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Profiting from arbitrage and odds biases of the European football gambling market

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ABSTRACT: A gambling market is usually described as being inefficient if there are one or more betting strategies that generate profit, at a consistent rate, as a consequence of exploiting market flaws. This paper examines the online European football gambling market based on 14 European football leagues over a period of seven years, from season 2005/06 to 2011/12 inclusive, and takes into consideration the odds provided by numerous bookmaking firms. Contrary to common misconceptions, we demonstrate that the accuracy of bookmakers' odds has not improved over this period. More importantly, our results question market efficiency by demonstrating high profitability on the basis of consistent odds biases and numerous arbitrage opportunities.

Keywords: betting market, favourite-longshot bias, football betting, profit margin, soccer betting, sports betting, sports gambling

1 Introduction

The enormous popularity of Association Football (hereafter referred to as simply *football*), along with increasing interest in gambling (particularly after its introduction online) means that great attention is now paid to its betting odds. While betting interest in horse racing has decreased, betting on football has increased so that is now by far the biggest sport in terms of turnover through the online bookmakers (Finnigan & Nordsted, 2010). For example, in 1998 the turnover for British football betting was close to £2bn (Global Betting and Gaming Consultants, 2001), and even then football betting was described as the fastest growing sector in British gambling (Mintel Intelligence Report, 2001). In comparison, the turnover reported by just a single bookmaker (bwin Group, 2009) for 2008 was around £2.92bn, which represented an astonishing 31.4% increase from the previous year.

For any large scale gambling market (and this includes financial markets) the question of *efficiency* is paramount. If there is a betting strategy that is consistent in generating profit against a gambling market, then such a market is normally described as being *inefficient*. The possibility of profiting because of market flaws is clearly both important and exciting. Because of the explosion of interest in football betting, increasingly researchers have turned their attention to evaluating the efficiency of this particular betting market. (Sauer, 1998) suggested that if a betting market is to be considered efficient, then such a market should encompass all the relevant information available in order to eliminate bettors exploiting opportunities within the market and achieving profits. Intuitively, many assume that the market is efficient. Yet, indications of an inefficient football betting market are becoming increasingly popular within the academic literature.

(Dixon & Coles, 1997) concluded that the UK football betting market is inefficient after a rather simple bivariate Poisson distribution model was able to earn positive returns under specific high-discrepancy trading rules during the English Premier League (EPL) season 1995/96. Similar

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conclusions have been reported in (Rue & Salvesen, 2000; Kuypers, 2000; Dixon & Pope, 2004). Further, in 2004 (Goddard & Asimakopoulos, 2004) found that the betting market is inefficient at the start[2] and, most notably, at the end of a football season, whereas (Forrest & Simmons, 2008) concluded that published odds appear to be influenced by the number of fans of each club in a match after observing that popular teams are offered more favourable terms on their wagers. More recently, (Constantinou et al., 2012) demonstrated how a Bayesian network model was able to generate profitability against the various published market odds by incorporating subjective information along with relevant historical data, whereas in (Constantinou et al., 2013) an improved Bayesian network model questioned market efficiency by demonstrating even greater profitability. In (Constantinou & Fenton, 2013) a novel and simple rating approach, called the *pi-rating*, provided further evidence of market inefficiency by demonstrating profitability over a period of five English Premier League (EPL) seasons. Yet, perhaps the primary reason why the football betting market is considered by many to be inefficient is the strong evidence of a *favourite-longshot* bias (see Section 4) as reported in (Cain et al., 2000, Forrest & Simmons, 2001; 2002).

In contrast to the studies above, other researchers came to opposite conclusions. In 1989 (Pope & Peel, 1989) investigated the *ex post* inefficiency of the fixed odds provided between bookmaking firms and concluded that no profitable betting strategies could have been implemented *ex ante* at that time. (Forrest et al., 2005) demonstrated how the efficiency of the market increased over a five year period with the help of an ordered profit model and showed that their model was unable to make profitable returns against the bookmakers. More recently, (Graham & Stott, 2008) introduced two forecast models; one based on football results, which is similar to that of (Forrest et al., 2005), and another based on past bookmaking odds in an attempt to compare the bookmaking opinion of various UK teams with the ratings generated by the football results based model. They showed that bookmaking prices were rational and not significantly different than those generated by the model, even though in some cases systematic bookmaking odds biases were observed which could not have been explained. Possibly the strongest evidence of efficiency are reported in studies in which researchers have attempted to outperform bookmakers' odds by introducing their own forecast models (ranging from very simple to rather sophisticated models), but failed to do so. As a result, other relevant studies have concluded and/or assumed that the betting market is efficient (Peel & Thomas, 1988; 1992; 1997; Vecer et al., 2009).

While this paper is focused on fixed-odds football betting markets, it is worth noting that there are various other studies within the academic literature which focus on sport betting markets that encompass significant differences in betting behaviour[3]. Discussions regarding such distinct betting markets can be found in (Vergin & Scriabin, 1978; Hausch et al., 1981; Asch et al., 1984; Zuber et al., 1985; Sauer et al., 1988; Thaler & Ziemba, 1988; Golec & Tamarkin, 1991; Shin H., 1991; Shin R. E., 1992; Shin H., 1993; Woodland & Woodland, 1994; Peel & Thomas, 1997; Vaughan Williams & Paton, 1997; Golec & Tamarkin, 1998; Henery, 1999; Jullien & Salanie, 2000; Woodland & Woodland, 2001; Levitt, 2004; Paton & Vaughan Williams, 2005).

In this paper we evaluate the modern behaviour of the online football gambling market on the basis of the odds provided by numerous bookmaking firms for 14 European football leagues and over a period of seven football seasons (i.e. from season 2005/06 to season 2011/12 inclusive). The data used for this study is available at www.football-data.co.uk. Throughout this paper (to be consistent with other studies) we restrict our attention to just the three possible outcomes of any match: H (home win), D (Draw) and A (Away win). The paper is structured as follows: Section 2 analyses profit margin variability and resulting arbitrage opportunities, Section 3 assesses bookmakers' accuracy, Section 4 examines predetermined biases in published odds, and we provide our concluding remarks and implications in Section 5.

[2] This agrees with (Forrest et al., 2005), in which authors showed that over a five-year period, their benchmark statistical model was outperforming bookmaking odds at the very start of the season. However, in all cases the model eventually failed to outperform bookmakers' odds. No claims were made of an inefficient market.

[3] Notably, other markets include *pari-mutuel betting* where published odds are determined solely by the behaviour of the bettors (e.g. horse racing), *spread betting* where the returns are based on the accuracy of the bettor (e.g. NFL); *betting exchange* where one bettor can bet against another bettor (e.g. horse racing – this has also recently emerged in UK football betting (betfair, 2000)).

