Evaluating Optimal Farm Management of Phosphorus Fertilizer Inputs with Partial Observability of Legacy Soil Stocks*

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Abstract

This study explores the dynamic management of legacy phosphorus (P) in agricultural systems, focusing on the decision-making processes of farmers applying P fertilizer and soil sampling under legacy soil P state uncertainty. Employing a Mixed Observability Markov Decision Process framework, we examine how the dynamics of partially observable legacy P stocks, coupled with Epstein-Zin preferences for risk aversion and elasticity of intertemporal substitution, influence optimal management decisions. Our findings reveal that higher risk aversion leads to greater P fertilizer application across all levels of legacy P bioavailability. Additionally, as the elasticity of intertemporal substitution increases, indicating a higher preference for intertemporal smoothing, there is a shift toward higher rates of soil sampling to manage uncertainty in legacy P levels more effectively. These findings suggest that farmer risk preferences significantly shape their management strategies, guiding development and policies that could enhance sustainable agricultural practices by optimizing the use of legacy P while mitigating environmental impacts.

JEL Codes: Q15, Q24, C61, C63
Keywords: Legacy Phosphorus, Risk Preference, State Uncertainty, Epstein-Zin Preference, Mixed-Observability Markov Decision Processes

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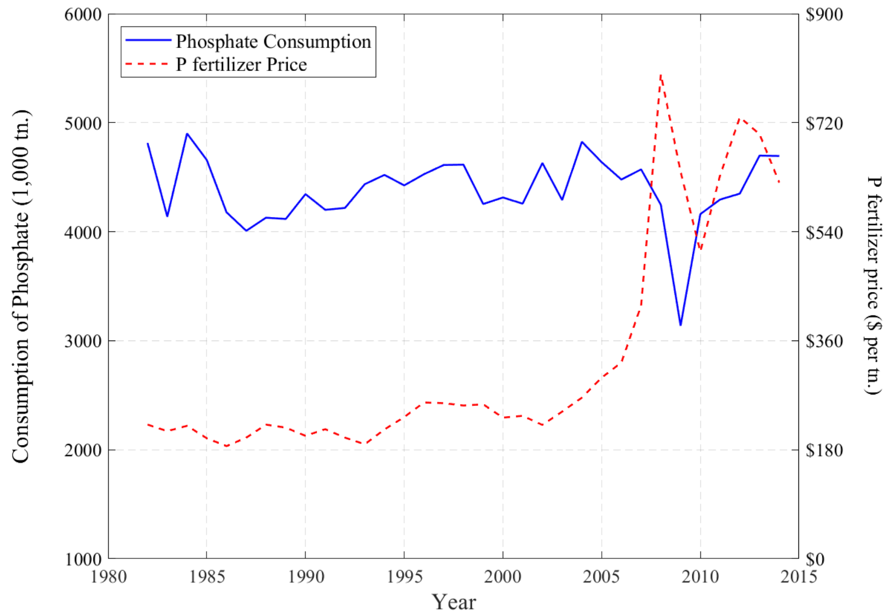
1 Introduction

Economically efficient management of the agricultural nutrient phosphorus (P) has become a critical global issue for ensuring sustainable crop production and environmental protection. P is imbalanced in the global food system, with some regions lacking sufficient access to synthetic or organic P fertilizers that could boost yields and rural incomes, leaving producers to rely on limited P stocks in nutrient-deficient soils (Zou et al. 2022). In the United States (and in other advanced economies) the main social challenge in P management is the excessive application of P fertilizer in farmland, which contributes to water quality degradation. In addition, there are concerns that P fertilizer overuse in advanced economies depletes mineral P stocks and increases prices. However, as illustrated in Figure 1, P fertilizer consumption by US farmers has remained relatively stable over the last few decades and has evidently responded only temporarily to recent and persistent price increases, suggesting relatively price inelastic demand for P in US cropping systems (Denbaly and Vroomen 1993).

Notably, unlike nitrogen fertilizer, P fertilizer application residuals after crop take-up can accumulate in soils. This accumulating soil stock of P – referred to as 'legacy P' – can be stored in non-bioavailable reserve, be taken up by future crop plantings or be mobilized by subsequent precipitation events, flowing into water bodies. A significant extent of US agricultural land has been estimated to have accumulated legacy P stocks over decades of continuous cultivation application of P from synthetic and organic sources (e.g., annually, $>$ 1,000 tonnes of P have accumulated in the Vermont agricultural region) (Wironen et al. 2018, Ringeval et al. 2018). Phoshorus runoff into water bodies catalyzes eutrophication, which can lead to hypoxic 'dead zones' and greenhouse gas (GHG) emissions (Arrow et al. 2018, Conley et al. 2009, Iho and Laukkanen 2012, Rabotyagov et al. 2014, Paudel and Crago 2020). Downing et al. (2021) estimate a substantial cost associated with GHG emission from eutrophication in freshwater system globally (equivalent to 20% of the emission from global fossil fuel consumption).

Various policies have been proposed to mitigate environmental issues arising from the overuse of P fertilizers, including the Numeric Nutrient Criteria under Clean Water Acts and Binational Phosphorus Reduction Strategy in Lake Erie (US EPA 1995, Lake Erie LaMP 2011), with one notable proposal focusing on getting farmers to substitute legacy, soil-bound P stocks for P fertilizer and to reduce P fertilizer applications (Sattari et al. 2012, USDA 2020). Properly managed, bioavailable legacy P stocks can substitute for P fertilizer, reducing costs and environmental impacts from fertilizer-intensive crop operations (Sattari et al. 2012).

Figure 1: U.S. Phosphorus consumption and phosphorus fertilizer price



Notes: The graph shows the relationship between P fertilizer consumption and P fertilizer prices from 1982 to 2014. The blue solid line represents P consumption, measured in 1,000 short tons on the left y-axis and the red dashed line indicates the price of P fertilizer, measured in dollars per short ton on the right y-axis.

Yet this policy idea begs the question of why farmers currently appear for the most part not to utilize legacy P stocks at all (given their accumulation over time), given the potential cost savings to farmers from doing so.

This paper studies this question using a model-based approach to analyze farmers' dynamic incentives to utilize these stocks in a setting of imperfect information and risk aversion. The complexity of managing legacy P stocks poses significant challenges for farmers, and the economic payoffs from different strategies recommended by agricultural extension are uncertain. Recent analysis suggests that farmers may not fully account for these residual P stocks in their P fertilizer decisions, due to a lack of high-quality information and the inherent uncertainty about the condition and bioavailability of legacy P stocks across their farmland. Particularly when accounting for farmer risk aversion, the uncertainty surrounding legacy P could contribute to its underutilization. This paper explores how these factors shape the dynamic incentives for legacy P utilization and examines whether improved access to enhanced monitoring of legacy P stocks could reduce P fertilizer application.

To address the management of legacy P accumulated in the soil and its losses to surface water, previous studies have analyzed the optimization of fertilizers in farmland along with P control or conservation policies. Schnitkey and Miranda (1993) analyze the optimal steady-state application of fertilizer under various policy settings which limit the soil P level. Goetz and Zilberman (2000) examine the intertemporal and spatial optimal application of mineral fertilizer levels given P concentrations in bodies of water associated with agricultural land for optimal lake restoration policy. Innes (2000) explains that environmental impact of nutrient runoff from livestock production can be mitigated by regulating facility size, implementing waste policies based on cleanup costs, and combining fertilizer taxes with subsidies for manure spreading equipment. Lötjönen et al. (2020) provide a theoretical spatial modeling framework to study climate and water policies for P mineral and manure fertilizer use in dairy farm management . While the models in these studies account for optimal fertilizer usage decisions to manage P accumulation in soils and to reduce P loss to the surface water, they do not incorporate the observational uncertainty related legacy P, and thus cannot answer the question we address here.

Farmers in the US do typically have some baseline information about soil P. In the US, farmers commonly employ standard soil sampling, provided by state agencies or extension services and by private soil testing service labortories at nominal fees, to gauge legacy P availability. These tests, usually conducted at a few spots within fields, offer preliminary insight into soil P content, serving as noisy indicators of the actual bioavailable legacy P stock across a field (Austin et al. 2020). While more comprehensive sampling options exist, offering clearer information, they come at a higher cost, presenting a trade-off between accuracy and expense (Austin et al. 2020, Gatiboni et al. 2022).

Economically, this situation can be described as one in which the agent – here, the farmer –optimizes their utilization of an uncertain resource stock – here, legacy P – in which they may dynamically update their beliefs about these fluctuating stocks based on costly monitoring. Generically, this a common situation in the resource management literature, referred to as a Partially Observable Markov Decision Process or POMDP (Clark 2010, Fackler and Pacifici 2014, Fackler 2014). Previous applications of POMDP models and extensions in resource management have included invasive species control (Haight and Polasky 2010, Rout et al. 2014, Kling et al. 2017), forestry (Sloggy et al. 2020), environmental conservation (White 2005), erosion prevention (Tomberlin and Ish 2007), and infectious diseases (Chadès et al. 2011).

To our knowledge POMDP methods have yet to be applied either in a depletable resource context or in farm production economics (though Sloggy et al.'s forestry application is adjacent to such a setting), reflecting one contribution of this paper. Previous agricultural economics studies have addressed the partial observability and monitoring problem using more heuristic optimization methods that separate inference about unobserved state variables from the optimization. For example, Fan et al. (2020) employ such an approach using state-space models to analyze efficient monitoring of an agricultural pest, but they specifically note the theoretical superiority of a POMDP approach for their application were it not for the computational difficulty of these methods.

Additionally, as far as we are aware, agent risk preferences have not previously been included in POMDP or optimal applications, at least in agricultural or resource economics. It is natural to conjecture that risk aversion could strongly affect demand for monitoring and the utilization of uncertain stock dynamics. Our analysis of that general conjecture represents another contribution. Because standard discounted expected utility in dynamic economic models conflates preference parameters for risk aversion and intertemporal substitution, we employ a widely used recursive utility Epstein-Zin specification to disentangle these effects in our analysis (Epstein and Zin 1991).

We develop our model's empirical foundation through econometric analysis of North Carolina field data on legacy P abundance, fertilizer application, and yield response in a corn-farming context spanning six years. We also account for stochastic crop and P fertilizer price dynamics, which we jointly estimate using publicly available USDA data. This extends the model into what are known as Mixed Observability Markov Decision Processes (MOMDPs) (Kovacs et al. 2012, Sloggy et al. 2020). Inclusion of these dynamics increases the robustness of our analysis, given that previous studies show that stochastic price dynamics have important effects on other dynamic farm resource management problems, such as crop rotation and cover crop planting (Livingston et al. 2015, Chen 2022).

Including all the elements described above is a significant computational challenge. In particular, POMDPs involve stochastic dynamic programming in which the agents possess belief states that specify their current subjective probability distributions about imperfectly observed biophysical states, with these beliefs states updated via Bayes' Rule. The specification introduces a high-dimensional state space (i.e. a space of probability distributions) that imposes considerable barriers on numerical computation. To address this problem, we closely follow recently applied density projection methods (e.g. Zhou et al. 2010, Springborn and Sanchirico 2013, MacLachlan et al. 2017, Kling et al. 2017, Sloggy et al. 2020) that reduce the dimensionality of the belief states, while avoiding some of the restrictions and pitfalls of prior methods (e.g. use of conjugate priors or coarse discretization of the unobserved state). We also use an econometric approach in estimating price dynamics that aids numerical tractability in the MOMDP optimization that is still informed by the empirical analysis: We first econometrically estimate a Markov regime-switching model for the price dynamics (supported by statistical tests), and then in the dynamic programming impose a conditional, within-regime equilibrium assumption to limit the dimensionality of the state space.

We find that higher risk aversion among optimizing farmers in the model generally decreases the reliance on legacy P stocks in favor of fertilizer application. Demand for enhanced monitoring is also affected by the farmer's degree of risk aversion, but in nonmonotonic ways that also interact with preferences for intertemporal smoothing. Utilization of legacy P and demand for monitoring also tends to increase with farmer tolerance for intertemporal substitution, especially when risk aversion is low. Furthermore, sensitivity analysis with much higher fertilizer prices (e.g. from a sustained global market disruption or a tax on fertilizer) or much lower monitoring costs (e.g. from a subsidy for more intensive sil testing) do not induce much substitution from fertilizer to legacy P use. These results raise questions about the potential effectiveness of proposed price-based instruments to correct externalities associated with agricultural fertilizer.

This paper's sections proceed as follows. First, the dynamics of legacy P are described, detailing the model's structure and equations, which capture both the accumulation and bioavailability of legacy P. Next, the economic and management problems are discussed, outlining how farmers can evaluate the recursive expected utility of their controls, P fertilizer application, and soil sampling in the face of stochastic prices and the unobservable state of legacy P. Then, the methodological framework and specification are presented, including price dynamics and the density projection approach for managing Bayesian belief updating. The application of this model to the corn market provides a practical example of how it can be used to guide decision-making in agriculture. Finally, the results of the model are discussed and are integrated with Epstein-Zin preferences, highlighting the implications of risk preferences in shaping farmers' P fertilizer application and soil sampling decisions.

