

Program Analysis

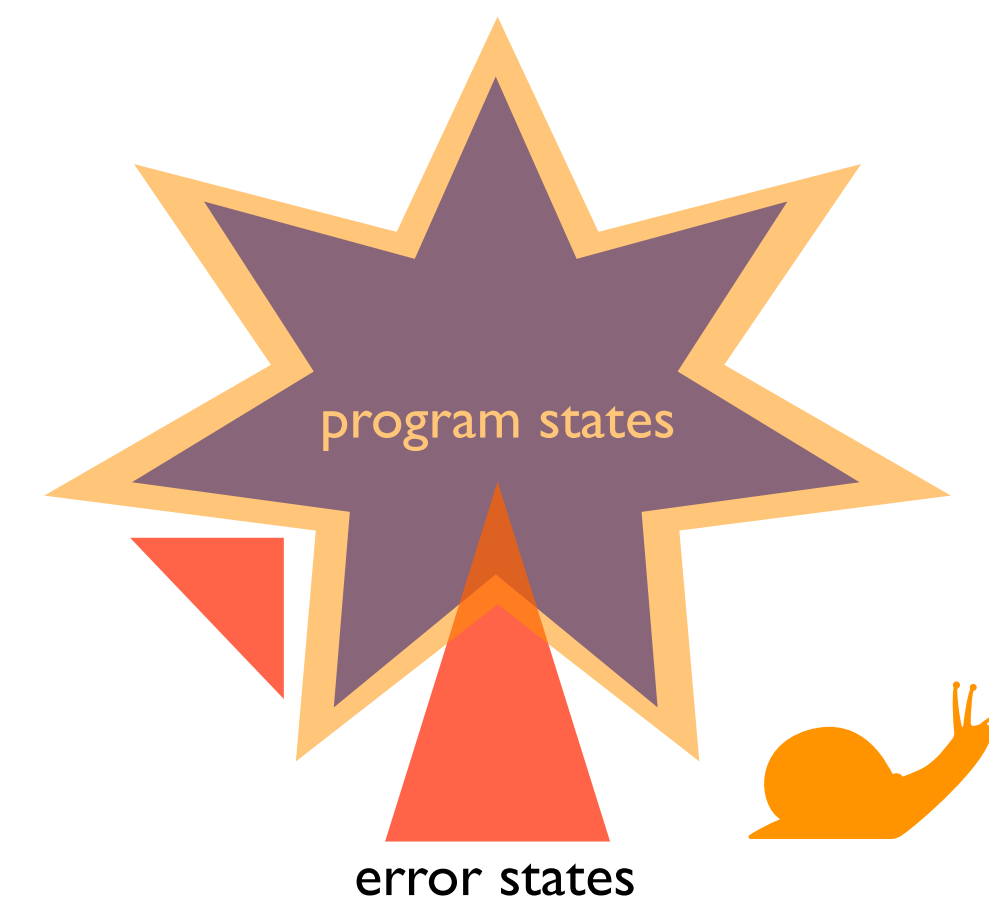
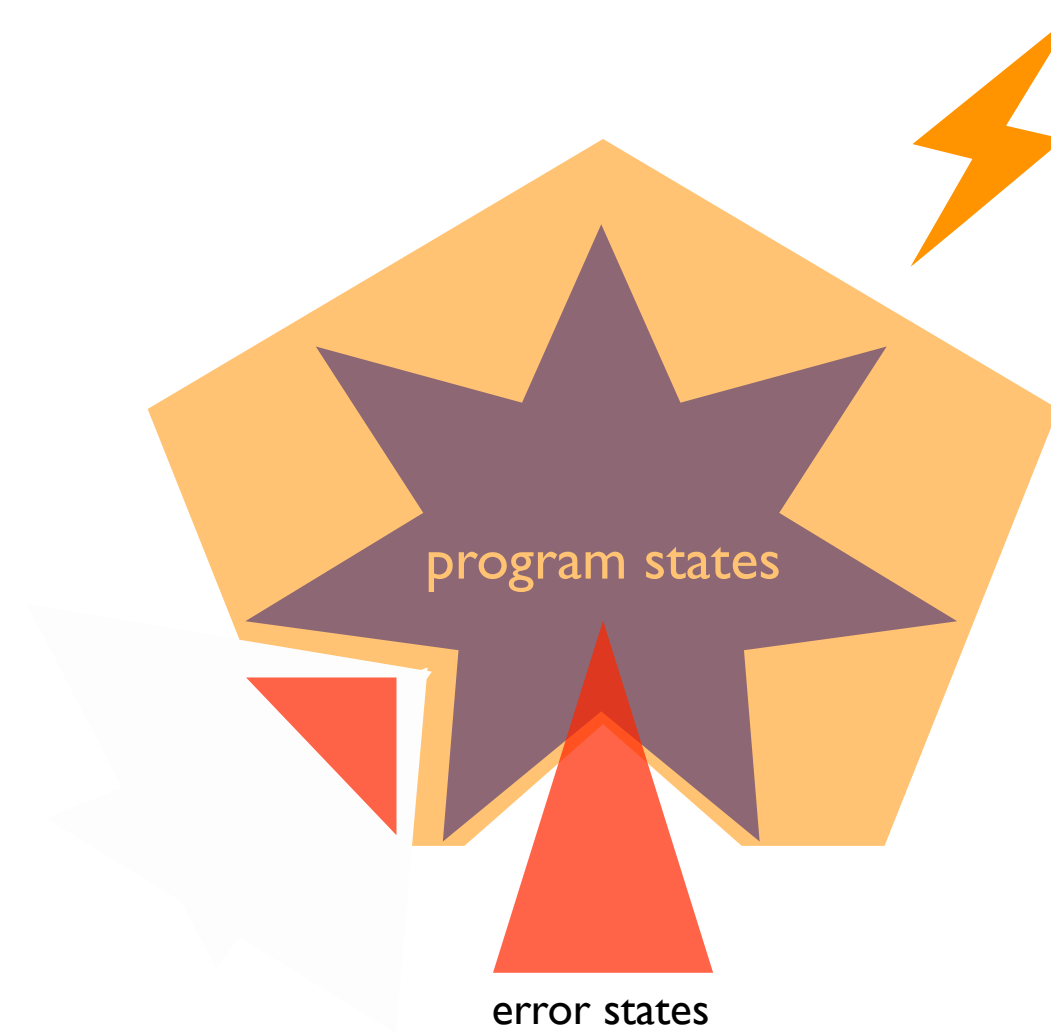
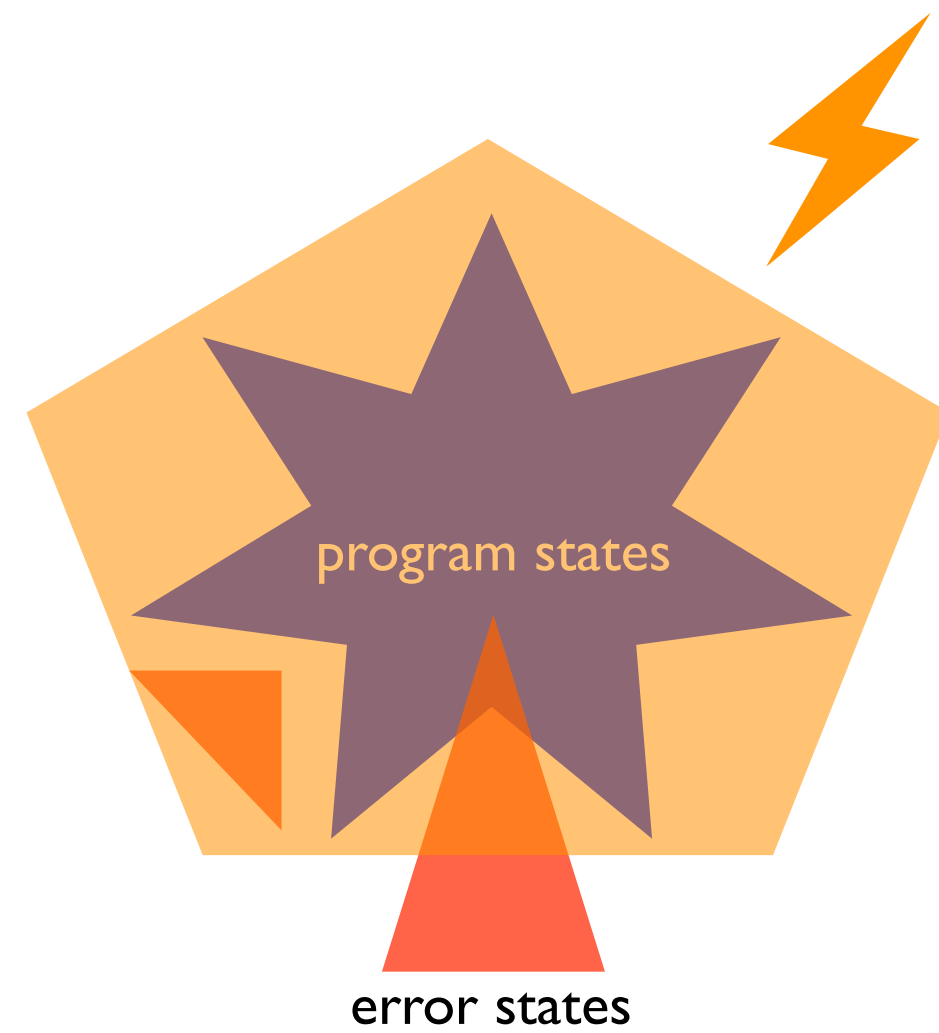
12. Selective X-sensitivity

