Q1. (1) B.

令 $f_1(t)=(t-1)u(t-1)$, $f_0(t)=tu(t) \xleftrightarrow{\mathcal{L}} F_0(s)=\frac{1}{s^2}$, $Re\{s\}>0$

$\therefore f_1(t)=f_0(t-1) \xleftrightarrow{\mathcal{L}} F_1(s)=e^{-s}\cdot F_0(s)=\frac{e^{-s}}{s^2}$, $Re\{s\}>0$

$f(t)=tu(t-1)=f_1(t)+u(t-1) \xleftrightarrow{\mathcal{L}} F_1(s)+\frac{e^{-s}}{s}=\frac{e^{-s}}{s^2}+\frac{e^{-s}}{s}$

$=\frac{s+1}{s^2}e^{-s}$

$Re\{s\}>0$

(2) D

Q2. (1) $X(0^+)=\lim_{s\to\infty}sX(s)=\lim_{s\to\infty}\frac{s(2s+1)}{s(s^2+3s+2)}=0$

$X(\infty)=\lim_{s\to 0}sX(s)=\lim_{s\to 0}\frac{s(2s+1)}{s(s^2+3s+2)}=\frac{1}{2}=0.5$

(2) $x(t)=e^{2t}$

$x(t)=e^{2t} \longrightarrow y(t)=H(s)\big|_{s=2}\cdot e^{2t}$

$H(s)\big|_{s=2}=\frac{1}{s^2+5s+6}\Big|_{s=2}=\frac{1}{20}$

$\therefore y(t)=\frac{1}{20}e^{2t}$

H(s) Rational,

ROC: $Re\{s\}>-2$ 在最右极点右边, 则系统稳定 stable. (Yes)

(3) 两边 LT:

$sY(s)+5Y(s)=X(s)\cdot Z(s)-X(s)$

$Z(s)=3\cdot\frac{1}{s+1}+2=\frac{3+2(s+1)}{s+1}=\frac{2s+5}{s+1}$

$\therefore sY(s)+5Y(s)=X(s)\cdot\frac{2s+5}{s+1}-X(s)=X(s)\cdot\frac{s+4}{s+1}$

$\therefore H(s) = \frac{Y(s)}{X(s)} = \frac{s+4}{(s+5)(s+1)}$. causual. ROC: $Re\{s\} > -1$

$\therefore H(j\omega) = \frac{j\omega+4}{(j\omega+5)(j\omega+1)}$

$\therefore h(t) = \mathcal{L}^{-1}\{H(s)\}$

$H(s) \xrightarrow{\text{部分展开式}} \frac{A}{s+5} + \frac{B}{s+1}$

$A = H(s)\cdot(s+5)\Big|_{s=-5} = \frac{s+4}{s+1}\Big|_{s=-5}$

$H(s) = \frac{1}{4}\cdot\frac{1}{s+5} + \frac{3}{4}\cdot\frac{1}{s+1}$

$B = H(s)(s+1)\Big|_{s=-1} = \frac{s+4}{s+5}\Big|_{s=-1} = \frac{3}{4}$

$\therefore h(t) = \frac{1}{4}e^{-5t}u(t) + \frac{3}{4}e^{-t}u(t)$

Q3. From clues 1 and 2.

$X(s) = \frac{A}{(s+a)(s+b)}$

Furthermore. we are given that one of the poles $X(s)$ is $-1+j$. Since $x(t)$ is real. $X(s)$'s poles must occur in conjugate reciprocal pairs. There, $a=1-j$ and $b=1+j$ and $H(s) = \frac{A}{(s+1-j)(s+1+j)}$

From clue 5, we know that $X(0)=8$. Therefore, we may deduce that $A=16$ and $H(s) = \frac{16}{s^2+2s+2}$

Therefore, $Re\{s\} < -1$ or $Re\{s\} > -1$, we will now use clue 4 to pick one. $y(t) = e^{2t}x(t) \xleftrightarrow{\mathcal{L}} Y(s) = X(s-2)$

Since it's given that $y(t)$ is not absolumely integrable, the ROC of $Y(s)$ should not include the $j\omega$-axis. This is possible only if R is $Re\{s\} > -1$

(13.2) Solution:

(a) Zeros: $\gamma_1 = 1$

Poles: $\lambda_1 = -2$, $\lambda_2 = -1$.