By addressing both the economic decisions associated with agricultural production and the biophysical aspects of legacy P management, this paper provides valuable insights for policymakers and researchers interested in P sustainability in agricultural systems.

2 Model Description

A model of optimal management for legacy P is presented that incorporates uncertainties of both the legacy P status and stochastic pricing. Farmers' primary objective is to maximize the crop yields through actions that involve two control variables: P fertilizer application and soil sampling. During each decision period, farmers observe current legacy P levels and crop prices and then choose P fertilizer application levels and decide whether to employ soil sampling. The information derived from the controls is used for updating beliefs about the status of legacy P.

2.1 Legacy Phosphorus Dynamics Model

The model in this paper closely follows that of Iho and Laukkanen (2012), with the addition of stochastic carry-over properties. The model describes the average accumulated legacy P per hectare given by L_t with the change in legacy P status shown in equation (1):

$$L_{t+1} = \rho_t L_t + (\gamma_1 + \gamma_2 L_t) \underbrace{\left[F_t - \overbrace{(\gamma_3 \log(L_t) + \gamma_4)}^{\text{Concentration on Yield}} Y(L_t, F_t) \right]}_{\text{Legacy P Surplus}} \tag{1}$$

where ρ_t denotes the carry-over parameter of legacy P, F_t represents the amount of P fertilizer input, and $Y(L_t, F_t)$ is the crop yields at time t. The terms $(\gamma_3 \log(L_t) + \gamma_4)$ defines the legacy P concentration of the crop yield, which increases logarithmically with L_t. As L_t increases, the legacy P concentration also rise, initially leading to augmented yields. However, despite ongoing increases in L_t, the marginal yield gains attribute to each additional unit of legacy P progressively diminish. The term $(\gamma_1 + \gamma_2 L_t)$ is the legacy P balance scaling factor that scales the effect of the legacy P balance on the change in L_{t+1} (Ekholm et al. 2005). The parameter values of γ are summarized in Table 3.

While there are several ways to introduce uncertainty into dynamics resource allocation models, our dynamics model concentrates on introducing uncertainty into the carry-over of legacy P. In a deterministic environment, the carry-over parameter is less than one, which characterizes the gradual reduction of the P reserve over time (Ekholm et al. 2005, Iho and Laukkanen 2012). Here, we assume that the carry-over parameter is a diminishing stochastic multiplier with resource abundance:

$$\rho_t = \exp\left[\left(\mu_\rho - \frac{\sigma_\rho^2(L_t)}{2} + \sigma_\rho(L_t)W_t\right)\right], \qquad \text{with } W_t \sim \mathcal{N}(0,1), \tag{2}$$

where μ_ρ is the log mean percentage rate of growth/decay in L_t and $\sigma_\rho(L_t)$ is the standard deviation of this log percentage growth rate. We assume $\mu_\rho < 0$ meaning the legacy P stock L_t stochastically decay without added P fertilizer or uptake by crop. We specify the degree of stochasticity in growth rate as $\sigma_\rho^2(L_t) = \ln(1 + \varsigma^2/L_t^2\mathrm{M}_\rho^2)$ where ς is an uncertainty coefficient and $\mathrm{M}_\rho = \mathbb{E}[\rho_t|L_t]$, which keeps the stochastic behavior of the legacy P stock more realistic, by preventing the conditional variance in next-period stock from growing without bound as L accumulates (Loury 1978, Gilbert 1979, Melbourne and Hastings 2008, Sims et al. 2017, Sloggy et al. 2020).[1] We also consider a fixed variance scenario ($\sigma_\rho(L_t) = \varsigma$) in the Appendix.

2.2 Economics and Management Problem

Annual payoffs in the model are evaluated as the profit determined by crop yields and stochastic prices. Formally the expected (partial) profit is specified as the per hectare production function $Y(L_t, F_t)$ and stochastic prices:

$$\pi(L_t, F_t, P_{t+1}^Y, P_t^F, s_t) = P_{t+1}^Y Y_t(L_t, F_t) - P_t^F F_t - c_s s_t, \tag{3}$$

where P_{t+1}^Y and P_t^F are prices for the crop and P fertilizer, respectively, and c_s is a soil sampling cost. The variable s_t reflects the adoption of soil sampling at time t, where a value of one indicates soil sampling adoption and zero indicates no adoption. Fertilizer application decisions are based on the known fertilizer price P_t^F at the time of application, whereas the crop price P_{t+1}^Y will only be realized at the end of the season, is not yet observed at the time of applying fertilizer. This means that the decision to apply fertilizer is informed by the current fertilizer price and the last harvest's crop price, with the upcoming season's crop price P_{t+1}^Y yet to be realized by the end of the season.

In research on resource management utilizing stochastic prices, it is common to model the price dynamics as an autoregressive process to describe certain time series models,

[1] Given that $W_t \sim \mathcal{N}(0,1)$, using the moment generating function of the normal distribution, the $\mathbb{E}[\rho_t|L_t]$ is given by $\mathbb{E}[\rho_t|L_t] = \exp(\mu_\rho)$. This result shows that M_ρ simplifies to a constant that does not depend on L_t when considering the expectation with respect to the normal distribution of W_t

due to their simplicity and efficiency in handling linear relationships. However, a range of economic and environmental factors significantly influence crop and fertilizer prices, making traditional autoregression processes insufficient due to their constant parameterization and limitation in capturing the non-linear dynamics characteristic of the market. For instance, favorable weather conditions, which lead to crop abundance, typically result in lower volatility, thereby keeping prices relatively constant. Conversely, adverse weather, global trade, or geopolitical conditions may lead to supply scarcity and result in higher volatility, increasing price fluctuations. These varying conditions and their impact on crop and P fertilizer prices demonstrate the non-linear and regime-dependent nature of the process.

Markov-Switching Dynamic Regression (MSDR) models, with their ability to incorporate multiple regimes within their processes, provide a more robust framework using regime-dependent parameters. A MSDR model allows for the probability of switching between different regimes according to the Markov process (Hamilton 1989), each with its own distinct economic and environmental characteristics. When the process is in regime r_{t+1} at time $t+1$, we model the crop and P fertilizer prices as:

$$\begin{aligned}\ln(P_{t+1}^Y) &= \alpha_{0,r_{t+1}} + \alpha_{1,r_{t+1}} \ln(P_t^Y) + \alpha_{2,r_{t+1}} \ln(P_t^F) + \epsilon_{t+1} \\ \ln(P_{t+1}^F) &= \beta_{0,r_{t+1}} + \beta_{1,r_{t+1}} \ln(P_t^F) + \beta_{2,r_{t+1}} \ln(P_t^Y) + v_{t+1}\end{aligned} \tag{4}$$

where $\alpha_{0,r_{t+1}}, \beta_{0,r_{t+1}}$ are the intercepts for price regime r_{t+1} and ϵ_{t+1}, v_{t+1} are the identical distribution (i.i.d.) normal errors with mean zero and regime-dependent variance $\sigma^2_{\epsilon,r_{t+1}}, \sigma^2_{v,r_{t+1}}$, respectively. In the MSDR, the probability of regime r_{t+1} can be specified as $p_{ij} = \Pr(r_{t+1} = i|r_t = j)$ where p_{ij} represents the probability of transition from regime j at time t to regime i at time $t+1$ (Hamilton 1989). To maintain tractability, we assume that farmers perfectly observe the price regime they are in, despite the theoretical complexity of this assumption adding another layer of partial observability to the model's dynamics.

Incorporating the dynamic interdependencies between crop price and P fertilizer price into the MSDR model, more accurately reflects intertemporal relationships in price processes. Crop price changes, influenced by the price of P fertilizer, necessitate the inclusion of P_t^F as input cost. Similarly, recognizing the financial planning cycle in agriculture, in which the decision to purchase P fertilizer is based on profits from one years prior, we incorporate P_t^Y to show the economic cycle of P fertilizer purchase decisions.

Legacy P is not perfectly observed, but farmers can receive the information both by soil sampling and adjusting P fertilizer application and observing crop response. Famers obtain the observation O_t about legacy P stocks as they make their decisions. In this paper, we consider two kinds of soil sampling: standard soil sampling and point sampling. Standard soil sampling, typically provided by state agencies or extension services at nominal fees, involves collecting samples from a few spots within fields. These tests offer preliminary insights into soil P content but serve as noisy indicators of the actual bioavailable legacy P stock across a field (Austin et al. 2020). Point sampling, on the other hand, involves collecting multiple samples at specific grid points or random locations within grid cells, providing more precise information on legacy P bioavailability but at a higher cost (Austin et al. 2020, Gatiboni et al. 2022). We consider these two types of sampling to reflect the farmer's incentive to pay for more accurate information, which can improve decision-making and optimize P fertilizer application. We exclude the no sampling case because, in practice, farmers in the US typically have some baseline information about soil P and commonly employ standard soil sampling to gauge legacy P availability. Given the exclusion of the no soil sampling adoption, the cost of soil sampling, previously denoted as $c_s s_t$, is now adjusted to c_{s_t}, where $s_t = \{ss_t, p_t\}$. In this formulation, s_t represents the specific soil sampling method adopted by the farmer, with ss_t indicating standard soil sampling and p_t representing point soil sampling. The cost c_{s_t} thus varies depending on the chosen soil sampling method, reflecting the associated expenses for either standard or point sampling.

The observation O_t is determined as follows:

$$O_t^s = \begin{cases} L_t + \lambda_t^s & \text{if } L_t > -\lambda_t^s \\ 0 & \text{if } L_t \leq -\lambda_t^s \end{cases} \qquad \text{where } \lambda_t^s \sim \mathcal{N}(0, \sigma_s^2). \tag{5}$$

This specification truncates the observation at the zero from below. It also keeps the variance of the observation error conditional on L_t from growing with the level L_t. Together, these assumptions are meant to stylistically reflect the nature of soil sampling, including minimum detectable level and at sufficiently high levels of L an absolute margin of error that is effectively fixed. Here the $s = \{ss, p\}$ denotes the controls, representing standard soil sampling (s) and point sampling (p). λ_t^s is an i.i.d. sequence of observation error given by the controls:

$$\lambda_t^s = \begin{cases} \lambda_t^{ss} & \text{if standard soil sampling} \\ \lambda_t^p & \text{if point sampling} \end{cases} \tag{6}$$

Our observation equation employs the methodology of Zhou et al. (2010) with the addition of the non-negative observation properties and we assume that the error variance is greater when famers adopt standard soil sampling, as indicated by $\sigma_{ss} > \sigma_p$.

The farmer's beliefs about the distribution of legacy P are denoted as $b_t(L_t)$, representing a subjective probability distribution over the unobserved L_t, conditional upon the history of controls and resulting observations (Kling et al. 2017). Bayesian updating of these beliefs, combining each period's prior belief with new information, is given by:

$$b_{t+1}(L_{t+1}) \propto p(O^s_{t+1} \mid L_{t+1}, s_t) \int p(L_{t+1} \mid L_t, F_t) b_t(L_t) dL_t \tag{7}$$

with a given $b_0(L_0)$ specifying the prior beliefs about initial stocks and where $p(O^s_{t+1}|L_{t+1}, s_t)$ is the conditional probability distribution of the observation, as determined from equations (5) and (6). The Markovian properties ensure that the next period beliefs only depend on the current beliefs, controls, and information gained in the current period. Figure 2 illustrates how the farmer updates their belief state b_t based on observations O_{t+1} and actions F_t, s_t to manage the unobservable legacy P state L_t over time. The schematic captures the dynamic interaction between the farmer's actions, belief updates, and the evolution of the unobservable state.[2]

The Bellman equation for the recursive expected utility function is follows:

$$V(\boldsymbol{S}_t) = \max_{F,s} \; C(\boldsymbol{S}_t, F_t, s_t) + \beta \mathbb{E}\big\{V(\boldsymbol{S}_{t+1}) \mid \boldsymbol{S}_t, F_t, s_t\big\}, \tag{8}$$

