

# Evolutionary Computation for Dynamic Optimization Problems: Fundamentals

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# Lecturer: Shengxiang Yang

- Education and career history:
  - PhD, Northeastern University, China, 1999
  - Worked at King's College London, University of Leicester, Brunel University, 1999-2012
  - Joined De Montfort University (DMU) as Professor in Computational Intelligence (CI) in July 2012
  - Director, Centre for Computational Intelligence (CCI), since 2013
- Research interests:
  - Evolutionary Computation (EC) and nature-inspired computation
  - Dynamic optimisation and multi-objective optimisation
  - Relevant real-world applications
- Over 290 publications and over £2M funding
- AE/Editorial Board Member for 8 journals
- Chaired two IEEE CIS Task Forces
  - EC in Dynamic and Uncertain Environments (2011-2017)
  - Intelligent Network Systems (2012-2017)

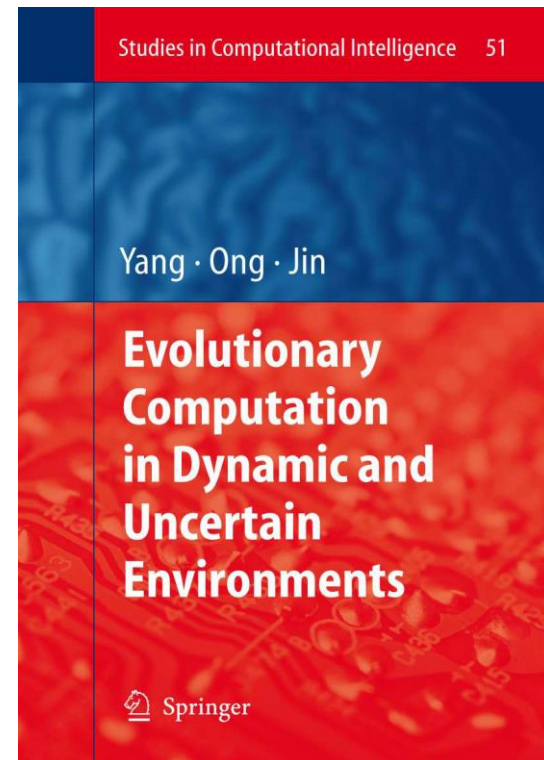
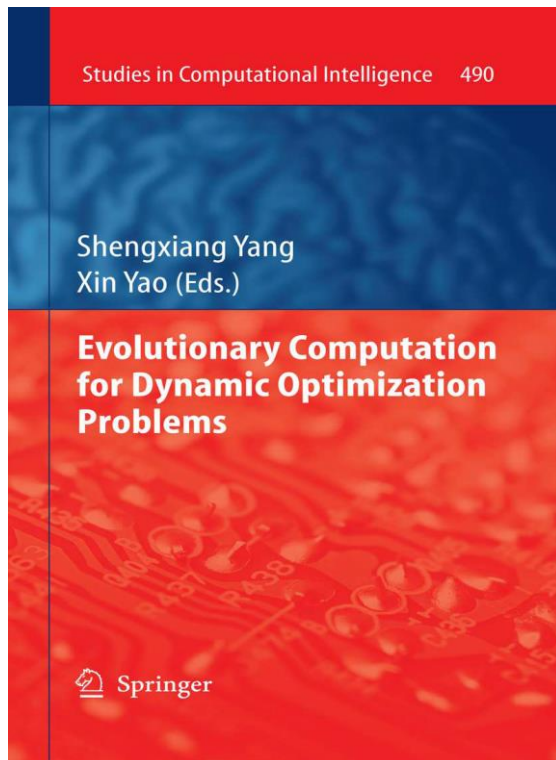
# Centre for Computational Intelligence, DMU



- CCI ([www.cci.dmu.ac.uk](http://www.cci.dmu.ac.uk)):
  - **Mission:** Developing fundamental theoretical and practical solutions to real-world problems using a variety of CI paradigms
  - Members: 16 staff, research fellows, ~30 PhDs, **visiting researchers**
  - Themes: EC, fuzzy logic, neural networks, data mining, robotics, game ...
- Funding:
  - Research Councils/Charities: EPSRC, ESRC, EU FP7 & Horizon 2020, Royal Society, Royal Academy of Eng., Innovate UK, KTP, Nuffield Trust ...
  - Government: Leicester City Council, DTI
  - Industries: Lachesis, EMDA, RSSB, Network Rail, etc.
- Collaborations:
  - Universities: UK, USA, Spain, and China
  - Industries and local governments
- Teaching/Training:
  - DTP-IS: University Doctor Training Programme in Intelligent Systems
  - MSc: Intelligent Systems (& Robotics); Data Analytics; BI & Data Mining
  - BSc: AI with Robotics; Computer Game Programming; Math; Comp Sci
- YouTube page: <http://www.youtube.com/thecci>

# Two Relevant Books

- S. Yang and X. Yao (eds.),  
Evolutionary Computation for  
Dynamic Optimization Problems,  
in the series Studies in  
Computational Intelligence, vol.  
490, Springer, 2013
- S. Yang, Y.-S. Ong, Y. Jin (eds.),  
Evolutionary Computation in  
Dynamic and Uncertain  
Environments, in the series Studies  
in Computational Intelligence, vol.  
51. Springer, March 2007

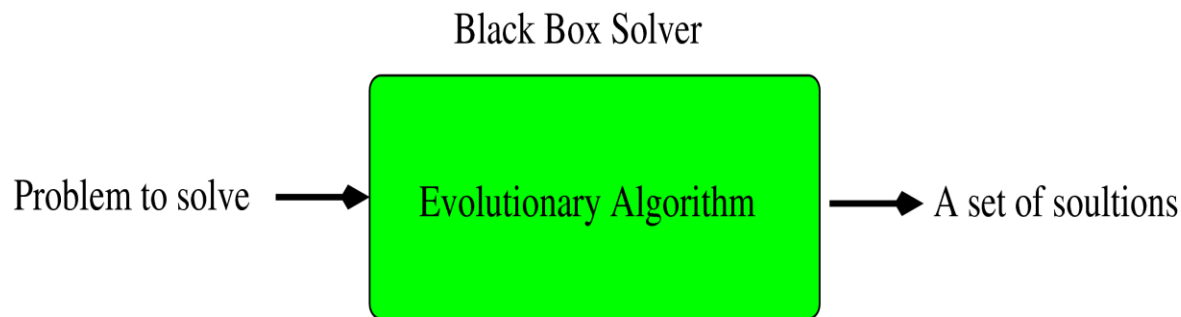


# Outline of the Lecture

- Introduction to evolutionary computation (EC)
- EC for dynamic optimization problems (DOPs): Concept and motivation
- Benchmark and test problems
- Performance measures
- EC approaches for DOPs
- Summary

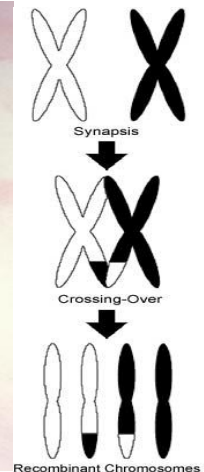
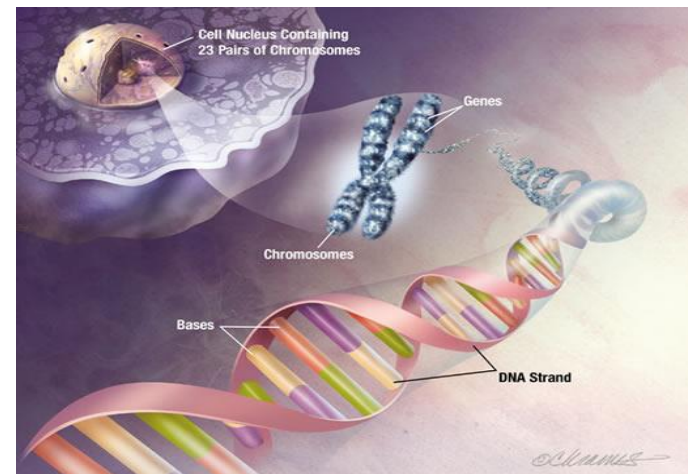
# What is Evolutionary Computation (EC)?

