CS 165 Winter 2023 *Last updated:*February 28, 2023

Homework 3

Due March 8th, 2023

This assignment is a continuation of the last assignment.

1 Environment setup

You will need to use GPU for this assignment. You can use the provided Nvidia GPU instance, Google colab, or your personal desktop. If you opt for Nvidia GPU instance:

```
Sign up and enroll in the course:
https://courses.nvidia.com/courses/course-v1:DLI+S-OV-04+V1/
Click Enter Promo Code and enter:
DLITEACH0223_11_NWXB_32
Enter your billing address and submit.
```

Restarting the GPU instance will reset any changes you made, so save your work frequently. You can also request access to the Caltech cluster by contacting the TA.

First, let's run the notebook 1-setup.ipynb given in the Bootcamp repo `https://github.com/NeuralOperator/bootcamp/`. This notebook will install all the packages and download the darcy flow data we need for this assignment.

2 Tensorized Fourier neural operator

Tensorized Fourier neural operator (TFNO) parameterizes the weights of the Fourier layers with a low-rank tensor factorization. Specifically, TFNO impose a coupling between all the weights in the Fourier space by jointly parameterizing them with a single tensor, learned in a factorized form such as Tucker or Canonical-Polyadic. This coupling allows TFNO to maintain FNO's expressivity with fewer parameters. The goal of this assignment is to use the tensorized Fourier neural operator to simulate the Darcy equation.

The Darcy Flow equation is a 2nd-order Elliptic equation

$$\begin{aligned} -\nabla \cdot (a(x)\nabla u(x)) &= f(x) \qquad x \in (0,1)^2 \\ u(x) &= 0 \qquad\quad\;\; x \in \partial(0,1)^2 \end{aligned} \tag{1}$$

This PDE has numerous applications including modeling the pressure of the subsurface flow, the deformation of linearly elastic materials, and the electric potential in conductive materials. We are interested in learning the operator mapping the diffusion coefficient a to the solution u. $G : a \mapsto u$. The force $f = 1$ is fixed with a zero Dirichlet boundary.

You can use the same training pipeline we used in previous assingment (notebook for training FNO).

1. Use the configuration file provided in tfno_darcy_config.yaml to train TFNO on the given datasets. Report the test L^2 and H^1 errors on resolutions 32 and 64. Report the number of parameters of TFNO.
2. Based on the provided original yaml, vary the number of modes of TFNO from 8 to 32 (You can use the same number for height and width). Plot the test L^2 and H^1 error on resolutions 32 and 64 change versus number of modes.
3. Use the best number of modes you found. Now vary the rank of tensor factorization from 0.01 to 0.9. Plot the test L^2 and H^1 error on resolutions 32 and 64 change versus the number of paramters of TFNO.