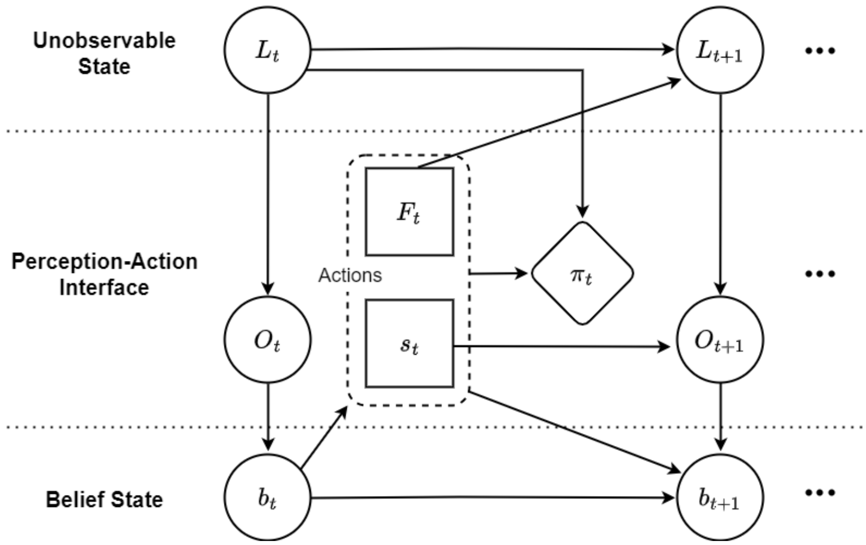
where β is the discount factor and $\boldsymbol{S}_t \equiv [b_t(L_t), P^Y_t, P^F_t]$ is the vector, which includes the current belief distribution of legacy P stock and the prices of crop and fertilizer at time t. Specially, P^Y_t represents the last observed crop price at the beginning of the growing season, i.e. last season's crop price at harvest. $C(\boldsymbol{S}_t, F_t, s_t)$ is the certainty-equivalent payoff for over uncertain end-of-season profits $\pi(L_t, F_t, P^Y_{t+1}, P^F_t, s_t)$:

$$C(\boldsymbol{S}_t, F_t, s_t) \equiv \iint \pi(L_t, F_t, P^Y_{t+1}, P^F_t, s_t) b_t(L_t) f(P^Y_{t+1} \mid P^Y_t, P^F_t) dL_t dP^Y_{t+1} \tag{9}$$

where $f(P^Y_{t+1}|P^Y_t, P^F_t)$ is conditional probability density function of crop price P^Y_{t+1} at the

[2]We also consider a two-stage belief updating process within one season. In this process, farmers receive information before applying P fertilizer by adopting soil sampling and then receive additional information after harvesting, based on the crop yield. However, since the yield signal does not provide sufficient information, we primarily focus on soil sampling information. The details of the two-stage belief updating process are provided in the Appendix.

Figure 2: Schematic of Partially Observable Markov Decision Process



Notes: The farmer infers the unobservable state L_t through observations O_t and update its belief state b_t. Phosphorus fertilizer application F_t and soil sampling s_t influence both the state transition L_{t+1} and future observations O_{t+1}.

upcoming harvest, given the last observed harvest price P_t^Y and the current fertilizer price P_t^F. In addition, we develop the MOMDP model for managing legacy P by integrating Epstein-Zin preferences into the dynamic programming mapping process, allowing for a more precise depiction of risk premiums distinct from an individual's intertemporal substitution preferences (Lybbert and McPeak 2012). The Bellman equation for the recursive expected utility function, given Epstein-Zin preferences, is as follows:

$$V_{EZ}(\boldsymbol{S}_t) = \max_{F,s} \left[(1-\beta) C_{EZ}(\boldsymbol{S}_t, F_t, s_t)^{1-\psi^{-1}} + \beta \mathbb{E}\left\{ V_{EZ}(\boldsymbol{S}_{t+1})^{1-\eta} \mid \boldsymbol{S}_t, F_t, s_t \right\}^{\frac{1-\psi^{-1}}{1-\eta}} \right]^{\frac{1}{1-\psi^{-1}}}, \tag{10}$$

where $C_{EZ}(\boldsymbol{S}_t, F_t, s_t)$ is the certainty-equivalent payoff of Epstein-Zin preferences:

$$C_{EZ}(\boldsymbol{S}_t, F_t, s_t) \equiv \left(\iint \pi(L_t, F_t, P_{t+1}^Y, P_t^F, s_t)^{1-\eta} b_t(L_t) f(P_{t+1}^Y \mid P_t^Y, P_t^F) dL_t dP_{t+1}^Y \right)^{\frac{1}{1-\eta}} \tag{11}$$

and where η and ψ indicate, respectively, relative Risk Aversion (RA) and Elasticity of Intertemporal Substitution (EIS).

Note that one of the state variables in this model is the entire belief function $b_t(\cdot)$, which makes the dynamic programming problem intractable in its current form. Various methods have been proposed to reduce the dimensionality of beliefs in POMDP, the simplest of which is to use conjugate prior $b_t(\cdot)$ and likelihood function $p(\cdot)$, so that $b_{t+1}(\cdot)$ belongs to the same family as $b_t(\cdot)$ (e.g. normal distribution), reducing the beliefs to simply the parameters of that family (e.g. mean and variance of normal distribution). However, the conjugacy assumption is overly restrictive for most modern resource management problems. The prevailing alternative in the resource economics literature is the density projection method described below.

2.3 Density Projection Approach and Particle Filtering

Continuous state POMDP has challenges due to an infinite-dimensional belief space and because approximating belief states by discretization can lead to computational issues. Exact evaluation of the posterior distribution is difficult to address, and even structuring the belief updating process in discretized space is often infeasible. To address this challenge, a density projection technique suggested by Zhou et al. (2010) and employed by Kling et al. (2017) in economics is utilized.

Density projection projects the infinite-dimensional belief space onto a low-dimensional parameterized family of densities.[3] Projection mapping from the belief state $b(L)$ to exponential family of density $f(L;\theta)$, where θ is a natural parameter, is achieved by minimizing the *Kullback-Leibler* (KL) divergence between $b(L)$ and $f(L;\theta)$ as:

$$\begin{aligned} b^P(L) &\triangleq \arg\min_f \; D_{KL}(b \parallel f) \\ &\text{where } D_{KL}(b \parallel f) \triangleq \int b(L) \log \frac{b(L)}{f(L;\theta)} dL \\ &\forall L, b(L) > 0 \leftrightarrow f(L;\theta) > 0 \end{aligned} \tag{12}$$

and thus belief $b(L)$ and its projection $f(L;\theta)$ satisfies:

$$\mathbb{E}_b[T_j(L)] = \mathbb{E}_\theta[T_j(L)] \quad \text{for } j = 1, 2, ..., J \tag{13}$$

where $T(L)$ is the sufficient statistics of the probability density (Zhou et al. 2010).

[3]Technical interpretation of density projection and particle filtering hereafter closely follows Zhou et al. (2010).

Bayesian updating of projected belief state is implemented adopting a particle filtering, which uses a Monte Carolo simulation approach to estimate the belief state with a limited set of particles (samples) and simulates the transition of the belief state (De Freitas 2001, Arulampalam et al. 2002). In the particle filtering, particles L_t^i for $i = 1, 2, ..., Z$ are drawn from $b_t(L_t)$ and L_{t+1}^i from the propagation $p(L_{t+1}|L_t, F_t, s_t)$. This allows for the approximation of $b_{t+1}(L_{t+1})$ by the probability mass function (Zhou et al. 2010):

$$b_{t+1}(L_{t+1}) \approx \sum_{i=1}^{Z} \tau_{t+1}^i \phi(L_{t+1} - L_{t+1}^i) \tag{14}$$

where $\tau_{t+1}^i \propto p(O_{t+1}^l | L_{t+1}^i, F_t, s_t)$, denoting the associated weight and ϕ represent the Kronecker delta function. Substituting equation (14) into (13), the approximation becomes:

$$\begin{aligned}
\mathbb{E}_{b_{t+1}}[T_j(L_{t+1})] &= \int T_j(L_{t+1}) b_{t+1}(L_{t+1}) dL_{t+1} \\
&\approx \int T_j(L_{t+1}) \left[\sum_{i=1}^{Z} \tau_{t+1}^i \phi(L_{t+1} - L_{t+1}^i) \right] dL_{t+1} \\
&= \sum_{i=1}^{Z} \tau_{t+1}^i T_j(L_{t+1}^i) \\
&= \mathbb{E}_{\theta_{t+1}}[T_j(L_{t+1})]
\end{aligned} \tag{15}$$

simplified by the properties of the Kronecker delta function. Thus, if the particles L_t^i are drawn from the projected belief state $b_t^P = f(\cdot; \theta_t)$ and their propagation L_{t+1}^i satisfy the $\sum_{i=1}^{Z} \tau_{t+1}^i T_j(L_{t+1}^i) = \mathbb{E}_{\theta_{t+1}}[T_j(L_{t+1})]$, the transition probability of θ_t to θ_{t+1} can be calculated.

Density projection effectively reduces infinite-dimensional density to low-dimensional, parameter-defined density, transforming the belief Markov decision proces (MDP) into a more manageable and solvable form referred to as 'projected belief MDP'. In this paper, the legacy P states are defined as the natural parameters of log-normal distribution and transform to the θ in the 'projected belief MDP' calculation (Kling et al. 2017). The parameter transformation and particle filtering algorithms are noted in the Appendix. The utilization of the log-normal distribution in parameterized density is particularly advantageous, primarily due to its tractability to positive-valued state variables and its parametric simplicity characterized by two parameters: mean and coefficient variation (Sloggy et al. 2020).

While there are numerous ways to solve the projected belief MDP, we follow Kling et al.

(2017) and discretize the projected belief MDP space into a discrete-state space. Because the value function in equations (8) and (10) is a function both of the belief and price states, we then compute the value function on a grid of all discretized possible belief and price state combinations. The details of dynamic programming and numerical algorithm are described in the Appendix.

3 Application to a Representative NC Corn Research Trial

The previous sections presented the model for optimal management of legacy P under unobservable bioavailability to influence crop yield. For the numerical simulation, we apply this model to representative eastern North Carolina corn farm using field trial data (Morales et al. 2023). To specify crop yield response to fertilizer and legacy P input, we adopt a functional form introduced by Myyrä et al. (2007), and then estimate the parameters of this function using the field trial data:

$$Y_{i,t}(L_{i,t}, F_{i,t}) = \underbrace{\kappa_1\left[1 - \kappa_2 \exp(-\kappa_3 L_{i,t})\right]}_{\text{Response to Legacy P}} + \underbrace{(\chi_1 - \chi_2 L_{i,t})\sqrt{F_{i,t}} + \frac{(\chi_3 - \chi_4 F_{i,t})F_{i,t}}{L_{i,t}}}_{\substack{\text{Response to P Application} \\ \text{(mediated by Legacy P)}}} + \upsilon_i + \epsilon^Y_{i,t}, \quad (16)$$

where i denotes the soil samples and the samples are collected from 30 plots within the North Carolina Tidewater region. The yield response function is comprised of two parts, including the parameters κ and χ, each part reflecting a different aspect of corn yield response. The first part represents the influence of legacy P status on yield, which can explain the independent impact of current legacy P levels. The second part describes the response to the current P application. Specifically, this part is also a function of legacy P, illustrating diminishing marginal productivity with respect to legacy P status.[4] Thus, as legacy P levels increase, the marginal effects of additional P applications on yield tend to decrease. This aspect reflects the non-linear aspect of nutrient application dependent on existing soil conditions.

To estimate the parameters in the yield response function, the data from 5 years of field experiments (2010, 2012, 2014, 2021, and 2022) that examined soil sampling, legacy P bioavailability measured by Mehlich 3 method and reported in milligrams per cubic centimeter of soil (mg/dm^3), P fertilizer application (kg/ha), and yield response of corn production (kg/ha) in the North Carolina Tidewater Research Station were used.

[4] Figure B1 shows the diminishing marginal return and interaction effect of corn yield function ($\partial Y_{i,t}/\partial F_{i,t} > 0$, $\partial^2 Y_{i,t}/\partial F_{i,t}^2 < 0$, and $\partial^2 Y_{i,t}/\partial F_{i,t}\partial L_{i,t} < 0$) based on estimated parameters in this section.

Table 1: Summary Statistics of North Carolina Tiedwater Data

Variable	Obs	Mean	Median	IQR	SD	Min	Max
Legacy P (mg/dm^3)	139	63.986	46	37–66.75	50	28	279
P application (kg/ha)	139	47.036	22	11–67	53.948	0	168
Corn yield (kg/ha)	139	4751.9	4442	2266.7–6517.9	2950.3	131	13712

Notes: Interquartile Range (IQR) is a measure of statistical dispersion, being equal to the difference between the 75th and 25th percentiles. It represents the range within which the central 50% of the data lie.

Table 1 provide the summary statistics and insights into the distribution of legacy P levels, P application rates, and corn yield. For legacy P, the data includes 139 observations with a mean value of 63.986 mg/dm^3 and a median of 46 mg/dm^3. The IQR is 37–66.75 mg/dm^3, indicating that the central 50% of the data falls within this range. The standard deviation is 50 mg/dm^3, with minimum and maximum values of 28 mg/dm^3 and 279 mg/dm^3, respectively. For phosphorus application rates, the data shows a mean application of 47.036 kg/ha and a median of 22 kg/ha. The IQR is 11–67 kg/ha, with a standard deviation of 53.948 kg/ha, and the minimum and maximum values range from 0 to 168 kg/ha. These statistics illustrate a wide variation in phosphorus application practices among the observations. Corn yield, measured in kg/ha, has a mean value of 4751.9 kg/ha and a median of 4442 kg/ha. The IQR for corn yield is 2266.7–6517.9 kg/ha, with a standard deviation of 2950.3 kg/ha. The minimum and maximum corn yields are 131 kg/ha and 13712 kg/ha, respectively.