- EC encapsulates a class of **stochastic optimisation algorithms**, dubbed **Evolutionary Algorithms (EAs)**
- An EA is **an optimisation algorithm** that is
  - **Generic**: a black-box tool for many problems
  - **Population-based**: evolves a population of candidate solutions
  - **Stochastic**: uses probabilistic rules
  - **Bio-inspired**: uses principles from biological evolution



# Principles of Biological Evolution

- For each organism, a **population of individuals** exists in a **natural environment** with limited resources and evolves as:
  - Individuals compete for resources: **selection of the fittest** to **reproduce offspring**
  - They reproduce offspring by **genetic inheritance with variation** (**recombination** and **mutation**)
  - Offspring have their **fitness evaluated** by the environment and compete for survival
- Over time, natural selection drives the population to be better and better



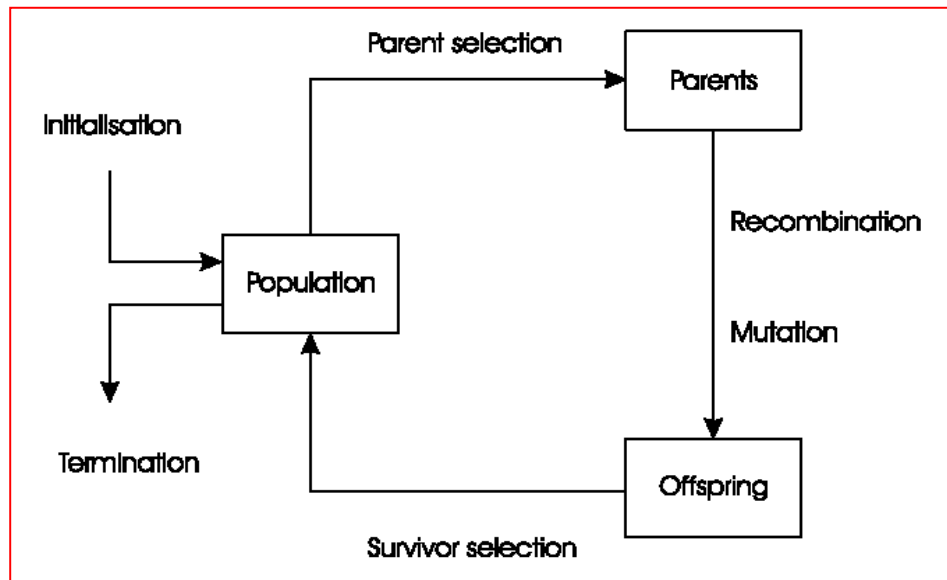
# Design and Framework of an EA

- Given a problem to solve, key things to consider:

- Representation of solution into individual
- Evaluation or fitness function

- EA Framework

- Initialization of population
- Evolve the population
  - Selection of parents
  - Variation operators (recombination, mutation)
  - Selection of offspring into next generation
- Termination condition: a given number of generations



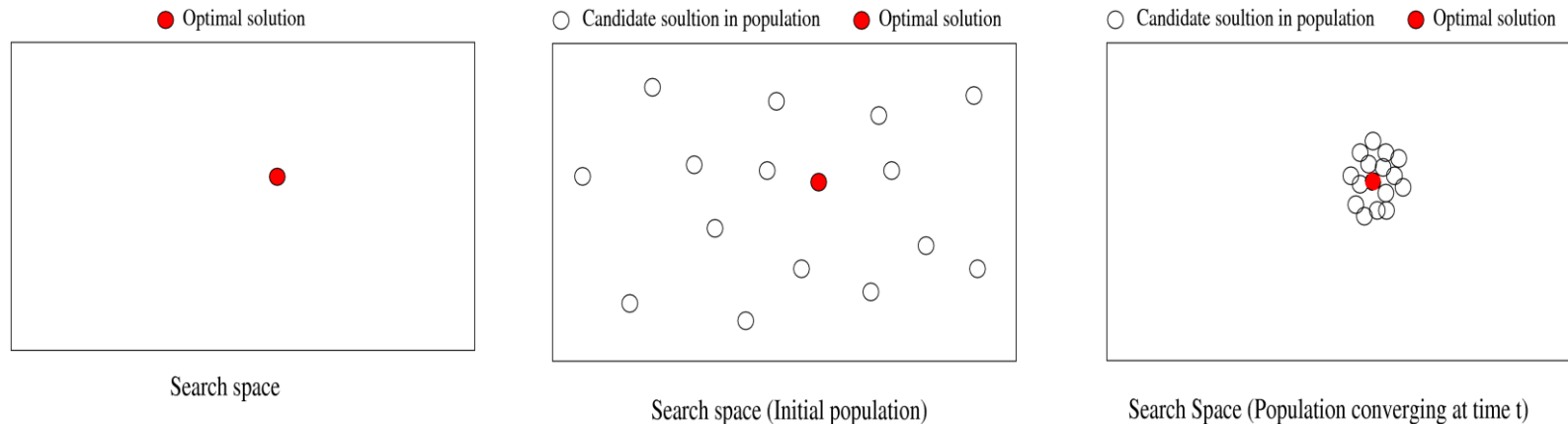


# EC Applications

- Easy to use: No strict requirements to problems
- Widely used for optimisation and search problems
  - Search-based software engineering
  - Financial and economical systems
  - Transportation and logistics systems
  - Automatic programming, art and music design
  - Industrial engineering
  - .....

# EC for Dynamic Optimisation Problems: Motivation

- Traditionally, EAs are mainly applied for static problems
  - Aim: find optimum **quickly** and **precisely** in the search space



- But, many real-world problems are **dynamic optimisation problems (DOPs)**, where changes occur over time
  - Transport systems: travel time between nodes may change
  - Logistics: customer demands may change

# What Are DOPs?

- In general terms, “optimisation problems that change over time” are called **dynamic problems** or **time-dependent problems**:

$$F = f(X, S, t)$$

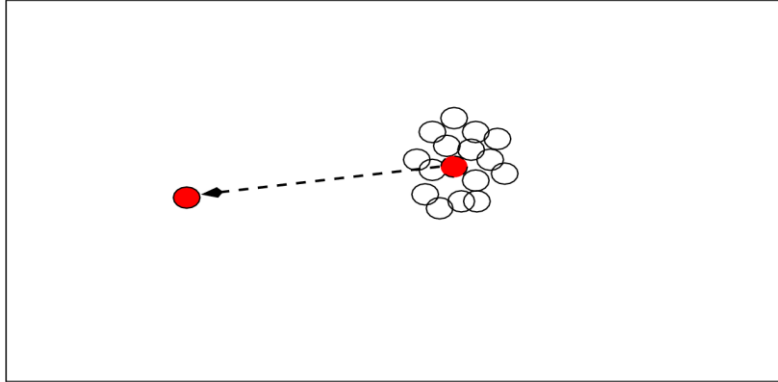
where X: decision variable(s); S: parameters; t: time

- DOPs: a special class of dynamic problems that are **solved online by an algorithm** as time goes by

# Why DOPs Challenging EC?

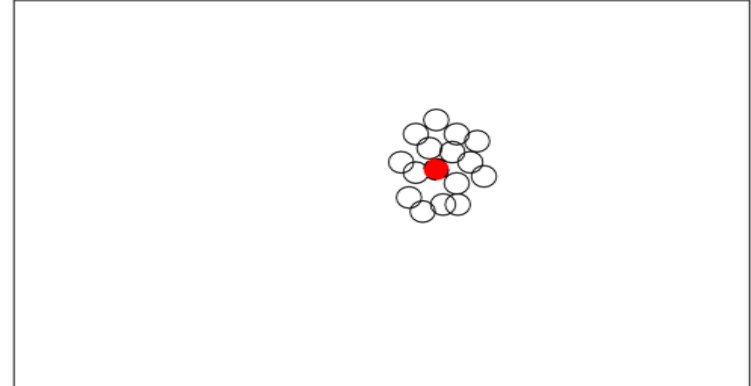
- For DOPs, optima may move over time in search space
  - We need to track the moving optima over time
- DOPs challenge traditional EAs
  - Once converged, hard to escape from the old optimum

