Program Analysis

10. Advanced Iteration Techniques

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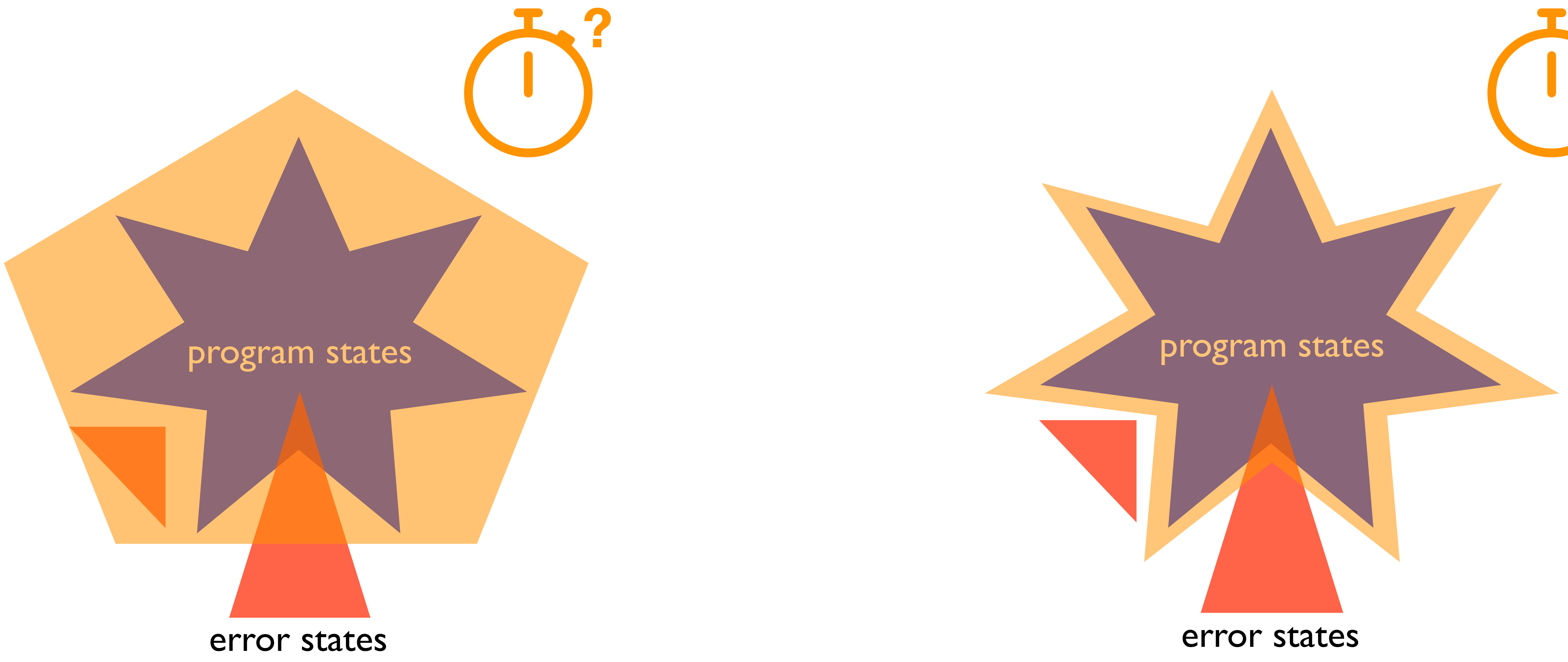
Advanced Analysis Techniques

- So far, our focus most has been **sound** abstract semantics
- From now on, we will cover several advanced techniques to achieve **efficient** and **accurate** analysis



Iteration Strategies

- **Loop invariant inference**: sequences of abstract iterations
  - Compute **weaker** and **weaker** abstract states until stabilization (via join and widening)
- "***Loop is evil***": a main source of imprecision in static analysis
- Needs for techniques to improve the precision

Problem 1: Overused Widening

- Recall the worklist algorithm

$X : \mathbb{L} \to \mathbb{M}^\sharp$

$F^\sharp : (\mathbb{L} \to \mathbb{M}^\sharp) \to (\mathbb{L} \to \mathbb{M}^\sharp)$