To estimate the parameters in equation (16), a non-linear least square (NLS) was performed using the field trial data. The estimated parameters are summarized in Table 2 and biological and other economic parameters are presented in Table 3. The equation (16) includes plot-level effect υ_i and an error term ϵ_{it}^{Y} The maximum yield parameter κ_1 was calibrated to corresponding to the maximum yield of North Carolina Tidewater samples (Iho and Laukkanen 2012).

The use of fixed effects for plots is justified by the non-random distribution of legacy P, which is influenced by historical management practices and environmental conditions unique to each plot. Fixed effects control for these unobserved, time-invariant characteristics, allowing for more accurate estimation of the impact of P on yield. We estimate NLS with plot-level fixed effects for the purpose of simplifying the MOMDP, the fixed effects can either be disregarded or averaged, focusing on the dynamic management of P application and legacy P bioavailability rather than plot-specific characteristics. This approach ensures that

Table 2: Nonlinear least square estimation for corn yield

Corn Yield (kg/ha)			
Corn yield response to legacy P		Corn yield response to P application	
κ_1	13712	χ_1	-2594.112***
	(Max yield)		(428.609)
κ_2	0.511	χ_2	-5.125***
	(0.758)		(1.618)
κ_2	0.032	χ_3	10925.06***
	(0.0503)		(316.377)
		χ_4	26.345*
			(14.055)
Plot Fixed Effect	Yes		
Observation	139		
Adjusted R-squared	0.577		
Root MSE	1955.089		

Notes: Clustered standard errors in parentheses. The standard errors are adjusted for clustering in soil sampling plots. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

the model accurately reflects the essential dynamics of P management while controlling for plot-specific factors that could bias the results.

Standard soil sampling typically involves collecting one soil sample per 2½ acres, costing around \$10 per acre (Austin et al., 2020). Point sampling is recommended at a spacing of 100 to 200 feet, where four composite samples are collected per acre with 104 feet (¼-acre grid), resulting in approximately ten samples for 2½ acres (Austin et al., 2020). Thus, point sampling provides more precise information on legacy P bioavailability but is a more expensive methodology to implement. Based on this information, we assumed that the observation error variance of point sampling (σ_p) was smaller than standard sampling ($\sigma_{ss} > \sigma_p$), and the cost was ten times higher than standard sampling ($c_p = 10 \cdot c_{ss}$). The values for the observation errors are denoted in Table 3.

Figure 3 displays the simulation results of legacy P accumulation (mg/dm^3) over 50 years without P fertilizer application, illustrating the range of stochastic paths.[5] The solid green

[5]The results depicted in Figure 2 were generated from 10,000 simulations.

Table 3: Parameters and description

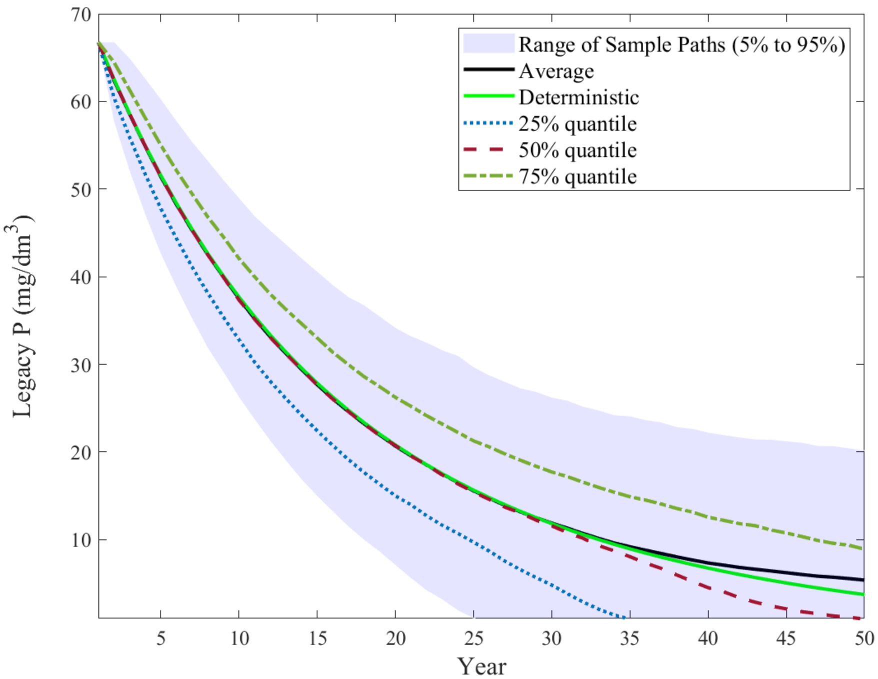
	Value	Desccription
Biological Parameters		
μ_ρ	-0.02	Average rate of growth (Myyrä et al. 2007)
ς	3	Uncertainty coefficient
γ_1	0.0032	Legacy P balance parameters (Ekholm et al. 2005)
γ_2	0.00084	
γ_3	0.000186	Legacy P surplus parameters (Iho and Laukkanen 2012)
γ_4	0.003	
Economic Parameters		
c_{ss}	\$24.715	Standard soil sampling cost per ha. (\$10 per acre, Austin et al. 2020)
c_p	$c_p = 10 \cdot c_{ss}$	Point soil sampling cost per hectare
β	0.9345	Discount factor with 8% discount rate (Duquette et al. 2012)
σ_{ss}	0.4	Observation error of standard soil sampling
σ_p	0.05	Observation error of point soil sampling

Notes: Soil sampling cost varies depending on the institute. This paper uses the North Carolina case (Austin et al. 2020, \$10 per acre).

line represents the deterministic path with 2% decay rate that assumes no uncertainty in legacy P dynamics. The shaded area represents the range of simulation sample paths from the 10% to 90% quantile, which becomes broader as the legacy P extends further into the future. Quantile lines for the 25% (blue dots), 50% (red dash-dots), and 75% (green dashes) show the distribution of accumulation, with the 50% quantile also indicated as the median path. The black line represents the average of all simulation results.

The stochastic trend of legacy P dynamics follows closely to the deterministic path, suggesting that the parameters used in modeling legacy P dynamics and stochasticity do not deviate significantly from the deterministic trend. This consistency indicates that our model parameters effectively capture the essential dynamics of legacy P without substantial stochastic deviations. The light blue shaded area illustrates the variability and uncertainty in legacy P levels due to stochastic factors, showing a steady decline in legacy P, showing the gradual depletion of P reserves in the soil over time.

Figure 3: Legacy phosphorus accumulation without phosphorus fertilizer application



Notes: For the deterministic legacy P accumulation (green solid line), we employ a constant carry- over parameter $\rho_t = \rho = 0.98$ (2% decay rate) as adopted by Myyrä et al. (2007). The initial value is the 75th percentile (66.75 mg/dm^3) of legacy P in the North Carolina Tidewater data.

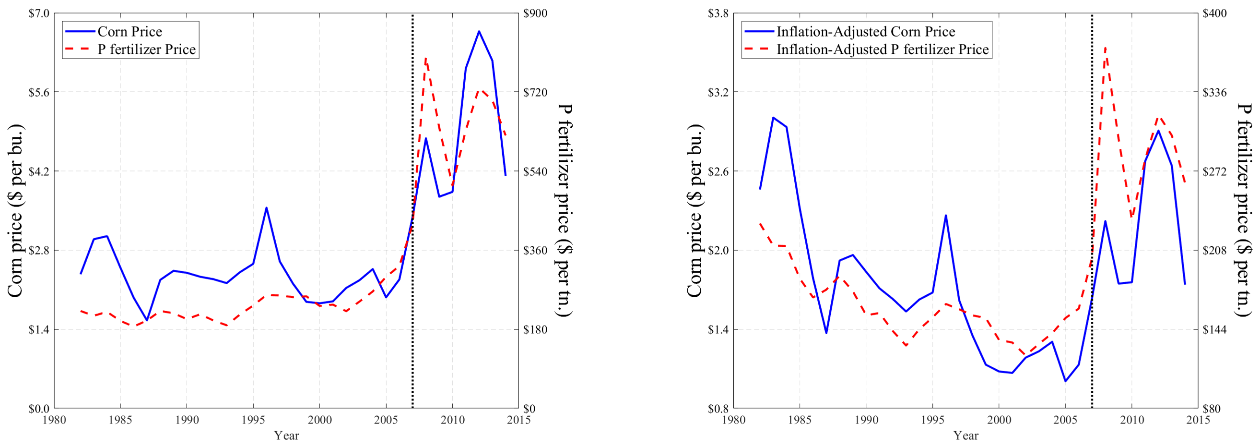
4 Results

In this section, the solution corresponding to the management model introduced in the previous sections, including state uncertainty and price stochasticity, is presented. First, we present results from empirical analysis of prices and from estimation of yield response.

4.1 Price Dynamics and Regime Switching

To analyze price stochasticity, we used P fertilizer (44%-46% phosphate) price data from the USDA "Fertilizer Use and Price" report and sourced corn price data from the USDA's "U.S. Bioenergy Table" (USDA 2024a, USDA 2024b). The P fertilizer and corn price data spanned 33-years (1982-2014). Figure 4 shows the nominal and real prices over the observed periods. The solid blue lines represent the corn price per bushel (per bu.), and the dashed red lines indicate the P fertilizer price per short ton (per tn.). The price shows a notable correlation.

Figure 4: Corn and phosphorus fertilizer price dynamics



Notes: Inflation-adjusted prices are adjusted using the Consumer Price Index (CPI) for all urban consumer (index 1983=100), with data sourced from the Federal Reserve Bank of Minneapolis (2024.04).

As shown in the left panel of Figure 4, both nominal prices remain relatively low during the moderate period of 1982 to 2007, after which both prices increase sharply due to surge in the global commodity prices and economic recession.

The right panel of Figure 4, which shows the inflation-adjusted prices, more clearly illustrates the changing dynamics. Before 2007, the inflation-adjusted prices of both corn and P fertilizer show a decreasing trend, reflecting a period of declining real prices that suggest a market characterized by an oversupply relative to earlier years. After 2007, this trend sharply reversed, with the corn and P fertilizer prices beginning to rise significantly, indicating a shift to a high-price regime. This rise aligns with the global increase in commodity prices and marks a distinct change in market dynamics. This information can support the rationale for implementing a two-regime MSDR model, effectively distinguishing between the earlier period of declining prices and the subsequent period of rapid price escalation.

Thus, Figure 4 suggests that the two-regime MSDR model, $r_t \in \{\text{moderate, high}\}$, is suitable for capturing the non-linear and regime-dependent nature of the dynamics underlying corn and P fertilizer prices. We use inflation-adjusted prices in the MSDR model to ensure that the analysis reflects real market dynamics undistorted by inflation. This enables the model to accurately identify different market regimes based on real economic changes rather than nominal price shifts. Tables 4 and 5 present the results of the MSDR model and the transition probabilities of the inflation-adjusted corn and P fertilizer prices, respectively. Table 4 demonstrates how the impact of current prices on future prices varies depending on

Table 4: Markov switching dynamics regression for corn and phosphorus fertilizer prices

	Corn ($\ln(P^Y_{t+1})$)		Phosphorus fertilizer ($\ln(P^F_{t+1})$)	
	Moderate	High	Moderate	High
$\ln(P^F_t)$	0.091	0.763***	0.947***	–1.347***
	(0.186)	(0.284)	(0.151)	(0.251)
$\ln(P^Y_t)$	0.633***	0.280	–0.034	2.234***
	(0.199)	(0.205)	(0.097)	(0.470)
Const.	–0.410	–3.408**	0.275	11.866***
$(\alpha_{0,r_t}, \beta_{0,r_t})$	(0.923)	(1.448)	(0.723)	(1.862)
Std Dev.	0.109		0.075	
$(\sigma_{\epsilon,r_t}, \sigma_{v,r_t})$	(0.014)		(0.009)	
Log-likelihood	12.309		31.502	
AIC	–0.207		–1.406	

Notes: Robust standard errors are in parentheses. In the regression, constant standard deviation $\sigma^2 = \sigma_i^2 = \sigma_j^2$ is assumed for $r_t \in \{i, j\}$, $i \neq j$. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

whether the market is in a moderate or high regime. In the moderate regime, the current corn price at the time significantly influences future price, indicating price persistence. However, the current P fertilizer price exerts a statistically nonsignificant effect on the corn price. In contrast, during the high regime, the influence of the current corn price is nonsignificant, whereas the effect of the P fertilizer price on the corn price is statistically significant, suggesting that input costs begin affecting corn prices more substantially.