○ Candidate solution in population      ● Optimal solution



Search Space (Optimum moved at time  $t+1$ )

○ Candidate solution in population      ● Optimal solution



Search Space (Population converging at time  $t$ )

# Why EC for DOPs?

- Many real-life problems are DOPs
  - Desirable to present solutions to decision makers over time
- EAs, once properly enhanced, are good choice
  - Inspired by biological behaviour, always in dynamic environments
  - Intrinsically, should be fine to deal with DOPs
- Research on EC for DOPs rises recently
  - Books, PhD Theses
  - Journal special issues
  - Workshops and conference special sessions
  - IEEE Symposium on CIDUE (2011, 2013-2020)
  - IEEE Competitions: within 2009 & 2012 IEEE CEC

# Benchmark and Test DOPs

- Basic idea: change base static problem to create DOPs
- Real space:
  - Switch between different functions
  - Move/reshape peaks in the fitness landscape
- Binary space:
  - Switch between  $\geq 2$  states of a problem: knapsack
  - Use binary masks: XOR DOP generator (Yang & Yao'05)
- Combinatorial space:
  - Change decision variables: item weights/profits in knapsack problems
  - Add/delete decision variables: new jobs in scheduling, nodes added/deleted in network routing problems

# Moving Peaks Benchmark (MPB) Problem

- Proposed by Branke (1999)
- The MPB problem in the D-dimensional space:

$$F(\vec{x}, t) = \max_{i=1, \dots, p} \frac{H_i(t)}{1 + W_i(t) \sum_{j=1}^D (x_j(t) - X_{ij}(t))^2}$$

–  $W_i(t)$ ,  $H_i(t)$ ,  $X_i(t) = \{X_{i1} \dots X_{iD}\}$ : height, width, location of peak  $i$  at  $t$

- The dynamics:

$$H_i(t) = H_i(t - 1) + \text{height\_severity} * \sigma$$

$$W_i(t) = W_i(t - 1) + \text{width\_severity} * \sigma$$

$$\vec{v}_i(t) = \frac{s}{|\vec{r} + \vec{v}_i(t - 1)|} ((1 - \lambda)\vec{r} + \lambda\vec{v}_i(t - 1))$$

$$\vec{X}_i(t) = \vec{X}_i(t - 1) + \vec{v}_i(t)$$

- $\sigma \sim N(0, 1)$ ;  $\lambda$ : correlated parameter
- $\vec{v}_i(t)$ : shift vector, which combines random vector  $\vec{r}$  and  $\vec{v}_i(t - 1)$  and is normalized to the shift length  $s$

# Dynamic Knapsack Problems (DKPs)

- Static knapsack problem:

- Given  $n$  items, each with a weight and a profit, and a knapsack with a fixed capacity, select items to fill up the knapsack to maximize the profit while satisfying the knapsack capacity constraint

- The DKP:

- Constructed by changing weights and profits of items, and/or knapsack capacity over time as:

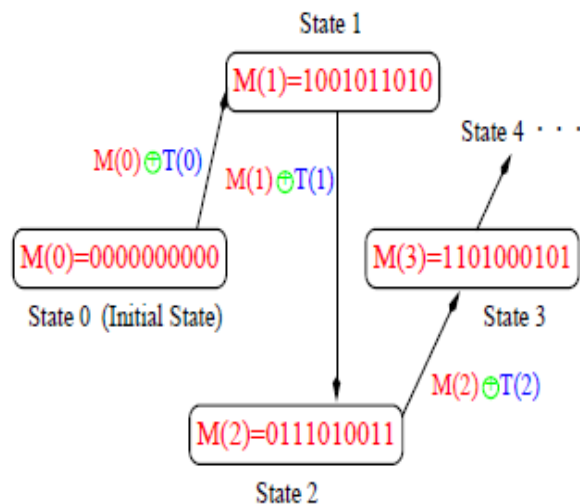
$$\text{Max } f(\vec{x}(t), t) = \sum_{i=1}^n p_i(t) \cdot x_i(t), \quad \text{s. t. : } \sum_{i=1}^n w_i(t) \cdot x_i(t) \leq C(t)$$

- $\vec{x}(t) \in \{0, 1\}^n$ : a solution at time  $t$
- $x_i(t) \in \{0, 1\}$ : indicates whether item  $i$  is included or not
- $p_i(t)$  and  $w_i(t)$ : profit and weight of item  $i$  at  $t$
- $C(t)$ : knapsack capacity at  $t$



# The XOR DOP Generator

- The **XOR DOP generator** can create DOPs from any binary  $f(\vec{x})$  by an XOR operator “ $\oplus$ ” (Yang, 2003; Yang & Yao, 2005)
- Suppose the environment changes every  $\tau$  generations
- For each environmental period  $k = \lfloor t/\tau \rfloor$ , do:



1 Create a template  $T_k$  with  $\rho * l$  ones

2 Create a mask  $\vec{M}(k)$  incrementally

$$\vec{M}(0) = \vec{0} \text{ (the initial state)}$$

$$\vec{M}(k+1) = \vec{M}(k) \oplus \vec{T}(k)$$

3 Evaluate an individual:

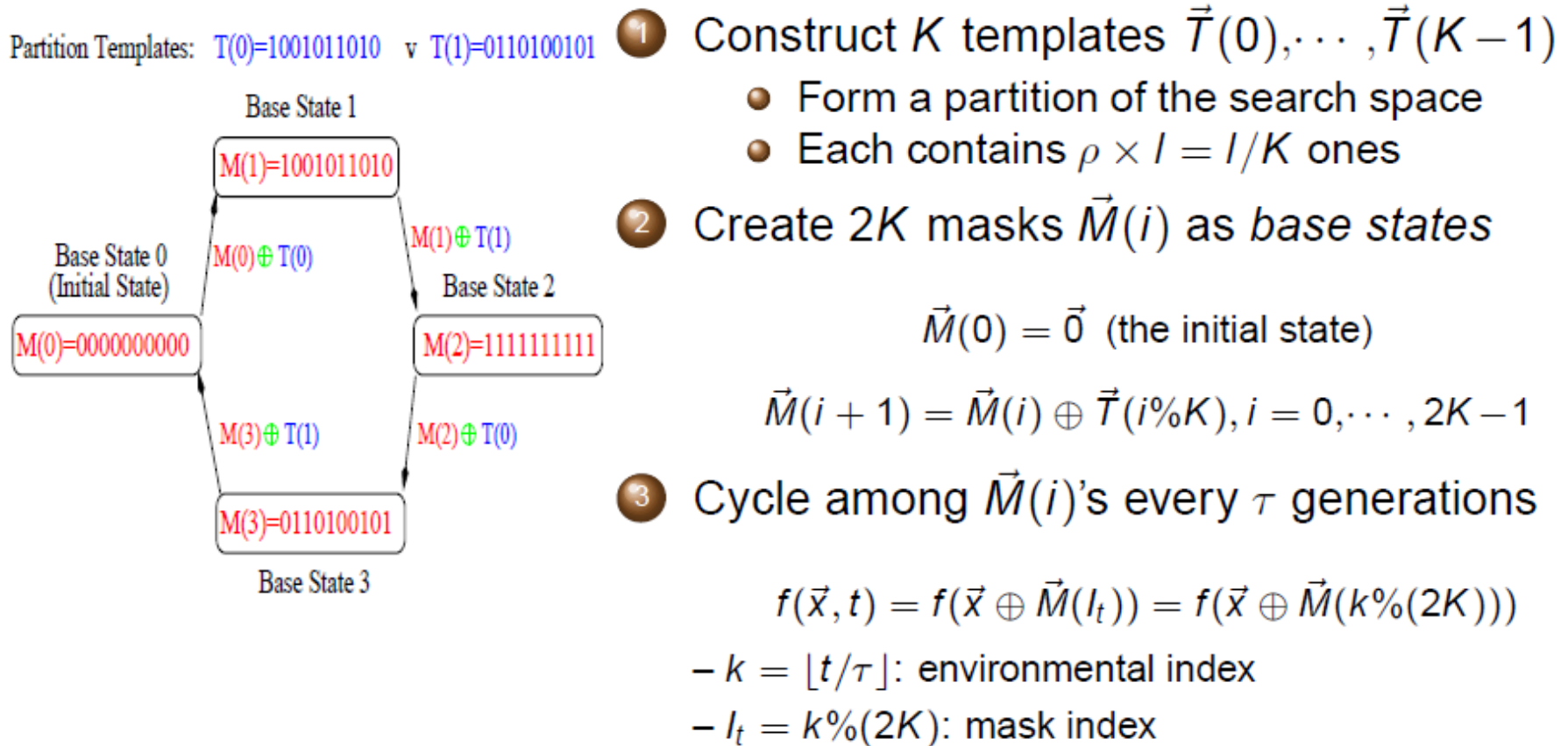
$$f(\vec{x}, t) = f(\vec{x} \oplus \vec{M}(k))$$

- $\tau$  and  $\rho$  controls the speed and severity of change respectively

S. Yang and X. Yao. Experimental study on population-based incremental learning algorithms for dynamic optimization problems. Soft Computing, 9(11): 815-834, November 2005.