2 PROFIT MARGIN & ARBITRAGE

The bookmakers' profit margin, also known as 'over-round', refers to the margin by which the sum of the probability odds of the total outcomes exceeds 1 (thus, making the odds unfair for the bettor). A lower profit margin results in less-unfair published odds. In short, the profit margin indicates the precise profit a bookmaker expects to receive if bets are distributed such that the bookmaker pays identical amounts of winnings whatever the match outcome. Since it is highly unlikely that the bets are distributed as specified above, the profit margin is just an approximation of a bookmaker's expected profit.

On the other hand an arbitrage opportunity is simply an opportunity whereby profit is guaranteed on the basis of a negative profit margin which results by combining the odds published by the various bookmaking firms. In particular, arbitrage opportunities depend on two factors: a) the divergence in outcome probabilities between bookmaking firms and b) the profit margin by each bookmaker. Negative profit margin is simply a scenario where a set of HDA probabilities is found (for a single match instance) in which the sum of the probabilities within that set is < 1. Hence, profit for the bettor can be guaranteed if the bets are placed such that the return is identical whatever the outcome.

For example, if we find that the best (lowest) probabilities for the bettor for a specific match instance, over a number of bookmaking firms, are $p(H) = 0.45$, $p(D) = 0.29$ and $p(A) = 0.25$, the sum of probabilities is just $s = 0.99$; corresponding to the respective decimal odds of 2.222, 3.4482 and 4. For this scenario the arbitrage size is 1% and we can guarantee a profit of 1.0101% $\left(\frac{1}{99}\right)$. If we want to bet $b = £100$, then the bet has to be distributed on the three outcomes as follows (rounded to the nearest penny):

- £45.45 on outcome H
- £29.29 on outcome D
- £25.25 on outcome A

using the equation $\frac{\left(\frac{b}{s}\right)}{o}$ for each case of HDA, where o represents the odds of H while calculating the bet to be placed on outcome H and so forth.

Academic evidence that demonstrate arbitrage opportunities date back to the 1980s, where (Pope & Peel, 1989) reported many such cases by considering the odds offered by four bookmakers, on a pre-tax basis, from 1980 to 1982. However, more recent studies (Dixon & Pope, 2004; Forrest et al., 2005) found no such opportunities in modern betting and concluded that there have been far less divergences in odds in recent years than in earlier periods. In particular, (Forrest et al., 2005) performed similar tests for the EPL seasons 1998 to 2003 using information from five bookmakers with introduced profit margins within the range of 10% and 12%. They showed that the minimum possible profit margin attained over the five seasons was averaging close to 6.6% and as expected, they reported no cases of positive arbitrage returns during that period.

2.1. Results

For determining the profit margin we have considered the average odds provided by the different bookmaking firms, whereas for determining arbitrage opportunities we have considered the maximum odds (best available for the bettor). The number of bookmaking firms taken into consideration for computing averages and maximums varies between 25 and 60 for each match instance (for details refer to Table A.1).

Table 1 presents the average profit margin introduced, from average bookmaking odds, for the specified football league and season. The results are not surprising since they clearly indicate that the profit margin has decreased over time and this outcome is in agreement with (Hvattum & Arntzen, 2010) who concluded that the competitiveness of the football betting market has increased during the period 2000 to 2008 on the basis of similar market behaviour.

It is interesting that the reduction in profit margin for the most recent seasons (i.e. 2009/10 onwards) is much stronger. Table 1 also demonstrates how lower divisions suffer from increased profit margins relative to the top divisions. It is also interesting to note that the odds provided for match instances of the English League II (i.e. 4^{th} division in England) suffer from a lower profit margin than those published for top divisions in Holland, Belgium and Greece. (Rue & Salvesen, 2000) suggested that it is natural for the bookmakers to provide better odds for the Premier League than for the lower divisions as the majority of the bettors bet on the Premier League; if their hypothesis is correct then we should conclude that more bettors bet on the 4^{th} division in England rather than on top divisions in Holland, Belgium and Greece. But with no further evidence it is very difficult to accept such a conclusion and hence, even though we believe that the market-forces assumption of (Rue & Salvesen, 2000) is rational, it is possible that other important factors that would allow us to formulate such a conclusion are not available for assessment. For example we do not have sufficient information whether the following kind of scenarios occur: if 100 bookmakers publish odds for league A and 500 bookmakers publish odds for league B, then it may be possible that the published odds of league B suffer from a lower profit margin than those of A due to the higher competitiveness between bookmaking firms for league B.

Table 1. Average profit margin introduced, from average bookmaking odds, for the specified league and season.

League	Season							
	2005-06	**2006-07**	**2007-09**	**2008-09**	**2009-10**	**2010-11**	**2011-12**	**Overall**
Eng. Premier Leag.	9.45%	8.98%	8.29%	7.26%	6.67%	6.09%	6.08%	7.55%
Eng. Champ.	10.97%	10.42%	9.69%	8.51%	8.13%	7.45%	7.38%	8.94%
Eng. League I	11.17%	10.85%	10.30%	9.08%	8.63%	7.89%	8.07%	9.43%
Eng. League II	11.19%	10.96%	10.48%	9.24%	8.74%	7.99%	8.13%	9.53%
Eng. Conf. League	11.47%	11.20%	11.02%	10.51%	9.98%	9.41%	9.40%	10.43%
Germ. Bundesliga I	10.15%	9.60%	8.79%	8.03%	7.45%	6.62%	6.52%	8.17%
Germ. Bundesliga II	11.20%	10.56%	10.03%	9.37%	8.86%	8.48%	8.30%	9.54%
Italian Serie A	10.17%	9.80%	8.96%	8.26%	7.63%	7.02%	6.65%	8.36%
Italian Serie B	11.15%	11.00%	10.72%	10.17%	9.74%	9.68%	9.13%	10.23%
Sp. La Liga Primera	10.19%	9.74%	8.94%	8.03%	7.36%	6.63%	6.59%	8.21%
Sp. La Liga Segunda	11.76%	11.68%	11.03%	10.29%	9.53%	9.22%	9.04%	10.36%
Dutch Eredivisie	11.15%	10.98%	10.17%	9.49%	8.82%	8.32%	8.22%	9.59%
Belgian Jupiler	11.29%	11.19%	10.32%	9.82%	9.25%	9.09%	9.00%	9.99%
Greek Ethniki Kat.	11.40%	11.26%	10.87%	10.48%	9.51%	8.85%	8.98%	10.19%

Table 2 presents the average profit margin in the EPL per specified bookmaker over the same period. The results here show that the profit margin can be significantly different between bookmaking firms; suggesting that the published odds of one bookmaker cannot be representative of the whole market. This outcome contradicts the assumptions of (Forrest & Simmons, 2002; Forrest et al., 2005) who based their conclusions on the notion that the odds of a single bookmaker are representative of the whole market and that little information is lost by concentrating on just one bookmaker.

