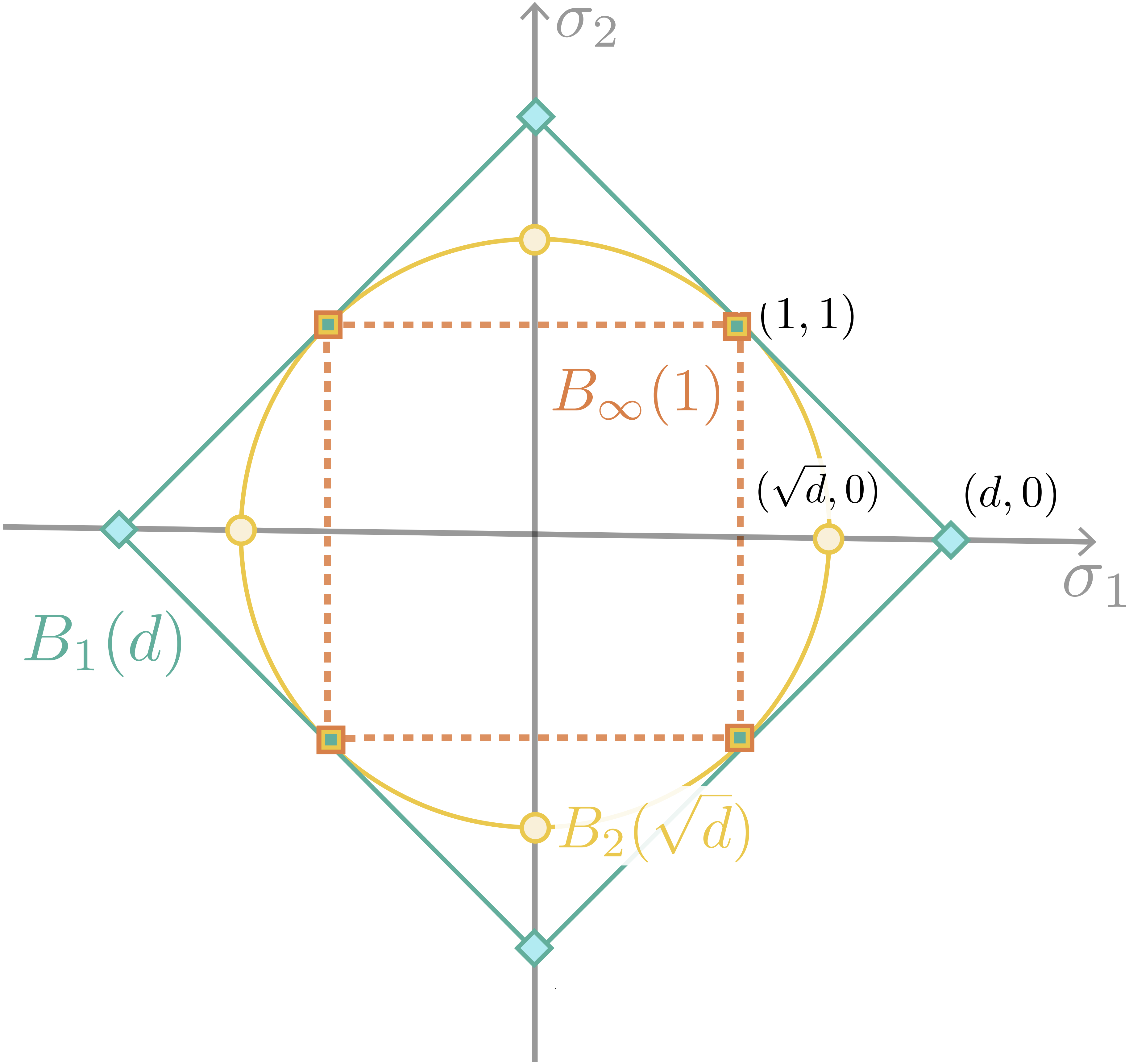
[redacted]

