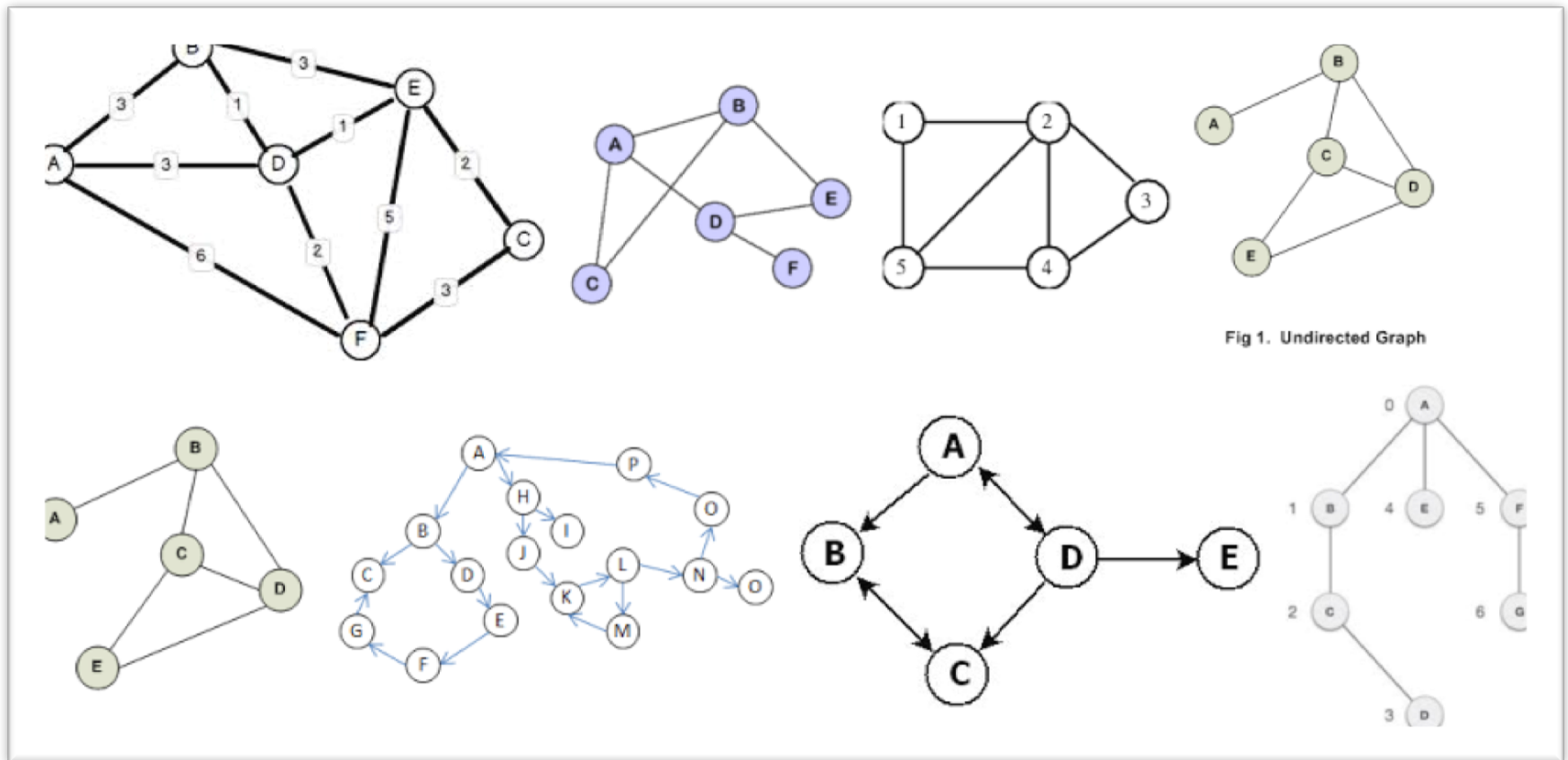


Graph

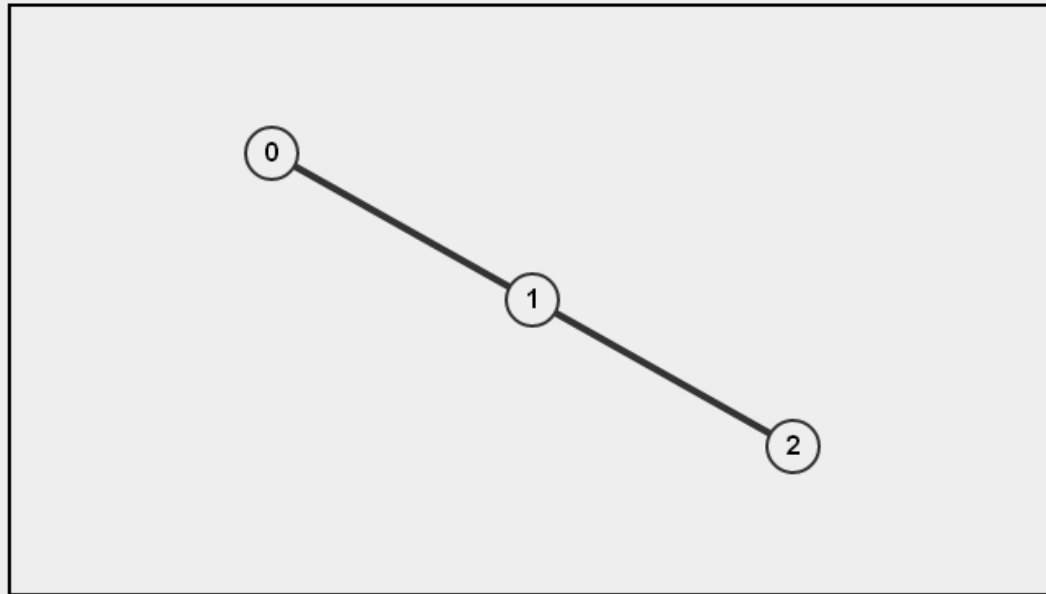
Dr. Seung Chul Han
Dept. Computer Engineering
Myongji University

