Introduction to Program Analysis

11. Sparse Analysis

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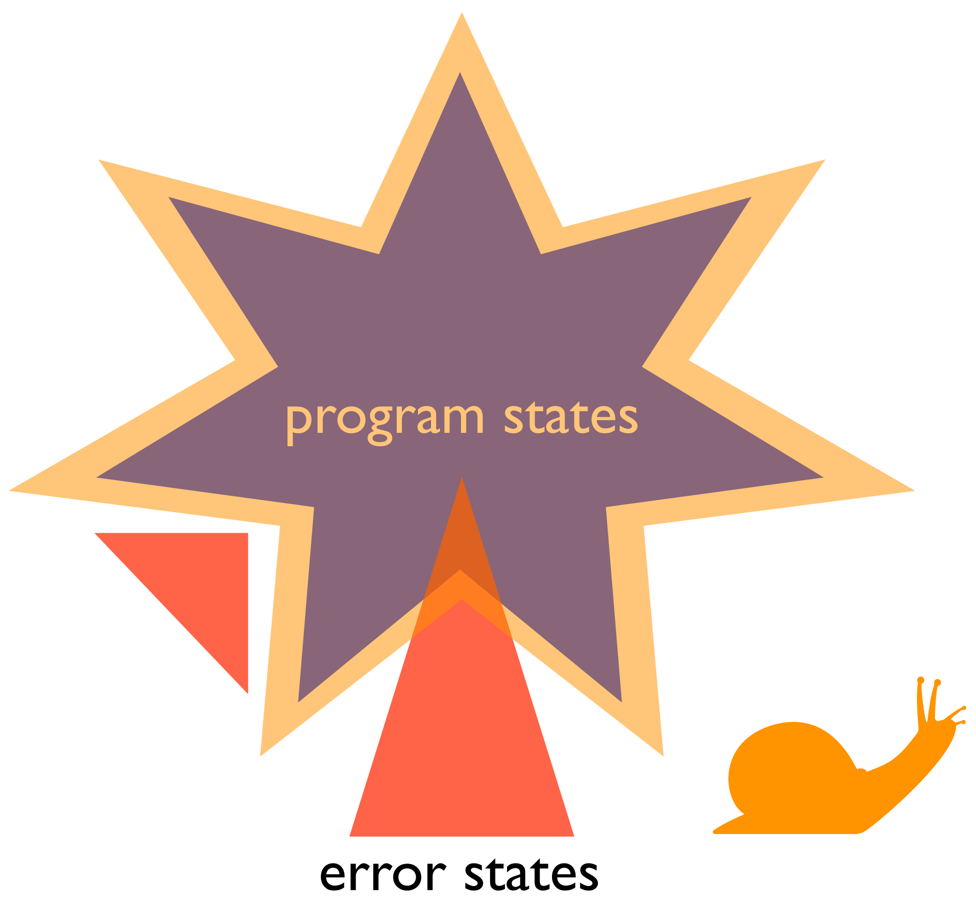
Cost Reduction Techniques

- How to **reduce** the analysis cost **without sacrificing** the analysis precision?
  - In terms of memory and time consumption



Software Complexity

less-382 (23,822 LOC)