Figures A.1 and A.2 provide further evidence that discredit the notion of a single representative bookmaker by demonstrating how distinct profit margins per match of the EPL seasons 2010/11 and 2011/12 can further vary considerably. The results show that the profit margin for distinct match instances of the same league and bookmaker can still vary considerably.

Table 2. Average profit margin introduced per specified bookmaker and EPL season.

Bookmaker	Season						
	2005/06	**2006/07**	**2007/08**	**2008/09**	**2009/10**	**2010/11**	**2011/12**
William Hill	12.49%	12.49%	12.37%	7.01%	7.35%	6.50%	6.70%
BET365	7.91%	7.95%	5.98%	5.31%	5.43%	5.44%	5.46%
Bwin	10.13%	10.07%	10.06%	10.07%	8.30%	8.01%	6.42%
Gamebookers	8.11%	8.04%	7.45%	7.29%	7.75%	7.68%	7.67%

Interwetten	12.33%	11.35%	11.39%	10.21%	8.36%	10.13%	10.07%
Ladbrokes	12.27%	12.32%	12.19%	9.26%	7.48%	6.49%	6.65%
Sportingbet	8.14%	10%	10.13%	10.14%	10.12%	10.12%	7.66%

By taking into consideration a relatively small number of different bookmaking firms, previous studies (Dixon & Pope, 2004; Forrest et al., 2005) demonstrated no evidence of arbitrage opportunities in recent years. However, that situation has now changed with the increase in number of bookmakers. Indeed, this increase, together with the reduced bookmaker profit margins, means there are extensive arbitrage opportunities and high return rates.

Figure 1(a) demonstrates the significant increase in arbitrage size per match instance as we move towards season 2011/12; in particular, for the two most recent seasons the potential profit from arbitrage has doubled. Figure 1(b) demonstrates the average profit per match instance generated through arbitrage over all leagues and seasons for each country and shows that profitability from arbitrage can vary significantly between leagues of different countries. Figure A.3 presents the number of arbitrage instances discovered per 100 match instances for the specified league and season, clearly demonstrating how arbitrage has become common over the more recent seasons. Table A.1 provides detailed information regarding a) the number of arbitrage opportunities discovered, b) the average number of bookmaking firms taken into consideration, c) the average negative profit margin attained and d) the average profit generated, for each football league and season.

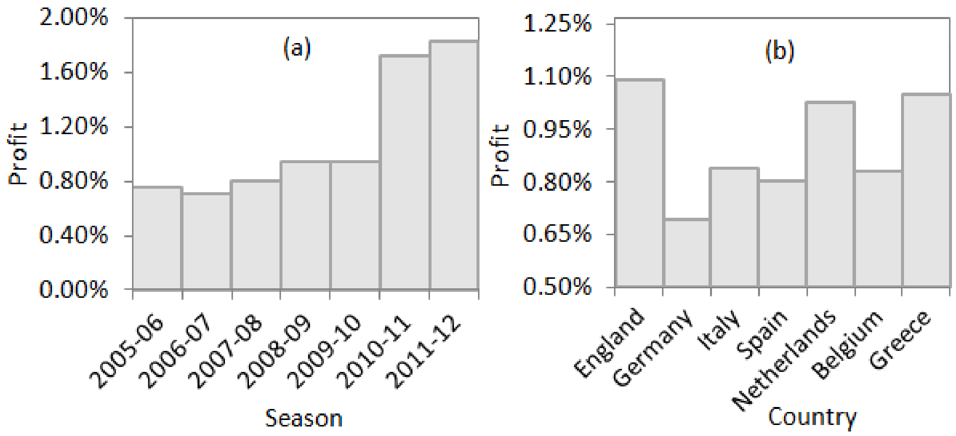

Figure 1. Average profit generated through arbitrage per match instance over all leagues and per football season (graph (a)), and over all leagues and seasons for each country (graph (b)).

When it comes to different levels of football divisions, Figure 2 demonstrates weak evidence of increased profitability from arbitrage as we move towards lower divisions. However, the number of bookmaking firms taken into consideration for computing maximums cannot explain this behaviour. In fact, we can observe from Table A.1 that the average number of bookmaking firms taken into consideration, per match instance, for the English divisions were (starting from top division) 47.03, 41.26, 37.73, 36.96 and 32.69 respectively. In reality, we would have expected the profitability generated for the lower divisions to decrease since the number of bookmaking firms taken into consideration for arbitrage in those divisions is lower than in top divisions. As a result, we can only infer that the divergence in odds provided by the various bookmaking firms is much higher for lower divisions than it is for higher divisions, which would explain the higher profitability generated via odds combination.

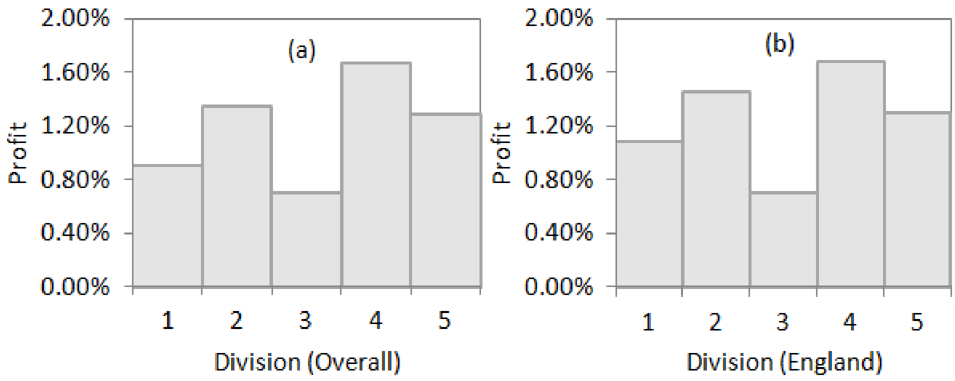

Figure 2. Average profit generated through arbitrage per match instance over all leagues and per division level (graph (a)), and for the different division levels in England (graph (b)). Note that the results for divisions 3, 4 and 5 are practically identical for both graphs since no data for weaker divisions was available in countries other than England.

3 BOOKMAKERS' ACCURACY

Since bookmakers increase profitability by encouraging bettors to place as many bets as possible, their profit is not only determined by the profit margin, but also by the accuracy of their published odds which should, therefore, represent a good approximation of the 'true' probabilities of any particular match without introducing biases. In this section we examine the degree of variation between bookmakers with regards to the accuracy of their normalised[4] odds. For this assessment we have considered the average bookmaking odds.

