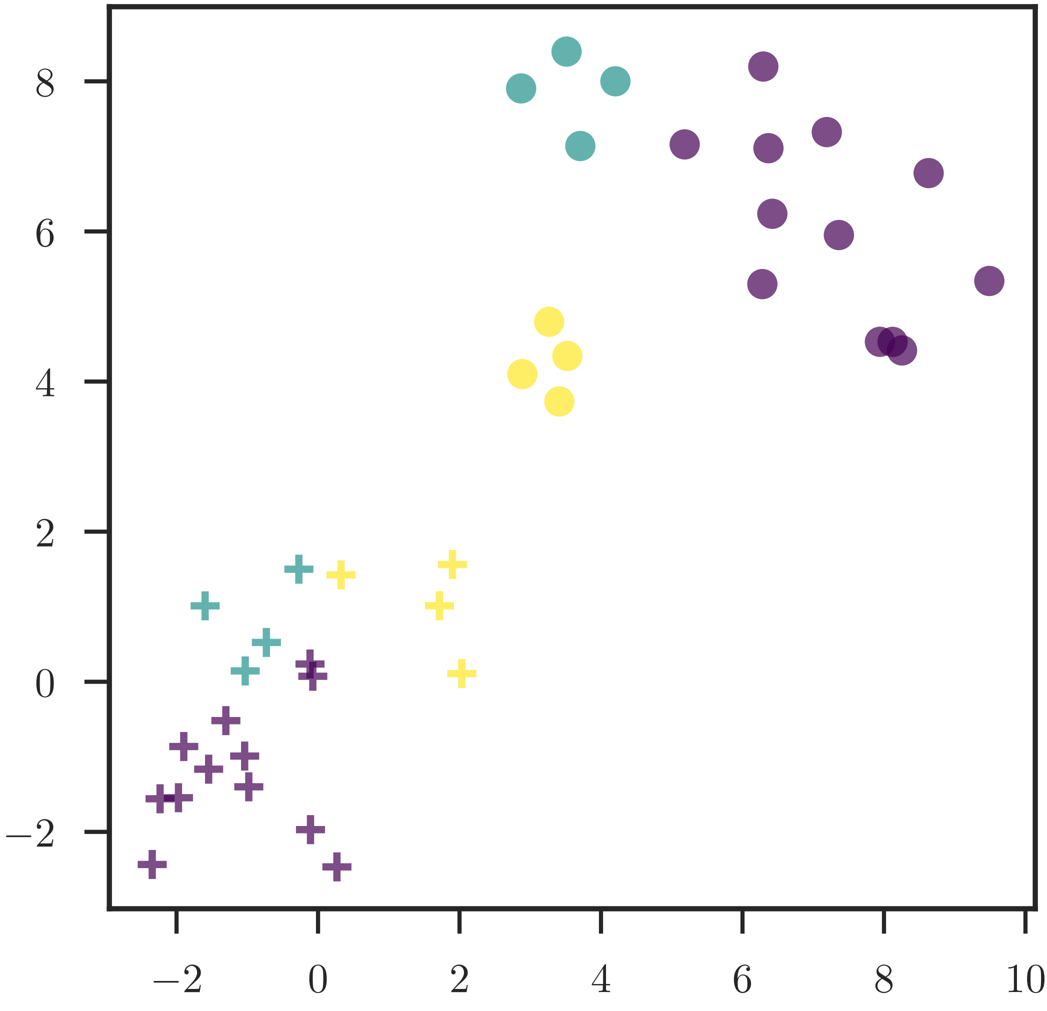
Synthetic Experiments


8
6
4
2
0
−2
−2
0
2
4
6
8
10

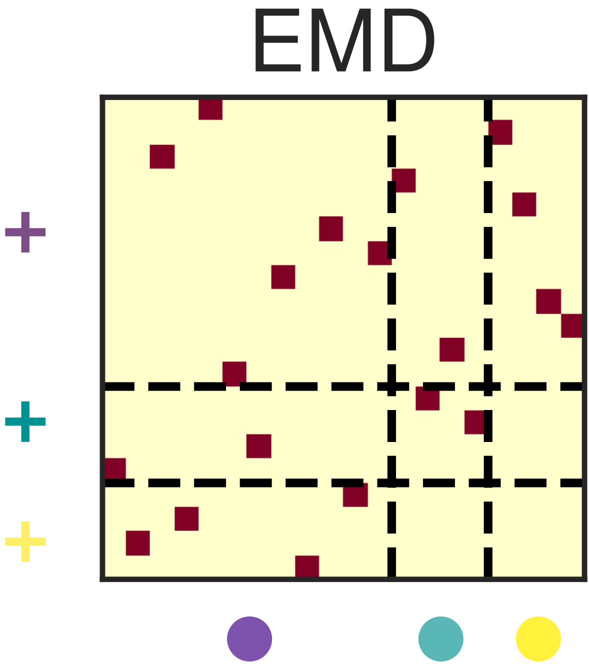
Structured OT

77

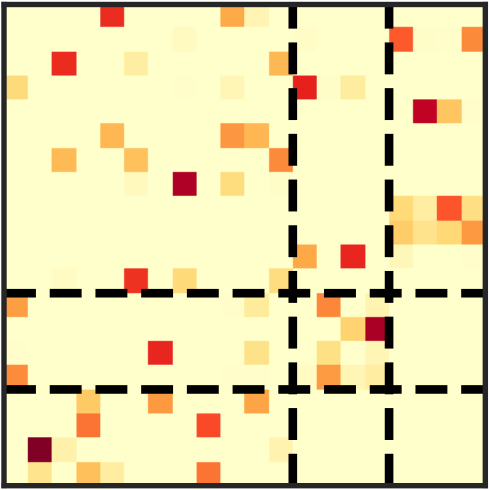
- Two point clouds, three clusters

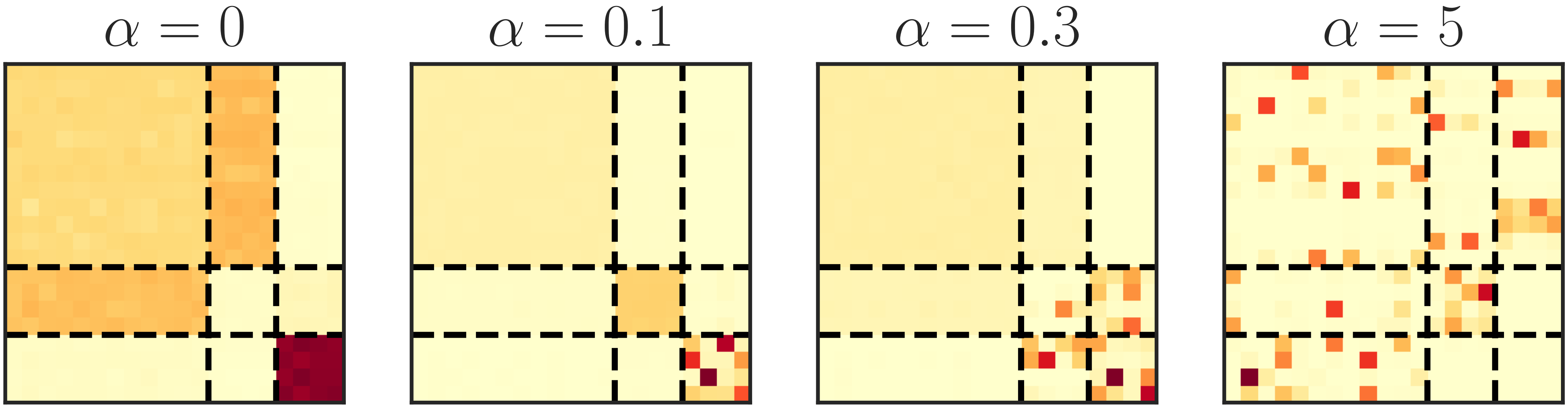
- Groups:

