Evolutionary Computation for Many-objective Optimization

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Outline of the Lecture

- Concept of many-objective optimization
- Challenges
- Evolutionary algorithms for many-objective optimization
- Summary

Many-objective Optimization Problems

Many-objective optimization problems (MaOPs) are multi-objective optimization problems (MOPs) where the number of objectives is larger than three

- MaOPs can be modeled as follows:

  $$Min/Max \quad F(x) = (f_1(x), f_2(x), \dots, f_m(x))$$

  $$\text{Subject to } g_j(x) \geq 0,\ h_k(x) = 0,\ x^L \leq x \leq x^U$$

  where $m \geq 4$
- Correspondingly, MOPs have 2 or 3 objectives

Single, Multi- & Many-Objective Optimization

Single-Objective Optimisation

- Often only one optimal solution
- Main work on Search

Multi-Objective Optimisation

- A number of Pareto optimal solutions
- Apart from Search, some work on Performance Assessment

Many-Objective Optimisation

- A huge number of Pareto optimal solutions
- Visualization is also important work

Visualisation | **Search** | **Performance Assessment**

Evolutionary Many-Objective Optimization

- Many EMO algorithms are effective for 2- or 3-objective optimization problems
- But they do not scale up well on MaOPs

Reasons?

Difficulties in Handling Many Objectives

- A larger fraction of population is non-dominated
- Recombination operation may be inefficient
- Representation of Pareto front is difficult
- Performance indicators are computationally expensive
- Visualization is hard
- Time or space requirement rapidly increases in some algorithms
- Setting of parameters is difficult in some algorithms

Many-Objective Optimization Applications

- Water resource engineering
  - Pierro et al, 2007, Kasprzyk et al, 2009
- Industrial scheduling problems
  - Sulflow et al, 2008
- Control system design
  - Fleming et al, 2005, Herrero et al, 2009
- Molecular design
  - Kruisselbrink et al, 2009
- Software product line
  - Sayyad et al, 2013, Olaechea et al, 2014, Henard et al, 2015
- Etc…

Evolutionary Many-Objective Optimisation: Challenges

- Common existence of optimization problems with 4 or more objectives in industrial and engineering design leads MaOPs to be a hot research topic in the EMO community
- However, some intrinsic characteristics of the high dimension landscape bring great challenges for algorithmic designers and practitioners
- Challenge I: Visualization
  - Infeasible to directly observe the solutions with four or more objectives
- Challenge II: Performance assessment
  - Difficulty of the substitution of the Pareto front
  - Complexity of the storage and running time
- Challenge III: Search methods
  - Ineffectiveness of the Pareto dominance relation
  - Misleading estimation of individuals' density in the population

Challenge I: Visualization General Methods

- Visualize a set of vectors in the objective space
  - Directly display objective vectors without modifications, e.g., parallel coordinate, bar chart, and star coordinate methods
    - Useful, but no information on Pareto dominance relation between vectors
  - Mapping high-dimensional objective vectors to 2-D/3-D vectors. Key concerns:
    - Maintenance of the Pareto dominance relation between vectors
    - Reflection of their location information in the population
- Introduce special test problems to help visualize search process
  - Distance minimization problems (Koppen & Yoshida, 2007) and extensions (Ishibuchi et al, 2010)
  - Weakness: Fails to exactly reflect the performance of objective vectors, i.e., their convergence and distribution regarding the Pareto front

Visualization: the Rectangle Problem

- Recently, we proposed a test problem, called **Rectangle problem**, to aid the visual investigation of many-objective search
- It minimizes the Euclidean distance from a solution to 4 lines (x1 = a1, x1 = a2, x2 = b1, and x2 = b2) parallel to the coordinate axes

$$\min \begin{array}{l} f_1(\boldsymbol{x}) = |x_1 - a_1| \\ f_2(\boldsymbol{x}) = |x_1 - a_2| \\ f_3(\boldsymbol{x}) = |x_2 - b_1| \\ f_4(\boldsymbol{x}) = |x_2 - b_2| \end{array}$$

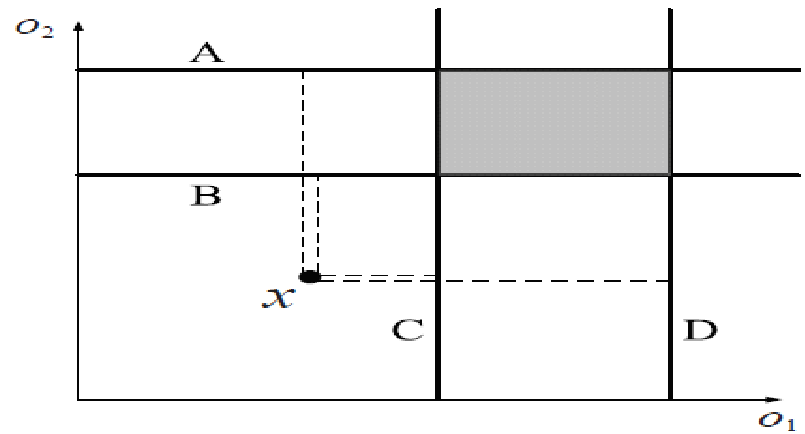


Figure: Illustration of a Rectangle problem whose Pareto optimal region is determined by 4 lines

M. Li, S. Yang, and X. Liu. A test problem for visual investigation of high-dimensional multi-objective search. Proceedings of the 2014 IEEE Congress on Evol Comput, 2140-2147, 2014

Visualization: the Rectangle Problem

- Key features:
  - Pareto optimal solutions lies in a rectangle in the 2-variable decision space
  - Pareto optimal solutions are similar (in the sense of Euclidean geometry) to their images in the objective space
- So, easy to understand the distribution of objective vectors by observing their position and crowding degree in the rectangle in the 2-D decision space
- This work won the 2014 IEEE CEC Best Student Paper Award

Test Results on the Rectangle Problem

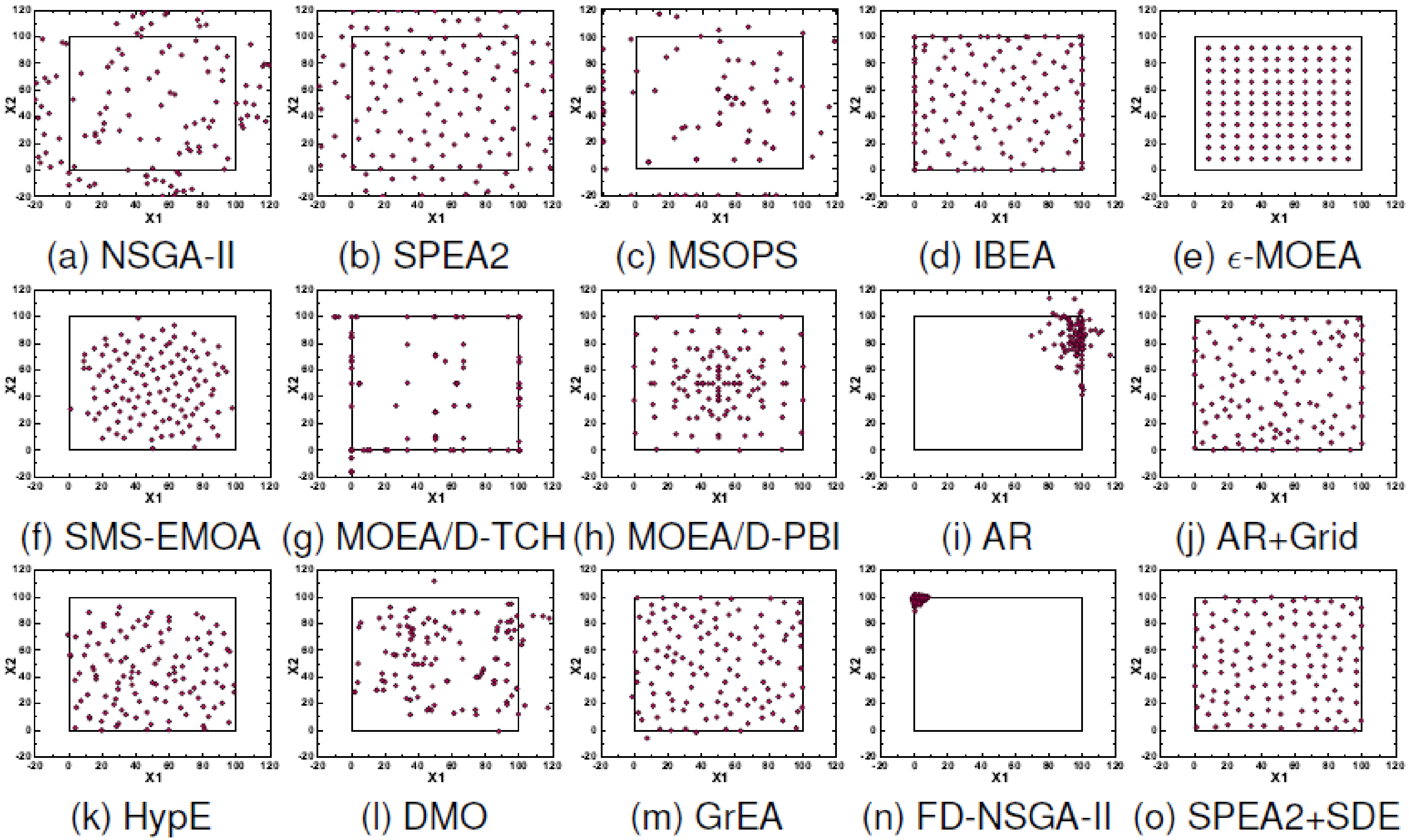


Figure: Final solutions of 15 MOEAs on the Rectangle prob. ($x_1, x_2 \in [-20, 120]$)