$$p = 1$$

$$p = \infty$$

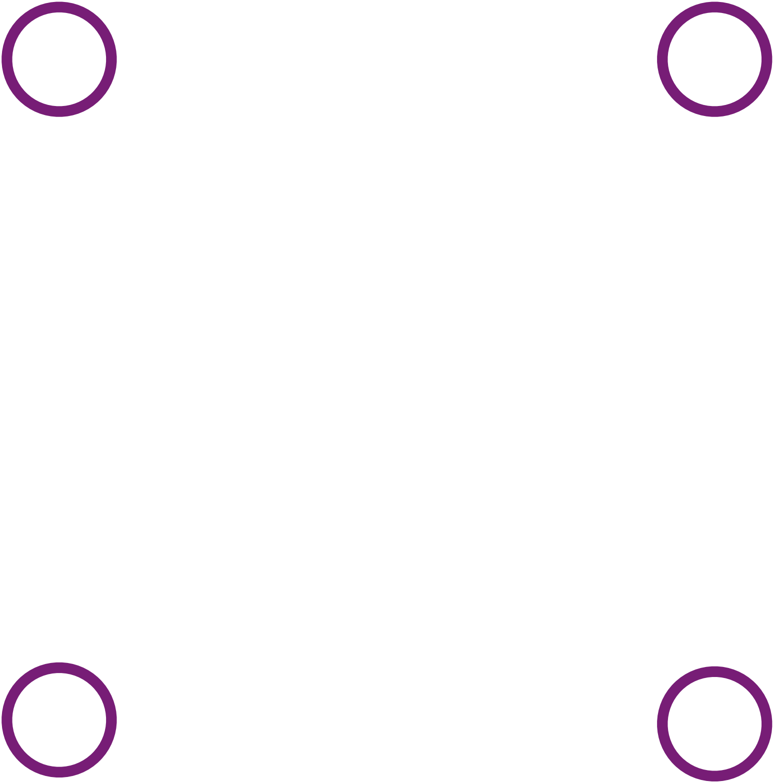


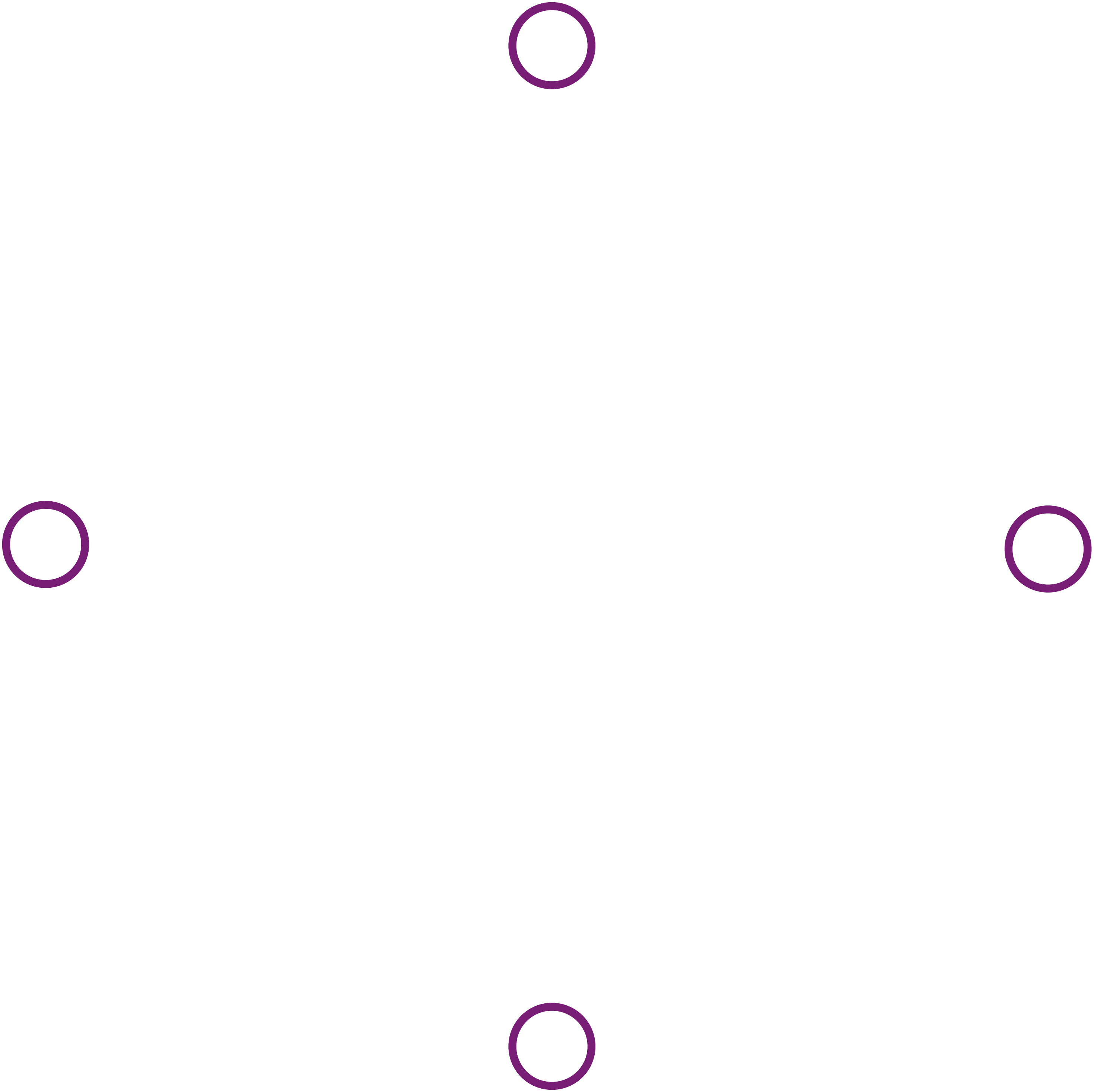
σ_2
$(1, 1)$
$B_\infty(1)$
$(\sqrt{d}, 0)$
$(d, 0)$
σ_1
$B_1(d)$
$B_2(\sqrt{d})$

$$B_\infty(1) \leftrightarrow \mathcal{F}_\infty$$

$$B_2(\sqrt{2}) \leftrightarrow \mathcal{F}_2$$

$$B_1(d) \leftrightarrow \mathcal{F}_1$$





$$\mathscr{F}_p = \left\{ \mathbf{P} \in \mathbb{R}^{d \times d} \mid \|\mathbf{P}\|_p \leq k_p \right\}$$

$$\|\mathbf{P}\|_p = \|\sigma(\mathbf{P})\|_p$$

with Schatten-balls

Invariance

40

- Idea: model invariances with linear operators with bounded Schatten-p norm

- Modeling advantage: clear interpretation

– : sparse spectra (projections)