3.1. Methodology

For forecast assessment we make use of the Rank Probability Score (RPS), a scoring rule introduced in (Epstein, 1969), and which has been described to be particularly appropriate in assessing both interval and ordinal scale probabilistic variables (Murphy, 1970). (Strumbelj & Sikonja, 2010) have already used the RPS to assess bookmakers' football match odds and (Constantinou & Fenton, 2012) explain why RPS is the most rational scoring rule of those that have already been proposed and used for assessing probabilities assigned to football outcomes. In brief, this scoring rule represents the difference between the observed and forecasted cumulative distributions in which a higher difference leads to a higher penalty (Wilks, 1995) which is subject to a negative bias that is strongest for small ensemble size (Jolliffe & Stephenson, 2003). RPS is both strictly proper and sensitive to distance (Murphy, 1969; Murphy, 1970) and for a single match instance is defined as

$$RPS = \frac{1}{r-1}\sum_{i=1}^{r-1}\left(\sum_{j=1}^{i}(p_j - e_j)\right)^2$$

where r is the number of potential outcomes (three in this case), and p_j and e_j are the forecasts and observed outcomes at position j (so, for example, p_j is the forecast probability of H and e_j is 1 if the outcome was a home win and 0 otherwise). A lower score indicates a more accurate forecast (less error). This methodology is defined in detail, along with relevant examples, in (Constantinou & Fenton, 2012).

3.2. Results

Table 2 presents the accuracy scores, over all bookmakers, for each league and season. The results are rather surprising. Whereas (Forrest et al., 2005) concluded that bookmakers provided more

[4] The odds are normalised such that the profit margin is eliminated and the sum of the probabilities over the possible events is equal to 1.

accurate odds over time, our results show no indications of forecast improvement in bookmaking performance over the period of seven seasons and this is true for almost all of the 14 distinct leagues. Overall, the results tend to suggest that the accuracy of the odds provided for top division leagues is, to some extent, higher than lower division leagues.

Table 2. RPS assessment for each league over the seven seasons. Lower score indicates greater accuracy.

	Season							
League	**2005-06**	**2006-07**	**2007-08**	**2008-09**	**2009-10**	**2010-11**	**2011-12**	**Overall**
Eng. Premier Leag.	0.1949	0.1953	0.178	0.1917	0.1828	0.2006	0.2014	0.1921
Eng. Champ.	0.2083	0.2233	0.2178	0.215	0.2075	0.22	0.2176	0.2156
Eng. League I	0.2209	0.2206	0.2158	0.2195	0.2007	0.2203	0.2082	0.2151
Eng. League II	0.2186	0.2185	0.2289	0.2166	0.223	0.2176	0.2167	0.22
Eng. Conf. League	0.2161	0.2254	0.2105	0.2092	0.2094	0.2047	0.2105	0.2123
Germ. Bundesliga I	0.1918	0.2169	0.2055	0.2049	0.2068	0.228	0.2032	0.2082
Germ. Bundesliga II	0.2235	0.2094	0.2012	0.2072	0.2192	0.2071	0.2007	0.2098
Italian Serie A	0.1783	0.1829	0.1912	0.1954	0.1931	0.2008	0.1986	0.1915
Italian Serie B	0.1897	0.1896	0.1962	0.2041	0.2084	0.2026	0.2105	0.2002
Sp. La Liga Primera	0.2101	0.2101	0.2135	0.2025	0.1812	0.1907	0.1897	0.1997
Sp. La Liga Segunda	0.2148	0.2176	0.2106	0.2001	0.2085	0.2103	0.2083	0.21
Dutch Eredivisie	0.1956	0.1863	0.2048	0.1939	0.1709	0.1846	0.1893	0.1893
Belgian Jupiler	0.2028	0.1893	0.1993	0.2011	0.2042	0.1925	0.1901	0.197
Greek Ethniki Kat.	0.1639	0.1846	0.1818	0.1803	0.1784	0.2016	0.1929	0.1834

Table 3 presents the average RPS values generated for each specified bookmaker over this period. Clearly, the accuracy of the odds between bookmakers turns out to be extremely consistent. These results are also in agreement with (Forrest et al., 2005) who claimed that the variation in the predictability of match results from season to season is much larger than the variation in forecasting performance between bookmakers for the same season.

Table 3. Average RPS for each bookmaker and EPL season. Lower score indicates greater accuracy.

	Season						
Bookmaker	**2005/06**	**2006/07**	**2007/08**	**2008/09**	**2009/10**	**2010/11**	**2011/12**
William Hill	0.1952	0.1953	0.1799	0.1914	0.1832	0.2002	0.2010
BET365	0.1949	0.1955	0.1769	0.1915	0.1824	0.2003	0.2013
Bwin	0.1956	0.1960	0.1782	0.1921	0.1843	0.2010	0.2022
Gamebookers	0.1953	0.1949	0.1777	0.1918	0.1833	0.2014	0.2016
Interwetten	0.1967	0.1963	0.1798	0.1916	0.1832	0.2008	0.2014
Ladbrokes	0.1957	0.1953	0.1799	0.1927	0.1846	0.2004	0.2010
Sportingbet	0.1951	0.1954	0.1786	0.1921	0.1836	0.2006	0.2013

4 ODDS BIASES

In gambling markets, the favourite-longshot bias refers to the tendency for bets at short odds to generate a higher return than bets at long odds. Possibly the strongest hypothesis behind this phenomenon is the preference of the bettor in backing risky outcomes, which are also referred to as longshots, and this phenomenon is observed in many different markets (Ali M., 1977; Quandt, 1986; Thaler & Ziemba, 1988; Shin H., 1991, Shin R. E., 1992; Shin H., 1993; Woodland & Woodland, 1994; Vaughan Williams & Paton, 1997; Golec & Tamarkin, 1998; Jullien & Salanie, 2000), and various theories exist, such as risk-loving behaviour, on why people are willing to bet on such uncertain propositions (Sobel & Raines, 2003; Snowberg, 2010).

For example, consider a game between a top team playing at home against a very weak team. Suppose the 'true' probability of a home win is 0.9. Then even if a bookmaker offered the perfectly fair odds of 1.11 for a home win a typical bettor is reluctant apparently to place £100 bet to win only £11. However, at the other extreme, if the 'true' probability of an away win in this match is 0.05 then a typical bettor would be prepared to bet at less than fair odds of, say 15, because for a very small bet

£10 they stand to win £150. Bookmakers are believed to exploit these types of behaviour and increase profitability by offering more-than-fair odds for 'safe' outcomes, and less-than-fair odds for 'risky' outcomes.

