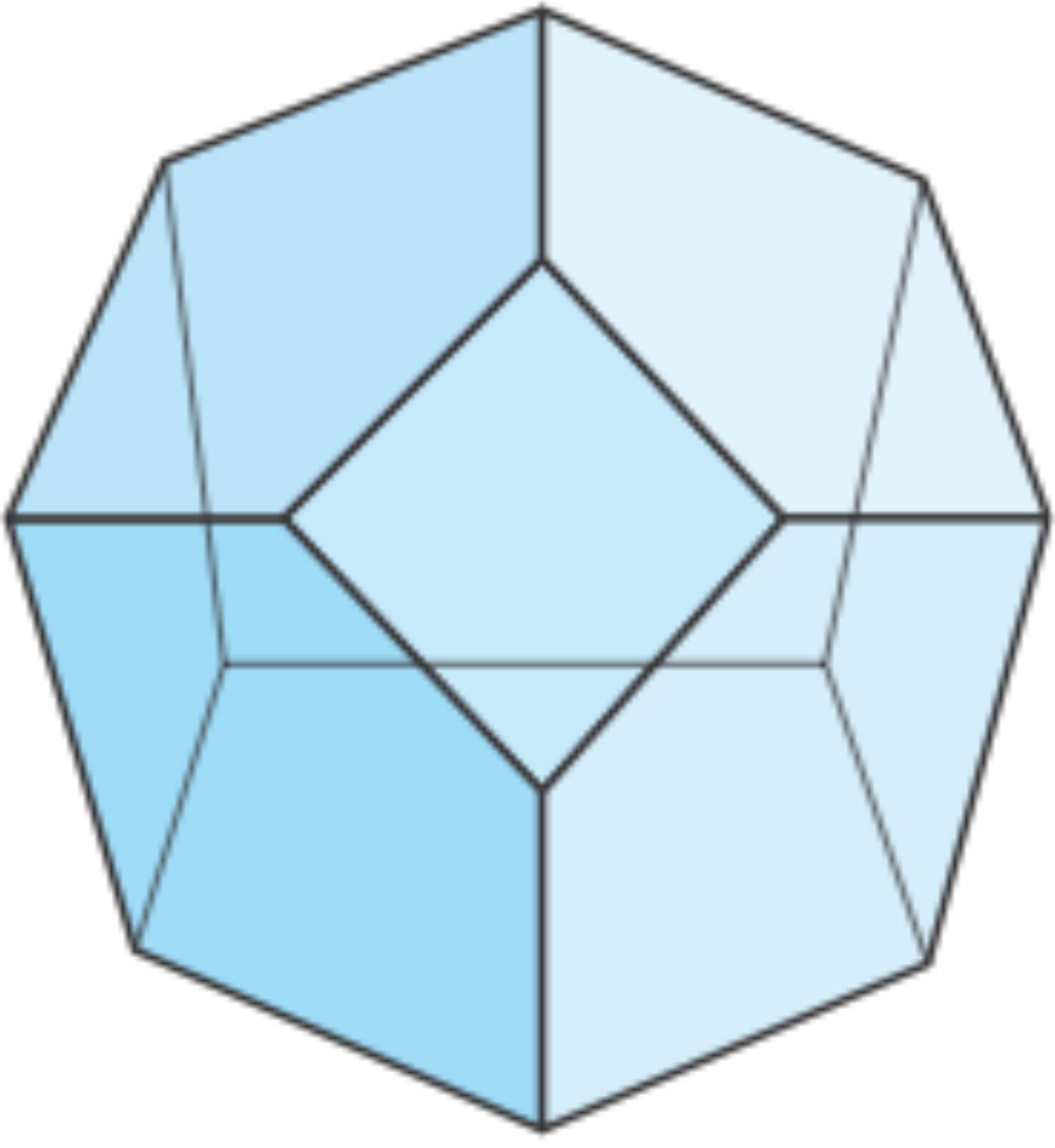
