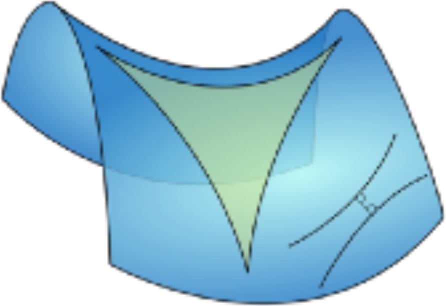


In a nutshell

Hyperbolic Embeddings

56

- Hyperbolic geometry: non-Euclidean geometry, studies spaces of constant negative curvature

- Various models, here we focus on the **Poincaré disk**

- Some Properties:

- volume grows exponentially with radius (like in trees!)

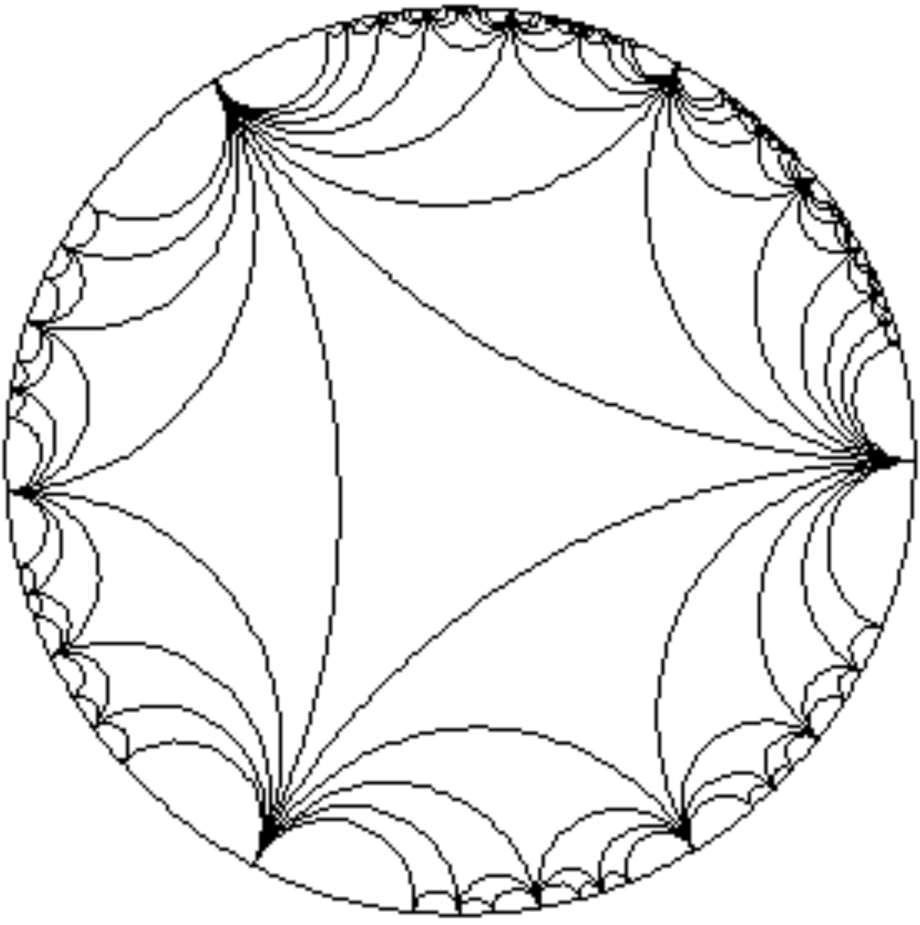
- geodesics are circles perpendicular to boundary

- can embed trees with arbitrary low distortions in (!= Euclidean space) [Gromov 1987, De Sa 2018]

$$d_{\mathbb{D}} = \cosh^{-1}\left(1 - 2\frac{\|x-y\|^2}{(1-\|x\|^2)(1-\|y\|^2)}\right)$$

Image Credits: (1) MathWorld-Wolfram, (2) By Trevorgoodchild at English Wikipedia

