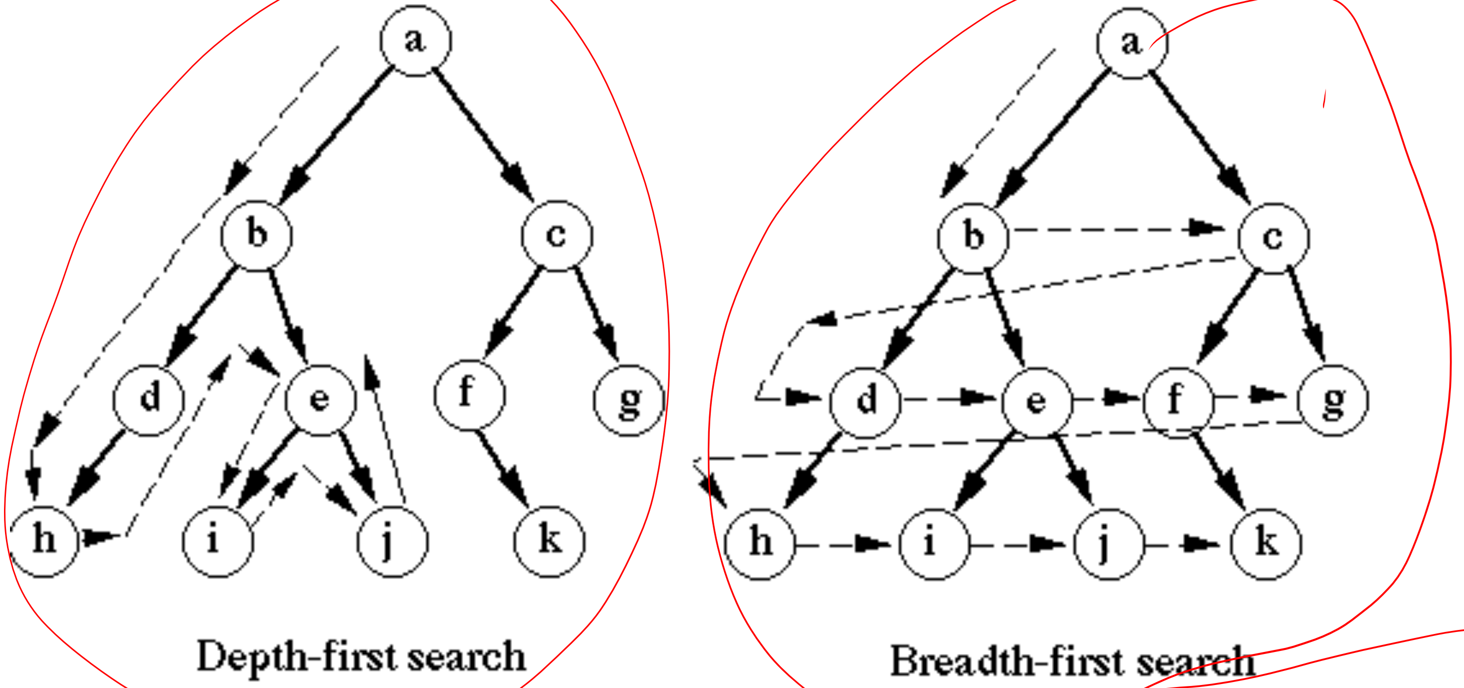
TREE (part 2)

Dr. Seung Chul Han
Dept. Computer Engineering
Myongji University

Tree Search

- Depth-First Search / Breadth-First-Search



Depth-first search

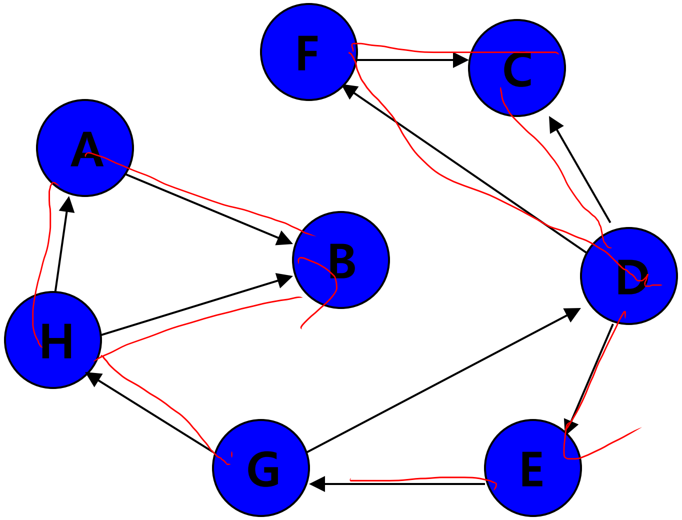
Breadth-first search

사이클 확인, stack을 사용

최단거리 확인, queue 사용

Example: DFS

사이클확인.

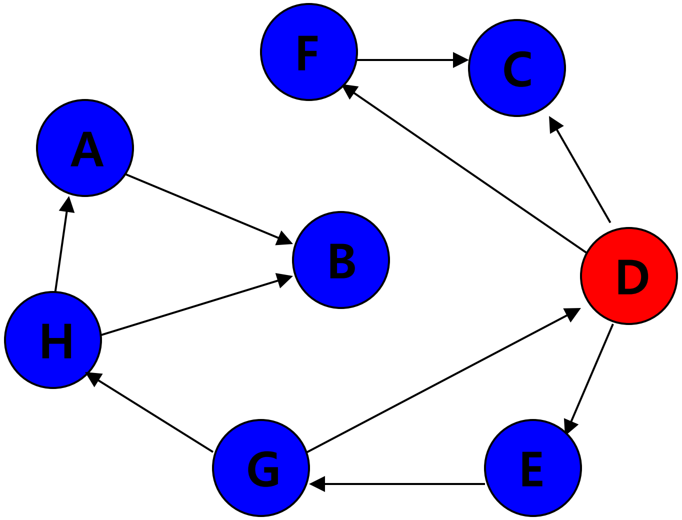


Visited Array

A	✓
B	✓
C	✓
D	✓
E	✓
F	✓
G	✓
H	✓

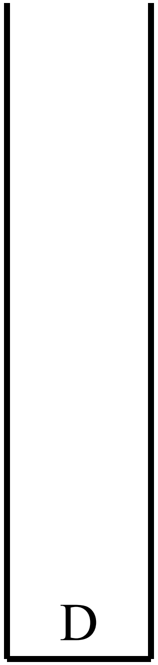
Task: Conduct a depth-first search of the graph starting with node D

Walk-Through



Visited Array

A	
B	
C	
D	√
E	
F	
G	
H	

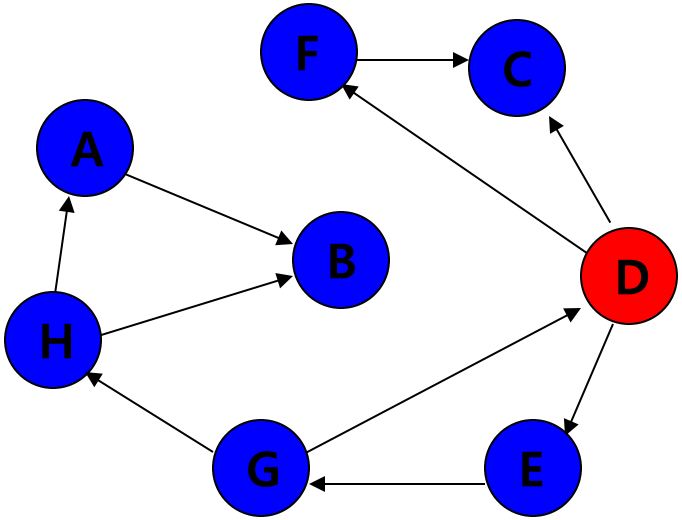

D

The order nodes are visited:

D

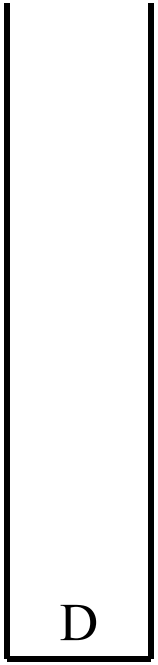
Visit D

Walk-Through



Visited Array

A	
B	
C	
D	√
E	
F	
G	
H	

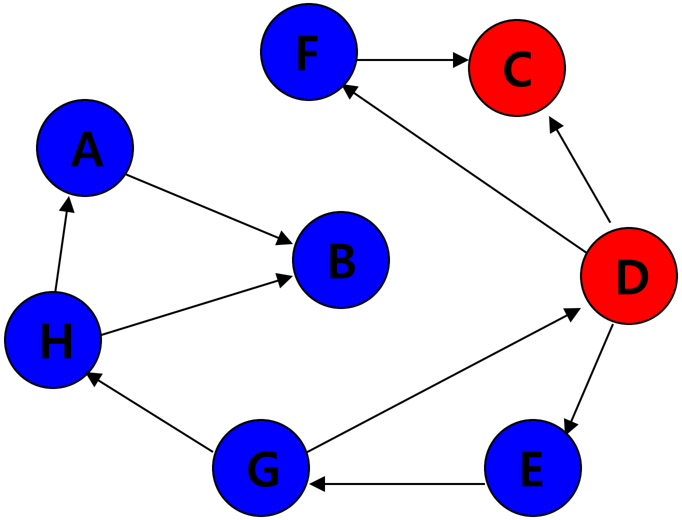


The order nodes are visited:

D

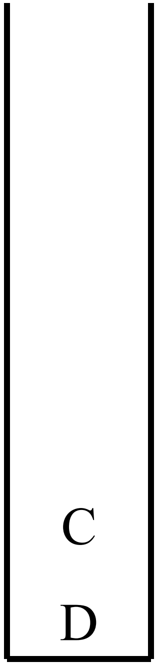
Consider nodes adjacent to D, decide to visit C first (Rule: visit adjacent nodes in alphabetical order)

Walk-Through



Visited Array

A	
B	
C	√
D	√
E	
F	
G	
H	

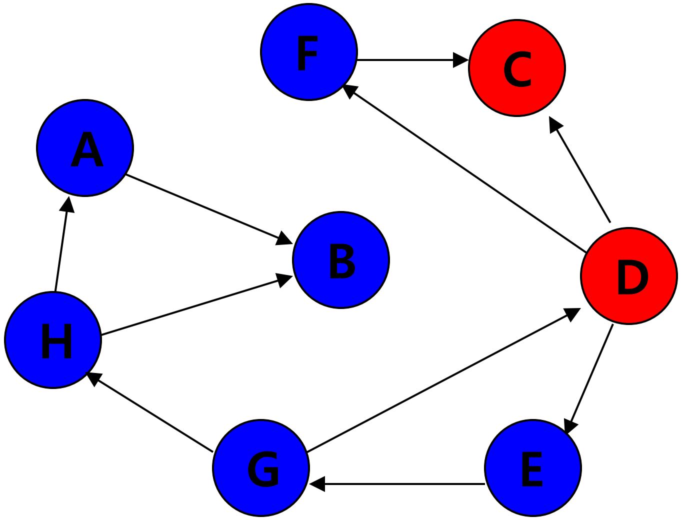


Visit C

The order nodes are visited:

D, C

Walk-Through



Visited Array

A	
B	
C	√
D	√
E	
F	
G	
H	

C

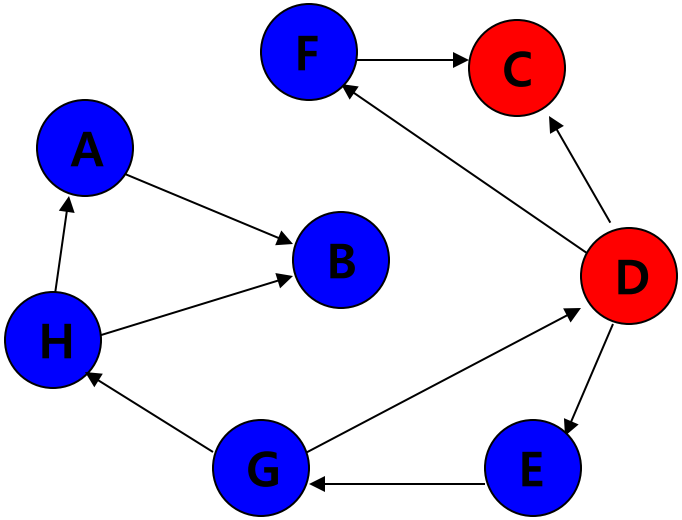
D

The order nodes are visited:

D, C

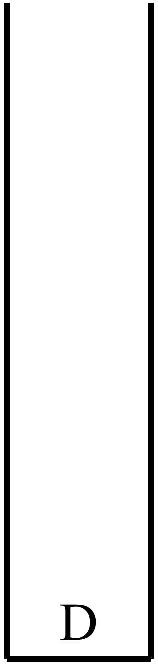
No nodes adjacent to C; cannot continue ➔ *backtrack*, i.e., pop stack and restore previous state

Walk-Through



Visited Array

A	
B	
C	√
D	√
E	
F	
G	
H	

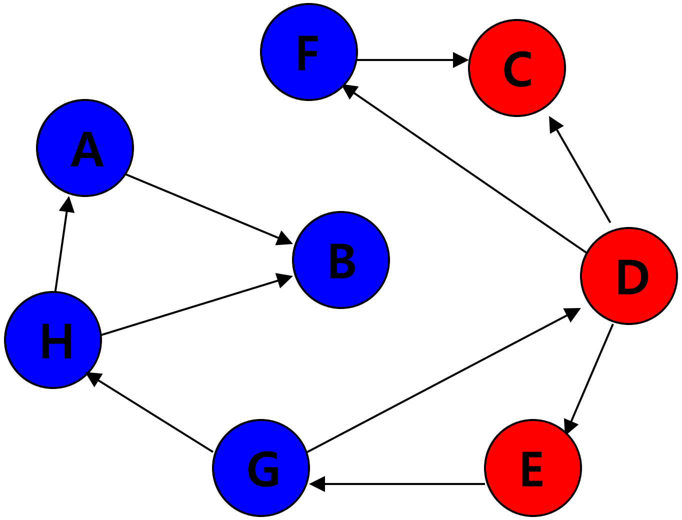


The order nodes are visited:

D, C

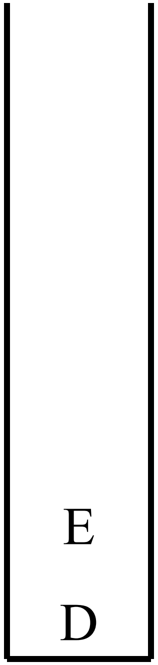
Back to D – C has been visited, decide to visit E next

Walk-Through



Visited Array

A	
B	
C	√
D	√
E	√
F	
G	
H	

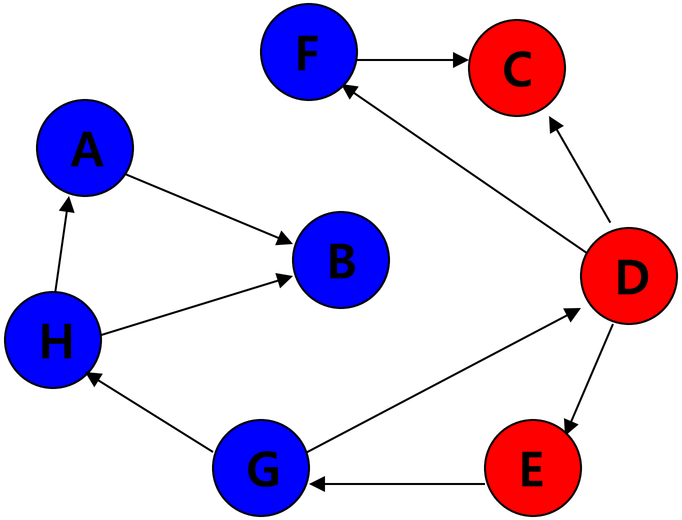


The order nodes are visited:

D, C, E

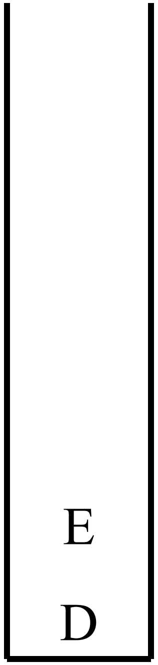
Back to D – C has been visited, decide to visit E next

Walk-Through



Visited Array

A	
B	
C	√
D	√
E	√
F	
G	
H	


E

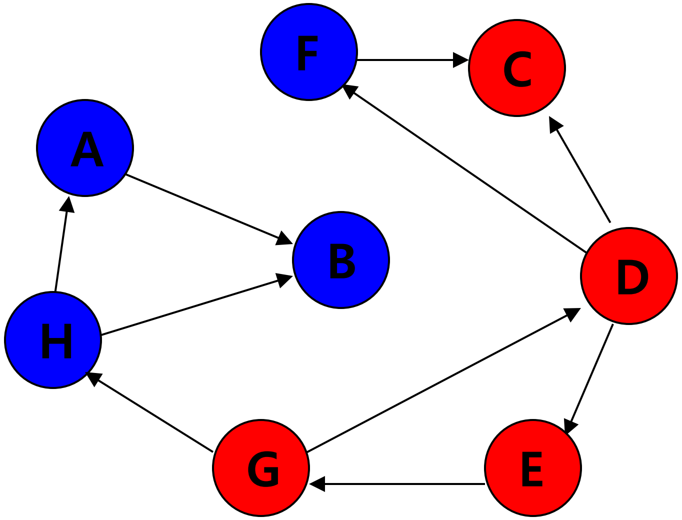
D

The order nodes are visited:

D, C, E

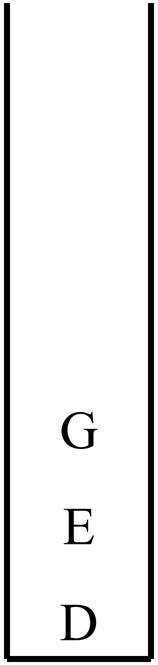
Only G is adjacent to E

Walk-Through



Visited Array

A	
B	
C	√
D	√
E	√
F	
G	√
H	



G

E

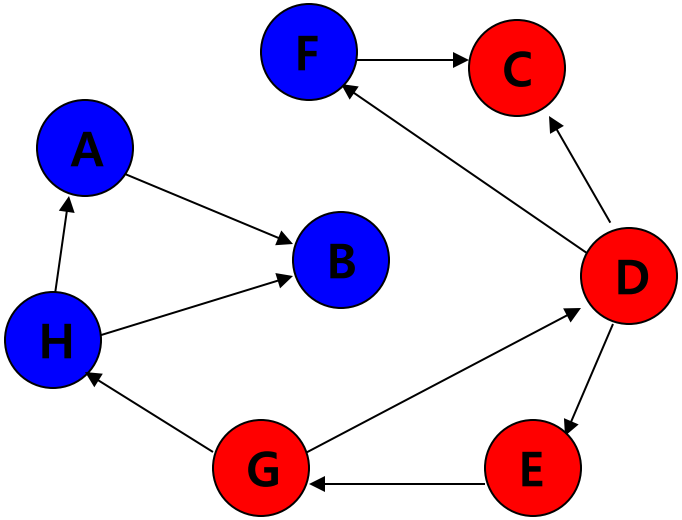
D

Visit G

The order nodes are visited:

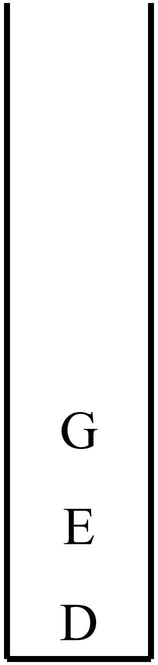
D, C, E, G

Walk-Through



Visited Array

A	
B	
C	√
D	√
E	√
F	
G	√
H	


G

E

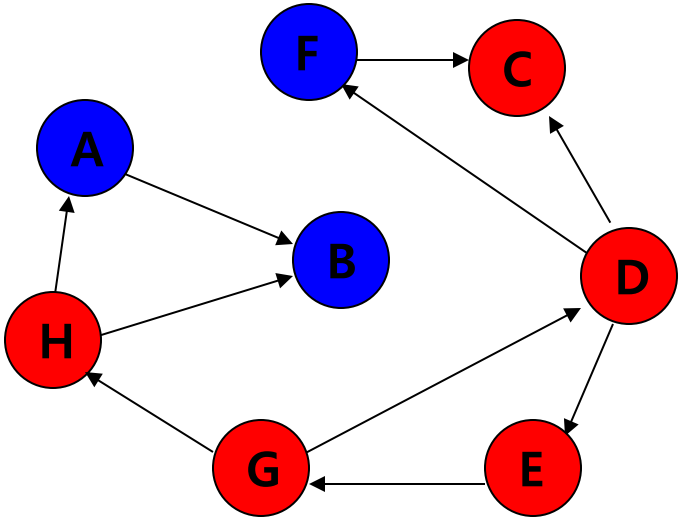
D

The order nodes are visited:

D, C, E, G

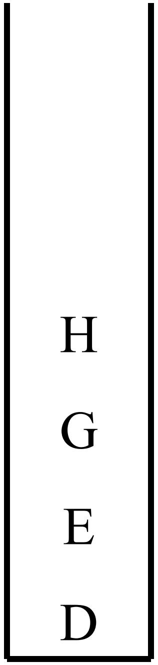
Nodes D and H are adjacent to G. D has already been visited. Decide to visit H.