$\mathit{Worklist} : \wp(\mathbb{L})$

begin

  $\mathit{Worklist} \leftarrow \mathbb{L}$

  $X \leftarrow \bot$

  repeat

    $(w, \mathit{Worklist}) \leftarrow$ po

    $m^\sharp_{old} \leftarrow X(w)$

    $m^\sharp_{new} \leftarrow \bigsqcup\{m^\sharp_{in}$

    $m^\sharp_{new} \leftarrow m^\sharp_{old} \,\triangledown\, m^\sharp_{new}$

    if $m^\sharp_{new} \not\sqsubseteq m^\sharp_{old}$ then

      $X(w) \leftarrow m^\sharp_{new}$

      $\mathit{Worklist} \leftarrow \mathit{Worklist} \cup \{l \mid \langle w, m^\sharp_{new}\rangle \hookrightarrow^\sharp \langle l, \_\rangle\}$

    endif

  until $\mathit{Worklist} = \emptyset$

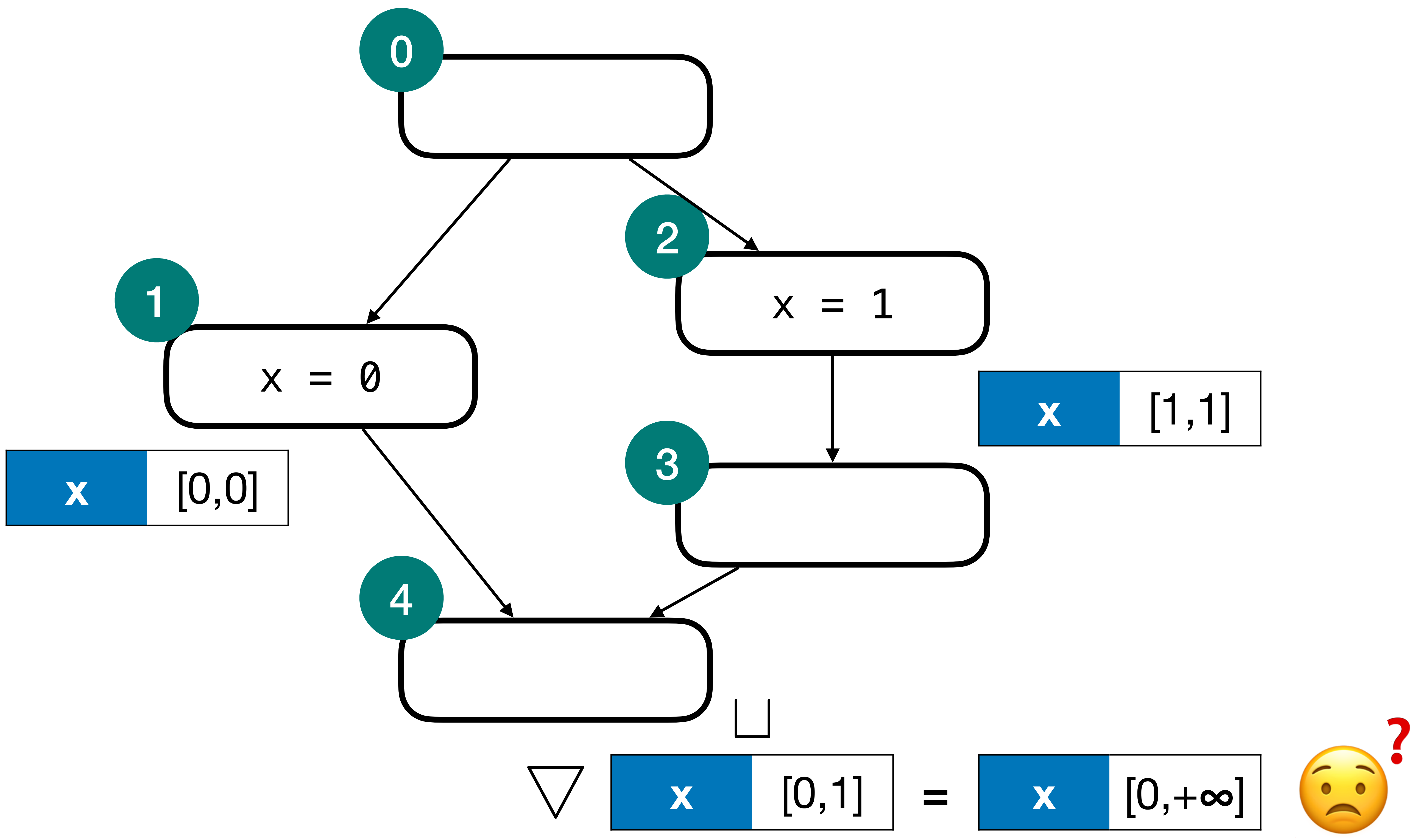
  return X

end

Widening Everywhere?

