CSAIL

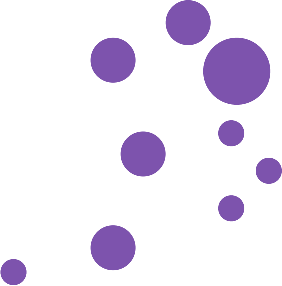
BACKGROUND

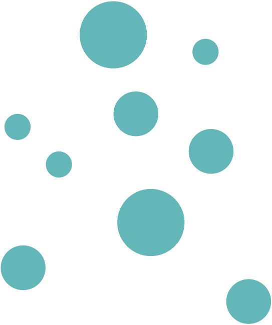
OPTIMAL TRANSPORT

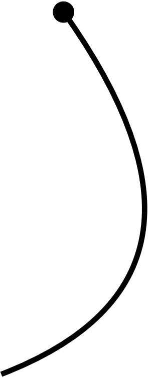
Discrete Optimal Transport

- Idea 1 [Monge]:

- Idea 2 [Kantorovich]: allow "mass splitting" -

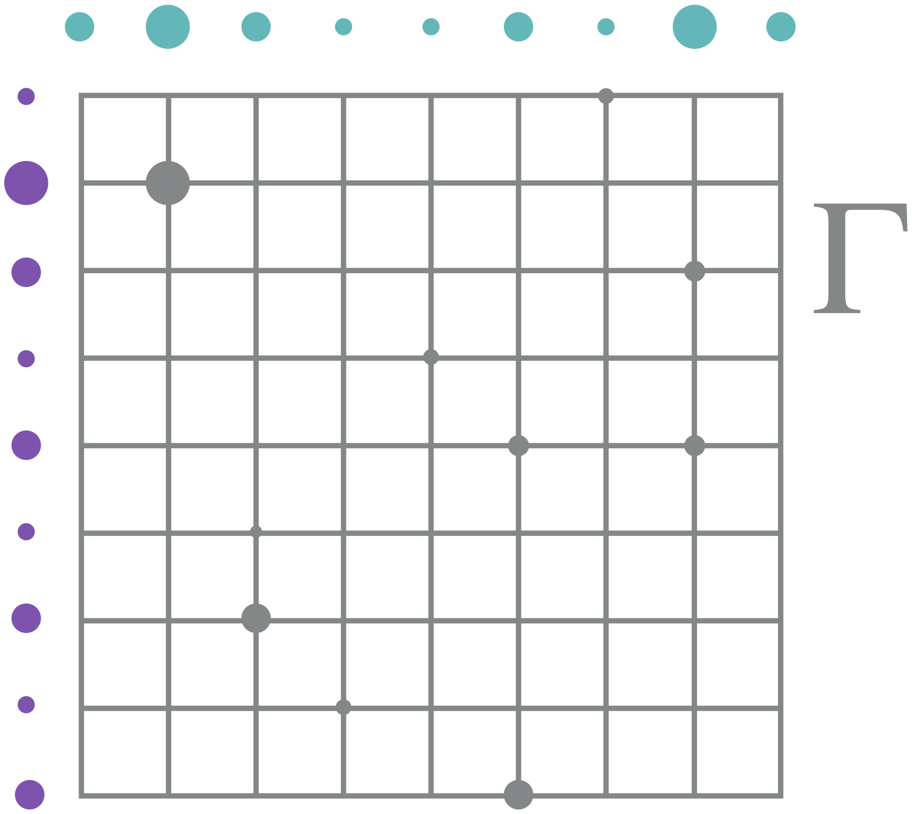






Hard! Solution might not exist

Discrete OT
a.k.a Earth Mover's Distance


Γ

$$\{(\mathbf{x}^{(i)}, p_i)\}_{i=1}^{n}$$

$$\{(\mathbf{y}^{(j)}, q_j)\}_{j=1}^{m}$$

$$\overleftrightarrow{C(\mathbf{x}^{(i)}, \mathbf{y}^{(j)})}$$

$$\min_{T} \sum_{i} C\left(\mathbf{x}^{(i)}, T(\mathbf{x}^{(i)})\right)$$