Walk-Through



Visited Array

A	
B	
C	√
D	√
E	√
F	
G	√
H	√



H

G

E

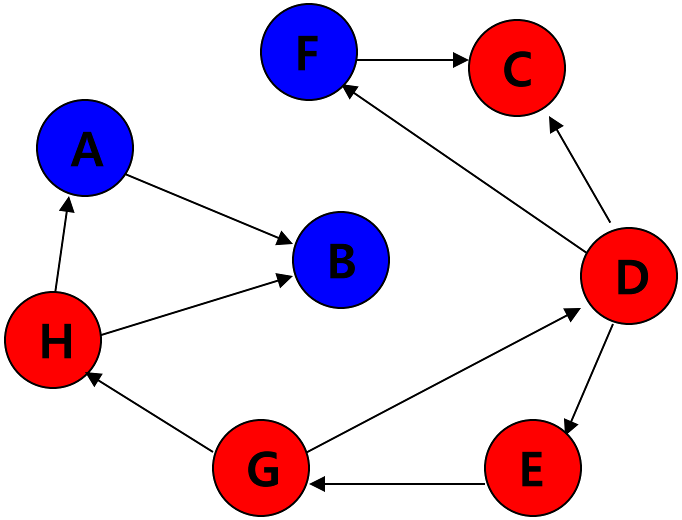
D

Visit H

The order nodes are visited:

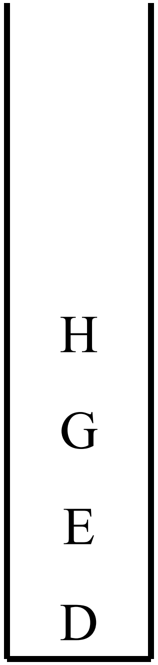
D, C, E, G, H

Walk-Through



Visited Array

A	
B	
C	√
D	√
E	√
F	
G	√
H	√



H

G

E

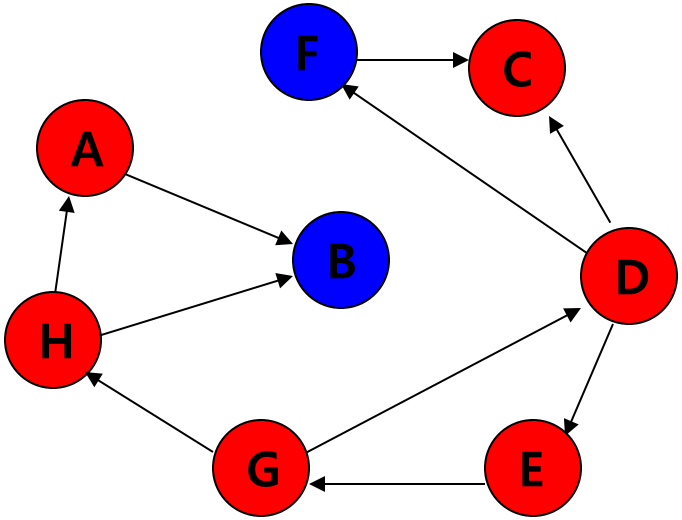
D

The order nodes are visited:

D, C, E, G, H

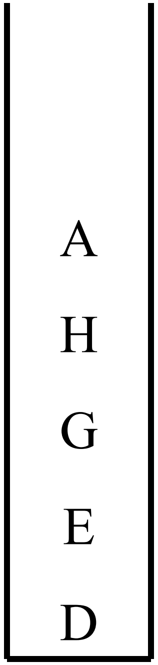
Nodes A and B are adjacent to F.
Decide to visit A next.

Walk-Through



Visited Array

A	√
B	
C	√
D	√
E	√
F	
G	√
H	√


A

H

G

E

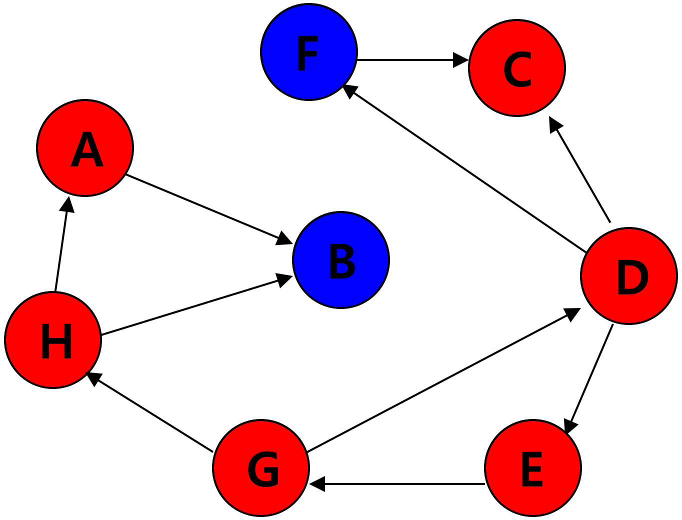
D

Visit A

The order nodes are visited:

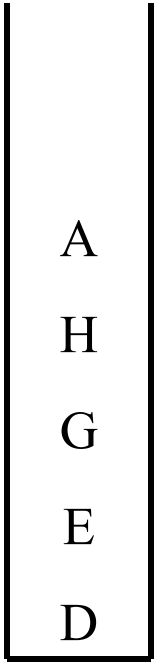
D, C, E, G, H, A

Walk-Through



Visited Array

A	√
B	
C	√
D	√
E	√
F	
G	√
H	√



A

H

G

E

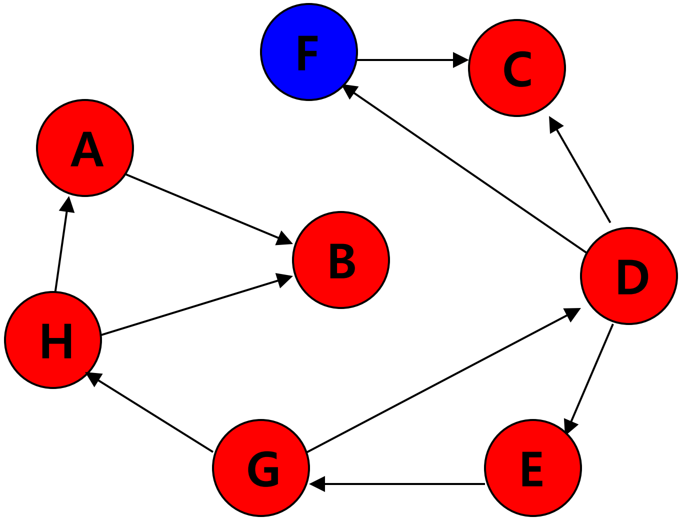
D

The order nodes are visited:

D, C, E, G, H, A

Only Node B is adjacent to A.
Decide to visit B next.

Walk-Through



Visited Array

A	√
B	√
C	√
D	√
E	√
F	
G	√
H	√

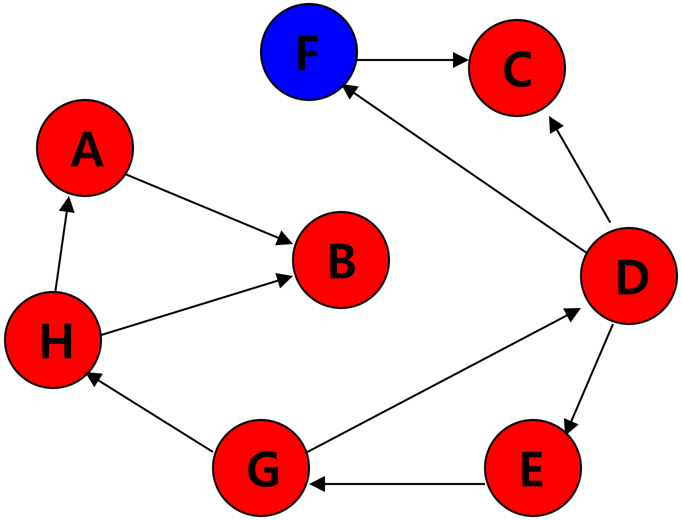
B
A
H
G
E
D

Visit B

The order nodes are visited:

D, C, E, G, H, A, B

Walk-Through



Visited Array

A	√
B	√
C	√
D	√
E	√
F	
G	√
H	√

A

H

G

E

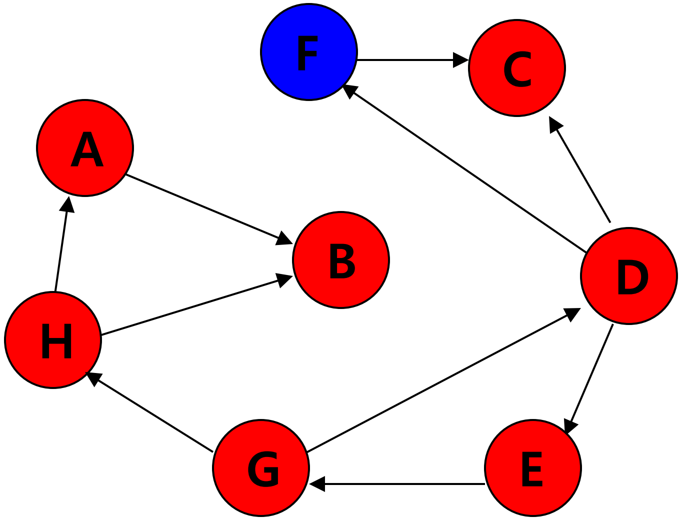
D

The order nodes are visited:

D, C, E, G, H, A, B

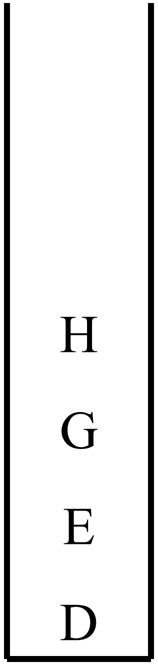
No unvisited nodes adjacent to B. Backtrack (pop the stack).

Walk-Through



Visited Array

A	√
B	√
C	√
D	√
E	√
F	
G	√
H	√



H

G

E

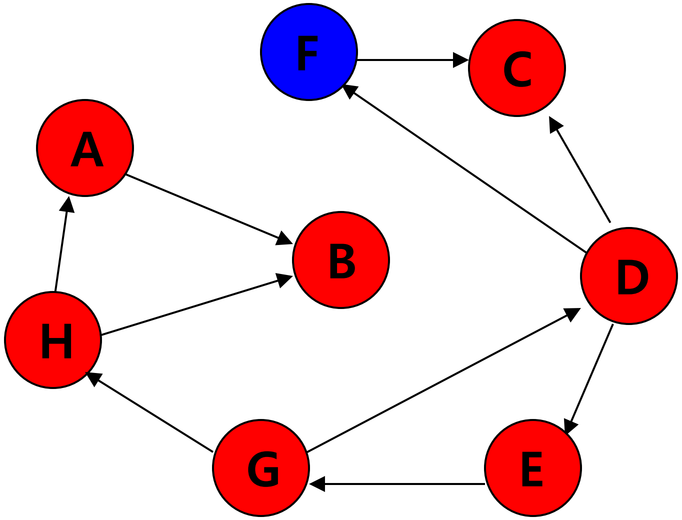
D

The order nodes are visited:

D, C, E, G, H, A, B

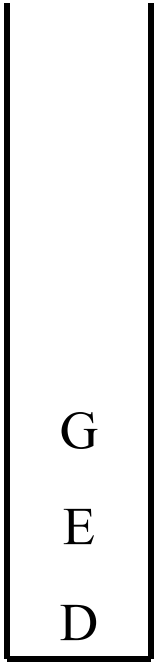
No unvisited nodes adjacent to A. Backtrack (pop the stack).

Walk-Through



Visited Array

A	√
B	√
C	√
D	√
E	√
F	
G	√
H	√



G

E

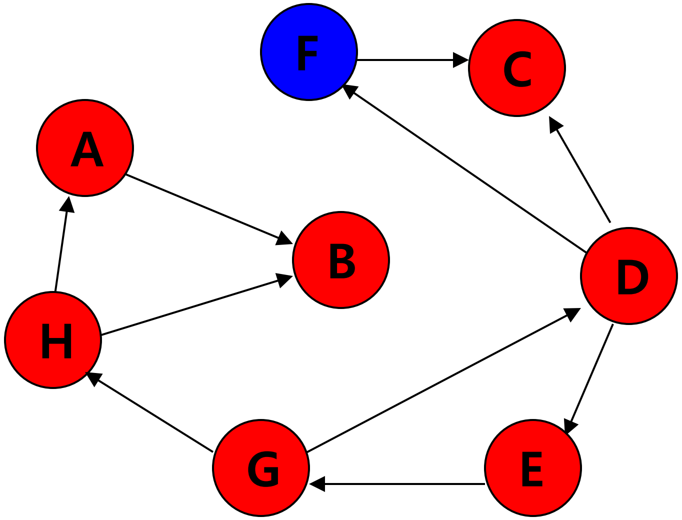
D

The order nodes are visited:

D, C, E, G, H, A, B

No unvisited nodes adjacent to H. Backtrack (pop the stack).

Walk-Through



Visited Array

A	√
B	√
C	√
D	√
E	√
F	
G	√
H	√

E

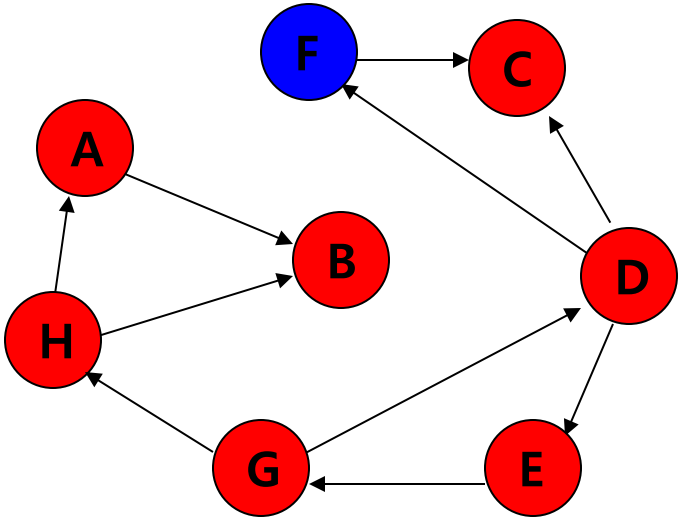
D

The order nodes are visited:

D, C, E, G, H, A, B

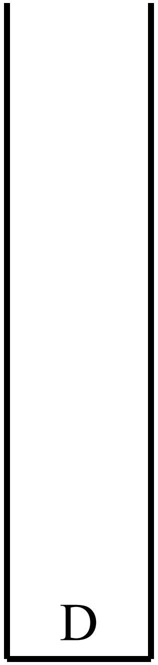
No unvisited nodes adjacent to G. Backtrack (pop the stack).

Walk-Through



Visited Array

A	√
B	√
C	√
D	√
E	√
F	
G	√
H	√

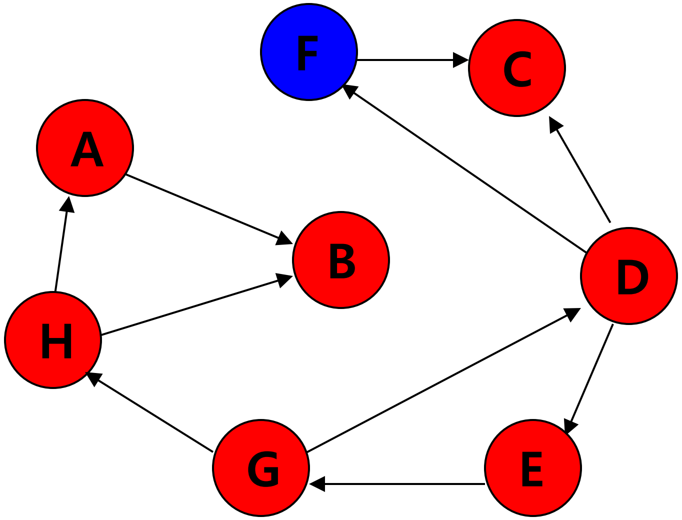


The order nodes are visited:

D, C, E, G, H, A, B

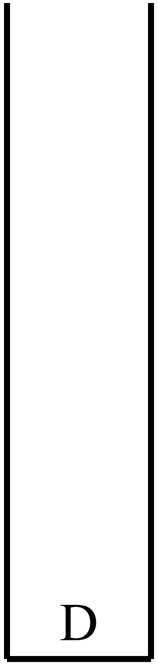
No unvisited nodes adjacent to E. Backtrack (pop the stack).

