note that since both $h_1[n]$ and $h_2[n]$ are causal. We may assume that g[n]=0 for n<0 .Therefore .

$$g[n] = \frac{B}{A}(-\frac{2}{3})^n u[n] .$$

Now ,clue 3slao states that $\sum_{k=0}^{\infty}| g[k]| = 3$ .Therefore .

$$\sum_{k=0}^{\infty}\frac{B}{A}(-\frac{2}{3})^k = 3$$

Or

$$\frac{B}{A}\frac{1}{1-2/3} = 3 \qquad \Rightarrow \qquad \frac{B}{A} = 1$$

Therefore ,

$$g[n] = (-\frac{2}{3})^n u[n] .$$

10.49 .(a) We may write the left side of eq. (p10.49-1)   as

$$\sum_{n=N_1}^{\infty}| x[n] |r_1^{-n} = \sum_{n=N_1}^{\infty}| x[n] |(r_0\frac{r_1}{r_0})^{-n} = \sum_{n=N_1}^{\infty}| x[n] |r_0^{-n}(\frac{r_1}{r_0})^{-n} \qquad (s10.49-1)$$

Since $r_1 \geq r_0$; the sequence $(r_1/r_0)^{-n}$ decays with increasing n . i . e .,as n->∞

$(r_1/r_0)^{-n}$ ->0 . Therefore , $(r_1/r_0)^{-n} \leq (r_1/r_0)^{-N_1}$ for $n \geq N_1$ .Substituting this in eq. $(s10.49-1)$, we get

$$\sum_{n=N_1}^{\infty}| x[n] |r_1^{-n} = \sum_{n=N_1}^{\infty}| x[n] |r_0^{-n}(\frac{r_1}{r_0})^{-n} \leq (\frac{r_1}{r_0})^{-N_1}\sum_{n=N_1}^{\infty}| x[n] |r_0^{-n}$$

Therefore , A= $(r_1/r_0)^{-N_1} = (r_0/r_1)^{N_1}$.

(b)The above inequality shows that if X(z) has the finite bound   B for $|z|=r_0$;thenX(z) has the finite bound $(r_0/r_1)^{N_1}$ B for $|z|= r_1 \geq r_0$. Thus ,X(z) converges for $|z|= r_1 \geq r_0$ .and Property 4 of Section 10.2 follows .

(c)Consider a left-sided sequence x[n] such that

$$x[n]=0 , n>N_2$$

and for which

$$\sum_{n=-\infty}^{\infty}| x[n] | r_0^{-n} = \sum_{n=-\infty}^{N_2} | x[n] | r_0^{-n}$$

Then we need to show that if $r_1 \leq r_0$,

$$\sum_{n=-\infty}^{N_2} |x[n]| r_1^{-n} \le P \sum_{n=-\infty}^{N_2} |x[n]| r_0^{-n} \qquad (s10.49\text{-}2)$$

When P is a positive constant .

We may write the left side of eq. (s10.49-2

$$\sum_{n=-\infty}^{N_2} |x[n]| r_1^{-n} = \sum_{n=-\infty}^{N_2} |x[n]| (r_0 \frac{r_1}{r_0})^{-n} = \sum_{n=-\infty}^{N_2} |x[n]| r_0^{-n} (\frac{r_1}{r_0})^{-n} \qquad (s10.49\text{-}3)$$

Since $r_1 \le r_0$;the sequence $(r_1/r_0)^{-n}$ decays with decreasing n , i.e., as

n->∞ $(r_1/r_0)^{-n}$ ->0 .Therefore , $(r_1/r_0)^{-n} \le (r_1/r_0)^{-N_2}$ for. $n \le N_2$

.Substituting this in eq (s10.49-3) .We get

$$\sum_{n=-\infty}^{N_2} |x[n]| r_1^{-n} = \sum_{n=-\infty}^{N_2} |x[n]| (r_0 \frac{r_1}{r_0})^{-n} \le (\frac{r_1}{r_0})^{-N_2} \sum_{n=-\infty}^{N_2} |x[n]| r_0^{-n}$$

Therefore , $P = (r_1/r_0)^{-N_2} = (r_0/r_1)^{N_2}$

The above inequality show that if X(z) has if X(z) has the finite bound B for $|z|=r_0$ .then X[z] has

the finite bound $(r_0/r_1)^{N_2}$ B for $|z|=r_1 \le r_0$ .Thus ,X(z) converges for $|z|= r_1 \le r_0$ and Property 5 of

Section 10.2 follows .

