Subook

TO WHICH IS ADDED MANY USEFUL ONE
LINERS AND CODE SO THAT
YOU CAN AWK LIKE A HAWK BY

Changxing Su

An Human

Contents

Part I

User Manual

Chapter 1

Introduction

Contents

As the year went on, I started typesetting my personal notes during class and realized that the LaTeXformat, while great for publications and lecture notes in general, was lacking a few small but useful template for me.

1.1 Required Packages

For *Subook,* the following packages are required

marginnote, sidenotes, fancyhdr, titlesec, geometry, and tcolorbox.

For a brief summary, the marginnote, sidenote, titlesec, and tcolorbox packages are used in creating the \part environment, the package geometry is used globally to set the page width, page height, and margin width, and finally, fancyhdr, which is overridden on the title page, the contents page, and the \part page, sets the header for the body.

1.2 License

This work may be distributed and/or modified under the conditions of the LaTeX Project Public License, either version 1.3 of this license or (at your option) any later version. The latest version of this license is found in http://www.latex-project.org/lppl.txt, and version 1.3 or later is part of all distributions of LaTeX version 2005/12/01 or later. The current maintainer of this work is Changxing Su.

1.3 Features

Subook includes the following:

1. Several mathematics and physics packages.

2. Margins and margin environments for tables, figures, and asides.

3. TEX shortcuts for various math scripts namely vector bold math, mathbb, math-frak, and mathcal.

4. amsthm integrations and special environments for theorems, lemmas, proofs, definitions, examples, and remarks.

5. Stylized support for the part environment.

6. A fullpage environment that spans across the text width and the margin for longer equations and horizontal figures.

Each of these will be discussed in the following subsections.

1.4   TEX Shortcuts

subook comes built in with a minimal set of keyboard shortcuts for a few special characters. All of these shortcuts can be found in subook.cls just under

```
% ------------------------------------------------------------------
%           User Created Commands
% ------------------------------------------------------------------
...
```

If one has their own macros then simply add it under this area.

Most people have their own shortcuts for commonly used mathematics, such as derivatives or integrals. For those looking for physics shortcuts, the excellent physics package (automatically included in *subook*) has possible everything that one can imagine.

1.5   amsthm **Environments**

amsthm environments are defined as usual being enclosed by \begin{environment}⋯ \end{environment} and most have been modified ostensibly from the original amsthm presets. Primarily, most environments, with the exception of the exercise environment, are now integrated with the wonderful tcolorbox package. Note that the counting for theorems and lemmas is distinct from the counting for definitions. Also note that the breakable for tcolorbox allows these environments to span multiple pages. All of these environment and the associated tcolorbox are provided by the code in subook.cls just under

```
% ------------------------------------------------------------------
%                     User Created Environments
% ------------------------------------------------------------------
...
%% ---------------------------- tcolorbox ----------------------------
...
```

Definition 1.1 (Test)

The definition environment ♣

Lemma 1.1 (Test)

The lemma environment ◇

Theorem 1.1 (Test)

The theorem environment ♡

Corollary 1.1 (Test)

The corollary environment ◇

Proposition 1.1 (Test)

The proposition environment ♠

Example 1   Test

The example environment

Proof. The proof environment   □

Remark. The remark environment

> **Some extra box environment**
>
> The something extra environment

1.5.1 tcolorbox **Environment and Known Issues**

The breakable should allow the proof environment to span multiple pages. If one wishes to change the color, simply modify the line which states borderline west={1pt} {0pt}{blue}. The first numeric value dictates the width of the line, the second dictates how close it is away from the *left* margin, while the last argument obviously dictates the color. This code could also be used to change any of the other amsthm environments.