# Constructing Cyclic Dynamic Environments

- Extend XOR DOP generator to create cyclic environments



# Dynamic Traveling Salesman Problem

- Stationary traveling salesman problem (TSP):
  - Given a set of cities, find the shortest route that visits each city once and only once
- Dynamic TSP (DTSP):
  - May involve dynamic cost (distance) matrix

$$D(t) = \{d_{ij}(t)\}_{n \times n}$$

$d_{ij}(t)$ : cost from city  $i$  to  $j$ ;  $n$ : the number of cities

- The aim: to find a minimum-cost route containing all cities at time  $t$
- DTSP can be defined as  $f(x, t)$ :

$$f(x, t) = \text{Min}(\sum_{i=1}^n d_{x_i, x_{i+1}}(t))$$

where  $x_i \in 1, \dots, n$ . If  $i \neq j$ ,  $x_i \neq x_j$ , and  $x_{n+1} = x_1$

# Performance Measures

- For EC for static problems, 2 key performance measures:
  - Convergence speed, Success rate of reaching optimality
- For EC for DOPs, over 20 measures (Nguyen et al., 2012)
  - Optimality-based performance measures
    - Collective mean fitness or mean best-of-generation
    - Accuracy
    - Adaptation
    - Offline error and offline performance
    - Mean distance to optimum at each generation
    - .....
  - Behaviour-based performance measures
    - Reactivity
    - Stability
    - Robustness
    - Satisfiability
    - Diversity measures
    - .....

# Performance Measures: Examples

- Collective mean fitness (mean best-of-generation):

$$\overline{F}_{BOG} = \frac{1}{G} \times \sum_{i=1}^{i=G} \left( \frac{1}{N} \times \sum_{j=1}^{j=N} F_{BOG_{ij}} \right)$$

- $G$  and  $N$ : number of generations and runs, resp.
- $F_{BOG_{ij}}$ : best-of-generation fitness of generation  $i$  of run  $j$

- Adaptation performance (Mori et al., 1997)

$$Ada = \frac{1}{T} \sum_{t=1..T} (f_{best}(t)/f_{opt}(t))$$

- Accuracy (Trojanowski and Michalewicz, 1999)

$$Acc = \frac{1}{K} \sum_{i=1..K} (f_{best}(i) - f_{opt}(i))$$

- $f_{best}(i)$ : best fitness for environment  $i$  (best before change)

# EC for DOPs: Things to Do

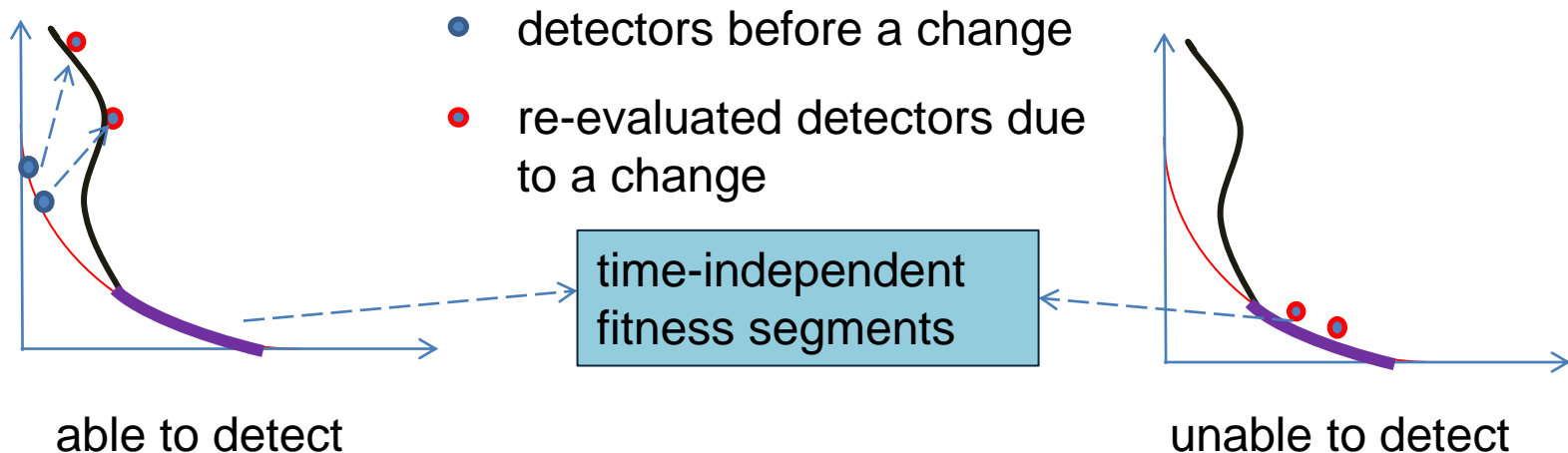
- **Change detection:** detect potential environmental changes
  - Success rate of detection
  - Cost of detection
- **Response or enhancing approaches:** track the changing optima
  - To expect a steady and fast change response
  - To reduce the cost of tracking (given the budget limit, i.e., time, memory)

# Change Detection Approaches

- Two ways of detecting changes:
  - Individual-level detection: fast but not robust
  - Population-level detection: slow but robust
  - Both methods could fail to detect changes (not 100% guaranteed)

# Individual-level Change Detection

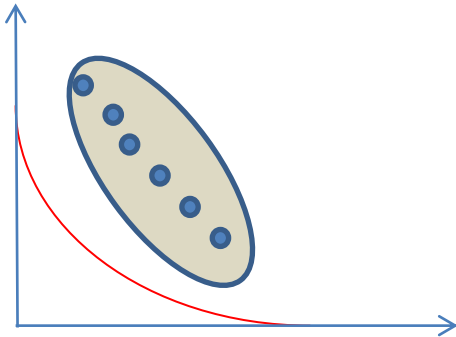
- Re-evaluate some individuals' fitness every generation
  - Check the discrepancy between their current and previous fitness
- Success rate of detection depends on
  - Detectability of environmental changes
  - Location of detectors placed



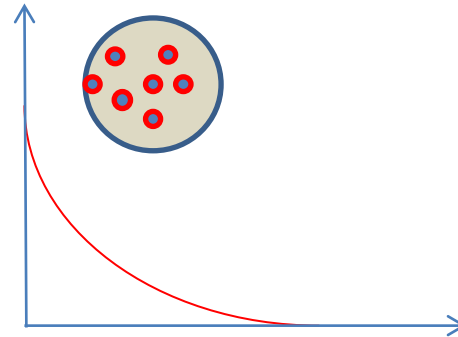


# Population-level Change Detection

- Population-related statistical information, i.e., distribution, is assessed in every generation
  - Check the significance of variation in statistical information



population distribution  
**before** a change



population distribution  
**after** a change

- Less sensitive to noise but possibly higher computational cost

# Response/Enhancing: First Thinking

- Recap: traditional EAs are not good for DOPs
- Goal: to track the changing optimum
- How about restarting an EA after a change?
  - Natural and easy choice
  - But, not good choice because:
    - It may be inefficient, wasting computational resources
    - It may lead to very different solutions before and after a change.  
For real-world problems, we may expect solutions to remain similar
- Extra approaches are needed to enhance EAs for DOPs