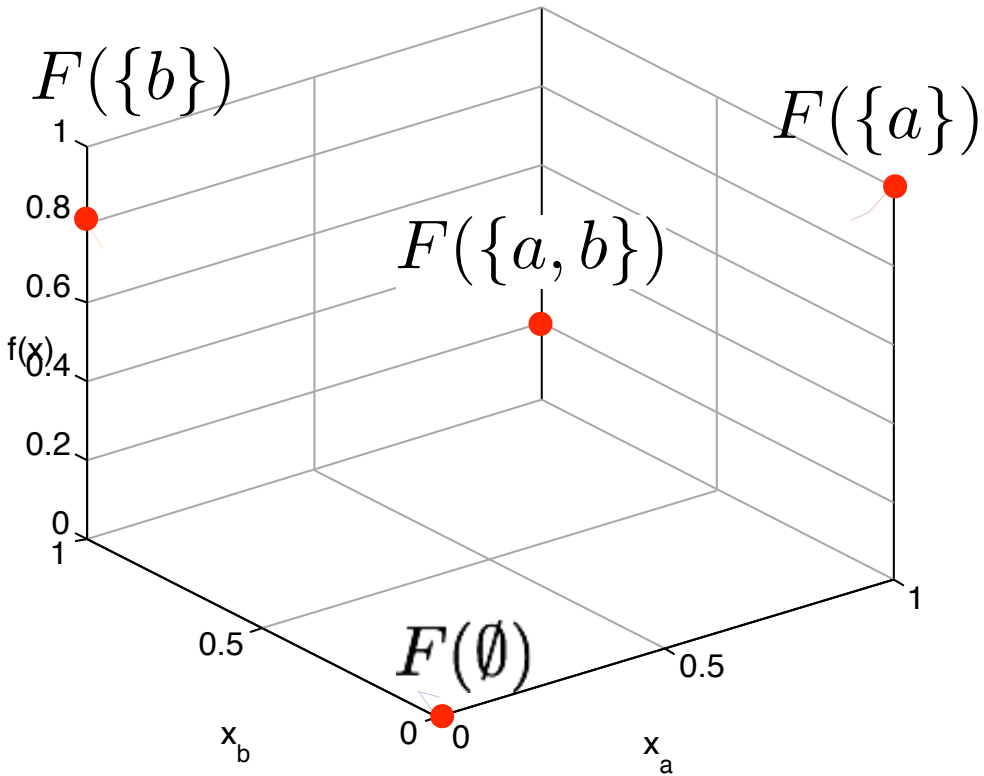
[redacted]