10.50.(a) From the given pole-zero plot ,we get

$$H(z)=A \frac{z^{-1} - a}{1 - az^{-1}} ,$$

Where A is some constant .Therefore

$$H(e^{j\omega})=A \frac{e^{-j\omega} - a}{1 - ae^{-j\omega}}$$

And

$$|H(e^{j\omega})|^2 = H(e^{j\omega}) H * e^{jw} = |A|^2 [\frac{e^{-j\omega} - a}{1 - ae^{-j\omega}}][\frac{e^{j\omega} - a}{1 - ae^{j\omega}}]$$

Therefore

$$|H(e^{j\omega})|^2 = |A|^2 \frac{1 - ae^{-j\omega} - ae^{j\omega} + a^2}{1 - ae^{-j\omega} - ae^{jw} + a^2} = |A|^2 .$$

This implies that $|H(e^{j\omega})|=|A|$=constant .

(b) We get $|v_1|^2 = 1 + a^2 - 2a\cos(\omega)$.

(c)We get

$$|v_2|^2 = 1 + \frac{1}{a^2} - \frac{2}{a} \cos\omega = \frac{1}{a^2}[a^2 + 1 + 2a \cos\omega] = \frac{1}{a^2} |v_1|^2$$

10.51. （a） We know that for a real sequence x[n],x[n]=x*[n] .Let us first find the z-transform of $x*[n]$ in

terms of X[z],the z-transform of x[n]. We have

$$Y(z)= \sum_{n=-\infty}^{\infty} y[n]z^{-n} = \sum_{n=-\infty}^{\infty} x*[n]z^{-n}$$

$$=[\sum_{n=-\infty}^{\infty} x[n](z^*)^{-n}]^* = [X(z^*)]^* .$$

Now since x[n]= $x*[n]$,we have Z{x[n]}=z{ $x*[n]$ }which in turn impels that X(z)=$X^* (z^*)$.

(b) If X(z) has o pole at $z=z_0$ .then $1/X(z_0)$=0.From the result of the previous part ,

We know that $\frac{1}{X^*(z_0^*)} = 0$.

Conjugating both sides ,we get $1/ X(z_0^*)$ =0. this implies that X(z) has a pole at z_0^* .

If X(z) has a zero at $z=z_0$ .then $X(z_0)$=0.From the result of the previous part ,

We know that

$$X^*(z_0^*)=0 .$$

Conjugating both sides, we get $X(z_0^*)=0$. This implies that $X(z)$ has a zero at z_0^* .

(c) (1) The z-transform of given sequence is

$$X(z)= \frac{1}{1-\frac{1}{2}z^{-1}}=\frac{z}{z-\frac{1}{2}} \quad , \quad |z|>1/2$$

Clearly, $X(z)$ has a pole at $z=1/2$ and a zero at $z=0$ and the property of part (b) holds .

(2) The z-transform of the given sequence is

$$X(z)= 1-\frac{1}{2}z^{1}+\frac{1}{4}z^{-2}=\frac{z^2-(1/2)z+(1/4)}{z^2},|z|>0.$$

$X(z)$ has two zero at $z=1/2$ and z zero at $z=0$ and the property of part(b) still holds

(d) Now, from part (b) of problem 10.43 we know that if $x[n]$ and $X(z)$ has a pole at $z_0= pe^{j\theta}$, then

$X(z)$ has a pole at $(1/z_0)=(1/z_0^*)=(1/\rho)e^{-j\theta}$

If $x[n]$ is real and $X(z)$ has a pole at $z_0= pe^{j\theta}$,then from part (b) we know that $X(z)$ must

have a pole at $z_0^*=\rho\ e^{-j\theta}$. ,then $X(z)$ must have a pole at $(1/z_0^*)=(1/\rho)e^{j\theta}$.