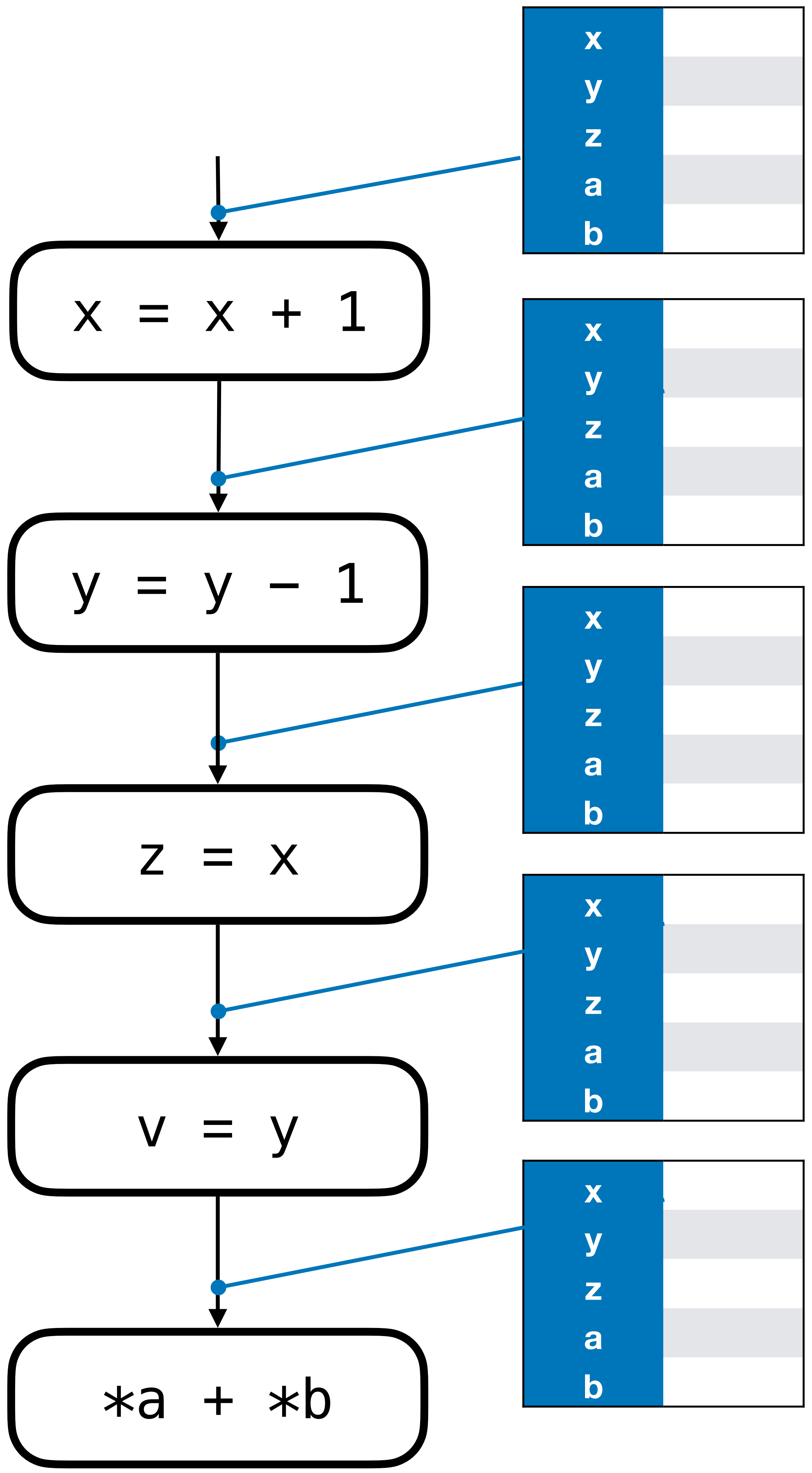
Key Idea: Sparsity

- "***Meaningful semantic effect is so sparse***"
- **Spatial sparsity**: each program portion (an expression, a statement, a block, etc) usually accesses only a small part of the whole memory
- **Temporal sparsity**: after the definition of a memory location, its use is not immediate but a while later
- Generally applicable to flow-sensitive analyses when a memory is a map from abstract locations to abstract values, e.g.,