However, while some researchers have focused their analysis on different bookmaking prices for determining favourite-longshot biases (Cain et al., 2000), others have considered all home and away wins serving as favourite and longshot outcomes respectively (Graham & Stott, 2008). When it comes to home and away match instances, it might be true that away wins can be seen as longshots on average (and vice versa), but the same does not hold for a significant proportion of matches. Therefore, while the analysis in (Cain et al., 2000) helps us to understand market behaviour when it comes to the favourite-longshot bias, the analysis in (Graham & Stott, 2008) could be best described as a home-away bias. To avoid confusion between the different types of biases, in this section we define and investigate the three following types of odds biases:

1. *Favourite-longshot bias*: as in (Cain et al., 2000), low probability outcomes can be considered as longshots (and vice versa). In this paper we focus our longshot bias analysis in grouped outcomes with less than 20%, 15% and 10% chance of occuring, and our favourite bias analysis in groups with more than 60%, 70% and 80% chance of occuring.

2. *Home-away bias*: as in (Graham & Stott, 2008), we assess home and away team biases by focusing on home and away wins respectively.

3. *Most-likely/least-ikely bias:* unlike the favourite-longshot bias which only considers high and low probability outcomes, here we investigate potential odds biases when it comes to the most likely and the least likely outcomes for each match instance (and therefore this takes into consideration all of the match instances).

Throughout this section we have considered the average match outcome odds, as provided by numerous bookmaking firms, for the top divisions of England, Germany, Italy and Spain and over the period of seven seasons; from 2005/06 to 2011/12. To investigate the biases we simulate £1 bets on the relevant outcomes and compare the respective cumulative/final returns.

4.1. Favourite-longshot bias

Figure 3 presents the average profit per bet as generated by simulating bets on match outcomes which satisfy the specified probabilistic conditions for favourite and longshot bets as specified above. The results clearly demonstrate how loss is increased while bets move towards longshot outcomes and returns break even while moving towards favourite outcomes. This implies that the bias was not strong enough to overcome the averaged bookmaking profit margin (i.e. most of the bookmaking firms falling between 5.5% and 12.5% from average bookmaking odds as demonstrated in Section 2).

Figure 3. Profit rates as generated by simulating £1 bets on match outcomes which satisfy the specified probabilistic favourite-longhshot conditions, given all of the top division match instances of England, Germany, Italy and Spain over the seven football seasons (from 2005/06 to 2011/12).

Figures B.1 and B.2 demonstrate how this outcome differs for each country-league and season respectively. With the exception of the German league, this phenomenon appears to be highly consistent. When it comes to different seasons, however, the bias appears to have diminished in recent seasons. In particular, the overall losses during season 2010-11 appear to be greater when betting on favourites, whereas for season 2011-12 the greater overall losses return back to longshot bets but still the losses seem to even out when compared to previous seasons. It should be noted that the unexpected outcome observed during season 2010-11 is due to a high number of unexpected longshot outcomes observed in the EPL.

4.2. Home-away bias

Figure 4 demonstrates the cumulative returns generated by simulating £1 bets on all home wins, draws and away wins, of match instances of England, Germany, Italy and Spain. Seasons are ordered such so that Season 1 represents season 2005/06 and Season 7 represents season 2011/12. The results demonstrate bias on returns generated from bets on the outcomes of a home win. In particular, betting on home wins yields the highest cumulative return under most of the scenarios. This phenomenon does also not appear to be particularly profitable but still, with the exception of the German league, home-away bias appears to be equally as strong as the favourite-longshot phenomenon. This is because, while bets on away wins generate a high cumulative loss, bets on home wins generate minimal cumulative loss over the whole period of seven seasons.

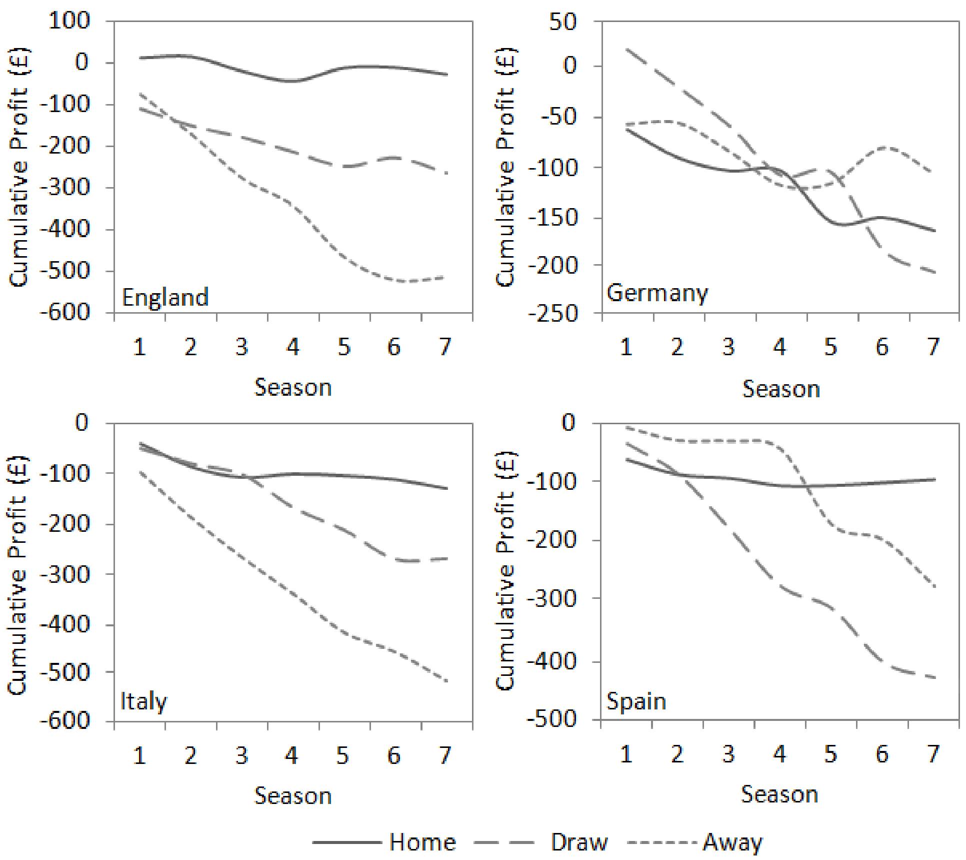

Figure 4. Cumulative returns generated by simulating £1 bets on all home wins, draws and away wins, of match instances of England, Germany, Italy and Spain and over the period of seven seasons (from 2005/06 to 2011/12).

4.3. Most-likely/least-likely bias

Figure 5 demonstrates the profitability generated over the period of seven seasons by simulating bets on all most-likely and least-likely outcomes for each country. Overall, the results appear to be in agreement with the favourite-longshot bias, whereby most-likely bets generate considerably higher returns than least-likely bets, and the results appear to be fairly consistent for the cases of England, Italy and Spain.