A similar argument may be constructed for zeros .

10.52 . We have

$$X_2 = \sum_{n=-\infty}^{\infty} x_2[n]z^{-n}$$
$$=\sum_{n=-\infty}^{\infty} x_1[-n]z^{-n}$$
$$=\sum_{n=-\infty}^{\infty} x_1[n]z^{n}$$
$$= X_1(z^{-1}) = X_1(1/z) \quad .$$

Using an argument similar to the one used on part (b) of problem 10.43 .we may argue that if $X_1(z)$ has a

pole (or zero) at $z= z_0$, then $X_2(z)$ which has a pole (or zero) at $z=1/z_0$.

10.53 .Let us assume that $x[n]$ is a sequence with z-transform $X(z)$ which has the ROC $\alpha <|z|<\beta$.

(a) (1) The z-transform of the sequence $y[n]=[n-n_0]$ is

$$Y(z)= \sum_{n=-\infty}^{\infty} y[n]z^{-n}$$
$$= \sum_{n=-\infty}^{\infty} x[n-n_0]z^{-n}$$

Substituting $m= n-n_0$ in the above equation ,we get

$$Y(z)= \sum_{m=-\infty}^{\infty} x[m]z^{-m-n_0} = = = z^{-n_0}\sum_{m=-\infty}^{\infty} x[m]z^{-m}$$
$$= z^{-n_0} X(z) .$$

Clearly, $Y(z)$ converges where $X(z)$ converges except for the addition or deletion of $z=0$

because of the z^{-n_0} term .Therefore ,the ROC is $\alpha <|z|<\beta$ .except for the possible addition of $z=0$ in the ROC .

(2) The z-transformer the sequence $y[n]= z_0^n x[n]$ is

$$Y(z)= \sum_{n=-\infty}^{\infty} y[n]z^{-n}$$
$$= \sum_{n=-\infty}^{\infty} z_0^n x[n]z^{-n}$$
$$= \sum_{n=-\infty}^{\infty} x[n](z/z_0)^{-n}$$
$$=X(z/z_0)$$

Since $X(z)$ converges for $\alpha <|z|<\beta$,$Y(z)$ converges for $\alpha <|z/z_0|<\beta$, Therefore ,

the ROC of Y(z) is $|z_0|\alpha <|z|<|z_0|\beta$.

(b) (1) From Problem 10.51 (a) ;we know that the z-transform of the sequence y[n]= $x^*[n]$ is Y(z)= $X^*[z^*]$ .The ROC of Y(z) is the same as the ROC of X(z).

(2)Suppose that the ROC of x[n] is $\alpha <|z|< \beta$,From subpart (2) of part (a).the z-transform of $y[n] = z_0^n x[n]$ is

$$Y(z)=X(z/z_0)$$

With ROC $|z_0|\alpha <|z|<|z_0|\beta$ .Therefore $R_y = |z_0|R_x$.

10.54. (a) Let x[n]=0 for n>0. Then ,

$$X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$= \sum_{n=-\infty}^{0} x[n]z^{-n}$$

$$=x[0]+x[-1]z+x[-2]z^2 +\ldots\ldots$$

Therefore

$$\lim_{z->0} X(z) = x[0].$$

(b) Let x[n]=0 for n<0 .Then ,

$$X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=0}^{\infty} x[n]z^{-n}$$

$$= x[0]+x[-1]z+x[-2]z^2 +\ldots\ldots$$

Therefore ,

$$\lim_{z->\infty} z(X(z) - x[0] = \lim_{z->\infty} z\{z[1]z^{-1} + x[-2]z^{-2} +\ldots..\} = x[1].$$

10.55. (a) From the initial value theorem ,we have

$$\lim_{z->\infty} X(z)=x[0] \text{non-zero and finite} .$$

Therefore ,as z->∞ ,X(z) tends to a finite non-zero value .This implies that X(z) has neither poles zeros at infinity .

