PhD Admissions Assessment
Part 1: Mathematical Reasoning

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Question 1

We are asked to compute $\dfrac{\partial \mathcal{J}}{\partial \lambda}$ using the definitions below.

$$\begin{aligned}
\Phi &= \frac{1}{1+e^{-\xi}},\\
\xi &= \lambda\rho + \delta,\\
\mathcal{J} &= \frac{1}{2}(\Phi - \tau)^2
\end{aligned}$$

Since $\mathcal{J}$ is a function of Φ, which is a function of ξ, which in turn is a function of λ, we can apply the chain rule to compute the derivative, such that:

$$\frac{\partial \mathcal{J}}{\partial \lambda} = \frac{\partial \mathcal{J}}{\partial \Phi} \cdot \frac{\partial \Phi}{\partial \xi} \cdot \frac{\partial \xi}{\partial \lambda}$$

To do this, we compute the following partial derivatives:

$$\begin{aligned}
\frac{\partial \mathcal{J}}{\partial \Phi} &= \Phi - \tau,\\
\frac{\partial \Phi}{\partial \xi} &= \Phi(1-\Phi),\\
\frac{\partial \xi}{\partial \lambda} &= \rho.
\end{aligned}$$

Therefore, by the chain rule:

$$\frac{\partial \mathcal{J}}{\partial \lambda} = \frac{\partial \mathcal{J}}{\partial \Phi} \cdot \frac{\partial \Phi}{\partial \xi} \cdot \frac{\partial \xi}{\partial \lambda} = (\Phi - \tau) \cdot \Phi(1-\Phi) \cdot \rho$$

Question 2

We are given:

$$\kappa = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 3 \end{bmatrix}, \quad \theta = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

and the function $\psi(u) = \max(0, u)$ applied elementwise.

To find ω, such that

$$\omega = \psi(\Sigma\kappa + \theta),$$

To do this, we first compute the product $\Sigma\kappa$ (weight matrix and input vector):

$$\begin{aligned} \Sigma\kappa &= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 \cdot 1 + 2 \cdot 0 + 0 \cdot 2 \\ -1 \cdot 1 + 1 \cdot 0 + 3 \cdot 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 5 \end{bmatrix}. \end{aligned}$$

Next, we add our result to θ (bias):

$$\begin{aligned} \Sigma\kappa + \theta &= \begin{bmatrix} 1 \\ 5 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 7 \end{bmatrix}. \end{aligned}$$

Finally, we apply the (activation) function $\psi(u) = \max(0, u)$ to each component of the resulting vector:

$$\begin{aligned} \omega &= \psi\left(\begin{bmatrix} 0 \\ 7 \end{bmatrix}\right) \\ &= \begin{bmatrix} \max(0, 0) \\ \max(0, 7) \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 7 \end{bmatrix}. \end{aligned}$$

Thus, we have that $\omega = \begin{bmatrix} 0 \\ 7 \end{bmatrix}$.