Kihong Heo



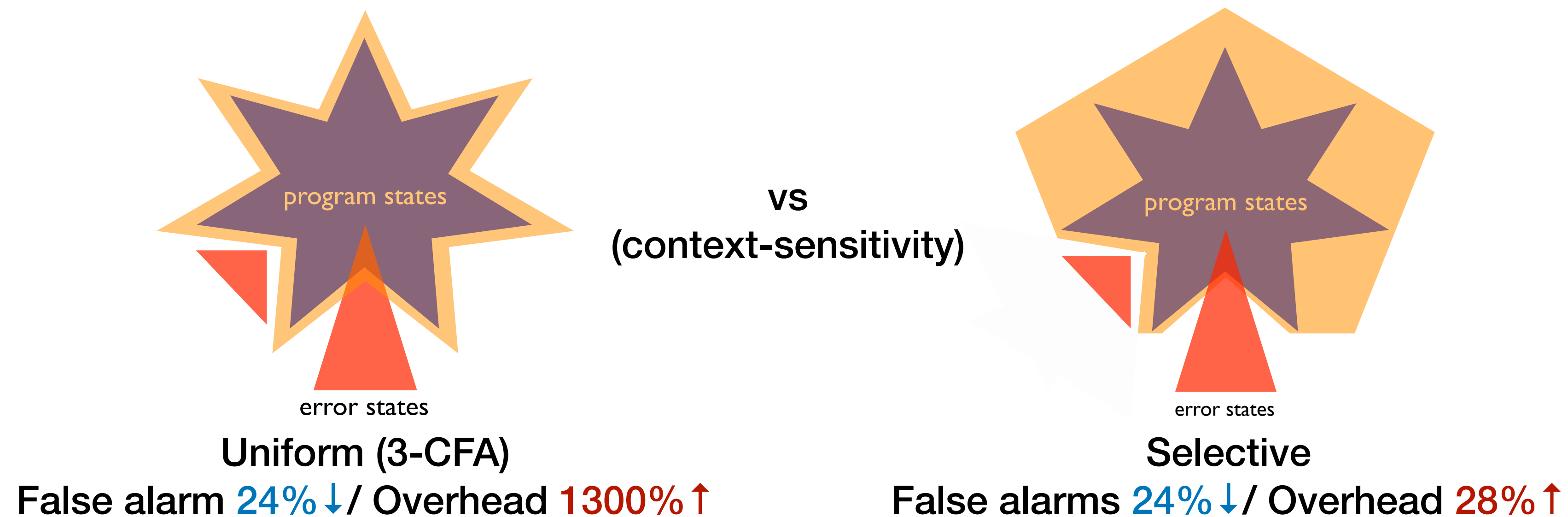
Cost-Accuracy Balance

- How to **strike a balance** between cost and accuracy?
- How to find a **minimal abstraction** to produce the **best result**?



Selective X-Sensitivity

- Selectively apply a precision-improving technique X-sensitivity
 - only when/where it matters
 - X : context, flow, variable relationship, etc



*Oh et al. Selective Context-Sensitivity Guided by Impact Pre-analysis, PLDI'14

Example: Context-Sensitivity

- Suppose an analysis with the interval domain

```
int h(n) { return n; }

void f(s) {
1:   p = h(s);
    assert(p > 1); // Q1: always true
2:   q = h(input());
    assert(q > 1); // Q2: not always true
}

3: void g() { f(8); }

void main(){
4:   f(4);
5:   g();
6:   g();
}
```

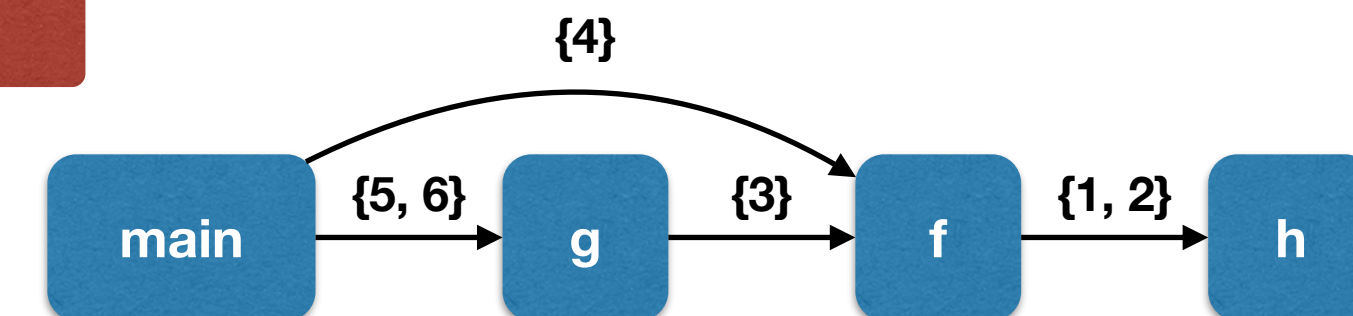
Example: Context-Sensitivity

- Context-insensitive analysis

```
int h(n) { return n; }  
void f(s) {  
1: p = h(s);  
   assert(p > 1); // Q1: always true  
2: q = h(input());  
   assert(q > 1); // Q2: not always true  
}  
  
3: void g() { f(8); }  
  
void main(){  
4: f(4);  
5: g();  
6: g();  
}
```