Graph

- $G=\{V,E\}$ V: set of vertex, E: set of edges
- Directed/undirected



Graph representation 1: Array



• Tree? **Yes**

• Complete? **No**

• Bipartite? **Yes**

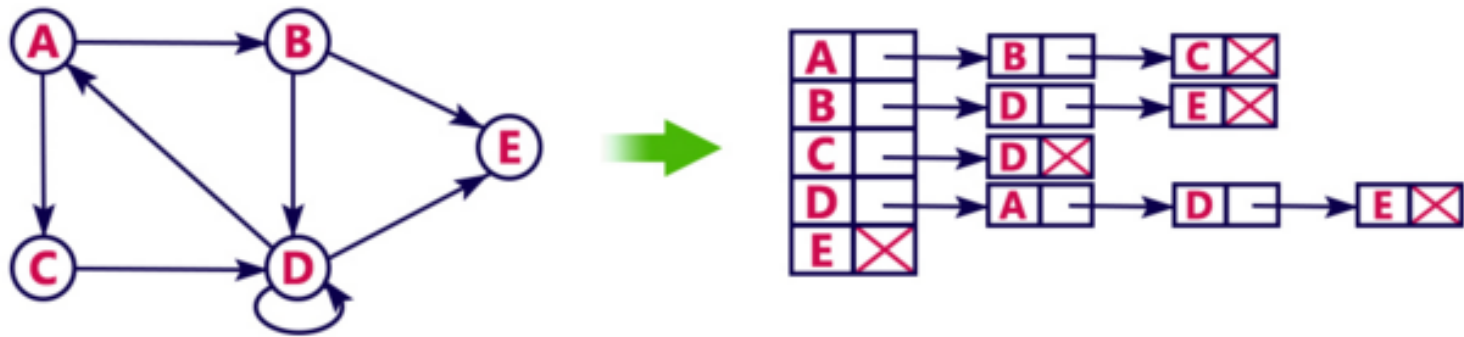
• DAG? **No**

Adjacency Matrix			
	0	1	2
0	0	1	0
1	1	0	1
2	0	1	0

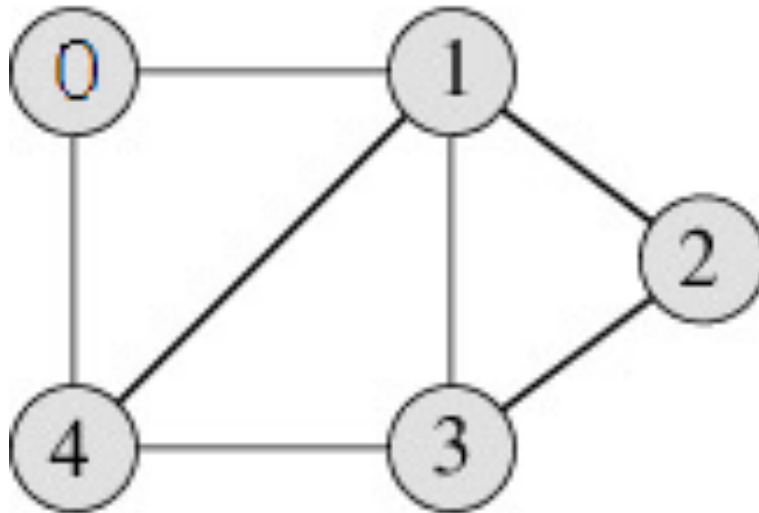