Test Results on the Rectangle Problem

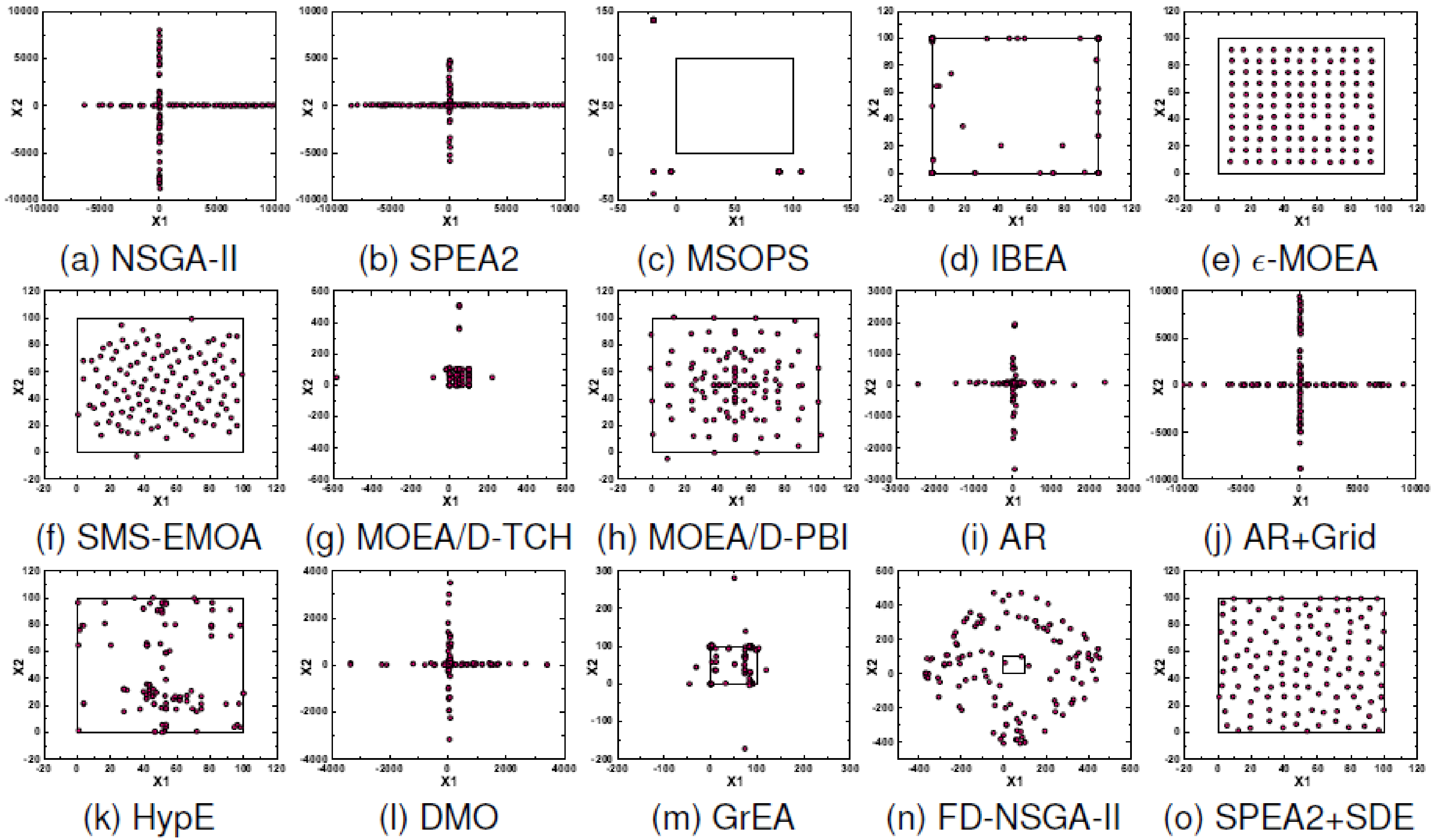


Figure: Final solutions of 15 MOEAs on the Rectangle prob. ($x_1, x_2 \in [-10^4, 10^4]$)

Challenge II: Performance Assessment

- For 2- or 3-objective optimization, numerous quality indicators, e.g., Hypervolume, IGD, and diversity measure, to evaluate the performance of Pareto front approximations obtained by MOEAs
  - Mainly concentrated on three aspects:
    - Convergence of the obtained approximation
    - Uniformity of the approximation
    - Spread (extensity) of the approximation

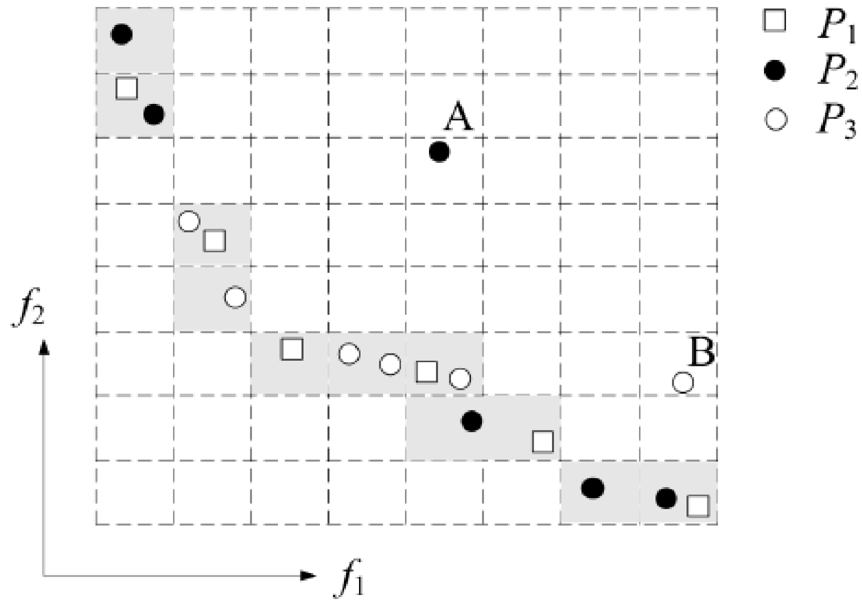
  The latter two are closely related, generally called **diversity** of an approximation
- But, for many-objective optimization, no much concern on quality indicators. Difficulties in comparing approximations:
  - Visual comparison: misleading or even impossible
  - Difficulty in generating the substitution of Pareto front (i.e., reference set)
  - Complexity of the storage and running time

Diversity Comparison Indicator (DCI)

- Recently, we developed a **Diversity Comparison Indicator (DCI)** to assess the diversity of solution sets obtained by MOEAs in the objective space in many-objective problems
- Basic idea: consider the contribution of different solution sets to the hyperboxes that have at least one non-dominated solution:
  - All solution sets are put into a grid environment so that there are some hyperboxes containing one or more non-dominated solutions
  - Depending on the contribution of a solution set to these hyperboxes, the diversity indicator of the set is defined
  - If the contributions of a solution set to all these hyperboxes are maximal, the best diversity value is achieved; if the contributions of a set to most of these hyperboxes are low, the diversity is poor

M. Li, S. Yang, X. Liu. Diversity comparison of Pareto front approximations in many-objective optimization. IEEE Transactions on Cybernetics, 44(12): 2568-2584, 2014

DCI Illustration



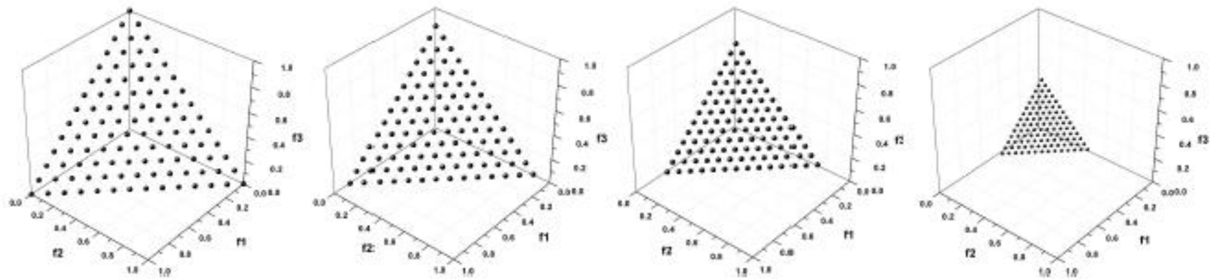
	$h^{0,7}$	$h^{0,6}$	$h^{1,4}$	$h^{1,3}$	$h^{2,2}$	$h^{3,2}$	$h^{4,2}$	$h^{4,1}$	$h^{5,1}$	$h^{6,0}$	$h^{7,0}$	DCI
P_1	2/3	1	1	2/3	1	2/3	1	2/3	1	2/3	1	0.848
P_2	1	1	0	0	0	1/3	2/3	1	2/3	1	1	0.606
P_3	0	0	1	1	2/3	1	1	2/3	1/3	0	0	0.515

Figure: The considered hyperbox $h^{i,j}$ is highlighted with a gray background, where i and j denote the coordinates in f_1 and f_2, resp. The number corresponding to an approximation P and a hyperbox h means the contribution degree of P to h (i.e., $CD(P, h)$)