(b) A rational z-transform is made up of factors of the form 1/(z-a) and (z-b) . Note that the factor 1/(z-b) has a pole at z=a and a zero at z=∞ .Also note that the factor (z-b) has a zero at z=∞ .From the result of part (a) ,we know that a causal sequence has no pole or zero at infinity .Therefore ,all zeros at infinity contributed by factors of the form 1/(z-a) .Consequently ,the number of zeros in the finite z-plane must equal the number of poles in the finite z-plane .

10.56. (a) The z-transform of $x_3[n]$ is

$$x_3(z) = \sum_{n=-\infty}^{\infty} x_3[n]z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} [\sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k]]z^{-n}$$

$$= \sum_{k=-\infty}^{\infty} x_1[k][\sum_{n=-\infty}^{\infty} x_2[n-k]z^{-n}]$$

$$= \sum_{k=-\infty}^{\infty} x_1[k]Z\{x_2[n-k]z^{-n}\}$$

$$= \sum_{k=-\infty}^{\infty} x_1[k]x_2(z)$$

(b) Using the time shifting property (10.5.2) ,we get

$$x_2(z) = Z\{x_2[n-k]\} = z^{-k}X_2(z),$$

Where $X_2(z)$ is the z-transform of $x_2[z]$ .Substituting in the result of part (a) ,we get

$$X_3(z) = X_2(z)\sum_{k=-\infty}^{\infty} x_1[k]z^{-k}$$

(c) Nothing that the z-transform of $x_1[n]$ may be written as

$$X_1(z) = \sum_{-\infty}^{\infty} x_1[k]z^{-k} ;$$

We may rewrite the result of part (b) as

$$X_3(z) = X_1(z)X_2(z).$$

10.57.(a) $X_1(z)$ is a polynomial of order N_1 in Z^{-1}. $X_1(z)$ is a polynomial of order N_2 in Z^{-1}.Therefore, $Y(z) = X_1(z)X_2(z)$ is a polynomial of order $N_1 + N_2$ in Z^{-1} . this implies that M= $N_1 + N_2$.

(b) By nothing that y[0] is the coefficient of Z^0 term in Y(z). y[1] is the coefficient of Z^{-1} term in Y(z),and y[2] is the coefficient of Z^{-2} term in Y(z). we get

$$y[0] = x_1[0]x_2[0],$$
$$y[1] = x_1[0]x_2[1] + x_1[1]x_2[0],$$
$$y[3] = x_1[0]x_2[2] + x_1[1]x_2[1] + x_1[2]x_2[0].$$

(c) we note the pattern that emerge form part (b). The k-th point in the sequence y[n] is the coefficient of Z^{-k} in Y(z), The Z^{-k} term of Y(z) is formed by the following sum :(the product of the Z^0 term of $X_1(z)$ with the Z^{-k} term of $X_2(z)$)+(the product of the Z^{-1} term of $X_1(z)$ with the Z^{-k+1} term of $X_2(z)$)+(the product of the Z^{-2} term of $X_1(z)$ with the Z^{-k+2} term of $X_2(z)$)+.......+(the product of the Z^{-N_1} term of $X_1(z)$ with the Z^{-k+N_1} term of $X_2(z)$).
Therefore,

$$y[k] = \sum_{m=-\infty}^{N_1} x_1[m]x_2[m-k]$$

Since $X_1[m]=0$ for m> N_1 and m<0, we may rewrite this as

$$y[k] = \sum_{m=-\infty}^{\infty} x_1[m]x_2[m-k].$$

10.58 Consider a causal and stable with system function $H(z)$.Let its inverse system have the function $H_i(z)$.The poles of $H(z)$ are the zeros of $H_i(z)$ and the zeros of $H(z)$ are the poles of $H_i(z)$.

