Dictionary

Abstract Data Types Seen So Far

	Insert	Min	Extract_Min	Union
Priority Queues	✓	✓	✓	X
Mergeable Priority Queues	✓	✓	✓	✓

Abstract Data Type: Dictionary

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- **Object** : Set S of (elements with) keys

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  - **Insert**(S, x): Inserts x in S

Abstract Data Type: Dictionary

- **Object** : Set S of (elements with) keys

- **Operations:**
  - **Search**(S, x): Returns element with key x, if x is in S; else return “Not Found”
  - **Insert**(S, x): Inserts x in S
  - **Delete**(S, x): Deletes x from S

Data Structures for Dictionaries

Linked List

Data Structures for Dictionaries

Linked List

- **Insert**

Data Structures for Dictionaries

Linked List

- **Insert :** $\Theta(1)$

Data Structures for Dictionaries

Linked List

- **Insert :** $\Theta(1)$
- **Search**

Data Structures for Dictionaries

Linked List

- **Insert :** $\Theta(1)$
- **Search :** $\Theta(n)$

Data Structures for Dictionaries

Linked List

- **Insert : $\Theta(1)$**
- **Search : $\Theta(n)$**
- **Delete :**

Data Structures for Dictionaries

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- **Insert :** $\Theta(1)$
- **Search :** $\Theta(n)$
- **Delete :** $\Theta(n)$

Data Structures for Dictionaries

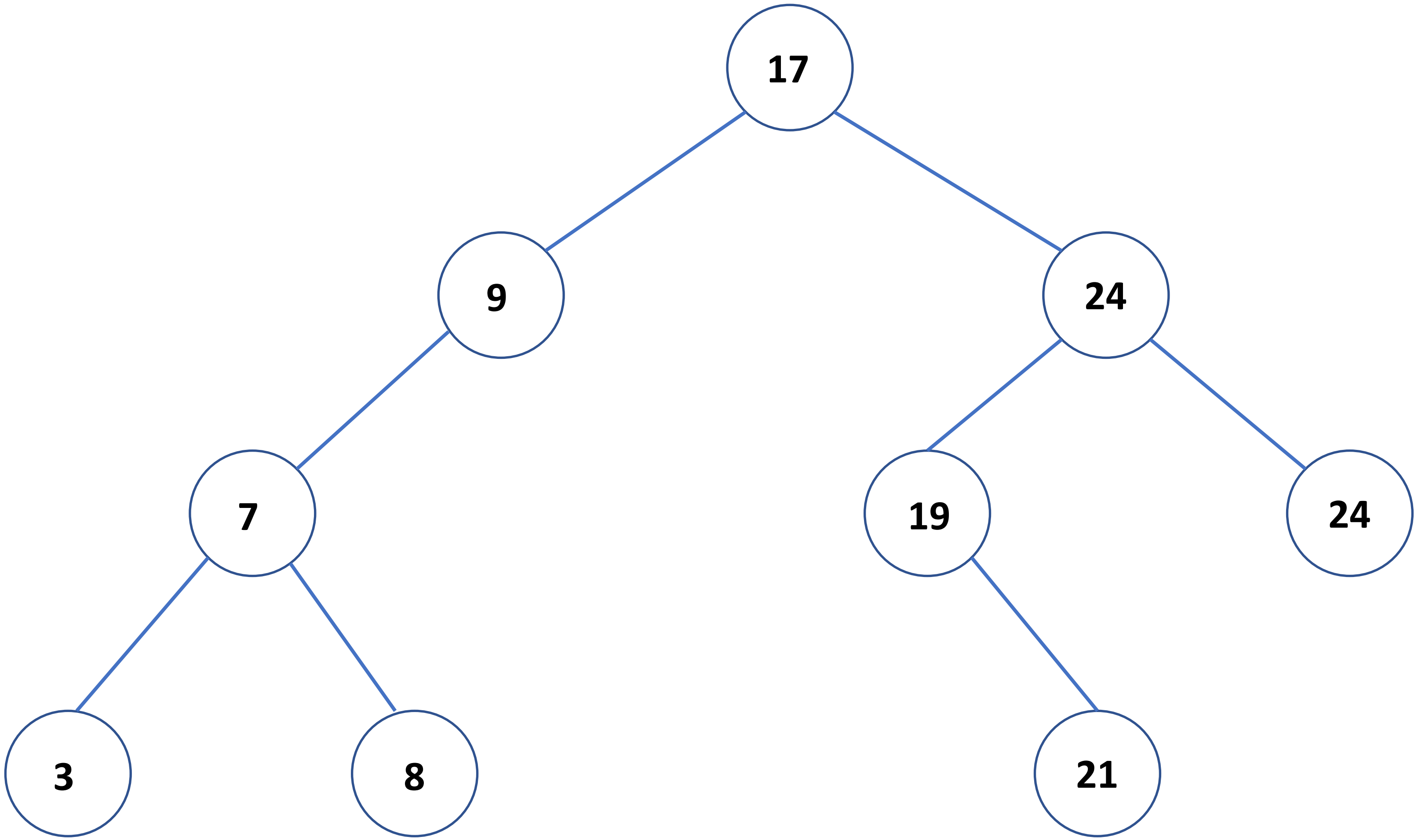
Linked List

- **Insert : ** $\Theta(1)$
- **Search : ** $\Theta(n)$
- **Delete : ** $\Theta(n)$

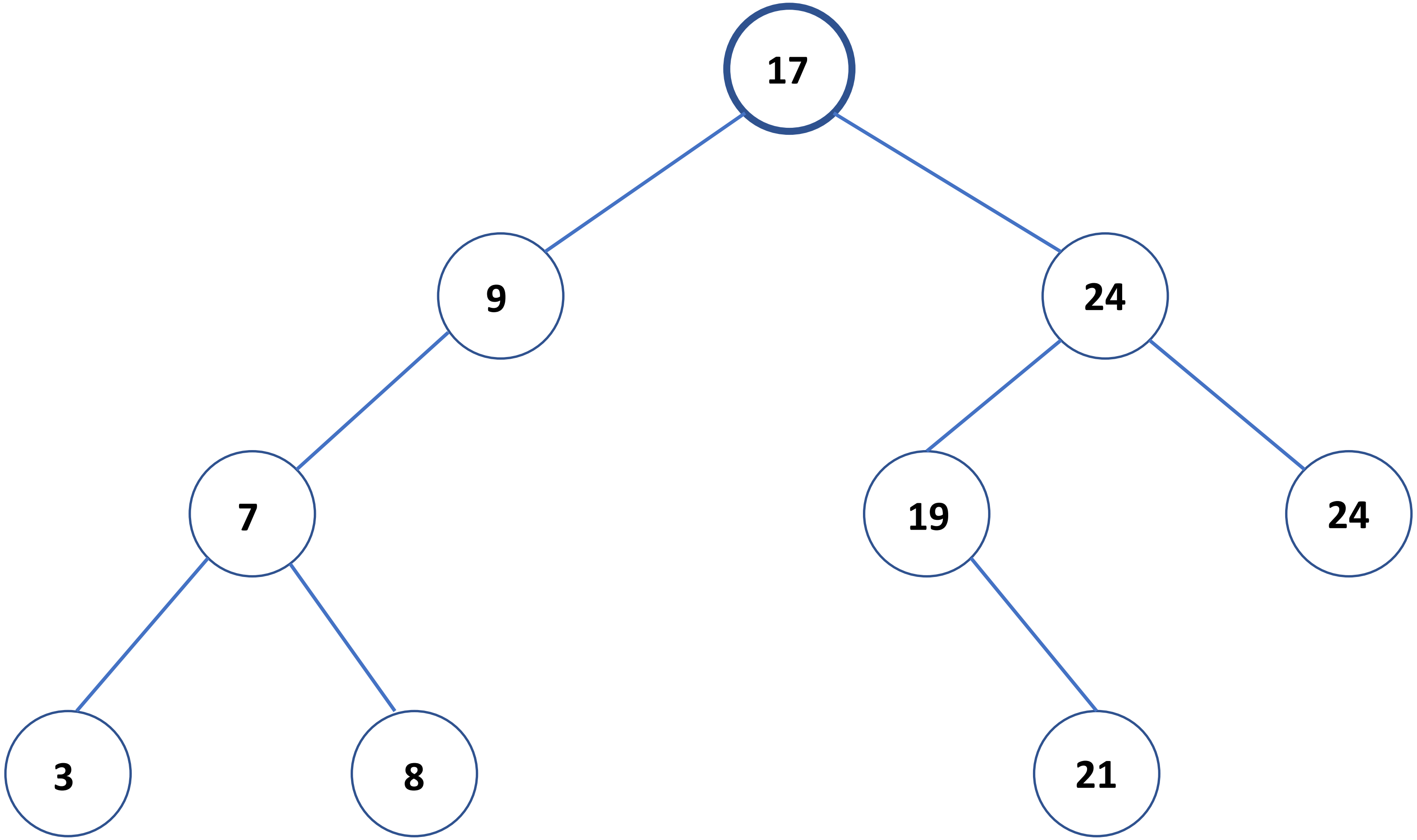
Goal: Data Structure that does each operation in $\Theta(\log n)$ time

Binary Search Trees

Binary Search Tree (BST)

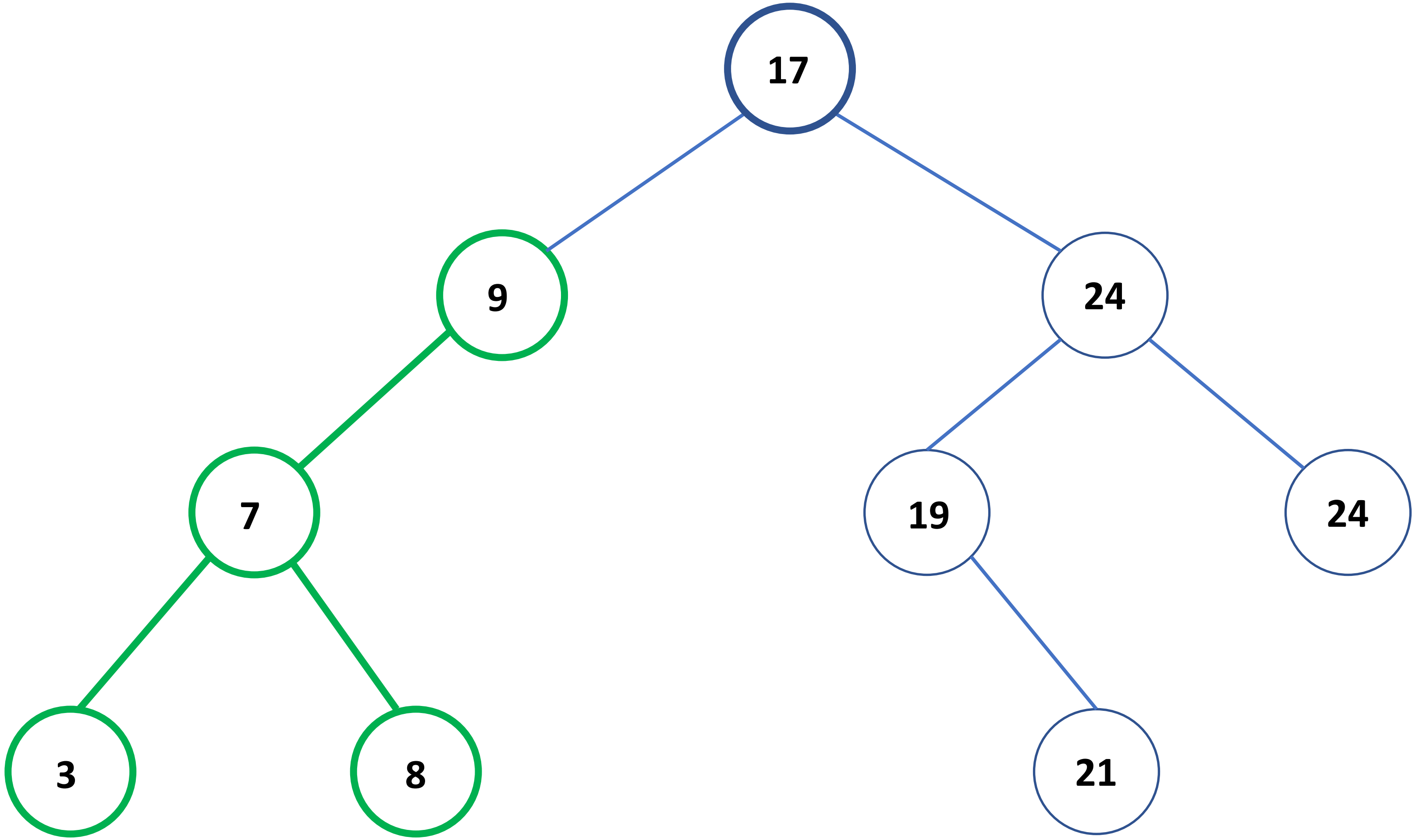


Binary Search Tree (BST)



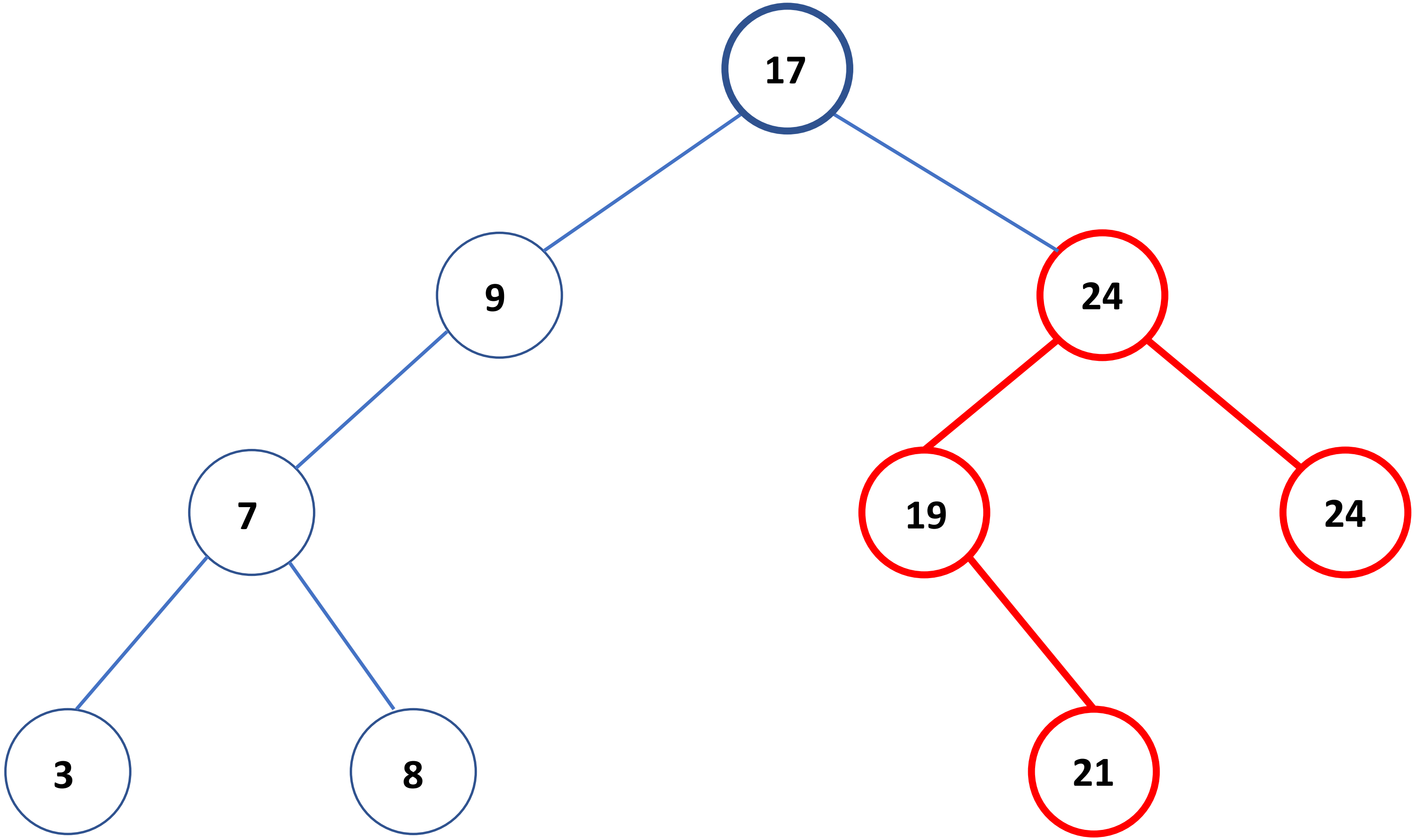
For each node

Binary Search Tree (BST)



For each node: keys in left subtree <= the node's key

Binary Search Tree (BST)



