CSC236 tutorial exercises, Week #3
best before Thursday evening

Work on small examples until you believe the claim. Try writing out the inductive hypothesis, without completing the inductive step, and discuss whether it is reasonable. Then write a complete proof.

For the first question, the solution should be in terms of an inequality, rather than proving and using an equality.

1. Full binary trees are binary trees where all internal nodes have 2 children (see page 34 of csc236 notes). Prove that any full binary tree with at least 1 node has more leaves than internal nodes. Use complete induction on the total number of nodes.

2. Use Complete Induction to show that postage of exactly n cents can be made using only 3-cent and 4-cent stamps, for every natural number n greater than k (you will have to discover the value of k).

3. Define function f of the natural numbers by:

$$f(n) = \begin{cases} 1 & \text{if } n = 0 \\ 3 & \text{if } n = 1 \\ 2\left(f(n-2) + f(n-1)\right) + 1 & \text{if } n > 1 \end{cases}$$

   Use Complete Induction to prove that $f(n) \leq 3^n$ for all $n \in \mathbb{N}$.