Walk-Through



Visited Array

A	✓
B	✓
C	✓
D	✓
E	✓
F	
G	✓
H	✓

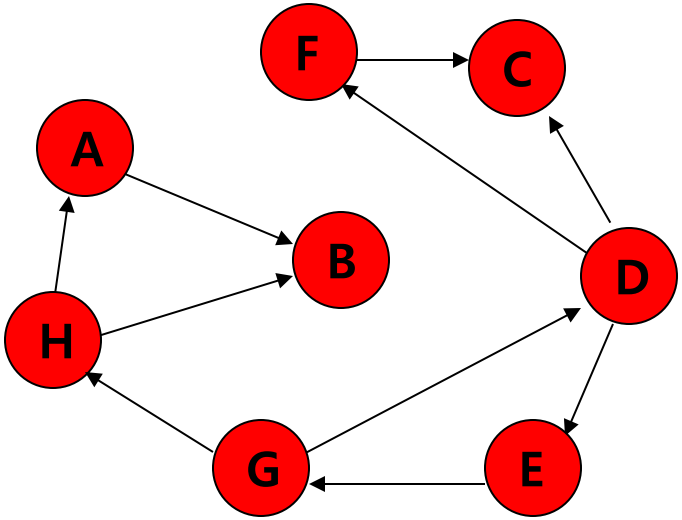


The order nodes are visited:

D, C, E, G, H, A, B

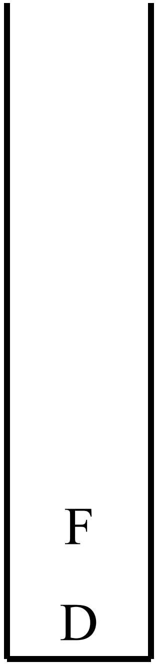
F is unvisited and is adjacent to D. Decide to visit F next.

Walk-Through



Visited Array

A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√



F

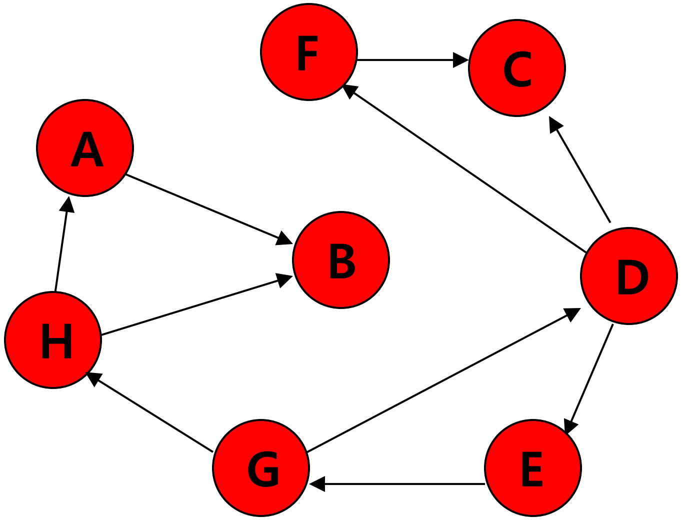
D

Visit F

The order nodes are visited:

D, C, E, G, H, A, B, F

Walk-Through

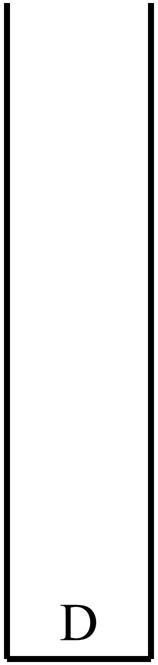


The order nodes are visited:

D, C, E, G, H, A, B, F

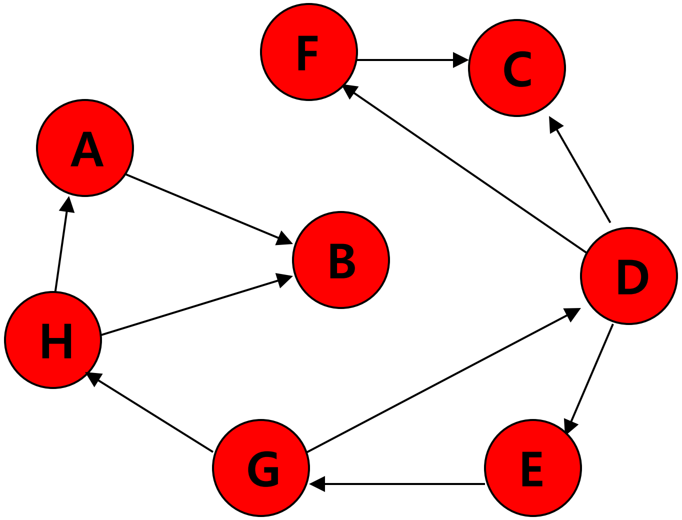
Visited Array

A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√



No unvisited nodes adjacent to F. Backtrack.

Walk-Through



Visited Array

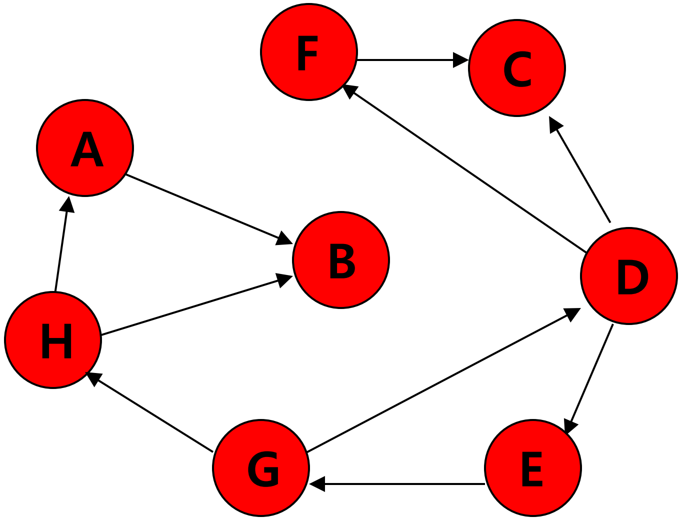
A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

The order nodes are visited:

D, C, E, G, H, A, B, F

No unvisited nodes adjacent to D. Backtrack.

Walk-Through



Visited Array

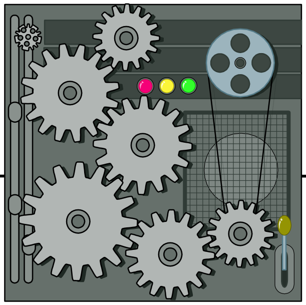
A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

The order nodes are visited:

D, C, E, G, H, A, B, F

Stack is empty. Depth-first traversal is done.

Analysis of DFS



- Setting/getting a vertex/edge label takes $O(1)$ time
- Each vertex is labeled twice
  - once as UNEXPLORED
  - once as VISITED
- Each edge is labeled twice
  - once as UNEXPLORED
  - once as DISCOVERY or BACK
- Method incidentEdges is called once for each vertex
- DFS runs in $O(n + m)$ time provided the graph is represented by the adjacency list structure
  - Recall that $\Sigma_v \deg(v) = 2m$

Path Finding

- We can specialize the DFS algorithm to find a path between two given vertices ***u*** and ***z*** using the template method pattern
- We call ***DFS*(*G*, *u*)** with ***u*** as the start vertex
- We use a stack ***S*** to keep track of the path between the start vertex and the current vertex
- As soon as destination vertex ***z*** is encountered, we return the path as the contents of the stack

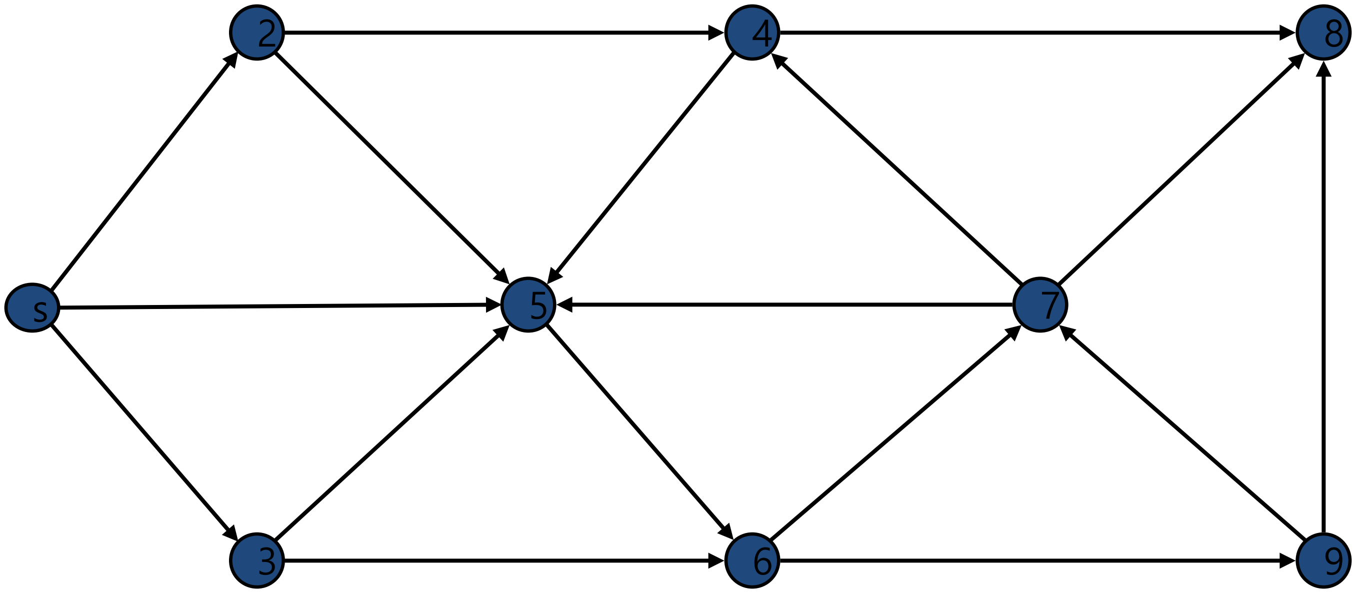
```
Algorithm pathDFS(G, v, z)
    setLabel(v, VISITED)
    S.push(v)
    if v = z
        return S.elements()
    for all e ∈ G.incidentEdges(v)
        if getLabel(e) = UNEXPLORED
            w ← opposite(v,e)
            if getLabel(w) = UNEXPLORED
                setLabel(e, DISCOVERY)
                S.push(e)
                pathDFS(G, w, z)
                S.pop(e)
            else
                setLabel(e, BACK)
    S.pop(v)
```

Cycle Finding

- We can specialize the DFS algorithm to find a simple cycle using the template method pattern
- We use a stack ***S*** to keep track of the path between the start vertex and the current vertex
- As soon as a back edge (***v***, ***w***) is encountered, we return the cycle as the portion of the stack from the top to vertex ***w***

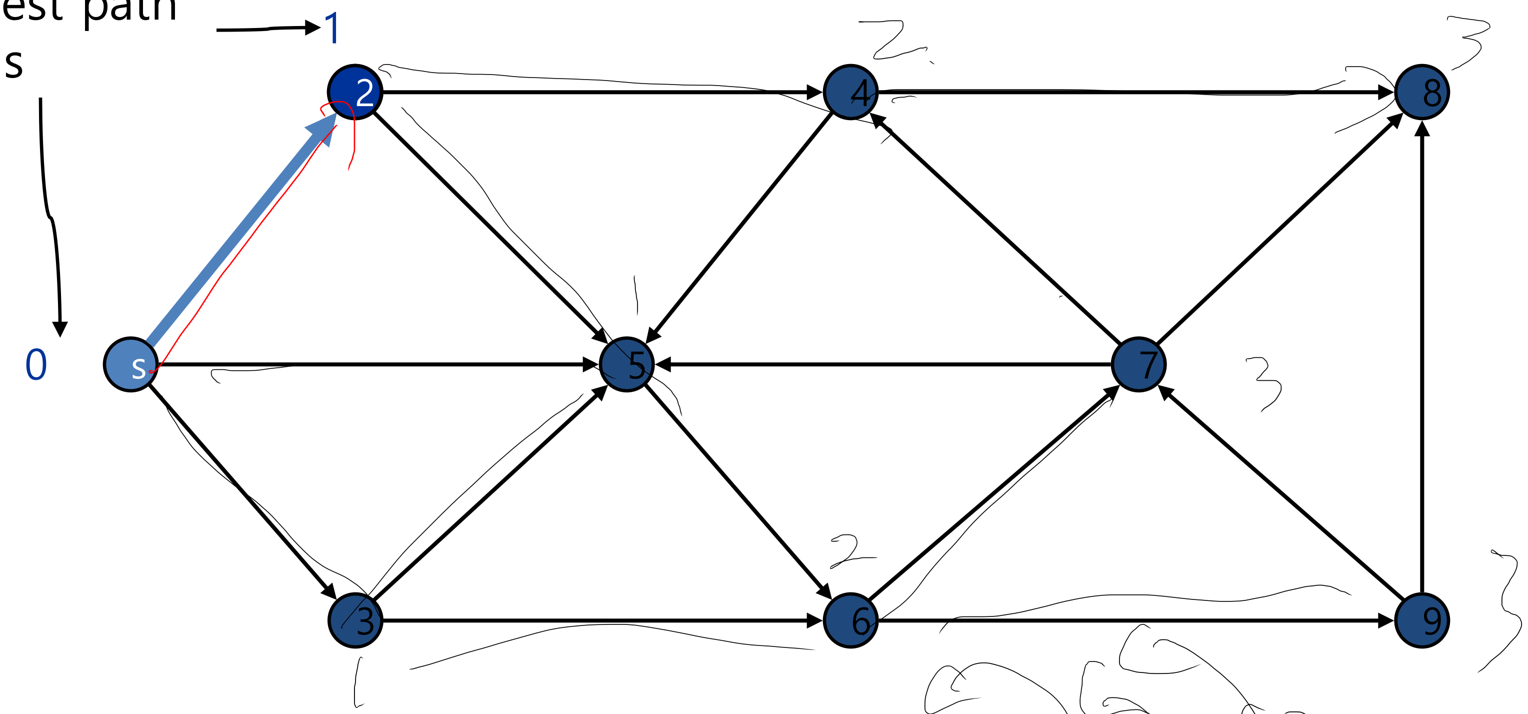
```
Algorithm cycleDFS(G, v, z)
    setLabel(v, VISITED)
    S.push(v)
    for all  $e \in$ G.incidentEdges(v)
        if  getLabel(e) = UNEXPLORED
            $w \leftarrow$ opposite(v,e)
            S.push(e)
            if getLabel(w) = UNEXPLORED
                setLabel(e, DISCOVERY)
                pathDFS(G, w, z)
                S.pop(e)
            else
                $T \leftarrow$ new empty stack
                repeat
                    $o \leftarrow$ S.pop()
                    T.push(o)
                until $o = w$
                return T.elements()
    S.pop(v)
```