Adjacency List		
0:	1	
1:	0	2
2:	1	

Edge List			
0:	0	1	
1:	1	2	

Graph representation 2: Linked List

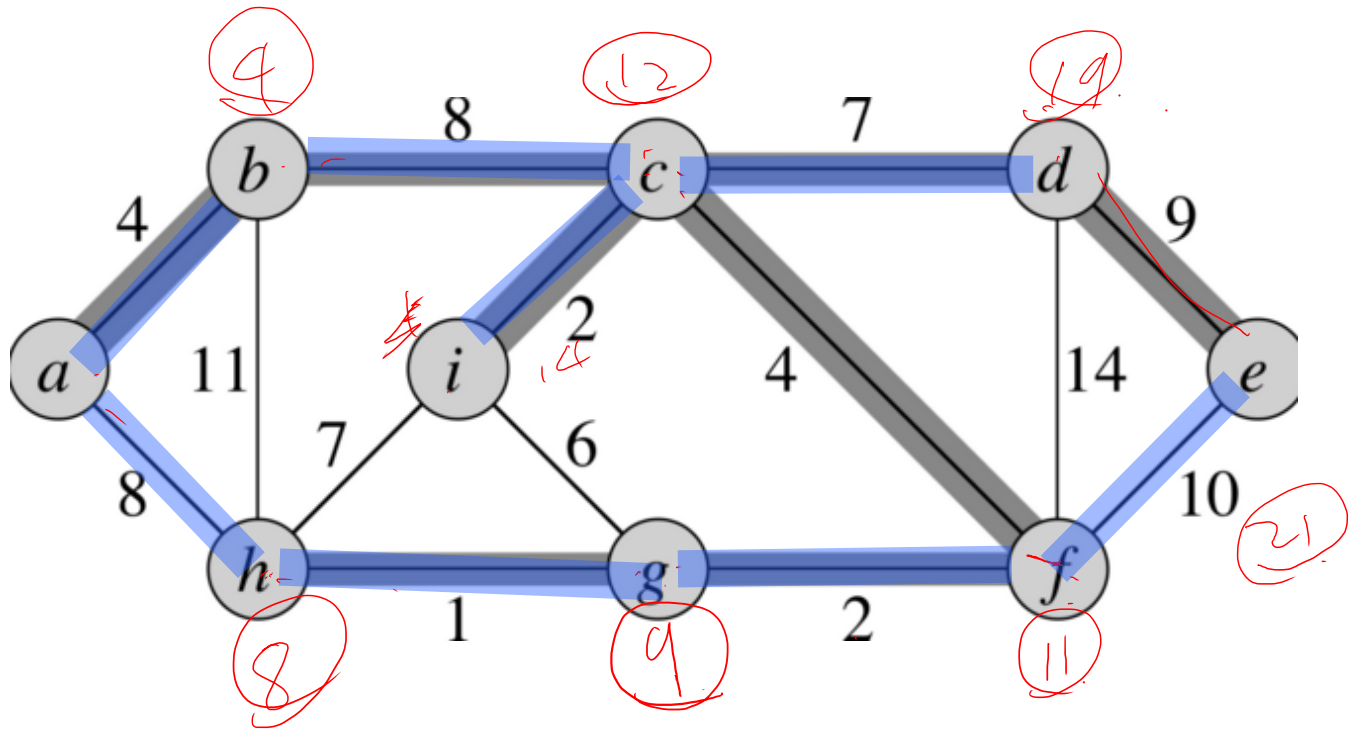


Quiz. 다음을 Array와 Linked List로 표현



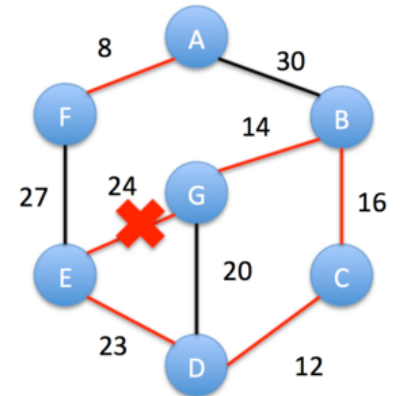
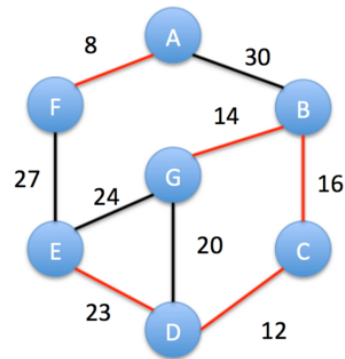
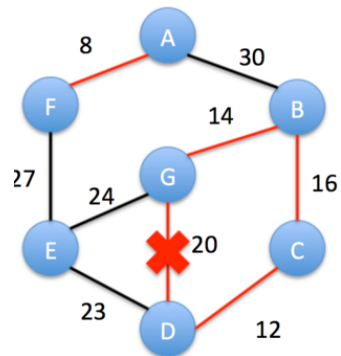
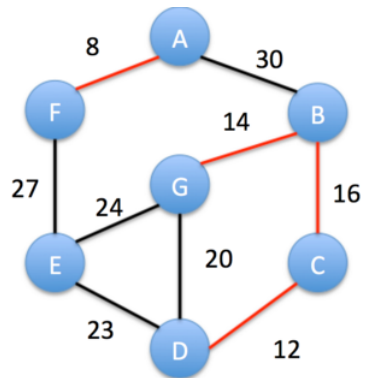
