CS 165 Winter 2023 *Last updated:* January 25, 2023

Homework 2

Due at 11:59 pm, Feb 2^{nd}

1 Simple binary hypotheses

Consider binary hypotheses

$$\mathcal{H}_0 : \; Y \sim f_0(y) = \begin{cases} 1 & y \in (0,1) \\ 0 & \text{otherwise} \end{cases} \quad \text{vs.} \quad \mathcal{H}_1 : \; Y \sim f_1(y) = \begin{cases} 2y & y \in (0,1) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Consider a Bayesian test with uniform cost, i.e., $C_{ii} = 0$, and $C_{ij} = 1$ for $i \neq j$. Find the Bayesian detector as a function of prior $\pi_0 = \Pr(\mathcal{H}_0)$ and the Bayesian risk.

2 Detection of mixture

Consider the following two hypotheses. For $\mathcal{H}_0$, the observation Y is drawn from a uniform distribution $f_0(y)$ in $[0,1]$. For $\mathcal{H}_1$, with probability 0.5, $f_0(y)$ is used, and and with probability 0.5, density $q(y) = 2y, y \in [0,1]$ is used. The distribution under $\mathcal{H}_1$ is called a mixture of $f_0(y)$ and $q(y)$.

1. Give the precise sketch of the likelihood ratio.
2. For the uniform cost, i.e., $C_{ij} = 0$ when $i = j$ and $C_{ij} = 1$ otherwise, find the Bayesian detector as a function of the prior $\pi = \Pr(\mathcal{H}_0)$.
3. Find the Bayesian risk expression $V(\pi)$ as a function of the prior $\pi = \Pr(\mathcal{H}_0)$.
4. **Optional:** Sketch $V(\pi)$ vs. π, and find the worst prior

$$\pi^* = \arg\max_{\pi} V(\pi)$$

   (You can use software if you cannot computed it directly)

3 Uniform noise

Suppose that a single observation Y is generated from

$$Y = \theta\lambda + N$$

where $N \sim \mathcal{U}(-1, 2)$ and $\lambda \in (0, 2)$ a known constant. The two hypotheses are given by

$$\mathcal{H}_0 : \theta = 0 \;\; \text{vs.} \;\; \mathcal{H}_1 : \theta = 1.$$

1. Find the NP detection rule for a given size $\alpha \in (0, 1)$.
2. Find the power of the detector as a function of α and λ. Sketch the ROC curve.