Figure 6 presents the profit rates from bets on most-likely and least-likely outcomes when partitioned into HDA outcomes. From this, we observe that:

a) For the cases of England and Germany, betting on a home win when home win is the least likely outcome appears to be highly profitable with respective overall profit rates of 6.51% and 5.12%, and the rate of profitability appears to be consistent. In contrast, for the cases of Italy and Spain this betting strategy generates significant losses with respective profit rates of -17.61% and -14.08%, and which are also rather consistent.

b) Overall, betting on a home least-likely win, rather than an away least-likely win, generates considerably higher returns. In particular, the average home win bet on least likely outcomes generated profits of 6.51%, 5.12%, -17.61%, and -14.08%, whereas the average away win bet generated profits of -26.22%, -3.66%, -25.28%, and -11.08%.

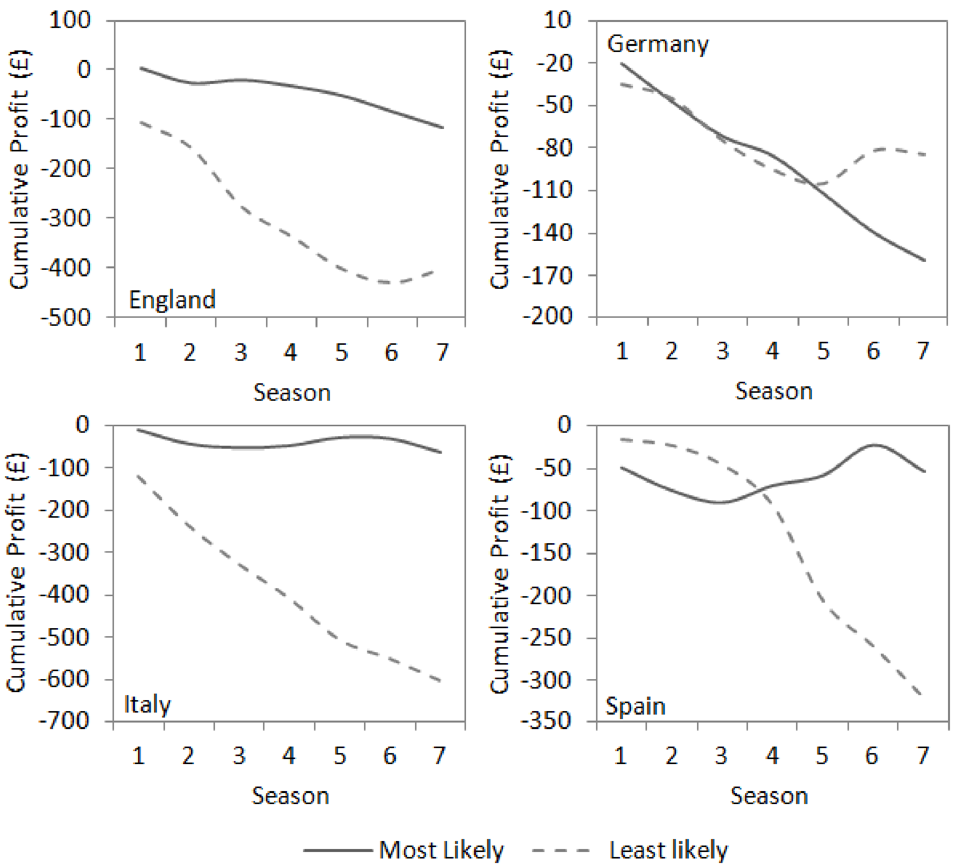

Figure 5. Cumulative returns generated by simulating £1 bets on all most-likely and all least-likely outcomes of match instances of England, Germany, Italy and Spain over the period of seven seasons (from 2005/06 to 2011/12).

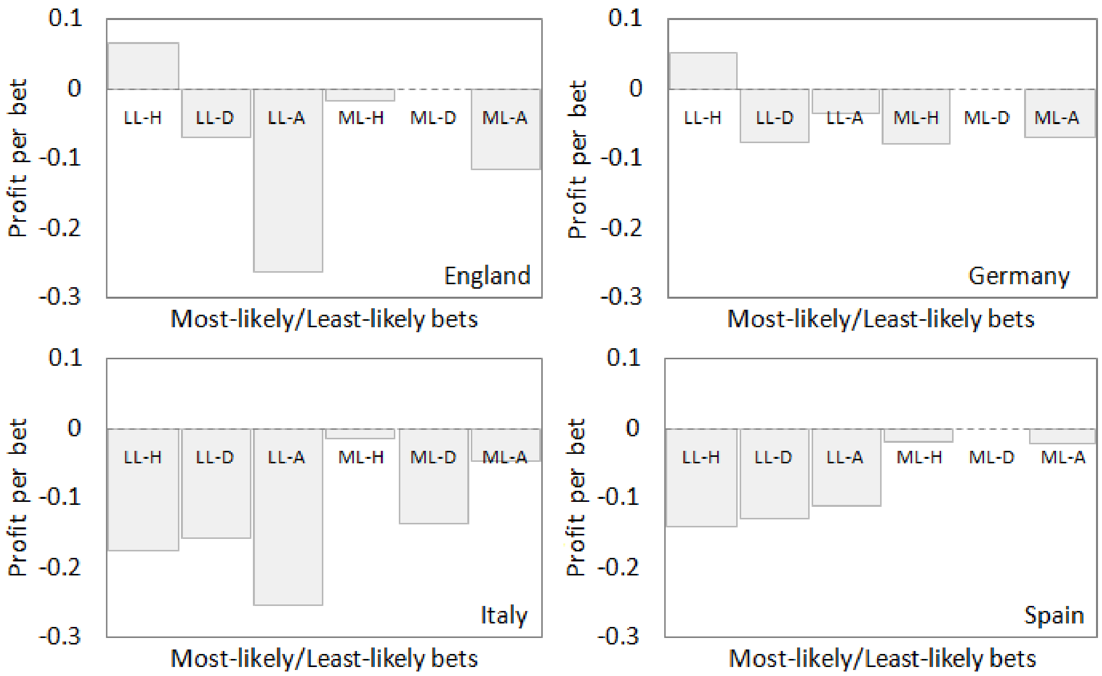

Figure 6. Profit rates generated by simulating £1 bets on all most-likely and least-likely outcomes, as partitioned into *HDA* outcomes and each country over the period of seven seasons.

