Building Blocks

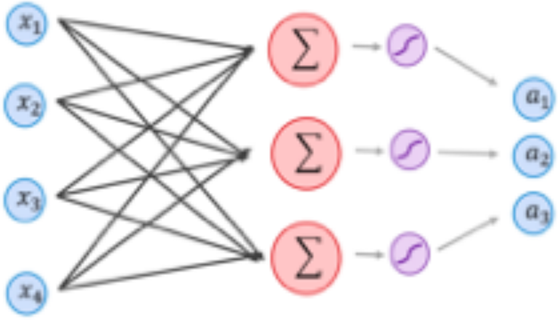
Hyperbolic Neural Networks

63

- Neural nets on the Poincaré Ball built from simple ingredients [Ganea et al. 2018]

- E.g. for a fully connected + activation layer , we need:

$$\mathscr{F} = \{\mathbf{P} : \mathbf{P}^\top \mathbf{P} = \mathbf{I}, \mathbf{P}\mathbf{P}^\top = \mathbf{I}\}$$


x_1
x_2
x_3
x_4
$\sum$
$\sum$
$\sum$
a_1
a_2
a_3

$$\mathbf{y} = \sigma(\mathbf{W}\mathbf{x} + \mathbf{b})$$

$$\mathbf{M} \otimes \mathbf{u} := \exp_0(\mathbf{M} \log_0(\mathbf{u}))$$

$$\mathbf{u} \oplus \mathbf{v} := \frac{(1 + 2\langle \mathbf{u}, \mathbf{v} \rangle + \|\mathbf{v}\|_2^2)\mathbf{u} + (1 - \|\mathbf{u}\|_2^2)\mathbf{v}}{1 + 2\langle \mathbf{u}, \mathbf{v} \rangle + \|\mathbf{u}\|_2^2 \|\mathbf{v}\|_2^2}$$

Matrix-vector product:

Vector-vector addition:

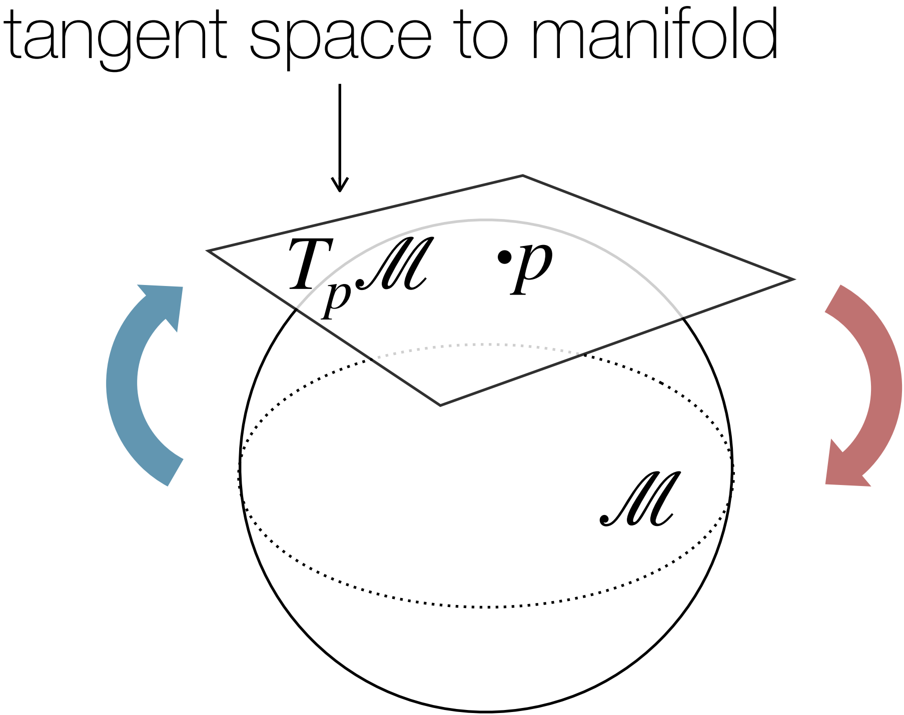
Applying a non-linearity:

$$\sigma(\mathbf{u}) := \exp_0(\sigma(\log_0(\mathbf{u})))$$

└→ Logarithmic map $\mathbf{log_p} : \mathscr{M} \mapsto T_{\mathbf{p}}\mathscr{M}$