For P fertilizer prices, the moderate regime reveals a strong dependence on its own current price, indicating that current prices are predictive of future prices under moderate conditions. The effect of the current corn price on the fertilizer price in this regime is nonsignificant. The high regime sees the dynamics change: The coefficient of the current P fertilizer price turns negative, signifying that increasing current prices may lead to reduced future prices, a reversal that can reflect unpredictable supply and demand dynamic under stress. Additionally, the influence of the corn price on the fertilizer price is significantly positive, showing that in a high regime, the price of corn can have an upward impact on P fertilizer prices, possibly due to increased production affecting the broader agricultural market. Table 5 shows, for example, that the corn price has a 71.1% likelihood of remaining at a moderate regime during the next period given that the process is moderate during the current period as well as a

Table 5: Transition probabilities of corn and phosphorus fertilizer prices

	Corn (p_{ij}^{Y})		Phosphorus fertilizer (p_{ij}^{F})	
	Moderate (t)	High (t)	Moderate (t)	High (t)
Moderate ($t+1$)	0.711	0.370	0.966	0.207
High ($t+1$)	0.289	0.630	0.034	0.793

Notes: State value of corn and phosphorus fertilizer prices, P^{Y} and P^{F}, for the moderate and high state are predicted and averaged from the price data and Markov switching dynamics regression results. $P^{Y}_{\text{Moderate}} = \$1.498$, $P^{Y}_{\text{High}} = \$2.030$ per bu. and $P^{F}_{\text{Moderate}} = \$177.328$, $P^{F}_{\text{High}} = \$433.547$ per tn.

Table 6: Statistical tests for coefficient equality across regime

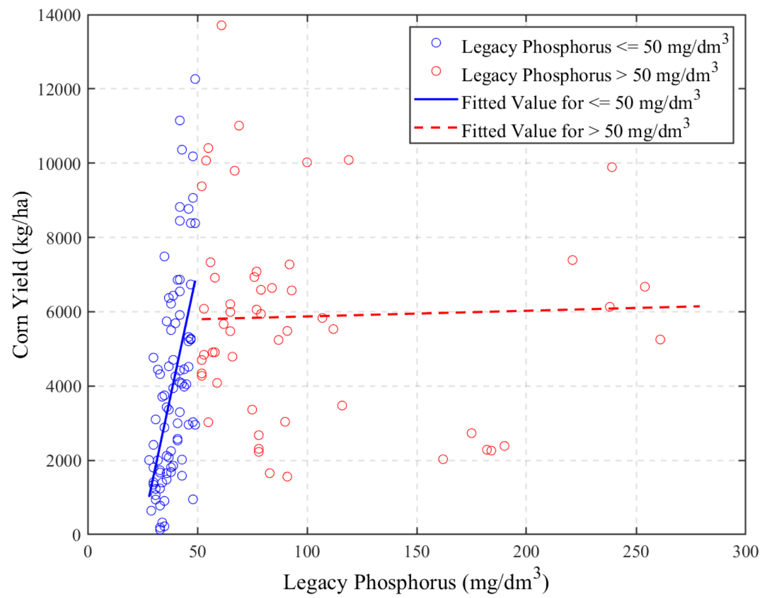
	Switching in corn price	Switching in phosphorus fertilizer price
$H_0 \ : \ \alpha_{1,\text{morderate}} = \alpha_{1,\text{high}}$	3.83**	
$H_0 \ : \ \alpha_{2,\text{morderate}} = \alpha_{2,\text{high}}$	7.19***	
$H_0 \ : \ \beta_{1,\text{morderate}} = \beta_{1,\text{high}}$		28.34***
$H_0 \ : \ \beta_{2,\text{morderate}} = \beta_{2,\text{high}}$		21.32***

Notes: The results of the Wald test and values indicate the chi-squared statistic. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

28.9% likelihood of moving to a high regime.

Table 6 presents the results of the specification tests for the MSDR model in relation to the equality of the inflation-adjusted price coefficients across the moderate and high regimes. The tests are designed to determine whether the impact of prices in time t on future $t+1$ prices differ significantly between the two regimes. The results of the tests on the equality of the corn price coefficient, $\alpha_{1,r_{t+1}}$, $\beta_{2,r_{t+1}}$, and the P fertilizer price coefficient, $\alpha_{2,r_{t+1}}$, $\beta_{1,r_{t+1}}$, across the regimes are highly significant. This strongly rejects the proportion of the equality hypothesis that coefficients are the same across regimes, suggesting that the dynamics underlying P fertilizer and corn prices differ depending on the regime characterizing the market. This also supports the notion that there are distinct regimes in the market with different price dynamics, again validating the use of the two-regime MSDR model to capture the differences in price changes over time.

Figure 5: Relationship Between Legacy Phosphorus Levels and Corn Yield



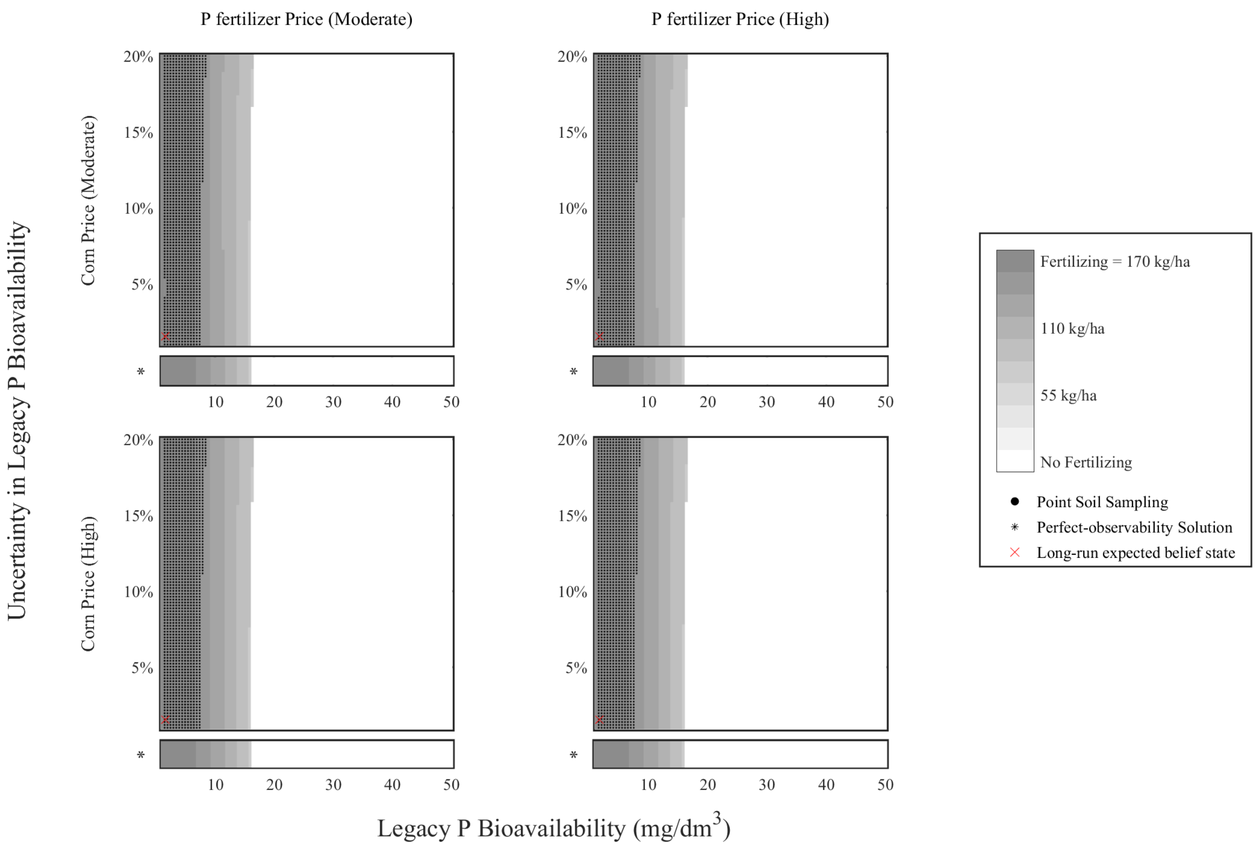
4.2 Optimal Policy and Dynamics of Legacy Phosphorus

According to the updated P fertilizer recommendations by Gatiboni et al. (2022) in North Carolina, extensive field trials have demonstrated that there is no significant yield response to P fertilizer applications when legacy P levels exceed 50 mg/dm^3. These trials, conducted across various crop types including corn, soybean, and small grains, consistently show that once legacy P levels reach approximately 50 mg/dm^3, additional P inputs do not result in higher crop yields. This saturation point indicates that plants' P uptake is maximized at this level, beyond which P availability is no longer a limiting factor for crop growth.

The concept of a P saturation point, as highlighted by Gatiboni et al. (2022), refers to the level at which legacy P is sufficiently abundant to meet crop needs, and any additional P does not enhance growth or yield. This critical point provides understanding the diminishing returns of P application. When legacy P exceeds this saturation point, it can lead to environmental concerns such as runoff and waterway pollution, without providing agricultural benefits.

Figure 5, based on samples from North Carolina Tidewater region, accompanied by linear fit lines for legacy P bioavailability less than or equal to 50 mg/dm^3 and greater than 50 mg/dm^3, visually illustrates the relationship between legacy P level and corn yield. The blue dots represent legacy P levels less than or equal to 50 mg/dm^3, while the red dots indicate

Figure 6: Optimal policy of P fertilizer application and soil sampling



legacy P levels above 50 mg/dm^3. The linear fit lines show a positive trend for legacy P levels up to 50 mg/dm^3, beyond which the trend flattens, indicating a minimal yield response to additional legacy P. Given the North Carolina samples and the agronomic evidence provided by Gatiboni et al. (2022), setting the maximum legacy P state variable to 50 mg/dm^3 in our MOMDP ensures that our model accurately reflects real-world conditions and provides relevant policy recommendations for optimal P management.

Figure 6 is composed of four graphs arranged in a two-by-two grid, each illustrating the optimal policy based on the bioavailability of legacy P, uncertainty, and the economic variables of the corn and P fertilizer prices. The horizontal axis measures legacy P bioavailability (mg/dm^3) within a range of 1 to 50 mg/dm^3. The vertical axis represents uncertainty, as measured by the coefficient of variation (CV) in L beleifs, from 1% to 20%.

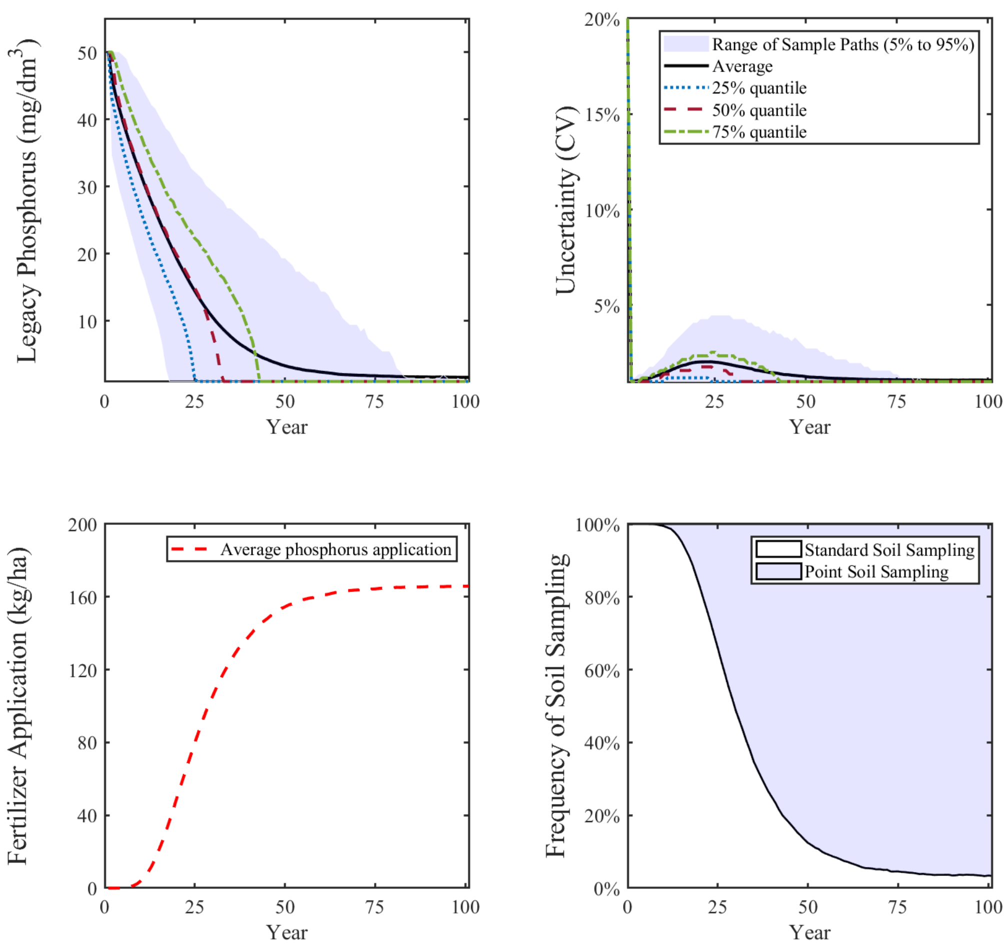
The figure illustrates the optimal application of P fertilizer for risk-neutral farmers. When uncertainty in legacy P bioavailability is high, risk-neutral farmers tend to apply more P fertilizer and are more likely to adopt soil sampling. The areas without dots indicate that farmers adopt standard soil sampling, while the dotted areas show where farmers opt for point sampling. When legacy P is low, farmers apply more P fertilizer to compensate for the low availability of P, ensuring sufficient nutrient supply for crop growth. Risk-neutral farmers particularly favor point sampling when legacy P level is low because it provides more accurate information, essential for making better-informed decisions about the optimal amount of P fertilizer to improve crop yield.

