





optimization

Invariant not



Algorithm\*: alternating minimization on  $\mathbf{I}, \mathbf{P}$

$$\min_{\Gamma \in \Pi(\mathbf{a}, \mathbf{b})} \min_{P \in \mathcal{F}_p} \sum_{ij} \Gamma_{ij} d(\mathbf{x}_i, P \mathbf{y}_j)$$

\*various other optimization approaches investigated in Thesis



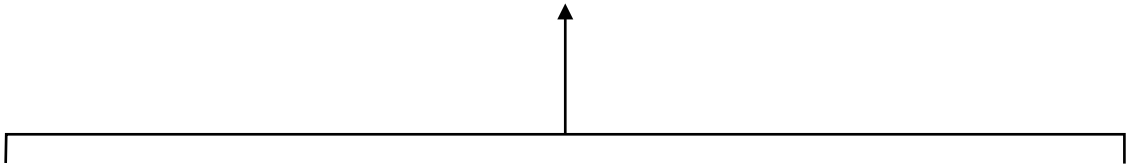


Repeat until  
convergence



→ p-Schatten norm

[Theorem] This has a closed form solution under simple conditions!



For a fixed  $\Gamma$ , obtain  $\mathbf{P}$  via SVD

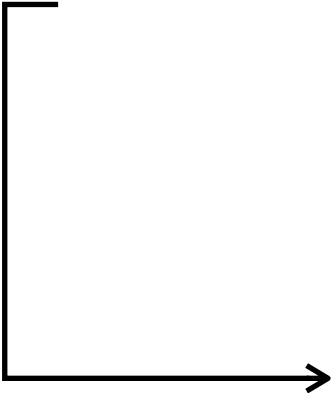
For a fixed  $\mathbf{P}$ , obtain (approximate)  $\Gamma$  via Sinkhorn-Knopp



This is still a classic OT problem on  $\Gamma$







# Invariant OT Optimization

[Theorem] This has a closed form solution under simple conditions!

$$\min_{\Gamma \in \Pi(\mathbf{a}, \mathbf{b})} \min_{\mathbf{P} \in \mathcal{F}_p} \sum_{ij} \Gamma_{ij} d(\mathbf{x}_i, \mathbf{P} \mathbf{y}_j)$$

$\downarrow$  This is still a classic OT problem on  $\Gamma$

$\rightarrow$  p-Schatten norm

Algorithm\*: alternating minimization on  $\Gamma, \mathbf{P}$

- $\rightarrow$  For a fixed  $\Gamma$ , obtain  $\mathbf{P}$  via SVD
- $\rightarrow$  For a fixed  $\mathbf{P}$ , obtain (approximate)  $\Gamma$  via Sinkhorn-Knopp

Repeat until  
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# Invariant OT

## toy dataset experiments

Classic OT

$\ell_{\infty}$  invariant OT