Breadth First Search



Breadth First Search

Shortest path from s



Undiscovered

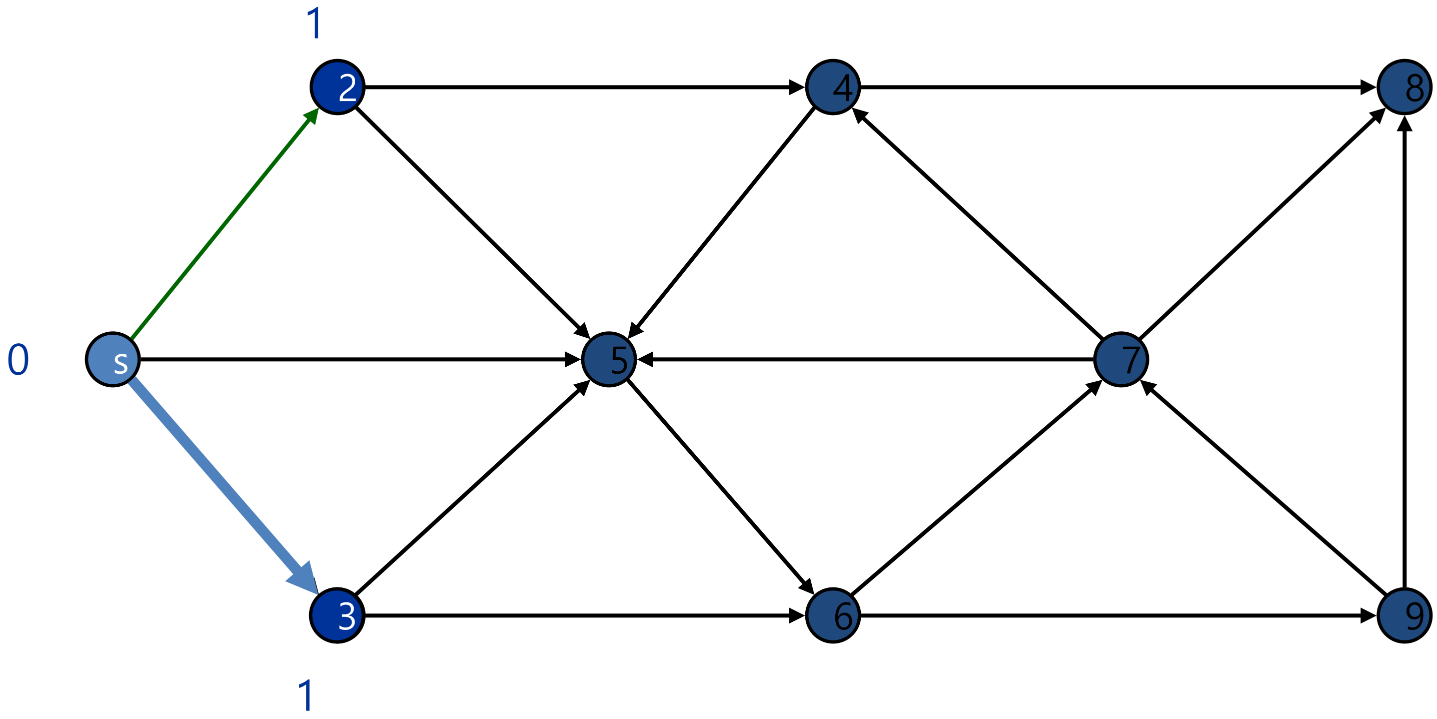
Discovered

Top of queue

Finished

Queue: s 2 3 5 4 6 8 7 9

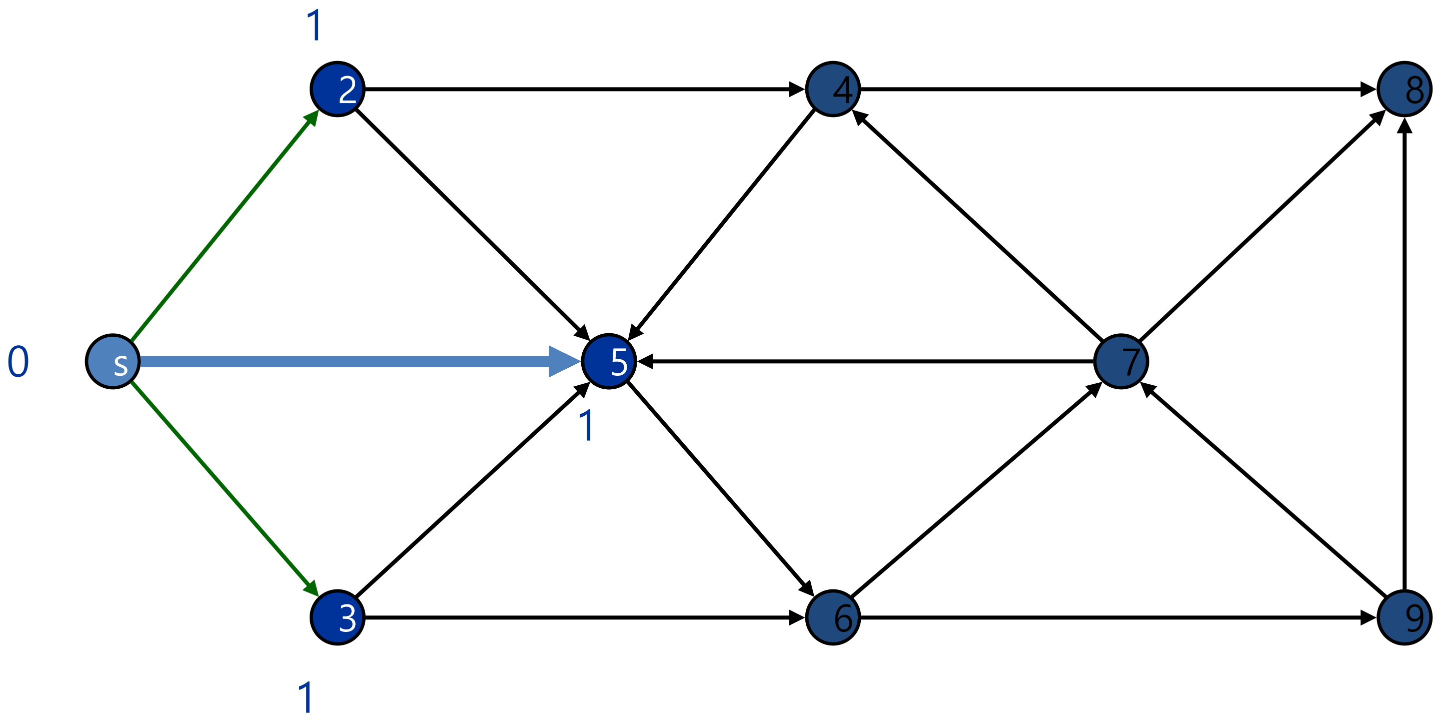
Breadth First Search



Undiscovered
Discovered
Top of queue
Finished

Queue: s 2

Breadth First Search



Undiscovered

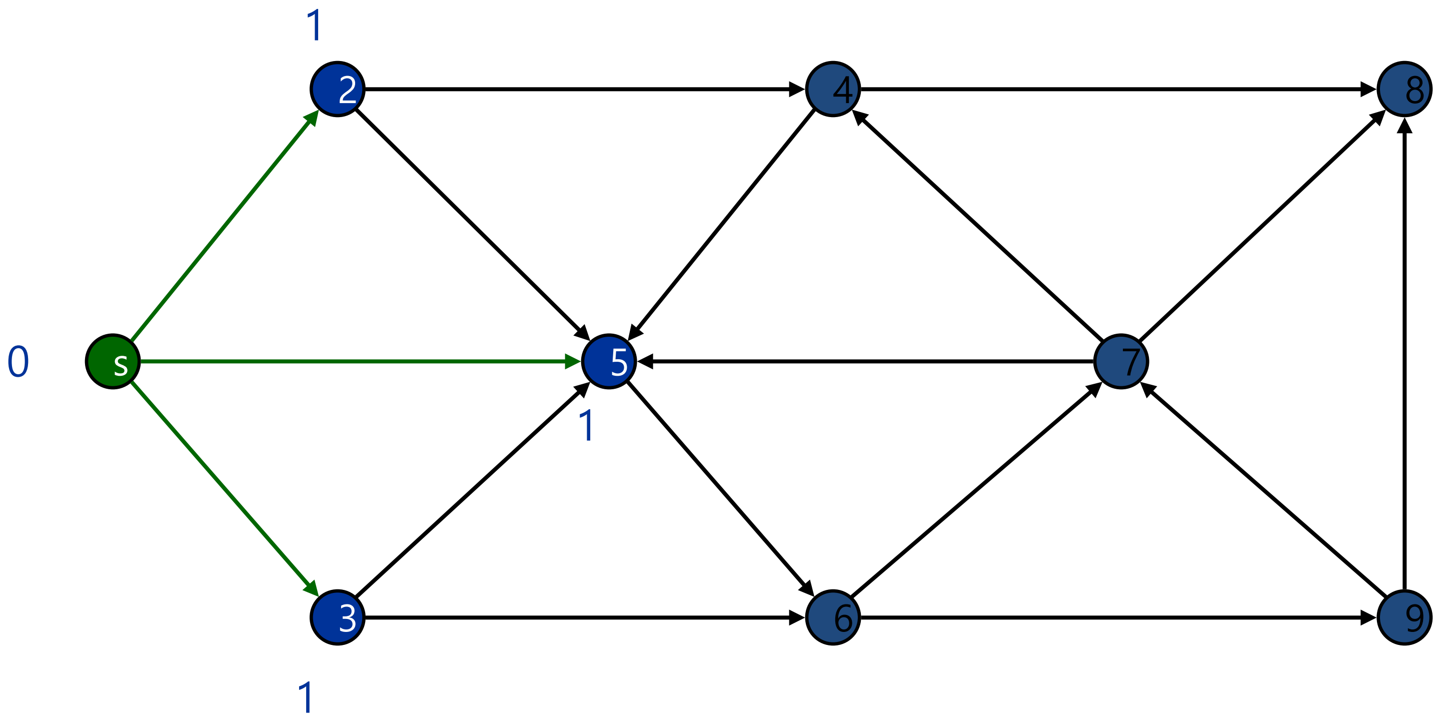
Discovered

Top of queue

Finished

Queue: s 2 3

Breadth First Search



Undiscovered

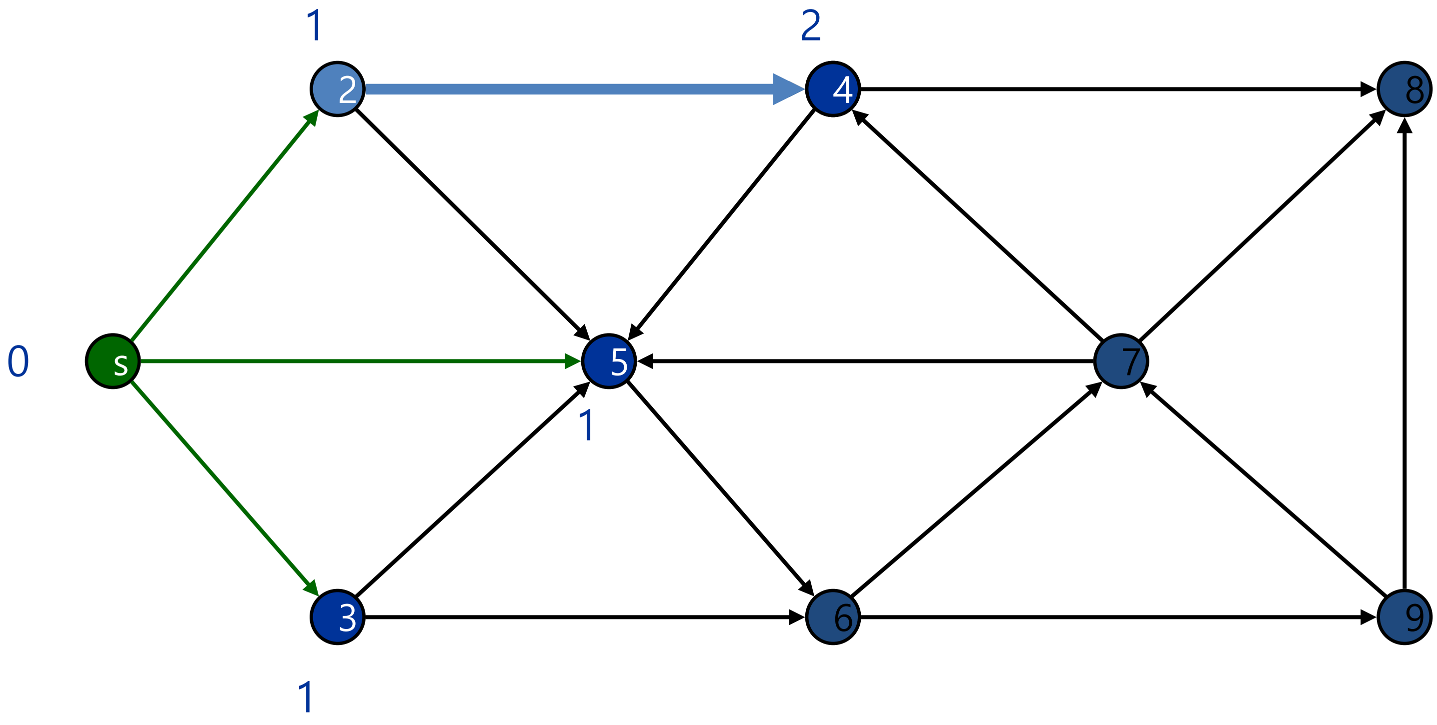
Discovered

Top of queue

Finished

Queue: 2 3 5

Breadth First Search



Undiscovered

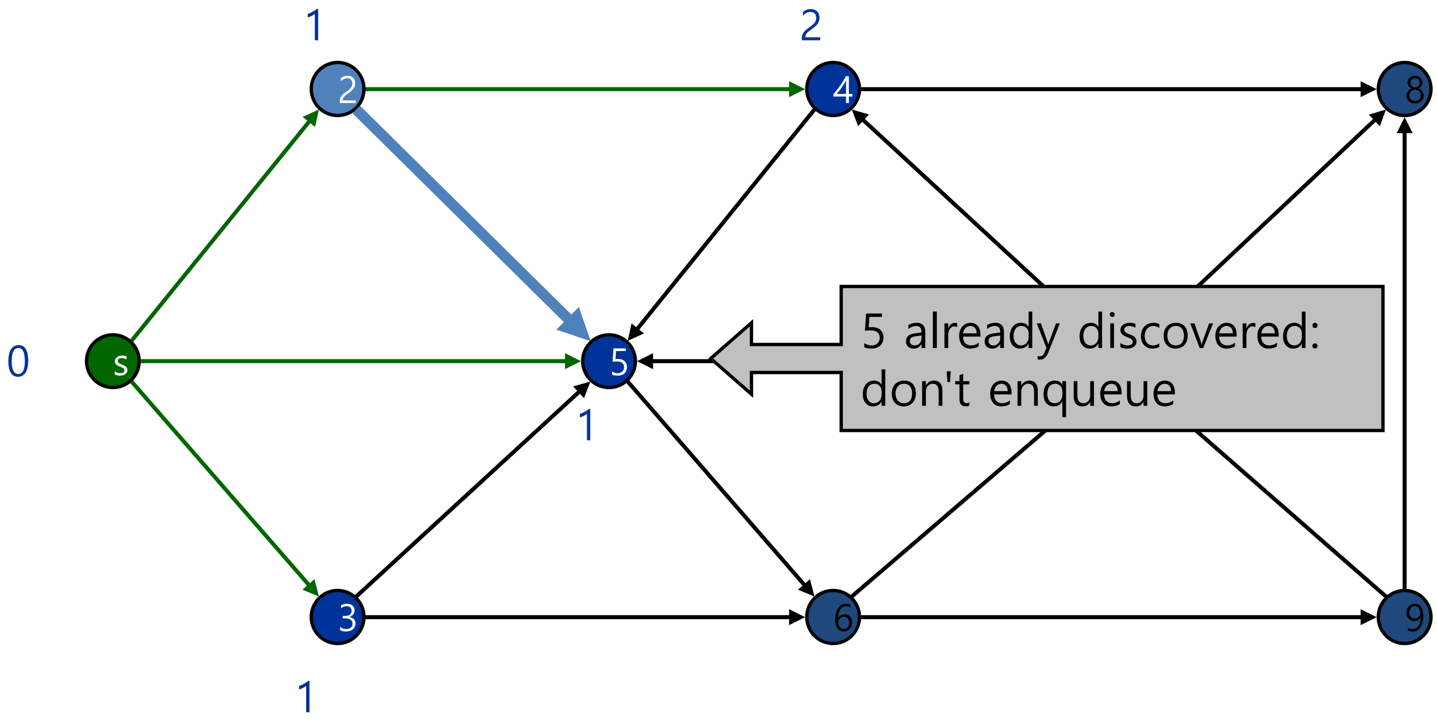
Discovered

Top of queue

Finished

Queue: 2 3 5

Breadth First Search

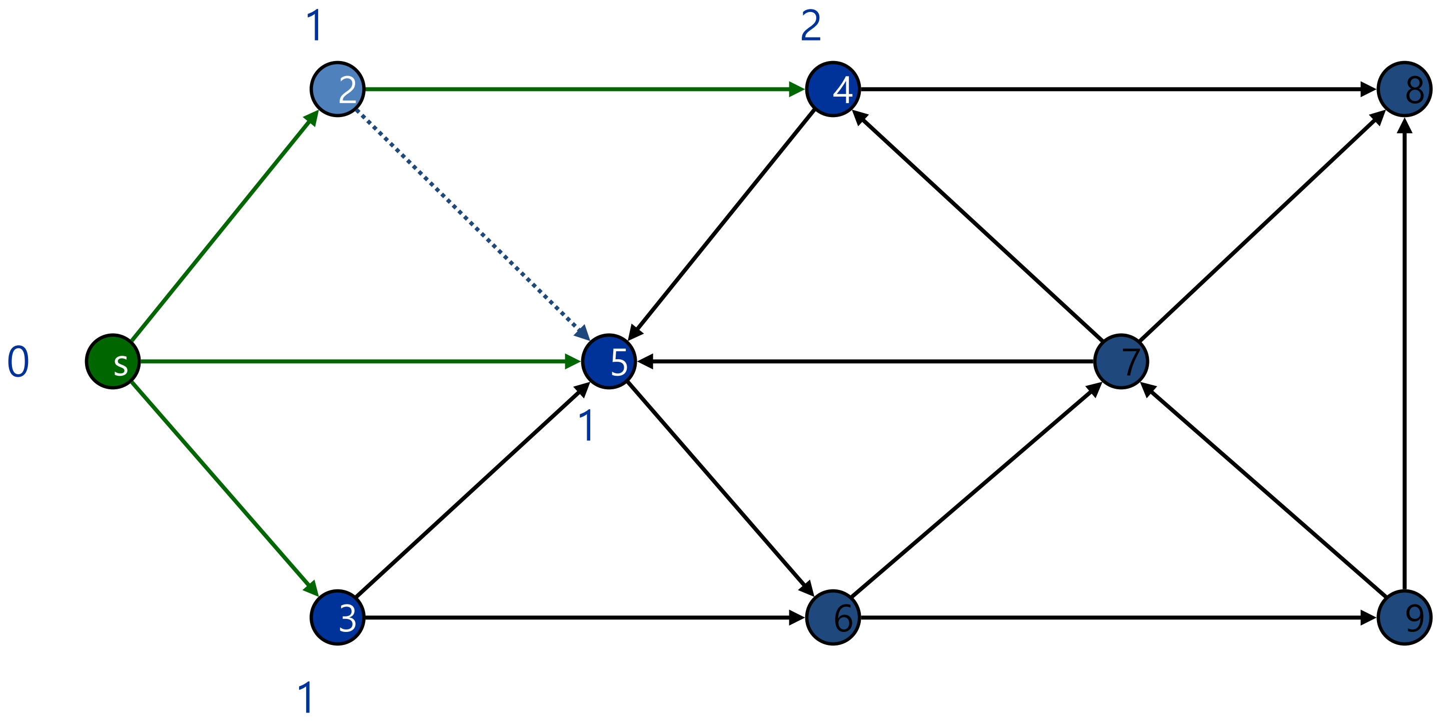


5 already discovered: don't enqueue

- Undiscovered
- Discovered
- Top of queue
- Finished

Queue: 2 3 5 4

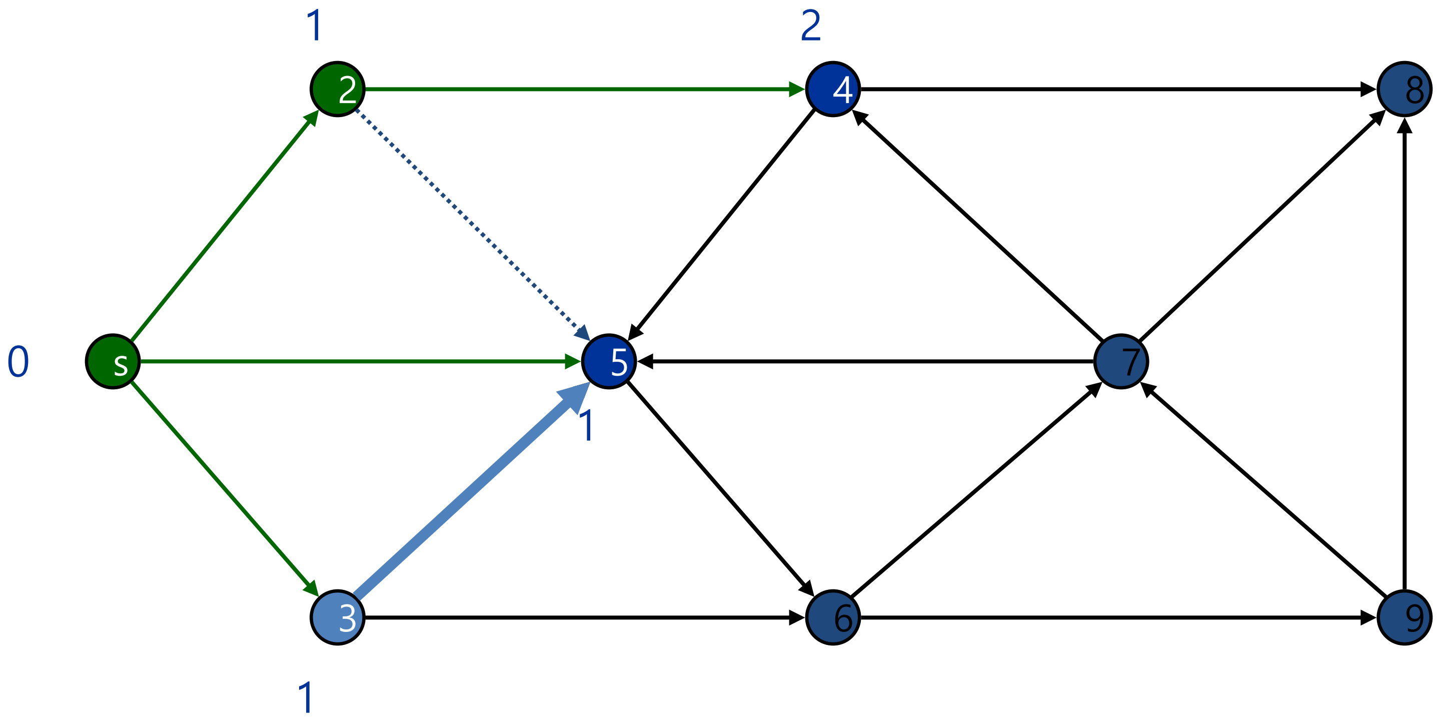
Breadth First Search



- Undiscovered
- Discovered
- Top of queue
- Finished

Queue: 2 3 5 4

Breadth First Search



Undiscovered