The perfect observability solution, marked by an asterisk, represents the scenario where farmers have perfect information about the amount of legacy P. The results for this scenario were derived using stochastic dynamic programming methods, which allow for optimal decision-making when the true state of legacy P is fully known. When the Mixed Observability Markov Decision Process (MOMDP) solution is applied in scenarios with very low uncertainty in legacy P bioavailability, the outcomes are similar to the perfect observability solution. This similarity occurs because, in cases of very low uncertainty, the farmer's belief about the legacy P state becomes highly accurate, almost equivalent to having perfect information. As a result, the decisions made under the MOMDP approach closely align with those made under perfect observability, as the need to account for uncertainty in the belief state diminishes, allowing the farmer to act almost as if they had complete knowledge of the legacy P levels.

The "+"mark represents the long-run expected belief state, where the process stabilizes over time. The position of this stable belief state is notably close to the minimum level of legacy P bioavailability and uncertainty. This pattern can be explained by considering the dynamics of legacy P: without the application of P fertilizer, legacy P naturally depreciates over time, diminishing its availability as a resource for crop yield. Given that legacy P is a resource for farmers, they have no reason or incentive to delay its use. This tendency to exhaust legacy P resources sooner rather than later, driven by its depreciation and essential role in crop production, explains why the long-term expected belief state aligns with low levels of legacy P bioavailability.

Additionally for the low level of uncertainty in the long-run expected belief state, as the process progresses, the information available to the farmers about legacy P becomes increasingly reliable. Over time, as farmers repeatedly observe the outcomes of their actions and adjust their practices, their belief about the amount of legacy P converges, leading to a reduction in uncertainty. This steady accumulation of knowledge and the diminishing variability in outcomes mean that farmers can predict the legacy P levels with a high degree of confidence, resulting in very low uncertainty at the steady state.

Figure 7: Dynamics simulation of stochastic growth



Notes: The initial values for the legacy phosphorus level and uncertainty are 50 mg/dm^3 and 20%, respectively. The initial conditions also include high corn price and high P fertilizer price. The figures were generated from 10,000 simulations

This pattern is expected to be reflected in the dynamic pattern, which further demonstrates the farmers' behavior in response to the legacy P dynamics over time. Figure 7 presents the controlled dynamics of the belief state and optimal policies over a 100-year period. In the top left panel, the legacy P level consistently decreases as the years progress. This decline in legacy P reflects its natural depreciation over time and the farmers' decision to utilize this resource promptly rather than delay its use. The reduction in legacy P availability leads to an increase in P fertilizer application, as shown in the bottom left panel. As legacy P diminishes, farmers compensate by applying more fertilizer to ensure sufficient nutrient availability for crop production.

In the top right panel, the uncertainty in legacy P bioavailability decreases rapidly in the early years and then stabilizes at a lower level. This behavior can be explained by the dynamics of legacy P and the corresponding sampling strategies adopted by farmers. Initially, when legacy P is abundant, there is relatively less uncertainty because the high levels of legacy P provide a lower variation in the state transitions. Farmers can effectively reduce uncertainty by adopting standard soil sampling, which is sufficient given the abundance of legacy P. As a result, uncertainty quickly decreases as farmers gain a clearer understanding of the soil's nutrient content.

However, as the legacy P levels begin to decline due to its use in crop production, the uncertainty in bioavailability starts to increase. This increase in uncertainty occurs because, with lower levels of legacy P, the variability in soil nutrient content becomes more pronounced, making it harder for farmers to predict future conditions accurately. In response to this rising uncertainty, farmers begin to adopt point sampling more frequently, as indicated in the bottom right panel of the figure. Point sampling provides more precise and localized information about the remaining legacy P, helping farmers to better manage their fertilizer application in the face of increasing variability.

As more farmers adopt point sampling, the additional and more accurate information gathered leads to a further reduction in uncertainty. Over time, this results in the uncertainty level decreasing again, as farmers' understanding of the legacy P bioavailability becomes highly refined. This pattern illustrates the adaptive behavior of farmers, who initially rely on standard sampling when legacy P is abundant but switch to more precise point sampling as legacy P becomes scarce and uncertainty increases.

4.3 Risk Analysis: Epstein-Zin Preference

Dynamic programming mapping is an efficient method for solving belief $\times$ price MDP by breaking the optimization problem down into a sequence of subproblems. However, it assumes a risk-neutral decision-maker. To understand the effects of risk preferences on the legacy P management problem, we extended our MOMDP model by incorporating an Epstein-Zin preferences (Epstein and Zin 1989).

Since we have no data on farmer risk preference over time in this context that would have permitted on estimation of η and ψ, we chose the range of estimated parameters from the literature on environmental and agricultural studies listed in Table 7. What is more important for our analysis than specific values is the effect of high or low RA and EIS on model results.

Table 7: Estimated value of risk aversion and elasticity of intertemporal substitution in literature

Literature		RA (η)	EIS (ψ)
Howitt et al. (2005)	California (US)	1.4	0.1
Lybbert and McPeak (2012)	Chalbi (Keyna)	0.5 (OLS) 0.8 (IV)	0.7(OLS) 0.9(IV)
	Dukana (Keyna)	13.5 (OLS) 12.5 (IV)	2.8(OLS) 3.3(IV)
Augeraud-Véron et al. (2019)		0.5-11	0.1-2
Cai and Lontzek (2019)		10	0.5, 1.5
Daniel et al. (2019)		1.1-15	0.6-1.2

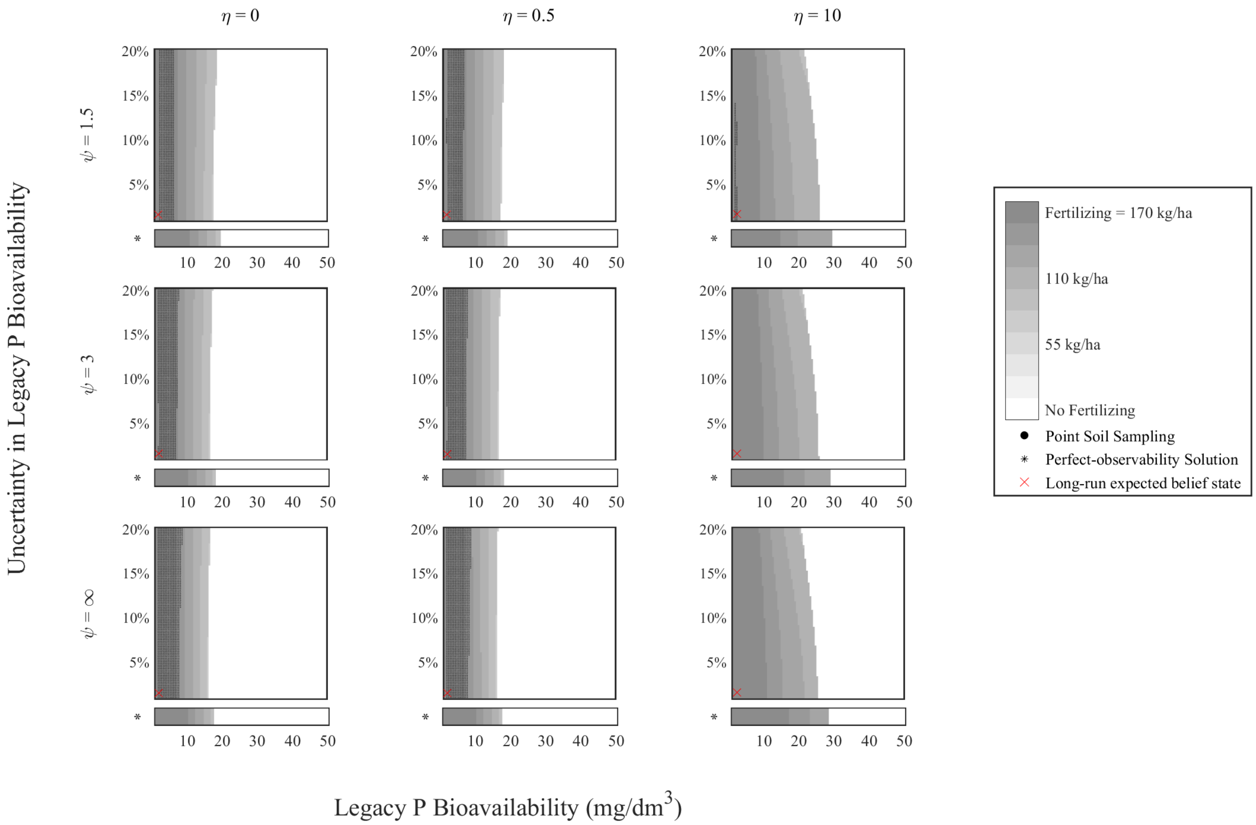
Notes: OLS and IV indicate Ordinary Least Squares regression and Instrumental variables estimation, respectively.

In the literature, the RA and EIS ranges are defined as $0.5 \leq \eta \leq 15$ and $0.1 \leq \psi \leq 3.3$, respectively. For our benchmark parameters, we choose multiple parameters across the ranges from which $\eta = (0.5, 10)$ and $\psi = (1.5, 3)$ were selected. In addition to benchmark parameters, the risk-neutral condition, $\eta = 0$, and the perfectly elastic intertemporal substitution, $\psi = \infty$, are considered. When $\eta = 0$ and $\psi = \infty$, the problem is reduced to the risk-neutral dynamic programming problem seeking to maximize the expected utilities.

Figure 8 shows how optimal policy changes with η and ψ. As η increased, indicating higher risk aversion, there is a noticeable shift toward higher P fertilizer application. This trend underscores the precautionary behavior adopted by risk-averse farmers who, facing uncertainty, prefer to ensure sufficient P fertilizer and legacy P levels for crop yield rather than risk potential yield losses. This decision is slightly modulated by ψ change, with lower ψ suggesting a preference for less intertemporal fluctuation in profit. This is demonstrated by a propensity to immediately apply more P fertilizer rather than defer application and potentially affect future profits.

When the farmer becomes more risk-averse, their preference shifts toward securing immediate and certain outcomes rather than uncertain future gains. This is because risk-averse individuals place a higher value on minimizing exposure to risks that could impact their short-term financial stability. Applying more P fertilizer offers a direct and immediate benefit by increasing the likelihood of a successful crop yield in the current season. For a risk-averse farmer, this immediate return is particularly attractive because it provides certainty in the

Figure 8: Epstein-Zin preferences and Optimal policy of P fertilizer application and soil sampling



Notes: Initial corn and phosphorus fertilizer price states are high. Other initial condition results are provided in the Appendix.

form of today's profit. This immediate profit helps the farmer meet essential financial needs, such as paying off debts, covering operational costs, or sustaining their household, thereby reducing their vulnerability to short-term financial pressures.

In contrast, while point sampling provides valuable information that could optimize fertilizer application in future seasons, it does not contribute directly to the current season's yield. The benefits of point sampling are realized over time, but they are contingent on future conditions, such as weather, market prices, and other variables that could influence the value of the information obtained. For a risk-averse farmer, the uncertainty associated with these future benefits makes them less appealing compared to the immediate and guaranteed returns from applying more fertilizer. Thus, as farmers become more risk-averse, they tend to prioritize strategies that offer immediate certainty, even if it means forgoing the potential long-term advantages of improved information.

5 Economic Sensitivity Analysis

A sensitivity analysis of economic conditions is also important for evaluating short-term productivity along with long-term agricultural sustainability when optimizing legacy P management using MOMDP. To understand the impact of varying economic conditions, we show the responses of the optimal policy to changes in the discount rate and exogenous shifts in P fertilizer prices (e.g. in response to a fertilizer tax).