$[-\infty, +\infty]$

cannot prove



Example: Context-Sensitivity

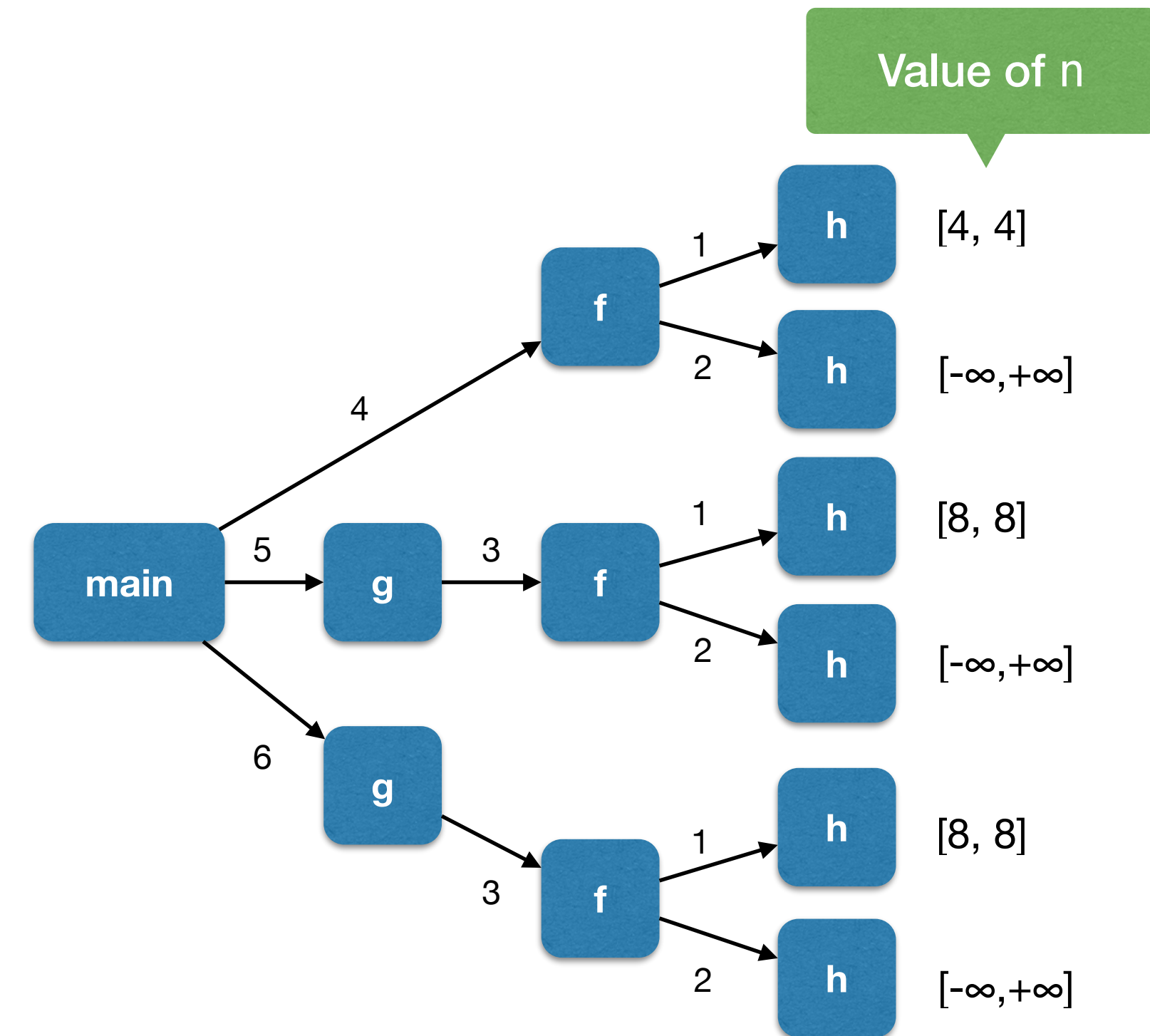
- Uniformly context-sensitive analysis (3-CFA)

```
int h(n) { return n; }

void f(s) {
1:   p = h(s);
    assert(p > 1); // Q1: always true
2:   q = h(input());
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3: void g() { f(8); }

void main(){
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```

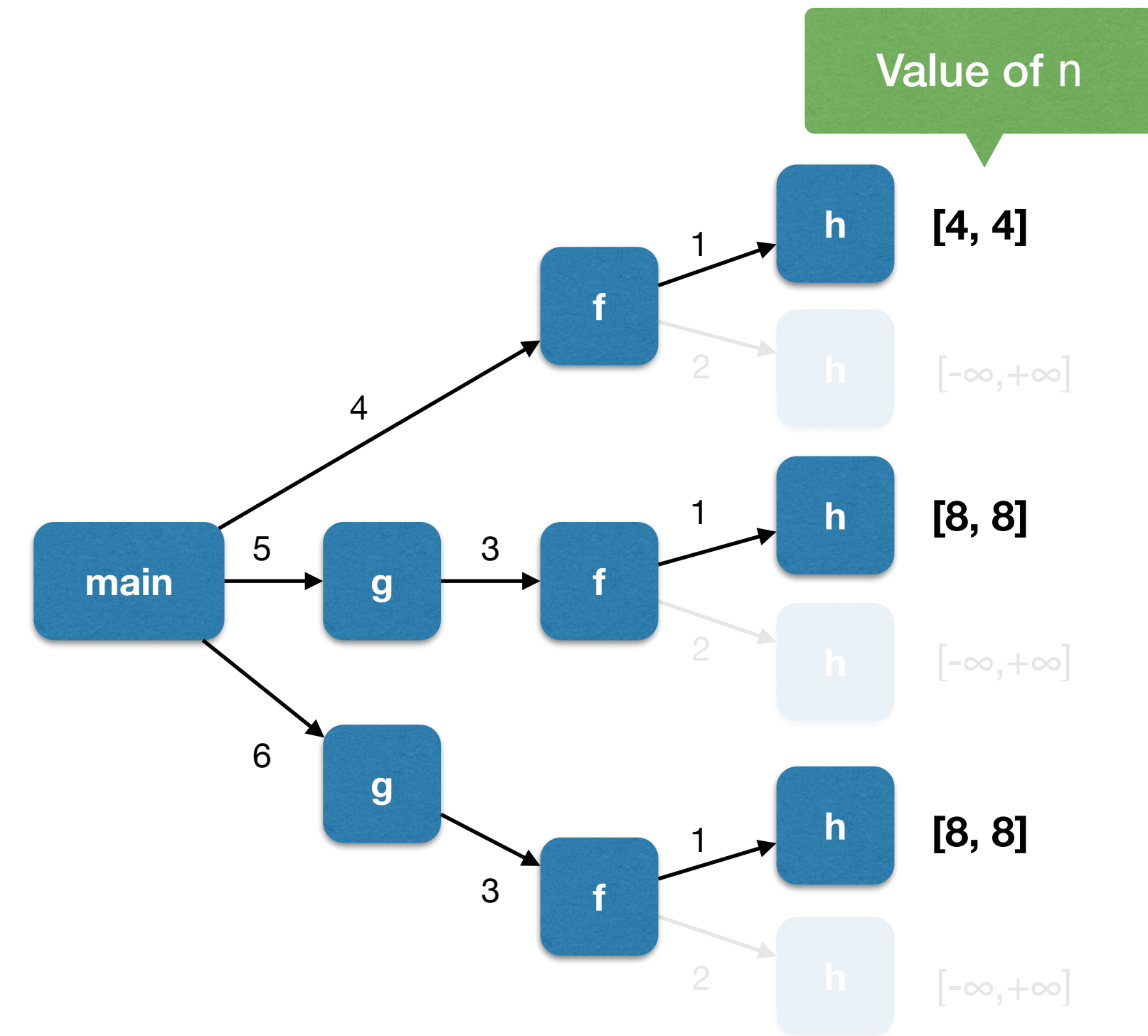


Example: Context-Sensitivity

- Uniformly context-sensitive analysis (3-CFA)

```
int h(n) { return n; }  
  
void f(s) {  
1:   p = h(s);  
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}  
  
3: void g() { f(8); }  
  
void main(){  
4:   f(4);  
5:   g();  
6:   g();  
}
```

Proved!



Example: Context-Sensitivity

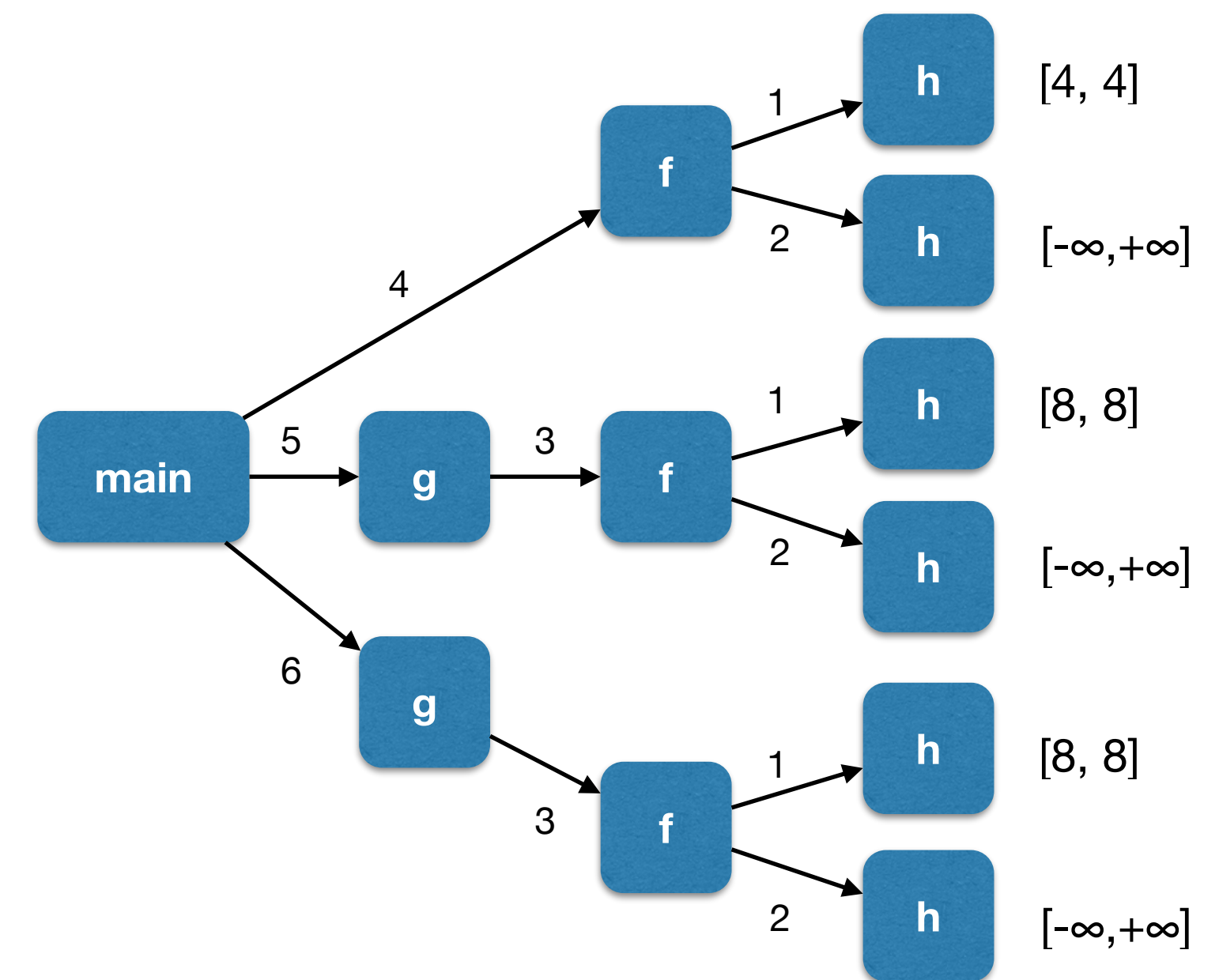
- Problem of uniformly sensitive analysis