$$\text{minimize} \quad \sum_{i,j} \Gamma_{ij} C_{ij}$$

$$\Gamma_{ij}$$

= HOW MUCH MASS IS MOVED FROM $\mathbf{x}^{(i)}$ TO $\mathbf{y}^{(j)}$

$$\Gamma \in \mathbb{R}^{n \times m}$$

Transport Plan

subject to

$$\sum_j \Gamma_{ij} = p_i \quad \forall i$$

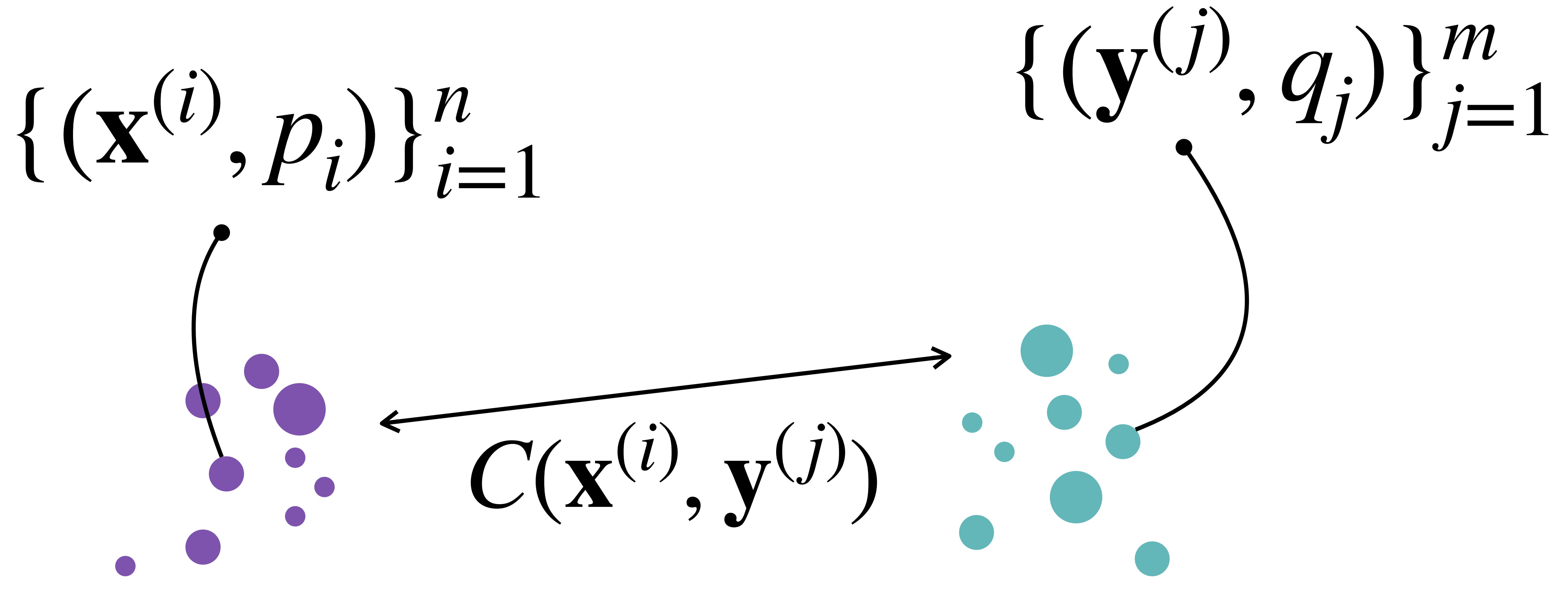
$$\sum_i \Gamma_{ij} = q_j \quad \forall j$$

$$q_1 \quad \cdots \quad q_j \quad \cdots \quad q_m$$

$$
\begin{matrix}
p_1 \\
\vdots \\
p_i \\
\vdots \\
p_n
\end{matrix}
$$

OPTIMAL TRANSPORT

- Discrete Optimal Transport



- Idea 1 [Monge]: $$\min_{T} \sum_{i} C\left(\mathbf{x}^{(i)}, T(\mathbf{x}^{(i)})\right)$$

