CSAIL

OUR APPROACH

THE GROMOV-WASSERSTEIN DISTANCE

- Generalizes OT to the non-registered case

- Main idea: compare **relations** instead of absolute **positions**

$$d(\mathbf{x}^{(i)}, \mathbf{y}^{(j)})$$

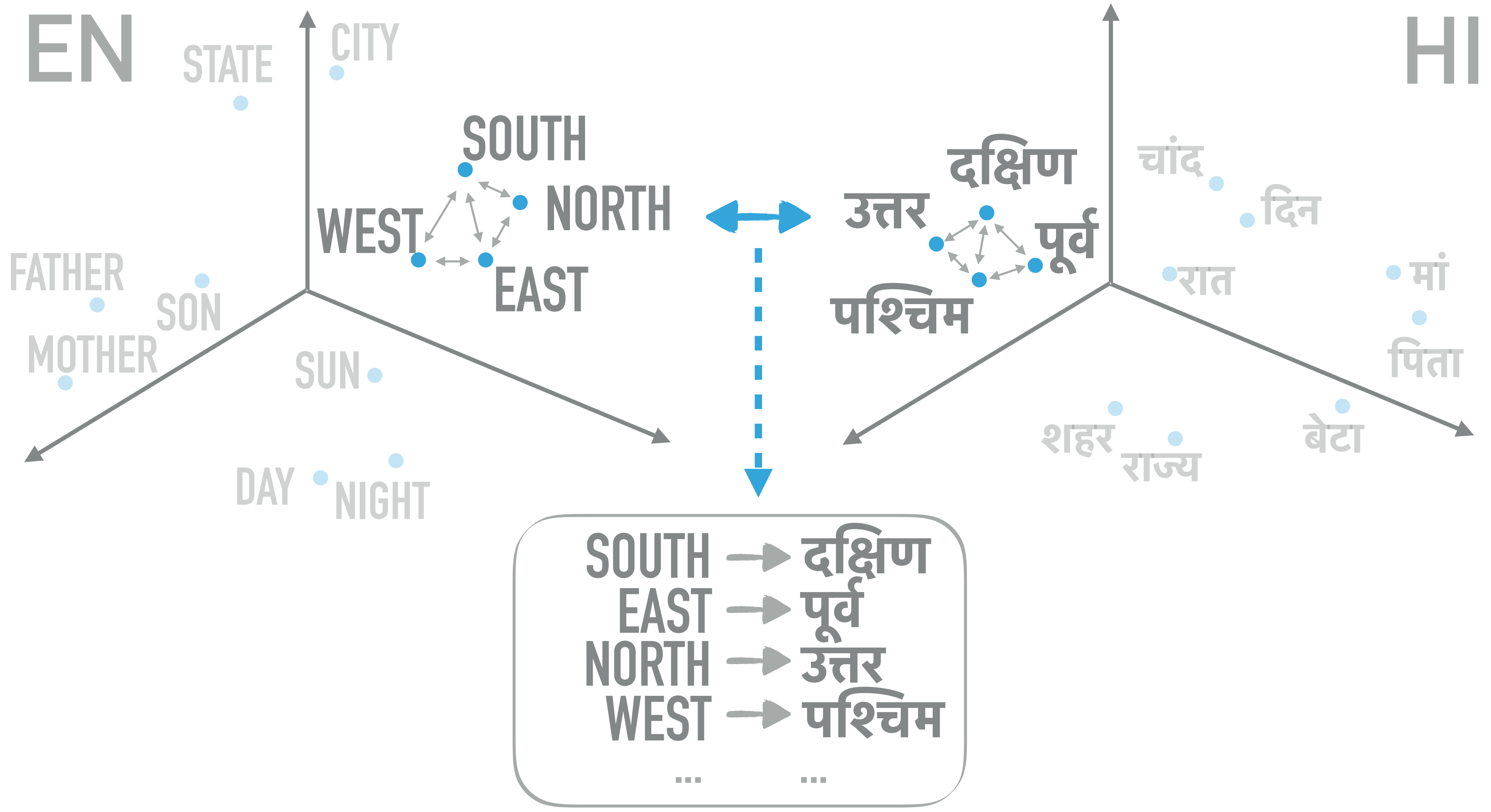
$$\mathscr{L}\left(d(\mathbf{x}^{(i)},\mathbf{x}^{(k)}),d(\mathbf{y}^{(j)},\mathbf{y}^{(l)})\right)$$