1.6 Fullpage Environment

The fullpage environment is defined by

$$\begin{fullpage}$$
$$\cdots$$
$$\end{fullpage}$$

with the width of the fullpage environment given by \textwidth+\marginparsep+\marginparwidth There are some clear benefits of having use of the full page at times. Suppose that one wants to place a figure that cannot fit into the margins, or if an equation is quite long and it bleeds into the margin, then the fullpage environment can both clearly separate these from the surrounding text and allot for the dimensions without hassle.
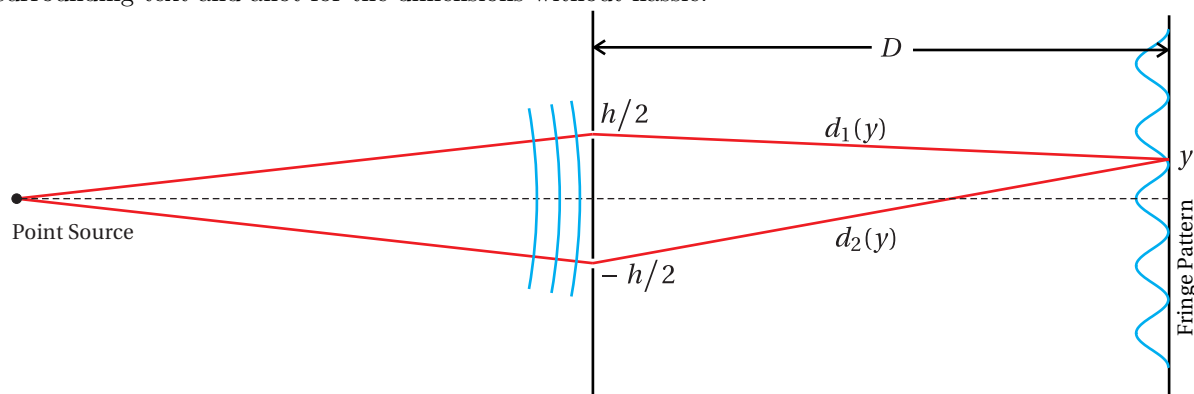

Figure 1.1: Figure caption

> **Definition 1.2 (big box)**
>
> dsdd ♣

Figure is a floating environment and minipage is, unfortunately, not. Therefore, if you put a floating object inside a non-floating minipage, you will get an error.One way is to avoid using figure entirely. This can be done with help of the caption package (with its captionof facility, so that you can have a caption for the figure):

```
\centering
    \includegraphics{img/f08Young.pdf}
    \captionof{figure}{Figure caption}
    \label{fig:example} % Unique label used for referencing the figure
```

Chapter 2

Test ss

*"Begin at the beginning," the King said gravely, "and go
on till you come to the end: then stop."*

— Lewis Carroll, *Alice in Wonderland*

Contents

2.1 test

The genomes of chimpanzees and humans are 99.9% identical, yet the differences between
the two species are vast. The relatively few differences in genetic endowment must explain
the possession of language by humans, the extraordinary athleticism of chimpanzees, dsdsdsdsd
and myriad other differences. Genomic comparison is allowing researchers to identify
candidate genes linked to divergences in the developmental programs of humans and the
other primates and to the emergence of complex functions such as language. The picture
will become clearer only as more primate genomes become available for comparison with
the human genome.

2.2 sdadsad

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luctus mauris.

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2.2.1   dsadsad

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> **Definition 2.1 (ds)**
>
> $$\mathcal{L}_{\mathrm{BCE}}(\hat{\boldsymbol{y}}, \boldsymbol{y}) = \sum_{i=1}^{N} -y^{(i)} \log(\hat{y}^{(i)}) - (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$
>
> $$= \sum_{i=1}^{N} \begin{cases} -\log(\hat{y}^{(i)}), & \text{for } y^{(i)} = 1 \\ -\log(1 - \hat{y}^{(i)}), & \text{for } y^{(i)} = 0 \end{cases} \tag{2.1}$$
>
> ♣

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> **Theorem 2.1 (ad)**
>
> abs
>
> ♡

assured

> **Lemma 2.1 (asd)**
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> sdsd
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❚ Proof. dsad □

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Example 2.1

I use my "examplebox" as usual text and margin figure work as they should be but I want my example box spread out to the right margin on an odd page, breakable, and spread out to the left margin on the even page.

I use my "examplebox" as usual text and margin figure work as they should be but I want my example box spread out to the right margin on an odd page, breakable, and spread out to the left margin on the even page.

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> **Corollary 2.1**
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> sadsd ◇

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> **Proposition 2.1 (dsaf)**
>
> dsadasd ♠

> **some extra box**
>
> dsadasd Similarly, the differences in genetic endowment among humans are vanishingly small compared with the differences between humans and chimpanzees, yet these differences account for human(**?**) variety—including differences in health and in susceptibility to chronic diseases. We have much to learn about the variability in genomic sequence among humans, and the availability of genomic information will almost certainly transform medical diagnosis and treatment. Several monumental studies in which the entire

A listings example:

Listing 2.1: Python example

```python
import numpy as np

def incmatrix(genl1,genl2):
    m = len(genl1)
    n = len(genl2)
    M = None #to become the incidence matrix
    VT = np.zeros((n*m,1), int)  #dummy variable

    #compute the bitwise xor matrix
    M1 = bitxormatrix(genl1)
    M2 = np.triu(bitxormatrix(genl2),1)

    for i in range(m-1):
        for j in range(i+1, m):
            [r,c] = np.where(M2 == M1[i,j])
            for k in range(len(r)):
                VT[(i)*n + r[k]] = 1;
                VT[(i)*n + c[k]] = 1;
                VT[(j)*n + r[k]] = 1;
                VT[(j)*n + c[k]] = 1;

                if M is None:
                    M = np.copy(VT)
                else:
                    M = np.concatenate((M, VT), 1)

                VT = np.zeros((n*m,1), int)
```

```
        return M
```