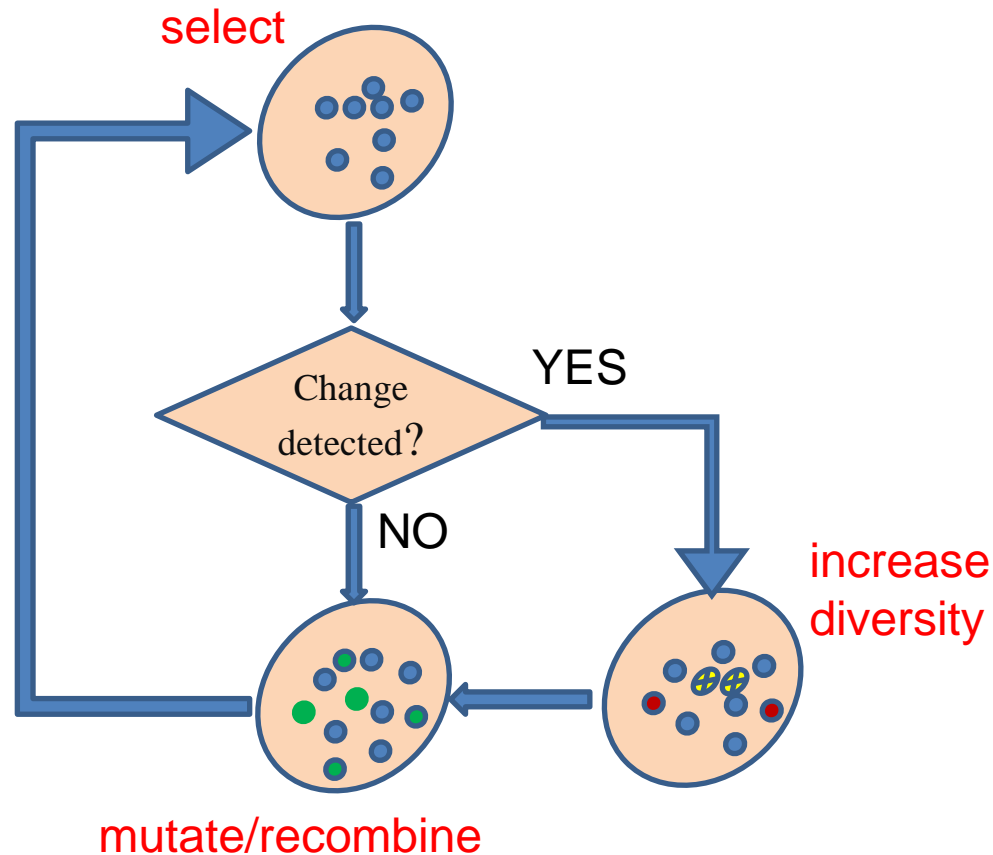
# Response/Enhancing: General Approaches

- Many approaches developed to enhance EAs for DOPs
- Typical approaches:
  - Diversity: handle convergence directly
  - Memory: store and reuse useful information
  - Multi-population: co-operate sub-populations
  - Adaptive: adapt generators and parameters
  - Prediction: predict changes and take actions in advance
- They have been applied to different EAs for DOPs

1. M. Mavrovouniotis, C. Li, and S. Yang. A survey of swarm intelligence for dynamic optimization: Algorithms and applications. *Swarm and Evolutionary Computation*, 33: 1-17, April 2017
2. T. T. Nguyen, S. Yang, and J. Branke. Evolutionary dynamic optimization: A survey of the state of the art. *Swarm and Evolutionary Computation*, 6: 1-24, October 2012

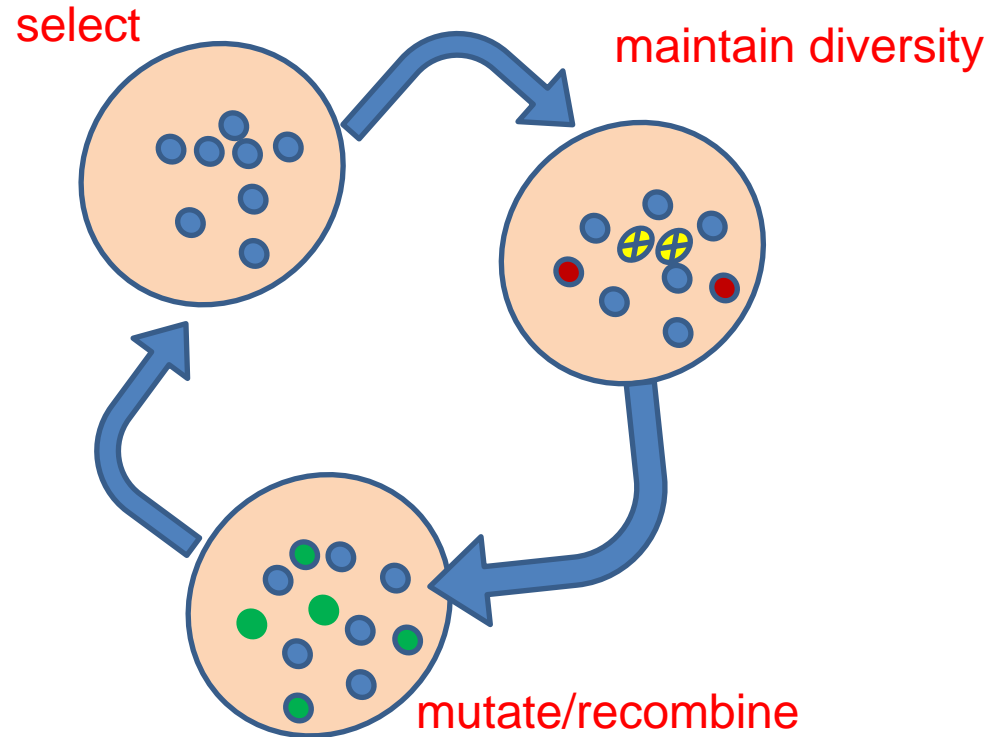
# Diversity Approaches

- **Diversity increase:** introduce diversity after a change
  - Partially random restart, hyper-mutation, variable local search



# Diversity Approaches

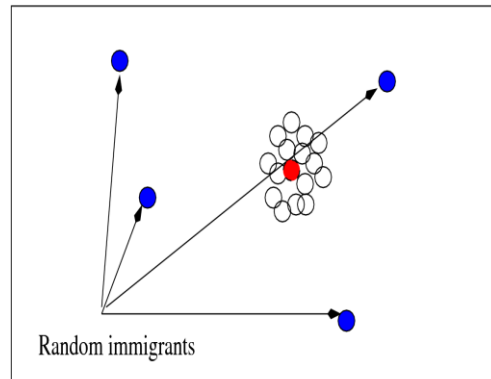
- **Diversity maintenance:** maintain diversity throughout the run (even if no change occurs)
  - Random immigrants



# Diversity Approaches: Random Immigrants

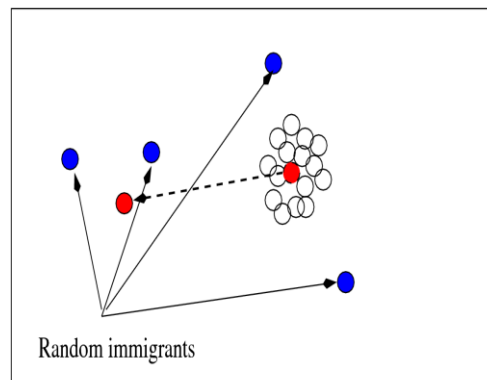
- Diversity approaches address convergence directly
  - Each generation, generate some random solutions (**random immigrants**) into the population to maintain the diversity
  - When optimum moves, random immigrants nearby take action
  - They will draw the population to the new optimum

○ Candidate solution in population    ● Optimal solution



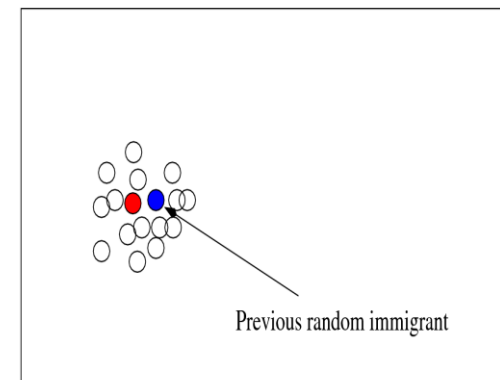
Search Space (Population converging at time  $t$ )

