Week 2 exercises, ODEs

At each task, visualize the situation by making a plot of the solution of the ODE.

1. Solve the ODE system below using
    - ODE numerical solver
    - the symbolic toolbox
    - Compare and visualize the solutions from both methods in a plot.

$$\begin{aligned} \frac{dx}{dt} &= z + 4, & x(0) &= 2 \\ \frac{dz}{dt} &= -3x, & z(0) &= 3 \end{aligned}$$

2. After oral ingestion some drug x enters the stomach and is then distributed into the blood from where the drug will either get metabolized or exits the body respectively. State equations thus describe the drug dynamics in each compartment as follows,

$$\begin{aligned} \frac{dx_1}{dt} &= -k_1 x_1 + a \\ \frac{dx_2}{dt} &= -k_1 x_1 - k_2 x_2 + a, \end{aligned}$$

   where x_1, x_2 represent the concentrations of drug x in each compartment (i.e in blood and stomach) in milligrams. a is the ingestion rate of the drug, measured in mg/min. k_1, k_2 are the rate constants for the drug distribution and drug exit from the body.

    - Solve the equation numerically for a suitable time interval with $k_1 = 0.06$, $k_2 = 0.03$, when the initial amounts are given by $x_1(0) = 0$, $x_2(0) = 0$. The ingestion rate of the drug a is $4mg/min$.
    - Solve again the equation using the symbolic toolbox.
    - Compare and visualize the solutions from both methods in a plot.

3. The "Monod" model for bioreaction kinetics can be expressed as

$$\frac{ds}{dt} = -\frac{ksx}{k_s + s}$$
$$\frac{dx}{dt} = y\frac{ksx}{k_s + s} - bx,$$

where, s = Growth limiting substrate concentration (Mol/L^3), x = Biomass concentration (Mol/L^3), k = Maximum specific uptake rate of the substrate (1/T) = 5, k_s = Half saturation constant for growth $(Mol/L^3) = 20$, y = Yield coefficient = 0.05, b = Decay coefficient = 0.01. The initial value of s_0 and x_0 is 2000 and 100 respectively. Solve the ODE system for the given system.

4. Solve the Oregonator ODE system below

$$\frac{dx}{dt} = s(y - xy + x - qx^2)$$
$$\frac{dy}{dt} = (-y - xy + fz)/s$$
$$\frac{dz}{dt} = w(x - z),$$

with the parameter values $s = 100$, $w = 3.835$, $q = 10^{-5}$, $f = 1.1$ and initial conditions $x(0) = 1$, $y(0) = 2$, $z(0) = 3$. Solve the ODE for the timespan $t = [0, 360]$. Make a 2D plot of the solution.

5. Solve the DAE below

$$\frac{dz}{dt} = -2z + y^2$$
$$5 = 2z - 100\log(y)$$

with the initial conditions $z(0) = 2$ and $y(0) = 0.99005$.