Part II

very big

Chapter 3

ds dsd

Contents

3.1   dsadsa

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Part III

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Chapter 4

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Part IV

probability

Chapter 5

Pro

5.1 Review and Overview

In this lecture we delineate a mathematical framework for supervised learning. We focus on regression problems and define the notion of the loss/risk associated with a model. We then analyze a particular loss function (the squared loss) in a general setting, then specialize our result to the case of linear models. We next define the notion of parameterized families of hypotheses and the maximum likelihood estimate, and we conclude with an asymptotic result relating the training MLE to the true maximum likelihood parameter.

5.2 Formulation of supervised learning

We begin by constructing a mathematical framework for prediction problems. Our framework consists of the following elements:

1. A space of possible data points $\mathcal{X}$.

2. A space of possible labels $\mathcal{Y}$.

3. A joint probability distribution P on $\mathcal{X} \times \mathcal{Y}$. We assume that our training data consists of n points
$$(x^{(1)}, y^{(1)}), \ldots, (x^{(n)}, y^{(n)}) \overset{\text{i.i.d.}}{\sim} P$$

    each drawn independently from P.

4. A prediction function/model $f : \mathcal{X} \to \mathcal{Y}$.

5. A loss function $\ell : \mathcal{Y} \times \mathcal{Y} \to \mathbb{R}$. We will usually assume that ℓ is bounded below by some constant, typically 0.

Given the prediction function f and the loss function ℓ, the loss of an example is $\ell(f(x), y)$. We can then define the *expected risk* (or *expected loss*, or *population risk*)

$$L(f) \overset{\triangle}{=} \mathbb{E}_{(x,y) \sim P}[\ell(f(x), y)].$$

Our goal will be to obtain a small expected loss. Often it will be infeasible to consider all possible models f, so we may restrict ourselves to a certain family of hypotheses $\mathcal{F}$. In this case, we define the *excess risk* of a model f as

$$L(f) - \inf_{g \in \mathcal{F}} L(g).$$

This gives us a measure of how well our model fits the data relative to the best we can hope to do within our set of options $\mathcal{F}$.

Within this framework, there are two main types of problems we will consider: *regression* problems, where the set of labels is $\mathcal{Y} = \mathbb{R}$; and *classification* problems, where the set of labels is some finite set $\mathcal{Y} = \{1, \ldots, k\}$. We will focus on regression problems in this lecture.

5.3 Regression problems and squared loss

We consider the regression problem of predicting y given x. We take as our loss function the *squared loss*

$$\ell(\hat{y}, y) = (\hat{y} - y)^2, \qquad L(f) = \mathbb{E}_{(x,y)\sim P}[(f(x) - y)^2].$$

In this setting, we can decompose the risk in a very informative way.

Lemma 5.1 (Decomposition of loss)

Under the squared loss, we have the decomposition

$$L(f) = \mathbb{E}_{x\sim P_x}[(f(x) - \mathbb{E}[y \mid x])^2] + \mathbb{E}_{x\sim P_x}[\mathrm{Var}(y \mid x)]$$

where P_x is the marginal distribution of x. ◇

The second term in this expansion is the intrisic variable of the label; it gives a lower bound on the loss we can achieve. Since the first term in the decomposition is nonnegative, it is an immediate corrolary that the optimal model is $f(x) = \mathbb{E}[y \mid x]$.

In order to prove Lemma 1, we make use of the following claim.

> If Z is a random variable and a is a constant, then
> $$\mathbb{E}[(Z - a)^2] = (\mathbb{E}[Z] - a)^2 + \mathrm{Var}(Z).$$

The proof of this claim is left as an exercise on HW 0. We are now ready to prove Lemma 1.

Proof. [Proof of Lemma 5.1] We have

$$
\begin{aligned}
L(f) &= \mathbb{E}[(f(x) - y)^2] \\
&= \mathbb{E}_{x\sim P_x}[\mathbb{E}_{P_{y \mid x}}[(f(x) - y)^2 \mid x]] && \text{(Law of total expectation)} \\
&= \mathbb{E}_{x\sim P_x}[(f(x) - \mathbb{E}[y \mid x])^2 + \mathrm{Var}(y \mid x)]. && \text{(Claim ??)}
\end{aligned}
$$

Note that Claim ?? holds in the third equation since $f(x)$ is a constant when we have conditioned on x. The desired result follows from linearity of expectation. □

Lemma 5.1 gives us a general lower bound on risk under squared loss. If we impose more structure on the set of hypotheses $\mathcal{F}$ from which we can select f, we can gain more information on the risk.