$$\mathbb{D}^\sharp = \mathbb{L} \to (\mathbb{X} \to \mathbb{V}^\sharp)$$

Example

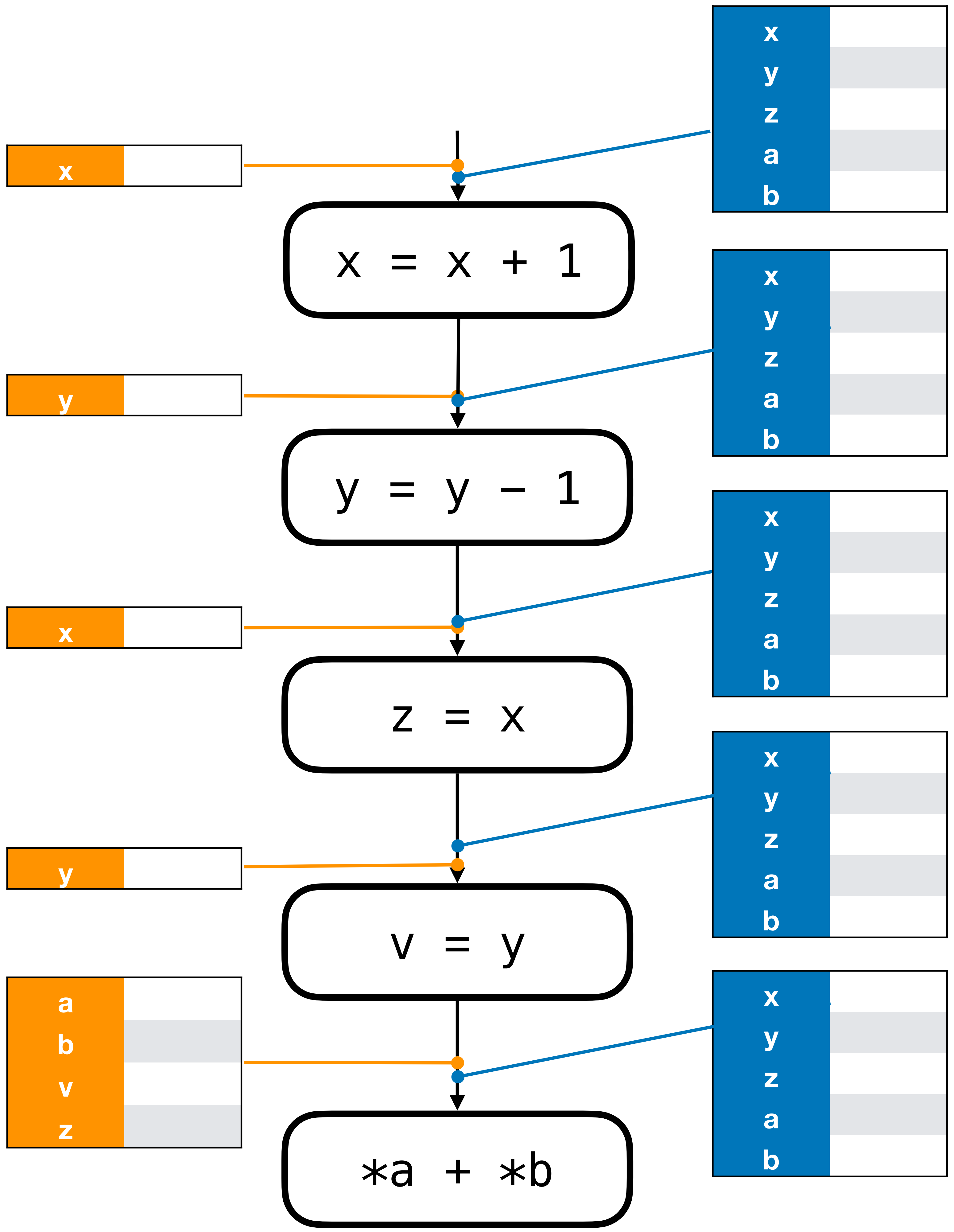
- Vanilla (non-sparse) analysis



*Assume a and b point to z and v

Example

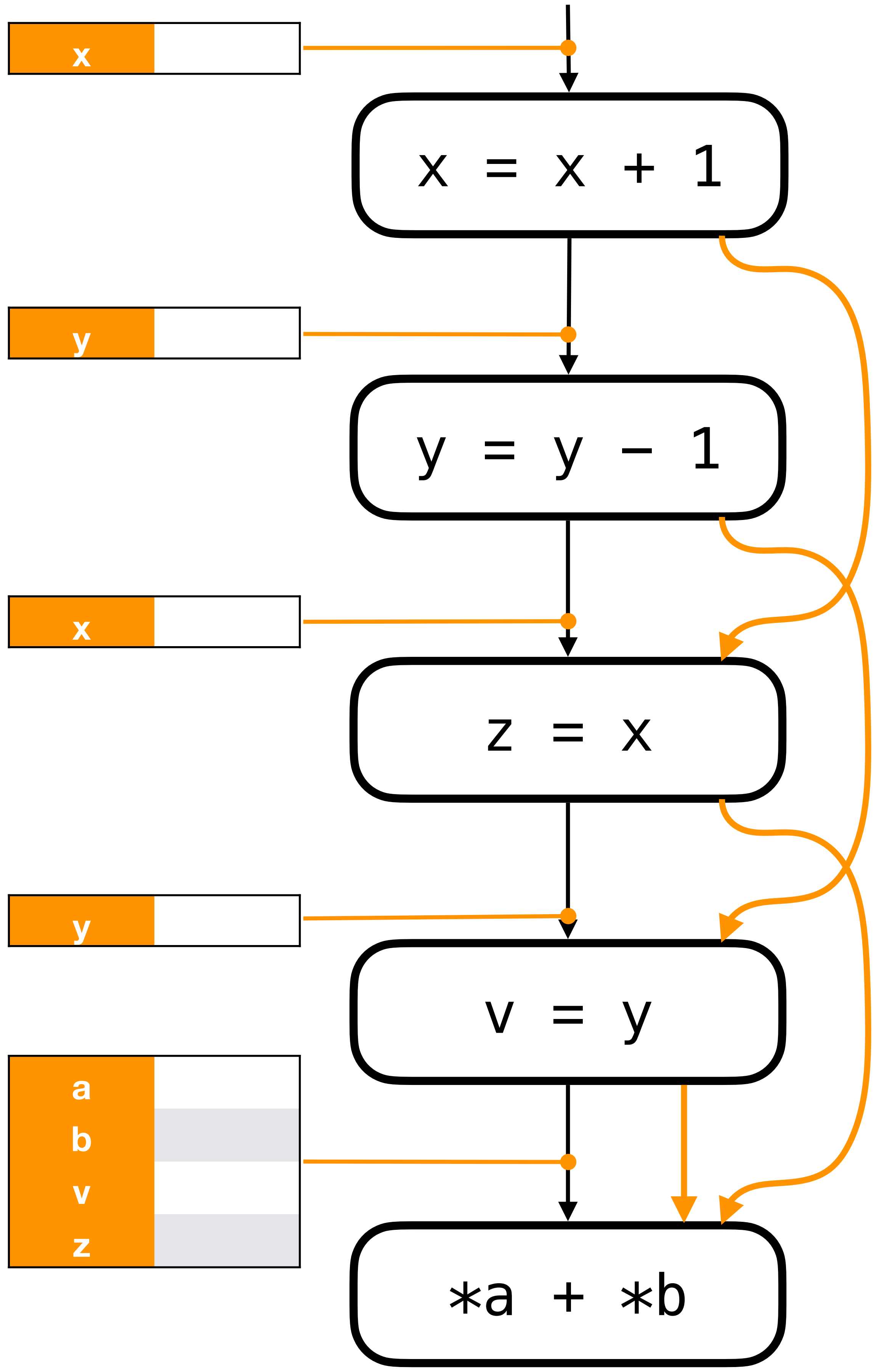
- Spatial sparsity



*Assume a and b point to z and v

Example

- Spatial + temporal sparsity



*Assume a and b point to z and v

Spatial Sparsity

- Only need the part of the memory **used** in that program portion
  - Otherwise, discard (so-called abstract garbage collection)
- The original abstract semantic function:

$$F^\sharp : (\mathbb{L} \to \mathbb{M}^\sharp) \to (\mathbb{L} \to \mathbb{M}^\sharp)$$

- The sparse version:

$$F^\sharp_{sparse} : (\mathbb{L} \to \mathbb{M}^\sharp_{sparse}) \to (\mathbb{L} \to \mathbb{M}^\sharp_{sparse})$$

$$\mathbb{M}^\sharp_{sparse} = \{m^\sharp \in \mathbb{M}^\sharp \mid dom(m^\sharp) = U^\sharp(l), l \in \mathbb{L}\} \cup \{\bot\}$$