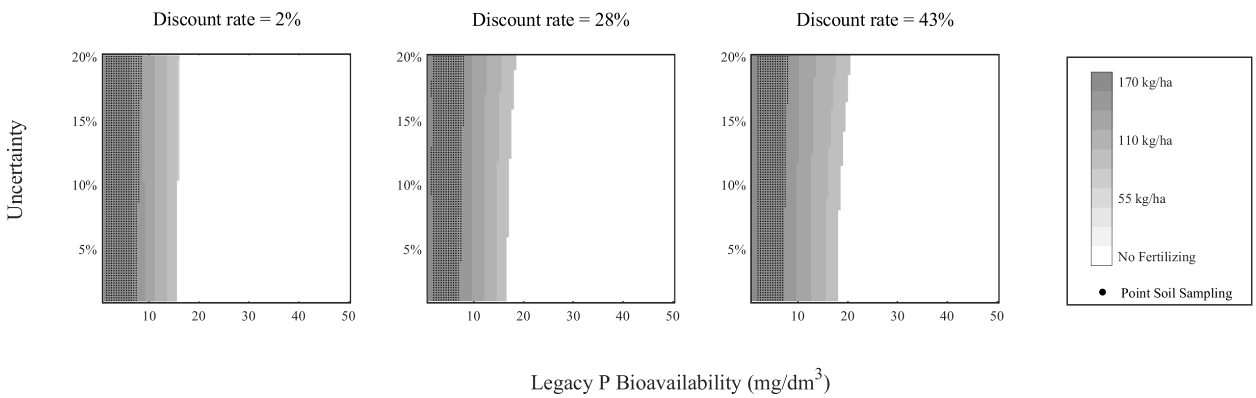
5.1 Discount Rate

In economic studies, particularly within agricultural and resource economics, the discount rate is a critical factor influencing farmers' decision-making processes. The discount rate essentially determines how much a farmer values future benefits compared to immediate gains. In addition to our benchmark discount rate of 8%, Duquette et al. (2012) also revealed that farmers often have relatively high discount rates, with some groups exhibiting rates as high as 43%, particularly among late adopters of new technologies, and others showing an average of 28%, especially among early adopters of best management practices. These rates are significantly higher than those typically used in benefit-cost analyses for federal programs.

In our sensitivity analysis, we selected three discount rates—2%, 28%, and 43%—to reflect a range of scenarios that align with both economic theory and empirical findings. The 2% discount rate represents the perspective of policymakers, based on the 30-year real interest rate on treasury notes and bonds (Young 2023), which reflects a long-term, low-discount environment where future benefits are highly valued. The 28% discount rate corresponds to the average rate found among early adopters of new agricultural practices in the study by Duquette et al. (2012), representing a middle-ground scenario where future benefits are still considered, but to a lesser extent. The 43% discount rate reflects the higher end of discount rates observed among farmers, particularly those who prioritize immediate returns over future gains.

Figure 9 illustrates how varying discount rates affect farmers' decisions regarding P fertilizer application and soil sampling. At a lower discount rate of 2%, farmers place greater value on future benefits, leading them to adopt point sampling more frequently and apply less P fertilizer, focusing on long-term profitability. As the discount rate increases to 28% and 43%, farmers increasingly favor immediate profits, resulting in reduced point sampling

Figure 9: Sensitivity analysis: Discount rate



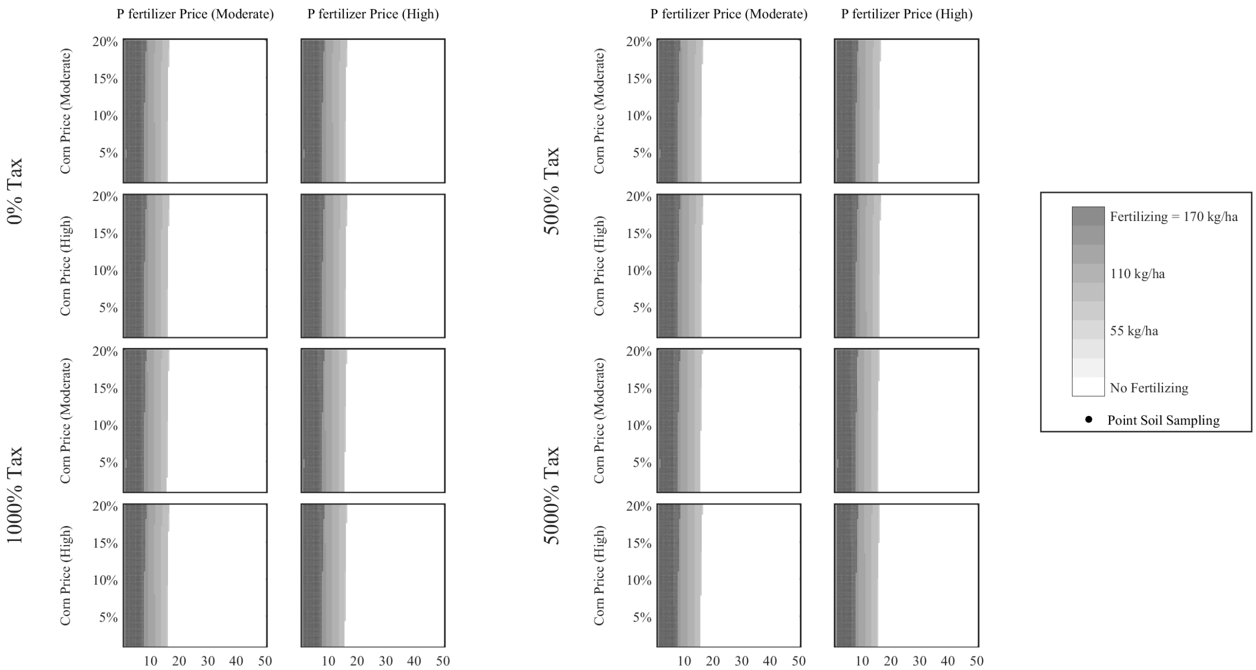
and more aggressive P fertilizer application. This shift is particularly pronounced at the 43% discount rate, where the emphasis is heavily on maximizing short-term yields at the expense of long-term soil management.

The behavior observed can be explained by two economic perspectives. First, the option value of information, the benefits of acquiring precise soil data through sampling before applying fertilizer—becomes more significant at lower discount rates. Farmers with a lower discount rate are more likely to invest in point sampling because they value the future flexibility and benefits that this information provides. Second, this behavior aligns with the concept of precautionary saving. By investing in point sampling, farmers improve their understanding of legacy P levels, thereby reducing the risk of future yield losses due to nutrient mismanagement. This strategic investment in information capital is more likely to occur when farmers place greater importance on future outcomes, as seen with lower discount rates.

5.2 Taxation on Phosphorus Fertilizer

Taxation on fertilizers to restrict chemical fertilization is a method to prevent water damage and this tool is incorporated by many states into their own environmental policies (Osteen and Kuchler 1986, Liang et al. 1998). However, the effectiveness of taxation on agricultural chemicals in reducing chemical fertilization is unclear. Liang et al. (1998) examined the effect of taxation on P and nitrogen on fertilizer use through two tax schemes, namely uniform and differentiated taxes. Their study revealed that a 500% tax reduced only 8% of on-farm fertilizer usage but caused at least a 30% reduction in agricultural labor.

Figure 10: Sensitivity analysis: taxation on phosphorus fertilizer

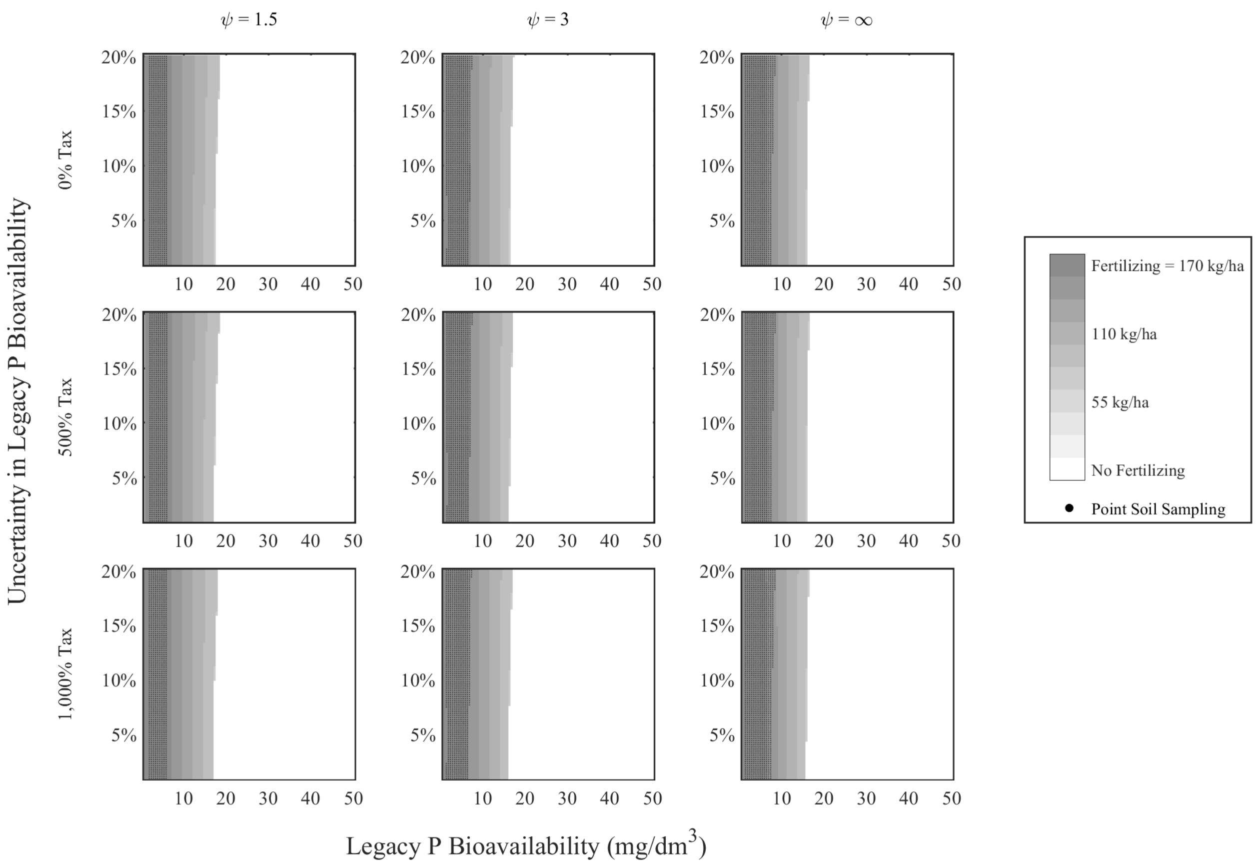


Notes: x-axis and y-axis indicate legacy P bioavailability and uncertainty in legacy P bioavailability, respectively.

This section recounts our investigation of possible explanations for inelastic fertilizer demand. For the general sensitivity analysis, a uniform tax scheme is considered with tax rates of up to 0%, 500%, 1,000%, and 5,000%. The uniform tax scheme can be defined as follows: $P^F_{\text{tax}} = P^F \cdot (1 + \textit{Tax Rate})$, where P^F is the producer price, and P^F_{tax} denotes the price of P fertilizer paid by farmers. Figure 10 represents how increased fiscal pressure on P fertilizer prices influences fertilizer application decisions within each taxation scenario. As taxation on P fertilizer intensified, farmers become more conservative and reduce P fertilizer application. However, despite the imposition of very high taxation rates on P fertilizer, no substantial decline in fertilizer application in farmland is observed. This trend can be attributed to several causes. Primarily, the demand for P fertilizer may be inelastic, with farmers considering it a necessary input for crop yield as it is an essential nutrient that cannot be replaced. In this particular cropping system, P fertilizer account for a relatively small share of the net revenue from total production costs on an annual basis, indicating room for farmers to absorb higher costs without forgoing positive profits.

Furthermore, farmers exhibit risk-averse behaviors, and may continue to apply fertilizer to minimize the risk of reduced yields due to P deficiencies, which can be financially more

Figure 11: Risk neutral farmer responses to P fertilizer tax



Notes: Initial corn and phosphorus fertilizer price states are high. Other initial condition results are provided in the Appendix.

damaging than the cost of P fertilizer. Figure 11 illustrates that as tax rates on P fertilizer rise, risk neutral ($\eta = 0$) farmers do not proportionally reduce their use of P fertilizer. The persistence in P fertilizer application despite high taxation reflects farmers' prioritization of long-term yield assurance over immediate cost implications. Additionally, with the higher EIS reflecting tolerance of intertemporal fluctuations in payoffs, farmers are more inclined to sustain their fertilizer application to secure long-term crop yields despite rising taxation.