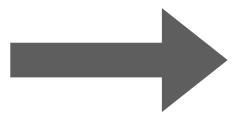


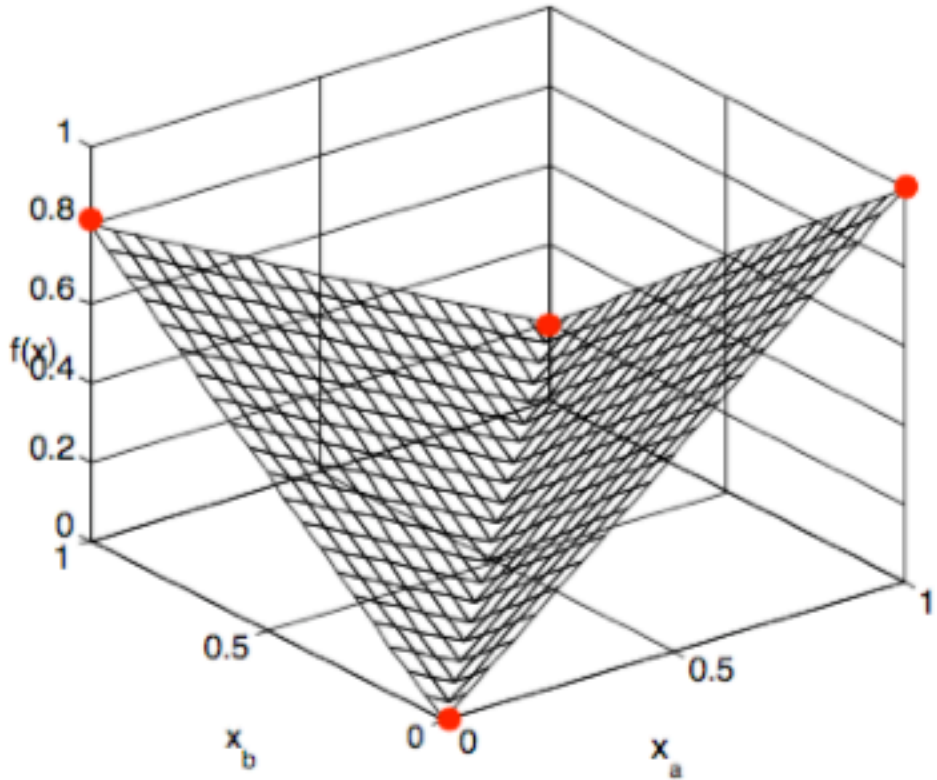
# The Lovász Extension

# Submodularity

26


$F(\{b\})$
$F(\{a\})$
$F(\{a,b\})$
$F(\emptyset)$
1
0.8
0.6
f(x)
0.4
0.2
0
1
0.5
0
0
0.5
1
$x_b$
$x_a$



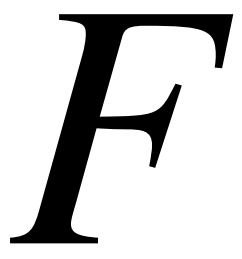

1
0.8
0.6
f(x)
0.4
0.2
0
1
0.5
0
0
0.5
1
$x_b$
$x_a$

$F \qquad \{0,1\}^n \qquad [0,1]^n$

- Extends the domain of from to

- For submodular:

- Convex!

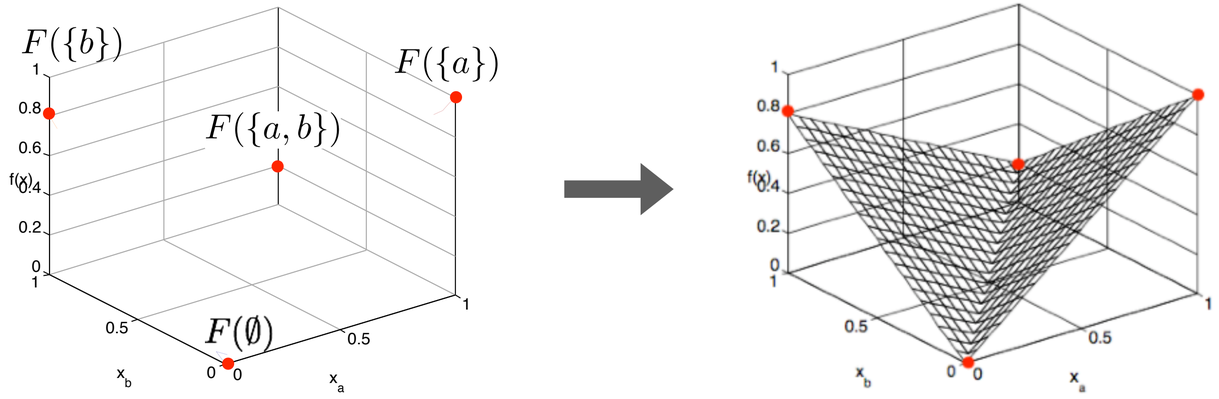


$$f(w) = \max_{x \in \mathcal{B}_F} w^\top x$$

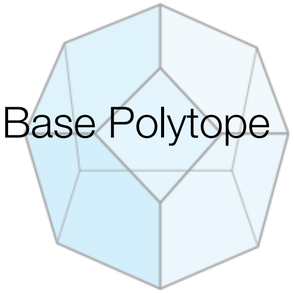
# Base Polytope

# Submodularity
## The Lovász Extension

- Extends the domain of $F$ from $\{0,1\}^n$ to $[0,1]^n$




- For $F$ submodular: $f(w) = \max_{x \in \mathscr{B}_F} w^\top x$ → Base Polytope



- Convex!

# Structured OT
## Relaxing the Objective

- So far: **deterministic** matches
- Want: **soft**, fractional assignments

| | Classic OT | Submodular OT |
|---|---|---|
| Strict Formulation | $\min_{M} \sum_{(i,j)\in M} C_{ij}$ | $\min_{M} F(M)$ |
| Relaxed Formulation | $\min_{\Gamma\in\Pi(\mathbf{a},\mathbf{b})} \sum_{(i,j)} C_{ij}\Gamma_{ij}$ | ? |