Abstract locations to be used for each label

Temporal Sparsity

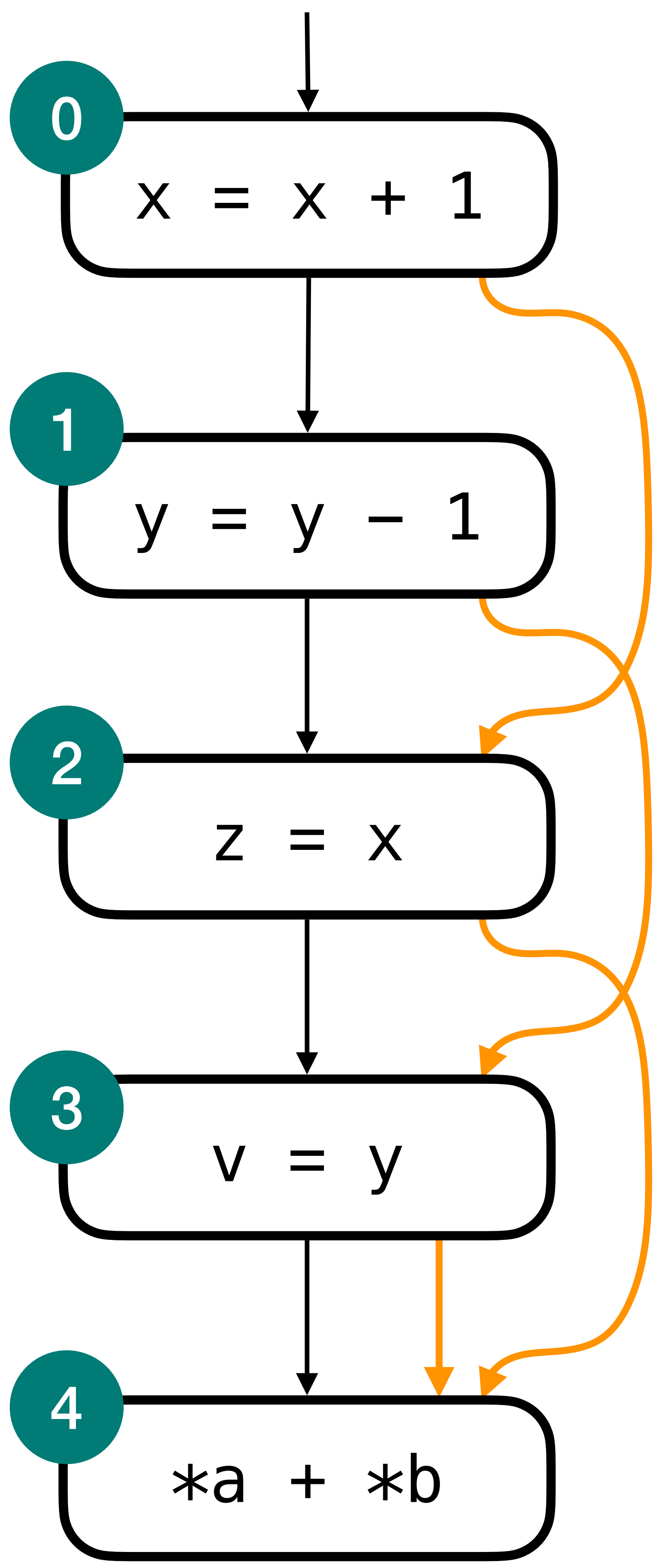
- Follow the **semantic dependency** to directly deliver the memory effect
  - Not blindly following the syntactic control-flow
- **Def-use chain**: for each label, defined locations are directly passed to its use labels

$$\langle l, m^\sharp \rangle \hookrightarrow^\sharp_{sparse} \langle l', m^{\sharp\prime} \rangle$$

Directly propagate defined locations to the use points

Example

- Def-use chain



$D^\sharp(0) = \{x\}$

$U^\sharp(0) = \{x\}$

$D^\sharp(1) = \{y\}$

$U^\sharp(1) = \{y\}$

$D^\sharp(2) = \{z\}$

$U^\sharp(2) = \{x\}$

$D^\sharp(3) = \{v\}$

$U^\sharp(3) = \{y\}$

$D^\sharp(4) = \{\}$

$U^\sharp(4) = \{a, b, v, z\}$

*Assume a and b point to z and v

Def-Use Chain

- How to formally define **semantic def and use sets**?
- Def and Use:

$$D^\sharp(l) = \{x \mid \exists m^\sharp \sqsubseteq \boxed{\mathbf{lfp}F^\sharp}(l).\ m^\sharp(x) \neq m^{\sharp\prime}(x), \langle l, m^\sharp\rangle \hookrightarrow^\sharp \langle \_, m^{\sharp\prime}\rangle\}$$
$$U^\sharp(l) = \{x \mid \exists m^\sharp \sqsubseteq \boxed{\mathbf{lfp}F^\sharp}(l).\ m_1^\sharp|_{D^\sharp(l)} \neq m_2^\sharp|_{D^\sharp(l)}, \langle l, m^\sharp\rangle \hookrightarrow^\sharp \langle \_, m_1^\sharp\rangle, \langle l, m^\sharp\backslash_x\rangle \hookrightarrow^\sharp \langle \_, m_2^\sharp\rangle\}$$