```
int h(n) { return n; }

void f(s) {
1:   p = h(s);
    assert(p > 1); // Q1: always true
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4:   f(4);
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Example: Context-Sensitivity

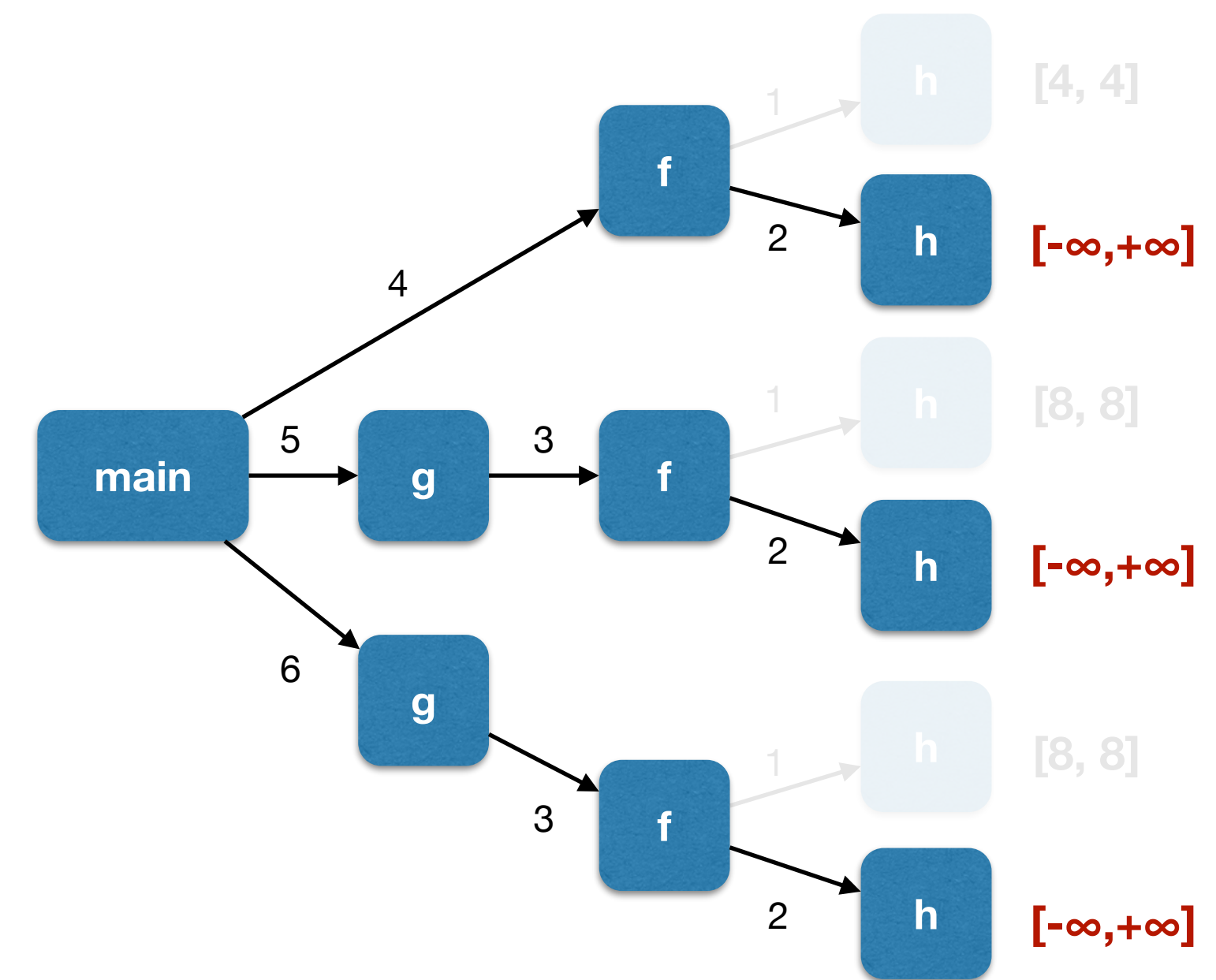
- Problem of uniformly sensitive analysis 1: useless sensitivity

```
int h(n) { return n; }

void f(s) {
1:   p = h(s);
    assert(p > 1); // Q1: always true
2:   q = h(input());
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3: void g() { f(8); }

void main(){
4:   f(4);
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```



Example: Context-Sensitivity

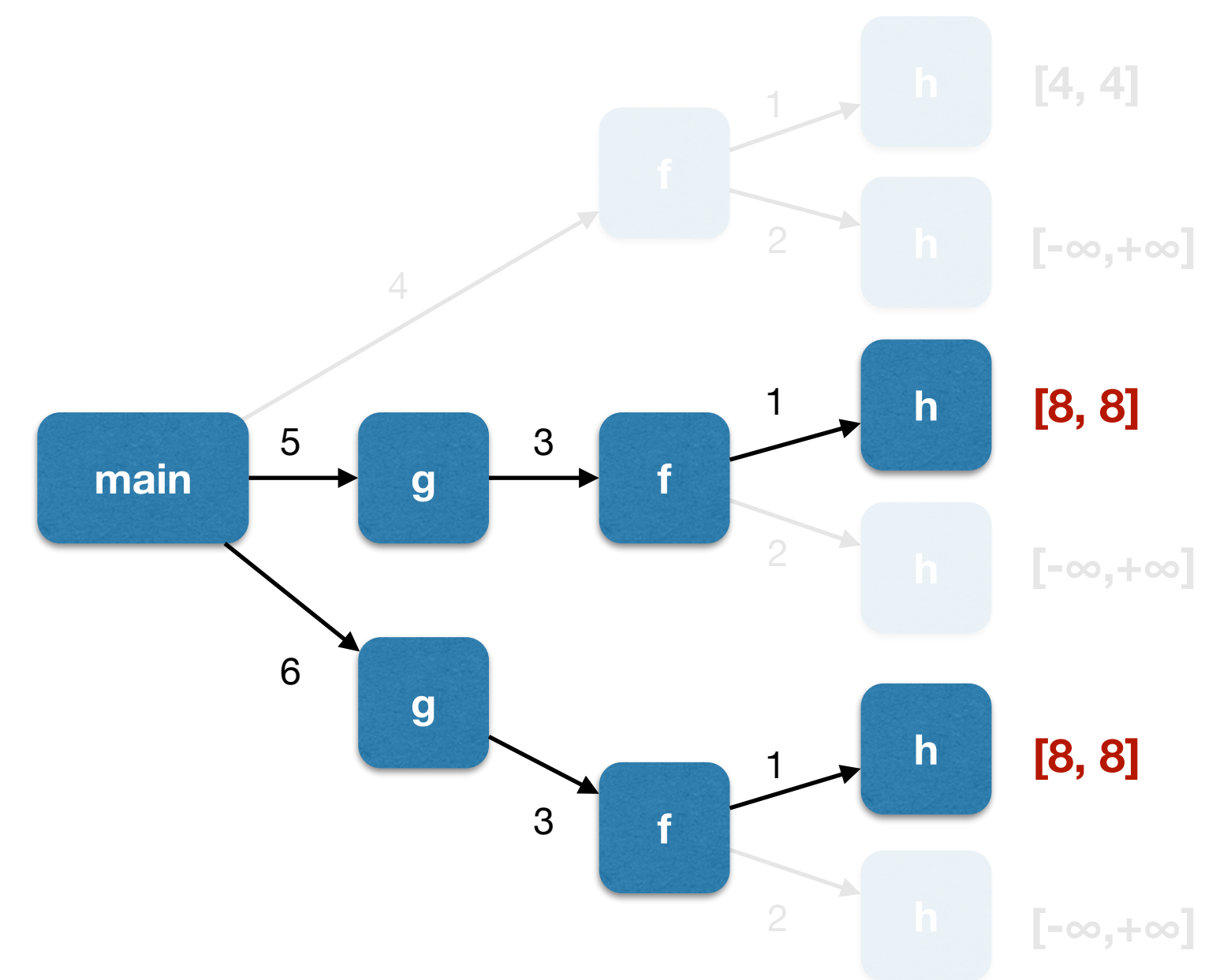
- Problem of uniformly sensitive analysis 2: too much sensitivity