DCI Characteristics

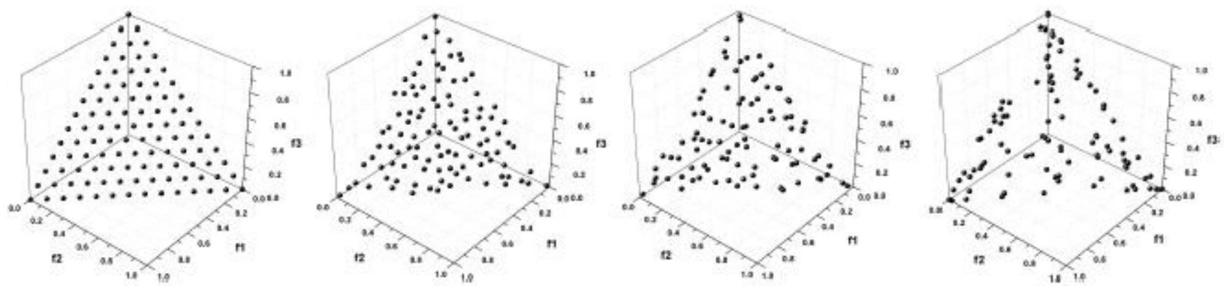
- DCI can identify any number of solution sets in a single run
- DCI has a quadratic computational complexity, with its computational cost within 1.0 second even if the number of objectives reaches 20
- DCI does not require a reference set that substitutes the Pareto front of a given problem
- DCI assesses the relative quality of different solution sets rather than providing an absolute measure of distribution for a single solution set

Artificial Example (Pareto Front f1+f2+f3=1)



(a) DCI = 0.800 (b) DCI = 0.782 (c) DCI = 0.703 (d) DCI = 0.403

Figure: DCI for artificial Pareto front approximations with different distribution ranges



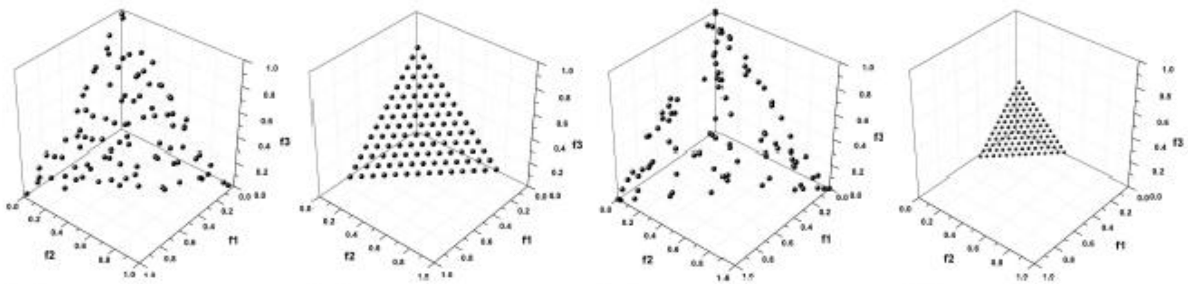
(a) DCI = 0.825 (b) DCI = 0.798 (c) DCI = 0.750 (d) DCI = 0.681

Figure: DCI for artificial Pareto front approximations with different uniformity degrees

- Note: if a set obtained the maximal value (DCI = 1), it does not mean it is uniformly distributed over the whole PF. Instead, it has a perfect advantage over other sets: it covers all hyper-boxes where the non-dominated solutions of other sets are located

Artificial Example (Pareto Front f1+f2+f3=1)

- The above two experiments have verified the correctness of the DCI in evaluating spread and uniformity separately
- How to compare solution sets with different distribution ranges and uniformity degrees? E.g., how to compare the diversity of two sets, if one performs better in spread but worse in uniformity?

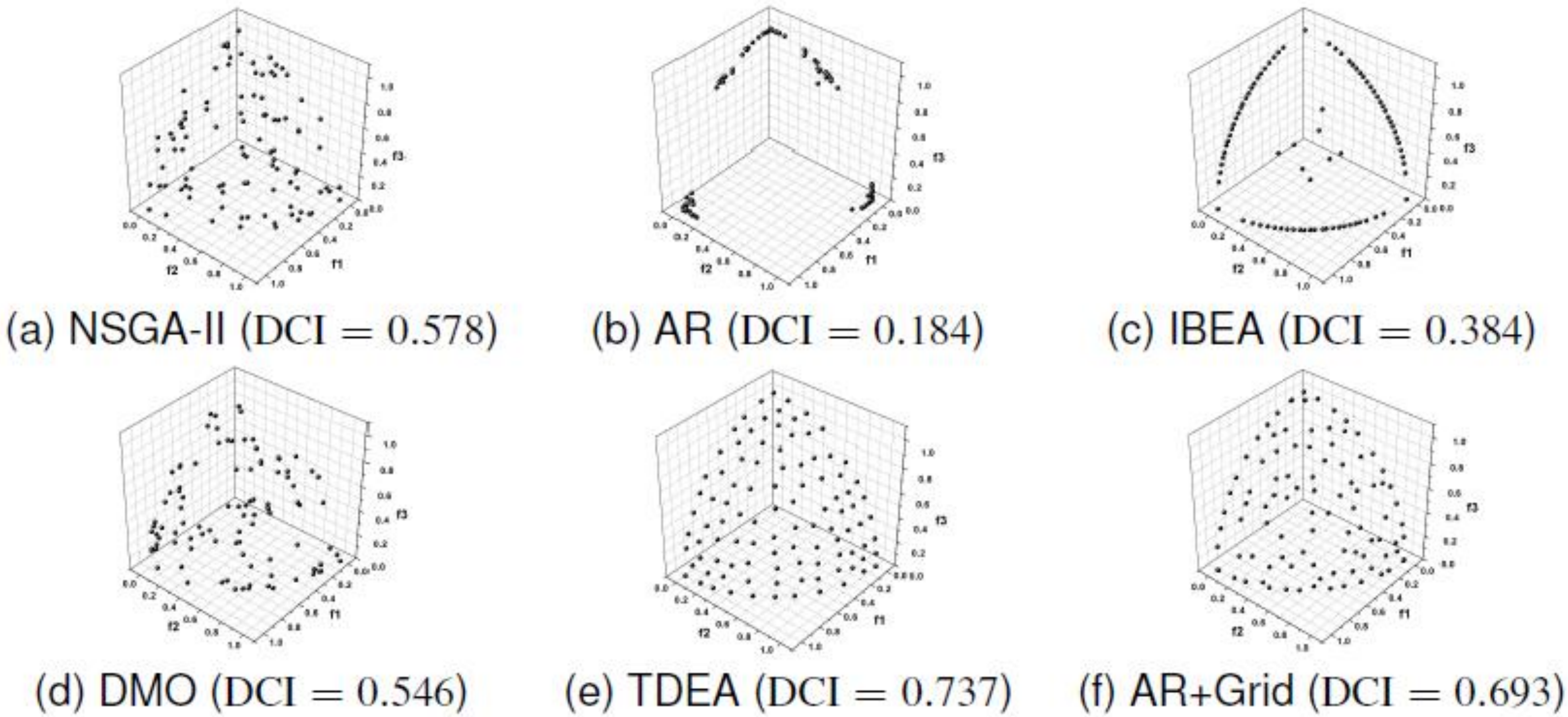


(a) DCI $= 0.746$ (b) DCI $= 0.697$ (c) DCI $= 0.633$ (d) DCI $= 0.438$

Figure: DCI for artificial Pareto front approximations with different distribution ranges and uniformity degrees

- DCI can be considered as a tradeoff evaluation between spread and uniformity. An solution with a great advantage over its competitors at one point will achieve a better DCI value, even though it performs slightly worse at the other point

Real Example: Comparing MOEAs



(a) NSGA-II (DCI = 0.578) (b) AR (DCI = 0.184) (c) IBEA (DCI = 0.384)

(d) DMO (DCI = 0.546) (e) TDEA (DCI = 0.737) (f) AR+Grid (DCI = 0.693)

Figure: Solution sets of 6 MOEAs and their DCI results on the 3-objective DTLZ7

- A solution set with a higher DCI value means that it performs better regarding the comprehensive performance in finding multiple Pareto optimal regions as well as maintaining solutions' uniformity and extensity in each region

Challenge III: Search Methods–General Comments

- Many MOEAs developed for multi-objective optimization
  - Basic idea: balancing convergence and diversity
- Three categories: based on the selection mechanisms
  - Pareto-based algorithms: comparing solutions based on dominance relation and density, e.g., NSGA-II, SPEA-II, PESA-II
  - Aggregation-based algorithms: MOEA/D
  - Indicator-based algorithms: IBEA, SMS-EMOA, MO-CMA-ES
- For MaOPs, most Pareto-based MOEAs do not work well
  - Reason: the proportion of non-dominated solutions in a population rises rapidly with the number of objectives
- Recently, rising efforts on designing EAs for MaOPs

Designing EMO Algorithms for MaOPs

- Decomposition-based or indicator-based algorithms
  - MSOPS [Hughes, 2003], NSGA-III [Deb and Jain, 2014], HypE [Bader and Zitzler, 2011]
- Modification of Pareto-based algorithms
  - S-CDA [Sato et al, 2007], POGA [Pierro et al, 2007], DMO [Adra and Fleming, 2011], SDE [Li et al, 2014]
- EMO algorithms designed specially for MaOPs
  - GrEA [Yang et al, 2013], PICEA [Wang et al, 2013], AGE-II [Wagner and Neumann, 2013], BiGE [Li et al, 2015]

