

输入

训练数据 $T = \{(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)\}$, 其中 $x_i = (x_i^{(1)}, x_i^{(2)}, \dots, x_i^{(n)})^T$, $x_i^{(j)}$ 是第 i 个样本的第 j 个特征, $x_i^{(1)} \in \{a_{j1}, a_{j2}, \dots, a_{jS_j}\}$, a_{jl} 是第 j 个特征可能取的第 l 个值, $j = 1, 2, \dots, n$, $l = 1, 2, \dots, S_j$, $y_i \in \{c_1, c_2, \dots, c_K\}$; 实例 x 。

输出

实例 x 的分类

步骤

(1) 计算先验概率及条件概率

$$P(Y = c_k) = \frac{\sum_{i=1}^N I(y_i = c_k)}{N}, k = 1, 2, \dots, K$$

极大似然估计

$$P(X^{(j)} = a_{jl} | Y = c_k) = \frac{\sum_{i=1}^N I(x_i^{(j)} = a_{jl}, y_i = c_k)}{\sum_{i=1}^N I(y_i = c_k)}, j = 1, 2, \dots, n; l = 1, 2, \dots, S_j; k = 1, 2, \dots, K$$

(2) 对于给定的实例 $x = (x^{(1)}, x^{(2)}, \dots, x^{(n)})^T$, 计算

$$P(Y = c_k) \prod_{j=1}^n P(X^{(j)} = x^{(j)} | Y = c_k), k = 1, 2, \dots, K$$

(3) 确定实例 x 的分类

$$y = \arg \max_{c_k} P(Y = c_k) \prod_{j=1}^n P(X^{(j)} = x^{(j)} | Y = c_k), k = 1, 2, \dots, K$$

解释

(1) 极大似然估计可替换为贝叶斯估计, 用来防止极大似然估计会出现估计的概率值为 0 的情况。

贝叶斯估计

$$P_{\lambda}(Y = c_k) = \frac{\sum_{i=1}^N I(y_i = c_k) + \lambda}{N + K\lambda}, k = 1, 2, \dots, K$$

$$P_{\lambda}(X^{(j)} = a_{jl} | Y = c_k) = \frac{\sum_{i=1}^N I(x_i^{(j)} = a_{jl}, y_i = c_k) + \lambda}{\sum_{i=1}^N I(y_i = c_k) + S_j\lambda}, j = 1, 2, \dots, n; l = 1, 2, \dots, S_j; k = 1, 2, \dots, K$$