- Def-use chain:

$$\begin{aligned} l_0 \overset{x}{\rightsquigarrow} l_n \iff & \\ & \langle l_0, \boxed{\mathbf{lfp}F^\sharp}(l_0)\rangle \hookrightarrow^\sharp \cdots \hookrightarrow^\sharp \langle l_n, \boxed{\mathbf{lfp}F^\sharp}(l_n)\rangle \\ & \wedge\ x \in D^\sharp(l_0)\ \wedge\ x \in U^\sharp(l_n)\ \wedge\ \forall i \in (0,n).\ x \notin D^\sharp(l_i) \end{aligned}$$

- Theorem:

$$\mathbf{lfp}F^\sharp = \mathbf{lfp}F^\sharp_{sparse}\ \text{modulo}\ D^\sharp$$

Computing Def-Use Chain

- The ideal def-use chain is available **only after** the main analysis
  - It requires the full abstract semantics of a given program
- But, we need def-use chain **before** the analysis to speed up
- Solution: compute **approximated def-use chain** by yet another analysis

Pre-Analysis

- A coarser (hence quicker) analysis than the main analysis
- Any sound approximation of the main analysis is eligible
- For example, the flow-insensitive version of the main analysis:

$$\mathbb{D}^\sharp = \mathbb{L} \to \mathbb{M}^\sharp \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \mathbb{D}^\sharp_{pre} = \mathbb{M}^\sharp$$

- Relationship between the pre and main analysis
  - The pre-analysis **controls the sparsity**, not the final precision

Precision-Preserving Def-Use Chain

- Safe def / use sets from pre-analysis must satisfy the following conditions:

1. The def and use sets from the pre-analysis over-approximate those of the original analysis:

$$\forall l \in \mathbb{L}.\ D^\sharp(l) \subseteq D^\sharp_{pre}(l) \quad \text{and} \quad U^\sharp(l) \subseteq U^\sharp_{pre}(l)$$

2. All spurious definitions from the pre-analysis are included in the use set from the pre-analysis:

$$\forall l \in \mathbb{L}.\ D^\sharp_{pre}(l) \setminus D^\sharp(l) \subseteq U^\sharp_{pre}(l)$$

- Approximated def-use chain by the pre-analysis

$$\begin{aligned} l_0 \overset{x}{\rightsquigarrow}_{pre} l_n \iff & \\ & \langle l_0, \mathbf{lfp} F^\sharp_{pre}(l_0)\rangle \hookrightarrow^\sharp \cdots \hookrightarrow^\sharp \langle l_n, \mathbf{lfp} F^\sharp_{pre}(l_n)\rangle \\ & \wedge\ x \in D^\sharp_{pre}(l_0)\ \wedge\ x \in U^\sharp_{pre}(l_n)\ \wedge\ \forall i \in (0, n).\ x \notin D^\sharp_{pre}(l_i) \end{aligned}$$

Example

- Def-use chain by the original analysis

$$\begin{tikzcd} x = \&y \arrow[r] \arrow[rr, bend left, "\{x\}"] & *p = \&z \arrow[r] & y = x \end{tikzcd}$$