Decomposition- or Indicator-based Algorithms

Decomposition-based algorithms for MaOPs

- **MSOPS** [Hughes, 2003]: Specify an individual with a number of weight vectors, which is unlike MOEA/D where an individuals corresponds to only one weight vector
- **NSGA-III** [Deb and Jain, 2014]: Combine the non-dominated sorting with a decomposition-based niching technique to balance convergence and diversity

Indicator-based algorithms for MaOPs

- **HypE** [Bader and Zitzler, 2011]: Adopt Monte Carlo simulation to approximate the exact hypervolume value and enable hypervolume-based search to be easily applied to MaOPs

Modification of Pareto-based Algorithms

- S-CDA [Sato et al, 2007]:
  - Control the degree of expansion or contraction of the dominance area of solutions
- POGA [Pierro et al, 2007]:
  - Use preference order-based approach (in the sense of Pareto dominance) as an optimality criterion to rank solutions
- DMO [Adra and Fleming, 2011]:
  - Tune the maintenance diversity operation according to the requirement of the evolutionary population

Convergence vs Diversity

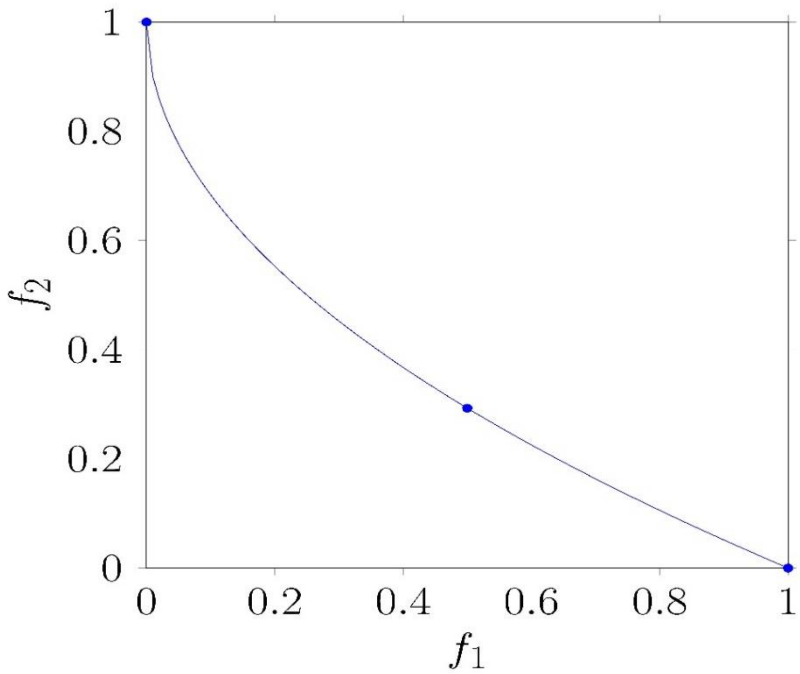
- Existing popular MOEAs
  - Pareto dominance based approaches (NSGA-II, SPEA2)
  - Indicator based approaches(ε-MOEA, SMS-EMOA, HyPE)
  - Decomposition-based approaches(MOEA/D, MSOPS)
  - Hybrid approaches (NSGA-III, KnEA)

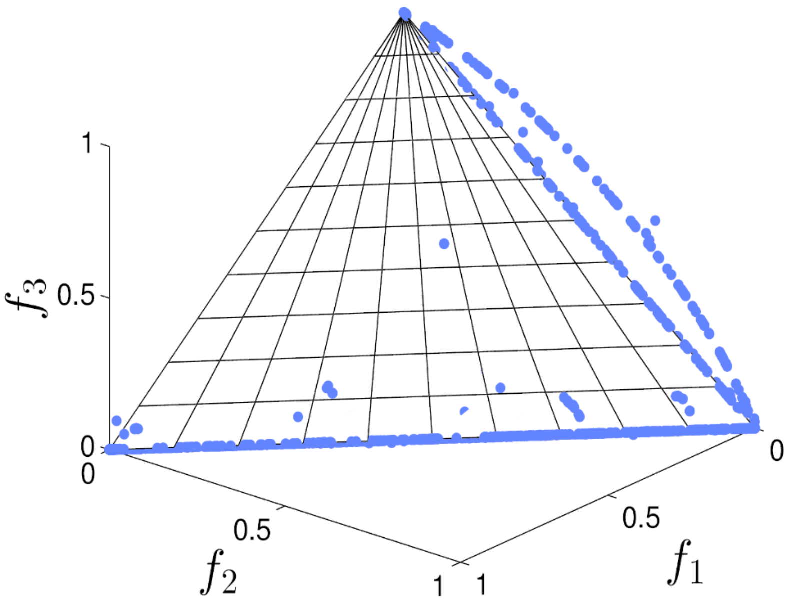
Highlight:

Most existing approaches conduct evolution in a convergence-first-and-diversity-second (CFDS) manner. That is, convergence is emphasized over diversity.

Convergence vs Diversity

NSGA-III (PICEAg-g, SPEA2, …) on two problems: MOP5 and MOP6

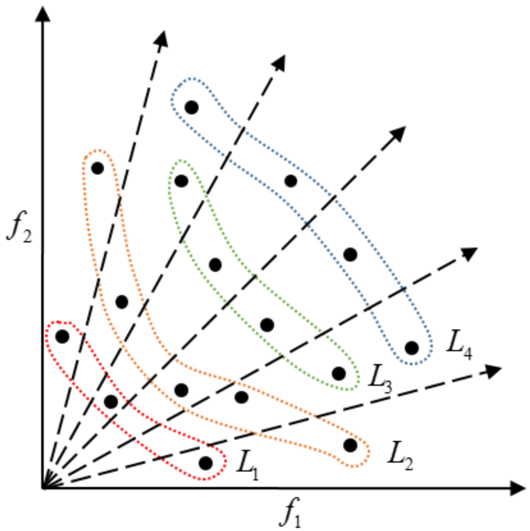


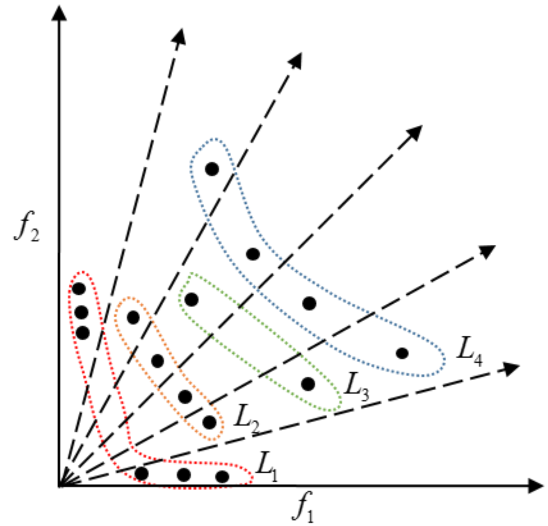


Remark:

If an MOEA is incapable of solving 2- and 3-objective problems, then there is no point studying it on many-objective optimization.

Non-dominated Sorting





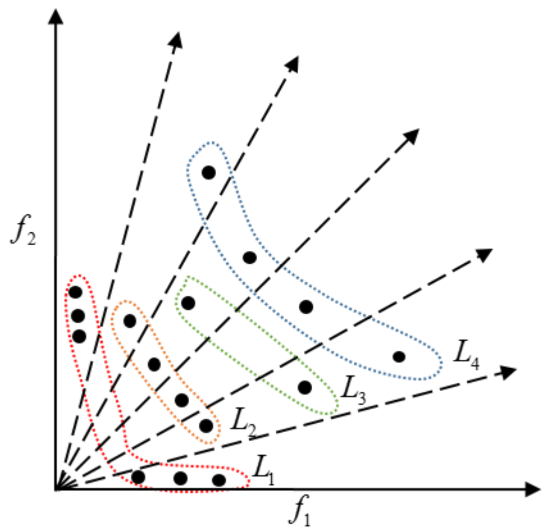
Imaginary distribution: population spreads well

(NSGA-III works)

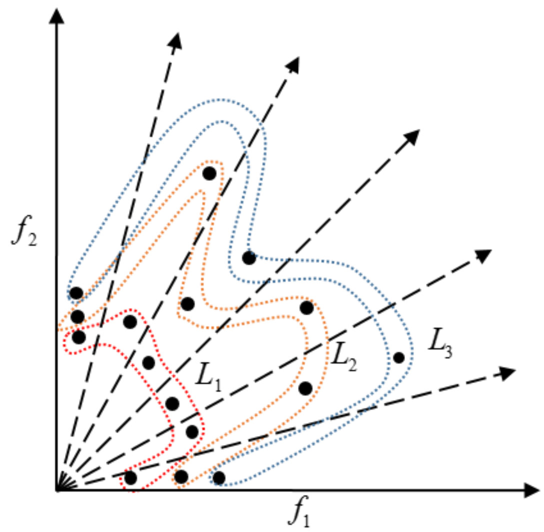
Actual distribution: crowded regions, particularly in many-objective problems

(NSGA-III fails)

Convergence vs Diversity Based Ranking



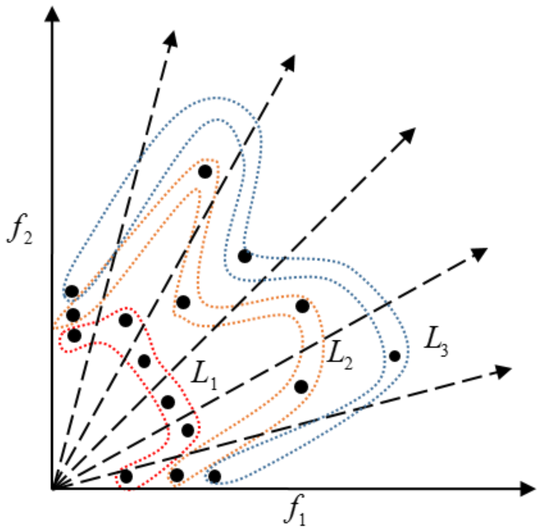
Convergence-based sorting
(NSGAII,SPEA2,…)



Diversity-based sorting

S. Jiang, S. Yang. Convergence versus diversity in multiobjective optimization. PPSN XIV, LNCS, vol. 9921, 984-993, 2016

SPEA/R: SPEA + Diversity Sorting



Diversity-based sorting

1, Reference directions;

2, Population partition (subpopulations);

3, Strength–based fitness assignment on the population and subpopulations;

4, Diversity–based sorting (L_1, L_2,...);

5, Solution selection on each front;

6, On L_r, filling up slots according to fitness.