5 SUMMARY AND CONCLUSIONS

Overall, the evidence presented in this paper, based on recent market trends, casts further doubt on the notion that the football betting market is efficient. We summarise our findings as follows:

Bookmakers' accuracy, profit margin and arbitrage

a) The market nowadays consists of a higher number of bookmaking firms who publish odds with significantly lower profit margins than even before. This has led to frequent arbitrage opportunities. The emergence of software to make it easier to spot arbitrage opportunities (for example, websites such as www.oddschecker.com[5]) and evolving systems that perform automated internet analysis in real time to spot arbitrage opportunities make the whole process much easier. If this trend continues then in the near future bookmakers may be exposed to substantial arbitrage risks.

b) Primarily based on English football data, evidence suggest that the profitability from arbitrage increases when betting on match instances of lower divisions and the results tend to indicate that the explanation for this is because the divergence in odds between bookmakers is much higher for lower divisions.

c) We have demonstrated that profit margins can vary significantly between bookmaking firms, and also for match instances of the same league offered by a single bookmaker. These observations contradict the popular assumption that the published odds of one bookmaker are representative of the whole market (Forrest & Simmons, 2002; Forrest et al., 2005).

d) Contrary to popular belief (Forrest et al., 2005; Graham & Stott, 2008) and based on a suitable measure of forecast accuracy, our results show no evidence of forecast accuracy improvement by bookmakers over a period of seven seasons and with 14 football leagues taken into consideration.

[5] A website that gives an overall view of the market and informs visitors about the best available odds by considering a large number of various online bookmakers.

Odds biases:

a) The presence of the widely accepted favourite-longshot bias is still strong. However, while (Cain et al., 2000) demonstrated this bias back 1991/92 with no evidence of potentially profitable strategies on favourites, our results in many cases show evidence of high profitability. For example, in Figure B.1 note the cases of England and Spain, and in Figure B.2 the cases of seasons 2005/06 and 2009/10, whereby bets on outcomes with >80% chance generated average profits of approximately 2%, 2%, 8% and 10% respectively. This outcome does not necessary imply that the favourite-longshot bias is now stronger than what it was two decades ago. The profitability could be explained by the recently reduced bookmaker profit margins. In fact, Figure B.2 demonstrates how this bias is diminished over the seven seasons under analysis, in which the average returns from each longshot bet with less than 10% chance were -55%, -64%, -68%, -33%, -83%, 20%[6] and -19% respectively (starting from season 2005/06 to 2011/12). Similar behaviour is observed for longshot outcomes with 15% and 20% chance of success. This reduction in loss, especially for the last two seasons, might also be explained by the recently reduced bookmaker profit margins; in particular, it may be possible that the bookmakers reduce the bias such so that the chance of profit when betting on favourites is not increased due to the continuous decrease in profit margins.
b) Based on match instances of England, Italy and Spain, our results demonstrate a clear home-away bias whereby the returns from bets on home win outcomes generate considerably higher returns, when compared to bets on away wins, and this result is in agreement with (Graham & Stott, 2008) who reported this outcome as evidence to support the favourite-longshot bias. In fact, our results show that this type of bias is almost equally as strong as the favourite-longshot bias.
c) Betting on the most likely outcome generates notably higher returns than betting on the least likely outcome. The bias in this case, however, does not appear to be profitable on the whole.
d) When considering all of the least likely outcomes, the returns from bets on home wins generate higher returns than returns from bets on away wins. While exploiting this bias, evidence of consistent profitability have been observed for the cases of England and Germany. In particular, over the period of seven seasons, the average profit from home win bets on least likely outcomes for England and Germany were 6.51% and 5.12%. In the case of bets on most-likely outcomes not strong bias has been discovered.

Due to the favourite-longshot bias observed over the previous seasons many have suggested that this bias was due to bookmakers taking dynamic positions against the presumed tendency of the bettors to underbet on favourites and to overbet on risky outcomes (Rossett, 1971; Snyder, 1978; Ali M. M., 1979; Asch et al., 1984; Levitt, 2004; Graham & Stott, 2008). On the basis of this assumption, which was also the case in various other gambling markets, a mixture of theories have been formulated such as risk-loving behaviour on why people are willing to bet on such uncertain propositions (Sobel & Raines, 2003; Snowberg, 2010). If the assumption of having bookmakers taking positions against bettors for maximising profit is correct, then bookmakers' odds are prices published with the intention of maximising profit, which would also support the above assumptions.

On the basis of the above theory, the additional odds biases demonstrated in this paper can only be explained by the theory as follows:

1. On the basis of evidence from (b) above we can infer that bettors prefer betting on an away win rather than on a home win.
2. On the basis of evidence from (c) above we can infer that bettors prefer betting on the least likely outcome, and this is in agreement with the favourite-longshot bias reasoning.
3. On the basis of evidence from (d) above we can infer that, when home and away wins are the least likely to occur, bettors still have preference betting on an away win rather than a home win.

[6] During season 2010/11 the returns from longshots with <10% chance generated highly profitable returns due to a large number of surprising results in the English Premier League.

Although we have suggested possible explanation for the phenomena that we have observed, the fact that the only data that we have available to work with is price data constrains our ability to infer the true underlying causal influences. Such inferences require important pieces of data on such (certainly unobserved) factors such as the proportion of bets placed on each outcome. Such data are not available to the public. Planned extensions of this research will examine how published football models which have already demonstrated profitability against market odds (Constantinou et al., 2012; 2013) might further benefit in betting effectiveness by formulating betting patterns which take into consideration the different types and degrees of biases presented in this and other recent relevant papers.

Acknowledgements

We acknowledge the financial support by the Engineering and Physical Sciences Research Council (EPSRC) for funding this research. We would also like to thank Dr. Xiao-Lu Yu from the Department of Physics and Astronomy, Mississippi State University, for indicating an error in an earlier version of our analysis.