○ Candidate solution in population    ● Optimal solution



Search Space (Optimum moved at time  $t+1$ )

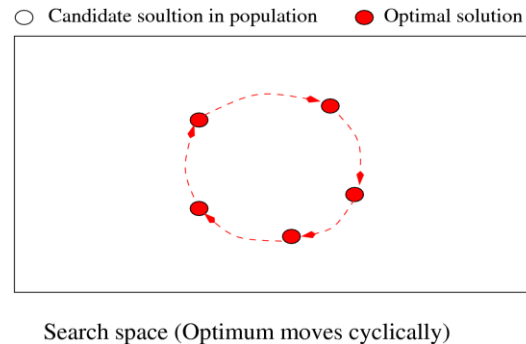
○ Candidate solution in population    ● Optimal solution



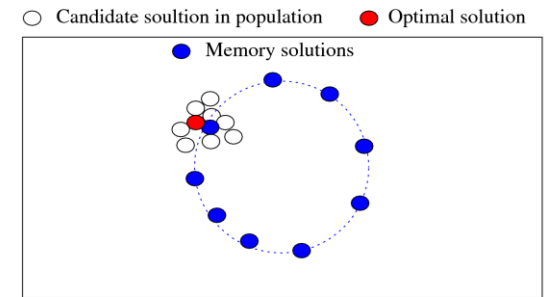
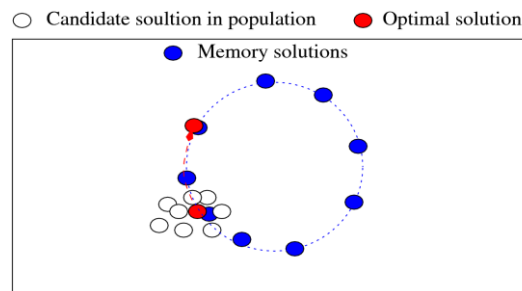
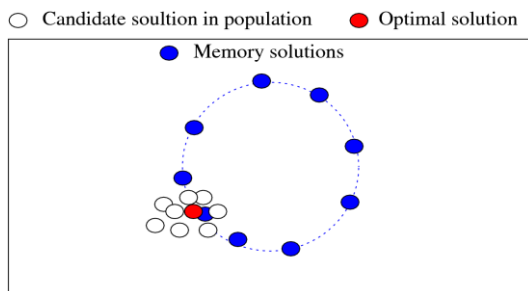
Search Space (Population moves to new optimum)

# Memory Approaches

- Cyclic DOPs: change cyclically among a fixed set of states

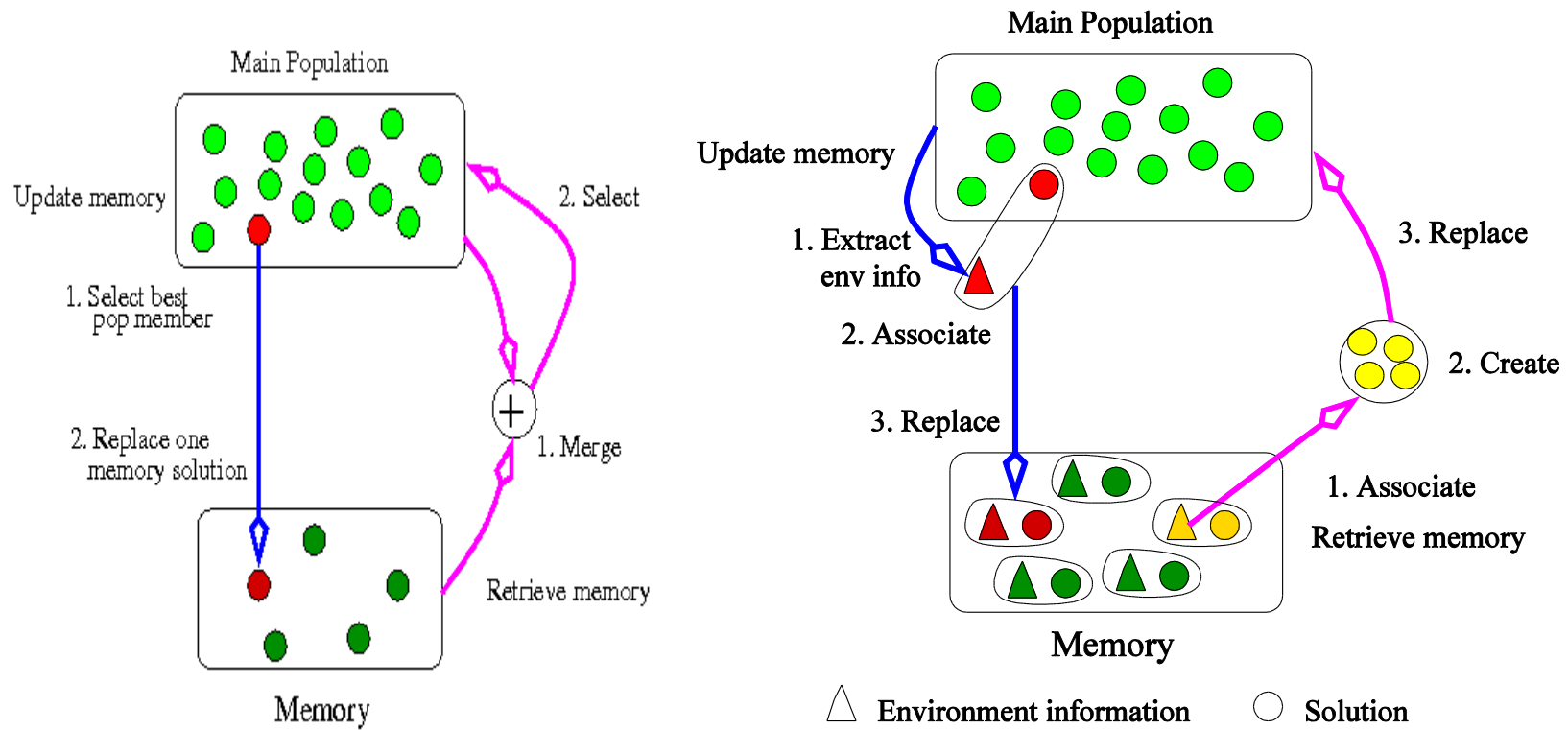


- For cyclic DOPs, use memory to store & reuse good solutions
  - With time, store the best solution of population into memory
  - When a change occurs, memory helps to track new optimum