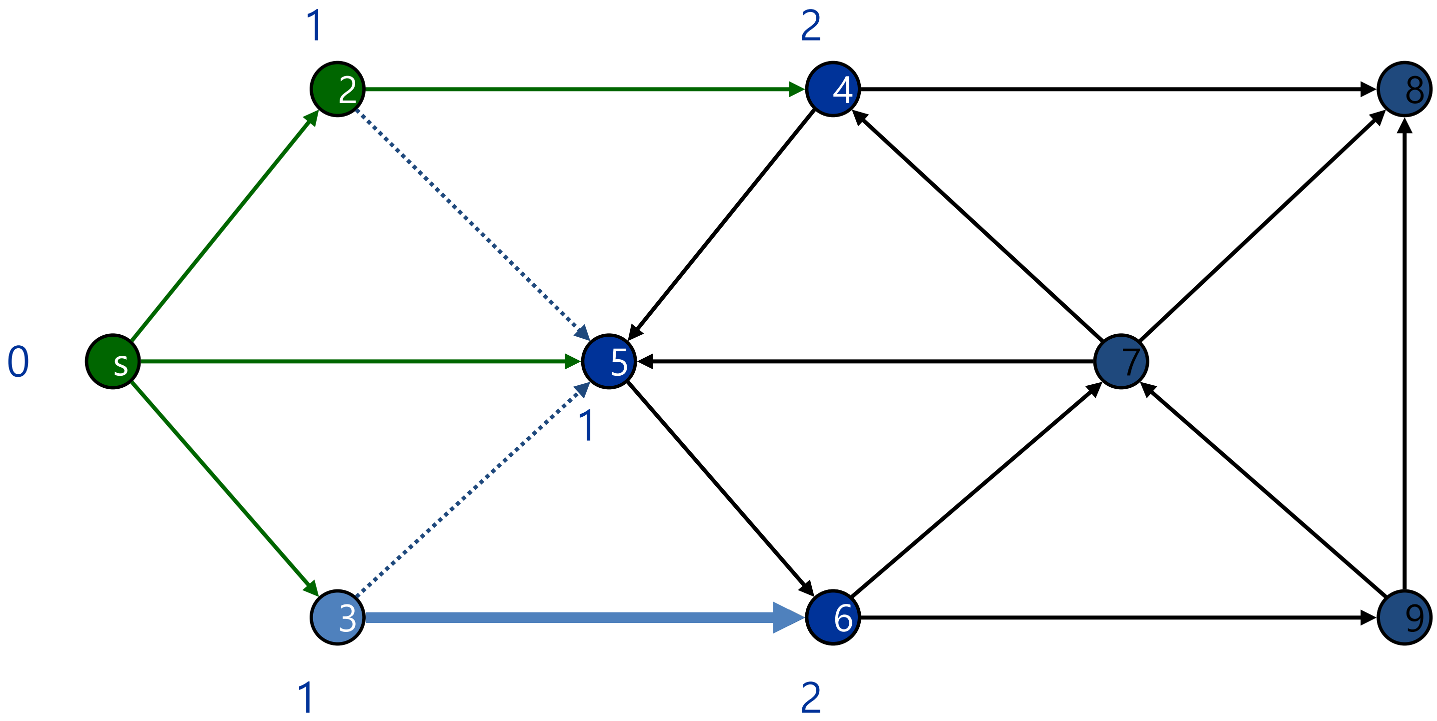
Discovered

Top of queue

Finished

Queue: 3 5 4

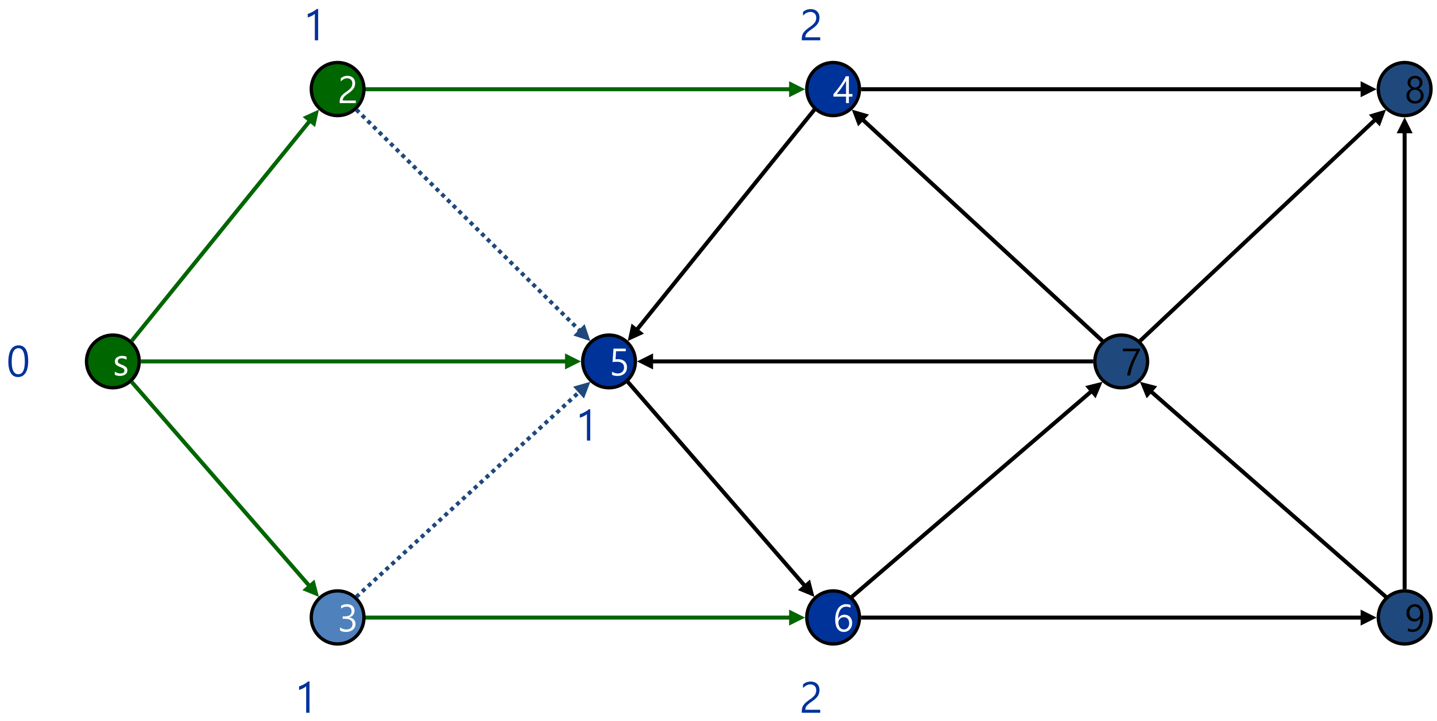
Breadth First Search



- Undiscovered
- Discovered
- Top of queue
- Finished

Queue: 3 5 4

Breadth First Search



Undiscovered

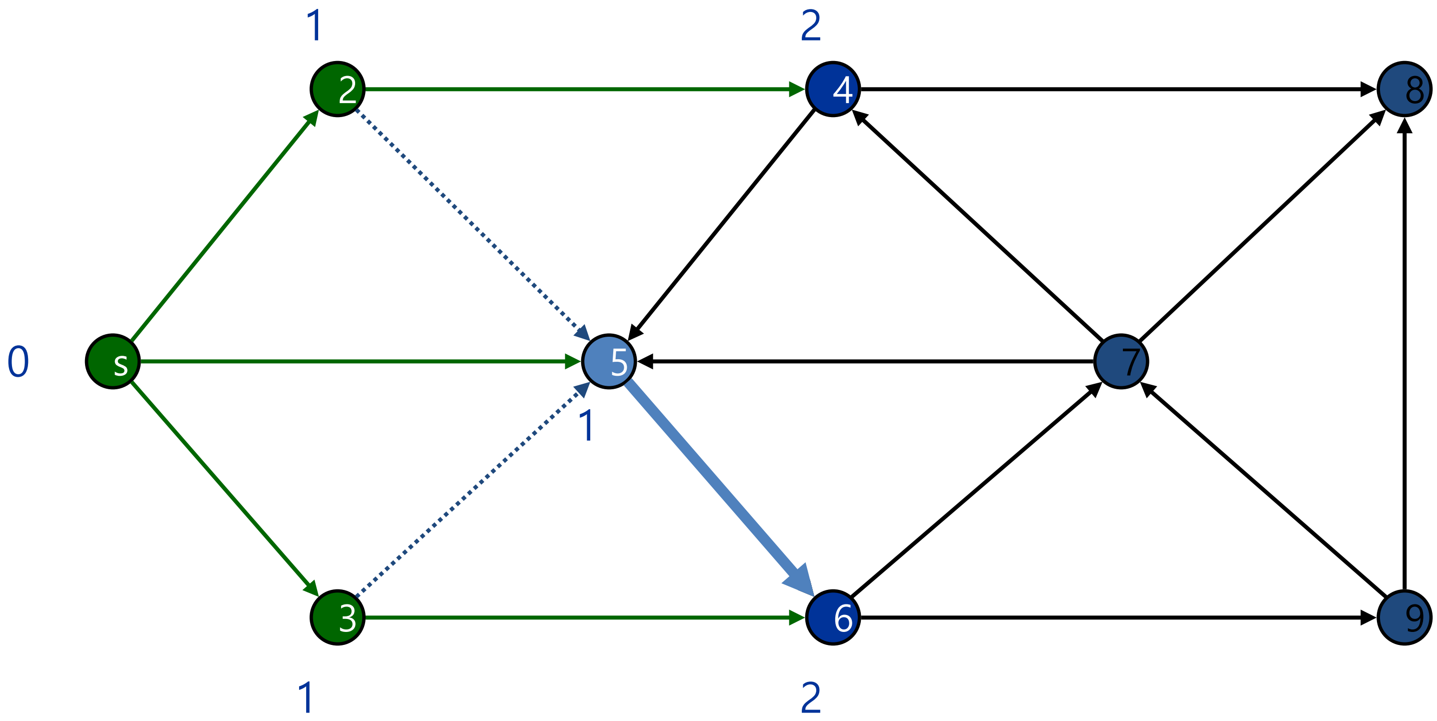
Discovered

Top of queue

Finished

Queue: 3 5 4 6

Breadth First Search



Undiscovered

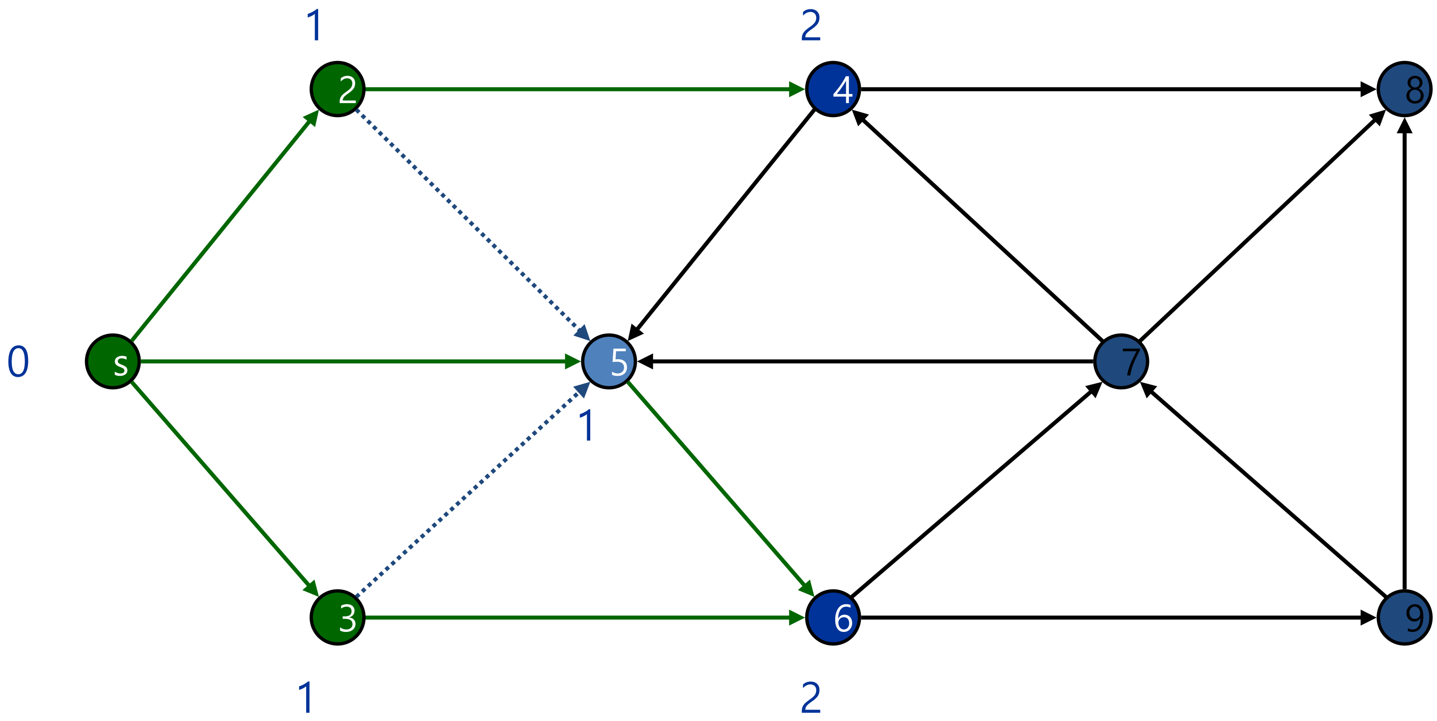
Discovered

Top of queue

Finished

Queue: 5 4 6

Breadth First Search



Undiscovered

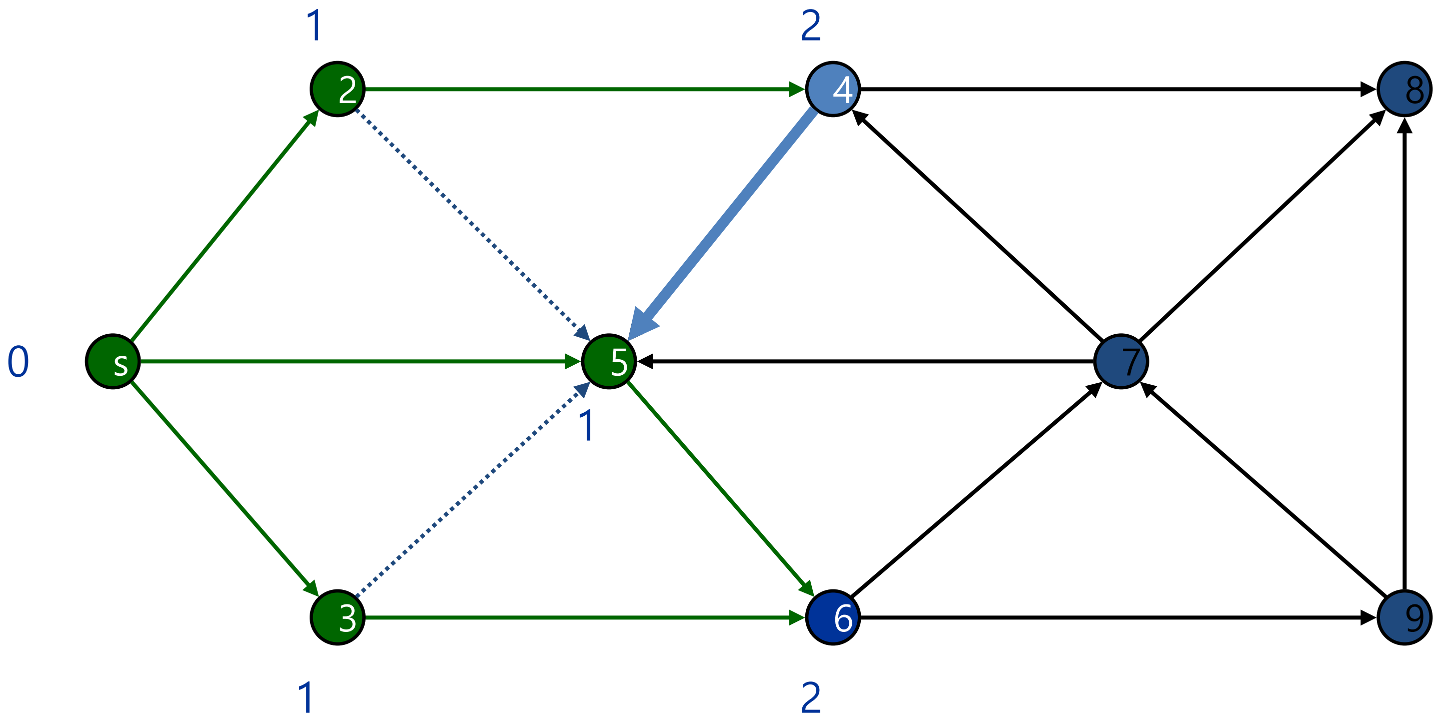
Discovered

Top of queue

Finished

Queue: 5 4 6

Breadth First Search



Undiscovered

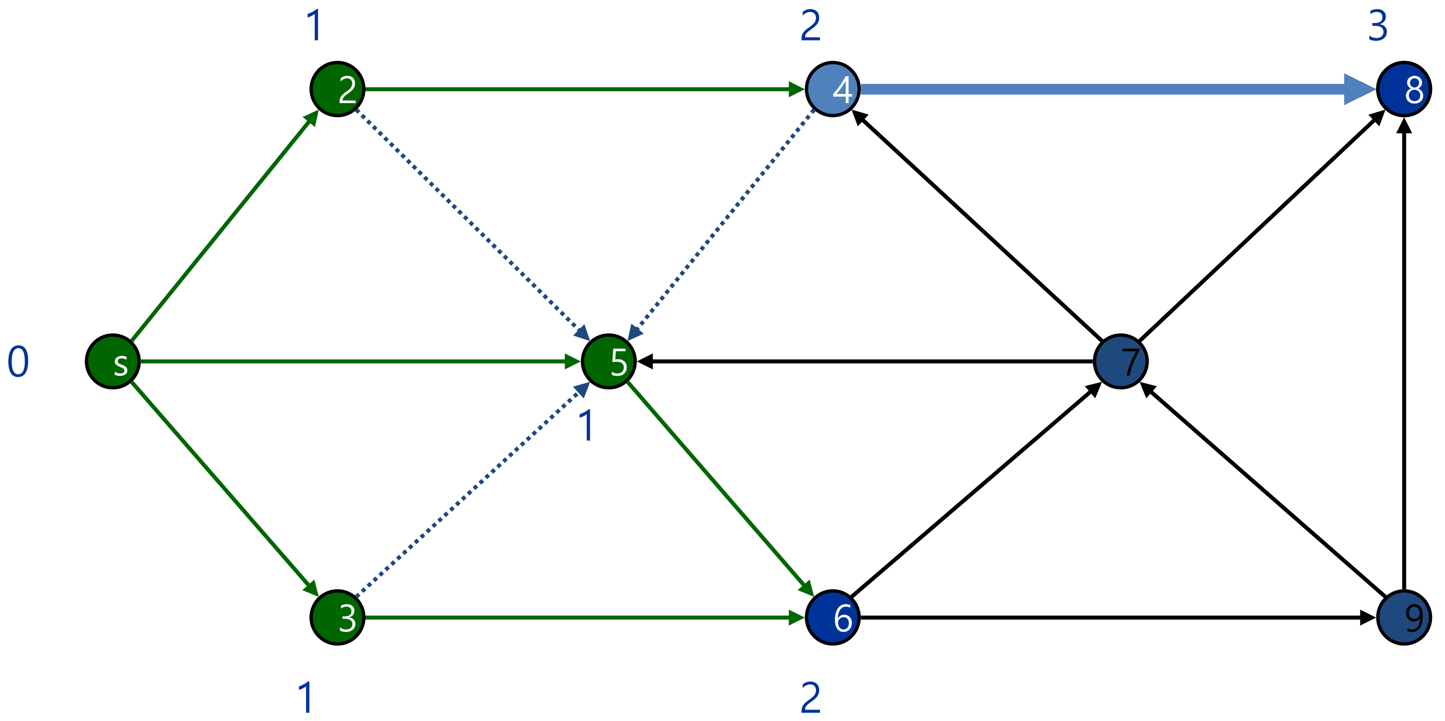
Discovered

Top of queue

Finished

Queue: 4 6

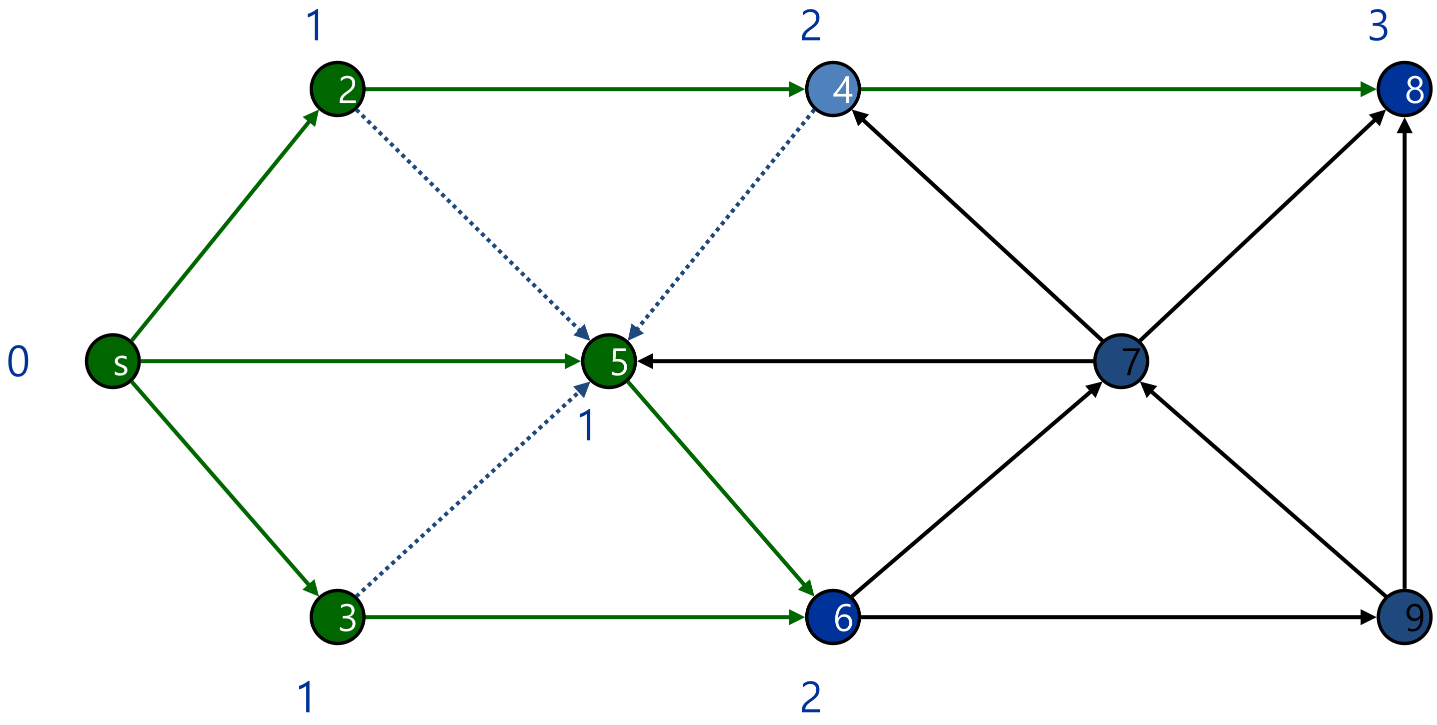
Breadth First Search



Undiscovered
Discovered
Top of queue
Finished

Queue: 4 6

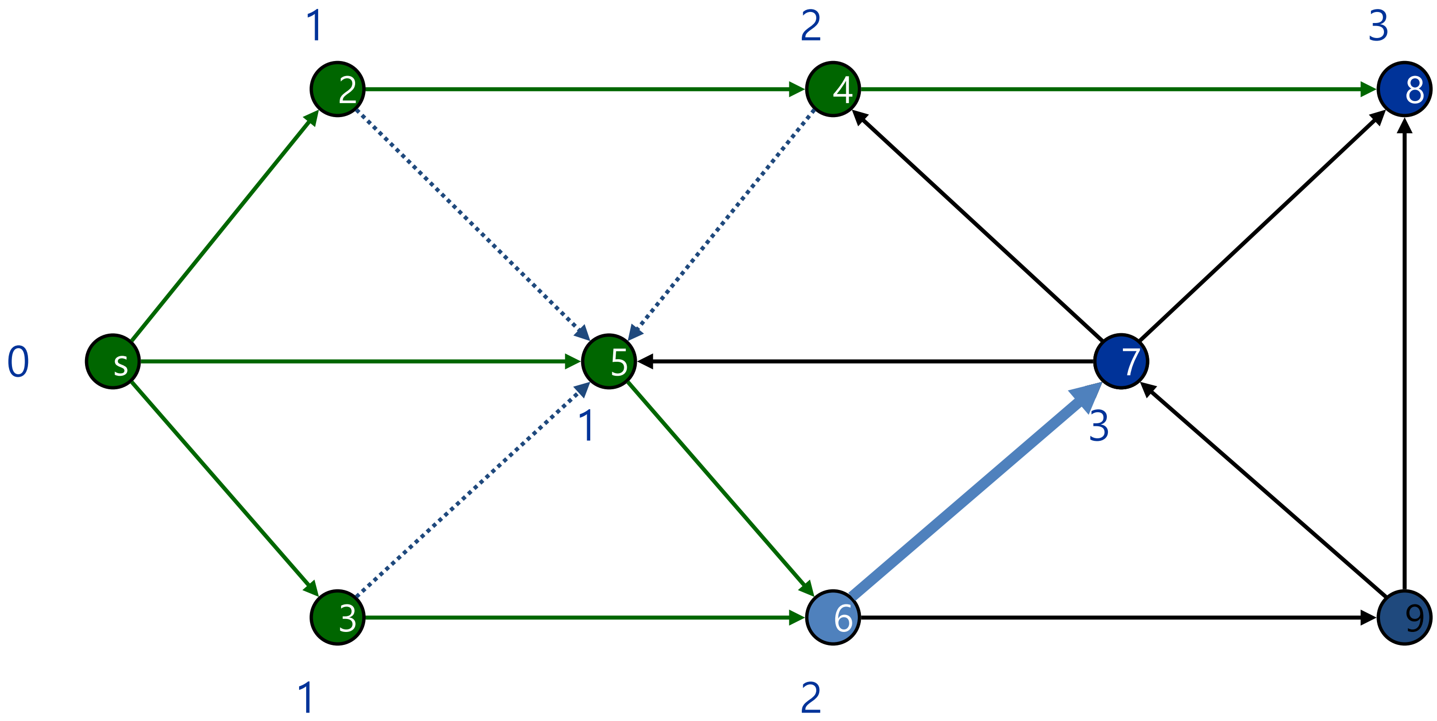
Breadth First Search



- Undiscovered
- Discovered
- Top of queue
- Finished

Queue: 4 6 8

Breadth First Search



Undiscovered

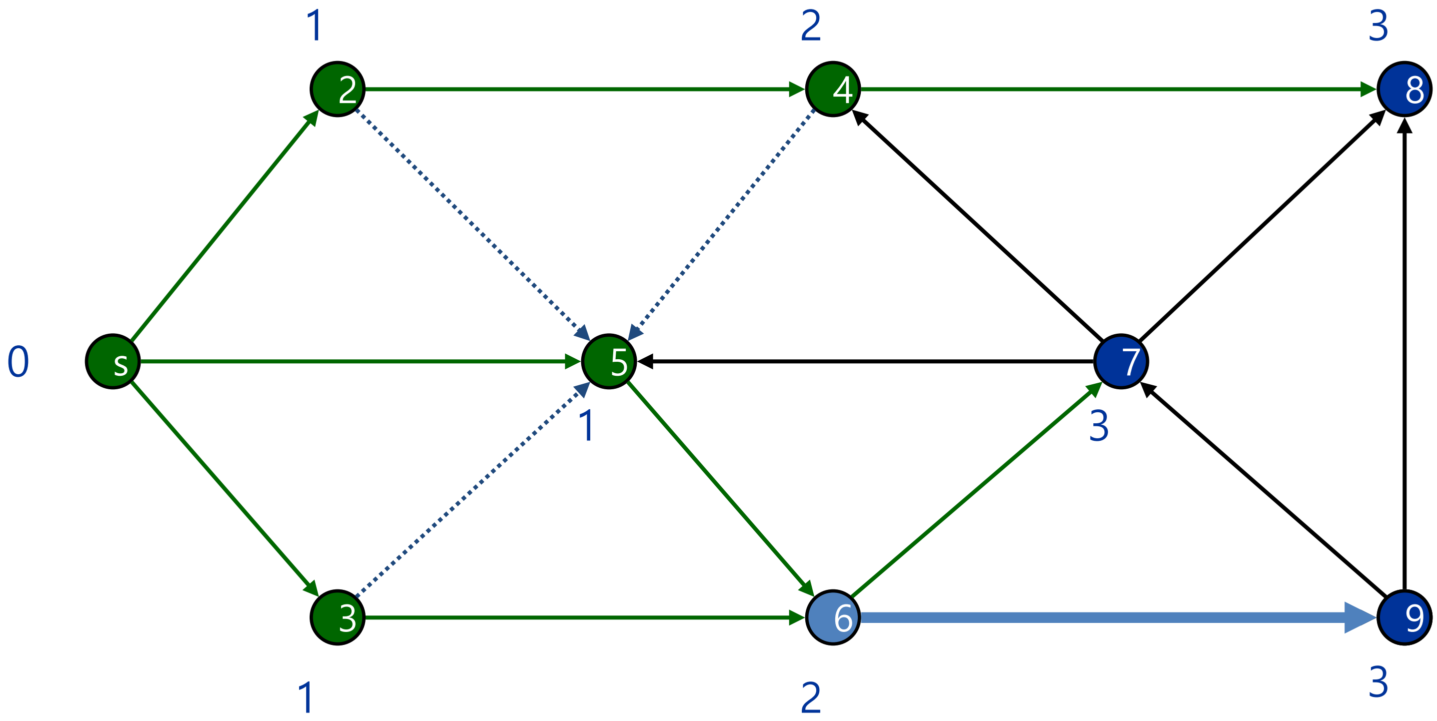
Discovered

