



CSAIL



OUR APPROACH

ALGORITHM: GROWING SPACES

Algorithm 1 Gromov-Wasserstein Computation for Word Embedding Alignment

Input: Source and target embeddings \mathbf{X} , \mathbf{Y} .
Regularization λ . Probability vectors \mathbf{p} , \mathbf{q} .

// Compute intra-language similarities

$$\mathbf{C}_s \leftarrow \cos(\mathbf{X}, \mathbf{X}), \quad \mathbf{C}_t \leftarrow \cos(\mathbf{Y}, \mathbf{Y})$$

$$\mathbf{C}_{st} \leftarrow \mathbf{C}_s^2 \mathbf{p} \mathbb{1}_m^\top + \mathbb{1}_n \mathbf{q} (\mathbf{C}_t^2)^\top$$

while not converged **do**

 // Compute pseudo-cost matrix (Eq. (9))

$$\hat{\mathbf{C}}_\Gamma \leftarrow \mathbf{C}_{st} - 2\mathbf{C}_s \Gamma \mathbf{C}_t^\top$$

 // Sinkhorn iterations (Eq. (7))

$$\mathbf{a} \leftarrow \mathbb{1}, \quad \mathbf{K} \leftarrow \exp\{-\hat{\mathbf{C}}_\Gamma / \lambda\}$$

while not converged **do**

$$\mathbf{a} \leftarrow \mathbf{p} \oslash \mathbf{K} \mathbf{b}, \quad \mathbf{b} \leftarrow \mathbf{q} \oslash \mathbf{K}^\top \mathbf{a}$$

end while

$$\Gamma \leftarrow \text{diag}(\mathbf{a}) \mathbf{K} \text{diag}(\mathbf{b})$$

end while

// Optional step: Learn explicit projection

$$\mathbf{U}, \Sigma, \mathbf{V}^\top \leftarrow \text{SVD}(\mathbf{X} \Gamma \mathbf{Y}^\top)$$

$$\mathbf{P} = \mathbf{U} \mathbf{V}^\top$$

return Γ, \mathbf{P}

**COMPUTE WITHIN-DOMAIN
PAIRWISE SIMILARITIES**

SINKHORN-KNOPP ITERATIONS

**(OPTIONALLY) LEARN EXPLICIT
PROJECTION VIA PROCRUSTES**

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EXPERIMENTAL RESULTS