Gradient Non-Linearity in B-Tensor Encoding

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1 Gradient Waveform

For a time-varying gradient waveform $\vec{G}(t) = [G_x(t)\, G_y(t)\, G_z(t)]^T$, we have the time-varying q-vector $\vec{q}(t)$

$$\vec{q}(t) = \gamma \int_0^t \vec{G}(t')\, \mathrm{d}t'$$

where γ is the ^{1}H gyromagnetic ratio ($2\pi \cdot 42.577 \times 10^6\, \mathrm{m^{-1}\, T^{-1}}$).

The gradients waveform has duration t_{tot} and is designed such that we have

$$\vec{G}(t) = \vec{0} \text{ for } t \notin]0, t_{tot}[$$

and

$$\vec{q}(t) = \vec{0} \text{ for } t \notin]0, t_{tot}[.$$

We decompose the time-varying q-vector into a time-varying q-vector norm ($q(t)$) and unit-norm orientation ($\vec{n}(t)$),

$$\vec{q}(t) = q(t) \cdot \vec{n}(t).$$

and we define the B-tensor ($\mathbf{B}$) has

$$\begin{aligned}\mathbf{B} &= \int_0^{t_{tot}} (\vec{q}(t') \otimes \vec{q}(t'))\,\mathrm{d}t' \\ &= \int_0^{t_{tot}} q^2(t')(\vec{n}(t') \otimes \vec{n}(t'))\,\mathrm{d}t'\end{aligned}$$

where $\otimes$ is the outer vector product.

2 Gradient Non-Linearity

We model the gradient non-linearity by a time-constant gradient non-linearity tensor ($\mathbf{L}$)

$$\mathbf{L} = \begin{bmatrix} L_{xx} & L_{xy} & L_{xz} \\ L_{yx} & L_{yy} & L_{yz} \\ L_{zx} & L_{zy} & L_{zz} \end{bmatrix}$$

acting upon the time-varying expected-gradient ($\vec{G}(t)$) to give us the time-varying actual-gradient ($\vec{G_a}(t)$)

$$\vec{G_a}(t) = \mathbf{L} \cdot \vec{G}(t).$$

Similarly, we can compute the time-varying actual-q-vector ($\vec{q_a}(t)$) as

$$\begin{aligned}\vec{q_a}(t) &= \gamma \int_0^t \vec{G_a}(t')\,\mathrm{d}t' \\ &= \gamma \int_0^t \mathbf{L} \cdot \vec{G}(t')\,\mathrm{d}t' \\ &= \mathbf{L} \cdot (\gamma \int_0^t \vec{G}(t')\,\mathrm{d}t') \\ &= \mathbf{L} \cdot \vec{q}(t)\end{aligned}$$

and we maintain the two key properties

$$\vec{G_a}(t) = \mathbf{L} \cdot \vec{G}(t) = \mathbf{L} \cdot \vec{0} = \vec{0} \text{ for } t \notin]0, t_{tot}[$$

and

$$\vec{q_a}(t) = \mathbf{L} \cdot \vec{q}(t) = \mathbf{L} \cdot \vec{0} = \vec{0} \text{ for } t \notin]0, t_{tot}[\,.$$

The actual B-tensor doesn't have a simple close-form formula from the gradient non-linearity tensor and the expected B-tensor.