Quiz 4.

1. Solutions.

(a) $X(z) = \frac{1}{1-z^{-1}} - \frac{1}{2}\frac{1}{1-z^{-1}}\cdot z^{-1}$, ROC: $|z|>1$

$Y(z) = +\frac{2}{3}\cdot\frac{1}{1+2z^{-1}} + \frac{1}{3}\frac{1}{1-z^{-1}}$, ROC: $1<|z|<2$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{2}{3}\frac{1}{1+2z^{-1}} + \frac{1}{3}\frac{1}{1-z^{-1}}}{\frac{1}{1-z^{-1}} - \frac{1}{2}\frac{z^{-1}}{1-z^{-1}}} = \frac{1}{1+\frac{3}{2}z^{-1}-z^{-2}} = \frac{4}{5}\frac{1}{1+2z^{-1}} + \frac{1}{5}\frac{1}{1-\frac{1}{2}z^{-1}}$$

$$= \boxed{\frac{z^2}{z^2+\frac{3}{2}z-1}}$$

poles: $-2, \frac{1}{2}$.

ROC: $\frac{1}{2}<|z|<2$.

IZT: $h[n] = -\frac{4}{5}(-2)^n u[-n-1] + \frac{1}{5}(\frac{1}{2})^n u[n]$.

(b) 由 $H(z)$ ROC 知, uncausal, stable.

(c) $x[n] = \cos\pi n = (-1)^n \longrightarrow \boxed{LTI} \longrightarrow y[n] = H(z)|_{z=-1}\cdot(-1)^n$

$$\therefore y[n] = H(-1)\cdot(-1)^n = \frac{1}{1+\frac{3}{2}(-1)^{-1}-(-1)^{-2}}(-1)^n = -\frac{2}{3}(-1)^n$$

$$= \boxed{-\frac{2}{3}\cos\pi n}.$$

$\therefore$

2. Solutions

(a) $H(s) = \frac{1}{s+1}\cdot e^{s}$ $\quad Re\{s\}>-1 \xleftrightarrow{ILT} h(t) = e^{-(t+1)}u(t+1)$

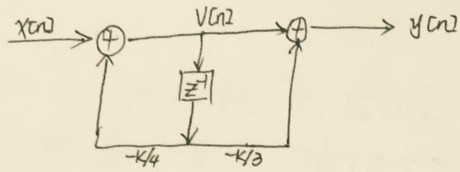
uncausal

(b) $H(z) = \frac{z^2+3z+1}{z+1} = \frac{(z+1)^2+z}{z+1} = z+1+\frac{z}{z+1} = z+1+\frac{1}{1+z^{-1}}$.

$\therefore h[n] = \delta[n+1] + \delta[n] + (-1)u[n]$

uncausal

3. Solutions:



8' (a) 由方框图知:

$$V(z) = X(z) - \frac{K}{4} V(z)\cdot z^{-1} \Rightarrow \frac{V(z)}{X(z)} = \frac{1}{1+\frac{K}{4}z^{-1}}$$

$$Y(z) = V(z) - \frac{K}{3} V(z)\cdot z^{-1} \Rightarrow \frac{Y(z)}{V(z)} = \frac{1-\frac{K}{3}z^{-1}}{1}$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1-\frac{K}{3}z^{-1}}{1+\frac{K}{4}z^{-1}}$$ 4'

因 causal, poles: $-\frac{K}{4}$ $\Big\} \Rightarrow$ ROC: $|z| > |\frac{K}{4}|$. 2'

$\therefore y[n] + \frac{K}{4} y[n-1] = x[n] - \frac{K}{3} x[n-1]$. 2'

8' (b) $H(z) = \frac{1}{1+\frac{K}{4}z^{-1}} - \frac{K}{3}\cdot\frac{1}{1+\frac{K}{4}z^{-1}}\cdot z^{-1}$, $|z| > \frac{|K|}{4}$.

