Homework 1: Analysis

Due Friday, April 8, at 11:59 PM

Asymptotic Notation

Specify whether f = O(g), or $f = \Omega(g)$ or both which would be $f = \Theta(g)$.

| | f(n) | g(n) |
|-----|----------------------|-------------------|
| (a) | n - 100 | n - 200 |
| (b) | $n^{rac{1}{2}}$ | $n^{\frac{2}{3}}$ |
| (c) | $100n + \log(n)$ | $n + (log(n))^2$ |
| (d) | nlog(n) | $10n \log(n)$ |
| (e) | $\log(2n)$ | log(3n) |
| (f) | $10 \log(n)$ | log(3n) |
| (g) | $n^{1.01}$ | n $log^2(n)$ |
| (h) | $\frac{n^2}{log(n)}$ | $n(log(n))^2$ |
| (i) | $n^{0.1}$ | $log(n)^{10}$ |
| (j) | $log(n)^{log(n)}$ | $rac{n}{log(n)}$ |
| (k) | \sqrt{n} | $log(n)^3$ |
| (1) | $n^{rac{1}{2}}$ | $5^{log_2(n)}$ |
| (m) | $n2^n$ | 3^m |

Iterative Substitution

Use a recursion tree to determine a good asymptotic upper bound on the recurrence. Use the substitution method to verify your answer.

1.
$$T(n) = T(n/2) + n^2$$

2.
$$T(n) = 4T(n/2) + n$$

Master's Theorem

When appropriate, use Master's theorem to solve the recurrence relations. If Master's theorem does not apply specify why.

1.
$$T(n) = 2T(n/2) + n^4$$

2.
$$T(n) = T(7n/10) + n$$

3.
$$T(n) = 16T(n/2) + n^2$$

4.
$$T(n) = 7T(n/3) + n^2$$

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5. T(n) = 7T(n/2) + n2
6. T(n) = 2T(n/4) + \sqrt{n}
7. T(n) = 7T(n-2) + n2
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Building Recurrence relation

For the following question you may assume that the function merge takes O(N) time.

- 1. Build the recurrence relation for the code shown below for BadSort.
- 2. Solve the recurrence relation
- 3. Use induction to prove the recurrence relation is correct.

```
def BadSort(A):
    if len(A) < 2:
        return A
    else:
        third = len(A)/3
        l = BadSort(A[:third])
        c = BadSort(A[third:2*third])
        r = BadSort(A[2*third:])
        return merge(1,c,r)</pre>
```