$D^\sharp$	**{x}**	**{v, w}**	**{y}**
$U^\sharp$	**{}**	**{p, v, w}**	**{x}**

Example

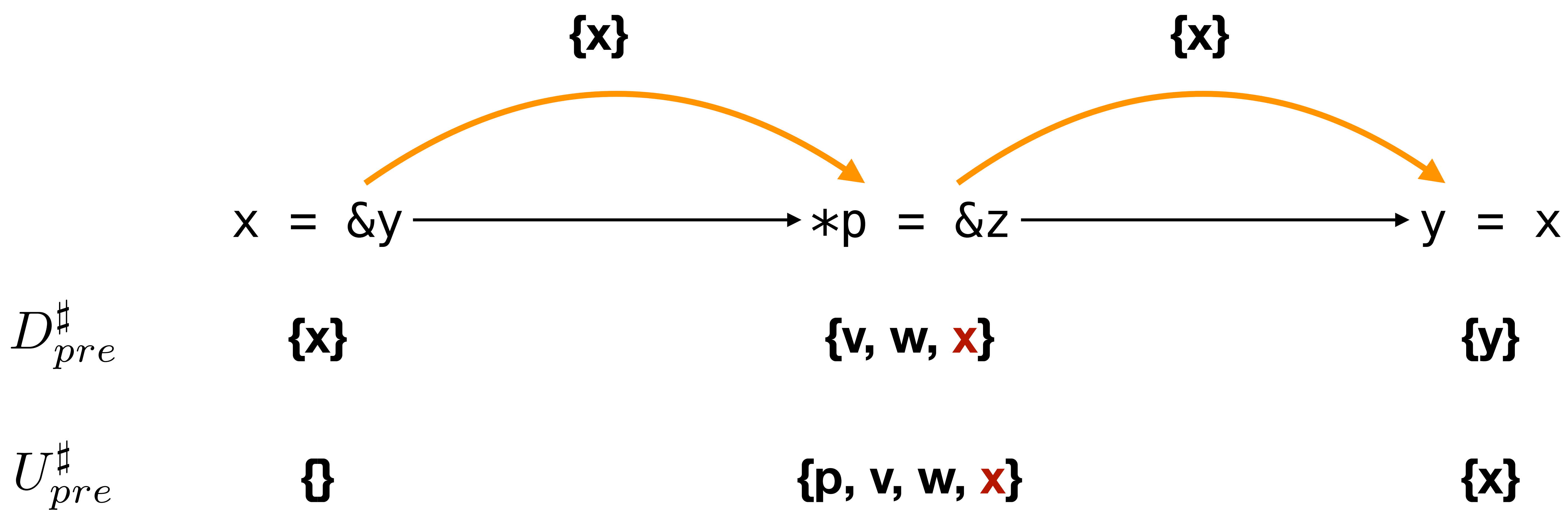
- Unsafe def-use edge by the pre-analysis

$$\begin{tikzcd} \texttt{x = \&y} \arrow[r] & \texttt{*p = \&z} \arrow[r] \arrow[r, bend left, "\{x\}"] & \texttt{y = x} \end{tikzcd}$$

	`x = &y`	`*p = &z`	`y = x`
$D^{\sharp}_{pre}$	**{x}**	**{v, w, x}**	**{y}**
$U^{\sharp}_{pre}$	**{}**	**{p, v, w}**	**{x}**

Example

- Safe def-use edge by the pre-analysis



Realizable Sparse Analysis

- The final sparse abstract semantic function:

$$\mathbb{M}^\sharp_{sparse^\sharp} = \{m^\sharp \in \mathbb{M}^\sharp \mid dom(m^\sharp) = U^\sharp_{pre}(l), l \in \mathbb{L}\} \cup \{\bot\}$$

$$F^\sharp_{sparse^\sharp} : (\mathbb{L} \to \mathbb{M}^\sharp_{sparse^\sharp}) \to (\mathbb{L} \to \mathbb{M}^\sharp_{sparse^\sharp})$$

- The sparse abstract state transitions (using approx. def-use chains):

$$\langle l, m^\sharp \rangle \hookrightarrow^\sharp_{sparse^\sharp} \langle l', m^{\sharp\prime} \rangle$$

- Theorem: $\mathbf{lfp} F^\sharp = \mathbf{lfp} F^\sharp_{sparse^\sharp} \text{ modulo } D^\sharp_{pre}$