Top of queue

Finished

Queue: 6 8

Breadth First Search



Undiscovered

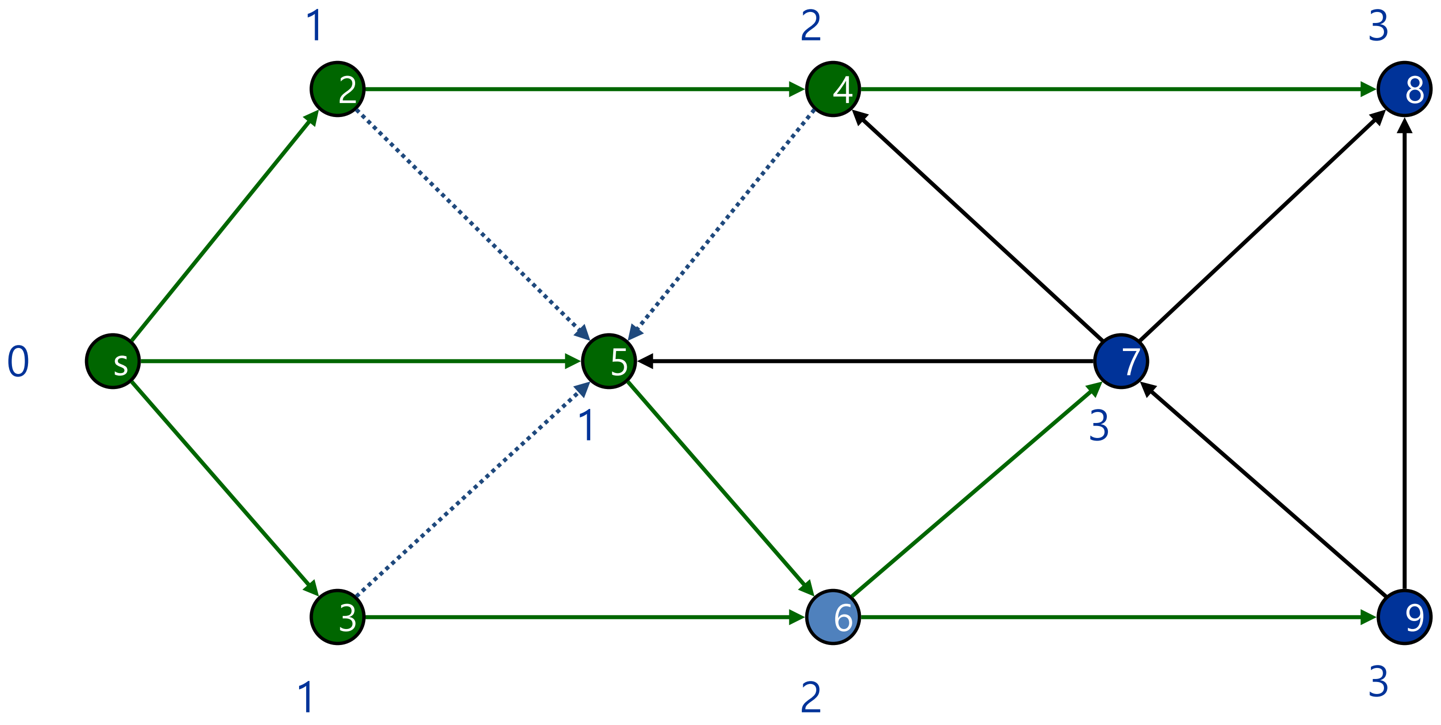
Discovered

Top of queue

Finished

Queue: 6 8 7

Breadth First Search



Undiscovered

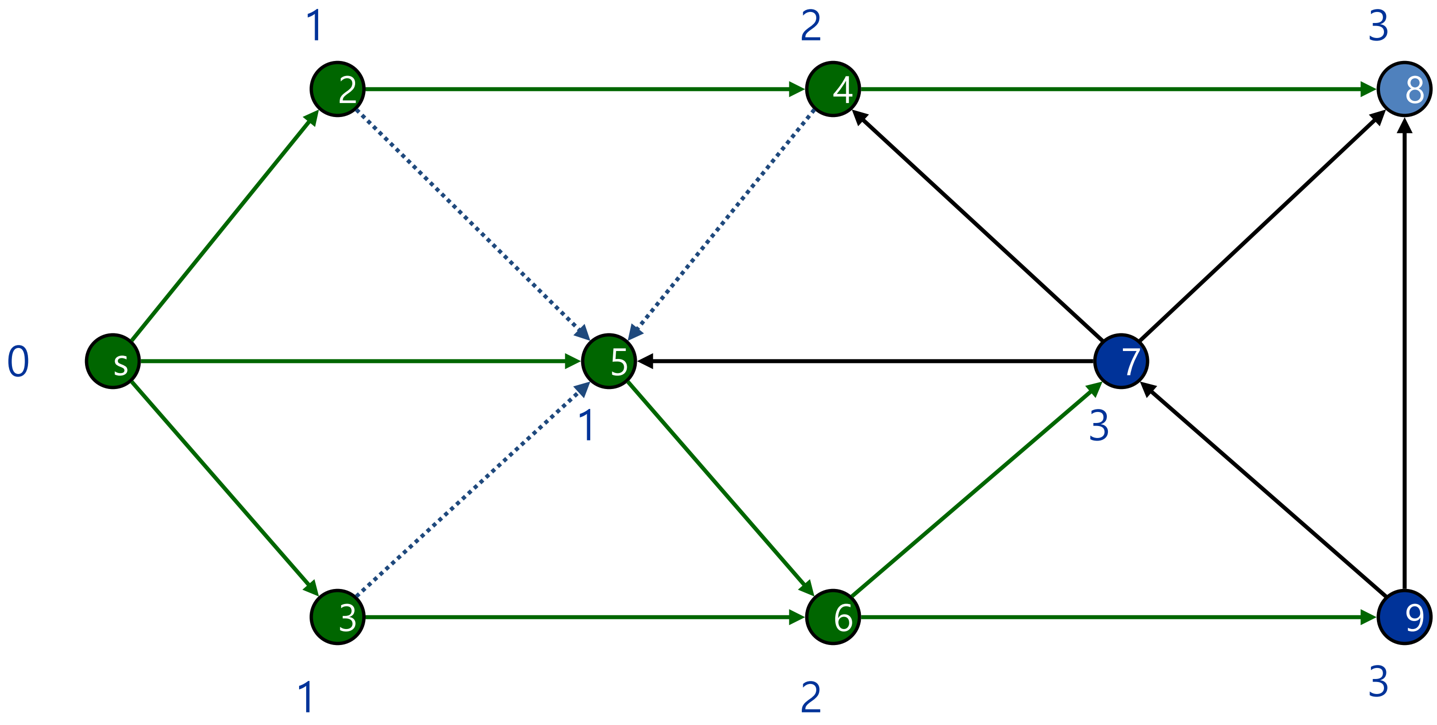
Discovered

Top of queue

Finished

Queue: 6 8 7 9

Breadth First Search



Undiscovered

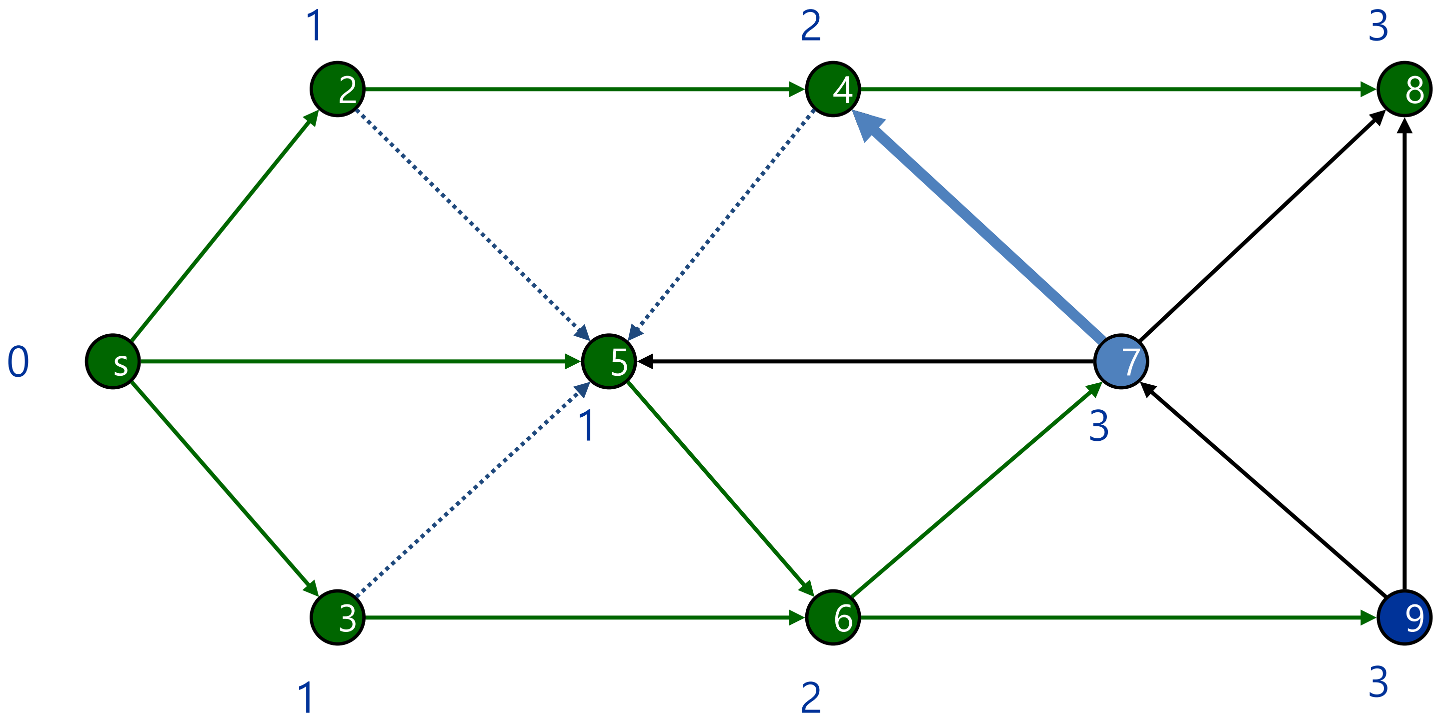
Discovered

Top of queue

Finished

Queue: 8 7 9

Breadth First Search



Undiscovered

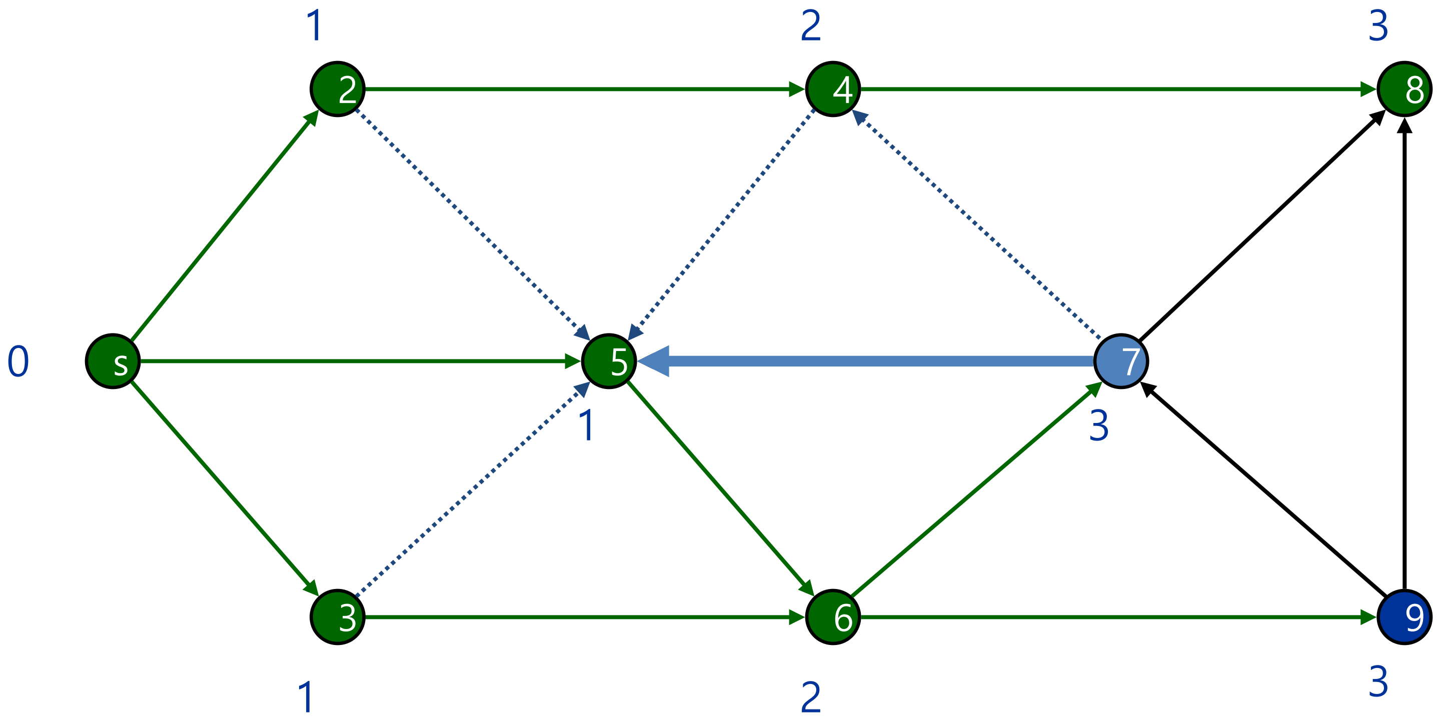
Discovered

Top of queue

Finished

Queue: 7 9

Breadth First Search



Undiscovered

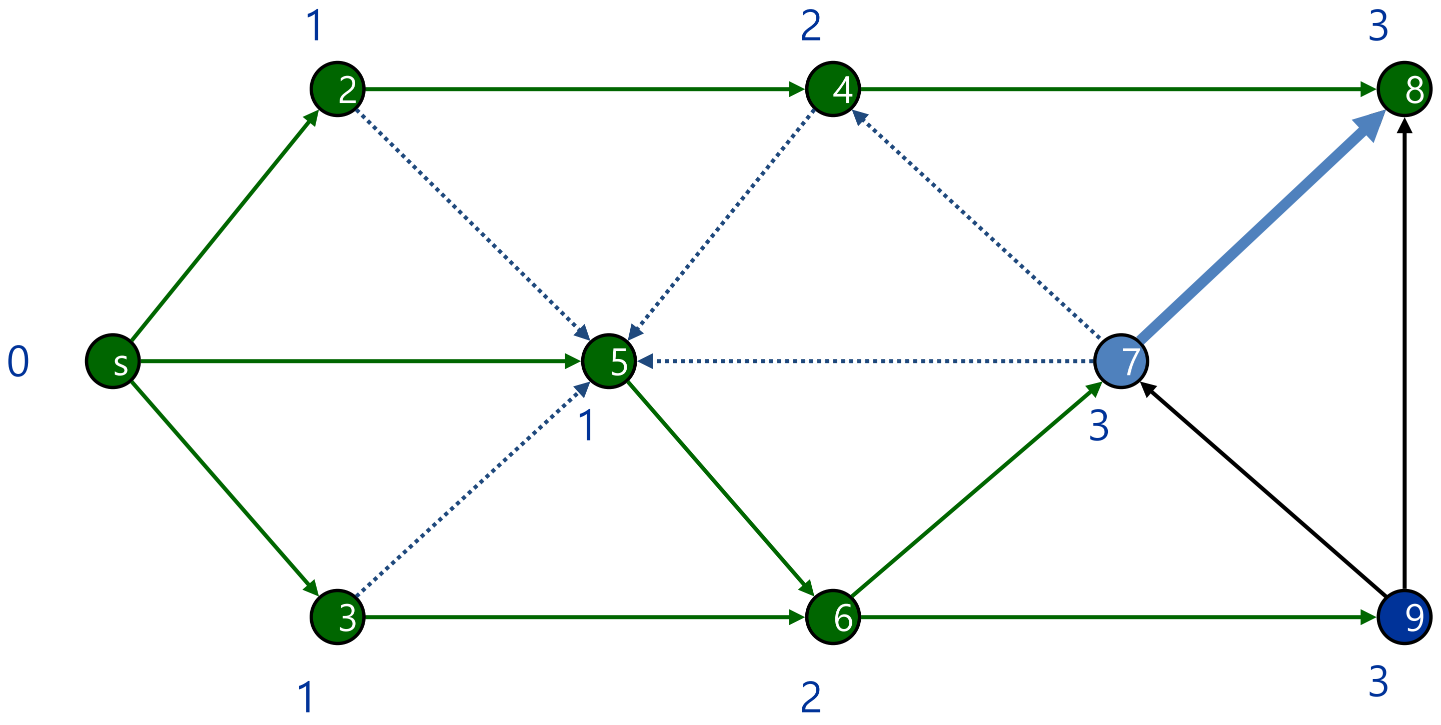
Discovered

Top of queue

Finished

Queue: 7 9

Breadth First Search



Undiscovered

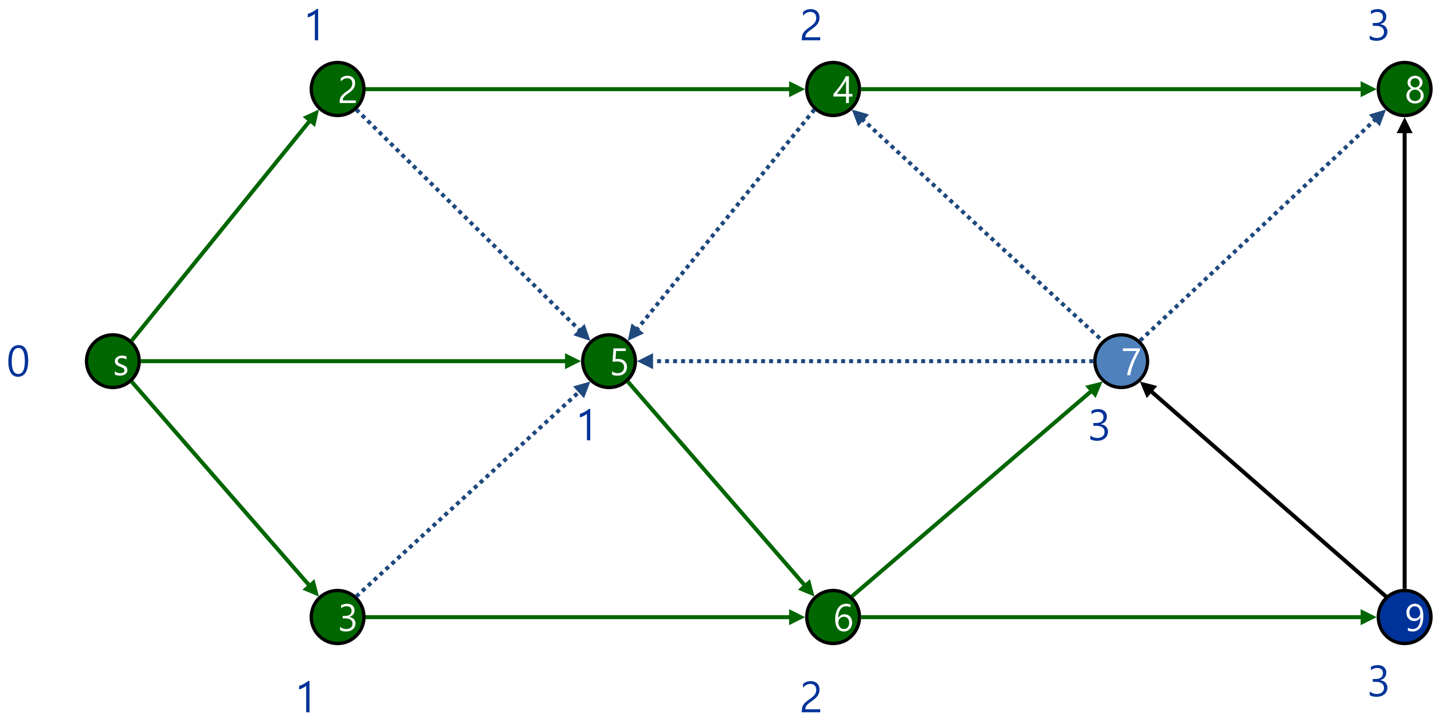
Discovered

Top of queue

Finished

Queue: 7 9

Breadth First Search



Undiscovered

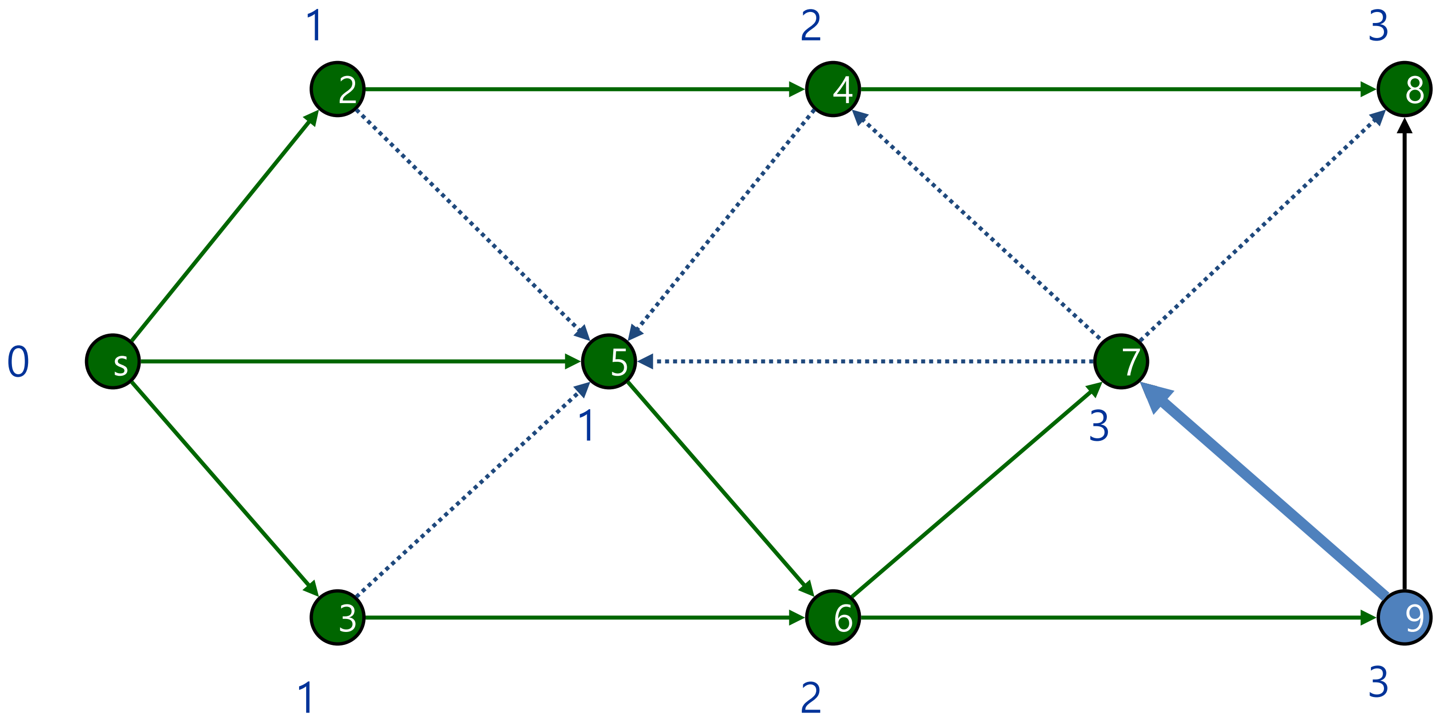
Discovered

Top of queue

Finished

Queue: 7 9

Breadth First Search



Undiscovered

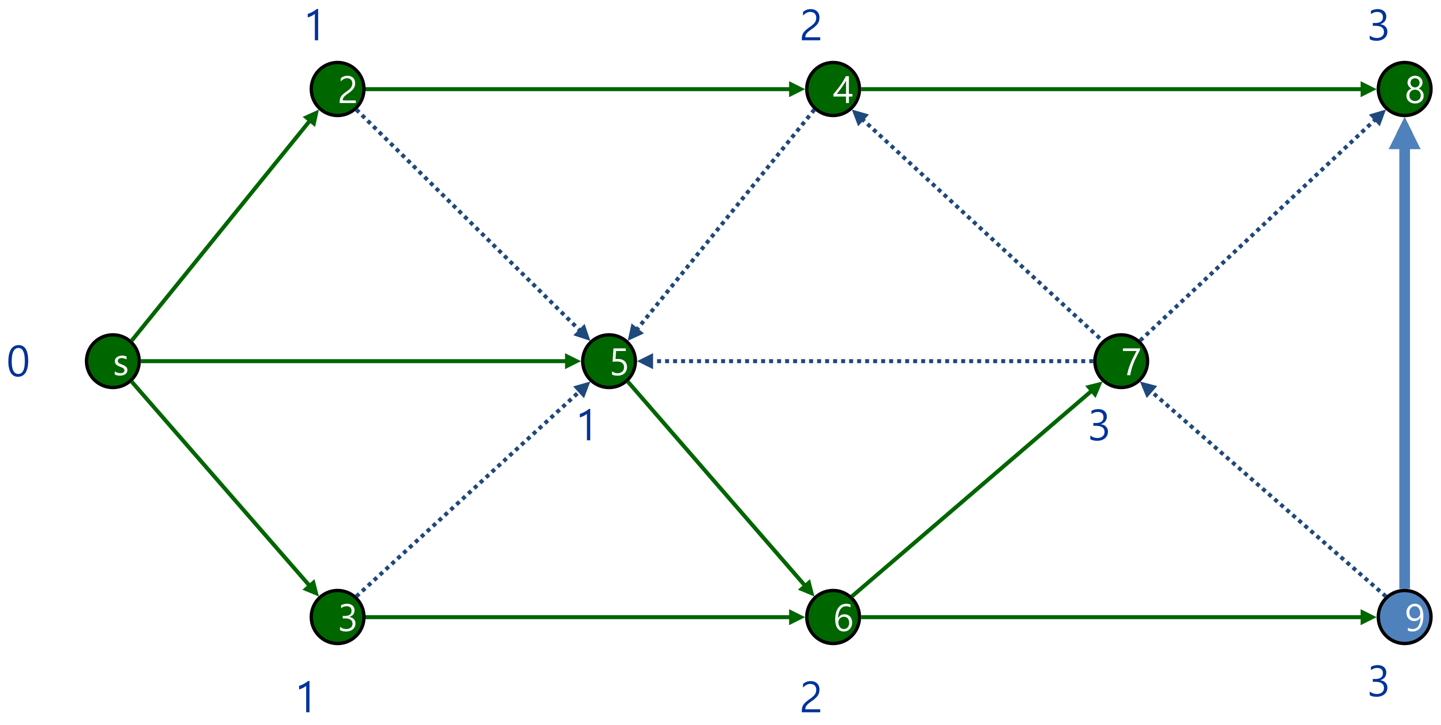
Discovered

Top of queue

Finished

Queue: 9