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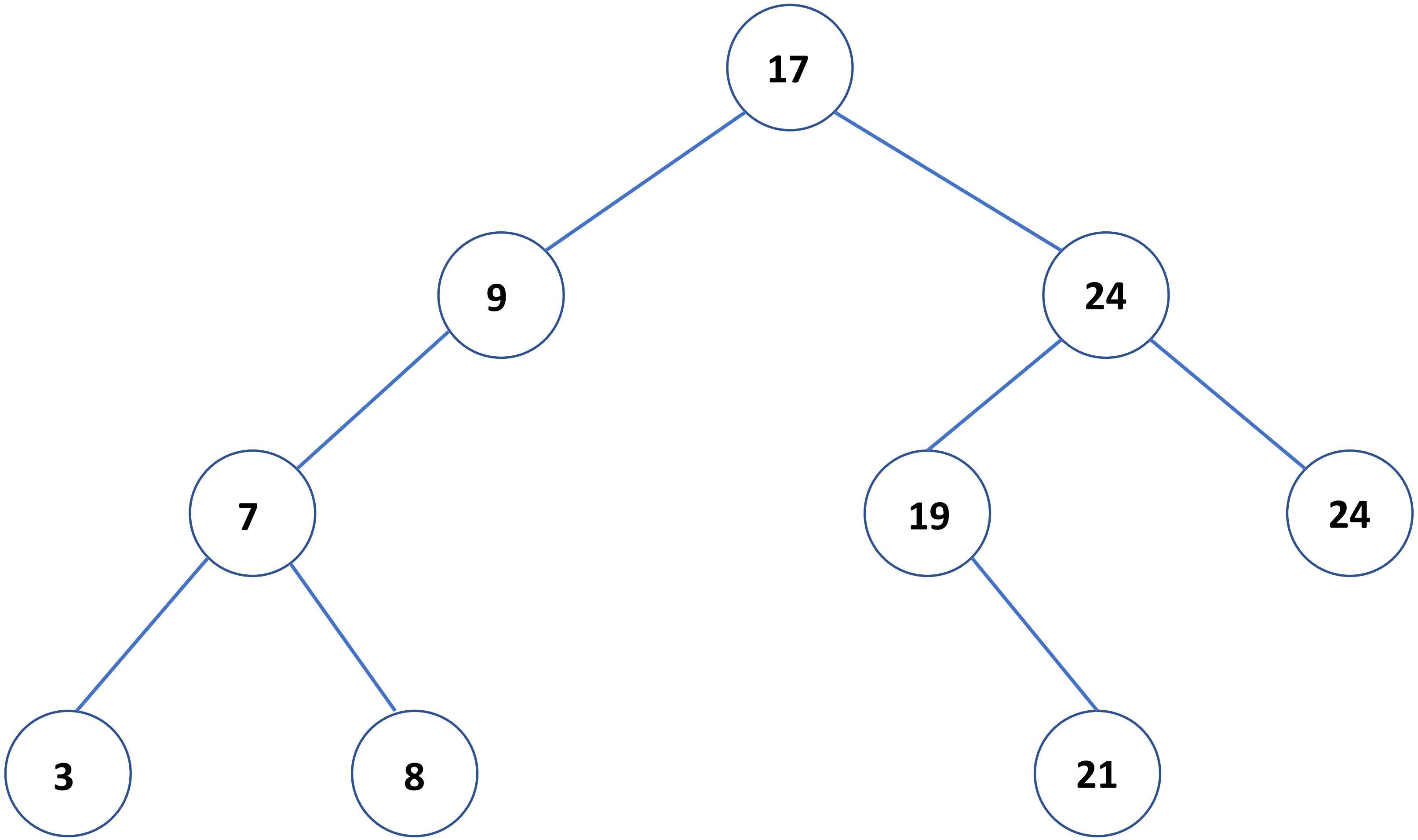
Binary Search Tree (BST)



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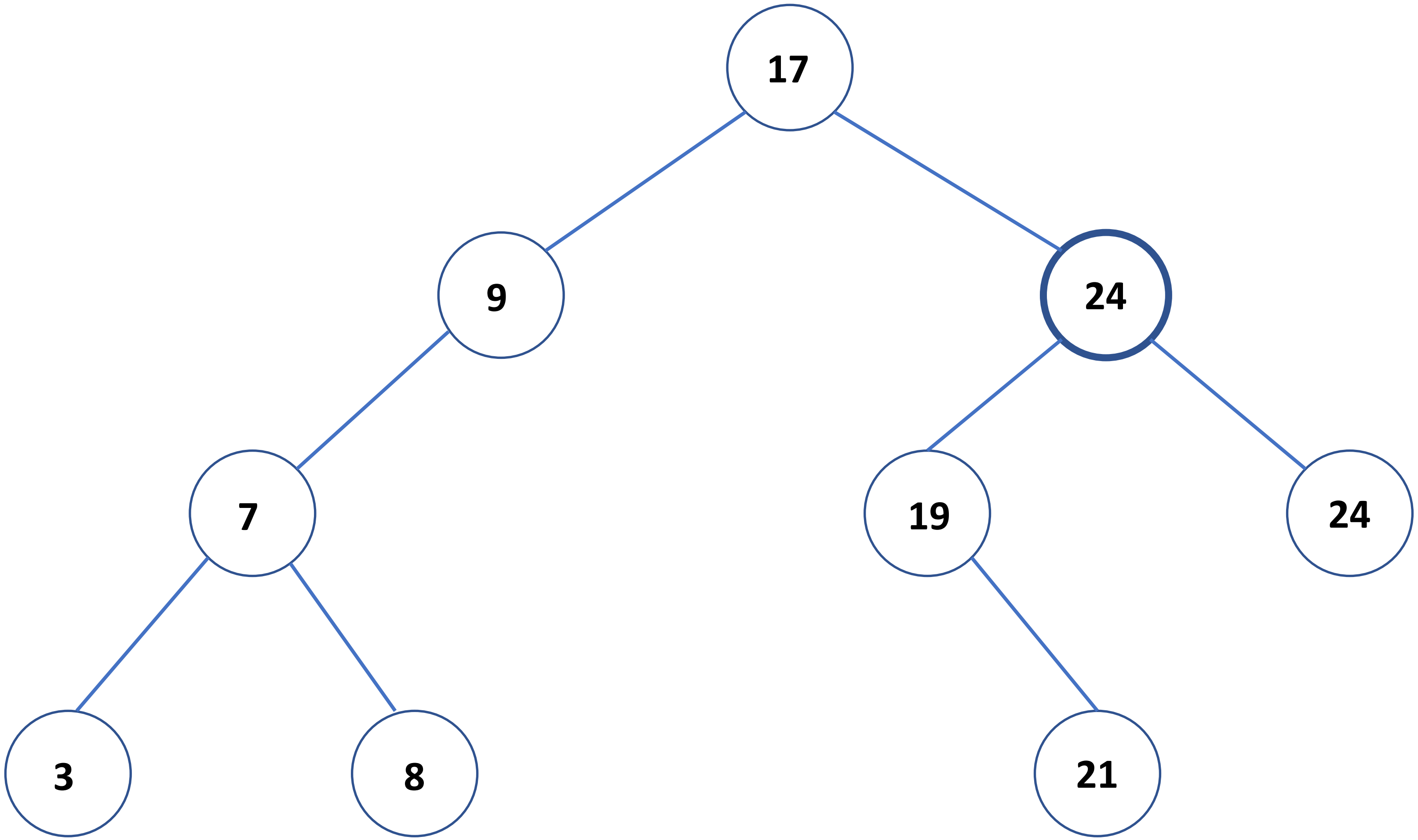

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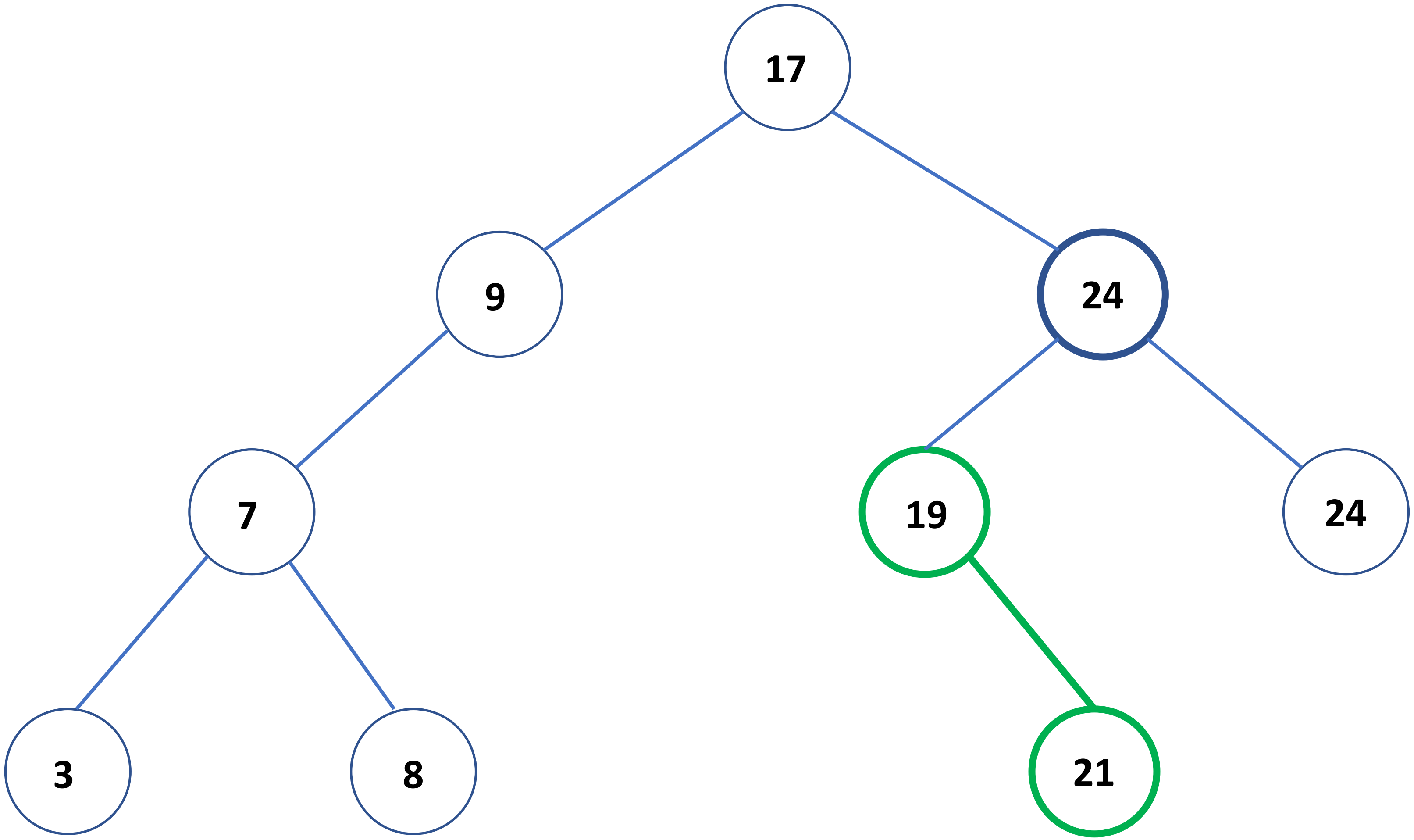

For each node: keys in left subtree <= the node’s key <= keys in right subtree

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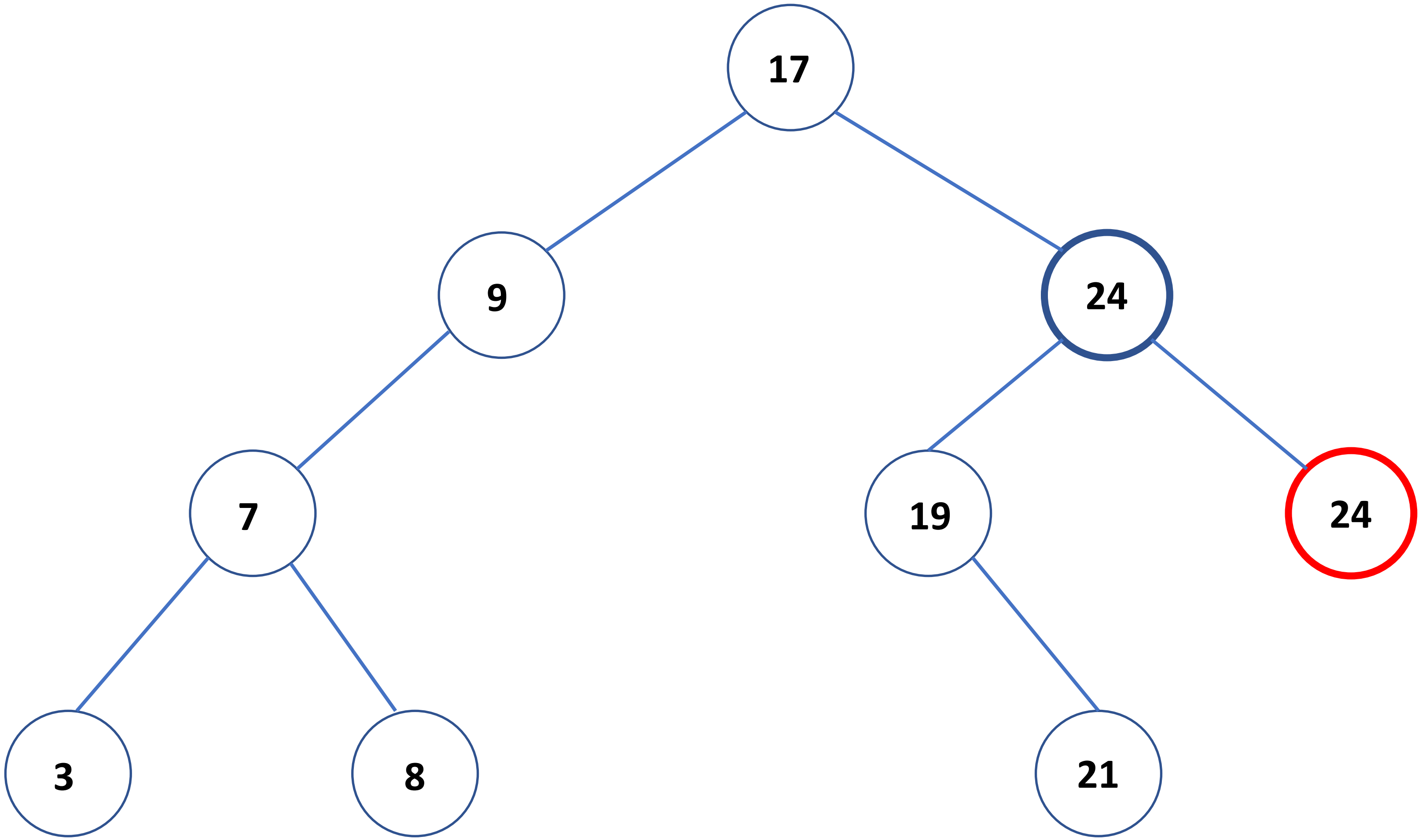

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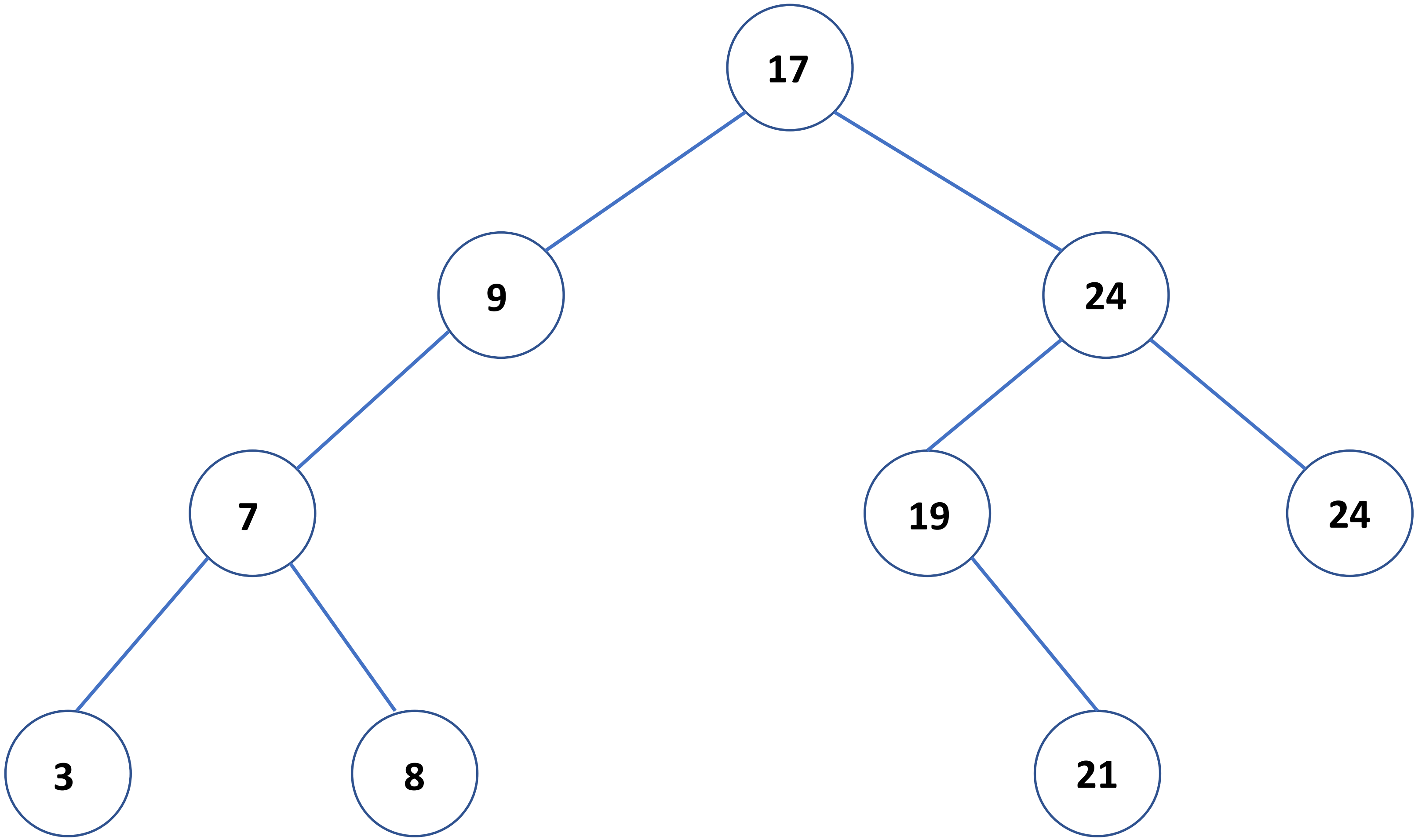
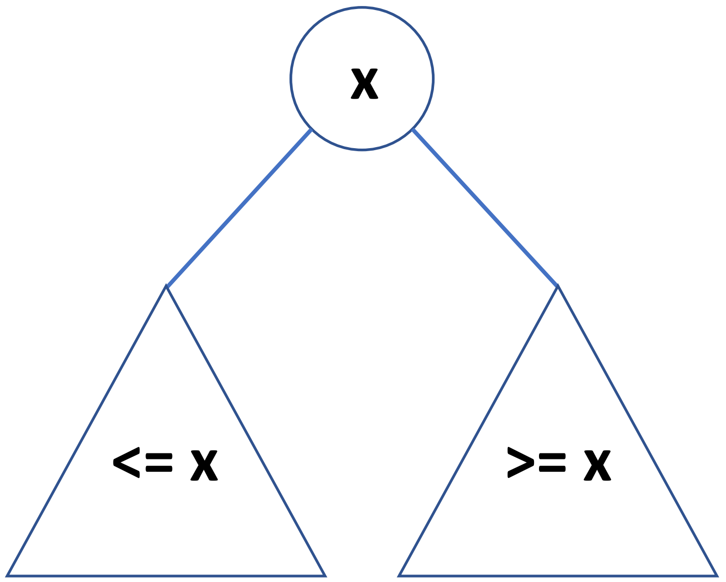


Keys are not necessarily unique!