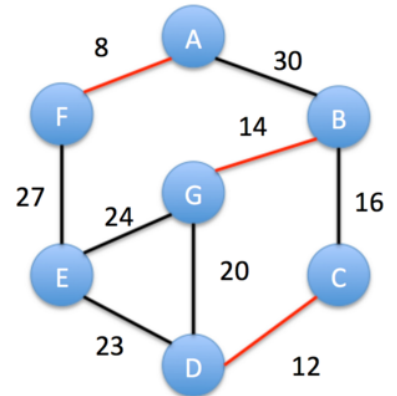
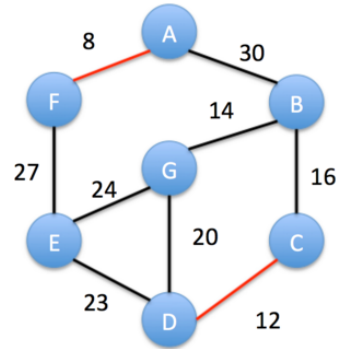
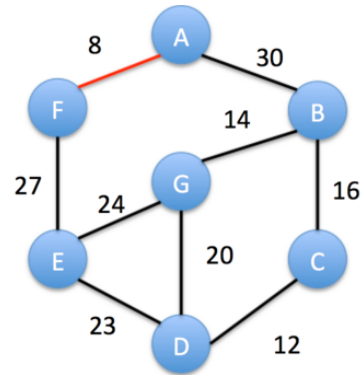
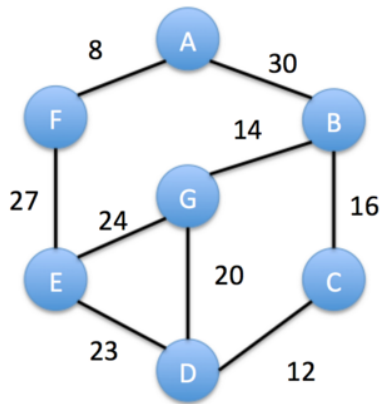
Spanning Tree

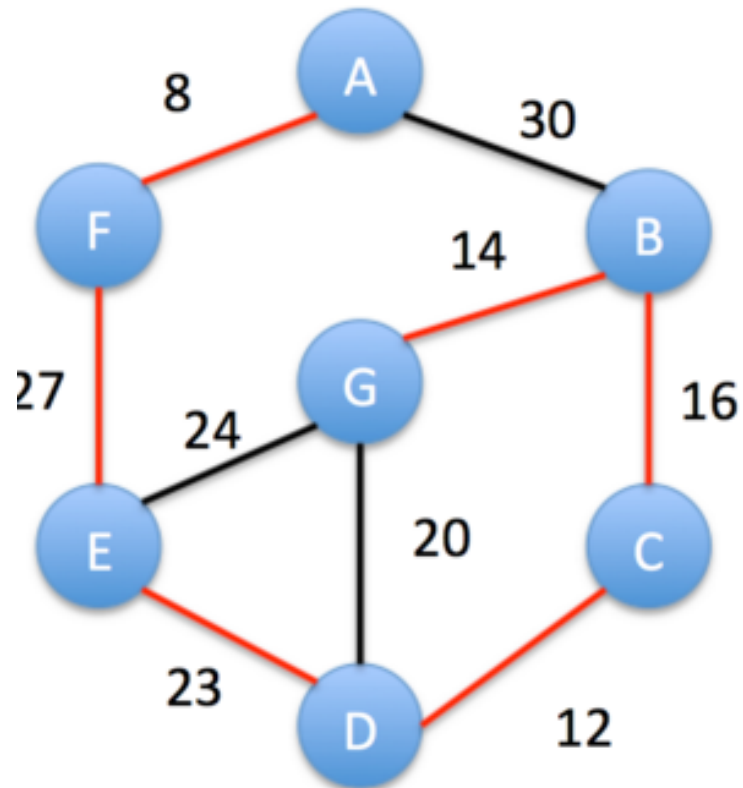
- $G=\{V,E\}$ undirected이고 각 edge (u,v) 에 값이 있을때, 모든 노드들을 연결하며 최소 edge값들을 갖도록 연결한 트리