S. Jiang, S. Yang. A strength Pareto evolutionary algorithm based on reference direction for multi-obective and many-objective optimization. IEEE Trans on Evol Comput, 21(3): 329-346, 2017

Environmental Selection with DFCS

1. Population is sorted using DFCS
2. Start selection from the highest front, stop until the new population is over the population size
3. In the last front considered, select individuals with the highest fitness to fill up the population

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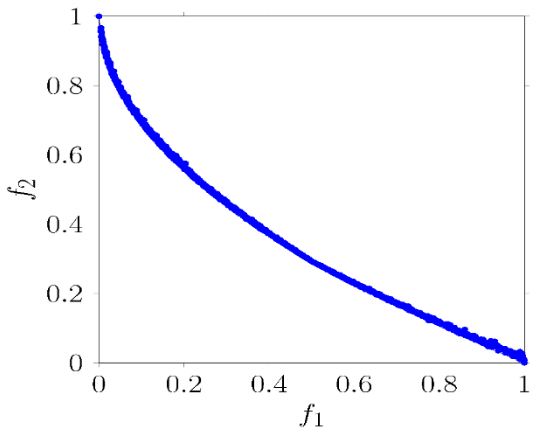
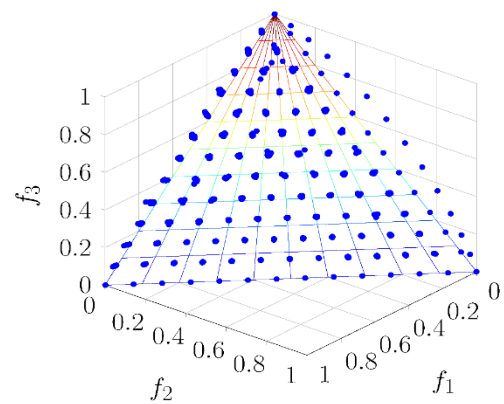
Algorithm 5.6: $\mathtt{DFCS}_{\mathtt{E}}\mathtt{nvironment\_Selection}(\overline{Q}, W)$

Input: N (population size), $\overline{Q}$ (combined population), W (reference direction set)
Output: P (new parent population).

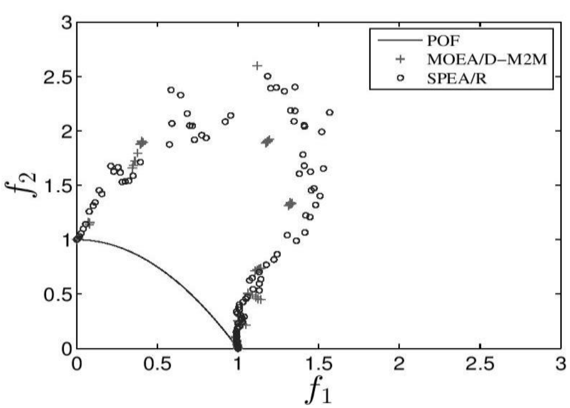
Set $P = \emptyset$;
$\{F_1, F_2, \dots\} = \mathtt{DFCS\_Sort}(Q)$; // DFCS sorting based on fitness;
$l \leftarrow 1$;
while $C(P \cup F_l) \leq N$ **do**
  $P \leftarrow P \cup F_l$;
  $l \leftarrow l + 1$;
end
Fill up P with the best $N - C(P)$ individuals in terms of fitness from F_l.

SPEA/R for 2-/3-objective MOPs

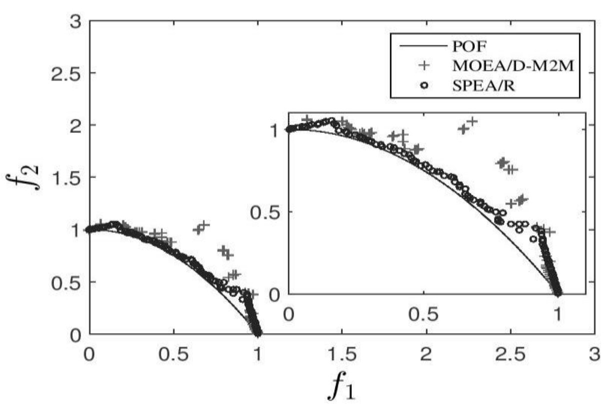




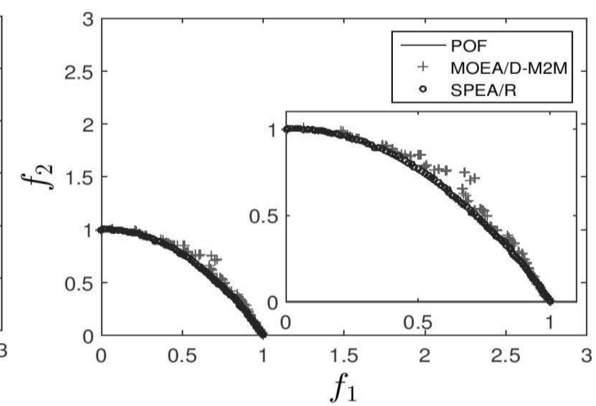
SPEA/R



50th genetation



500th genetation



1000th genetation

SPEA/R for MaOPs

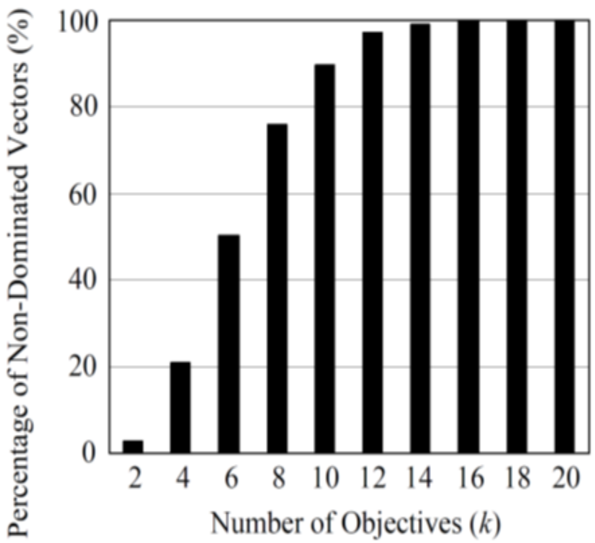
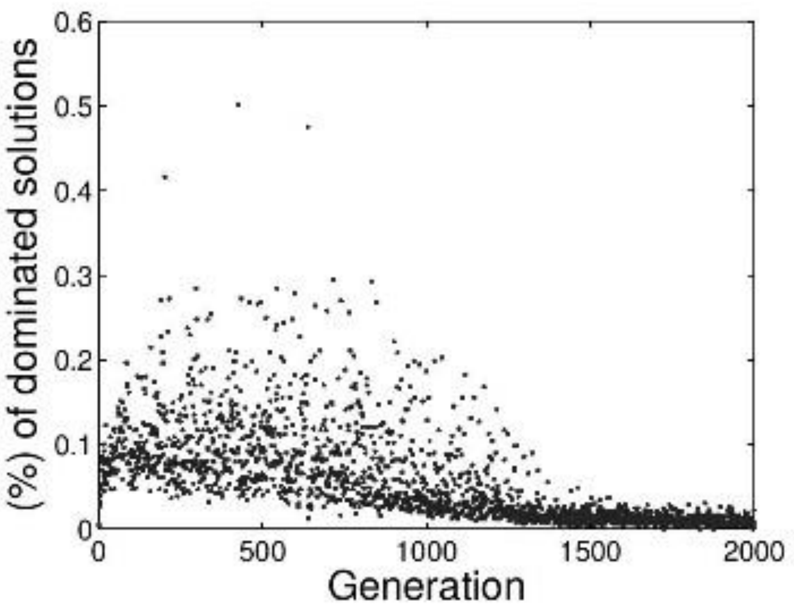


Fig. 1. Average percentage of non-dominated vectors among 200 vectors that are randomly generated in the k-dimensional unit hypercube $[0, 1]^k$.