$\longrightarrow$ A.k.a Möbius addition

Exponential map $\mathbf{exp_p} : T_\mathbf{p}\mathscr{M} \mapsto \mathscr{M}$



tangent space to manifold
$T_p\mathscr{M}$
$\bullet p$
$\mathscr{M}$

Hyperbolic Neural Networks
Building Blocks

- Neural nets on the Poincaré Ball built from simple ingredients [Ganea et al. 2018]

- E.g. for a fully connected + activation layer $\mathbf{y} = \sigma(\mathbf{W}\mathbf{x} + \mathbf{b})$, we need:

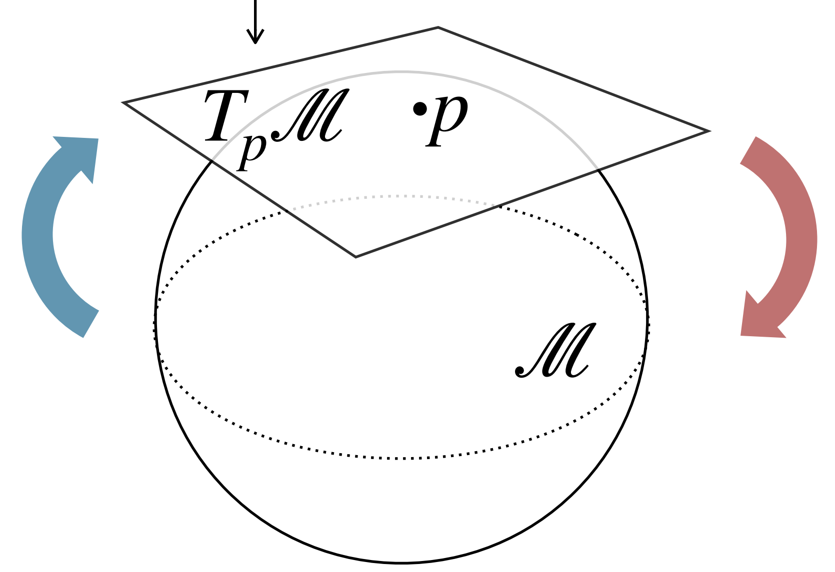
Vector-vector addition: $$\mathbf{u} \oplus \mathbf{v} := \frac{(1 + 2\langle \mathbf{u}, \mathbf{v} \rangle + \|\mathbf{v}\|_2^2)\mathbf{u} + (1 - \|\mathbf{u}\|_2^2)\mathbf{v}}{1 + 2\langle \mathbf{u}, \mathbf{v} \rangle + \|\mathbf{u}\|_2^2 \|\mathbf{v}\|_2^2}$$ $\longrightarrow$ A.k.a Möbius addition

Matrix-vector product: $$\mathbf{M} \otimes \mathbf{u} := \exp_0(\mathbf{M} \log_0(\mathbf{u}))$$

$\hookrightarrow$ Logarithmic map $\log_{\mathbf{p}} : \mathscr{M} \mapsto T_{\mathbf{p}}\mathscr{M}$

$\longrightarrow$ Exponential map $\exp_{\mathbf{p}} : T_{\mathbf{p}}\mathscr{M} \mapsto \mathscr{M}$

tangent space to manifold



Applying a non-linearity: $$\sigma(\mathbf{u}) := \exp_0(\sigma(\log_0(\mathbf{u})))$$

Aligning Hyperbolic Spaces
putting it all together

matching related embedded hierarchies with OT + deep nonlinear registration