HARD! SOLUTION MIGHT NOT EXIST

- Idea 2 [Kantorovich]: allow "mass splitting" -

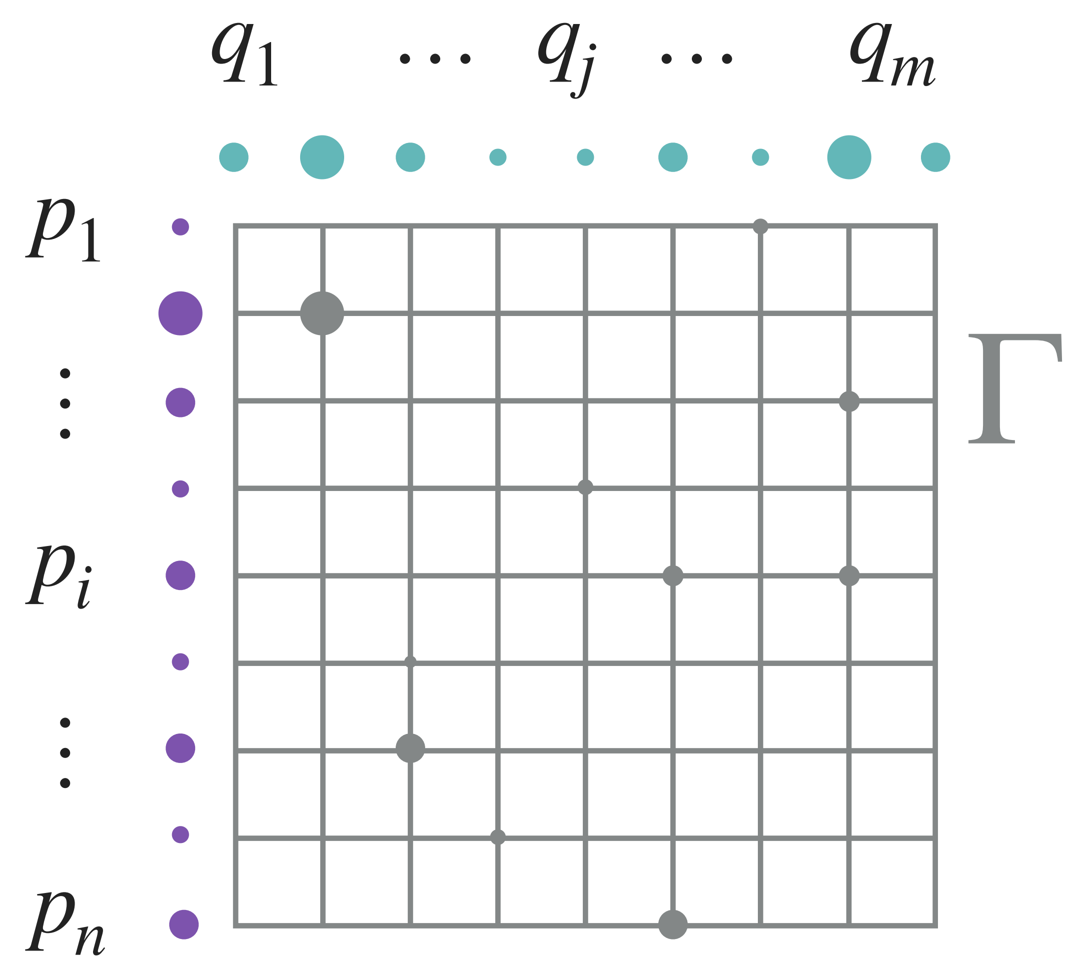
$$\text{minimize} \quad \sum_{i,j} \Gamma_{ij} C_{ij}$$

$$\text{subject to} \quad \sum_{j} \Gamma_{ij} = p_i \quad \forall i$$

$$\sum_{i} \Gamma_{ij} = q_j \quad \forall j$$

$\Gamma \in \mathbb{R}^{n \times m}$

Transport Plan



DISCRETE OT
A.K.A EARTH MOVER'S DISTANCE

OPTIMAL TRANSPORT BETWEEN WORD EMBEDDINGS