Example

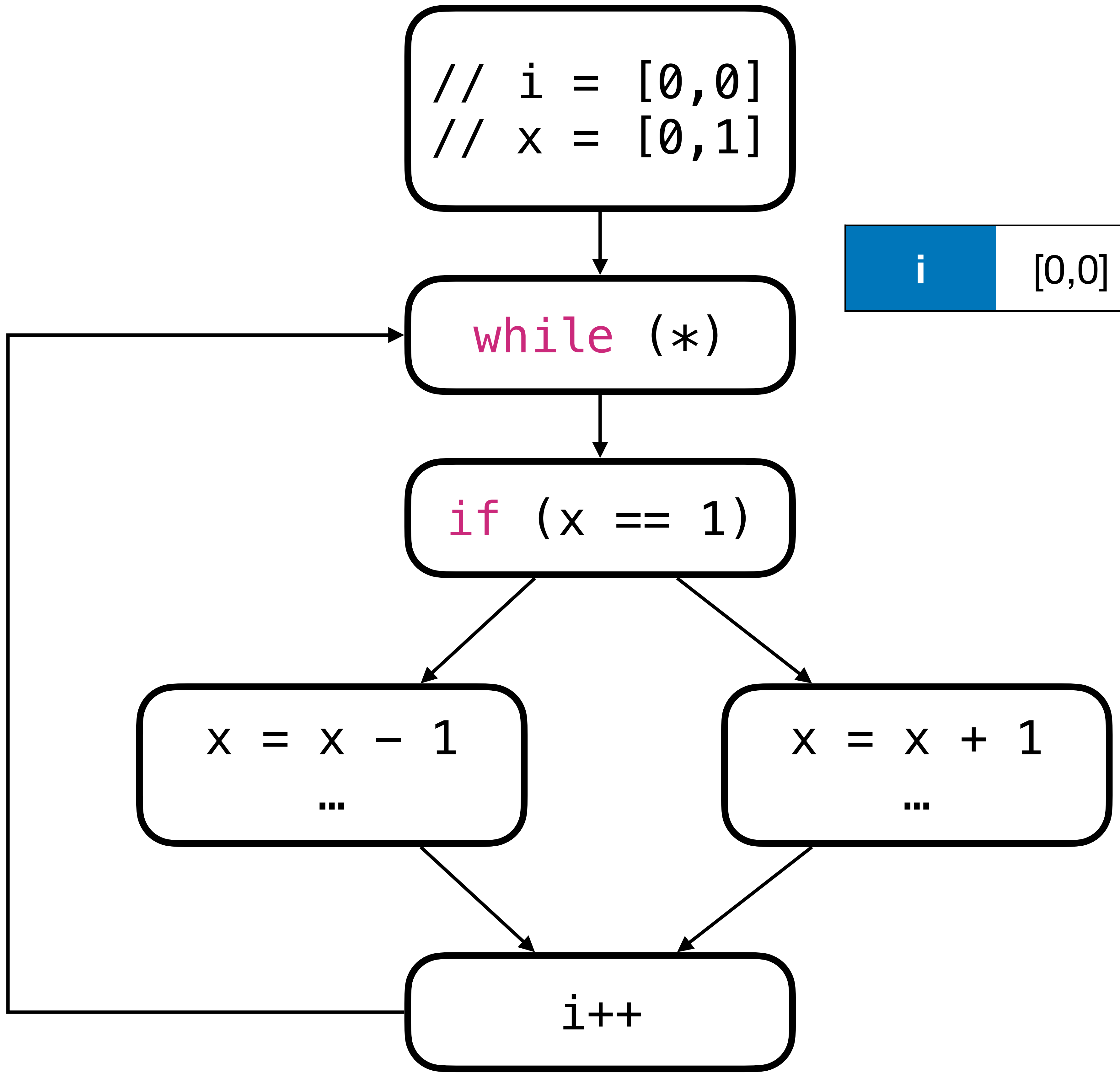
- Consider an analysis with the interval abstract domain