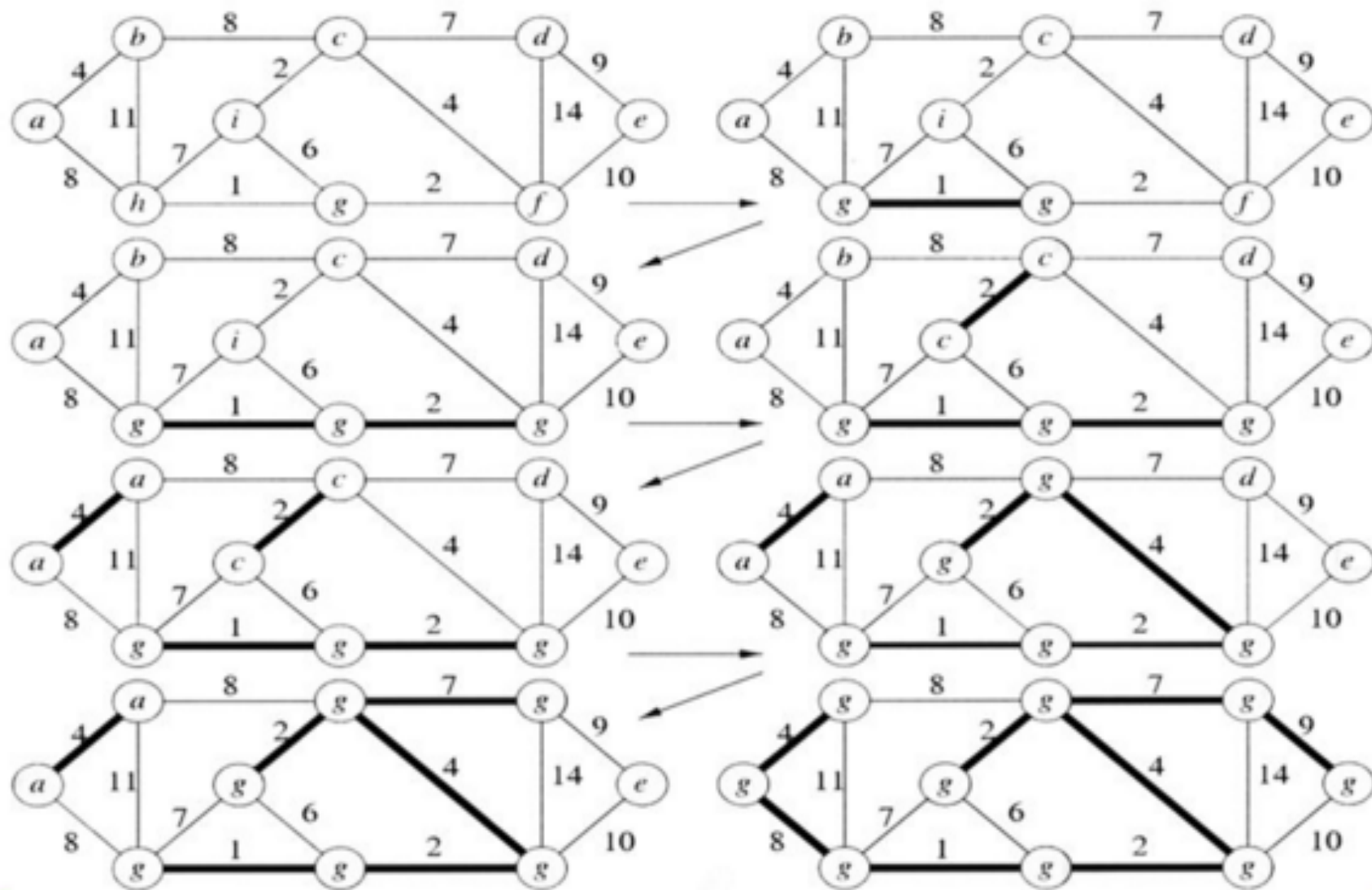


Kruskal's Algorithm

1. 모든 edge들을 작은 값부터 정렬;
2. 작은 값의 edge부터 하나씩 뽑아서, cycle을 만들지 않으면 포함;
3. 모든 vertex가 연결될 때까지 2를 반복;