For $H(z)$ to correspond to be a casual and stable system, all its poles must be within the unit circle. Similarly, for $H_i(z)$ to correspond to be a casual and stable system, all its poles must be within the unit circle. Since the poles of $H_i(z)$ are the zeros of $H(z)$,the previous statement implies that the zeros of $H(z)$ must be within the unit circle. Therefore, all poles and zeros of a minimum-phase system must be within the unit circle.

10.59 (a) From Figure S10.59, we have

$$W_1(z) = X(z) - \frac{k}{3}z^{-1}W_1(z) \quad \Rightarrow \quad W_1(z) = X(z)\frac{1}{1+\frac{k}{3}z^{-1}}$$

Also

$$W_2 = -\frac{k}{4}z^{-1}W_1(z) = -X(z)\frac{-\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}}$$

Therefore, $Y(z) = W_1(z) + W_2(z)$ will be

$$Y(z) = X(z)\frac{1}{1+\frac{k}{3}z^{-1}} - X(z)\frac{\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}}$$

Finally,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}}$$

Since $H(z)$ corresponds to a causal filter ,the ROC will be $|z|>|k|/3$.

(b) For the system to be stable, the ROC of $H(z)$ must include the unit circle . This is possible only if $|k|/3<1$.This is implies that $|k|$ has to be less than 3.

(c) If k=1,then
$$H(z)=\frac{\frac{1}{4}z^{-1}}{1+\frac{1}{3}z^{-1}}$$

The response to x[n]= $(2/3)^n$ will be of the form
$$y[n]=x[n]H(2/3)=\frac{5}{12}(2/3)^n .$$

10.60. The unilateral z-transform of y[n]=x[n+1] is
$$y[z]=\sum_{n=0}^{\infty}y[n]z^{-n}$$
$$= y(0)+y[1]z^{-1}+y[2]z^{-2}+......$$
$$= x[1]+x[2]z^{-1}+x[3]z^{-2}+......$$
$$= z\left\{x[0]+x[1]z^{-1}+x[2]z^{-2}+x[3]z^{-3}......\right\}-zx[0]$$
$$= z\chi(z)-zx[0].$$

10.61.(a) The unilateral z-transform of y[n]=x[n+3] is
$$y(z)=\sum_{n=0}^{\infty}y[n]z^{-n}$$
$$= \sum_{n=0}^{\infty}x[n+3]z^{-n}$$
$$= \sum_{n=0}^{\infty}x[n+3]z^{-n}-x[0]z^3-x[1]z^2-x[2]z$$
$$= \sum_{n=0}^{\infty}x[n]z^{-n+3}-x[0]z^3-x[1]z^2-x[2]z$$
$$= z^3\sum_{n=0}^{\infty}x[n]z^{-n+3}-x[0]z^3-x[1]z^2-x[2]z$$
$$= z^3\chi(z)-x[0]z^3-x[1]z^2-x[2]z$$

(b) The unilateral z-transform of y[n]=x[n+3] is
$$y(z)=\sum_{n=0}^{\infty}y[n]z^{-n}$$
$$= \sum_{n=0}^{\infty}x[n-3]z^{-n}$$
$$= \sum_{n=3}^{\infty}x[n-3]z^{-n}+x[-1]z^{-2}+x[-2]z^{-1}+x[-3]$$
$$= \sum_{n=0}^{\infty}x[n]z^{-n-3}+x[-1]z^{-2}+x[-2]z^{-1}+x[-3]$$
$$= z^{-3}\sum_{n=0}^{\infty}x[n]z^{-n}+x[-1]z^{-2}+x[-2]z^{-1}+x[-3]$$
$$= z^{-3}\chi(z)+x[-1]z^{-2}+x[-2]z^{-1}+x[-3]$$

(c) We have
$$y[z]=\sum_{k=-\infty}^{n}x[k]=\sum_{m=0}^{\infty}x[n-m]$$

Therefore
$$y[z]=\sum_{m=0}^{\infty}z^{-m}\chi(z)+\sum_{m=1}^{\infty}z^{-m}\sum_{l=1}^{m}x[-l]z^{l}$$
$$= \frac{\chi(z)}{1-z^{-1}}+\sum_{m=1}^{\infty}z^{-m}\sum_{m=0}^{m}x[-l]z^{l}$$