5.4 Linear regression under squared loss

A commonly used choice of hypotheses is the set of linear functions:

$$\mathcal{F} = \{f : \mathbb{R}^d \to \mathbb{R} \mid f(x) = w^\top x, \ w \in \mathbb{R}^d\}.$$

For $f \in \mathcal{F}$, we then have

$$L(f) = L(w) = \mathbb{E}[(w^\top x - y)^2].$$

Henceforth, we will denote $w^* \in \mathrm{argmin}_{w\in\mathbb{R}^d} L(w)$ and $\hat{w}$ will denote a model learned from training data.

One may ask why we have only allowed linear functions with 0 instead of allowing a nonzero intercept. Actually, the framework we have outlined above is enough to accomodate nonzero intercepts. If we wish to analyze the function $w^\top x + b$, we can simply set $\tilde{x} = (x, 1)$ and $\tilde{w} = (w, b)$. Then $\tilde{w}^\top \tilde{x} = w^\top x + b$ and we have reduced to the case of 0 intercept.

When we restrict to linear models, we can further decompose the risk under squared loss.

> **Lemma 5.2**
>
> With $w^* \in \operatorname{argmin}_{w \in \mathbb{R}^d} L(w)$, we have
>
> $$L(\hat{w}) = \mathbb{E}_x[\operatorname{Var}(y \mid x)] + \mathbb{E}_x[(\mathbb{E}[y \mid x] - w^{*\top} x)^2] + \mathbb{E}_x[(w^{*\top} x - \hat{w}^\top x)^2]. \quad (5.1) \quad \diamond$$

The second term in equation (5.1) can be thought of as the approximation error incurred by linear models. The third term can be interpreted as the estimation error we incur from having only a finite sample.

Proof. Define $g(\hat{w}) \triangleq \mathbb{E}[(\mathbb{E}[y \mid x] - \hat{w}^\top x)^2]$. By Lemma 5.1,

$$L(\hat{w}) = \mathbb{E}[\operatorname{Var}(y \mid x)] + g(\hat{w}). \quad (5.2)$$

Observe that since $w^* \in \operatorname{argmin} L(w)$, $\nabla L(w^*) = 0$. Furthermore, since $\mathbb{E}_x[\operatorname{Var}(y|x)]$ is a constant with respect to w, we have

$$\begin{aligned} \nabla L(w) &= \nabla g(w) \\ &= \mathbb{E}[\nabla_w (\mathbb{E}(y \mid x) - w^\top x)^2] \\ &= 2\mathbb{E}[(\mathbb{E}(y \mid x) - w^\top x)x]. \end{aligned}$$

Since $\nabla L(w^*) = 0$ we have

$$\mathbb{E}[(\mathbb{E}[y \mid x] - w^{*\top} x)x] = 0. \quad (5.3)$$

Next, we expand:

$$\begin{aligned} g(\hat{w}) &= \mathbb{E}[(\mathbb{E}[y \mid x] - \hat{w}^\top x)^2] \\ &= \mathbb{E}[((\mathbb{E}[y \mid x] - w^{*\top} x) - (\hat{w}^\top x - w^{*\top} x))^2] \\ &= \mathbb{E}[(\mathbb{E}[y \mid x] - w^{*\top} x)^2 + (\hat{w}^\top x - w^{*\top} x)^2] \\ &\quad - 2\mathbb{E}[(\mathbb{E}[y \mid x] - w^{*\top} x)(\hat{w}^\top x - w^{*\top} x)]. \end{aligned}$$

Finally, observe that

$$\mathbb{E}[(\mathbb{E}[y \mid x] - w^{*\top} x)(\hat{w}^\top x - w^{*\top} x)] = (\hat{w}^\top - w^{*\top})\mathbb{E}[(\mathbb{E}[y \mid x] - w^{*\top} x)x].$$

By equation (5.3), this quantity vanishes and it follows that

$$g(\hat{w}) = \mathbb{E}[(\mathbb{E}[y \mid x] - w^{*\top} x)^2] + \mathbb{E}[(\hat{w}^\top x - w^{*\top} x)^2]. \quad (5.4)$$

Combining equations (5.2) and (5.4) gives the desired result. $\qquad \square$

5.5 Parameterized families of hypotheses

Linear models are one type of *parameterized family* of hypotheses. In general, a parameterized family is given by a parameter space Θ. For each $\theta \in \Theta$ there is a hypothesis $f_\theta(x)$, sometimes written $f(\theta; x)$. In this case we may write the loss function as

$$\ell(f_\theta(x), y) = \ell((x, y), \theta).$$

In the special case of linear functions, our parameter space is $\Theta = \mathbb{R}^d$ and for $\theta \in \Theta$ we have $f_\theta(x) = \theta^\top x$.