$\therefore \quad H(s) = K\dfrac{(s-1)}{(s+1)(s+2)}$;

for Rational $H(s)$

Causal, ROC: $Re\{s\} > -1$.

$h(0^+) = 3 = \lim\limits_{s\to\infty} s \cdot H(s) = K = 3$.

$\therefore \quad K = 3$.

(b) $h(t) = \mathcal{L}^{-1}\{H(s)\}$.

$H(s) \overset{\text{部分分式展开}}{=\!=} 3\left[\dfrac{A}{s+1} + \dfrac{B}{s+2}\right]$

$A = H(s)[s+1]\big|_{s=-1} = \dfrac{s-1}{s+2}\Big|_{s=-1} = -2.$

$B = H(s)(s+2)\big|_{s=-2} = \dfrac{s-1}{s+1}\Big|_{s=-2} = 3.$

$\therefore H(s) = 3\left[\dfrac{-2}{s+1} + \dfrac{3}{s+2}\right]$, $Re\{s\} > -1$.

$\therefore \quad h(t) = -6\, e^{-t}u(t) + 9\, e^{-2t}u(t)$.

ROC 包含 $j\omega$-轴, 故 stable.

(c) $\quad x(t) = e^{t} \longrightarrow y(t) = H(s)\big|_{s=1} \cdot e^{t}$

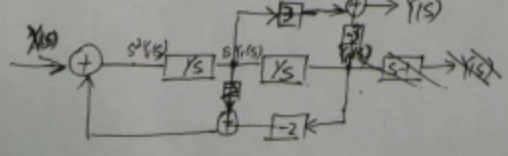
$= 0 \cdot e^{t} = 0$.

(d) $\dfrac{Y(s)}{X(s)} = H(s) = \dfrac{3(s-1)}{(s+1)(s+2)} = \dfrac{3(s-1)}{s^2+3s+2} \Rightarrow s^2Y(s) + 3sY(s) + 2Y(s) = 3[sX(s) - X(s)]$

$\therefore$ 两边 $\mathcal{L}^{-1}$, 得 LCCDE: $\dfrac{d^2y(t)}{dt^2} + 3\dfrac{dy(t)}{dt} + 2y(t) = 3\dfrac{dx(t)}{dt} - 3x(t)$

(e) $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{3(s-1)}{s^2+3s+2} = \dfrac{1}{s^2+3s+2} \cdot 3(s-1)$; 令 $H_1(s) = \dfrac{1}{s^2+3s+2}$

$H_2(s) = s-1$.



$\therefore Y_1(s) = H_1(s) \cdot X(s)$

$\therefore s^2Y_1(s) = X(s) - 3sY_1(s) - 2Y_1(s)$

$Y(s) = 3(s-1)Y_1(s)$

$= 3sY_1(s) - 3Y_1(s)$

Q5. (1) $e(t)$表示图中所示信号

$\because X(s)+E(s)\left(-\frac{5}{s}\right)+E(s)\left(-\frac{6}{s^2}\right)=E(s)$

$X(s)=E(s)\left(1+\frac{5}{s}+\frac{6}{s^2}\right)$

$\therefore H_1(s)=\frac{E(s)}{X(s)}=\frac{s^2}{s^2+5s+6}$

$E(s)\cdot\frac{1}{s}+E(s)\cdot\frac{1}{s^2}=Y(s)$

$\therefore H_2(s)=\frac{Y(s)}{E(s)}=\frac{s+1}{s^2}$

$\therefore H(s)=H_1(s)H_2(s)=\frac{s^2}{s^2+5s+6}\cdot\frac{s+1}{s^2}=\frac{s+1}{(s+2)(s+3)}$

$\because$系统为稳定系统

$\therefore H(s)$的收敛域包括虚轴

$\therefore H(s)$的收敛域为 $Re\{s\}>-2$

$\therefore H(s)=\frac{s+1}{(s+2)(s+3)}, Re\{s\}>-2$

(2) $H(s)=\frac{2}{s+3}-\frac{1}{s+2}$, $\therefore h(t)=2e^{-3t}u(t)-e^{-2t}u(t)=\left(2e^{-3t}-e^{-2t}\right)u(t)$

$\because t<0$时, $h(t)=0$

$\therefore$系统是因果的

(3) $y(t)=e^{-t}H(-1)=0$