The utilization of soil sampling provides detailed information on legacy P bioavailability in the soil, thereby allowing farmers to determine the optimal amount of P fertilizer needed for their crop. With precise knowledge of existing soil nutrients, farmers can apply the exact amount of P fertilizer required. This makes it challenging to significantly reduce P fertilizer usage, even with high tax rates, because the application is tailored directly to a crop's needs. Additionally, the perspective of legacy P as a long-term saving measure for soil fertility may outweigh concerns over the immediate cost increases associated with taxation. In on-farm management, the use of P fertilizer ensures crop quality and yield, further motivating farmers to maintain or slightly adjust rather than significantly decrease P fertilizer applications in response to tax increases.

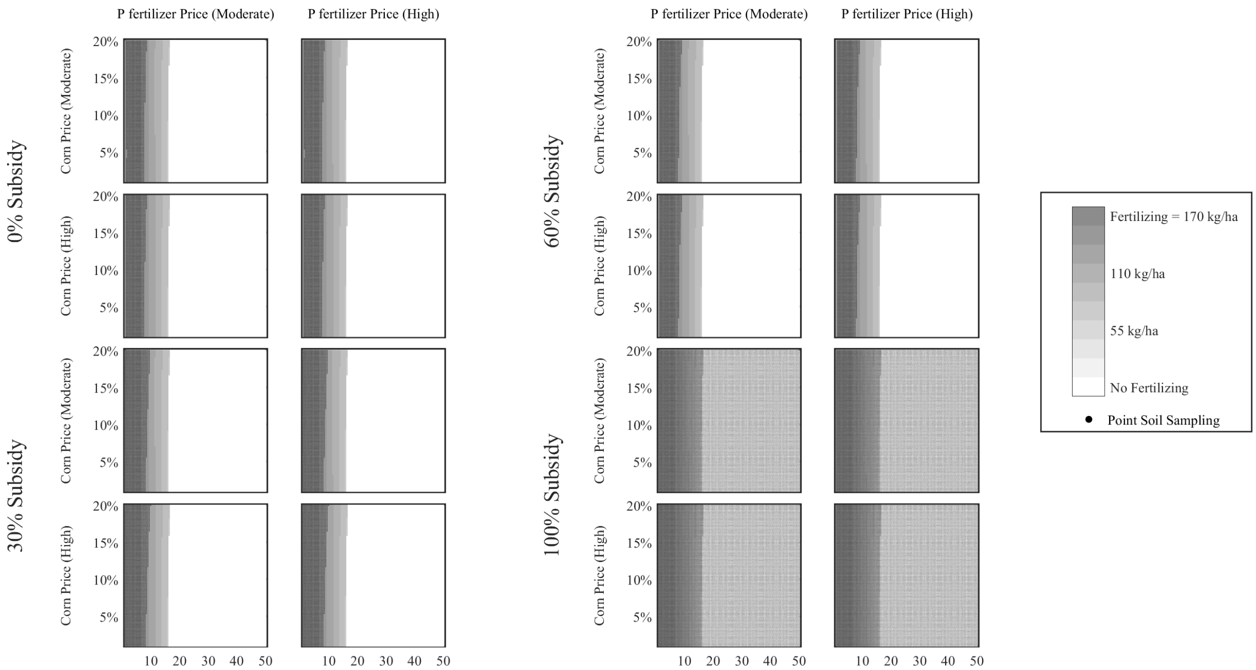
5.3 Subsidy on Soil Sampling

The adoption of soil sampling subsidies is a forward-looking agricultural policy instrument aimed at improving nutrient management practice among farmers. We study the potential impact of various levels of uniform subsidies on soil sampling rate, $c_s^{\text{subsidy}} = c_s(1 - \textit{Subsidy Rate})$, at 0%, 30%, 60% and full (100%) subsidization. The results presented in Figure 12 reflect a clear trend: As the subsidy rate increased, a corresponding rise occurs in the adoption of point sampling, particularly with full subsidy.

This propensity toward greater point sampling is indicative of a growing awareness and appreciation among farmers for the role of precise legacy P data in sustainable management. With subsidies easing financial loads, farmers are more inclined to assess the fertility of their soil, thus gaining valuable information that can inform their economic decisions. The increase in point sampling, driven by subsidies, offers significant potential for long-term shifts in P management practices. As farmers become increasingly informed with detailed data derived on soil sampling, we may observe a refinement in P fertilizer application strategies, tailored to the precise needs of crops.

Figure 13 illustrates the impact of soil sampling subsidies on farmers with $\psi = \infty$, mapped against RA levels as denoted by $\eta = (0, 0.5, 10)$. The analysis shows that as η increased, farmers tend to apply P fertilizer at higher rates, regardless of the subsidy levels for soil sampling. Risk-averse farmers prioritize securing crop yields by applying P fertilizer, even when subsidies are available to encourage soil sampling. This behavior underlines the precautionary actions taken by risk-averse individuals to mitigate the risk of yield loss due to insufficient nutrients. While financial incentives can encourage the adoption of point sampling, they do not necessarily lead to a proportional decrease in fertilizer usage among risk-averse farmers. This suggests that although subsidies make point sampling more accessible, the ingrained risk aversion and the perceived need to ensure crop yield stability drive continued high P fertilizer application rates. The results indicate that subsidies can effectively promote point sampling, but that their influence on reducing fertilizer application is moderated by a farmer's risk preferences. High-risk aversion diminishes the potential for subsidies to

Figure 12: Sensitivity analysis: subsidy on soil sampling



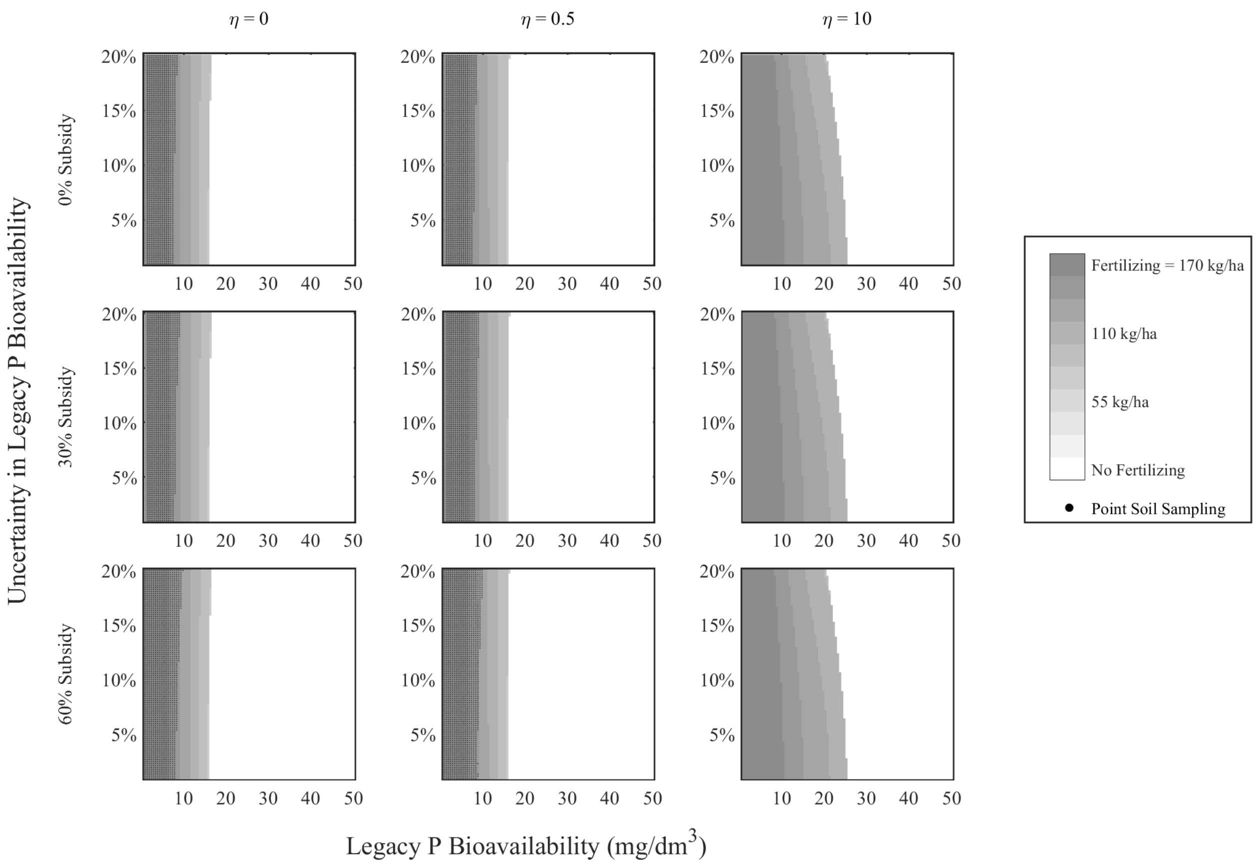
Notes: x-axis and y-axis indicate legacy P bioavailability and uncertainty in legacy P bioavailability, respectively.

significantly alter fertilizer application practices.

The findings presented in Figures 12 and 13 have important implications for policy design. Policy makers should consider structuring subsidy programs to not only reduce the cost of soil sampling but also address the underlying risk preferences of farmers. Combining financial incentives with risk management education and tools can enhance the overall effectiveness of such programs. Providing farmers with education and resources to better understand and manage risks associated with nutrient management can complement subsidy programs. By reducing the perceived risks related to crop yields, farmers may be more inclined to adjust their fertilizer application strategies.

Overall, while subsidies play a significant role in promoting soil sampling, addressing risk aversion through complementary measures is essential for achieving substantial change in fertilizer application practices. This comprehensive approach needs to be explored as a future project and can support sustainable P management, ensuring both agricultural productivity and environmental protection

Figure 13: Risk-averse farmer responses to soil sampling subsidy



Notes: Initial corn and phosphorus fertilizer price states are high. Other initial condition results are provided in the Appendix.

6 Discussion

The overuse of P fertilizer in agriculture causes significant surface water pollution, necessitating policy solutions that encourage farmers to use less P fertilizer while minimizing economic losses in agricultural production. Because of the dynamic and stochastic nature of P accumulation in soil, combined with state uncertainty about legacy P stocks, this research adopts a model-based approach to disentangling these dynamics and their effects on the fertilizer demand and soil sampling behaviors of risk-averse farmers. We apply methods developed for the resource management problems involving the partial observability of resource stocks and advance these methods to include agent risk and intertemporal smoothing preferences through the Epstein-Zin preferences. Accordingly, we reveal that risk aversion among farmers significantly contributes to the price-inelastic demand for fertilizer and their reluctance to rely on estimated legacy P stocks, despite extensive efforts to promote the utilization of these resources.

The focus of this research is understanding behavioral change among farmers rather than the environmental damage caused by P runoff. This distinction is critical because our primary objective is to analyze how farmers respond to different economic and informational incentives concerning legacy P management. Our findings provide important insights into why farmers may not fully exploit legacy P stocks and how their risk aversion shapes their P fertilizer application decisions.

While the environmental impacts of P runoff, such as eutrophication and greenhouse gas emissions, are important, our study specifically targets farmer behavior. By understanding the decision-making processes of farmers, we can better design policies that are more likely to be adopted and effectively reduce the overconsumption of P fertilizer. Behavioral focus advances the creation of more practical and applicable solutions tailored to the needs and preferences of farmers, ultimately leading to more sustainable agricultural practice. This focus on farmer behavior can be extended in future research to incorporate environmental factors more explicitly. For instance, expanding the model to consider the environmental and climate change implications of P management can provide a more comprehensive grasp of the overall impact of agricultural practices. Future studies can integrate spatial variability and explore interactions between farmland and adjacent areas, thus offering deeper insight into the collective economic and environmental outcomes of P fertilizer and soil sampling decisions.

Future research can also consider the multiple agents involved in the optimal management of the legacy P problem with additional areas. Currently, the environmental and resource economics literature using POMDP or MOMDP generally explores single agents in their models. Some researchers examine multiple agents, but they construct separate problems for each agent and disregard the interaction between the control exercised by each agent and the unobservable state problem. However, in a collective study of legacy P management, there will be multiple agents, in addition to farmers, that have their own observations and beliefs about the environmental state, which may also include beliefs about other agents' actions and strategies. By incorporating inter-agent dynamics into our POMDP model (Emery-Montemerlo et al. 2004), the POMDP may be constructed and extended as a 'Partially Observable Stochastic Game' (POSG) to solve for the optimal policy among multiple, competitive, or cooperative, agents' profits (Hansen et al. 2004).

This study demonstrates the significant influence of risk aversion on farmer behavior, highlighting the need for policies that only provide economic incentives but also address the underlying risk preferences of farmers. Our research centers on farmer related aspects of decision-making regarding P fertilizer application and soil sampling, laying the groundwork for future explorations that integrate environmental impacts and multi-agent dynamics, farmer associated factors, and government initiatives, offering an exhaustive approach to sustainable agricultural practices.

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