Solution: Selective Widening

- Apply widening only when the label is the target of a **cycling** control flow
    - e.g., while-loop heads, targets of cycling gotos, (spurious) call-cycle
- For other labels, apply the **join** operation instead

Case 1: Loop Heads



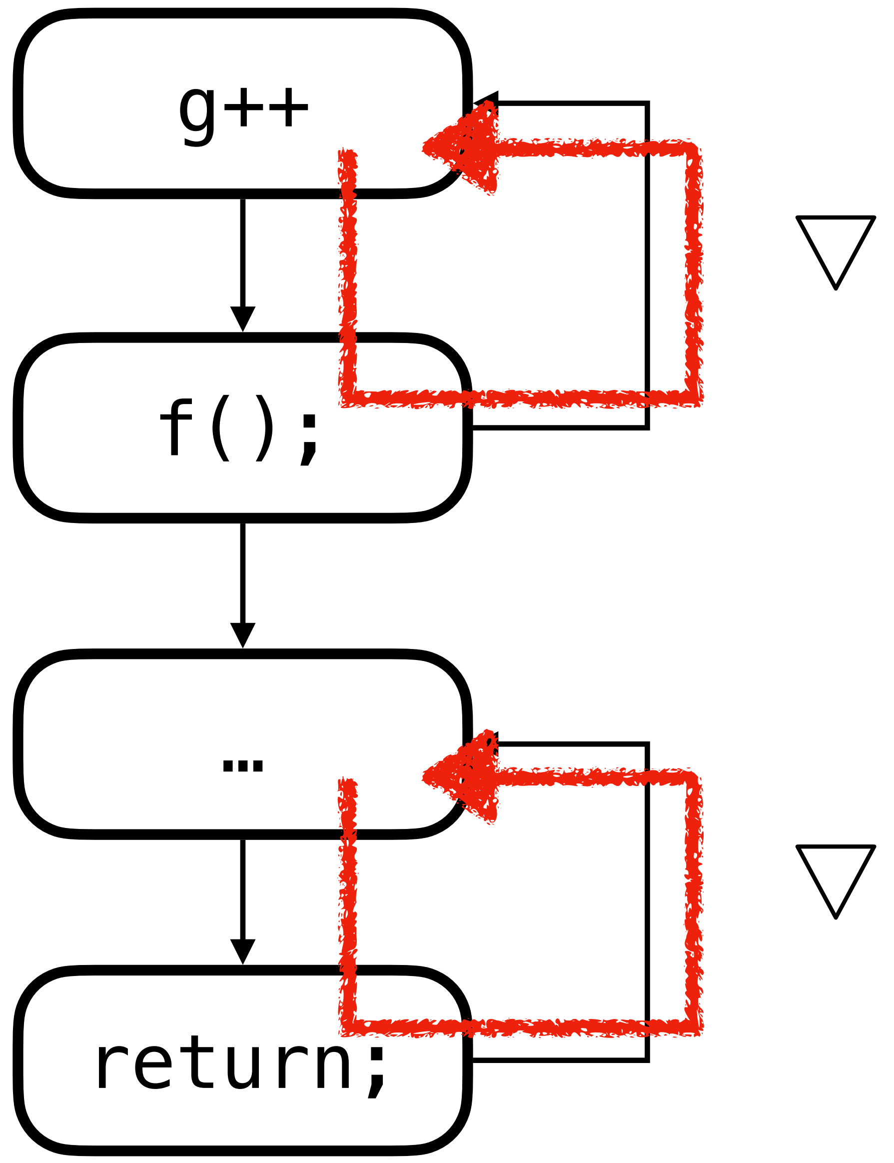
$\boxed{i \mid [0,0]} \;\triangledown\; \boxed{i \mid [0,1]} = \boxed{i \mid [0,+\infty]}$