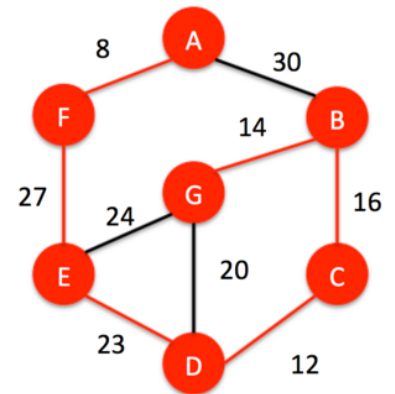
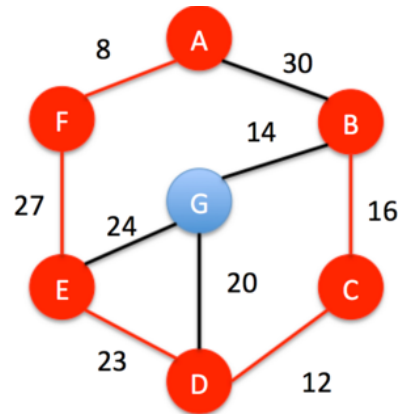
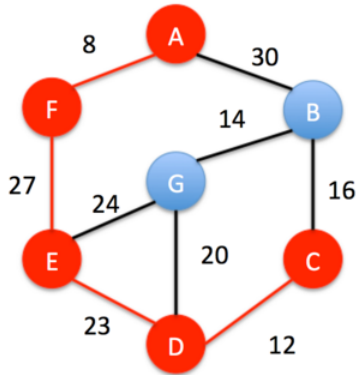
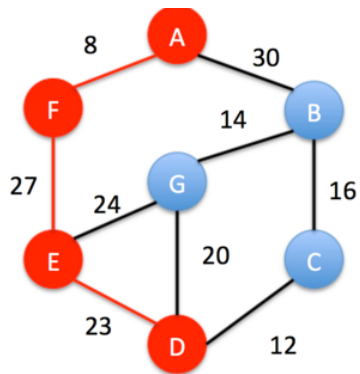
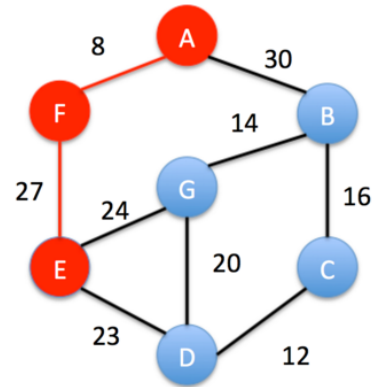
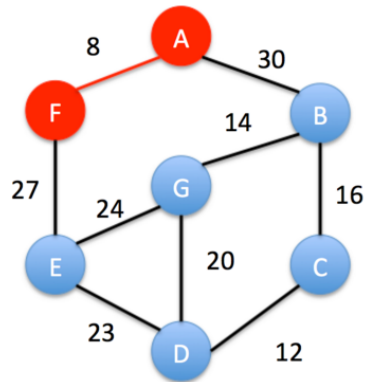
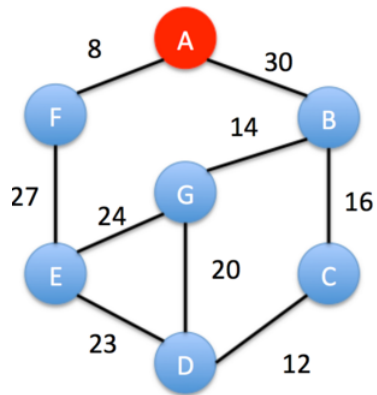
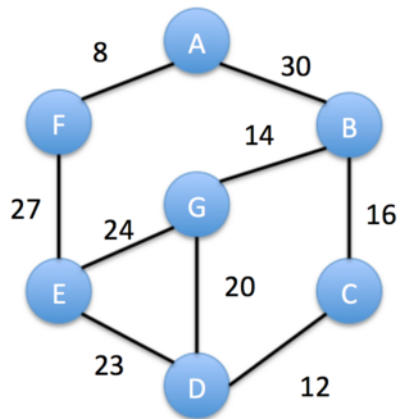