Less than 5% are dominated for 12 objectives

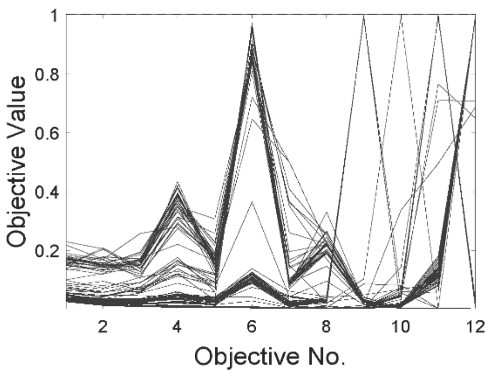
5%-50% are dominated before converging for 12-objective WFG5

Remarks:

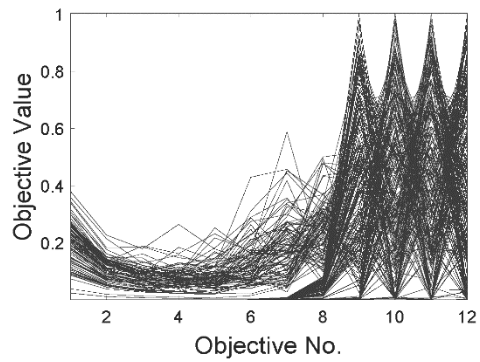
1. The percentage of dominated solutions in a merged population (parent & offspring) is more than we think.
2. Pareto dominance is usable for MaOPs if we can make most of dominated solutions

SPEA/R for MaOPs: Results

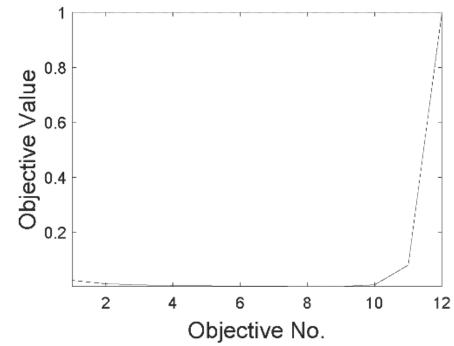
- WFG Problems (WFG4…)



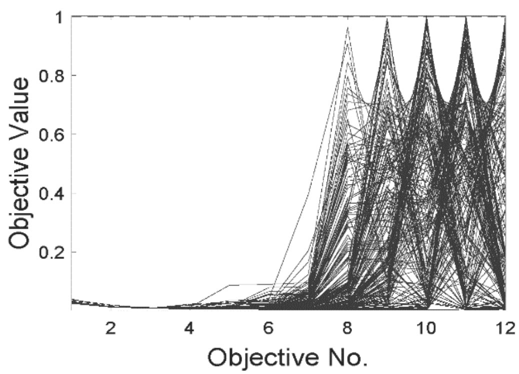
HypE



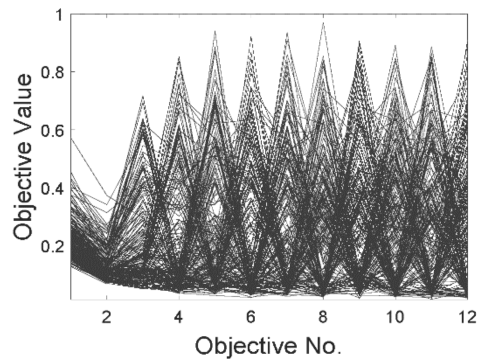
PICEA-g



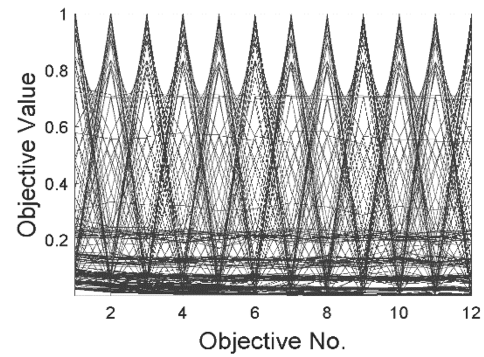
MOEA/D



NSGA-III



SPEA2+SDE



SPEA/R

SPEA/R vs NSGA-III

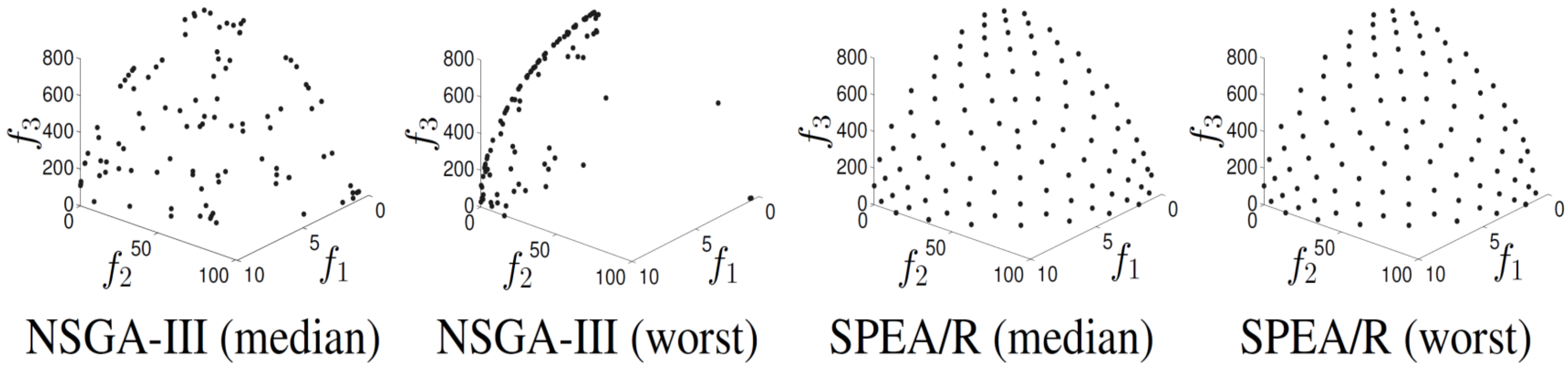


Fig. 5.15 PF approximations for scaled WFG5 in median and worst cases.

Results:

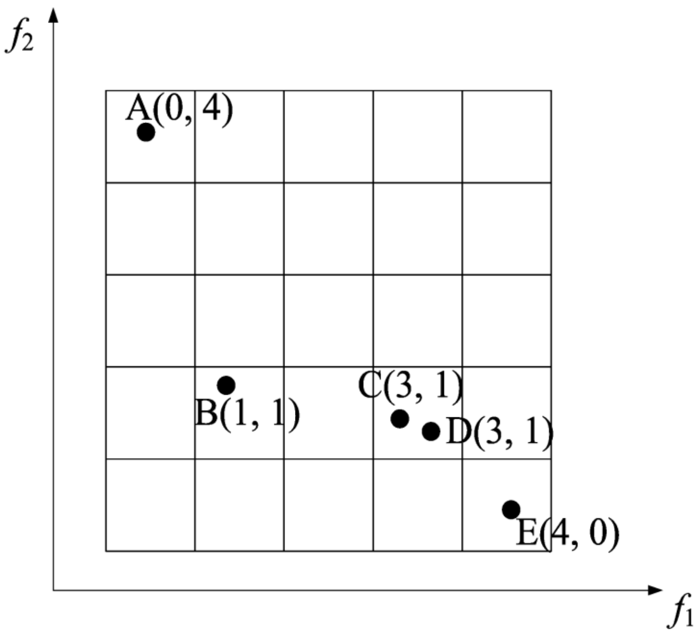
1. SPEA/R keeps diversity better
2. NSGA-III converges slightly faster

Specially Designed EMO Algorithms

- PICEA [Wang et al, 2013]:
  - Co-evolve a family of preferences simultaneously with the population of candidate solutions, where preferences are used to generate approximation sets for a *posteriori* decision making, rather than represent true articulations of decision-maker preferences
- AGE-II [Wanger and Neumann, 2013]:
  - Incorporate a formal notion of approximation on the basis of AGE [Bringmann et al, 2011] and also introduce an adaptive ε-dominance approach to balance the convergence speed and runtime
- BiGE [Li et al, 2015]:
  - Convert a multi-objective optimization problem into a bi-goal (objective) optimization problem regarding proximity and diversity, and then handles it using the Pareto dominance relation in this bi-goal domain

Grid-based EA (GrEA) for MaOPs

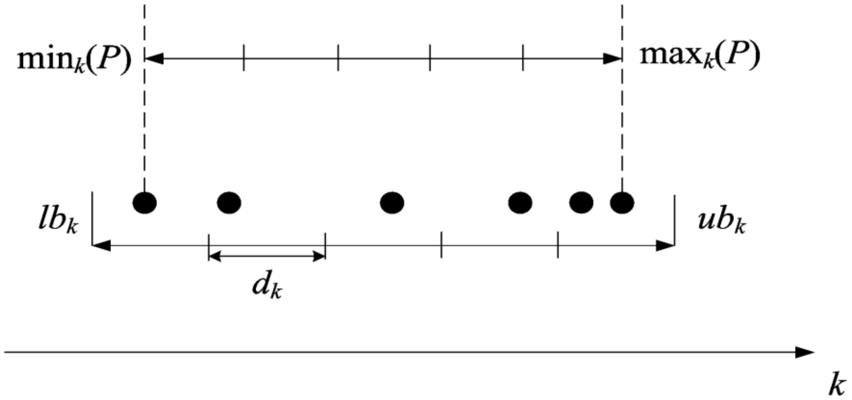
- A grid naturally reflects the distribution of solutions in the searching process by their grid locations (i.e., grid coordinates)
  - Distance: Solution **C** is farther away from **A** than **B**
  - Density: Solution **C** has a greater crowding degree than **A** and **B**
  - Convergence: **B** dominates **C** and **D**
  - Difference in the objective value: difference between **A** and **B** is greater in f2 than in f1