$\boxed{x \mid [0,0]} \;\sqcup\; \boxed{x \mid [0,1]} = \boxed{x \mid [0,1]}$ 👍

Case 2: Call-cycle

- Widening when a **recursive call-cycle** exists

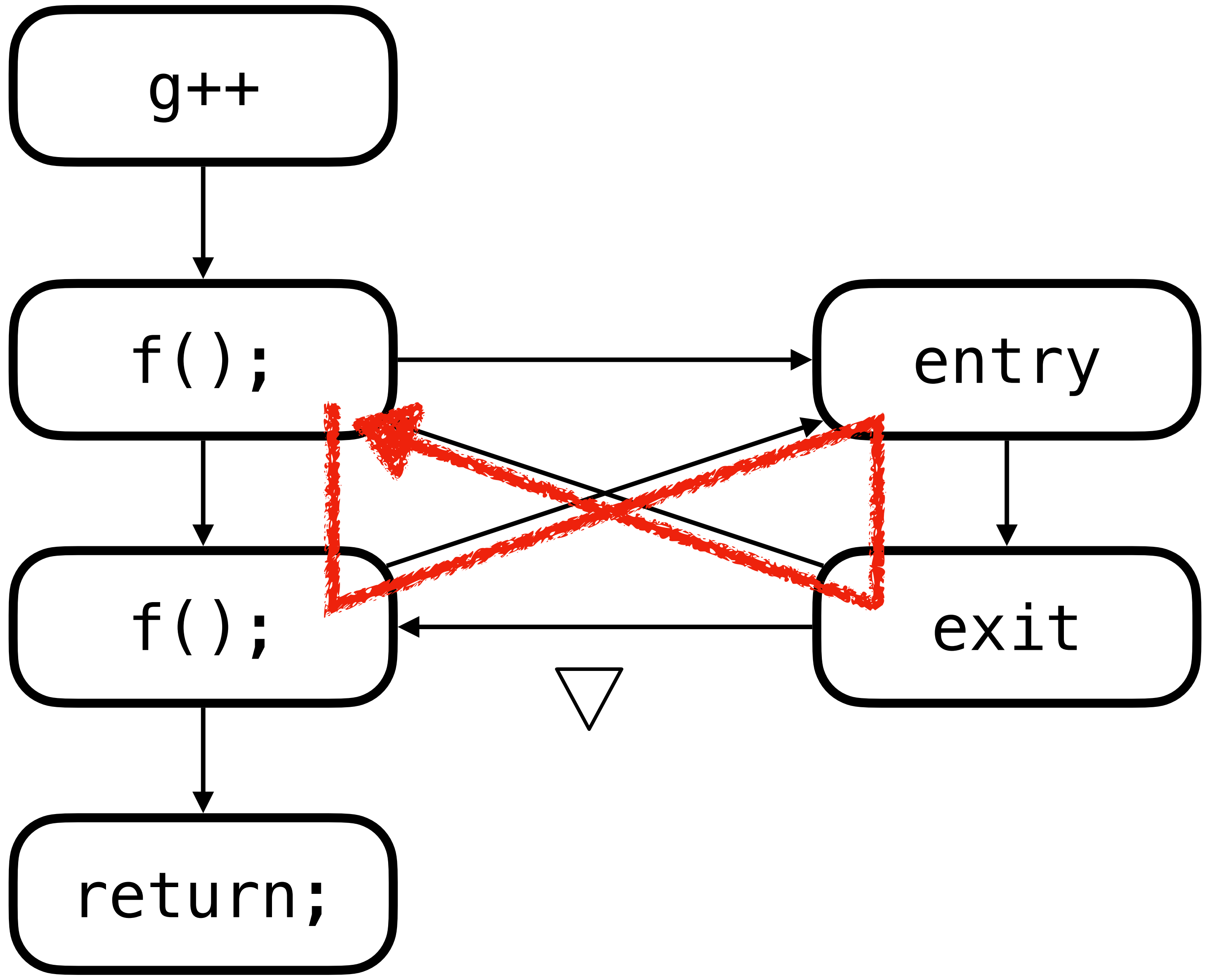
```
void f() {
  g++;
  f();
  …
  return;
}
```



Case 2: Call-cycle (Cont'd)