```
int h(n) { return n; }

void f(s) {
1:   p = h(s);
    assert(p > 1); // Q1: always true
2:   q = h(input());
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}

3: void g() { f(8); }

void main(){
4:   f(4);
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}
```



Example: Context-Sensitivity

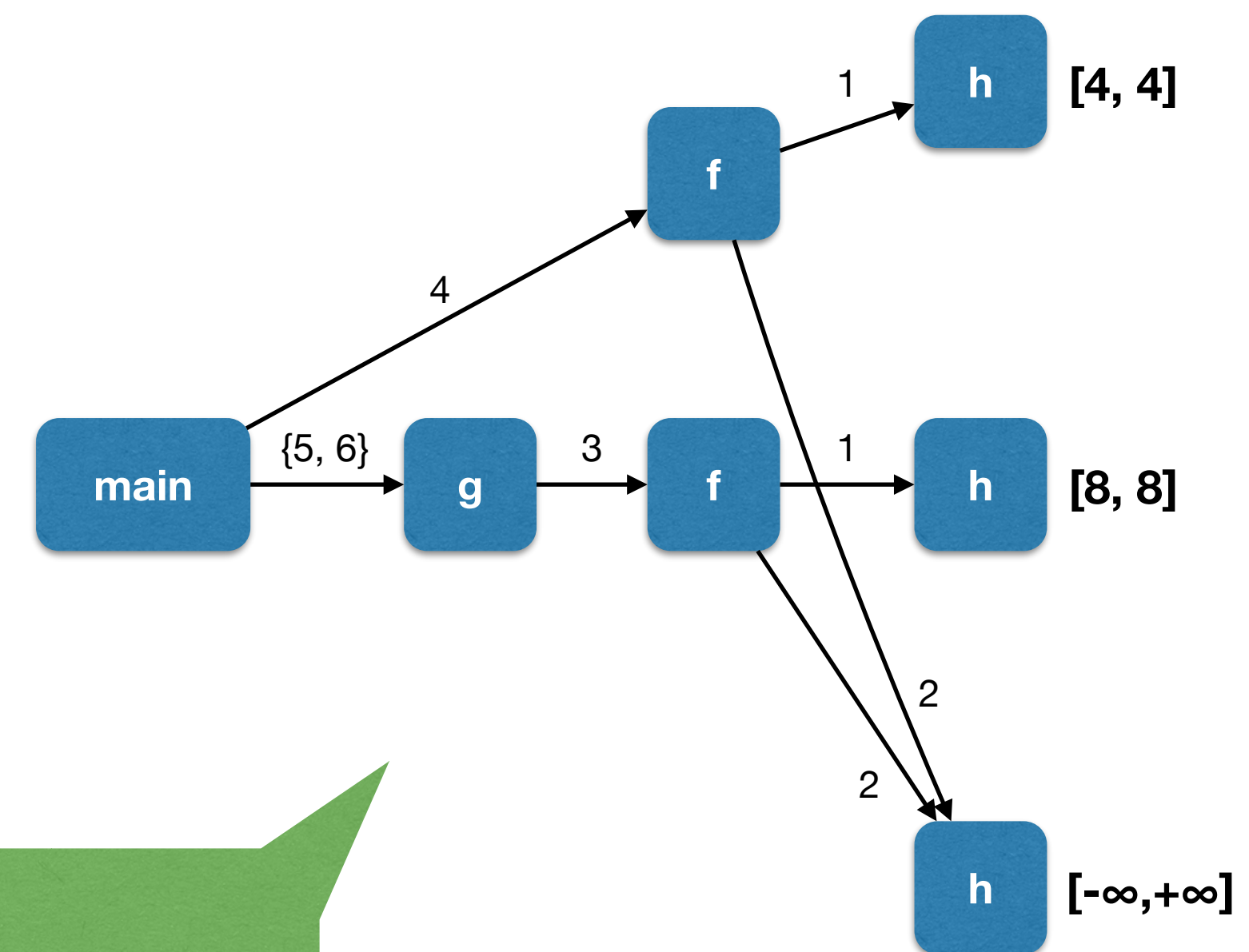
- Solution: **selective** context-sensitive analysis

```
int h(n) { return n; }

void f(s) {
1:   p = h(s);
    assert(p > 1); // Q1: always true
2:   q = h(input());
    assert(q > 1); // Q2: not always true
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3: void g() { f(8); }