S. Yang, M. Li, X. Liu, J. Zheng. A grid-based evolutionary algorithm for many-objective optimization. IEEE Transactions on Evolutionary Computation, 17(5): 721-736, 2013

GrEA: Grid Setting by a Given Population

Setting of the grid in the kth objective



$$lb_k = \min_k(P) - (\max_k(P) - \min_k(P))/(2 \times div)$$

$$ub_k = \max_k(P) + (\max_k(P) - \min_k(P))/(2 \times div)$$

$$d_k = (ub_k - lb_k)/div$$

$$G_k(\boldsymbol{x}) = \lfloor (F_k(\boldsymbol{x}) - lb_k)/d_k \rfloor$$

div: number of divisions in each objective dimension

d_k: hyperbox width in the kth objective

- Grid Dominance:
  Let $\boldsymbol{x}, \boldsymbol{y} \in P$, $\boldsymbol{x} \prec_{\text{grid}} \boldsymbol{y}$: $\Leftrightarrow$

$$\forall i \in (1, 2, ..., M) : G_i(\boldsymbol{x}) \leq G_i(\boldsymbol{y}) \wedge$$
$$\exists j \in (1, 2, ..., M) : G_j(\boldsymbol{x}) < G_j(\boldsymbol{y})$$

  where $\boldsymbol{x} \prec_{\text{grid}} \boldsymbol{y}$ denotes $\boldsymbol{x}$ grid-dominates $\boldsymbol{y}$

- Grid Difference:
  grid difference between $\boldsymbol{x}, \boldsymbol{y} \in P$ is:

$$GD(\boldsymbol{x}, \boldsymbol{y}) = \sum_{k=1}^{M} | G_k(\boldsymbol{x}) - G_k(\boldsymbol{y}) |$$

  where $G_k(\boldsymbol{x})$ is the grid coordinate of $\boldsymbol{x}$ in the kth objective

GrEA: Fitness Assignment

- GrEA presents three grid-based criteria to assign fitness:
  - Grid ranking (GR), Grid crowding distance (GCD), Grid coordinate point distance (GCPD)

$$GR(x) = \sum_{k=1}^{M} G_k(x)$$

$$GCD(x) = \sum_{y \in P, x \neq y, GD(x,y) < M} M - GD(x,y)$$

$$GCPD(x) = \sqrt{\sum_{k=1}^{M} \left(\frac{F_k(x) - (lb_k + Gk(x) \times d_k)}{d_k} \right)^2}$$

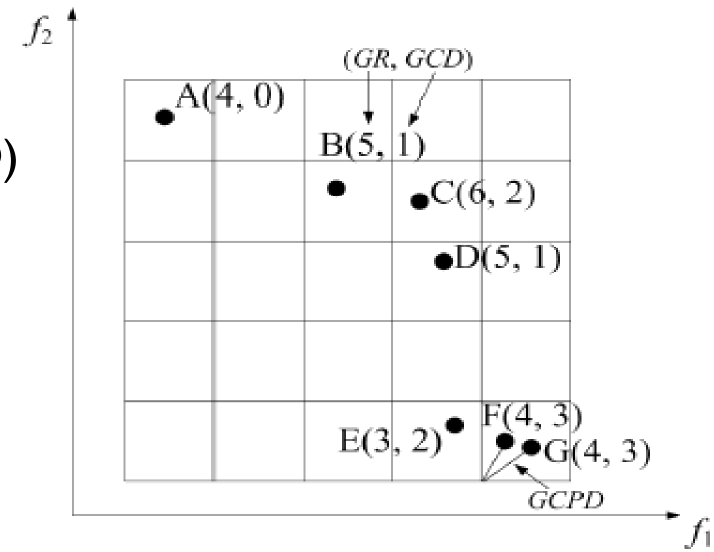


Illustration of fitness assignment for a minimization MOP

$G_k(x)$: grid coordinate of individual *x* in the *k*th objective
$F_k(x)$: actual objective value of individual *x* in the *k*th objective
GD(x, y): difference of the grid coordinates between *x* and *y*
lb_k: lower boundary of the grid in the *k*th objective
d_k: width of a hyperbox in the *k*th objective
M: the number of objectives

GrEA: Framework

Algorithm 1 Grid-Based EA (GrEA)

Require: P (population), N (population size)

$P \leftarrow Initialize(P)$
while termination criterion not met **do**
    $Grid\_setting(P)$
    $Fitness\_assignment(P)$
    $P' \leftarrow Mating\_selection(P)$
    $P'' \leftarrow Variation(P')$
    $P \leftarrow Environmental\_selection(P \bigcup P'')$
end while
return P

Algorithm 2 *Tournament Selection*

Require: individuals p, q randomly chosen

if $p \prec q$ **or** $p \prec_{grid} q$ **then**
    return p
else if $q \prec p$ **or** $q \prec_{grid} p$ **then**
    return q
else if $GCD(p) < GCD(q)$ **then**
    return p
else if $GCD(q) < GCD(p)$ **then**
    return q
else if $random(0,1) < 0.5$ **then**
    return p
else
    return q
end if

GrEA: Experimental Results

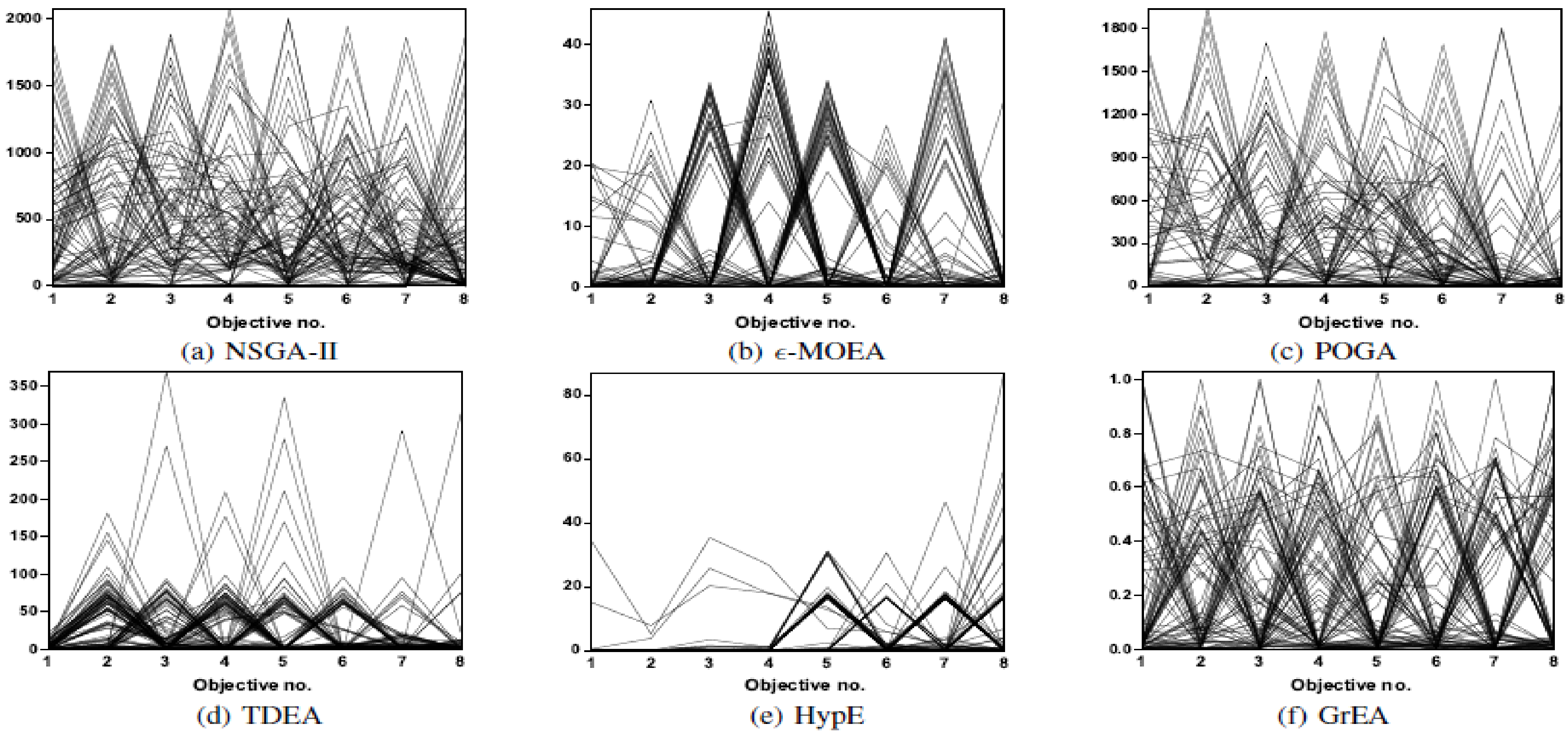
Figure: IGD results of MOEAs on DTLZ1, DTLZ3, and DTLZ6