# Direct Memory vs Associative Memory

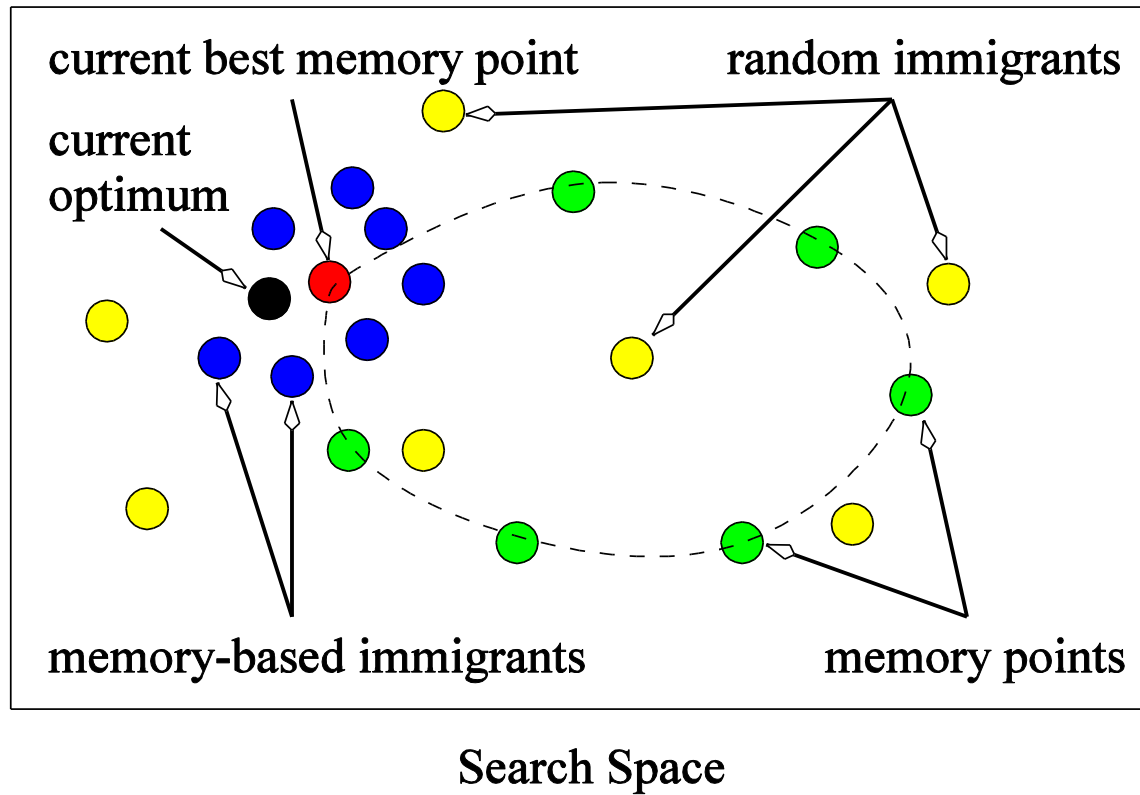
- **Direct memory**: store good solutions (Branke, CEC'99)
- **Associative memory**: store environmental information + good solutions





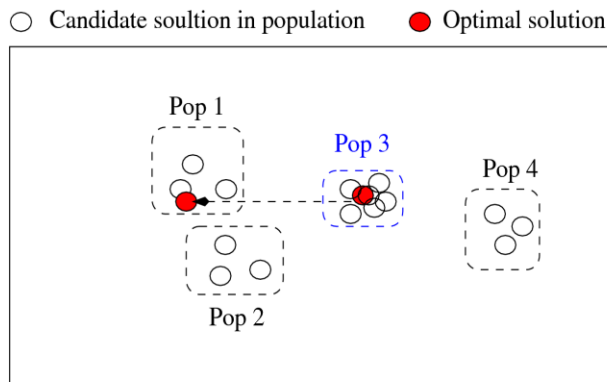
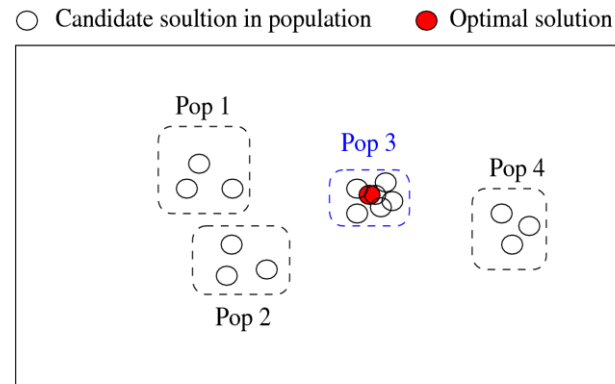
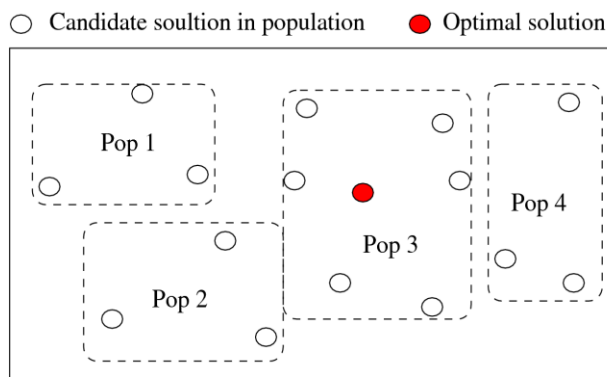
# Memory-Based Immigrants

- Use memory to guide immigrants toward current environment

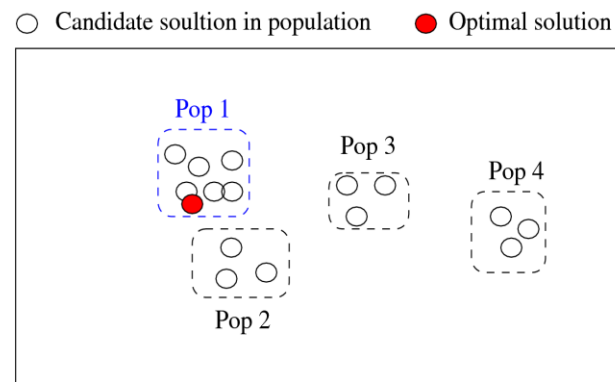


# Multi-population Approaches

- Idea: Use several cooperative populations
  - Populations evolve independently in different areas of search space
  - Populations exclude each other to avoid overlap
  - When optimum moves, nearby population will take action



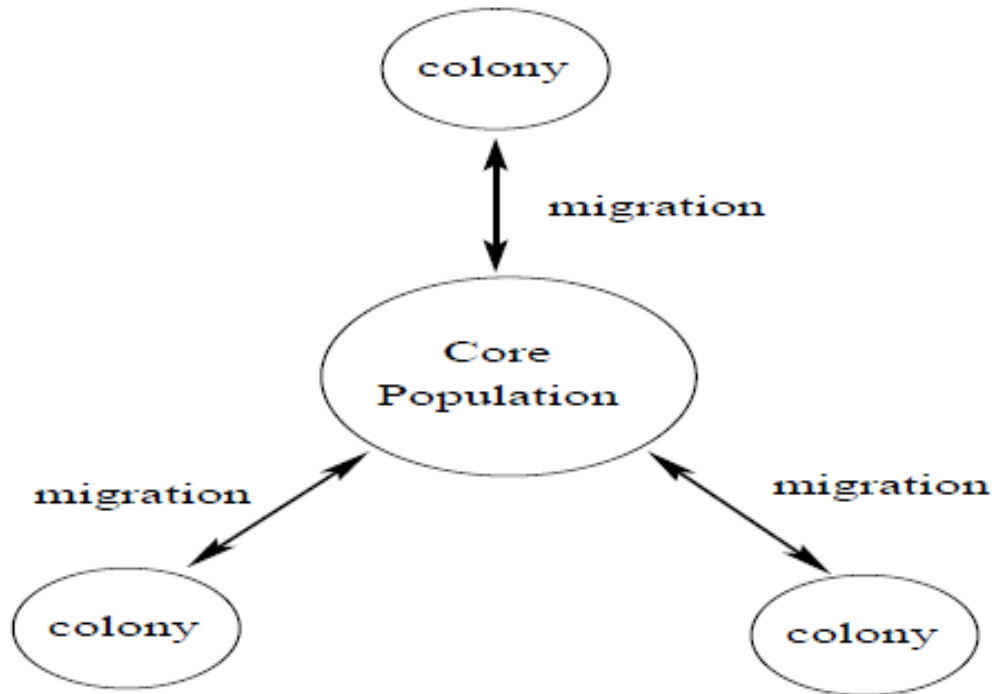
Search space (Optimum moves to near Pop 1)



Search space (Pop 1 becomes major pop)