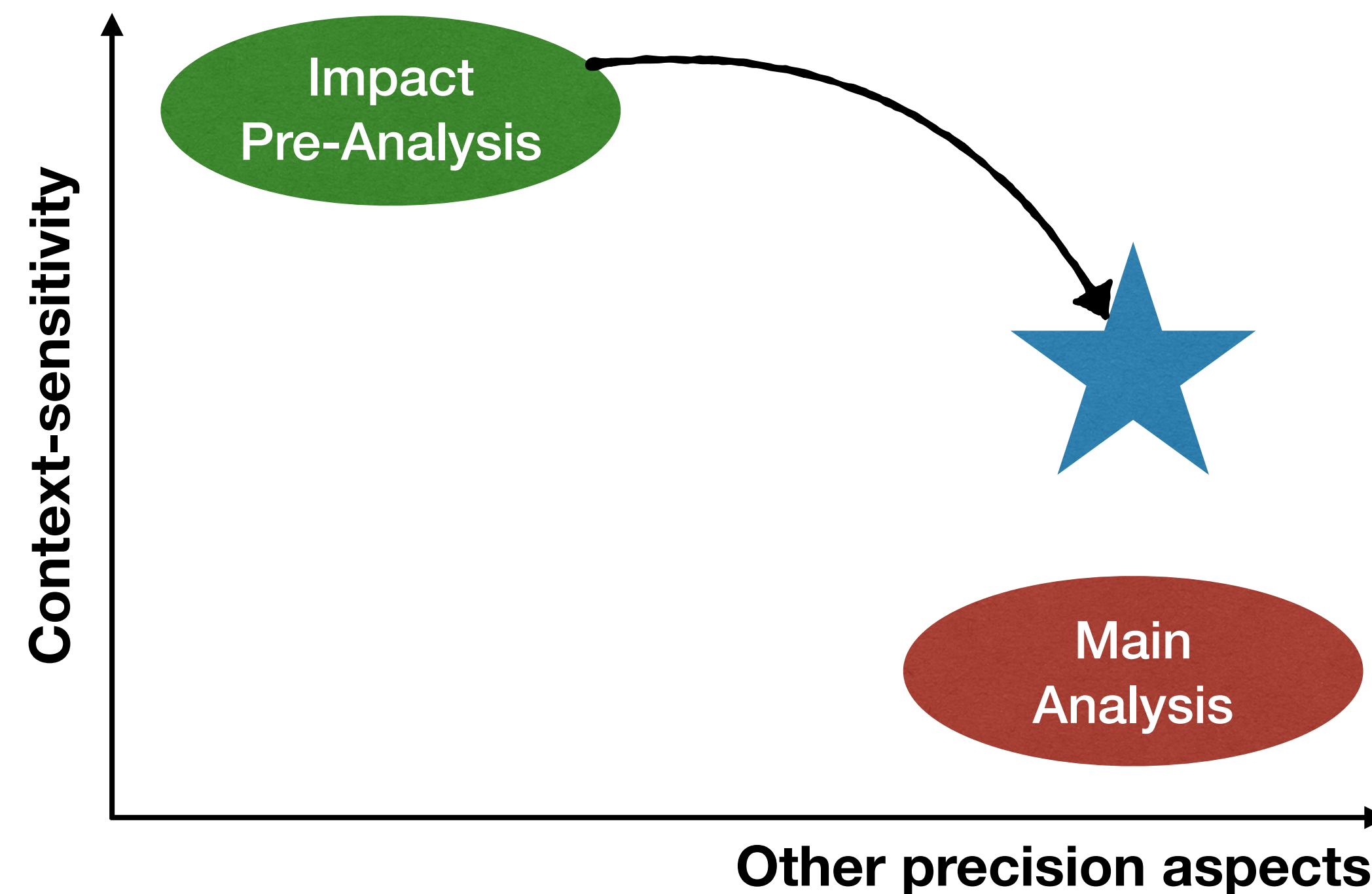
void main(){
4:   f(4);
5:   g();
6:   g();
}
```



Q: How to infer this selective context-sensitivity?

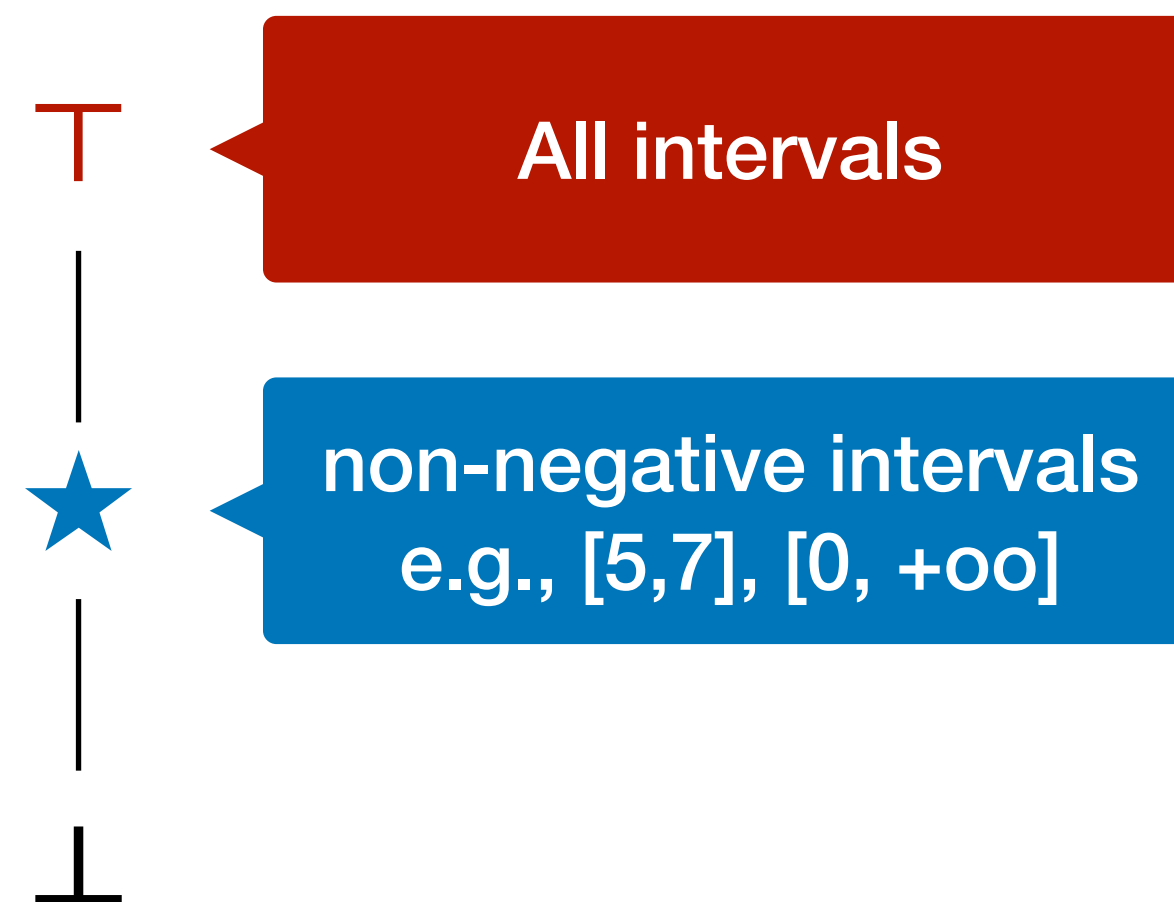
Key Idea: Impact Pre-Analysis

- Estimate the impact of X-sensitivity on main analysis
 - Fully X-sensitive, but approximated in other precision aspects



Design of Impact Pre-Analysis

- Main analysis: context-insensitive + interval domain
- Impact pre-analysis: fully context-sensitive + approximated interval domain



$$\wp(\mathbb{Z}^\#) \xrightleftharpoons[\alpha]{\gamma} \{\perp, \star, \top\}$$

Running Impact Pre-Analysis

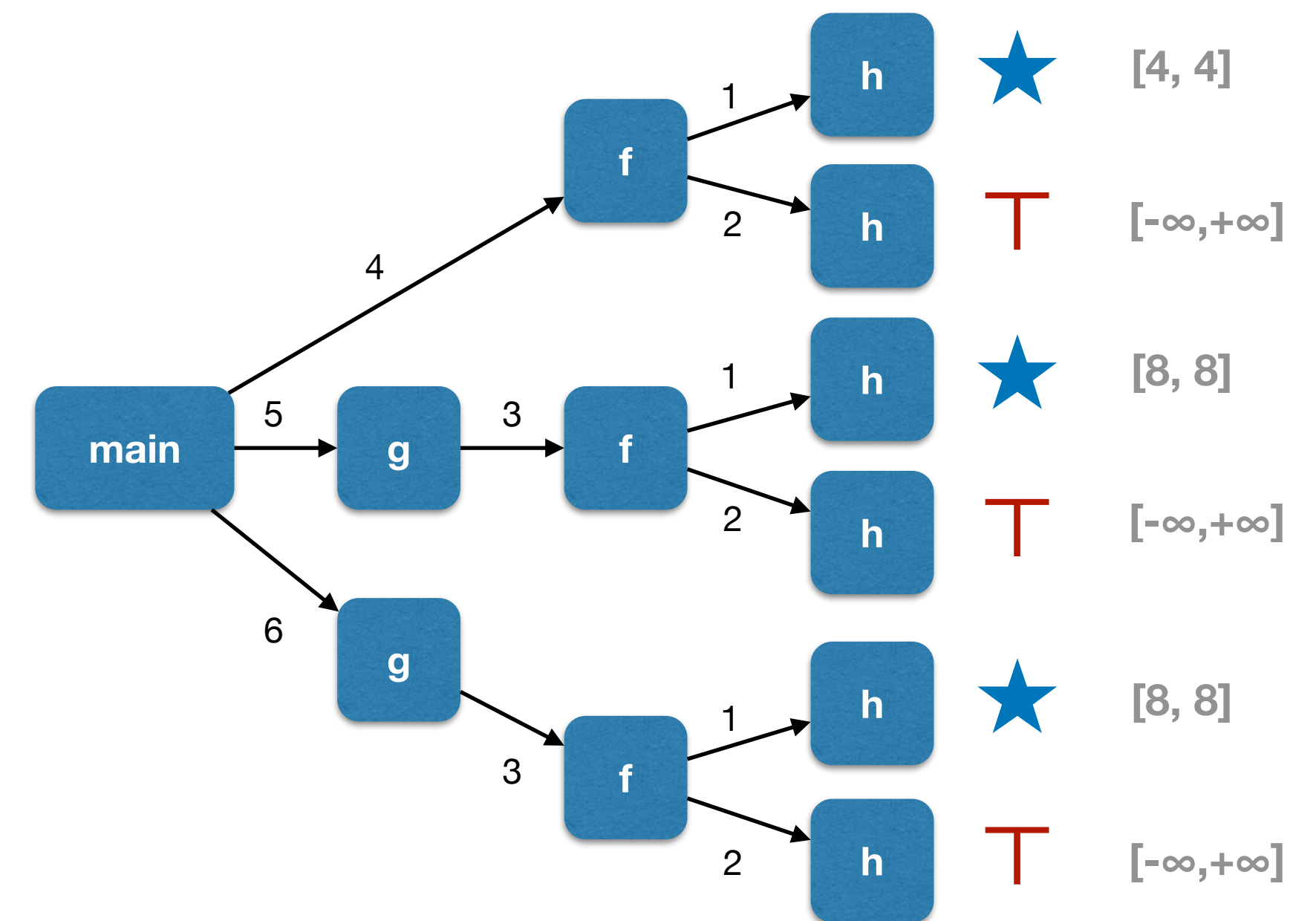
- Impact pre-analysis: fully context-sensitive + approximated interval domain

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3: void g() { f(8); }

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```



Constructing Selective Sensitivity

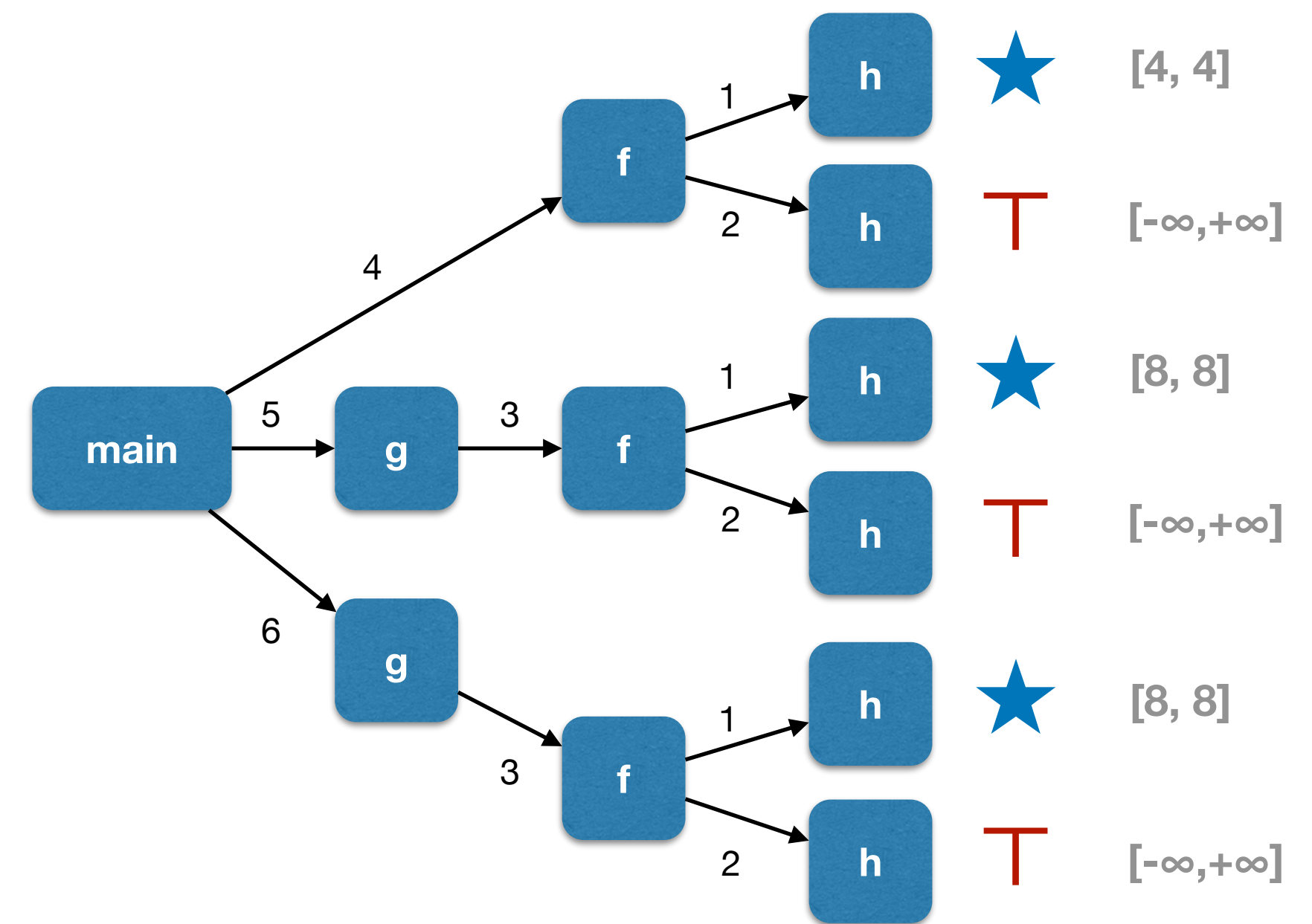
1. Collect queries whose expressions are assigned with ★