Problem	Obj.	ϵ-MOEA	POGA	HypE	MSOPS	MOEA/D	GrEA
DTLZ1	4	4.866E–2 (2.3E–3)	9.301E–1 (3.7E–2)	1.355E–1 (7.1E–2)	5.762E–2 (3.0E–3)	9.653E–2 (1.1E–4)	4.624E–2 (5.3E–3)
	5	6.930E–2 (8.1E–3)	4.470E+0 (4.3E+0)	3.107E–1 (3.8E–1)	8.673E–2 (3.8E–3)	1.192E–1 (1.3E–4)	6.257E–2 (7.5E–3)
	6	9.877E–2 (8.4E–2)	1.284E+1 (1.7E+1)	5.554E–1 (5.9E–1)	1.808E–1 (1.1E–1)	1.377E–1 (6.4E–3)	8.543E–2 (1.0E–2)
	8	3.069E–1 (3.4E–1)	1.012E+1 (6.9E+0)	1.017E+0 (1.4E+0)	7.770E–1 (6.8E–1)	1.852E–1 (5.8E–3)	1.060E–1 (4.8E–3)
	10	4.071E–1 (3.8E–1)	1.370E+1 (9.2E+0)	1.485E+0 (1.8E+0)	1.623E+0 (1.0E+0)	2.188E–1 (6.4E–3)	2.864E–1 (1.0E–1)
DTLZ3	4	1.417E–1 (8.9E–3)	1.317E+1 (5.7E+0)	1.029E+0 (7.1E–1)	1.072E+1 (6.8E+0)	2.130E–1 (1.7E–3)	1.522E–1 (4.7E–2)
	5	2.267E–1 (3.3E–2)	2.341E+1 (1.1E+1)	4.708E+0 (5.6E+0)	2.896E+1 (1.5E+1)	2.677E–1 (7.8E–4)	2.804E–1 (8.3E–2)
	6	4.578E–1 (1.4E–1)	3.239E+1 (1.2E+1)	2.689E+0 (1.7E+0)	4.665E+1 (1.6E+1)	4.085E–1 (3.2E–2)	4.368E–1 (1.5E–1)
	8	1.122E+1 (1.4E+1)	2.733E+1 (1.2E+1)	7.476E+0 (9.6E+0)	6.095E+1 (1.9E+1)	6.106E–1 (7.5E–2)	5.546E–1 (2.3E–1)
	10	2.040E+1 (2.7E+1)	3.157E+1 (1.4E+1)	6.226E+0 (6.2E+0)	6.312E+1 (1.8E+1)	6.599E–1 (5.5E–2)	7.743E–1 (2.5E–1)
DTLZ6	4	4.671E–1 (2.8E–2)	2.226E+0 (3.0E–1)	3.919E+0 (6.5E–1)	4.178E+0 (6.4E–1)	8.015E–2 (2.7E–2)	7.045E–2 (3.1E–2)
	5	1.678E+0 (1.5E–1)	1.801E+0 (4.5E–1)	5.431E+0 (5.6E–1)	6.560E+0 (5.0E–1)	1.194E–1 (3.8E–2)	1.429E–1 (4.3E–2)
	6	2.705E+0 (2.8E–1)	2.245E+0 (6.0E–1)	5.679E+0 (5.7E–1)	6.862E+0 (5.1E–1)	1.569E–1 (3.7E–2)	4.544E–1 (9.1E–2)
	8	1.987E+0 (1.3E+0)	5.807E+0 (3.9E+0)	6.165E+0 (6.2E–1)	6.813E+0 (4.4E–1)	1.830E–1 (2.8E–2)	5.971E–1 (3.8E–1)
	10	3.737E+0 (2.0E+0)	8.941E+0 (1.4E–1)	6.428E+0 (4.1E–1)	6.728E+0 (4.8E–1)	2.692E–1 (3.0E–2)	9.432E–1 (7.8E–1)

Figure: HV results of MOEAs on multiobjective TSP

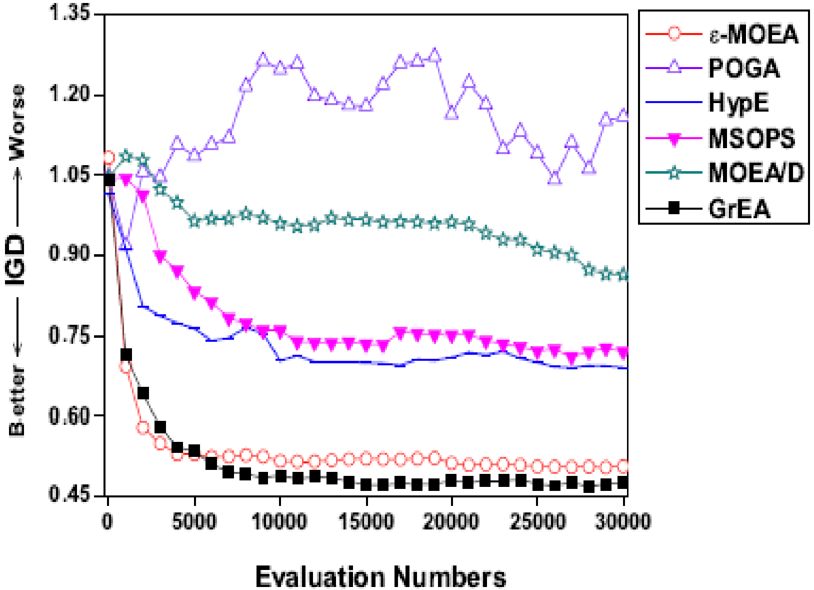
Obj.	$TSPcp$	ϵ-MOEA	POGA	HypE	MSOPS	MOEA/D	GrEA
5	–0.4	1.125E+6 (5.3E+4)	1.128E+6 (5.2E+4)	1.962E+5 (5.3E+4)	7.963E+5 (4.5E+4)	9.615E+5 (5.8E+4)	1.231E+6 (4.8E+4)
	–0.2	1.014E+6 (4.5E+4)	1.012E+6 (3.7E+4)	2.493E+5 (4.8E+4)	8.048E+5 (4.2E+4)	9.687E+5 (4.8E+4)	1.121E+6 (3.8E+4)
	0	9.211E+5 (2.7E+4)	8.445E+5 (3.2E+4)	3.596E+5 (4.6E+4)	7.837E+5 (3.0E+4)	8.729E+5 (2.8E+4)	9.844E+5 (3.5E+4)
	2	8.543E+5 (3.1E+4)	7.565E+5 (3.2E+4)	4.251E+5 (6.7E+4)	7.651E+5 (3.4E+4)	7.530E+5 (3.1E+4)	8.836E+5 (2.3E+4)
	4	8.177E+5 (2.1E+4)	7.206E+5 (4.8E+4)	5.259E+5 (5.5E+4)	7.737E+5 (2.5E+4)	7.255E+5 (4.3E+4)	8.466E+5 (2.4E+4)
10	–0.4	1.026E+11 (1.4E+10)	2.520E+11 (5.7E+10)	3.634E+09 (1.7E+09)	1.706E+11 (2.4E+10)	1.539E+10 (3.2E+09)	3.772E+11 (1.6E+10)
	–0.2	1.203E+11 (1.3E+10)	1.761E+11 (4.9E+10)	1.198E+10 (3.7E+09)	1.820E+11 (1.8E+10)	3.376E+10 (5.1E+09)	3.097E+11 (1.0E+10)
	0	1.260E+11 (1.5E+10)	1.181E+11 (4.1E+10)	2.386E+10 (7.0E+09)	1.704E+11 (1.2E+10)	5.458E+10 (8.8E+09)	2.551E+11 (8.3E+09)
	2	1.400E+11 (1.2E+10)	8.500E+10 (2.5E+10)	3.404E+10 (8.3E+09)	1.529E+11 (1.0E+10)	7.450E+10 (1.1E+10)	2.136E+11 (5.9E+09)
	4	1.467E+11 (1.1E+10)	8.095E+10 (1.7E+10)	5.298E+10 (9.2E+09)	1.447E+11 (6.8E+09)	1.068E+11 (1.3E+10)	1.818E+11 (6.8E+09)

GrEA: Experimental Results

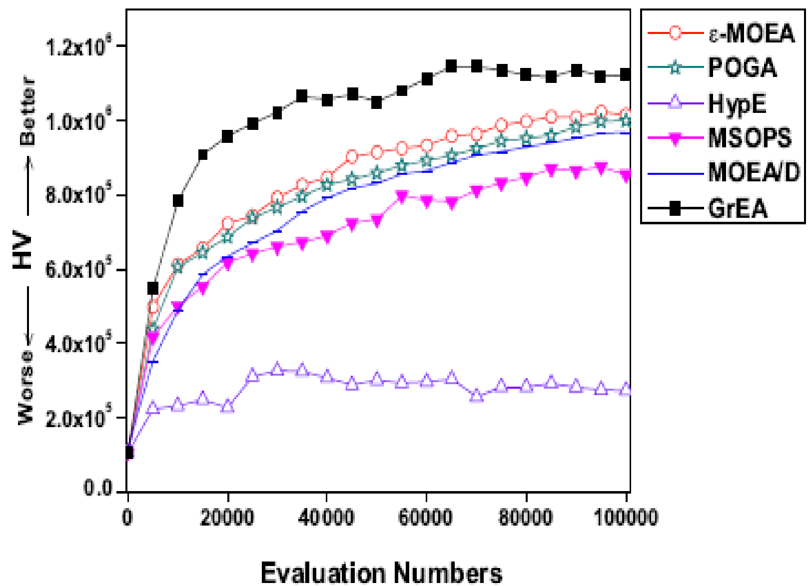


Solution sets of six MOEAs on the 8-objective DTLZ3 in parallel coordinates

Evolutionary Trajectories



Evolutionary trajectories of the performance metric IGD for six MOEAs on the 10-objective DTLZ2



Evolutionary trajectories of the performance metric Hypervolume for six MOEAs on the 10-objective TSP

Summary

- EC for MaOPs has become a hot topic nowadays
- Three challenging issues:
  - Visualization
  - Performance measures
  - Search methods
- This area is still young and needs much more efforts