CSAIL

OUR APPROACH

THE GROMOV-WASSERSTEIN DISTANCE

- Generalizes OT to the non-registered case
- Main idea: compare **distances** instead of absolute **positions**

$$\mathrm{GW}(\mathbf{C}, \mathbf{C}', \mathbf{p}, \mathbf{q}) = \min_{\Gamma \in \Pi(\mathbf{p}, \mathbf{q})} \sum_{i,j,k,l} \mathbf{L}_{ijkl} \Gamma_{ij} \Gamma_{kl}$$

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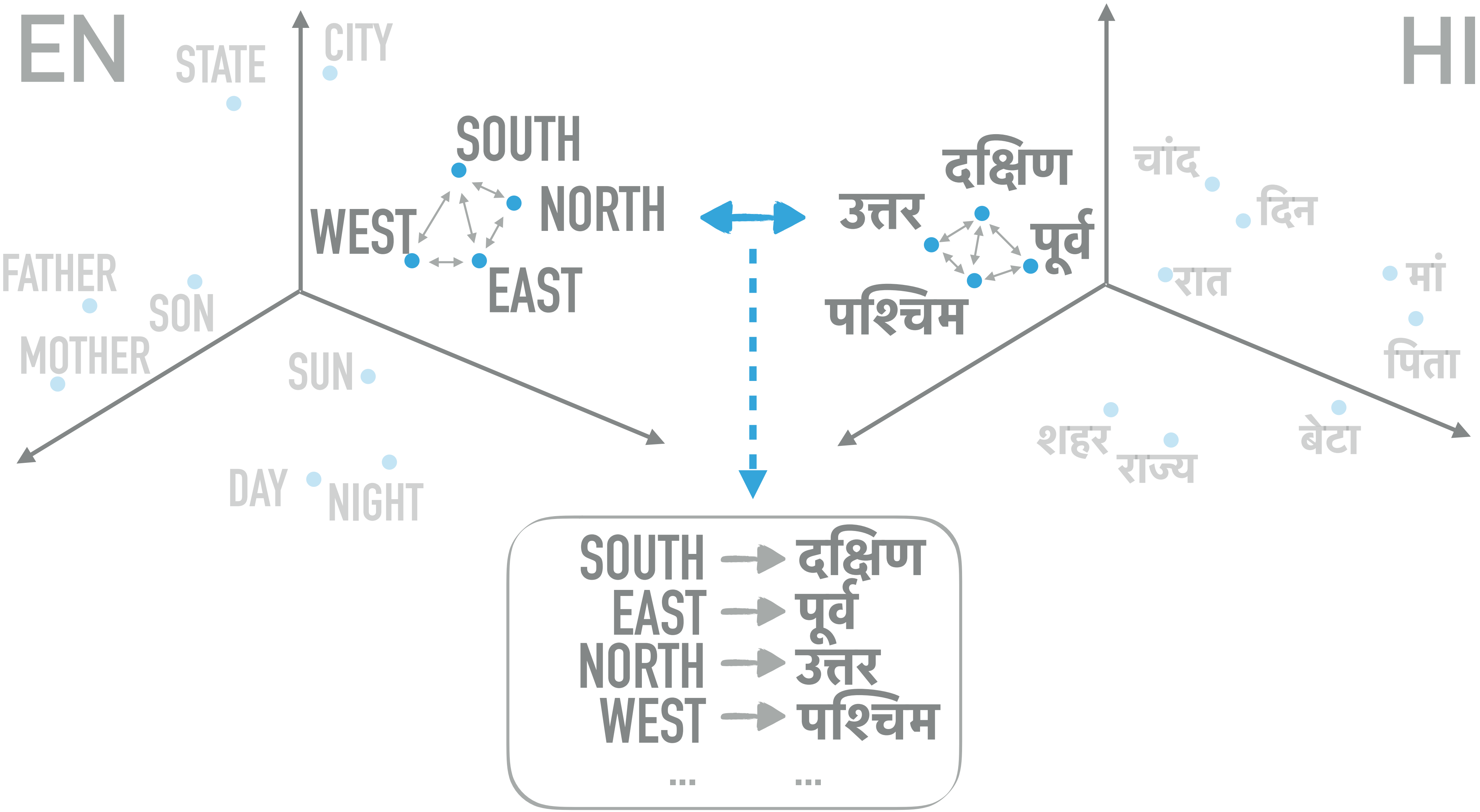
[Mémoli, 2011; Peyré et al. 2016]


EN
STATE
CITY
SOUTH
NORTH
WEST
EAST
FATHER
SON
MOTHER
SUN
DAY
NIGHT
HI
दक्षिण
उत्तर
पूर्व
पश्चिम
चांद
दिन
रात
मां
पिता
शहर
राज्य
बेटा
SOUTH → दक्षिण
EAST → पूर्व
NORTH → उत्तर
WEST → पश्चिम
... ...

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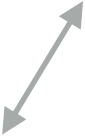
SOUTH
NORTH
WEST
EAST

NORTH

SOUTH



EAST

WEST

THEOREM (MÉMOLI, 2011)

With the choice $\mathcal{L} := L_2$, $\mathrm{GW}^{\frac{1}{2}}$ is a distance on the space of metric measure spaces

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OPTIMIZATION

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