Prim's Algorithm

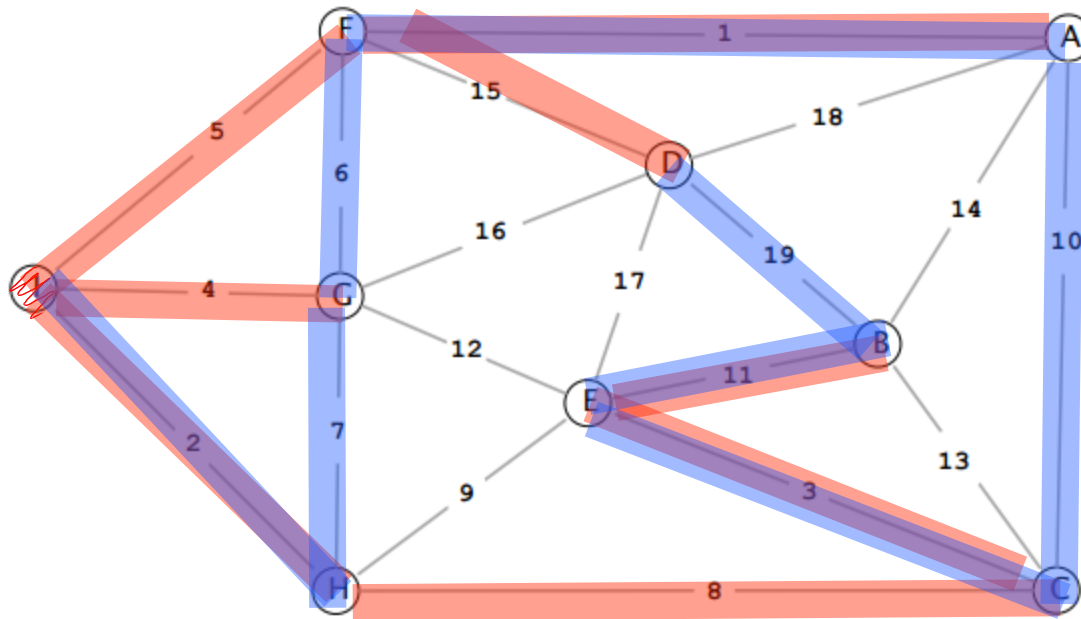
1. 임의의 vertex선택;
2. 선택된 vertex들 중 최소 값을 갖는 edge와 연결된 vertex를 선택;
3. 모든 vertex가 연결될 때까지 2를 반복;



Quiz. www.cs.princeton.edu/courses/archive

3. Minimum spanning trees. (8 points)

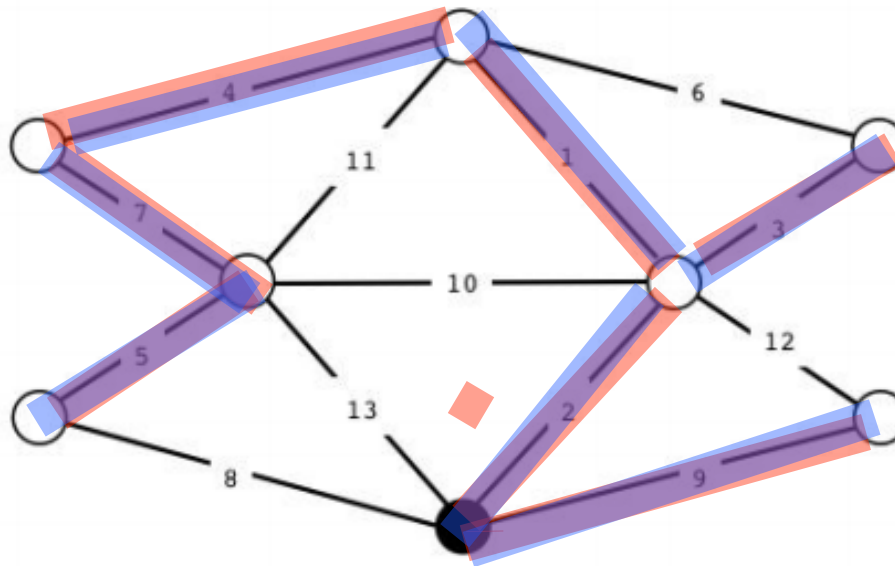
Consider the following edge-weighted graph with 9 vertices and 19 edges. Note that the edge weights are distinct integers between 1 and 19.



(a) Kruskal's algorithm

(b) Prim's algorithm

1. **MST** (10 points). Consider the following graph (numbers are edge weights):



Kruskal?