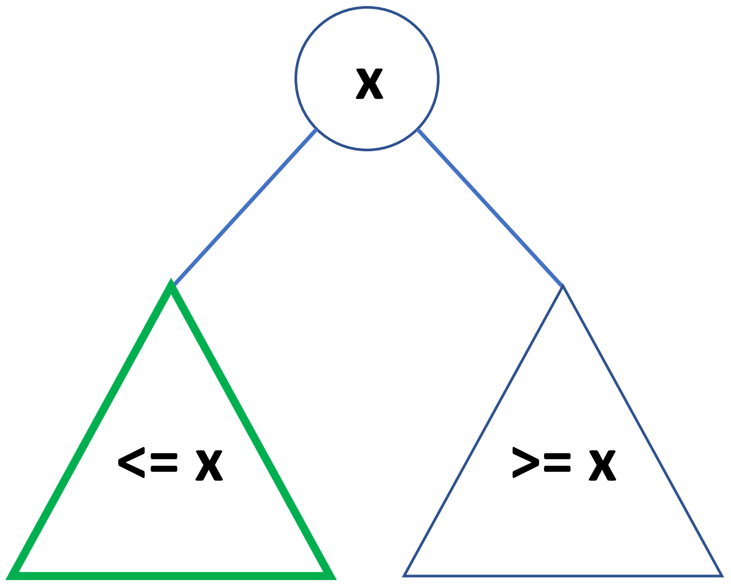
For each node: keys in left subtree <= the node's key <= keys in right subtree

In-order Traversal



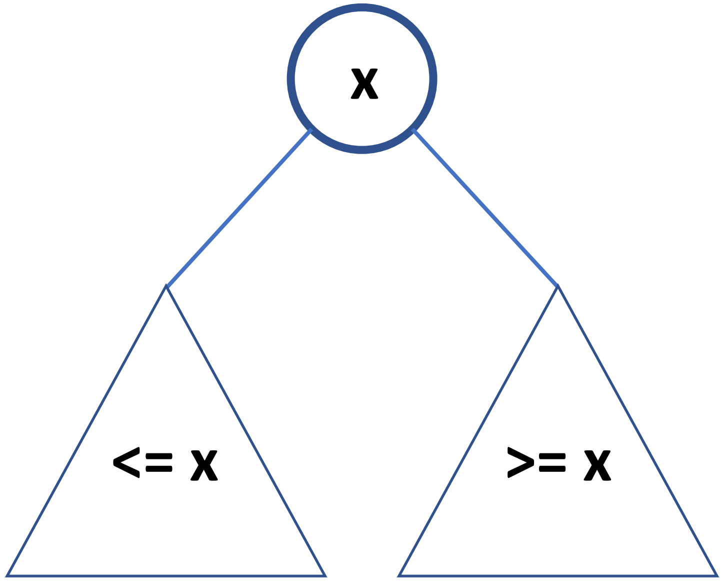
In-order Traversal

- Traverse node's left subtree recursively



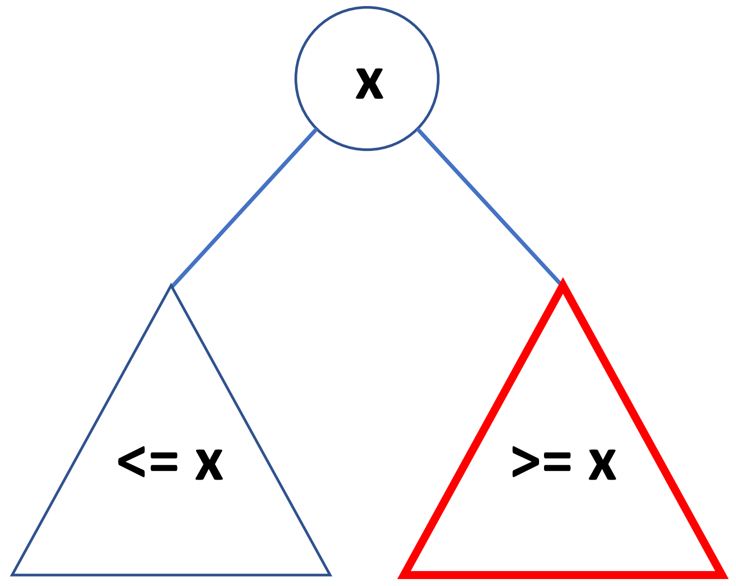
In-order Traversal

- Traverse node's left subtree recursively
- Visit node



In-order Traversal

- Traverse node’s left subtree recursively
- Visit node
- Traverse node’s right subtree recursively



In-order Traversal

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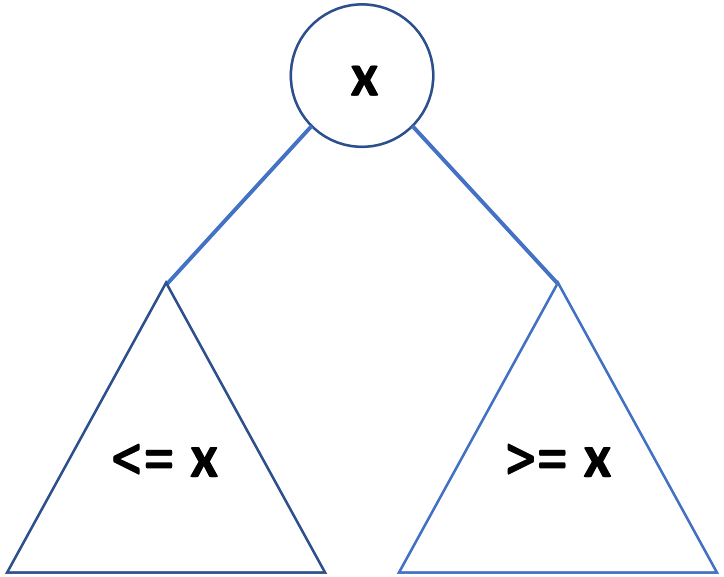

Fact: In-order traversal of a BST visits keys in ascending sorted order

Is the BST for a set S of keys unique?

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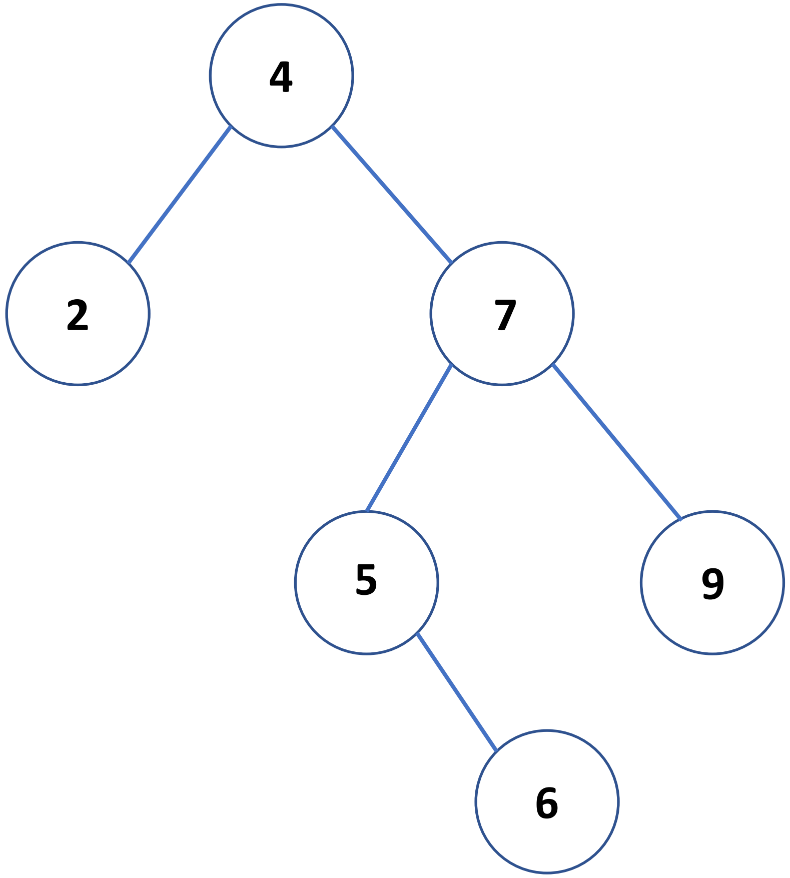

S : {2, 4, 5, 6, 7, 9}

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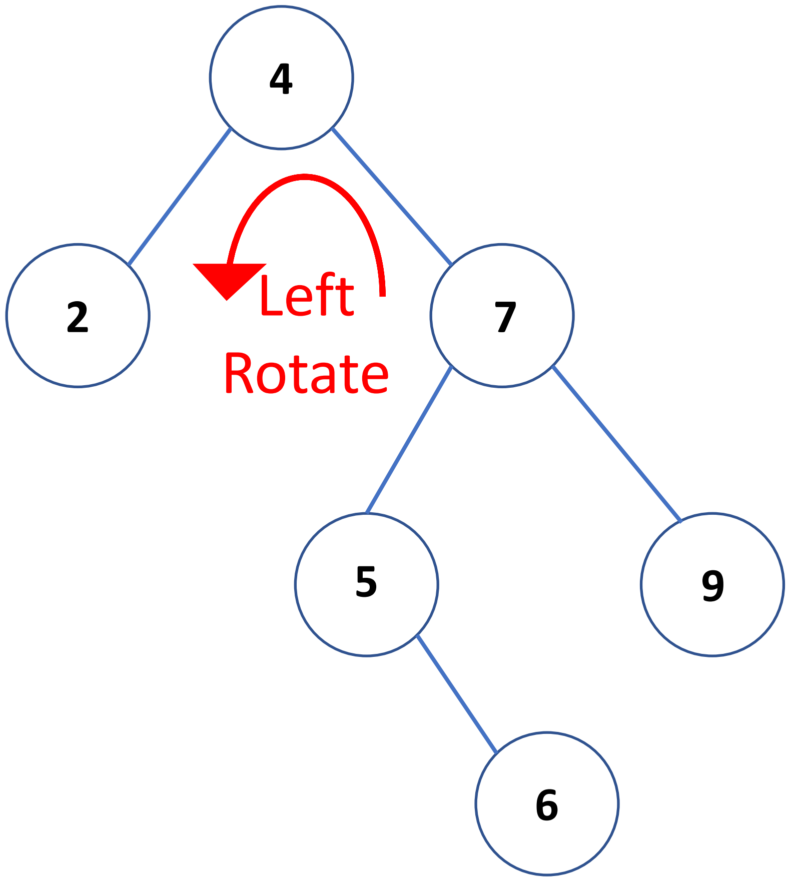

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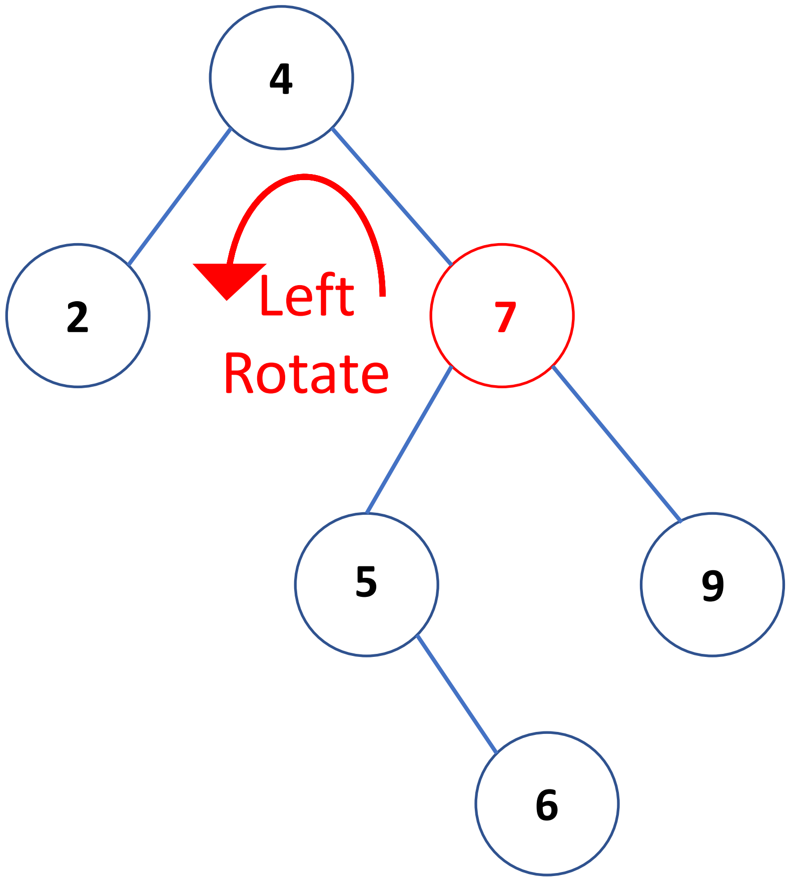

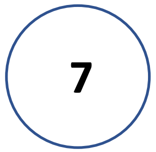

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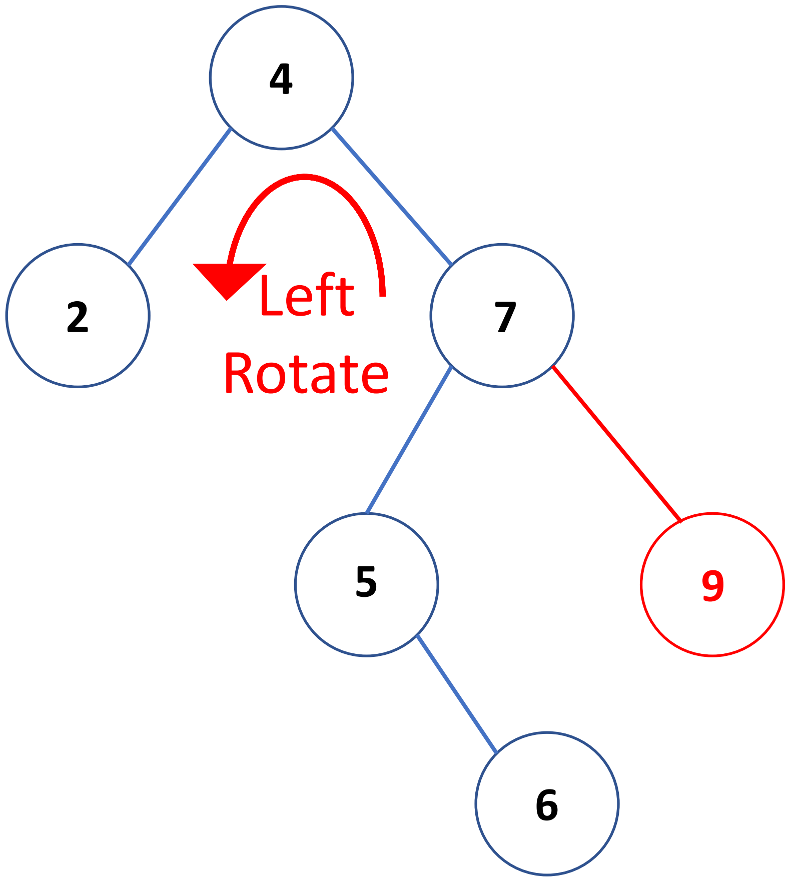

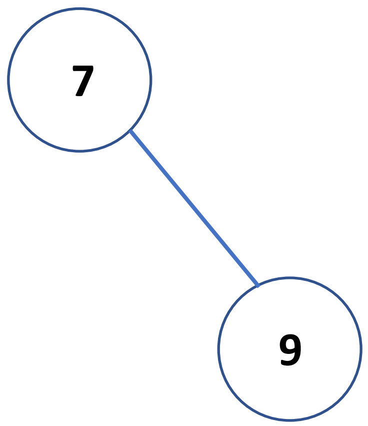

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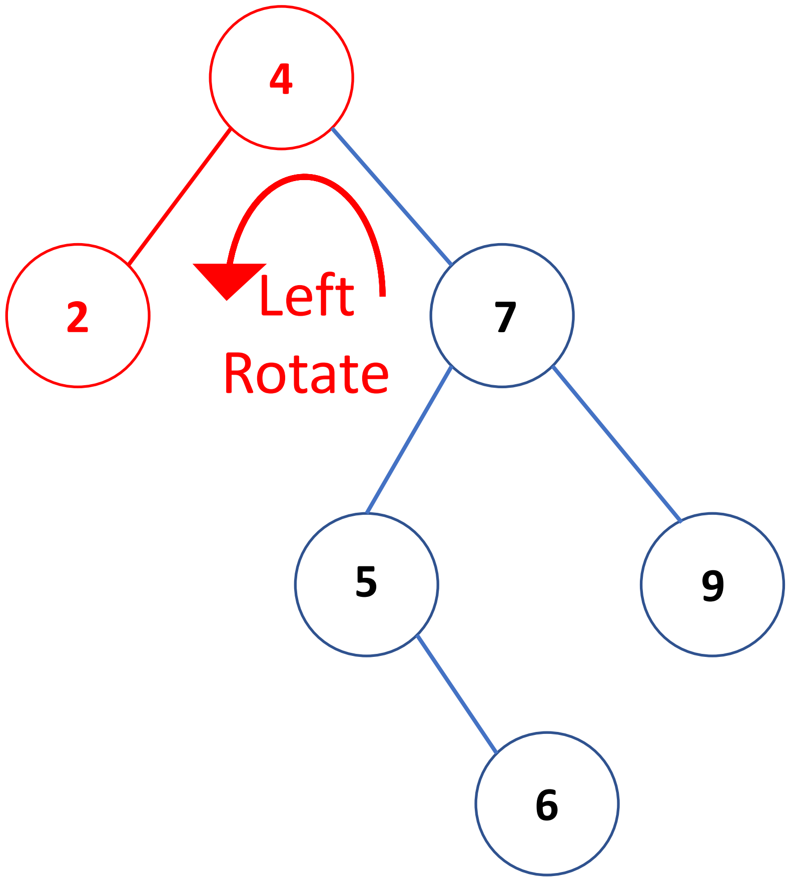