```
int h(n) { return n; }

void f(s) {
1:  p = h(s);
★ assert(p > 1); // Q1: always true
2:  q = h(input());
T assert(q > 1); // Q2: not always true
}

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void main(){
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```



Constructing Selective Sensitivity

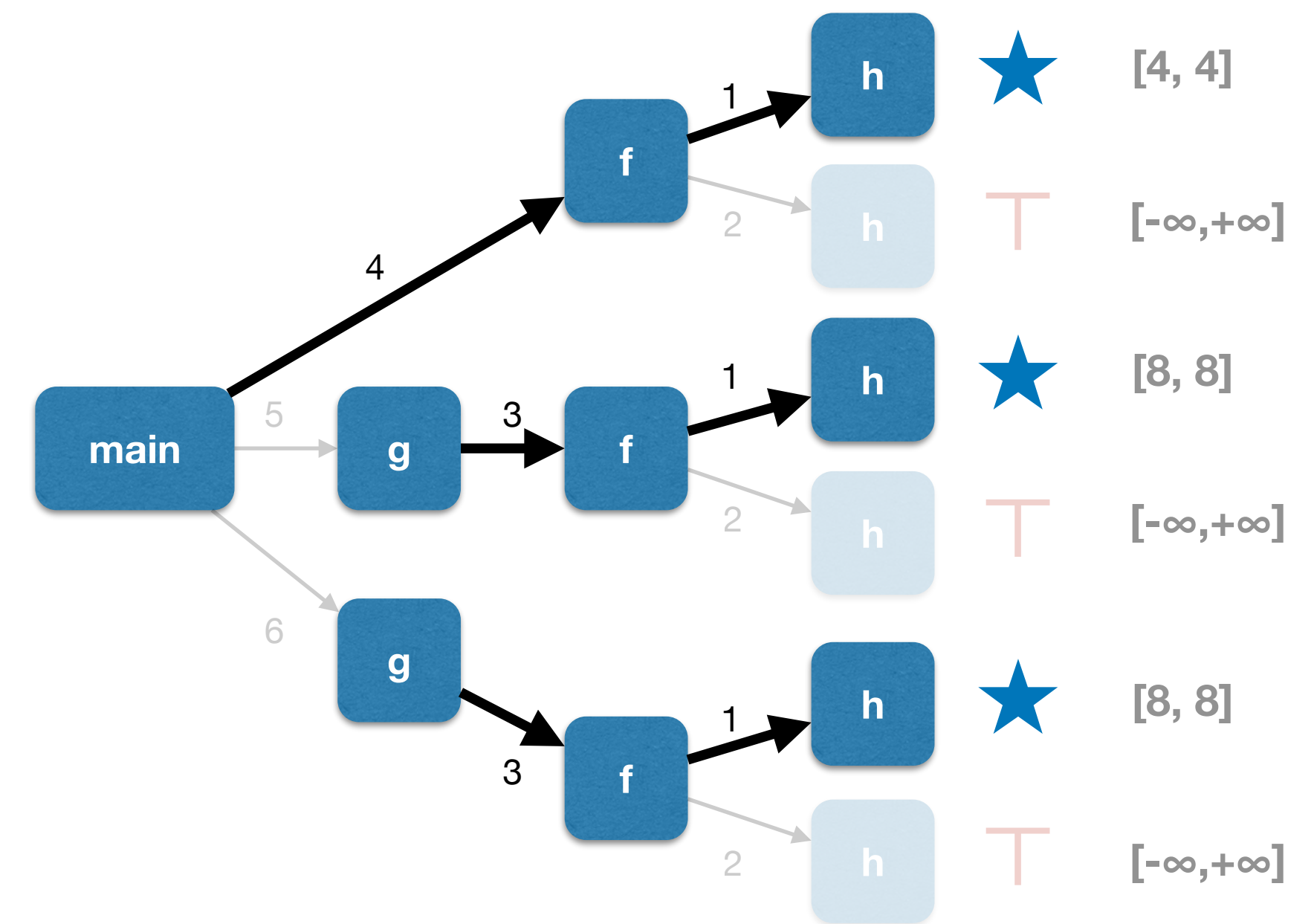
2. Find contexts that contribute to the selected queries

```
int h(n) { return n; }

void f(s) {
1:   p = h(s);
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3: void g() { f(8); }

void main(){
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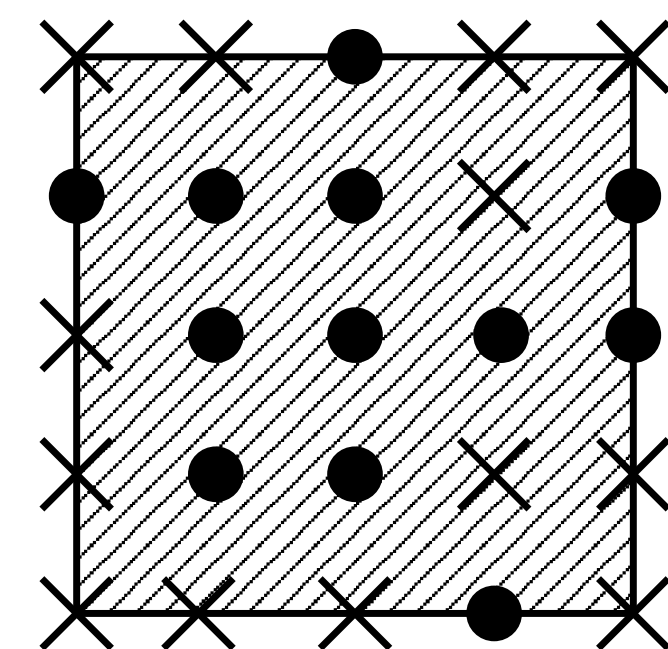
Selected contexts for function `h`: {4-1, 3-1}

Generality

- The same principle is applicable to other types of sensitivity
 - Running an impact pre-analysis
(full X-sensitivity + aggressive abstraction of other aspects)
 - Select queries that are judged promising by the pre-analysis
 - Construct X-sensitivity that contributes to the selected queries

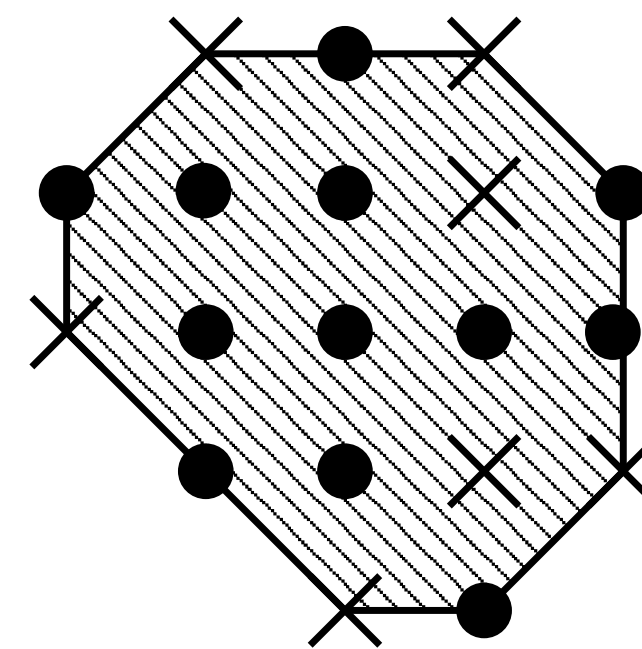
Example: Relational Analysis

- Keep track of relationships between variables in a certain form
 - E.g., octagon analysis: $(\pm x) - (\pm y) \leq c$

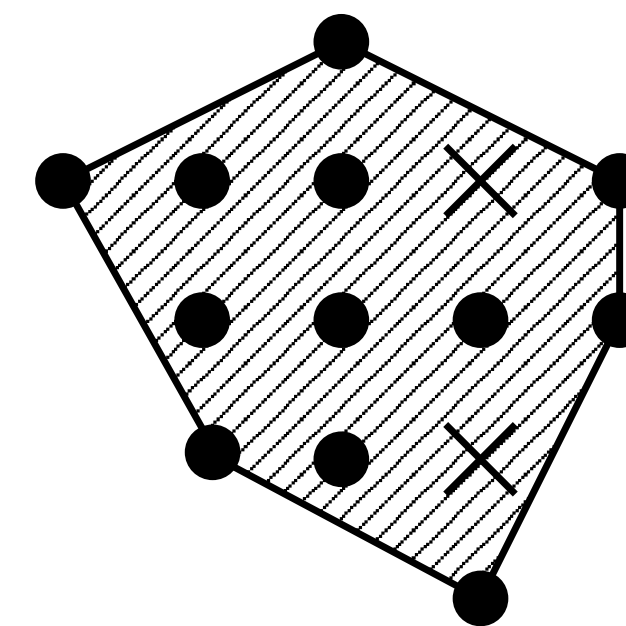


Intervals

Non-relational
Abstraction



Octagons



Polyhedra

Relational
Abstraction

*A. Mine, The Octagon Abstract Domain, HOSC'06

Example: Relational Analysis

- Non-relational analysis with the interval domain