Breadth First Search



Undiscovered

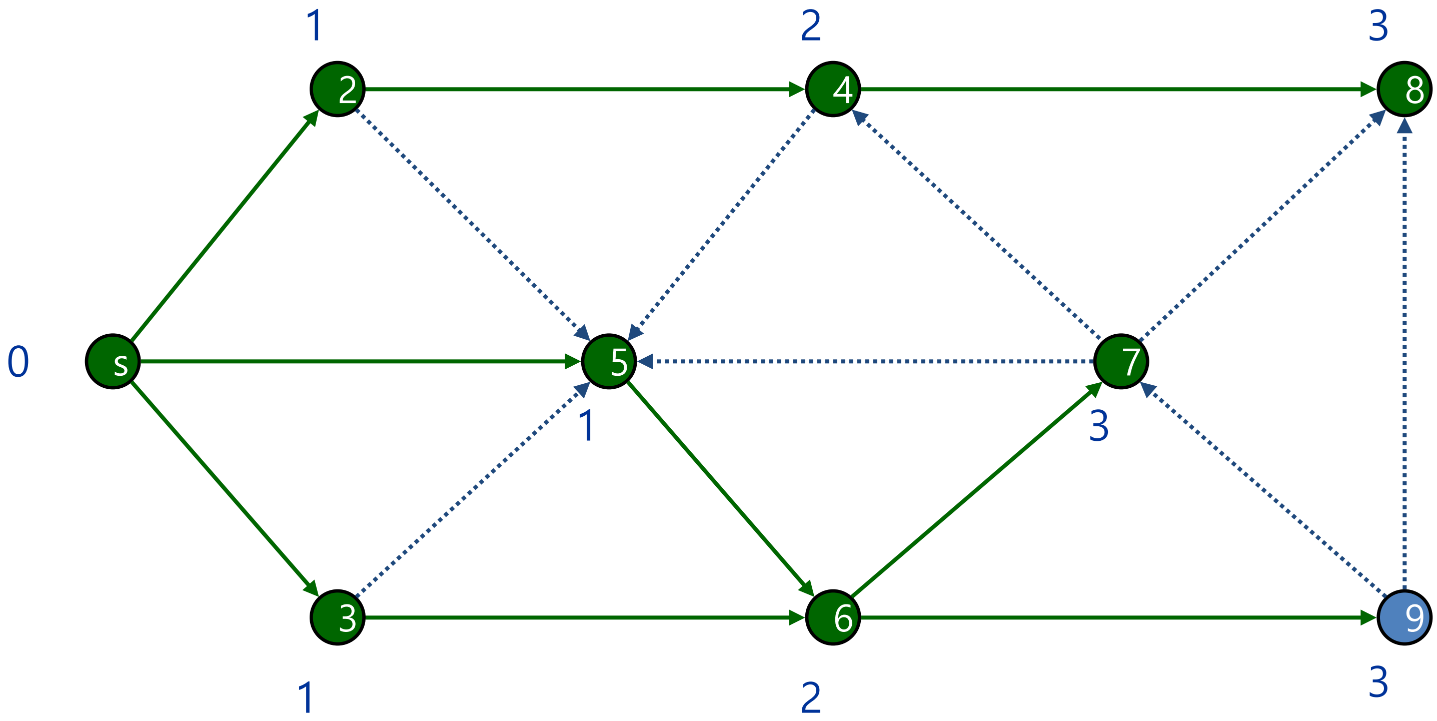
Discovered

Top of queue

Finished

Queue: 9

Breadth First Search



Undiscovered

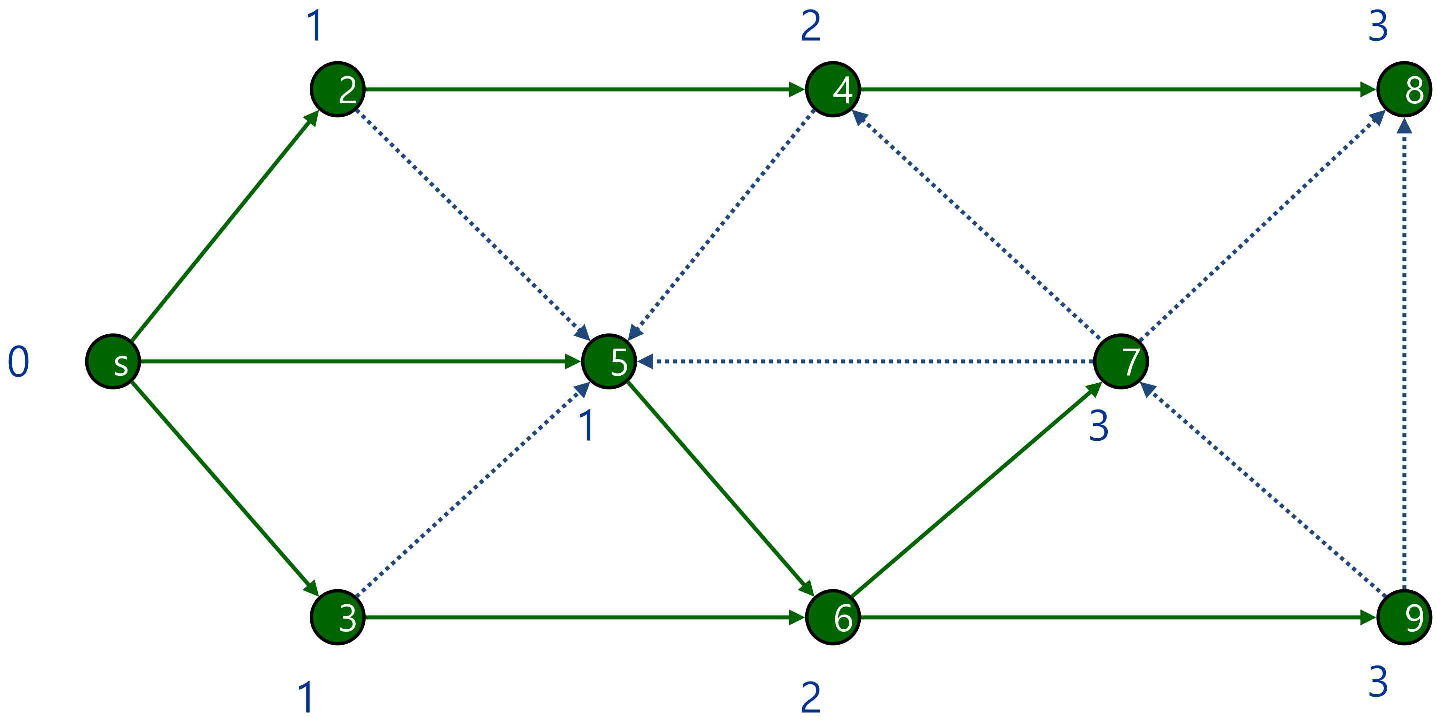
Discovered

Top of queue

Finished

Queue: 9

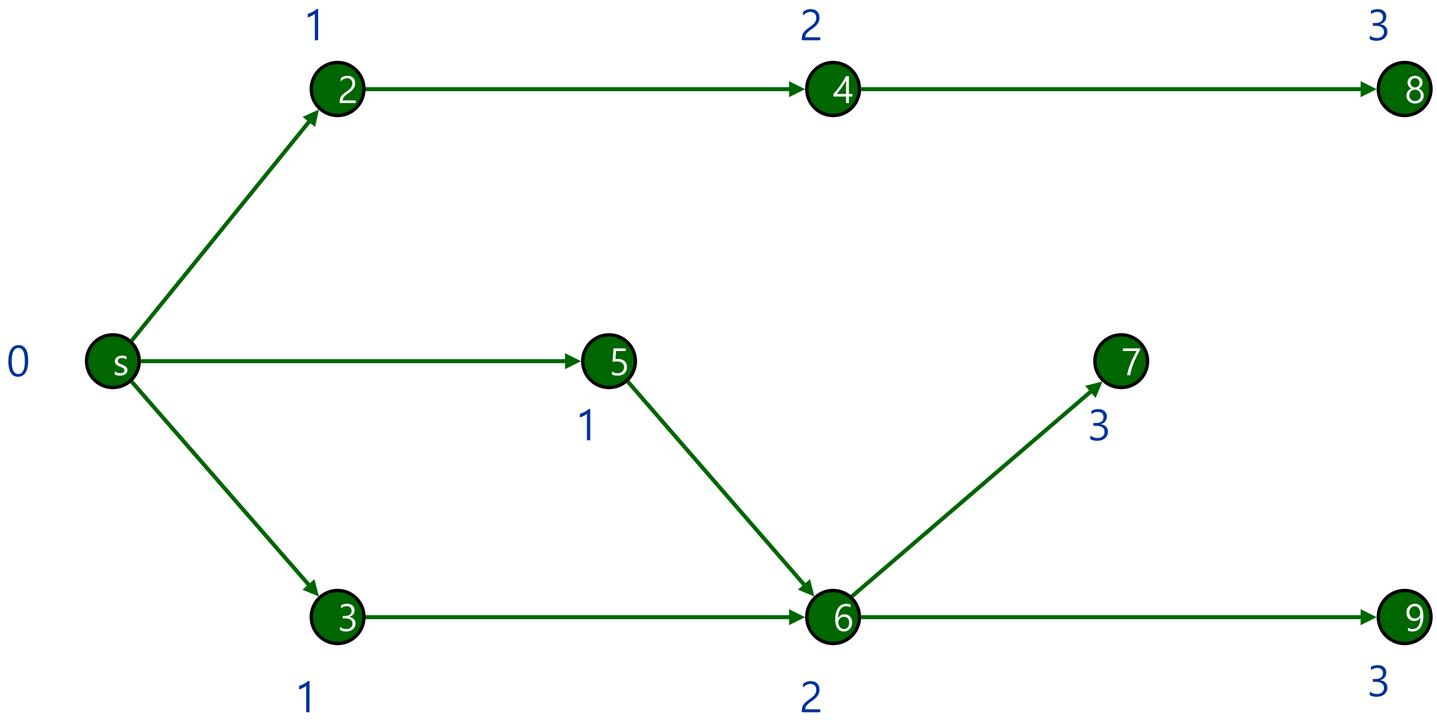
Breadth First Search



Undiscovered
Discovered
Top of queue
Finished

Queue:

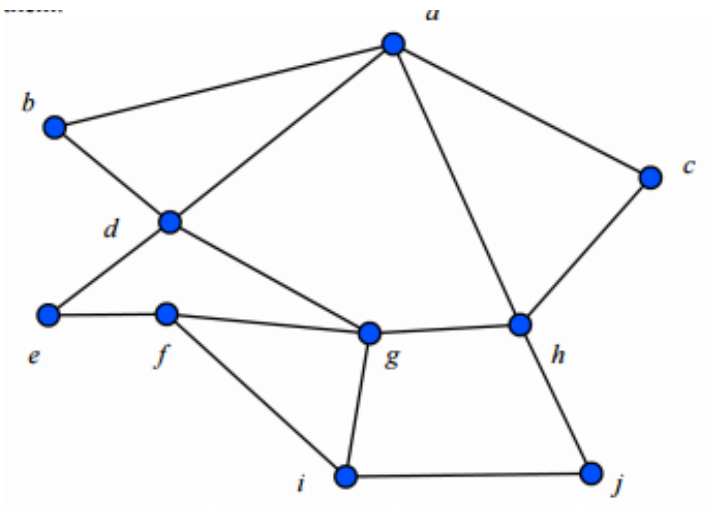
Breadth First Search



Level Graph

Psu.edu

- Do DFS and BFS starting with ‘a’



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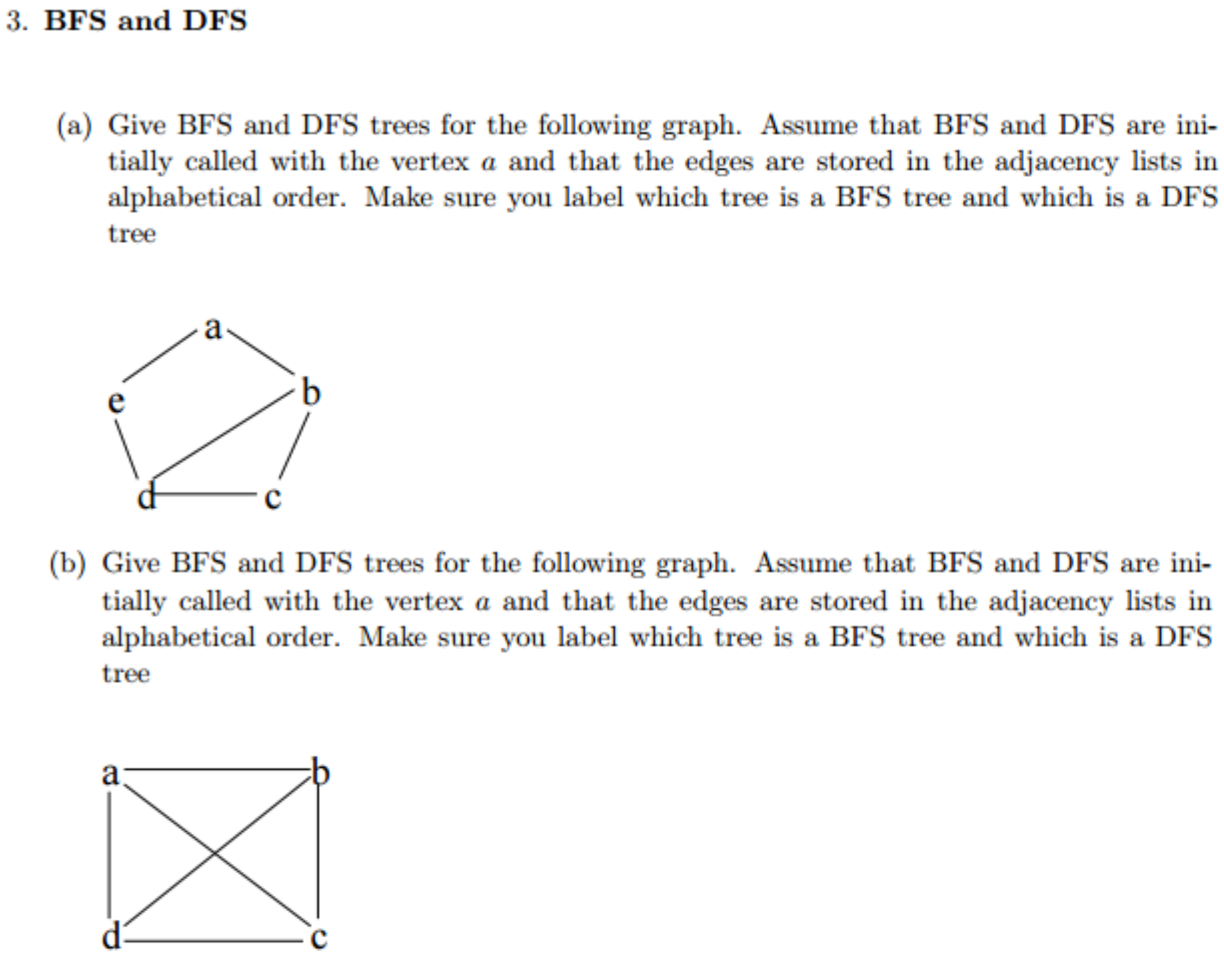


3. BFS and DFS

(a) Give BFS and DFS trees for the following graph. Assume that BFS and DFS are initially called with the vertex a and that the edges are stored in the adjacency lists in alphabetical order. Make sure you label which tree is a BFS tree and which is a DFS tree

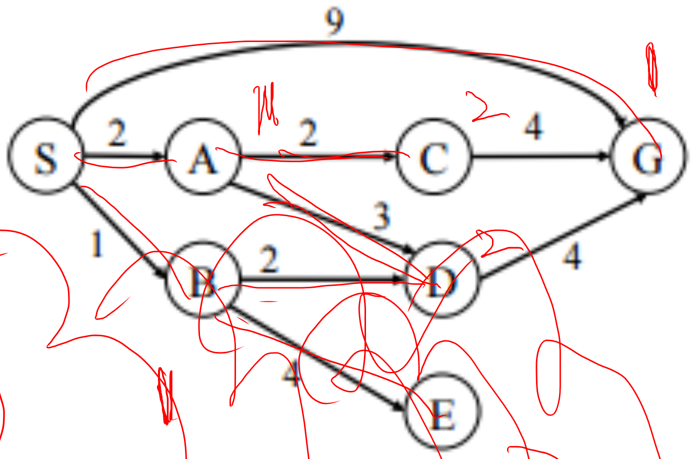
(b) Give BFS and DFS trees for the following graph. Assume that BFS and DFS are initially called with the vertex a and that the edges are stored in the adjacency lists in alphabetical order. Make sure you label which tree is a BFS tree and which is a DFS tree