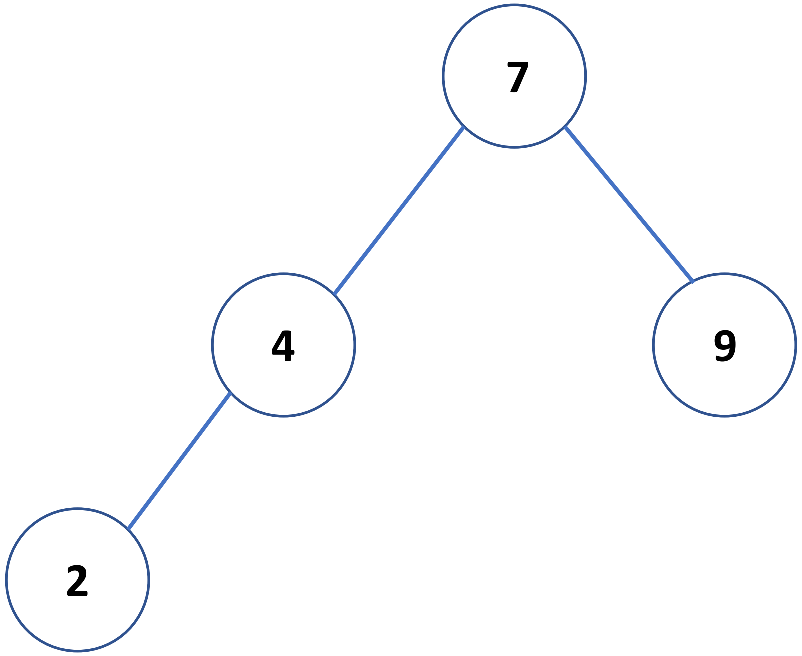

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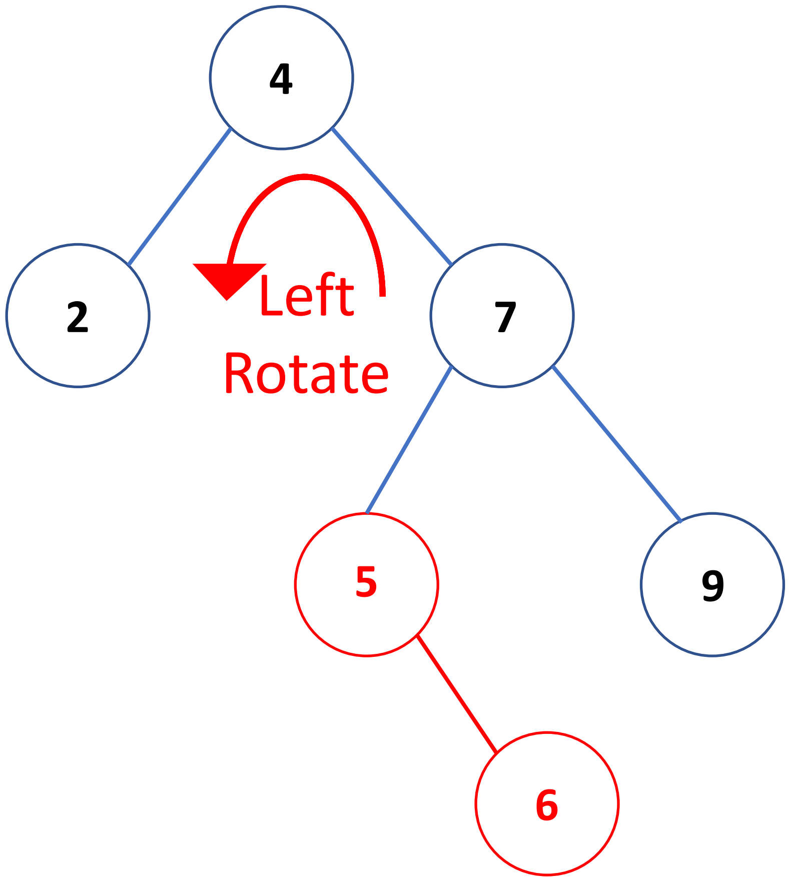

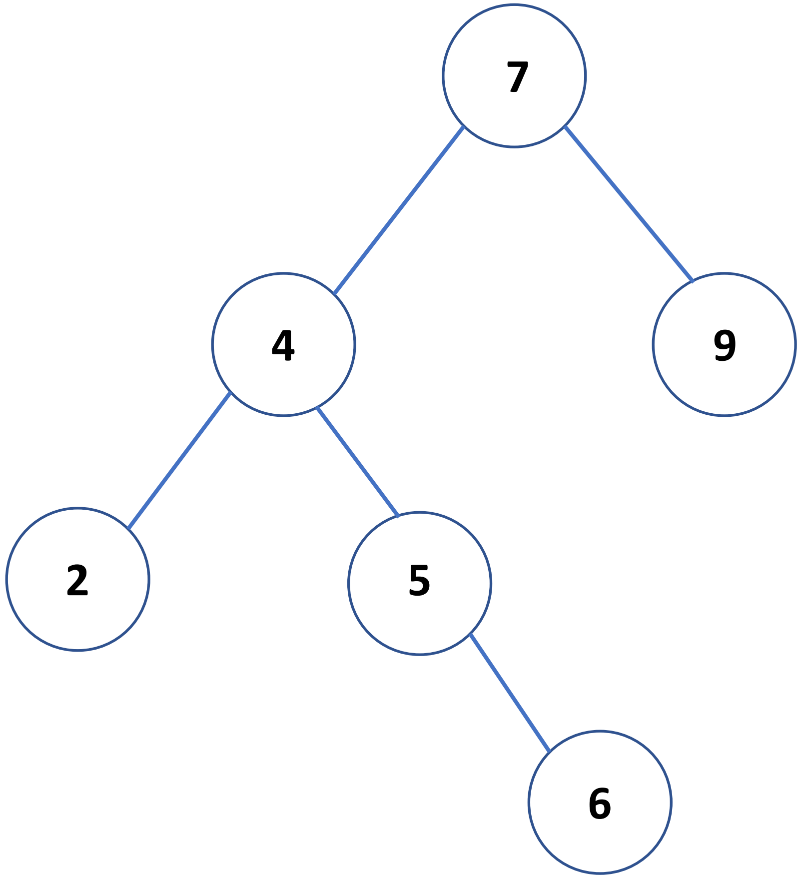

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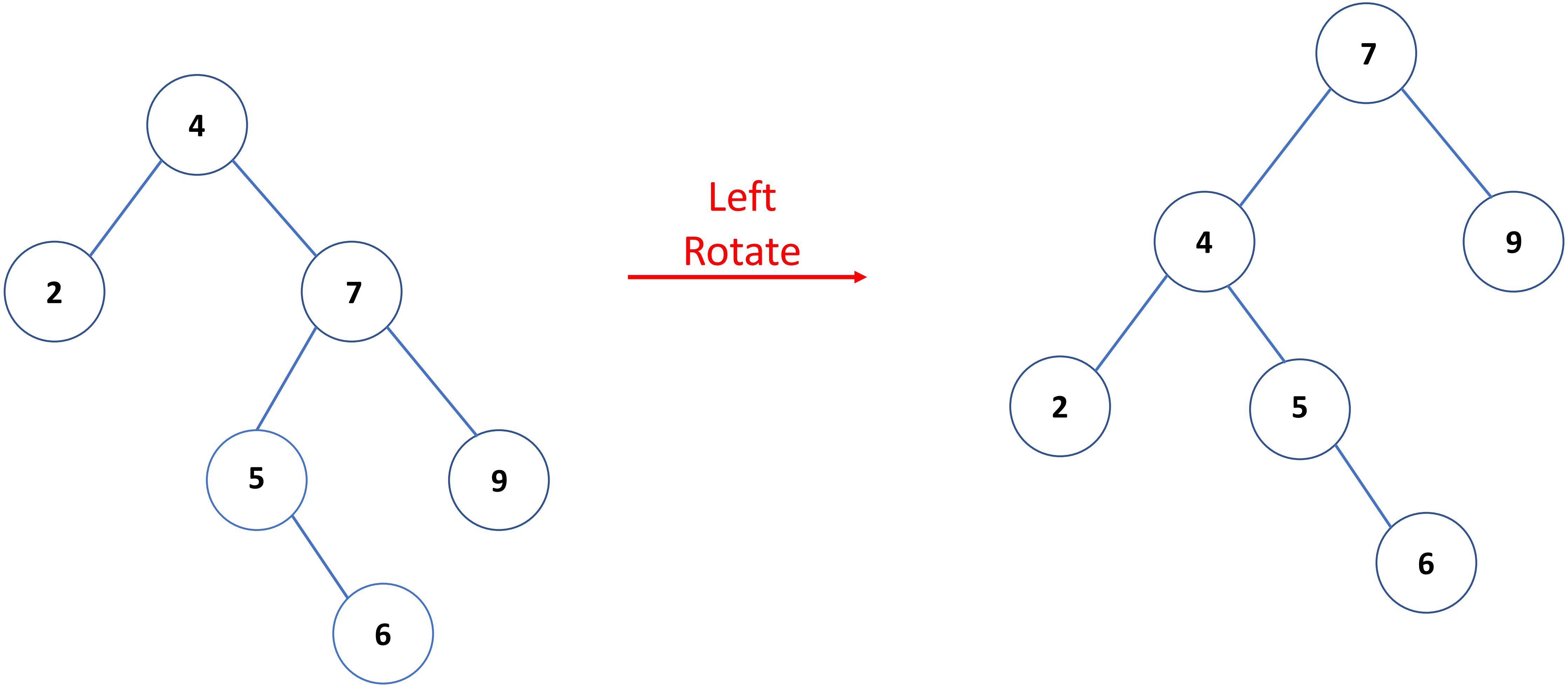

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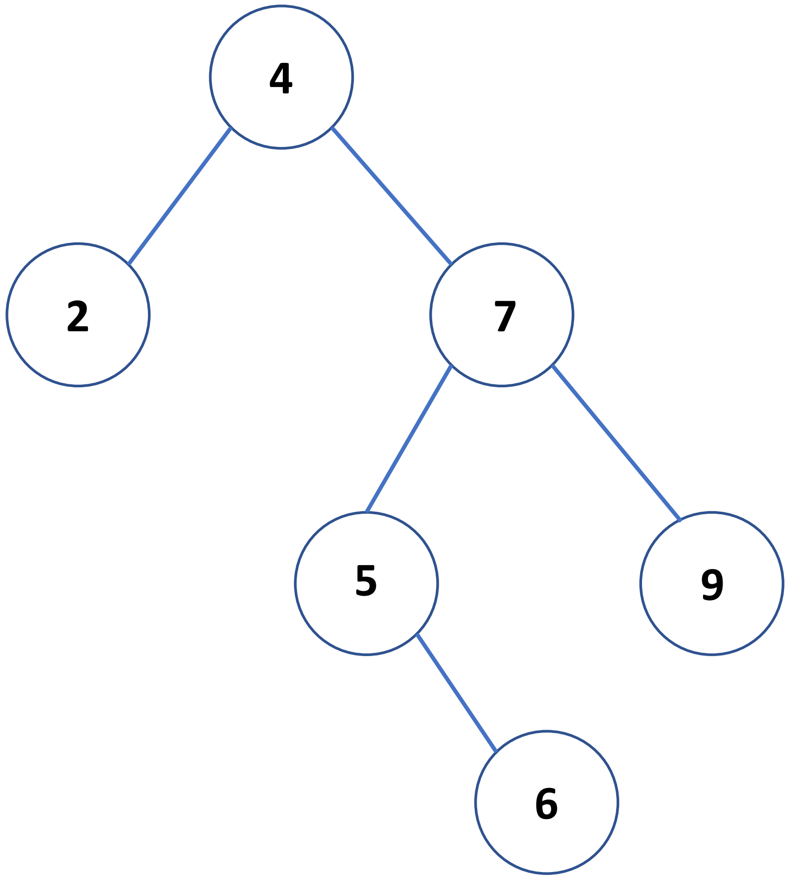
Operations on Binary Search Tree T

- **Search**(T, x)
- **Insert**(T, x)
- **Delete**(T, x)

We show how they work by examples.

Search(T, 5)



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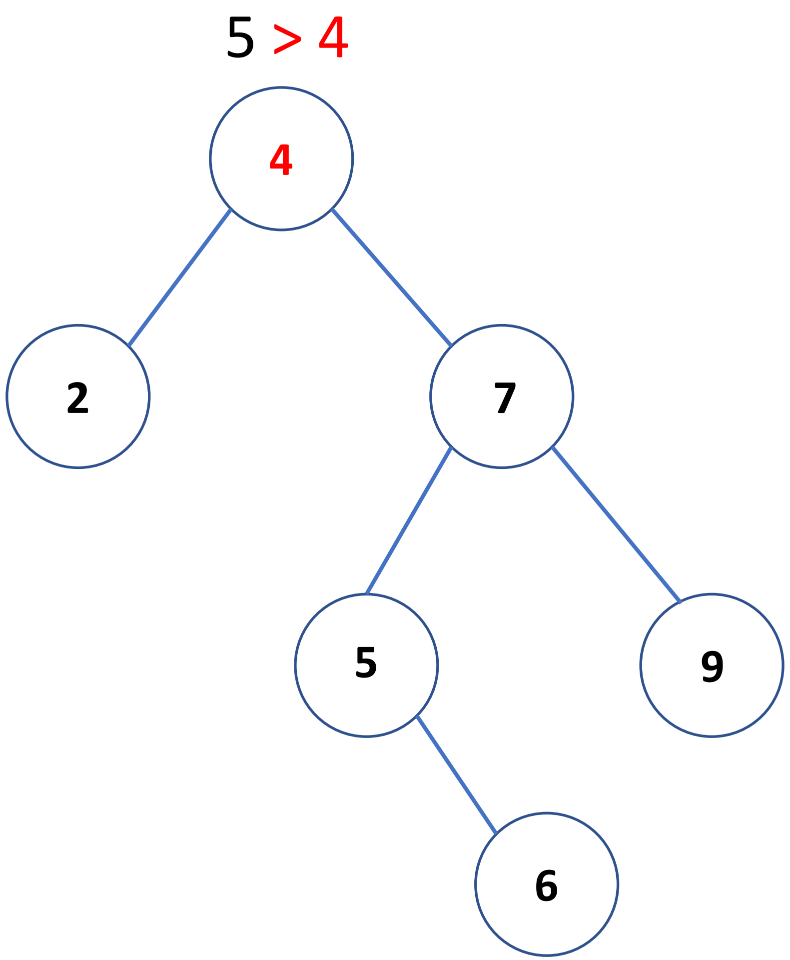

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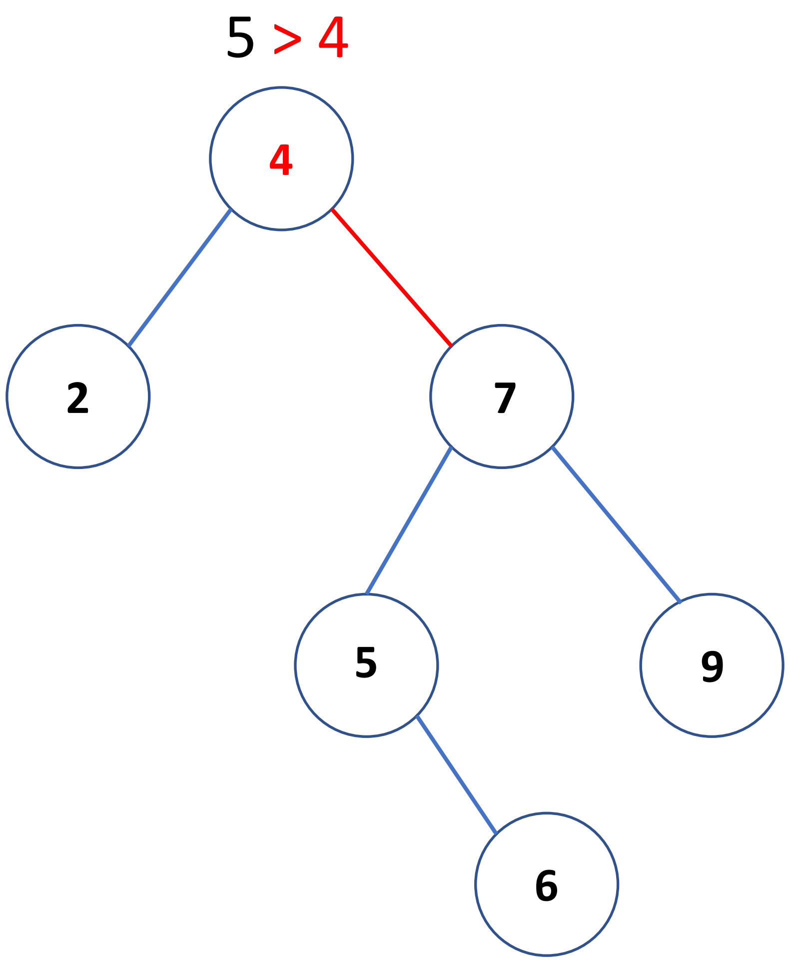

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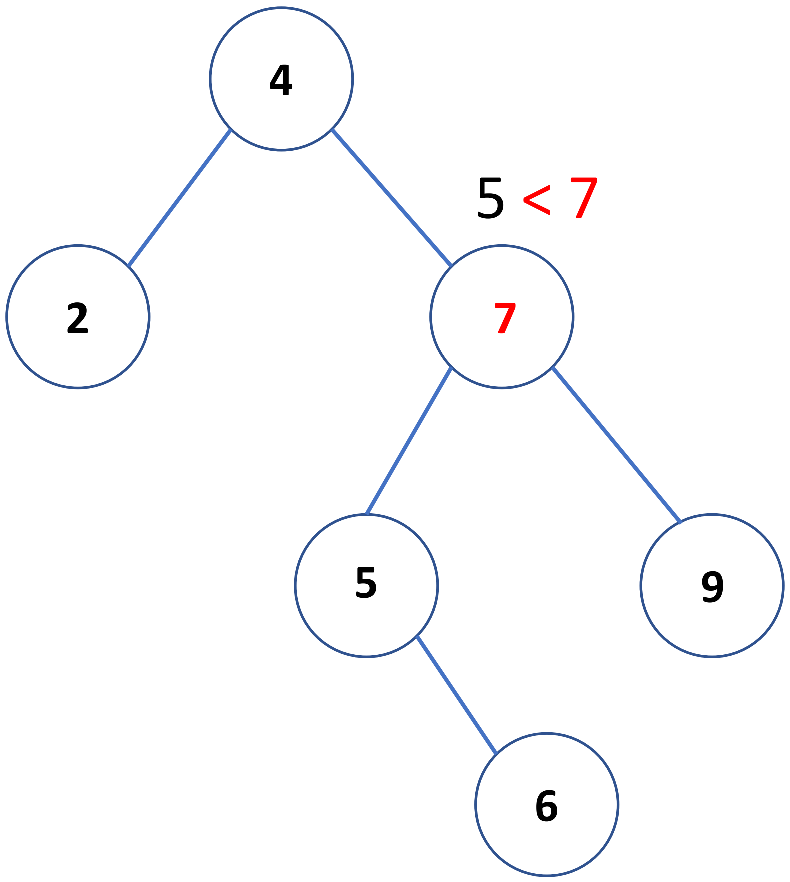