```
// b = [-∞, +∞]
1: int a = b;
2: int c = input();           // User input
3: for (i = 0; i < b; i++) {
4:     assert(i < a);          // Q1: always true
5:     assert(i < c);          // Q2: not always true
6: }
```

Var	Val
a	$[-\infty, +\infty]$
b	$[-\infty, +\infty]$
c	$[-\infty, +\infty]$
i	$[0, +\infty]$

Example: Relational Analysis

- Fully relational analysis with the octagon domain: $(\pm x) - (\pm y) \leq c$

```
// b = [-oo, +oo]
1: int a = b;
2: int c = input();           // User input
3: for (i = 0; i < b; i++) {
4:     assert(i < a);          // Q1: always true
5:     assert(i < c);          // Q2: not always true
6: }
```

	a	b	c	i
a	0	∞	∞	∞
b	∞	0	∞	∞
c	∞	∞	0	∞
i	∞	∞	∞	0

{a, b, c, i}

*Consider $x - y \leq c$ only, for simplicity

Example: Relational Analysis

- Fully relational analysis with the octagon domain: $(\pm x) - (\pm y) \leq c$

```
// b = [-∞, +∞]
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5:   assert(i < c);           // Q2: not always true
6: }
```

	a	b	c	i
a	0	0	∞	∞
b	0	0	∞	∞
c	∞	∞	0	∞
i	∞	∞	∞	0

$a - b \leq 0$ $b - a \leq 0$

{a, b, c, i}

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Example: Relational Analysis

- Fully relational analysis with the octagon domain: $(\pm x) - (\pm y) \leq c$

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3: for (i = 0; i < b; i++) {
4:   assert(i < a);           // Q1: always true
5:   assert(i < c);           // Q2: not always true
6: }
```

	a	b	c	i	
a	0	0	∞	∞	$c - a \leq \infty$
b	0	0	∞	∞	$c - b \leq \infty$
c	∞	∞	0	∞	
i	∞	∞	∞	0	

$a - c \leq \infty$
 $b - c \leq \infty$

{a, b, c, i}

*Consider $x - y \leq c$ only, for simplicity

Example: Relational Analysis

- Fully relational analysis with the octagon domain: $(\pm x) - (\pm y) \leq c$

```
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3: for (i = 0; i < b; i++) {
4:     assert(i < a);          // Q1: always true
5:     assert(i < c);          // Q2: not always true
6: }
```

	a	b	c	i
a	0	0	∞	∞
b	0	0	∞	-1
c	∞	∞	0	∞
i	∞	∞	∞	0

$i - b \leq -1$

{a, b, c, i}

*Consider $x - y \leq c$ only, for simplicity

Example: Relational Analysis

- Fully relational analysis with the octagon domain: $(\pm x) - (\pm y) \leq c$

```
// b = [-∞, +∞]
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2: int c = input(); // User input
3: for (i = 0; i < b; i++) {
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5:   assert(i < c); // Q2: not always true
6: }
```

	a	b	c	i
a	0	0	∞	-1
b	0	0	∞	-1
c	∞	∞	0	∞
i	∞	∞	∞	0

$b - a \leq 0$
 $\wedge i - b \leq -1$
 $\Rightarrow i - a \leq -1$

{a, b, c, i}

*Consider $x - y \leq c$ only, for simplicity

Example: Relational Analysis

- Problem of fully relational analysis: **useless** relationship

```
// b = [-∞, +∞]
1: int a = b;
2: int c = input(); // User input
3: for (i = 0; i < b; i++) {
4:   assert(i < a); // Q1: always true
5:   assert(i < c); // Q2: not always true
6: }
```

	a	b	c	i
a	0	0	∞	-1
b	0	0	∞	-1
c	∞	∞	0	∞
i	∞	∞	∞	0

{a, b, ~~c~~, i}

*Consider $x - y \leq c$ only, for simplicity

Example: Relational Analysis

- Solution: **selective** relational analysis

```
// b = [-∞, +∞]
1: int a = b;
2: int c = input();           // User input
3: for (i = 0; i < b; i++) {
4:     assert(i < a);          // Q1: always true
5:     assert(i < c);          // Q2: not always true
6: }
```

	a	b	i
a	0	0	-1
b	0	0	-1
i	∞	∞	0

{a, b, i}

+

Var	Val
c	[-∞, +∞]

{c}

*Consider $x - y \leq c$ only, for simplicity

Design of Impact Pre-Analysis

- Main analysis: non-relational analysis
- Impact pre-analysis: fully relational analysis + approximated upper bound

	a	b	c	i
a	0	0	∞	-1
b	0	0	∞	-1
c	∞	∞	0	∞
i	∞	∞	∞	0

Fully relational
octagon analysis

	a	b	c	i
a	★	★	T	★
b	★	★	T	★
c	T	T	★	T
i	T	T	T	★

Fully relational
impact pre-analysis

T
|
★

Maybe infinite

Definitely finite

Constructing Selective Sensitivity

1. Collect queries whose expressions are assigned with ★

```
// b = [-oo, +oo]
1: int a = b;
2: int c = input();
3: for (i = 0; i < b; i++) {
4:   assert(i < a) ★
5:   assert(i < c); T
6: }
```

// User input
// Q1: always true
// Q2: not always true

	a	b	c	i
a	★	★	T	★
b	★	★	T	★
c	T	T	★	T
i	T	T	T	★

Constructing Selective Sensitivity

2. Find variable relationships that contribute to the selected queries

```
// b = [-oo, +oo]
1: int a = b;
2: int c = input(); // User input
3: for (i = 0; i < b; i++) {
4:   assert(i < a); ★ // Q1: always true
5:   assert(i < c); T // Q2: not always true
6: }
```

	a	b	c	i
a	★	★	T	★
b	★	★	T	★
c	T	T	★	T
i	T	T	T	★

Selected variables: {a, b, i}

Summary

- Selective X-sensitivity: a framework for balancing between **cost & accuracy**
- Key idea: **“Apply X-sensitivity only when it matters”**
- Estimate the impact of X using the **impact pre-analysis**
 - Full X-sensitivity + aggressive approximation of other precision aspects
- Construct selective X-sensitivity from the guidance of the impact estimation