three views of the problem

Invariant OT

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- The problem can be expressed in three equivalent ways:

- Under simple assumptions, problem on has analytic solution (generalized Procrustes)

- Gist of alternating minimization algorithm:

- For a fixed , find via SVD decomposition

- For a fixed , find via Sinkhorn-Knopp algorithm

$$\min_{\Gamma \in \Pi(\mathbf{a},\mathbf{b})} \min_{\mathbf{P} \in \mathcal{F}} \sum_{ij} \Gamma_{ij}\, d(\mathbf{x}_i, \mathbf{P}\mathbf{y}_j)$$

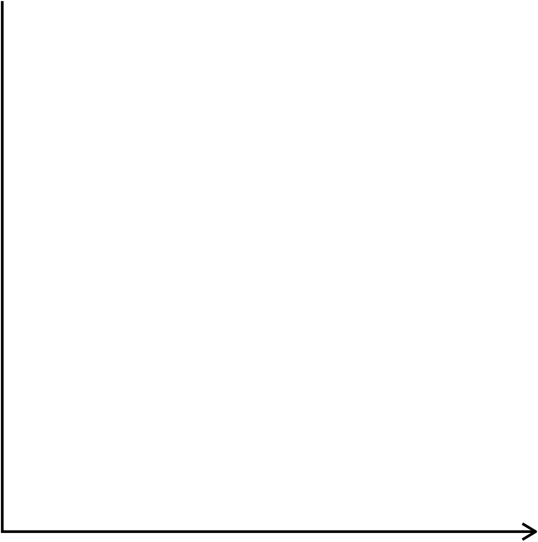
$$\min_{\Gamma \in \Pi(\mathbf{a},\mathbf{b})} \mathscr{L}(\Gamma)$$

$$\min_{\mathbf{P} \in \mathscr{F}} \quad \mathscr{L}(\mathbf{P})$$

projected gradient descent on Γ

(Stiefel) manifold optimization on $\mathbf{P}$

$$\min_{\Gamma \in \Pi(\mathbf{a},\mathbf{b})} \min_{\mathbf{P} \in \mathscr{F}} \mathscr{L}(\Gamma, \mathbf{P})$$



alternating minimization on $\boldsymbol{\Gamma}, \mathbf{P}$

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Repeat until convergence

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Proposed Opt. Algorithm

Invariant OT
three views of the problem

- The problem can be expressed in three equivalent ways:

$$\min_{\Gamma \in \Pi(\mathbf{a},\mathbf{b})} \min_{\mathbf{P} \in \mathscr{F}} \sum_{ij} \Gamma_{ij}\, d(\mathbf{x}_i, \mathbf{P}\mathbf{y}_j)$$

	Proposed Opt. Algorithm
$\min_{\Gamma \in \Pi(\mathbf{a},\mathbf{b})} \min_{\mathbf{P} \in \mathscr{F}} \mathscr{L}(\Gamma, \mathbf{P})$	$\boxed{\text{alternating minimization on } \Gamma, \mathbf{P}}$
$\min_{\Gamma \in \Pi(\mathbf{a},\mathbf{b})} \mathscr{L}(\Gamma)$	projected gradient descent on Γ
$\min_{\mathbf{P} \in \mathscr{F}} \mathscr{L}(\mathbf{P})$	(Stiefel) manifold optimization on $\mathbf{P}$

- Under simple assumptions, problem on $\mathbf{P}$ has analytic solution (generalized Procrustes)

- Gist of alternating minimization algorithm:
  - For a fixed Γ, find $\mathbf{P}$ via SVD decomposition
  - For a fixed $\mathbf{P}$, find Γ via Sinkhorn-Knopp algorithm

Repeat until convergence

Invariant OT
optimization method comparison