$$\mathrm{GW}(\mathbf{C}, \mathbf{C}', \mathbf{p}, \mathbf{q}) = \min_{\Gamma \in \Pi(\mathbf{p}, \mathbf{q})} \sum_{i,j,k,l} \mathbf{L}_{ijkl} \Gamma_{ij} \Gamma_{kl}$$

$$\mathrm{GW}(\mathbf{C}, \mathbf{C}', \mathbf{p}, \mathbf{q}) = \min_{\Gamma \in \Pi(\mathbf{p}, \mathbf{q})} \sum_{i,j,k,l} \mathbf{L}_{ijkl} \Gamma_{ij} \Gamma_{kl}$$

$$\mathrm{GW}(\mathbf{C}, \mathbf{C}', \mathbf{p}, \mathbf{q}) = \min_{\Gamma \in \Pi(\mathbf{p}, \mathbf{q})} \sum_{i,j,k,l} \mathcal{L}(\mathbf{C}_{ik}, \mathbf{C}'_{jl}) \Gamma_{ij} \Gamma_{kl}$$

[Mémoli, 2011; Peyré et al. 2016]

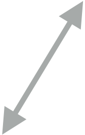

EN
STATE
CITY
SOUTH
NORTH
WEST
EAST
FATHER
SON
MOTHER
SUN
DAY
NIGHT
HI
दक्षिण
उत्तर
पूर्व
पश्चिम
चांद
दिन
रात
मां
पिता
शहर
राज्य
बेटा
SOUTH → दक्षिण
EAST → पूर्व
NORTH → उत्तर
WEST → पश्चिम
... ...

$$\mathrm{GW}(\mathbf{C}, \mathbf{C}', \mathbf{p}, \mathbf{q}) = \min_{\Gamma \in \Pi(\mathbf{p}, \mathbf{q})} \sum_{i,j,k,l} \mathcal{L}(\mathbf{C}_{ik}, \mathbf{C}'_{jl}) \Gamma_{ij} \Gamma_{kl}$$