5.5.1 Well-specified case and maximum likelihood

In the well-specified case, $P_\theta(y \mid x)$ is a family of distributions parameterized by $\theta \in \Theta$, and $y \mid x \sim P_{\theta^*}(y \mid x)$ is distributed according to some ground truth parameter θ^*. We define the *maximum likelihood* loss function by

$$\ell((x, y), \theta) = -\log P_\theta(y \mid x),$$

so that minimizing the loss function is equivalent to maximizing the likelihood of the data.

For example, suppose that $y \mid x$ is Gaussian distributed with mean $\theta^{*\top} x$ and variance 1, i.e. $y \mid x \sim N(\theta^{*\top} x, 1)$. The likelihood is then

$$
\begin{aligned}
\ell((x, y), \theta) &= -\log P_\theta(y \mid x) \\
&= -\log \exp\left(-\frac{(y - \theta^\top x)^2}{2}\right) + c \\
&= \frac{(y - \theta^\top x)^2}{2} + c
\end{aligned}
$$

where c is the log of the normalizing constant. This computation shows that in the Gaussian setting, minimizing the squared loss actually recovers the MLE.

5.6 Training loss

Often we do not know the true underlying distribution P with which to compute the expected loss. In these cases we need to use an approximation based on the data we do have. This motivates our definition of the *training loss*

$$
\hat{L}(\theta) \triangleq \frac{1}{n} \sum_{i=1}^{n} \ell((x^{(i)}, y^{(i)}), \theta).
$$

In the special case of maximum likelihood, we have $\hat{L}(\theta) = -\frac{1}{n} \sum_{i=1}^{n} \log p_\theta(y^{(i)} \mid x^{(i)})$. We define the *maximum likelihood estimator*

$$
\hat{\theta}_{\mathrm{MLE}} \in \operatorname{argmin}_{\theta \in \Theta} \hat{L}(\theta).
$$

This approximation is "good" in the sense that as $n \to \infty$, the minimizer of the training loss $\hat{\theta}_{\mathrm{MLE}}$ approaches the true maximum likelihood parameter θ^*. The following theorem quantifies this fact.

> **Theorem 5.1 (Asymptotic of MLE)**
>
> Assume $\nabla^2 L(\theta^*)$ is full rank. Let $\hat{\theta} = \hat{\theta}_{\mathrm{MLE}}$ and
>
> $$
> Q \triangleq \mathbb{E}_{(x,y) \sim P}[\nabla_\theta(\log p_\theta(y \mid x))(\theta^*) \nabla_\theta(\log p_\theta(y \mid x)(\theta^*)^\top].
> $$
>
> Assuming that $\hat{\theta} = \hat{\theta}_n \xrightarrow{p} \theta^\star$ (i.e. consistency) and under appropriate regularity conditions,
>
> $$
> \sqrt{n}(\hat{\theta} - \theta^*) \xrightarrow{d} N(0, Q^{-1}) \text{ and } n(L(\hat{\theta}) - L(\theta^*)) \xrightarrow{d} \frac{1}{2}\chi^2(p).
> $$
>
> as $n \to \infty$, where p is the dimension of θ and $\chi^2(p)$ is the distribution of the sum of the squares of p i.i.d. standard Gaussian random variables. ♡

Remark. The positive definiteness of the Hessian $\nabla^2 L(\theta^*)$ guarantees that identifiability holds *locally* (in a neighborhood of θ^*), but does not imply identifiability because of a lack of *global* information.

To give a counter-example, suppose θ^* is the global minimizer of L and the Hessian is positive definite, but there exists a sequence of θ_n such that $\|\theta_n - \theta^*\| = n$ but $L(\theta_n) = L(\theta^*) + 1/n$, then the identifiability is violated: the inf is not strictly greater than but equal to $L(\theta^*)$. The reason is that the Hessian does not reveal information about L outside an infinitesimal neighborhood of θ^*.

One way to exclude such adversarial case is to assume convexity: when L is convex, a local strong growth implies global growth and thus identifiability

Example 2   fasd

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