- Matching cost:


EMD

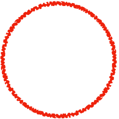
Entropy

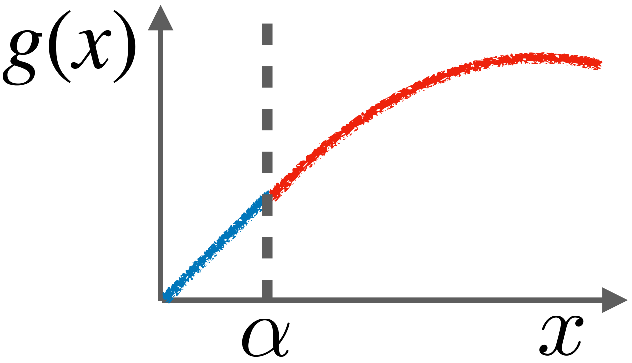



$\alpha = 0$
$\alpha = 0.1$
$\alpha = 0.3$
$\alpha = 5$

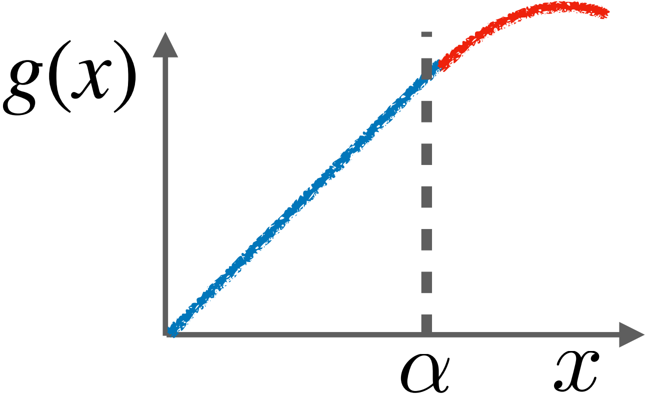
$$G_{(+,\bullet)} = \{(i, j) \in E \mid i \in U_{+},\ j = V_{\bullet}\}$$