SOUTH
NORTH
WEST
EAST

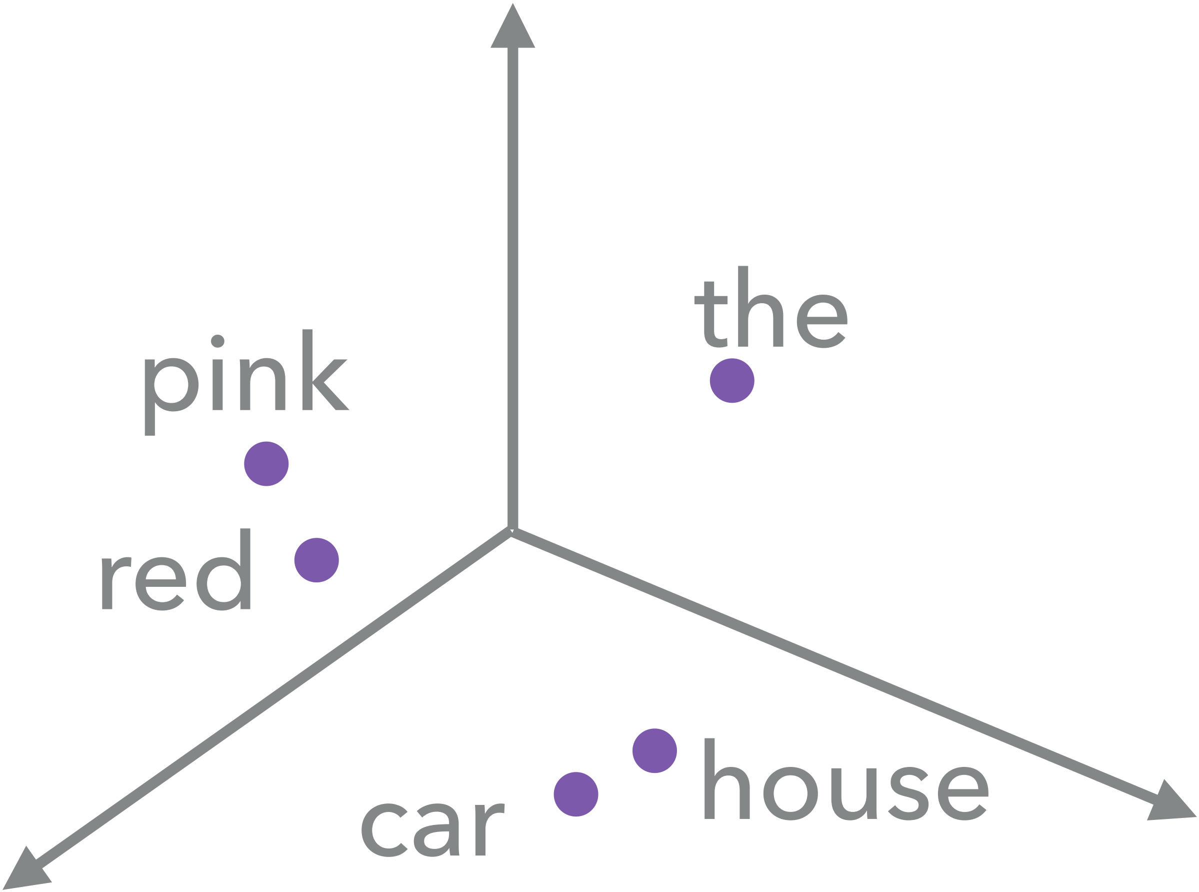
NORTH

SOUTH

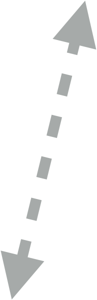


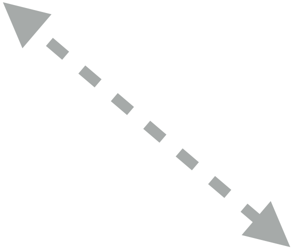
EAST

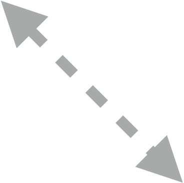
WEST

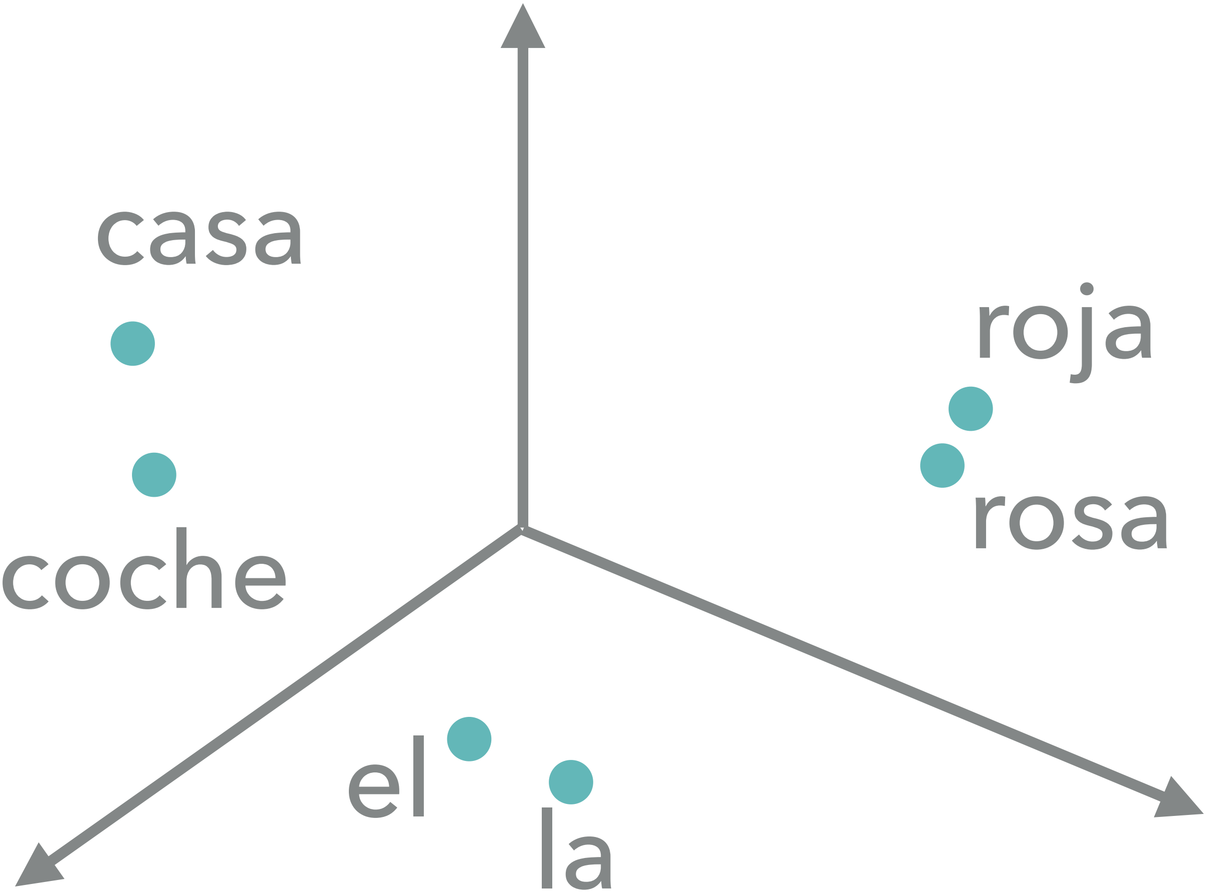


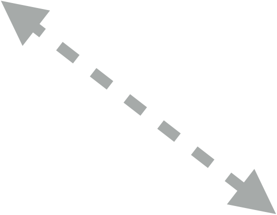
the
pink
red
house
car

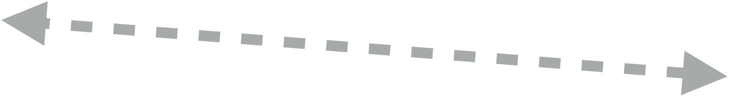


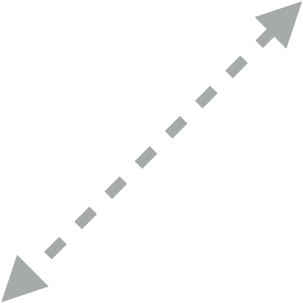


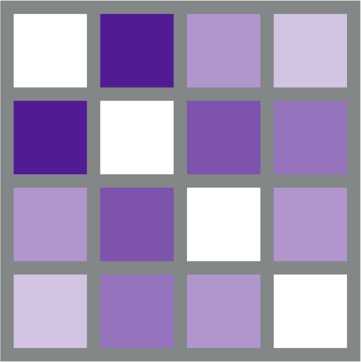


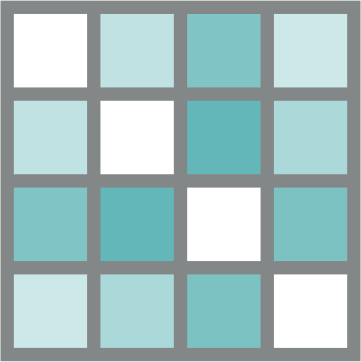

casa
roja
rosa
coche
el
la

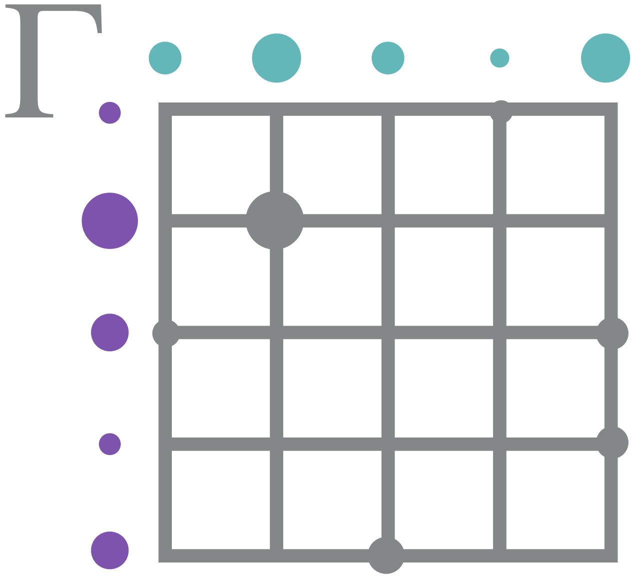











Γ

= cost of transporting one unit of mass from $\mathbf{x}^{(i)}$ to $\mathbf{y}^{(j)}$ ***and*** from $\mathbf{x}^{(k)}$ to $\mathbf{y}^{(l)}$

$$\mathcal{L}\Big(d(\mathbf{x}^{(i)}, \mathbf{x}^{(k)})), d(\mathbf{y}^{(j)}, \mathbf{y}^{(l)})\Big)$$

C

C'

$$\mathcal{L}(\mathbf{C}_{ik}, \mathbf{C}'_{jl})$$

- Main idea: compare **distances** instead of absolute **positions**

$$\mathrm{GW}(\mathbf{C}, \mathbf{C}', \mathbf{p}, \mathbf{q}) = \min_{\Gamma \in \Pi(\mathbf{p}, \mathbf{q})} \sum_{i,j,k,l} \mathcal{L}(\mathbf{C}_{ik}, \mathbf{C}'_{jl}) \Gamma_{ij} \Gamma_{kl}$$