ASTR615 Fall 2011 — Problem Set #3

Due Thu Nov 3rd, 2011

This problem set will give you some experience with root finding and/or least-squares fitting. This is a long assignment, you have two full weeks to solve it. Start early!

1. Consider two bodies of mass m_1 and m_2 separated by a distance d traveling in circular orbits around their mutual center of mass. Take the angular momentum vector of the system to be pointing in the $+z$ direction. In a frame that corotates with the orbital motion there are five locations (called the Lagrange points) where the effective acceleration felt by a test particle vanishes. The acceleration arises from the gravity due to m_1 and m_2 plus the rotation of the system as a whole. If the masses are at points $x_1 < x_2$ on the x axis (so $d = x_2 - x_1$), then by convention $L_3 < x_1 < L_1 < x_2 < L_2$. The L_4 and L_5 points lie off the x axis and form equilateral triangles with the points x_1 and x_2. Conventionally, L_4 is taken to be in the $+y$ direction, L_5 in the $-y$ direction.

   (a) Write a program to explore this system by computing the effective gravity (a vector) and potential (a scalar) at specific grid points in the xy-plane. Use units such that the gravitational constant $G \equiv 1$. The program should take as input the mass of both bodies and their separation. You may "hardwire" the grid dimensions into your code if you wish. The output should be the potential and x and y components of the acceleration at each grid point.

   (b) Use your favorite plotting program to plot vectors (for the effective acceleration) and contours (for the effective potential) for the cases where $m_1 = 3$, $m_2 = 1$, $d = 1$ and $m_1 = 100$, $m_2 = 1$, $d = 1$.

   (c) Use a simple root solver to determine the locations of the Lagrange points for both these cases, using your *a priori* knowledge of their approximate locations (i.e., first do a search along the x axis for the L_1, L_2, and L_3 points, then along the $x = (x_1 + x_2)/2$ axis for the L_4 and L_5 points. Watch out for singularities!...).

2. Write a program to fit first a Lorentzian

$$\phi(\nu) = \frac{1}{\pi} \frac{\alpha_L}{(\nu - \nu_0)^2 + \alpha_L^2} \tag{1}$$

and then a Gaussian

$$\phi(\nu) = \frac{1}{\alpha_D} \sqrt{\frac{\ln 2}{\pi}} e^{-(\ln 2)(\nu - \nu_0)^2/\alpha_D^2} \tag{2}$$

to the data in

`http://www.astro.umd.edu/~ricotti/NEWWEB/teaching/ASTR415/ps3.dat`

which is in the format ν ϕ e, where ν is the frequency, ϕ is the line strength, and e is the estimated error in each ϕ.

(a) What values of α_L and ν_0 do you get for the Lorentzian, and what values of α_D and ν_0 do you get for the Gaussian? Also report the error estimates for the fit parameters. Which model is a better fit to the data?

(b) Plot the data and the fits. Be sure to include the errorbars.