– : uniform spectra (orthogonal)

- Algebraic convenience:

- Unitary invariance, sub-multiplicative

- Easy characterization via duality

Nuclear (p=1)

Frobenius (p=2)

Spectral (p=∞)

Invariance
with Schatten-balls

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  - $p = \infty$: uniform spectra (orthogonal)

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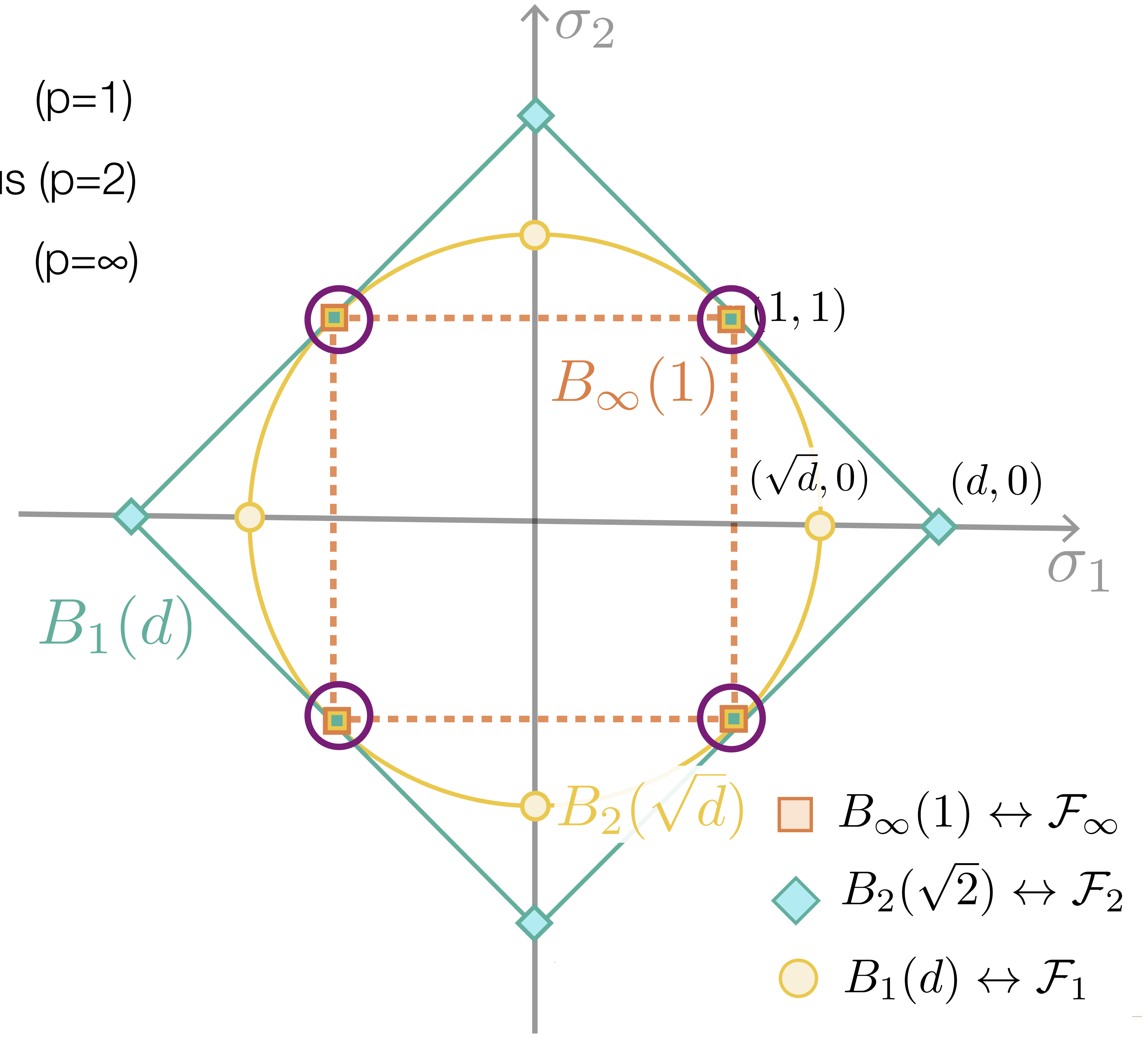

Invariant OT
Optimization

$$\min_{\Gamma \in \Pi(\mathbf{a},\mathbf{b})} \; \min_{\mathbf{P} \in \mathscr{F}_p} \; \sum_{ij} \Gamma_{ij} \, d(\mathbf{x}_i, \mathbf{P}\mathbf{y}_j)$$