10.62. Note that
$$\Phi_{xx}[n]=\sum_{k=-\infty}^{\infty}x[k]x[k+n]=x[n]*x[-n]$$

Now, applying the convolution property, the z-transform of $\Phi_{xx}[n]$ is
$$\Phi_{xx}(z)=X(z)Z\{x[-n]\}$$

From the time-reversal property we know that the z-transform of x[-n] is X(1/2). Therefore,

$$\Phi_{xx}(z) = X(z)X(1/2)$$

10.63. (a) Since the ROC is |z|<1/2, the sequence is   left-sided. Using the power-series expansion ,we get

$$\log(1-2z) = -\sum_{n=1}^{\infty}\frac{2^n z^n}{n} = -\sum_{n=-\infty}^{-1}-\frac{2^{-n}z^{-n}}{n}$$

Therefore ,

$$x[n]= \frac{2^{-n}}{n}u[\text{-}n\text{-}1].$$

(b) Since the ROC is |z|>1/2, the sequence is right-sided. Using the power-series expansion ,we get

$$\log(1-(1/2)z^{-1}) = -\sum_{n=1}^{\infty}\frac{(1/2)^n z^{-n}}{n}$$

Therefore ,

$$x[n]=-\frac{2^{-n}}{n}u[n\text{-}1].$$

10.64. Let us define Y(z) to be

$$Y(z) = -z\frac{d}{dz}X(z).$$

Then using the differentiation property of the z-transform ,we get

$$Y[n] = n\, x[n] .$$

(a) Now ,

$$Y(z) = -z\frac{d}{dz}X(z) = z\frac{2}{1-2z} = -\frac{1}{1-\frac{1}{2}z^{-1}}.$$

Nothing that the ROC of Y(z) is z<1/2(the   same   as the ROC is X(z)), we get

$$y[n]=\frac{1}{2}^n u[-n-1].$$

Therefore ,

$$x[n]=\frac{1}{n}(\frac{1}{2})^n u[-n-1]=\frac{2^{-n}}{n}u[-n-1].$$

This is the same as the answer obtained for Problem 10.63(a).
  (b) In this part,

$$Y(z) = -z\frac{d}{dz}X(z). = \frac{\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}}.$$

Nothing that the ROC is |z|>1/2, (the same   as the   ROC is X(z)), we get

$$y[n]=-\frac{1}{n}(\frac{1}{2})^n u[n-1]$$

Therefore,

$$x[n]=-\frac{1}{n}(\frac{1}{2})^n u[n-1]=\frac{2^{-n}}{n}u[n-1]$$

This is the same as the answer obtained for Problem 10.63(b).

10.65.(a) From the given H_c(s), we get

$$|H_c(j\omega)|=\frac{|a-j\omega|}{|a+j\omega|}=\frac{\sqrt{a^2+\omega^2}}{\sqrt{a^2+\omega^2}}=1$$

(b) Applying the bilinear transformation we get

$$H_d(z) = \frac{|a-\frac{1-z^{-1}}{1+z^{-1}}|}{|a+\frac{1-z^{-1}}{1+z^{-1}}|} = \frac{a-1}{a+1}[\frac{1+z^{-1}\frac{a+1}{a-1}}{1+z^{-1}\frac{a-1}{a+1}}]$$.

Therefore $H_d(z)$ has a pole at z= (a-1) / (a+1) and a zero at z= (a-1) / (a+1).
  Since a is real and positive ,

$$|\frac{a-1}{a+1}|\le 1 \quad \text{and} \quad |\frac{a-1}{a+1}|\ge 1$$

Therefore, the pole of $H_d(z)$ lies inside the unit circle and the zero of $H_d(z)$ lies outside the unit circle.
(c) $H_d(z)$ may be rewritten as

$$H(z) = \frac{a-1+z^{-1}(a+1)}{a+1+z^{-1}(a-1)}$$

Therefore ,

$$|H_c(e^{j\omega})| = |\frac{a-1+e^{-j\omega}(a+1)}{a+1+e^{-j\omega}(a-1)}| = |\frac{a-1+(\cos\omega - j\sin\omega)(a+1)}{a+1+(\cos\omega - j\sin\omega)(a-1)}|$$