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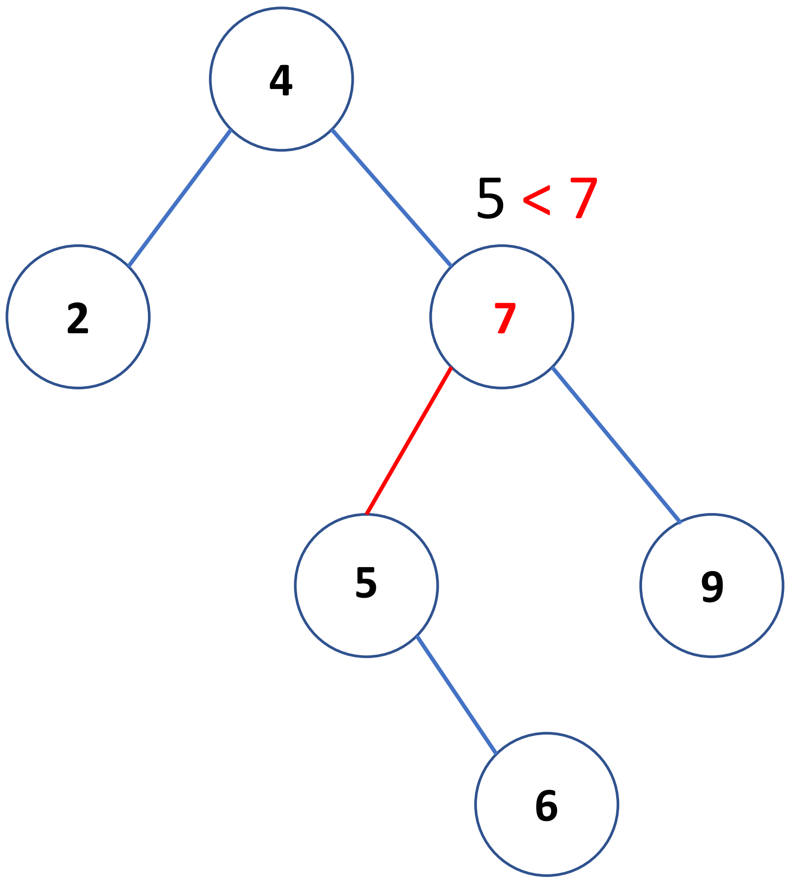

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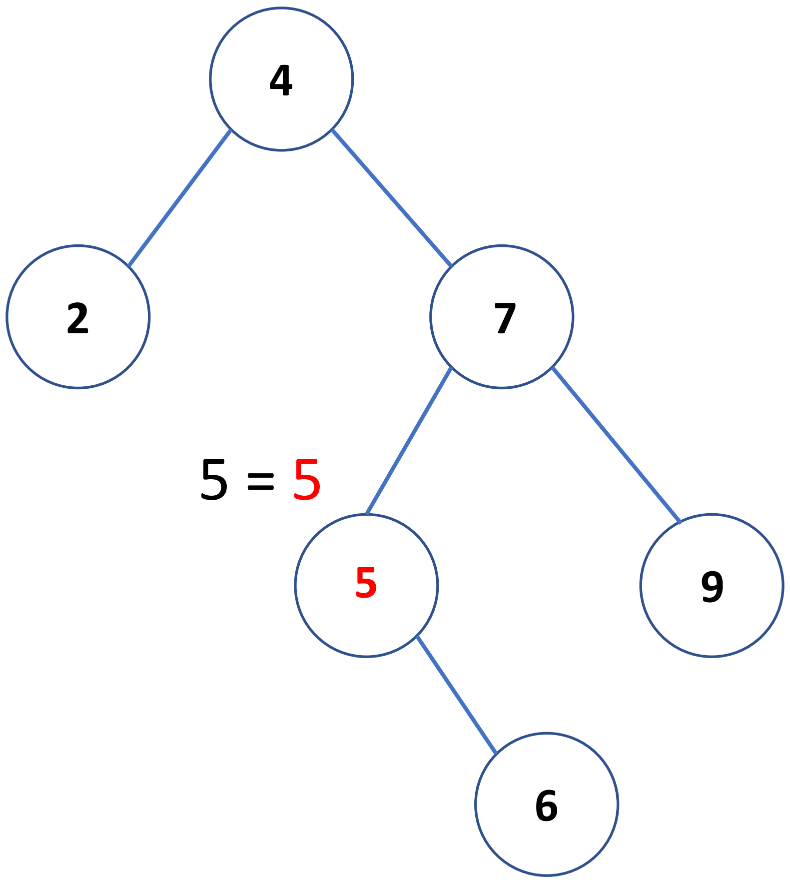

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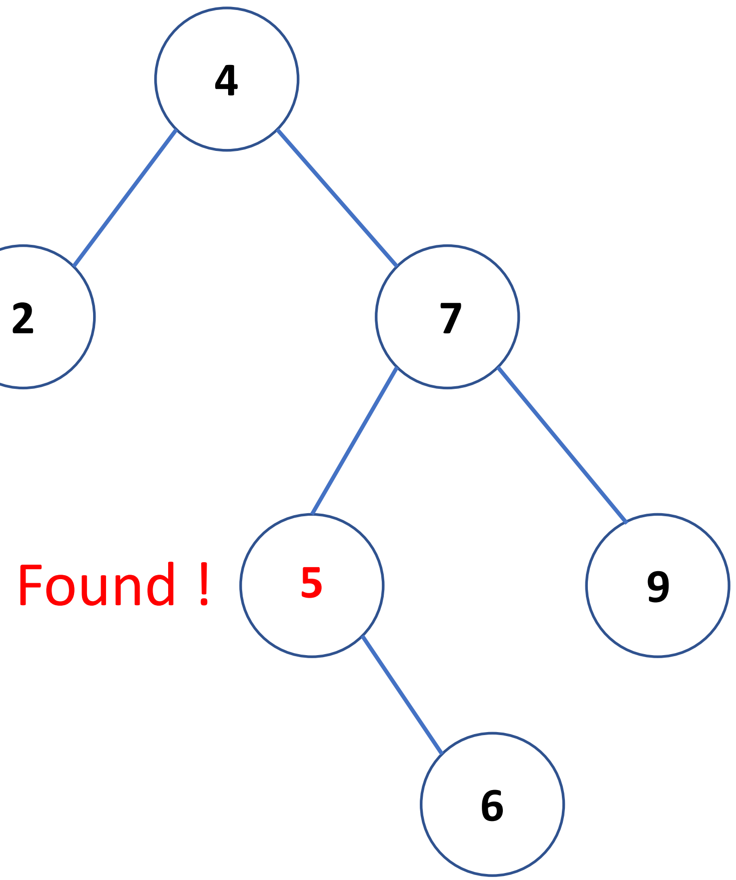
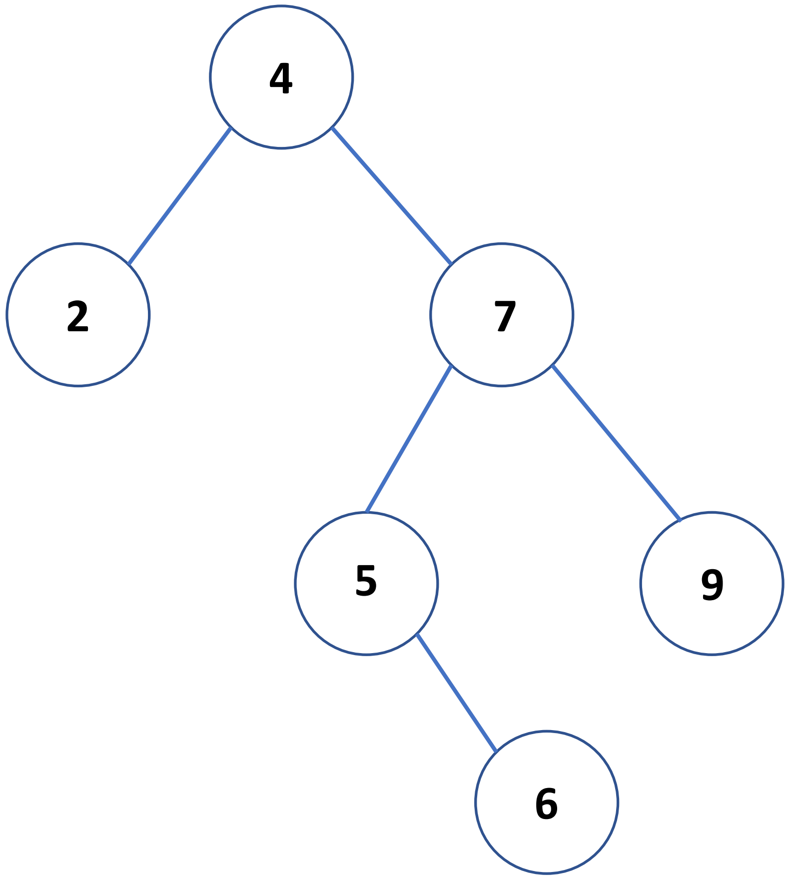
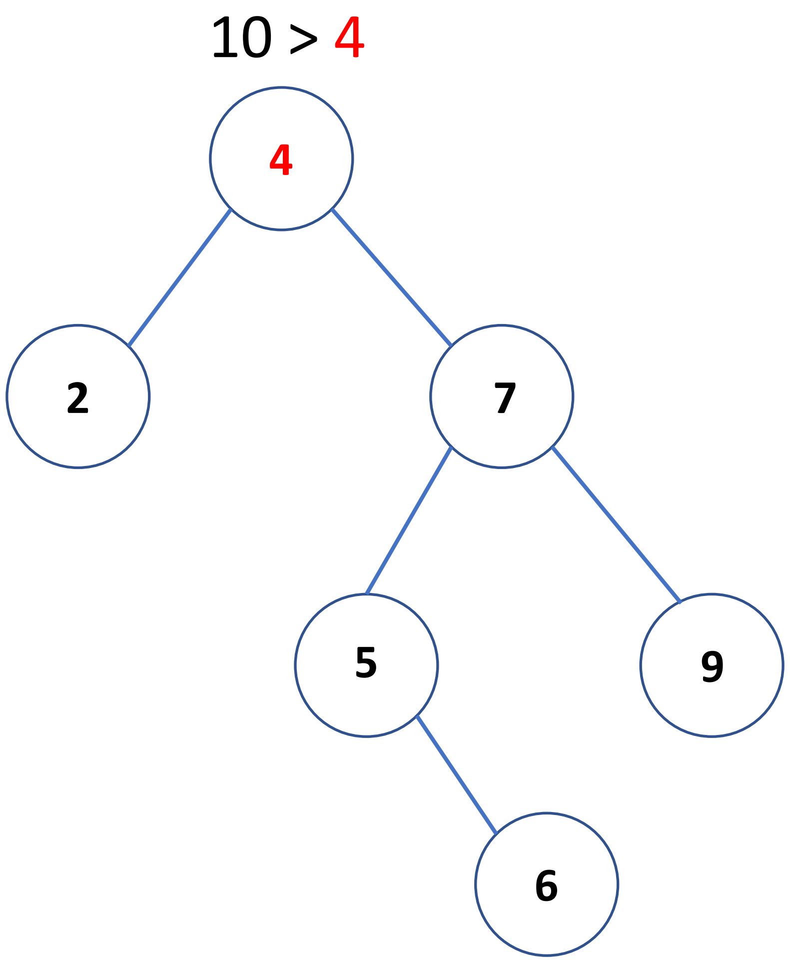


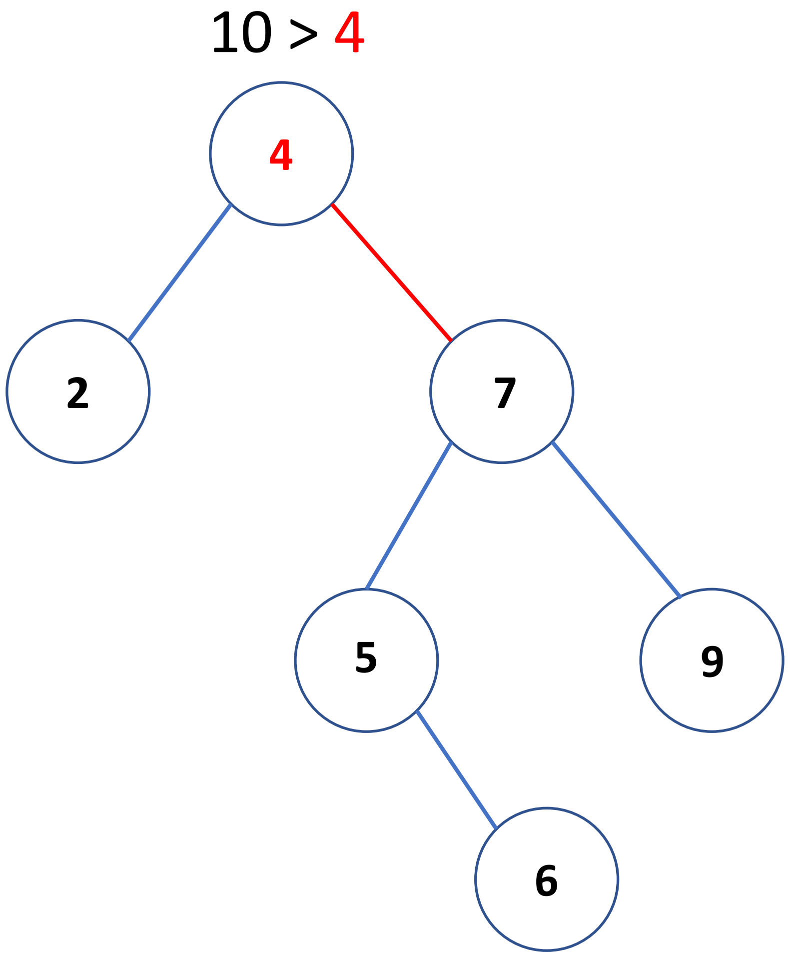
Insert(T, 10)



Insert(T, 10)



Insert(T, 10)



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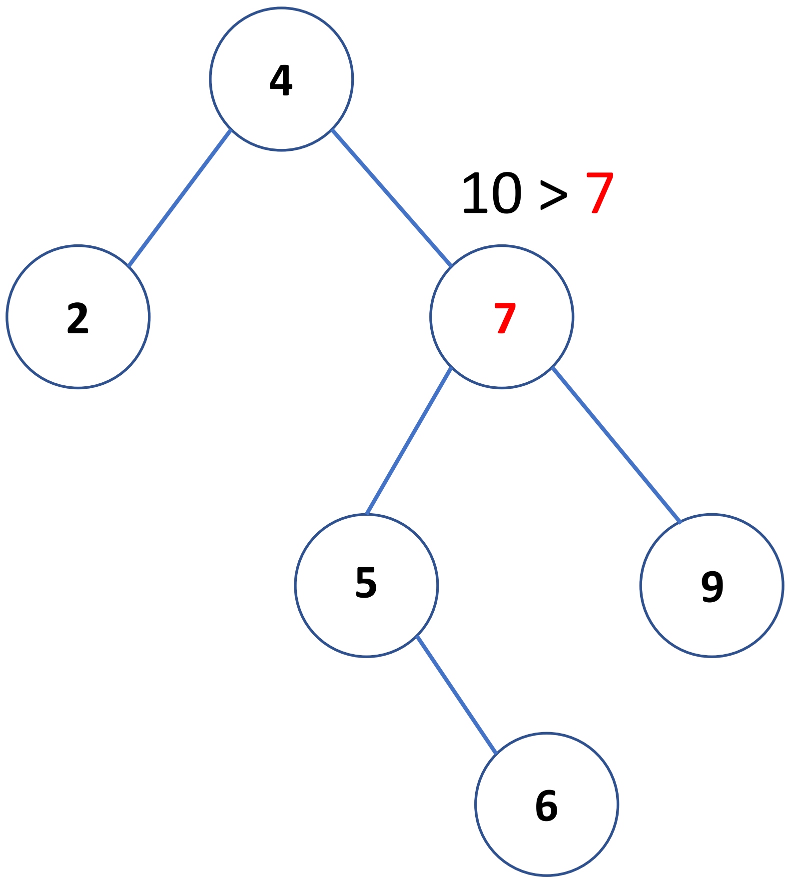

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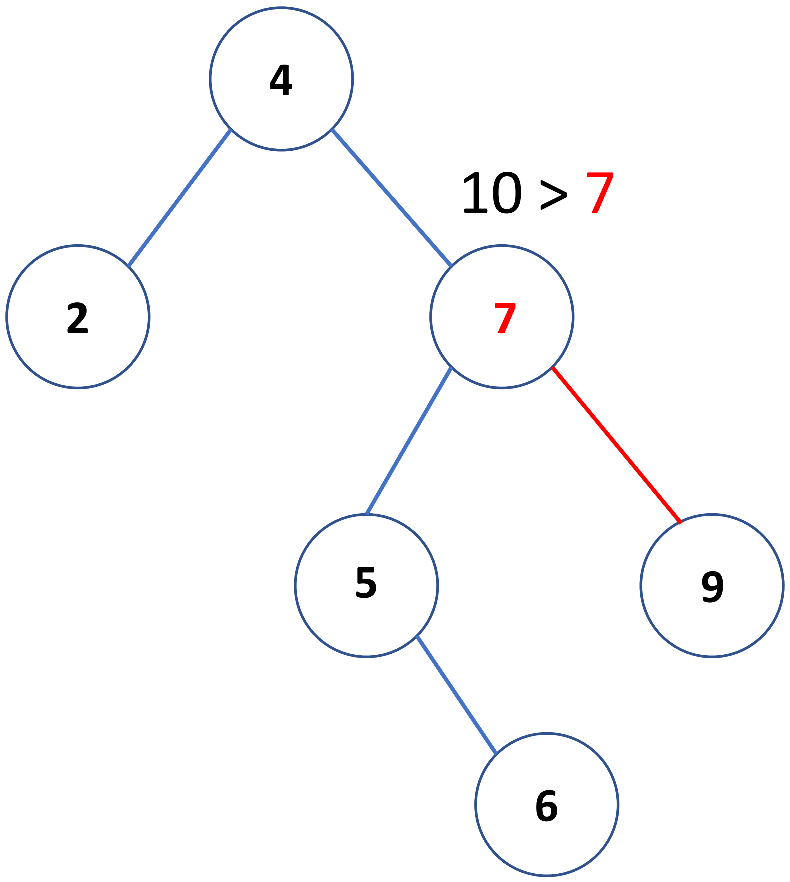