- Widening when even **spurious-cycle** happens
  - For example, context-insensitive analysis

```
int main() {
  g++;
  f();  // non-recursive
  f();
  return;
}
```



Caveat

- In general, cycle detection cannot be done before analysis
  - control-flow is **dynamic** (e.g., higher-order functions, exceptions, etc)
- Possible solutions:
  - online cycle-detection (during analysis): precise but costly
  - offline cycle-detection with pre-analysis (before analysis): imprecise but lightweight

Problem 2: Hasty Join

```
x = ?; // any value
i = 1;
while (i > 0) {
  if(x < 0 || x > 1000) {
    // [-∞, +∞]
    x = 0;
  } else {
    // [0, 1000]
    x = x + 1;
  }
  input(i);
}
// actually, x is in [0, 1001]
```

Initialization step

- The abstract value for x with a naive approach would be [-∞, +∞]
- Idea: **detach** the first iteration from the rest

Solution: Loop Unrolling

```
x = ?; // any value
i = 1;
while (i > 0) {
  if(x < 0 || x > 1000) {
    x = 0;
  } else {
    x = x + 1;
  }
  input(i);
}
// actually, x is in [0, 1001]
```

```
x = ?; // any value
i = 1;
if(x < 0 || x > 1000) {
  x = 0;
} else {
  x = x + 1;
}
```

first iter.

```
input(i);
// x is in [0, 1001]
while (i > 0) {
  if(x < 0 || x > 1000) {
    x = 0;
  } else {
    x = x + 1;
  }
  input(i);
}
```

rest

```
// x is in [0, 1001]
```

Problem 3: Hasty Widening

```
x = 0;
while (*) {
  if(*) {
    x = -1;
  } else {
    x = x + 1;
  }
}
// x >= -1
```

x	[0,0]

$\triangledown$

x	[-1,1]

=

x	[-∞,+∞]

- The abstract value of x with a naive approach would be [-∞, +∞]
- Idea: **delay** the application of widening for the first N iterations

Solution: Delayed Widening

```
x = 0;
while (*) {
  if(*) {
    x = -1;
  } else {
    x = x + 1;
  }
}
// x >= -1
```

Delayed widening where N = 1

x	[0,0]	$\sqcup$	x	[-1,1]	=	x	[-1,1]
x	[-1,1]	$\triangledown$	x	[-1,2]	=	x	[-1,+∞]
x	[-1,+∞]	$\triangledown$	x	[-1,+∞]	=	x	[-1,+∞]

Fixed Point!

Problem 4: Excessive Widening

```
x = 0;
while (x <= 100) {
  if(x >= 50) {
    x = 10;
  } else {
    x = x + 1;
  }
}
// actually, x is in [0, 50]
```

$[x \mapsto [0,0]] \;\triangledown\; [x \mapsto [0,1]] = [x \mapsto [0,+\infty]]$

- The abstract value of x with a naive approach is [0, +∞]
- Idea: use a **slower and more precise** widening

Solution: Widening with Thresholds

- Take several small steps and stops at pre-defined threshold values
- For example, consider only one threshold *B*:

A naive widening operator

$$[n,p] \ \triangledown \ [n,q] = \begin{cases} [n,p] & \text{if } p \geq q \\ [n,+\infty] & \text{if } p < q \end{cases}$$

A widening with thresholds

$$[n,p] \ \triangledown \ [n,q] = \begin{cases} [n,p] & \text{if } p \geq q \\ [n,B] & \text{if } p < q \leq B \\ [n,+\infty] & \text{if } B < q \end{cases}$$

*only the right bounds, for brevity

Widening with Thresholds

```
x = 0;
while (x <= 100) {
  if(x >= 50) {
    x = 10;
  } else {
    x = x + 1;
  }
}
```

Thresholds = {50}

x	[0,0]	$\triangledown$	x	[0,1]	=	x	[0,50]
x	[0,50]	$\triangledown$	x	[0,50]	=	x	[0,50]

Fixed Point!

Summary

- “***Loop is evil***”: one of the main source of imprecision
- Important to design effective iteration techniques
  - no universal solutions
  - depending on the target program’s characteristics
- Need for domain knowledge (human experts or learning techniques)