# Multi-Populations: Shifting Balance

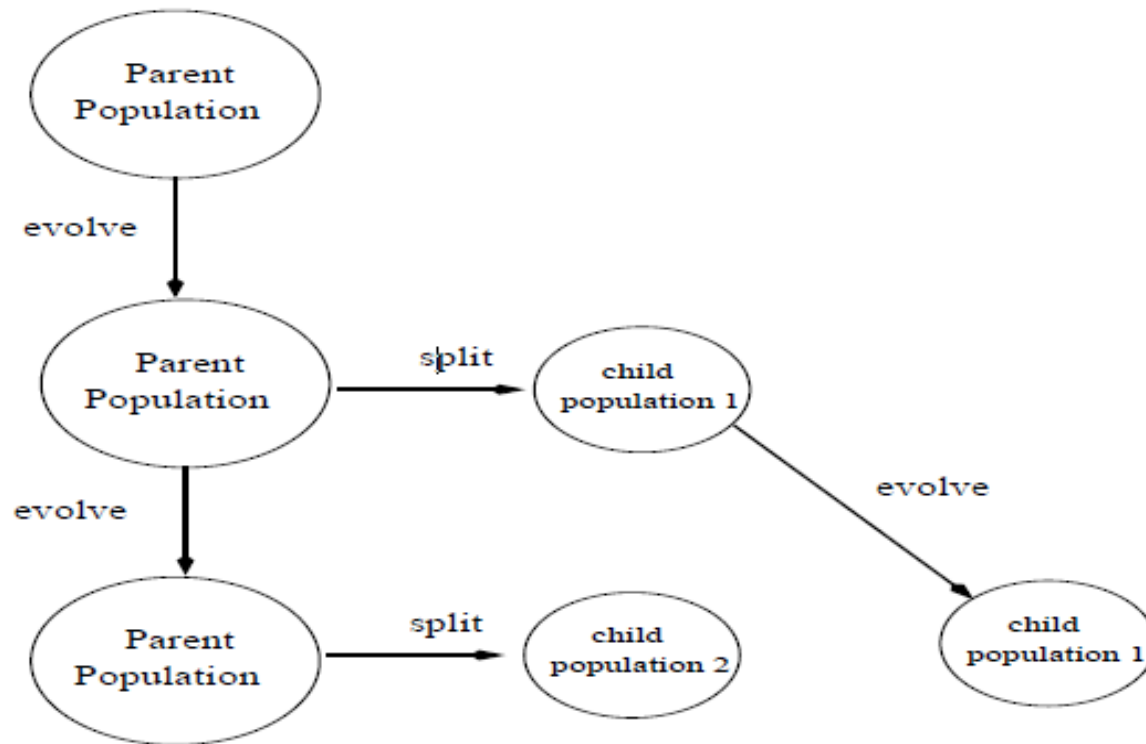
- Shifting Balance GA (Oppacher & Wineberg, 1999):
  - A core population exploits the promising area
  - Several colonies explore the search space



F. Oppacher, M. Wineberg (1999). The Shifting balance genetic algorithm: Improving the GA in a dynamic environment. GECCO'99, vol. 1, pp. 504–510

# Multi-Populations: Self-Organizing Scouts

- Self-organizing scouts (SOS) GA (Branke et al., 2000)
  - The parent population explores the search space
  - A child population is split under certain conditions
  - Child populations search limited promising areas



Branke J., Kaussler T., Smidt C., Schmeck H. (2000) A Multi-population Approach to Dynamic Optimization Problems. In: Parmee I.C. (eds) Evolutionary Design and Manufacture. Springer, London

# Prediction Approaches

- For some DOPs, changes exhibit predictable patterns
- Techniques (e.g., forecasting, Kalman filter) can be used to predict
  - The location of the next optimum after a change
  - When the next change may occur and which environment may appear

# Adaptive Approaches

- Aim: Adapt operators/parameters, usually after a change
  - **Hypermutation** (Cobb & Grefenstette'93): raise the mutation rate temporarily
  - Hyper-selection (Yang & Tinos'08): raise the selection pressure temporarily
  - Hyper-learning (Yang & Richter'09): raise the learning rate for Population-Based Incremental Learning (PBIL) temporarily

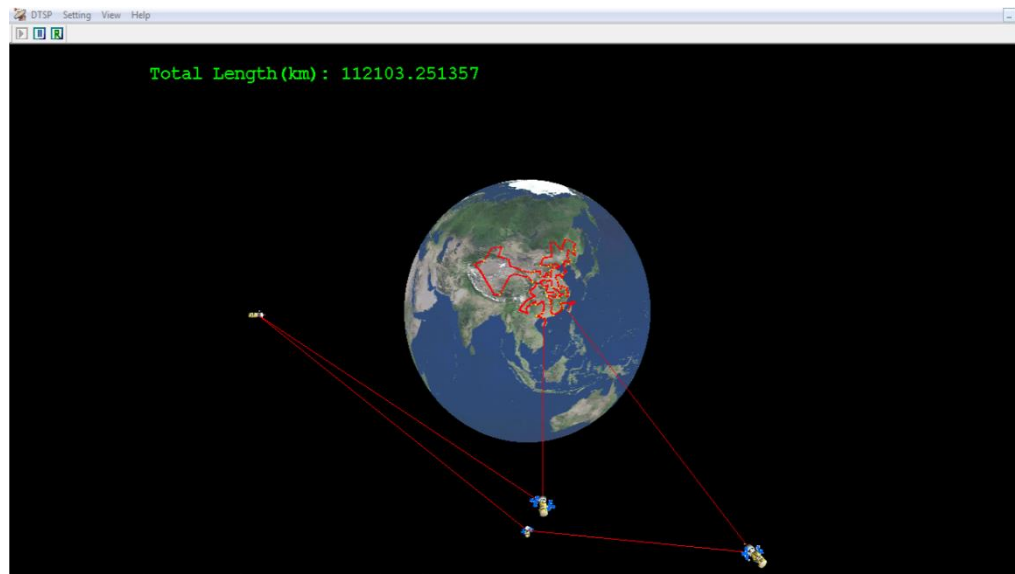
# Hybridization Approaches

- Idea: Using hybridization technique to improve the performance of EC for DOPs
- Hybridize EC with local search + diversity schemes, e.g.:
  - P-ACO: Hybridize ACO with local search and random immigrants
  - Multi-strategy ensemble PSO (MEPSO): Hybridize PSO with Gaussian local search + differential mutation
- Hybridize EC with other meta-heuristic methods
  - PSO + Cellular Automata
  - PSO + Fuzzy C-means

- M. Mavrovouniotis, S. Yang, A memetic ant colony optimization algorithm for the dynamic travelling salesman problem, *Soft Comput.* 15 (7) (2011) 1405–1425
- W. Du, B. Li, Multi-strategy ensemble particle swarm optimization for dynamic optimization, *Inf. Sci.* 178 (15) (2008) 3096–3109
- A. Hashemi, M. Meybodi, A multi-role cellular PSO for dynamic environments, in: 14th International Computer Conference (CSICC 2009), 2009, pp. 412–417
- M. Kamosi, A. Hashemi, M. Meybodi, A hibernating multi-swarm optimization algorithm for dynamic environments, in: 2010 2<sup>nd</sup> World Congress on Nature and Biologically Inspired Computing, 2010, 363–369

# Demo: Dynamic Travelling Salesman Problem

- Problem:
  - 144 Chinese cities, one geo-stationary satellite, and three mobile satellites
  - Find the path that cycles each city and satellite once with the minimal length **over time**
- Solver: EA with memory & immigrants schemes





# Remarks on Enhancing Approaches

- No clear winner among the approaches
- Memory is efficient for cyclic environments
- Multi-population is good for multimodal problems
  - Able to maintain diversity
  - The search ability will decrease if too many sub-populations
- Diversity schemes are usually useful
  - Guided immigrants may be more efficient
- **Thumb of rule:** balancing **exploration** & **exploitation** over time

# Summary

- EC for DOPs: important area
  - The domain is still young and active
  - Many challenges to be taken
- More young researchers are greatly welcome!



Thanks!

# Relevant Information

- IEEE CIS Task Force on EC in Dynamic and Uncertain Environments
  - <http://ieee-tf-ecidue.cug.edu.cn/>
- Source codes:
  - <http://www.tech.dmu.ac.uk/~syang/publications.html>
- Books and survey papers:
  - M. Mavrovouniotis, C. Li, and S. Yang. A survey of swarm intelligence for dynamic optimization: Algorithms and applications. *Swarm and Evolutionary Computation*, 33: 1-17, April 2017
  - T. T. Nguyen, S. Yang, J. Branke. Evolutionary dynamic optimization: A survey of the state of the art. *Swarm and Evolutionary Computation*, 6: 1-24, 2012
  - Y. Jin, J. Branke. Evolutionary optimization in uncertain environments—A survey. *IEEE Trans Evol Comput*, 9(3): 303–317, 2005
  - S. Yang, X. Yao. *Evolutionary Computation for Dynamic Optimization Problems*. Springer, 2013