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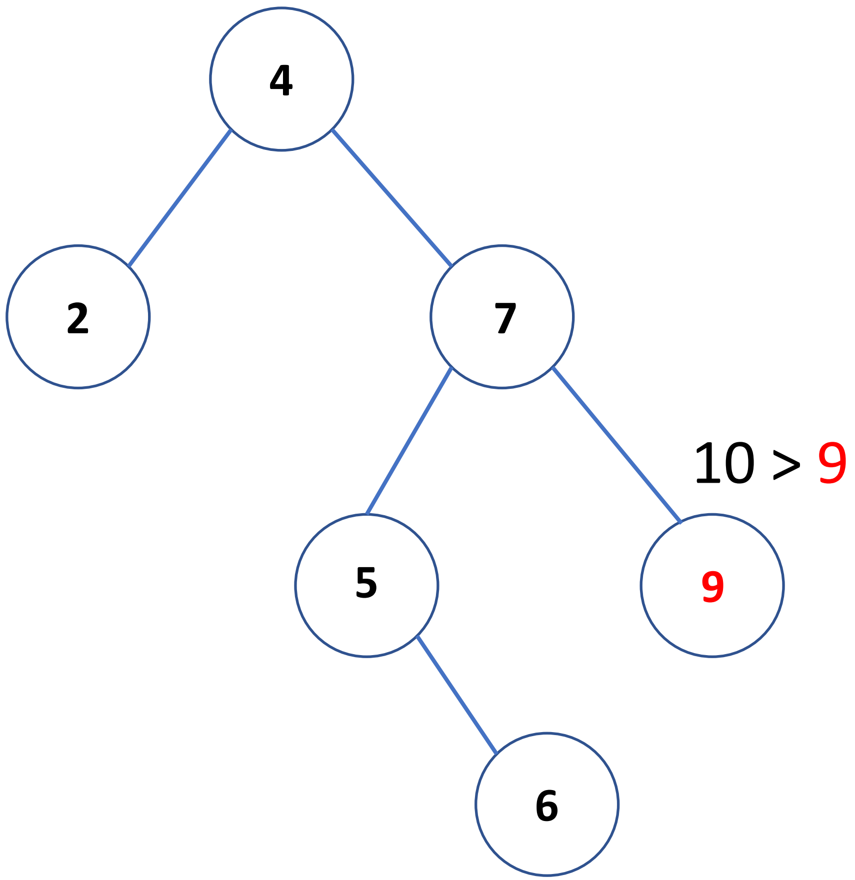

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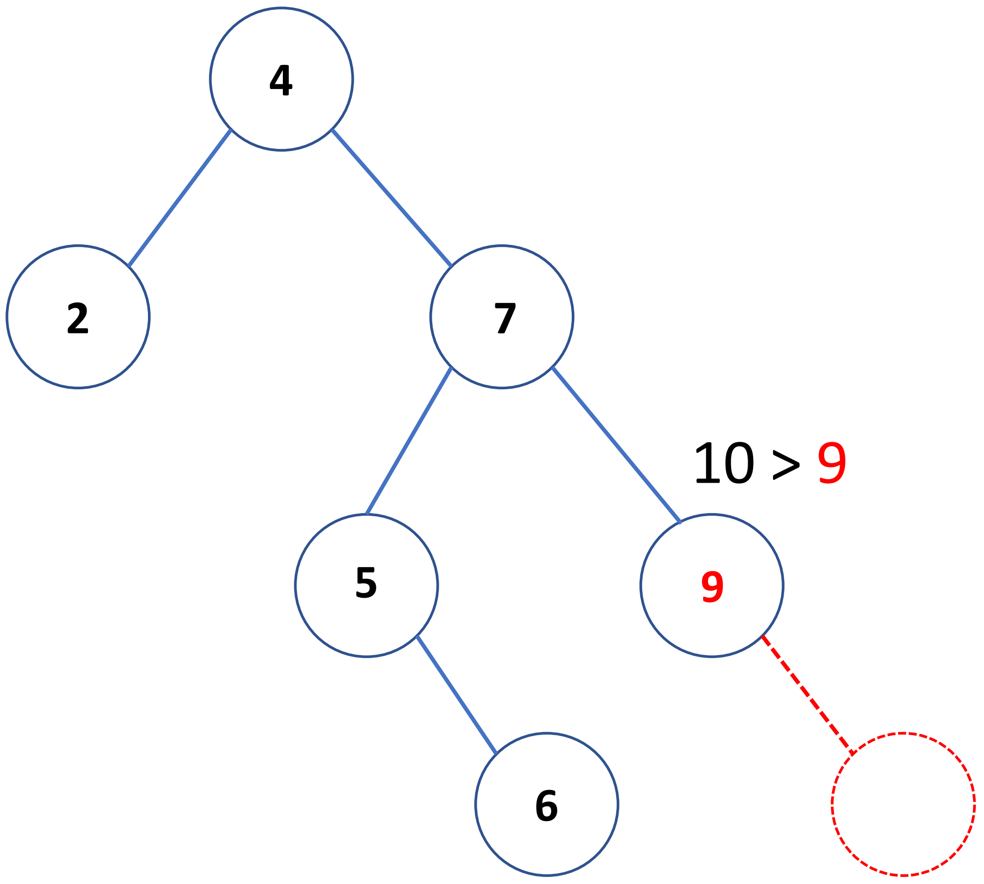

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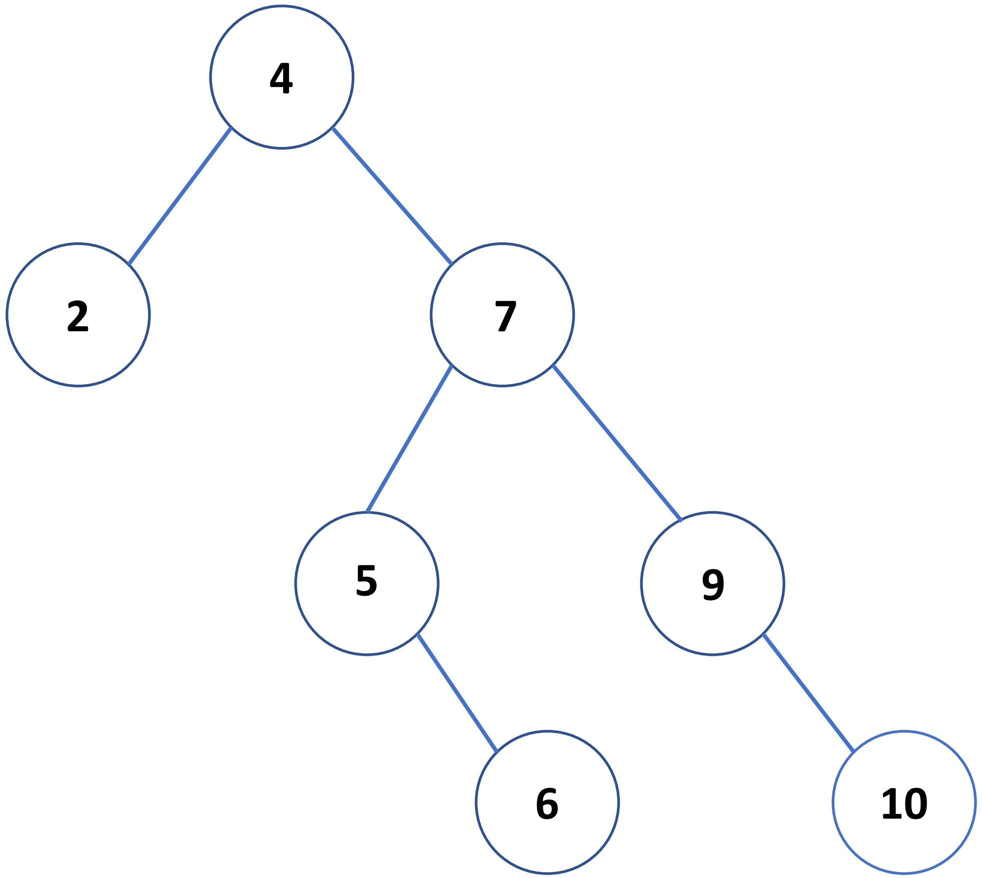
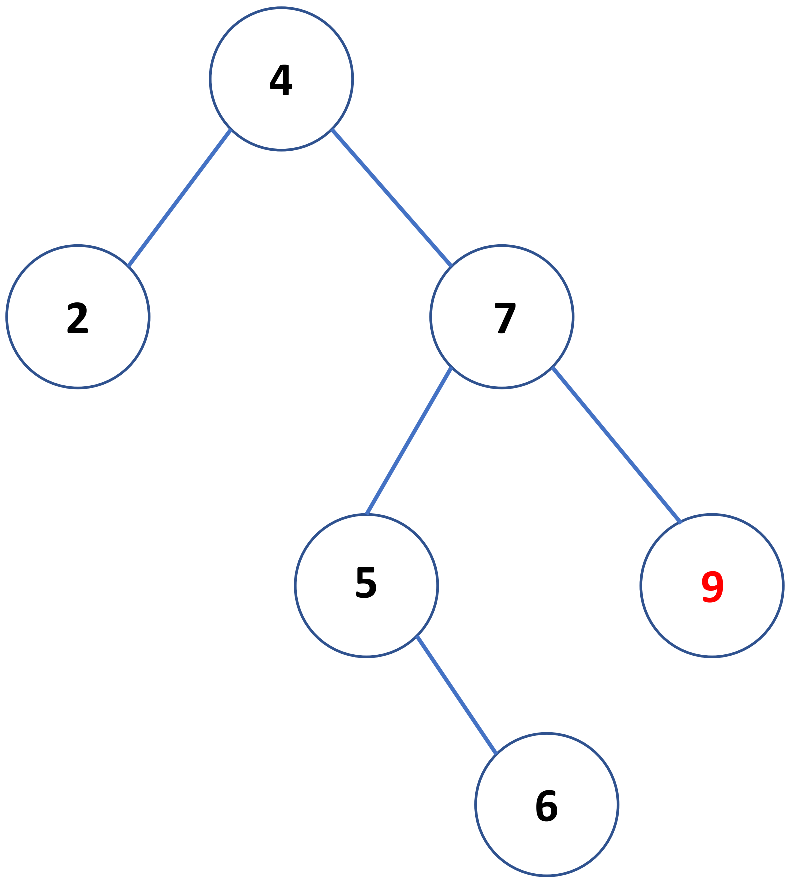


Delete(T,x)

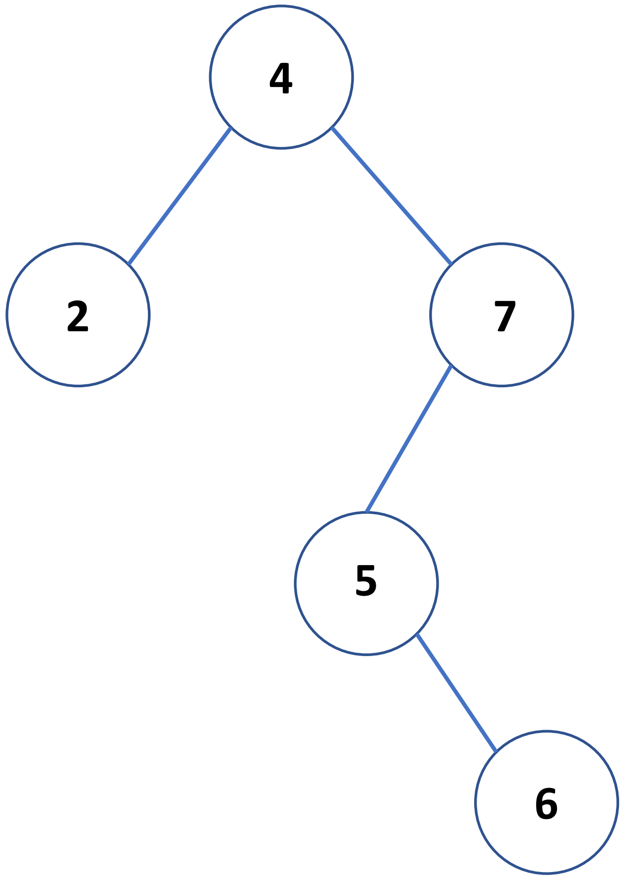
Several possible cases

Delete(T, 9)



If node to be deleted is a leaf

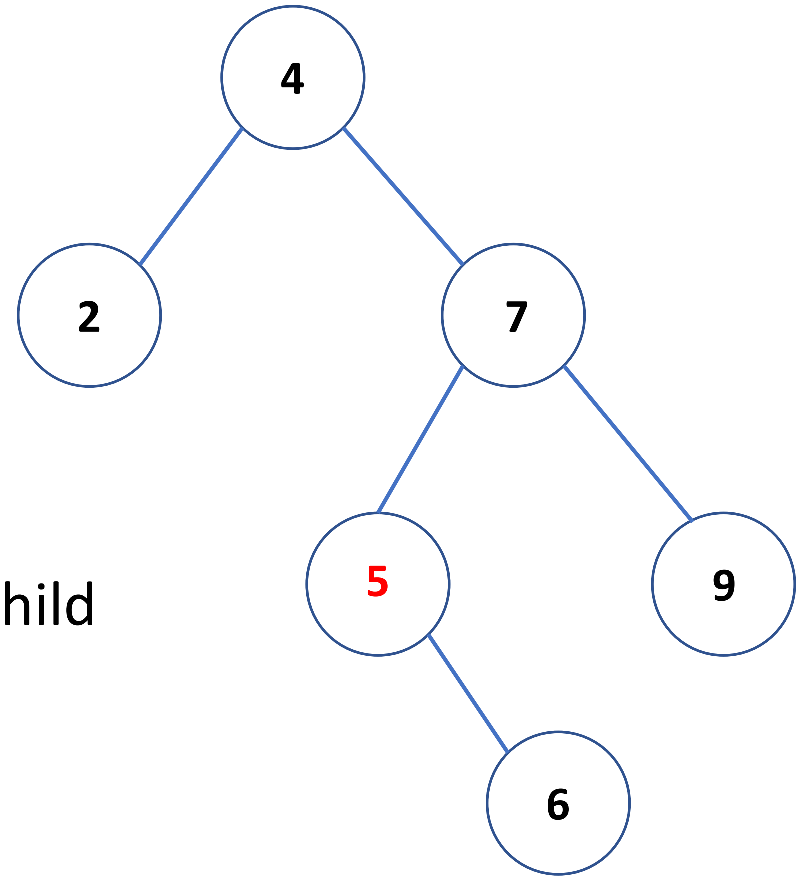
Delete(T, 9)



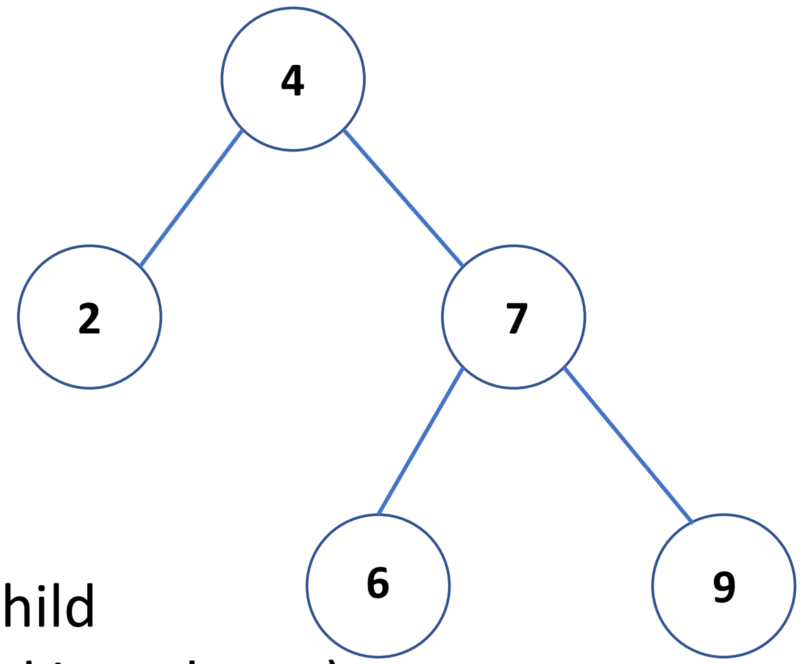
If node to be deleted is a leaf
remove the leaf

Delete(T, 5)

If node to be deleted has one child

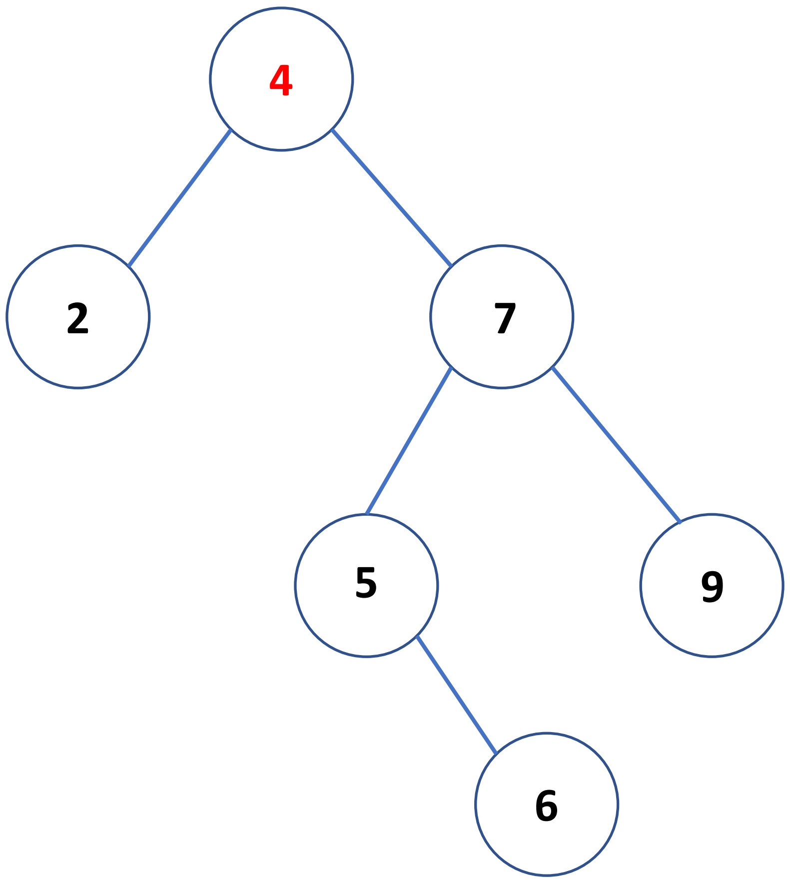


Delete(T, 5)