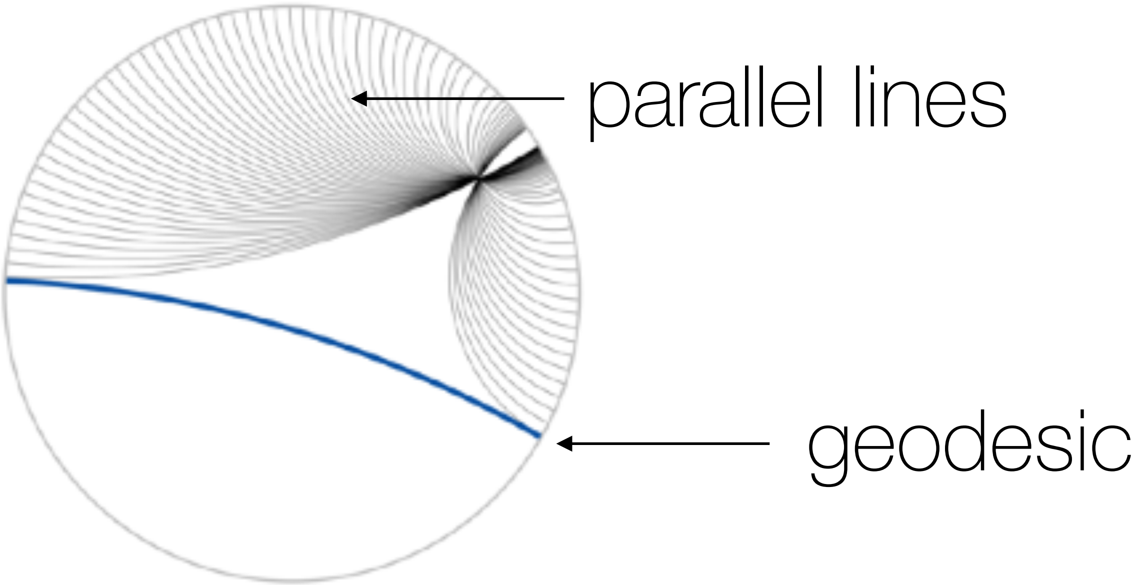
$$\mathbb{D}_d = \{x \in \mathbb{R}^d \; : \; \|x\| < 1\}$$



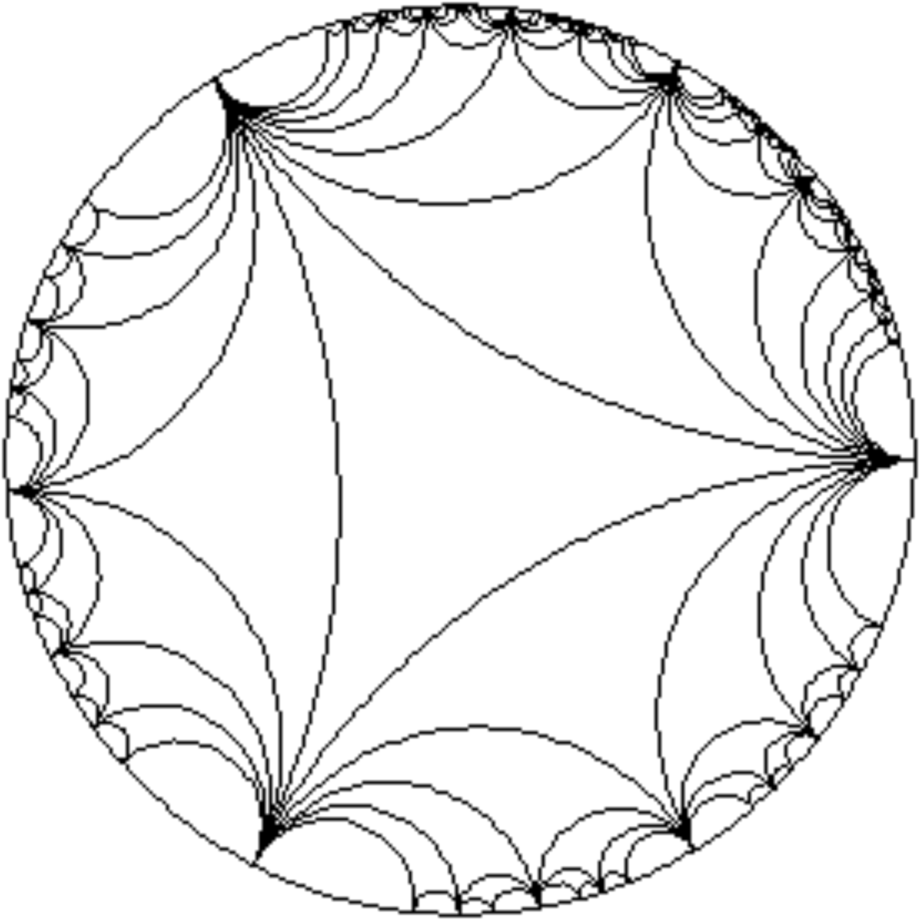
Manifold:

Distance:

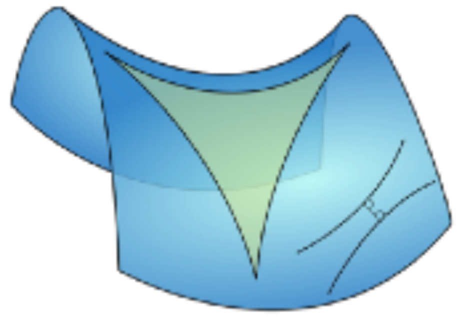
$\mathbb{D}_2$


parallel lines
geodesic

Poincaré disk $\mathbb{D}_2$



Hyperbolic Embeddings
In a nutshell



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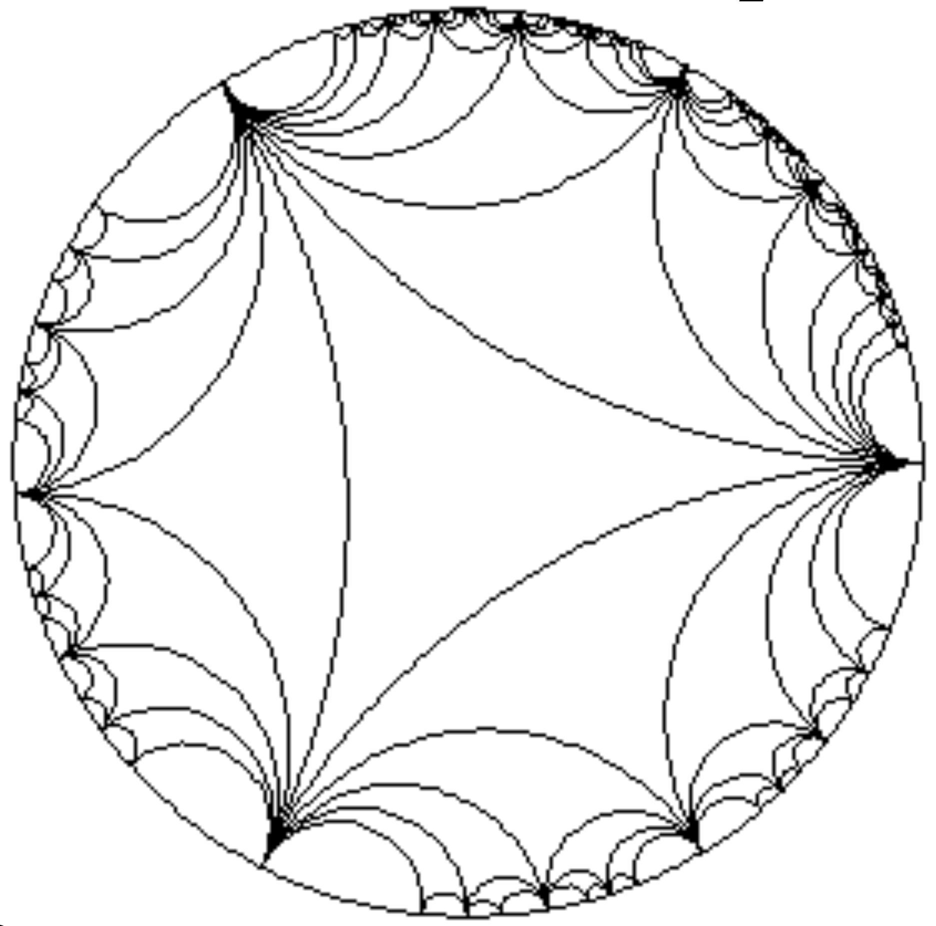
Poincaré disk $\mathbb{D}_2$



Manifold: $\mathbb{D}_d = \{x \in \mathbb{R}^d \; : \; \|x\| < 1\}$

Distance: $d_{\mathbb{D}} = \cosh^{-1}\left(1 - 2\frac{\|x-y\|^2}{(1-\|x\|^2)(1-\|y\|^2)}\right)$

- Some Properties:
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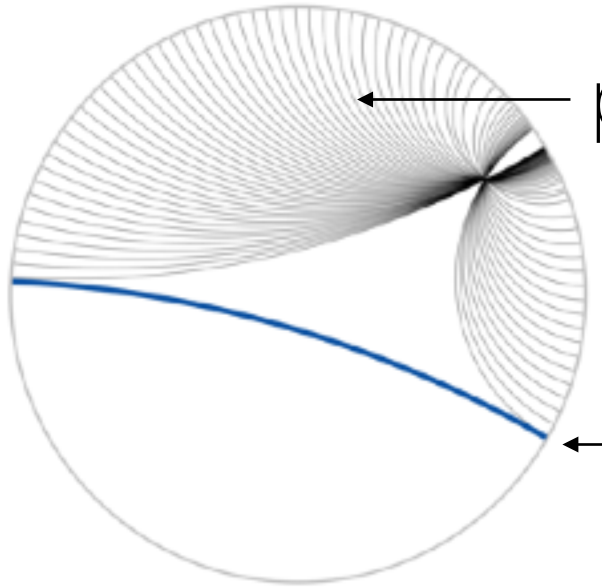

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Hyperbolic Embeddings

In a nutshell