$h[n] = (-\frac{K}{4})^n u[n] - \frac{K}{3}\cdot(-\frac{K}{4})^{n-1} u[n-1]$ 5'

stable, $|z| > \frac{|K|}{4}$ 包含单位圆 $\Rightarrow |K| < 4$. 3'

7' (c) $K=1$, $H(z) = \frac{1-\frac{1}{3}z^{-1}}{1+\frac{1}{4}z^{-1}}$, $|z| > \frac{1}{4}$.

$x[n] = (\frac{2}{3})^n \longrightarrow \boxed{LTI} \longrightarrow y[n] = H(z)\big|_{z=\frac{2}{3}} \cdot (\frac{2}{3})^n$.

$= \frac{4}{11}\cdot(\frac{2}{3})^n$.

7' (d) $K=2$, $H(z) = \frac{1-\frac{2}{3}z^{-1}}{1+\frac{1}{2}z^{-1}}$, $|z| > \frac{1}{2}$.

$X(z) = \frac{1}{1-\frac{2}{3}z^{-1}}$, $|z| > \frac{2}{3}$ $\Rightarrow Y(z) = \frac{1}{1+\frac{1}{2}z^{-1}}$, $|z| > \frac{1}{2}$.

$\therefore y[n] = (-\frac{1}{2})^n u[n]$.

4. Solutions:

(a) 两边 ZT:

$$\cancel{Y[n]}\ Y(z)\cdot z + Y(z) + \frac{1}{4}Y(z)\cdot z^{-1} = X(z)\cdot z - X(z).$$

$$\therefore\ H(z)=\frac{Y(z)}{X(z)}=\frac{z-1}{z+1+\frac{1}{4}z^{-1}}=\frac{1-z^{-1}}{1+z^{-1}+\frac{1}{4}z^{-2}}=\frac{1-z^{-1}}{(1+\frac{1}{2}z^{-1})^2}$$

poles: $-\frac{1}{2}, -\frac{1}{2}$.

Causal, $|z|>1/2$

(b). $\cancel{h[n]}\ H(z)=\frac{A}{(1+\frac{1}{2}z^{-1})}+\frac{B}{(1+\frac{1}{2}z^{-1})^2}=\frac{1-z^{-1}}{(1+\frac{1}{2}z^{-1})^2}$ $A=-2,\ B=+3$.

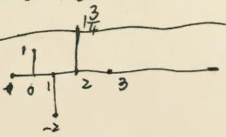
$$=(-2)\frac{1}{1+\frac{1}{2}z^{-1}}+3\cdot\frac{1}{(1+\frac{1}{2}z^{-1})^2}.\qquad |z|>1/2.$$

$$(-z)\left(\frac{1}{1+\frac{1}{2}z^{-1}}\right)'\cdot(-2z)$$

$$h[n] = -2\,(-\tfrac{1}{2})^{n}u[n] - 6(n+1)(-\tfrac{1}{2})^{n+1}u[n+1]$$

$(-\frac{1}{2})^{n-1}u[n]$ 或 $+3(n+1)(-\frac{1}{2})^{n}u[n+1]$

$(n+1)(-\frac{1}{2})^{n}u[n]-n(-\frac{1}{2})^{n-1}u[n-1]$



ROC 含单位圆，故 stable.

(c) $x[n]=\cos\pi n = (-1)^n \longrightarrow \boxed{LTI} \longrightarrow y[n] = H(z)|_{z=-1}\cdot(-1)^n$

$$=\frac{1-(-1)^{-1}}{1+\frac{1}{2}(-1)^{-1}}\cdot(-1)^n = 8\cdot(-1)^n.$$

(d). $H(z)=H_1(z)\cdot H_2(z)=\frac{1}{1+z^{-1}+\frac{1}{4}z^{-2}}\cdot(1-z^{-1})$

$$V(z)=X(z)\cdot H_1(z)=\frac{1}{1+z^{-1}+\frac{1}{4}z^{-2}}X(z).$$

$$Y(z)=V(z)\cdot H_2(z)=(1-z^{-1})V(z)=V(z)-V(z)\cdot z^{-1}.$$

$\therefore$ 构建 $X(z)\to V(z)$ 　　　　构建 $V(z)\to Y(z)$