$$F(M) = \sum_{k}^{K} g\left(\sum_{(i,j) \in M \cup G_k} C_{ij} \right)$$





$g(x)$
α
x



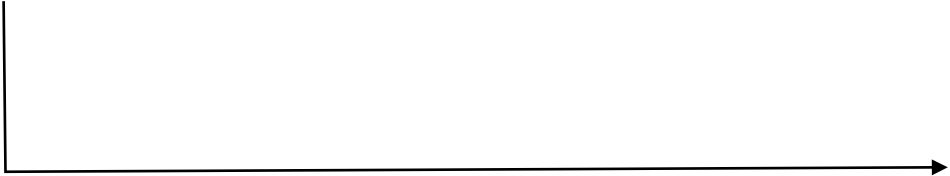
$g(x)$
α
x

Stronger cluster enforcement

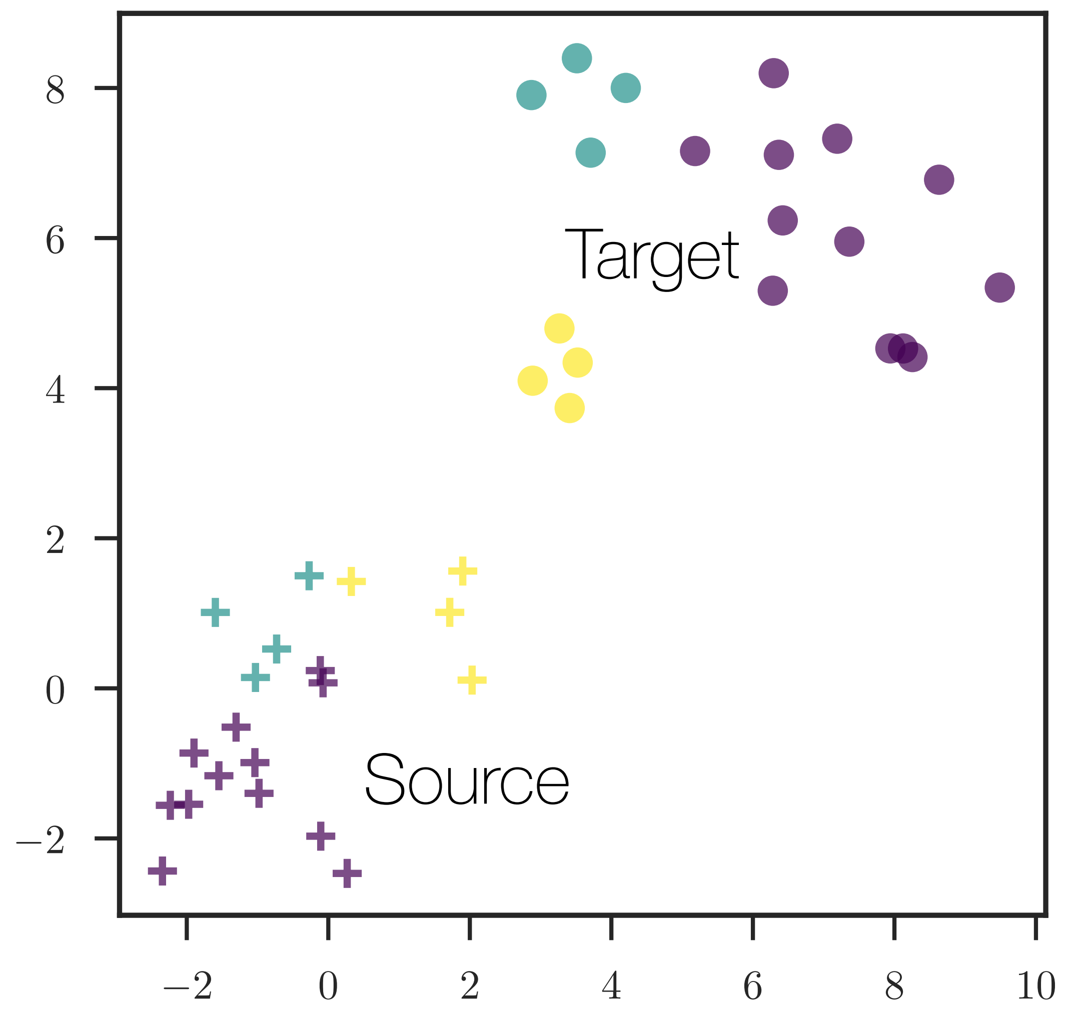
Recovers Entropy Sol

Source

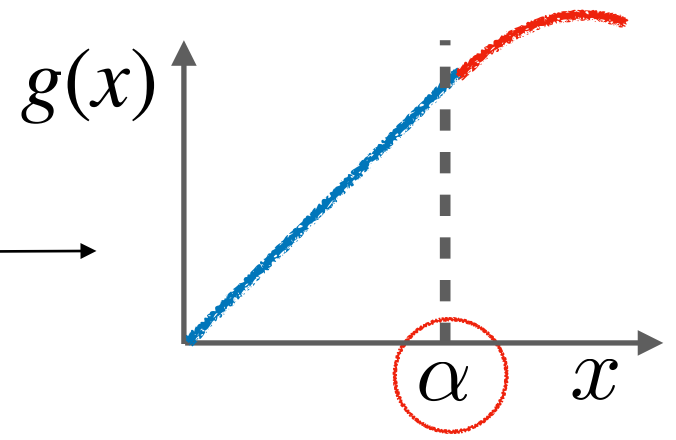
Target

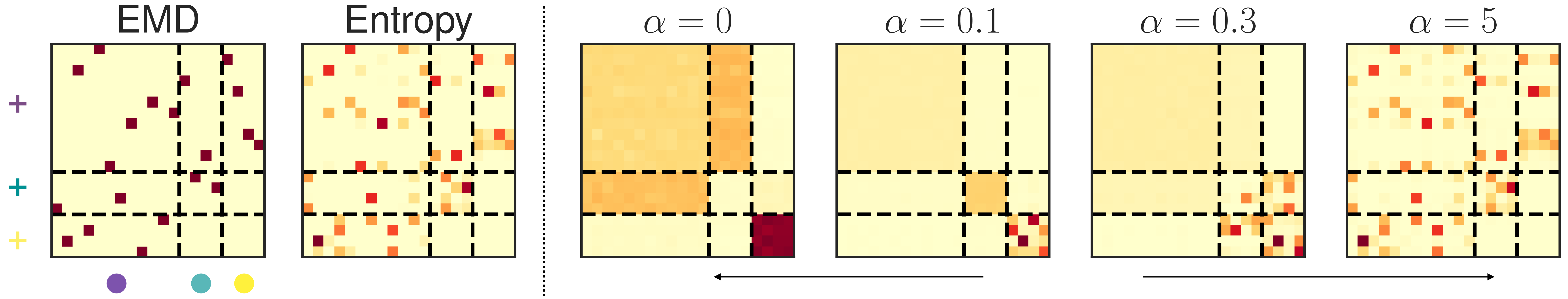


Structured OT
Synthetic Experiments



- Two point clouds, three clusters
- Groups: $G_{(+,\bullet)} = \{(i,j) \in E \mid i \in U_{+},\ j = V_{\bullet}\}$
- Matching cost: $F(M) = \sum_{k}^{K} g\left(\sum_{(i,j)\in M \cup G_k} C_{ij} \right)$





Stronger cluster enforcement

Recovers Entropy Sol

Structured OT
Optimization Sub-Routines

Subgradients of f.

$$\partial f(\Gamma) = \arg\max_{\kappa \in \mathscr{B}_F} \langle \Gamma, \kappa \rangle$$

i.e. linear optimization over the base polytope

solve by Edmond's greedy algorithm in $O(N \log N)$

Projections onto transport polytope

Mirror map: $\Phi_\Pi(\Gamma) = \sum_{ij} \Gamma_{ij} \log(\Gamma_{ij})$

Projection Step: $P_{\Pi(\mathbf{a},\mathbf{b})}(w) = \arg\min_{\Gamma \in \Pi(\mathbf{a},\mathbf{b})} \mathrm{KL}(\Gamma \| w)$

Solve by Sinkhorn-Knopp

Projections onto base polytope

Mirror map: $\Phi_{\mathscr{B}_F}(\kappa) = \frac{1}{2}\|k\|_2^2$

Projection Step: $P_{\mathscr{B}_F}(w) = \arg\min_{\Gamma \in \Pi(\mathbf{a},\mathbf{b})} \|\kappa - w\|_2^2$

Solve by Fujishige Min-Norm-Point Algorithm $O(n^6)$

For our decomposable fun's, can solve in $O(n \log n)$