Benchmarks

Program	LOC	Functions	Statements	Blocks	maxSCC	AbsLocs
gzip-1.2.4a	7K	132	6,446	4,152	2	1,784
bc-1.06	13K	132	10,368	4,731	1	1,619
tar-1.13	20K	221	12,199	8,586	13	3,245
less-382	23K	382	23,367	9,207	46	3,658
make-3.76.1	27K	190	14,010	9,094	57	4,527
wget-1.9	35K	433	28,958	14,537	13	6,675
screen-4.0.2	45K	588	39,693	29,498	65	12,566
a2ps-4.14	64K	980	86,867	27,565	6	17,684
sendmail-8.13.6	130K	756	76,630	52,505	60	19,135
nethack-3.3.0	211K	2,207	237,427	157,645	997	54,989
vim60	227K	2,770	150,950	107,629	1,668	40,979
emacs-22.1	399K	3,388	204,865	161,118	1,554	66,413
python-2.5.1	435K	2,996	241,511	99,014	723	51,859
linux-3.0	710K	13,856	345,407	300,203	493	139,667
gimp-2.6	959K	11,728	1,482,230	286,588	2	190,806
ghostscript-9.00	1,363K	12,993	2,891,500	342,293	39	201,161

*Oh et al., Design and Implementation of Sparse Global Analyses for C-like Languages, PLDI'12

Practical Impact

Programs	Interval$_{\text{vanilla}}$		Interval$_{\text{base}}$		Spd$\uparrow_1$	Mem$\downarrow_1$			Interval$_{\text{sparse}}$				Spd$\uparrow_2$	Mem$\downarrow_2$
	Time	Mem	Time	Mem			Dep	Fix	Total	Mem	$\hat{\mathrm{D}}(c)$	$\hat{\mathrm{U}}(c)$		
gzip-1.2.4a	772	240	14	65	55 x	73 %	2	1	3	63	2.4	2.5	5 x	3 %
bc-1.06	1,270	276	96	126	13 x	54 %	4	3	7	75	4.6	4.9	14 x	40 %
tar-1.13	12,947	881	338	177	38 x	80 %	6	2	8	93	2.9	2.9	42 x	47 %
less-382	9,561	1,113	1,211	378	8 x	66 %	27	6	33	127	11.9	11.9	37 x	66 %
make-3.76.1	24,240	1,391	1,893	443	13 x	68 %	16	5	21	114	5.8	5.8	90 x	74 %
wget-1.9	44,092	2,546	1,214	378	36 x	85 %	8	3	11	85	2.4	2.4	110 x	78 %
screen-4.0.2	∞	N/A	31,324	3,996	N/A	N/A	724	43	767	303	53.0	54.0	41 x	92 %
a2ps-4.14	∞	N/A	3,200	1,392	N/A	N/A	31	9	40	353	2.6	2.8	80 x	75 %
sendmail-8.13.6	∞	N/A	∞	N/A	N/A	N/A	517	227	744	678	20.7	20.7	N/A	N/A
nethack-3.3.0	∞	N/A	∞	N/A	N/A	N/A	14,126	2,247	16,373	5,298	72.4	72.4	N/A	N/A
vim60	∞	N/A	∞	N/A	N/A	N/A	17,518	6,280	23,798	5,190	180.2	180.3	N/A	N/A
emacs-22.1	∞	N/A	∞	N/A	N/A	N/A	29,552	8,278	37,830	7,795	285.3	285.5	N/A	N/A
python-2.5.1	∞	N/A	∞	N/A	N/A	N/A	9,677	1,362	11,039	5,535	108.1	108.1	N/A	N/A
linux-3.0	∞	N/A	∞	N/A	N/A	N/A	26,669	6,949	33,618	20,529	76.2	74.8	N/A	N/A
gimp-2.6	∞	N/A	∞	N/A	N/A	N/A	3,751	123	3,874	3,602	4.1	3.9	N/A	N/A
ghostscript-9.00	∞	N/A	∞	N/A	N/A	N/A	14,116	698	14,814	6,384	9.7	9.7	N/A	N/A

Non-sparse

Spatial Sparsity

Spatial + Temporal Sparsity

*Oh et al., Design and Implementation of Sparse Global Analyses for C-like Languages, PLDI'12

Summary

- Sparse analysis: a **general framework** for reducing the analysis cost while preserving the precision
  - Input: sound yet scalability-unattended static analysis
- Key idea: **"Right part at right moment"**
- Based on semantic **def-use chain** rather than syntactic control-flow
- Approximated def-use chain by **pre-analysis**
  - **Safety conditions** on def and use