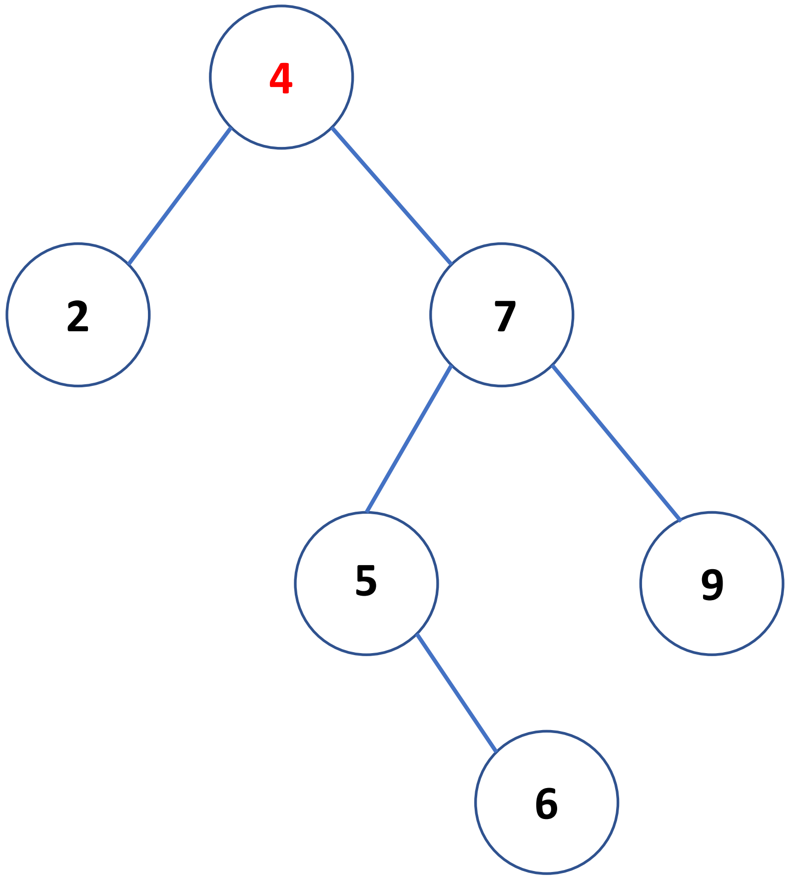
If node to be deleted has one child
replace the node with child (and its subtree)

Delete(T, 4)



Delete(T, 4)

If node to be deleted has two children:



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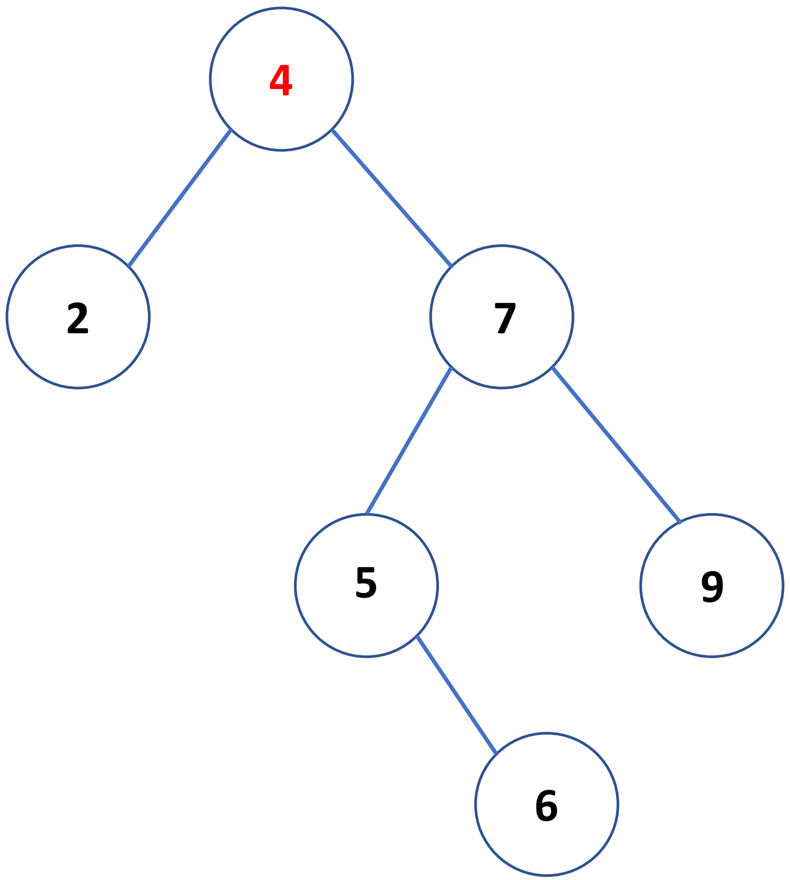

S : 2 4 5 6 7 9

Delete(T, 4)

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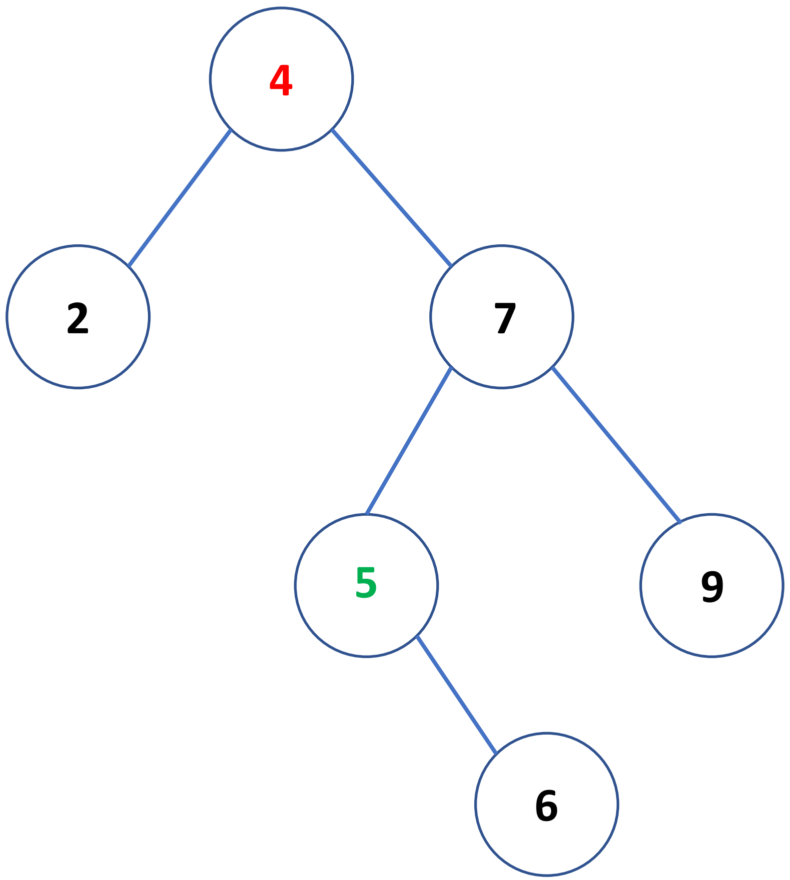
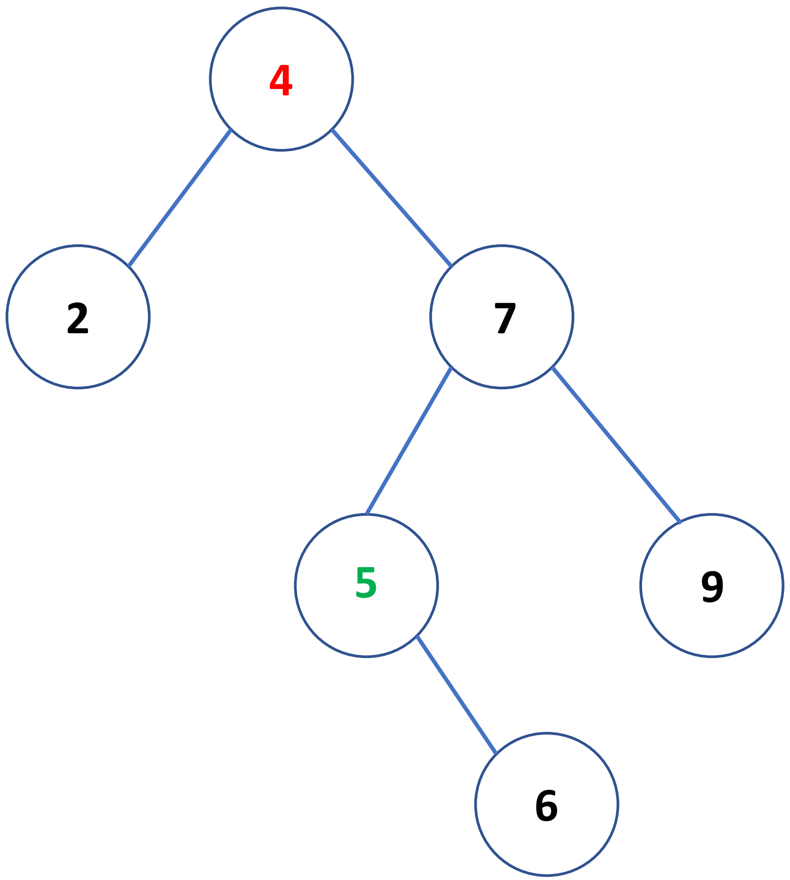


S : 2 4 5 6 7 9

Delete(T, 4)

If node to be deleted has two children:

- Find the "successor" of the node

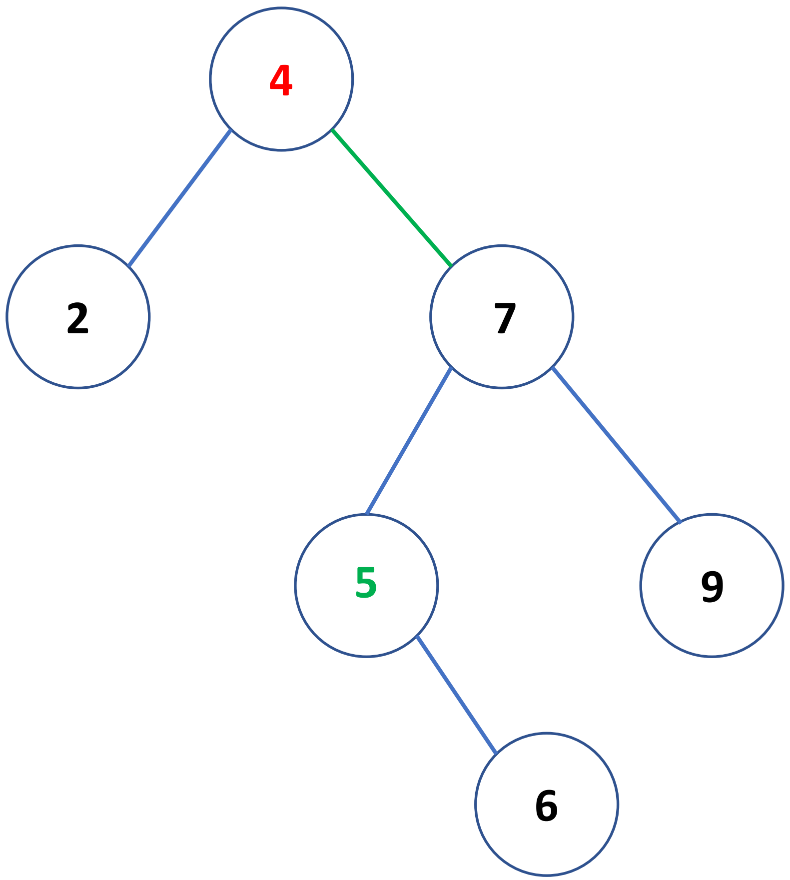


S : 2 4 5 6 7 9

Delete(T, 4)

If node to be deleted has two children:

- Find the “successor” of the node
  - Go **one step** down **right**



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Delete(T, 4)

If node to be deleted has two children:

- Find the "successor" of the node
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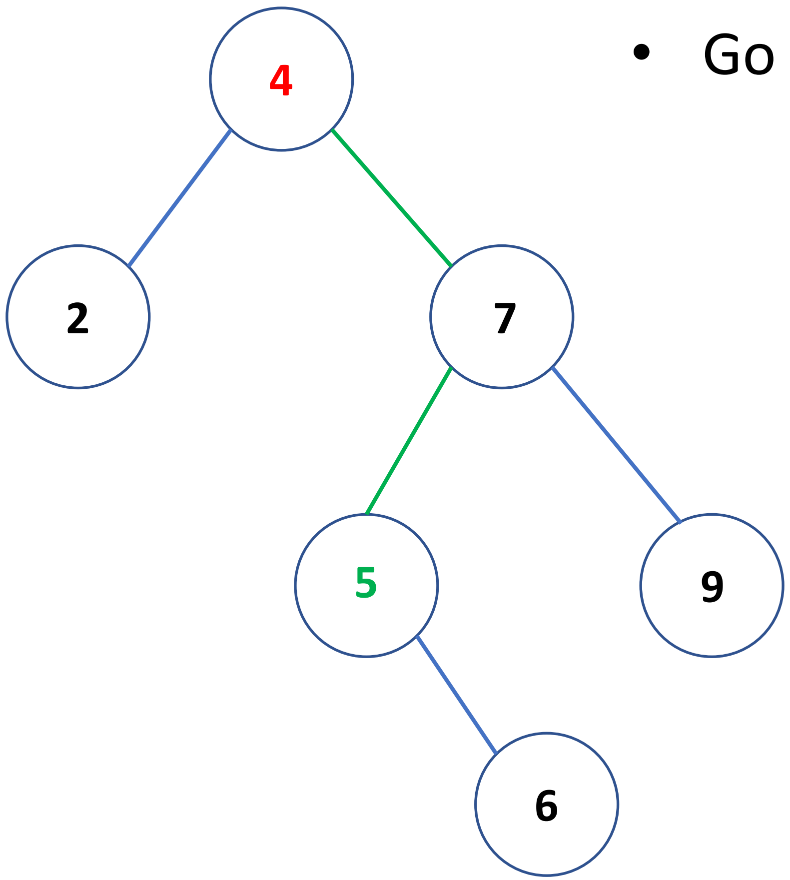
  - Go **all the way** down to the **left**



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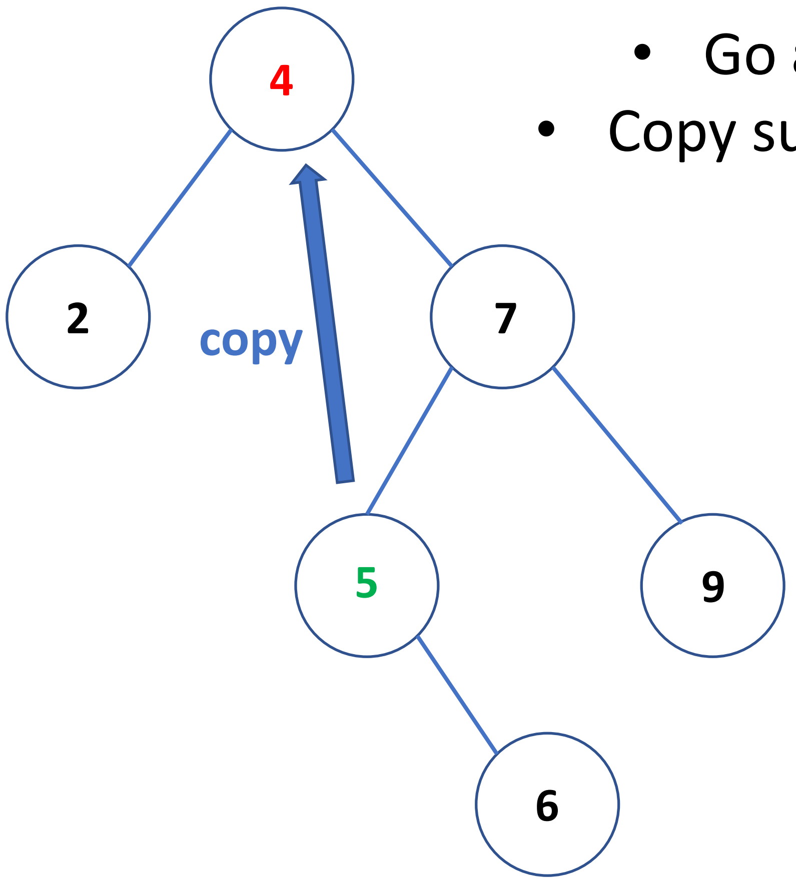
Delete(T, 4)

If node to be deleted has two children:

- Find the “successor” of the node
  - Go **one step** down **right**
  - Go **all the way** down to the **left**
- Copy successor’s key into node’s key