Prim

A. Give the list of edge weights in the MST in the order that *Kruskal's algorithm* inserts them.

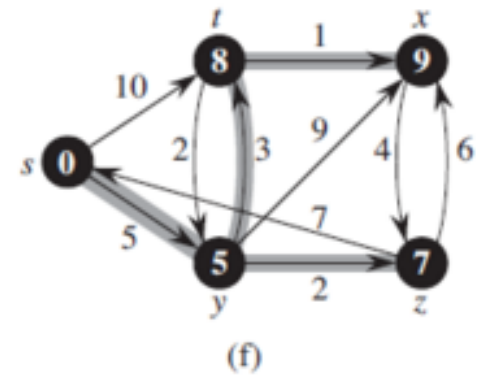
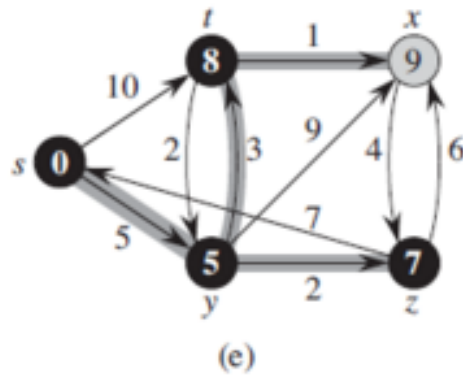
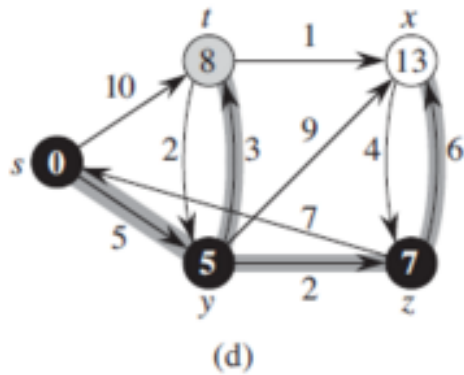
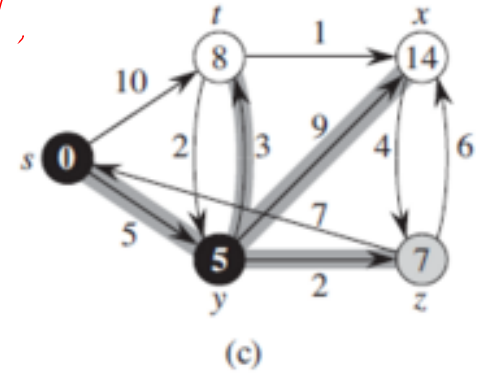
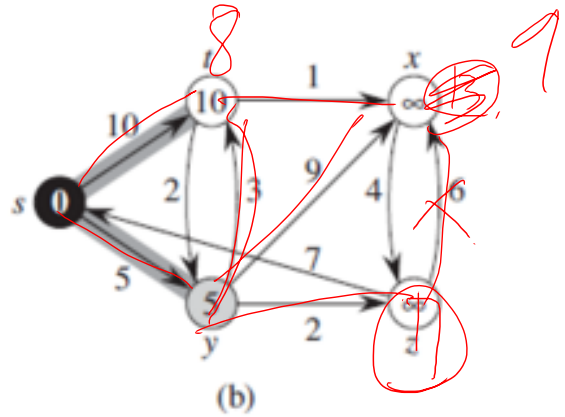
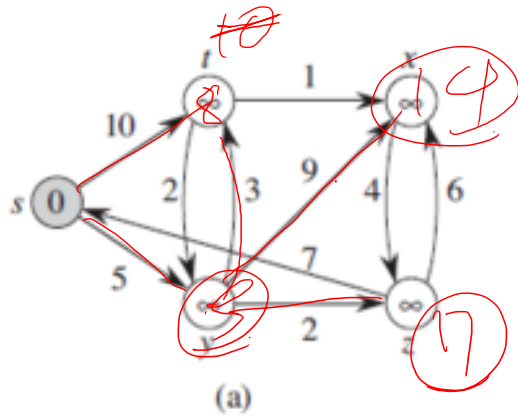
B. Give the list of edge weights in the MST in the order that *Prim's algorithm* inserts them, assuming that it starts at the black vertex.

Single Source Shortest Path

- Dijkstra's Algorithm
- Weighted, directed, positive graph

```
1 Initialization:  
2  $N' = \{u\}$   
3 for all nodes  $v$   
4   if  $v$  adjacent to  $u$   
5     then  $D(v) = c(u,v)$   
6   else  $D(v) = \infty$   
7  
8 Loop  
9   find  $w$  not in  $N'$  such that  $D(w)$  is a minimum  
10  add  $w$  to  $N'$   
11  update  $D(v)$  for all  $v$  adjacent to  $w$  and not in  $N'$  :  
12     $D(v) = \min( D(v), D(w) + c(w,v) )$   
13    /* new cost to  $v$  is either old cost to  $v$  or known  
14     shortest path cost to  $w$  plus cost from  $w$  to  $v$  */  
15 until all nodes in  $N'$ 
```







그 외 shortest path algorithms

- Bellman-Ford Algorithm
 - Edge 값이 negative도 가능
- Floyd-Warshall Algorithm
 - All pairs shortest paths, no negative