This may be written as

$$H_c(j\omega)| = \frac{\sqrt{(a-1)^2+\cos^2\omega(a+1)^2+2(a+1)(a-1)\cos\omega+(a+1)^2\sin^2\omega}}{\sqrt{(a+1)^2+\cos^2\omega(a-1)^2+2(a+1)(a-1)\cos\omega+(a-1)^2\sin^2\omega}}$$

$$= \frac{\sqrt{(a-1)^2+(a+1)^2+2(a+1)(a-1)\cos\omega}}{\sqrt{(a+1)^2+2(a+1)(a-1)\cos\omega+(a-1)^2}} = 1$$

10.66. (a) We are give that

$$H_d(z) = H_c(\frac{1-z^{-1}}{1+z^{-1}})$$

Therefore

$$H_d(e^{j\omega}) = H_c(\frac{1-e^{-j\omega}}{1+e^{-j\omega}}) = H_c(\frac{e^{j\omega/2}-e^{-j\omega/2}}{e^{j\omega/2}+e^{-j\omega/2}}) = H_c(j\tan\frac{\omega}{2})$$

(b) From the given H_c (s), we get

$$H_c(0) = \frac{1}{(e^{j\pi/4})(e^{-j\pi/4})} = 1$$

And

$$H_c(\infty) = \frac{1}{\lim_{s\to\infty}(s+e^{j\pi/4})(s-e^{-j\pi/4})} = 0$$

Now

$$|H_c(j\omega)| = |\frac{1}{(j\omega+e^{j\pi/4})(j\omega+e^{-j\pi/4})}| = |\frac{1}{-\omega^2+2\cos(\pi/4)j\omega+1}|$$

$$= \frac{1}{\sqrt{(1-\omega^2)^2+4\omega\cos^2(\pi/4)\omega^2+(a-1)^2}}$$

Clearly , $|H_c(j\omega)|$ decrease monotonically with increasing ω

(c) (1) We are given that

$$H_d(z) = H_c(\frac{1-z^{-1}}{1+z^{-1}}) \cdot$$

Therefore,

$$H_d(z) = \frac{1}{(\frac{1-z^{-1}}{1+z^{-1}}+e^{j\pi/4})(\frac{1-z^{-1}}{1+z^{-1}}+e^{-j\pi/4})}$$

This may be rewritten as

$$H_d(z) = \frac{1}{(1+e^{j\pi/4})(1+e^{-j\pi/4})}\frac{1}{[1-z^{-1}\frac{1+e^{j\omega/4}}{1-e^{-j\omega/4}}][1-z^{-1}\frac{1-e^{-j\omega/4}}{1+e^{-j\omega/4}}]}$$

Therefore , $H_d(z)$ has exactly two pole which lie at $z=-(1+e^{j\pi/4})/(1-e^{j\pi/4})$

And $z=-(1+e^{-j\pi/4})/(1-e^{-j\pi/4})$. It can be easily shown that both these poles lie inside the unit circle.

(2) From the result of part (a) , we have

$$H_d(e^{j0}) = H_c(j\tan 0) = H_c(j0) = 1$$

(3) We have

$$|H_c(e^{j\omega})| = |H_c(j\tan\frac{\omega}{2})| = \frac{1}{|1-\tan^2\frac{\omega}{2}+\sqrt{2}j\tan\frac{\omega}{2}|}$$

$$= \frac{1}{|(1-\tan^2\frac{\omega}{2})^2+2\tan^2\frac{\omega}{2}|} = \frac{1}{1+\tan^4\frac{\omega}{2}}$$

As ω increases from 0 to pi, $\tan\frac{\omega}{2}$ increases monotonically from 0 to ∞. Therefore , $|H_c(e^{j\omega})|$ decreases monotonically from 1 to 0 .