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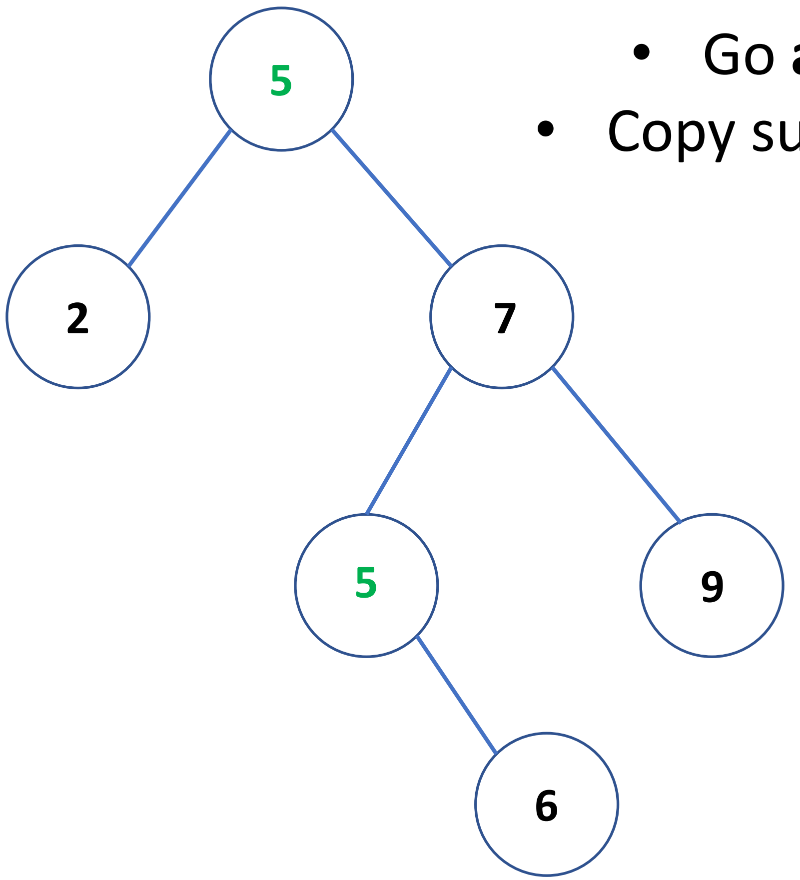

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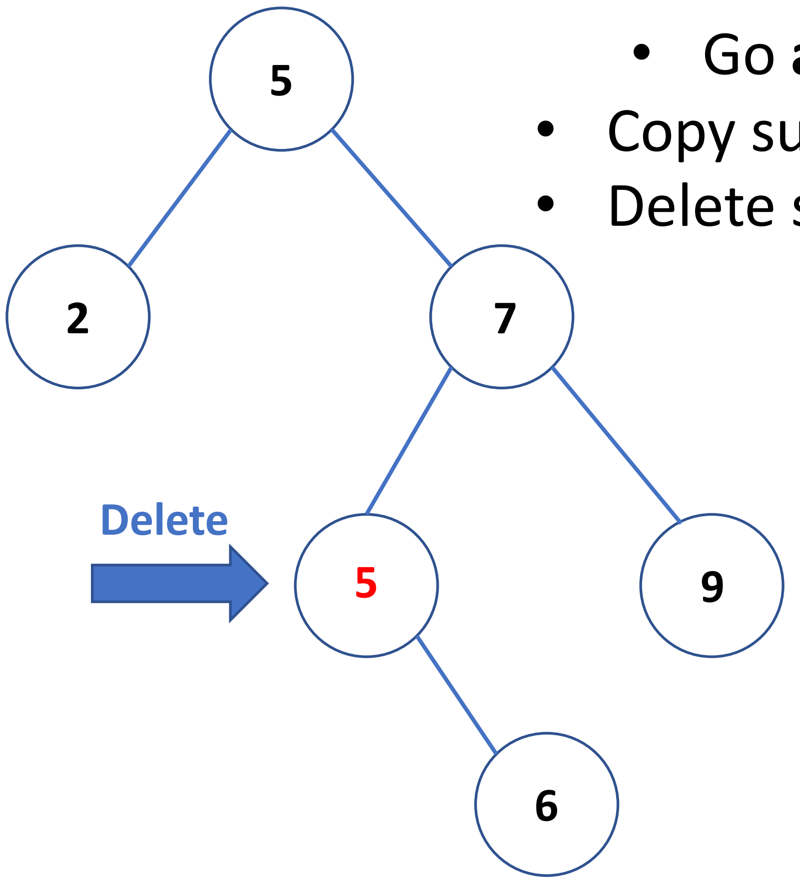
- Copy successor's key into node's key
- Delete successor node



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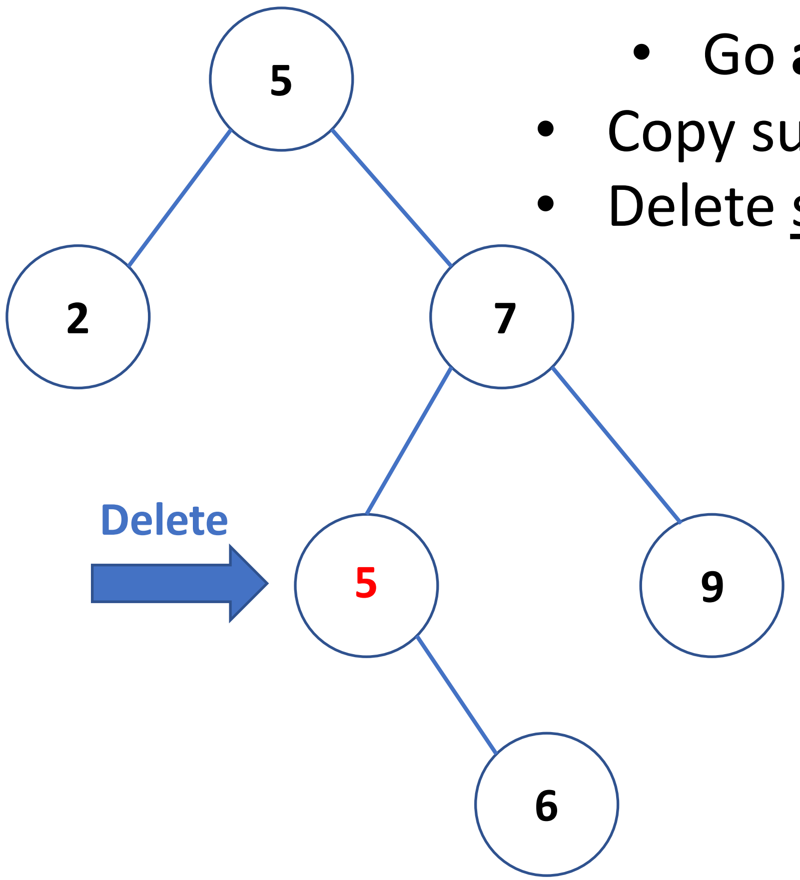
Has at most one child !



S : 2 5 5 6 7 9

Delete(T, 4)

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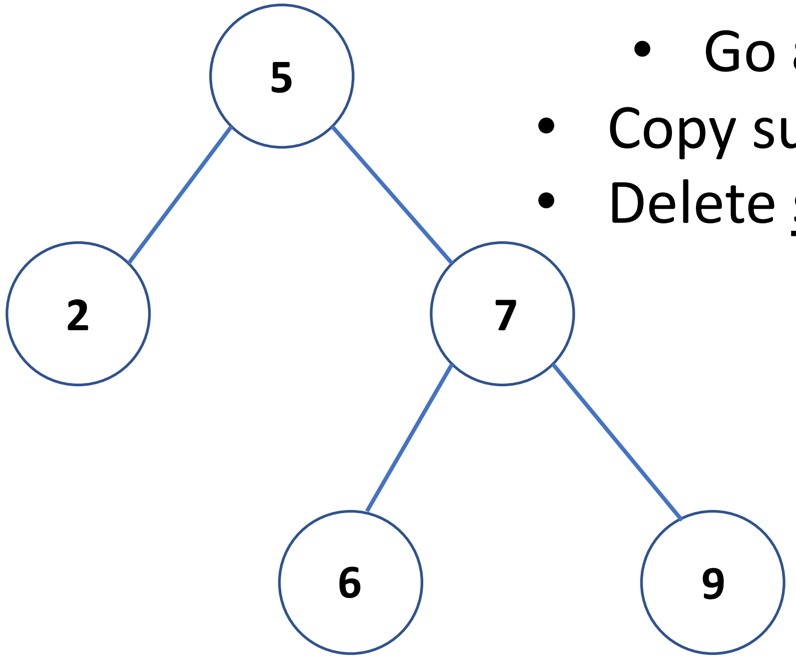


S : 2 5 6 7 9

Delete(T, x)

- Case 1: **x is a leaf** : Remove x
- Case 2: **x has one child** : Replace x with child and its subtree
- Case 3: **x has two children** :
    - y ← Successor(x) [Leftmost child in the right subtree of x]
    - Replace x with y
    - Delete successor node containing y (using Case 1 or 2)

Has at most
one child !

Worst-Case Time Complexity of **Search/Insert/Delete**

- Worst-Case time complexity of each operation is $\Theta($ $)$?

Worst-Case Time Complexity of **Search/Insert/Delete**

- Worst-Case time complexity of each operation is Θ(height of tree)

Worst-Case Time Complexity of **Search/Insert/Delete**

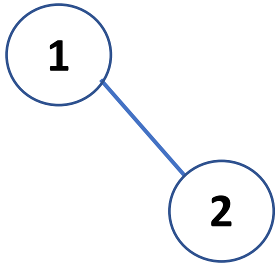
- Worst-Case time complexity of each operation is Θ(height of tree)
- Maximum height of a BST with n nodes?

Worst-Case Time Complexity of **Search/Insert/Delete**

- Worst-Case time complexity of each operation is Θ(height of tree)
- Maximum height of a BST with n nodes?

Worst-Case Time Complexity of **Search/Insert/Delete**

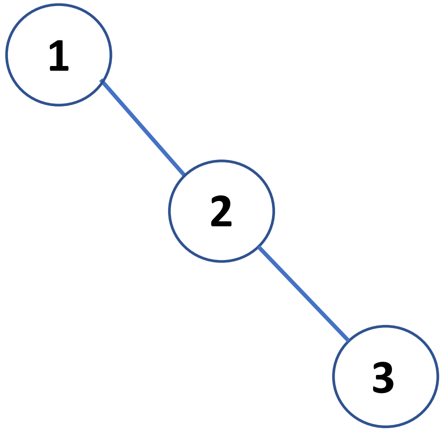
- Worst-Case time complexity of each operation is Θ(height of tree)
- Maximum height of a BST with n nodes?



Insert(T, 2)

Worst-Case Time Complexity of **Search/Insert/Delete**

- Worst-Case time complexity of each operation is Θ(height of tree)
- Maximum height of a BST with n nodes?

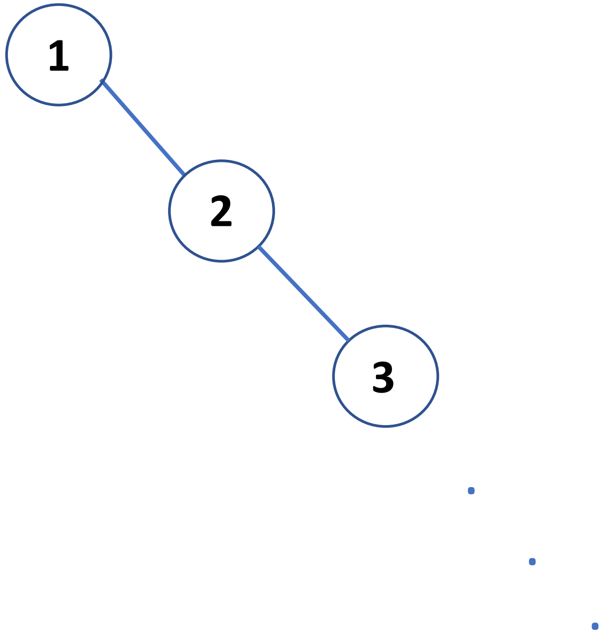


Insert(T, 2)

Insert(T, 3)

Worst-Case Time Complexity of **Search/Insert/Delete**

- Worst-Case time complexity of each operation is Θ(height of tree)
- Maximum height of a BST with n nodes?



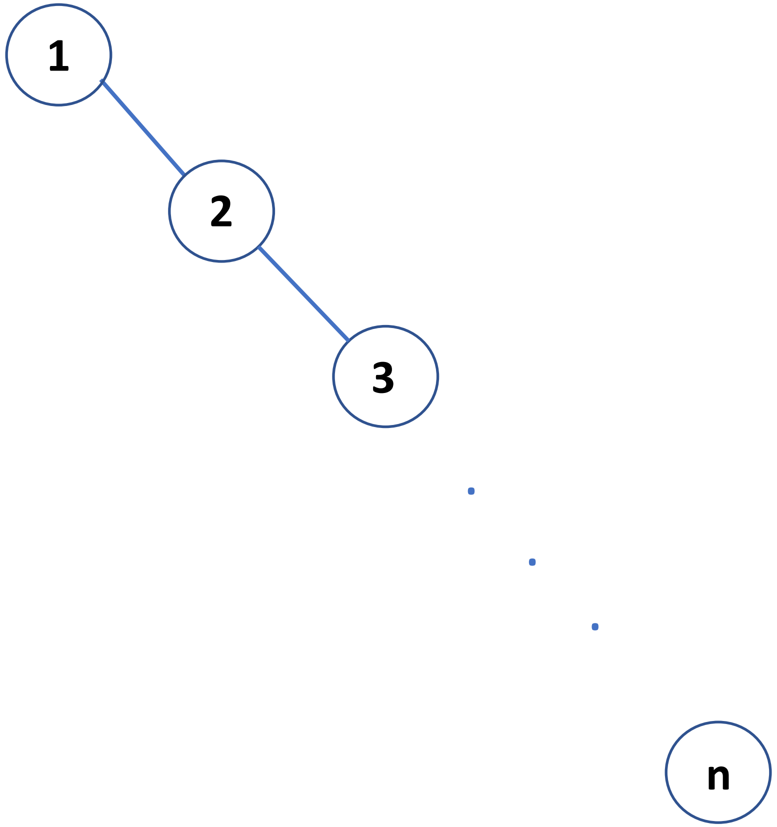
Insert(T, 2)

Insert(T, 3)

.
.
.

Worst-Case Time Complexity of **Search/Insert/Delete**

- Worst-Case time complexity of each operation is Θ(height of tree)
- Maximum height of a BST with n nodes?



Insert(T, 2)

Insert(T, 3)

.

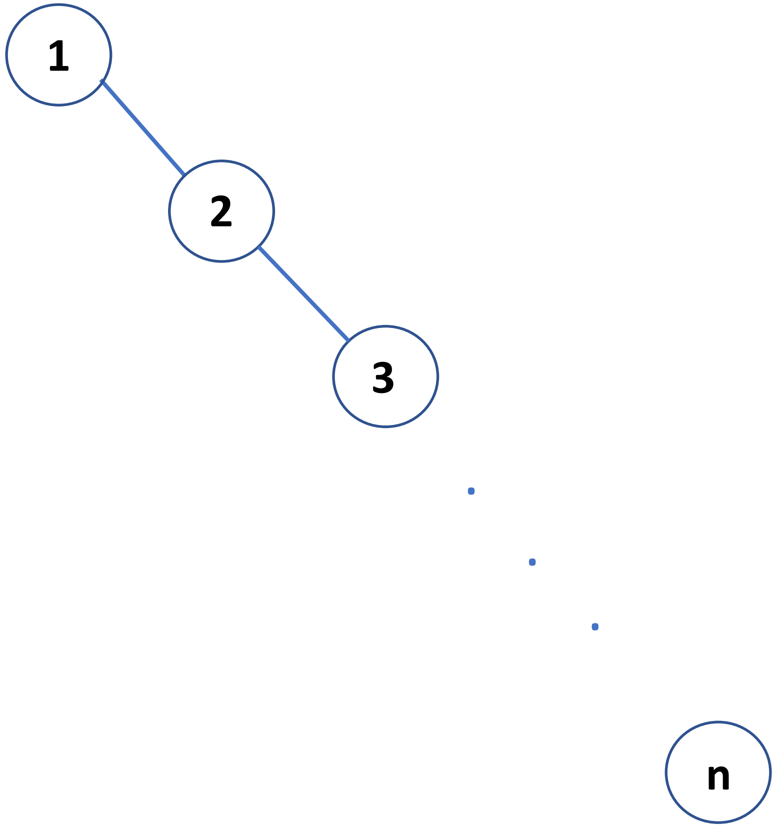
.

.

Insert(T, n)

Worst-Case Time Complexity of **Search/Insert/Delete**

- Worst-Case time complexity of each operation is Θ(height of tree)
- Maximum height of a BST with n nodes? n-1



Insert(T, 2)

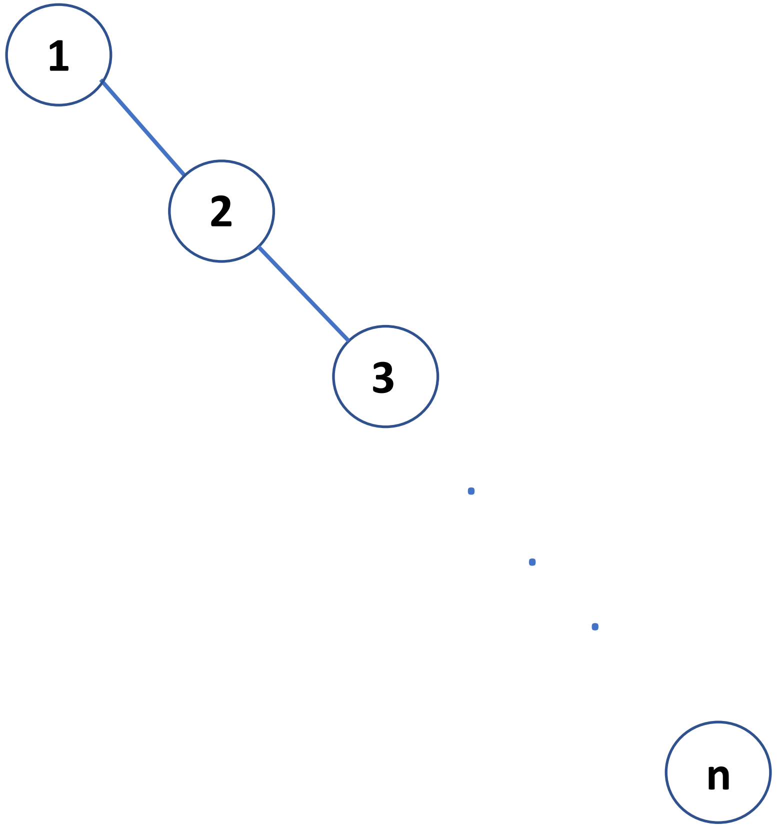
Insert(T, 3)

.
.
.

Insert(T, n)

Worst-Case Time Complexity of **Search/Insert/Delete**

- Worst-Case time complexity of each operation is Θ(n)
- Maximum height of a BST with n nodes? n-1



Insert(T, 2)

Insert(T, 3)

.

.

.

Insert(T, n)

BSTs are not very good in the worst case!

Next Week…

Variant of BST that ensures maximum height is O(log n) !