Solution: The edge set $\{(a,b),(b,c),(c,d)(d,e)\}$ *is the DFS tree. The edge set* $\{(a,b),(b,c),(b,d)(d,e)\}$ *is the BFS tree*

Solution: The edge set $\{(a,b),(a,c),(a,d)\}$ *is a BFS tree. The edge set* $\{(a,b),(b,c),(c,d)\}$ *is a DFS tree.*

Eecs.Berkeley.edu

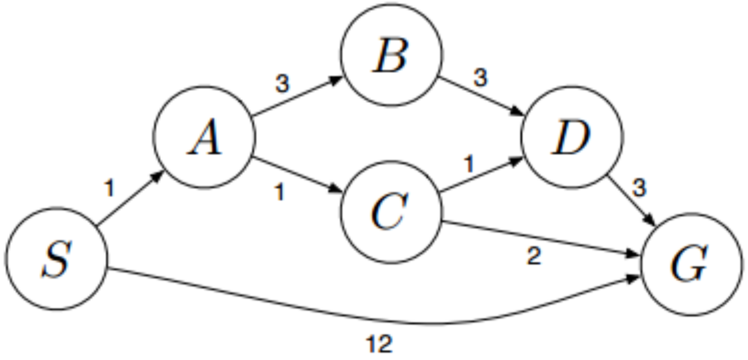
- Do DFS and BFS starting with s



A	✓
B	✓
C	✓
D	✓
E	✓
G	✓
S	✓

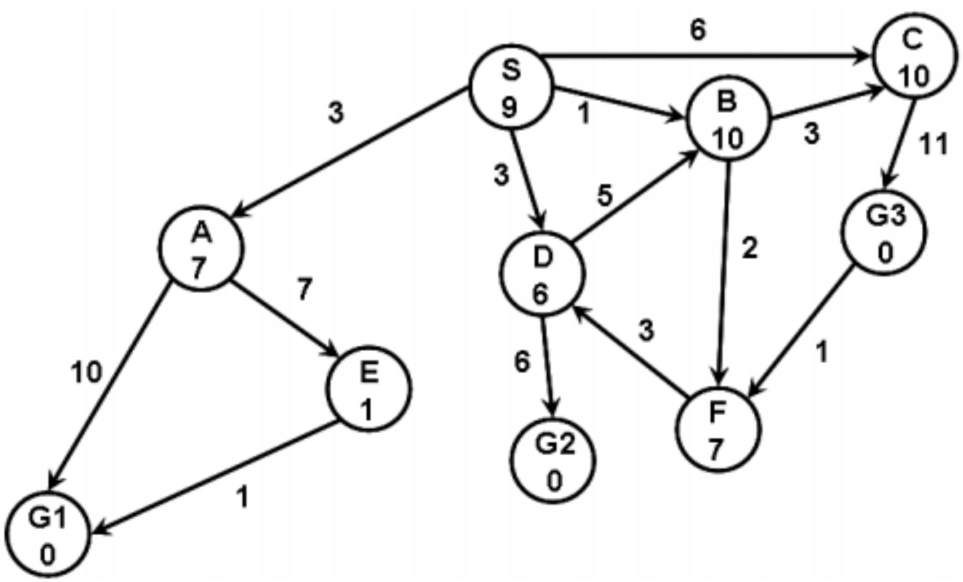
S A B G, C D E

- Do DFS and BFS starting with s



www.cs.cmu.edu

- Do DFS and BFS starting with s



4. For the graph below, do:

- **(a)** Do a DFS starting at vertex *a* (and using the "alphabetical rule"). Shade the edges of the resulting DFS spanning tree and show all vertex labels. Give the order vertices are added to T and P.
- **(b)** Do a BFS starting at vertex *a* (and using the "alphabetical rule"). Shade the edges of the resulting BFS spanning tree and show all vertex labels. Give the order vertices are added to the queue T.