Problem Set 5

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1) Timescales of structure formation

Structures of a given scale R form when:

$$\sigma^2(R) = \frac{1}{2\pi^2}\int P(k)W(K,R)K^2 dk = 1 \tag{1}$$

where

$$P = P_0(k)T^2(k) \tag{2}$$

$$T^2(k) = \frac{1}{1+\beta(\frac{k}{k_{eq}})^4} \tag{3}$$

$$P_0 = AK^n \tag{4}$$

$$W(R) = \Theta(R^{-1} - k) \tag{5}$$

and $n = 1$, $A = 5x10^{-5}$, and $B = 3x10^{-4}$. The integral will be evaluated over the interval 0 to $1/R$, as the heavside step function will make the integral constant after $k = \frac{1}{R}$.

$$\sigma^2(R) = \frac{A}{2\pi^2}\int_o^{1/R} \frac{K^3}{1+\beta(\frac{k}{k_{eq}})^4} dk = \frac{A}{2\pi^2}\frac{\log(\frac{\beta}{k_{eq}^4}k^4+1)}{\frac{4\beta}{k_{eq}}} \tag{6}$$

$$\sigma^2(R) = \frac{\log(\frac{\beta}{k_{eq}^4}k^4+1)}{\frac{r\beta}{k_{eq}}} \tag{7}$$

And from here I am not sure where to go. I don't see how to get this in terms of a time. It seems everything is in k, which I know can also be expressed in terms of R. But I don't see anywhere a time could come in.

2) E-Foldings of Inflation

Finding the number of e-foldings required to solve the horizon problem is described in section 7.8.2 of the textbook. From $\frac{\dot{a}}{a_0}^2 = H_0^2[\Omega_{o,w}(\frac{a_0}{a})^{1+3w} + (1-\Omega_{o,w})]$ and by requiring that the initial co-moving radius is much greater than than the co-moving radius in the current epoch, the number of e-foldings can be determined from eqn. 7.8.17 in the book as shown below.

$$N = \frac{60}{|1+3w|}[2.3 + \frac{1}{30}\ln(\frac{T_f}{T_p}) - \frac{1}{60}\ln(Z_{eq})] \tag{8}$$

With $w = 1$,$Z_{eq} = 3800$, and $T_f/T_p = \Delta E = 10^{28}$, the number of e-foldings is then given by:

$$N = \frac{60}{|4|}[2.3 + \frac{1}{30}\ln(10^{28}) - \frac{1}{60}\ln(3800)] = 65.83 \tag{9}$$