APPENDIX A: PROFIT MARGIN & ARBITRAGE OPPORTUNITIES

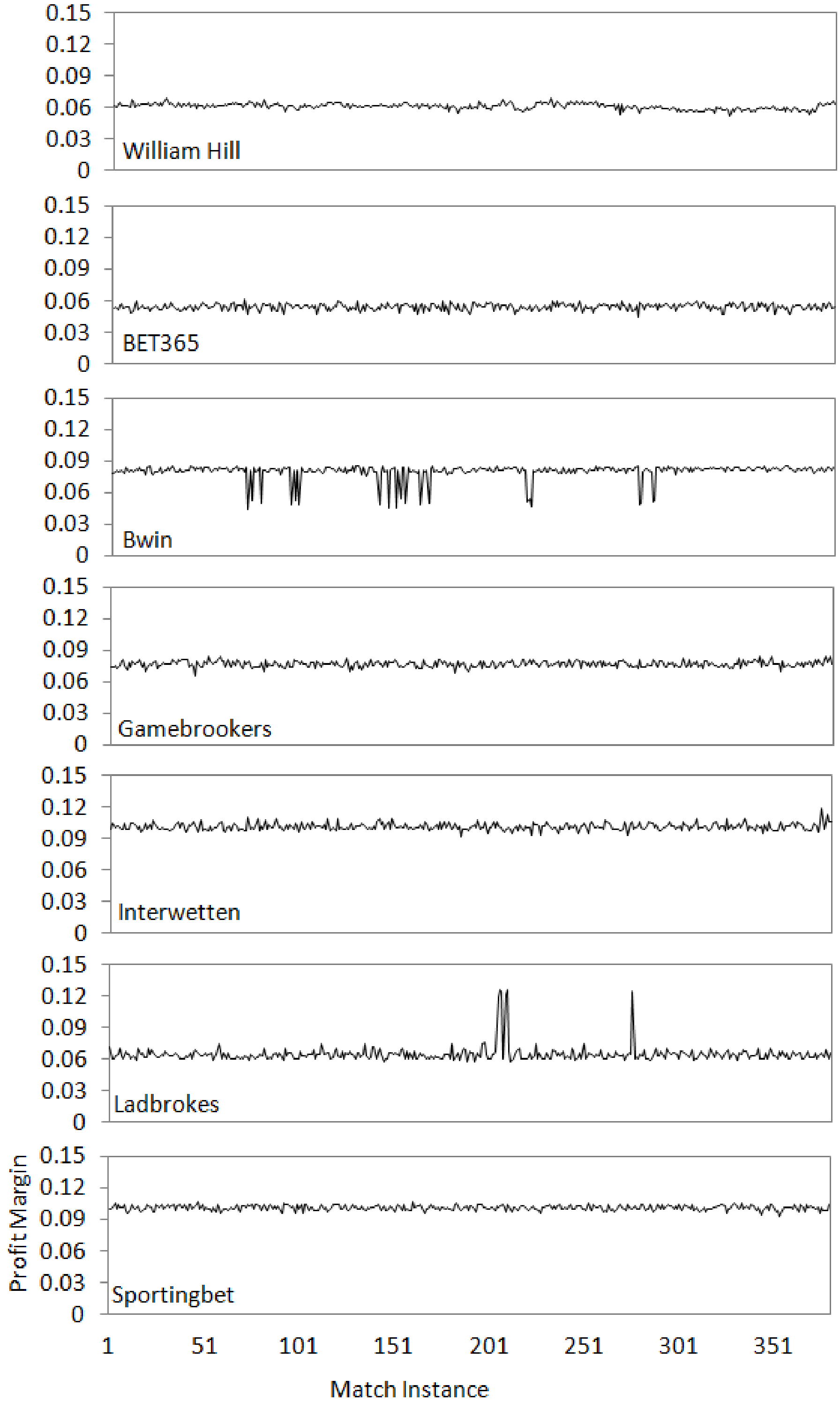

Figure A.1. Distinct profit margins, as introduced by the specified bookmaking firms, per match instance and over the EPL season 2010/11.

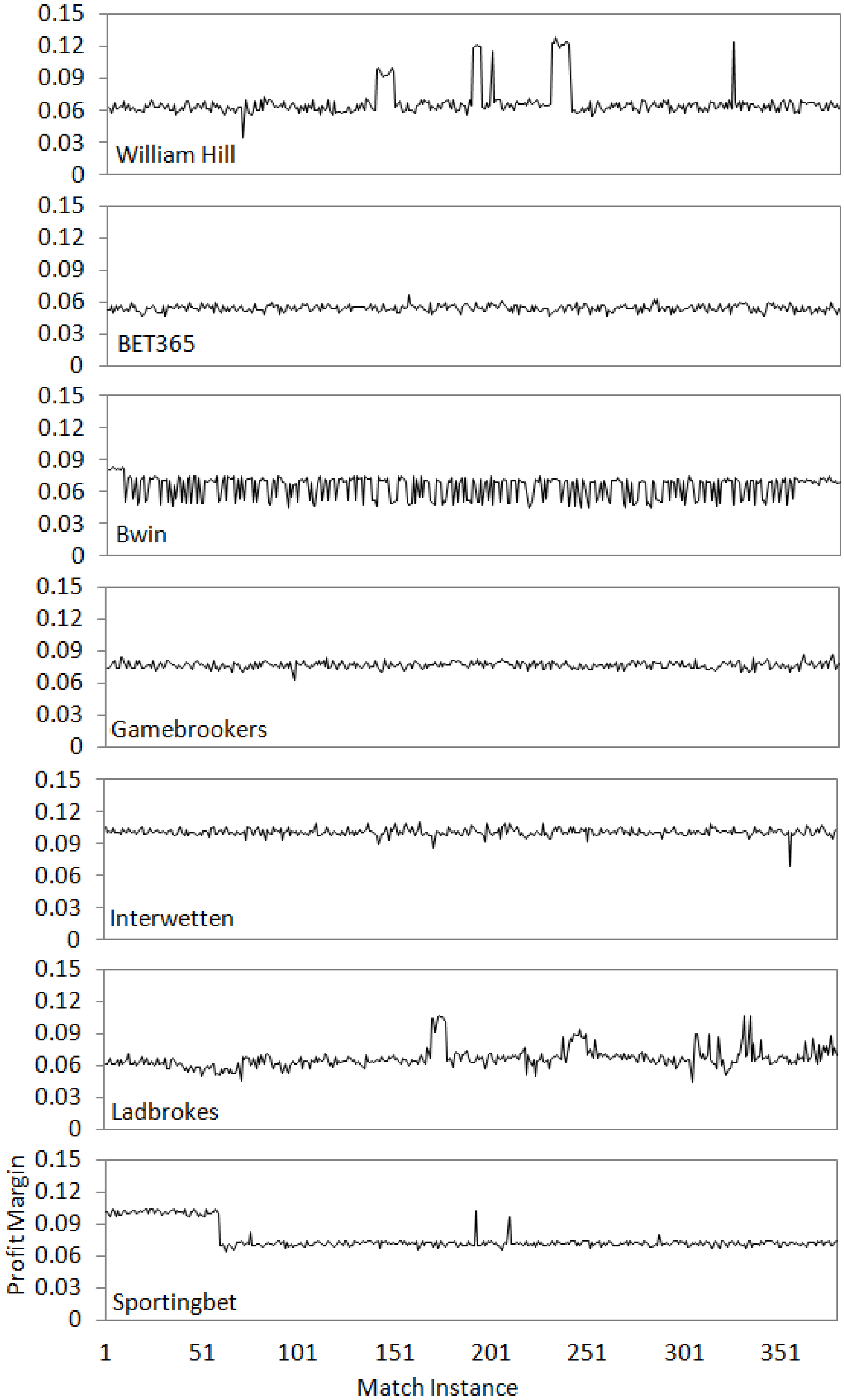

Figure A.2. Distinct profit margins, as introduced by the specified bookmaking firms, per match instance and over the EPL season 2011/12.

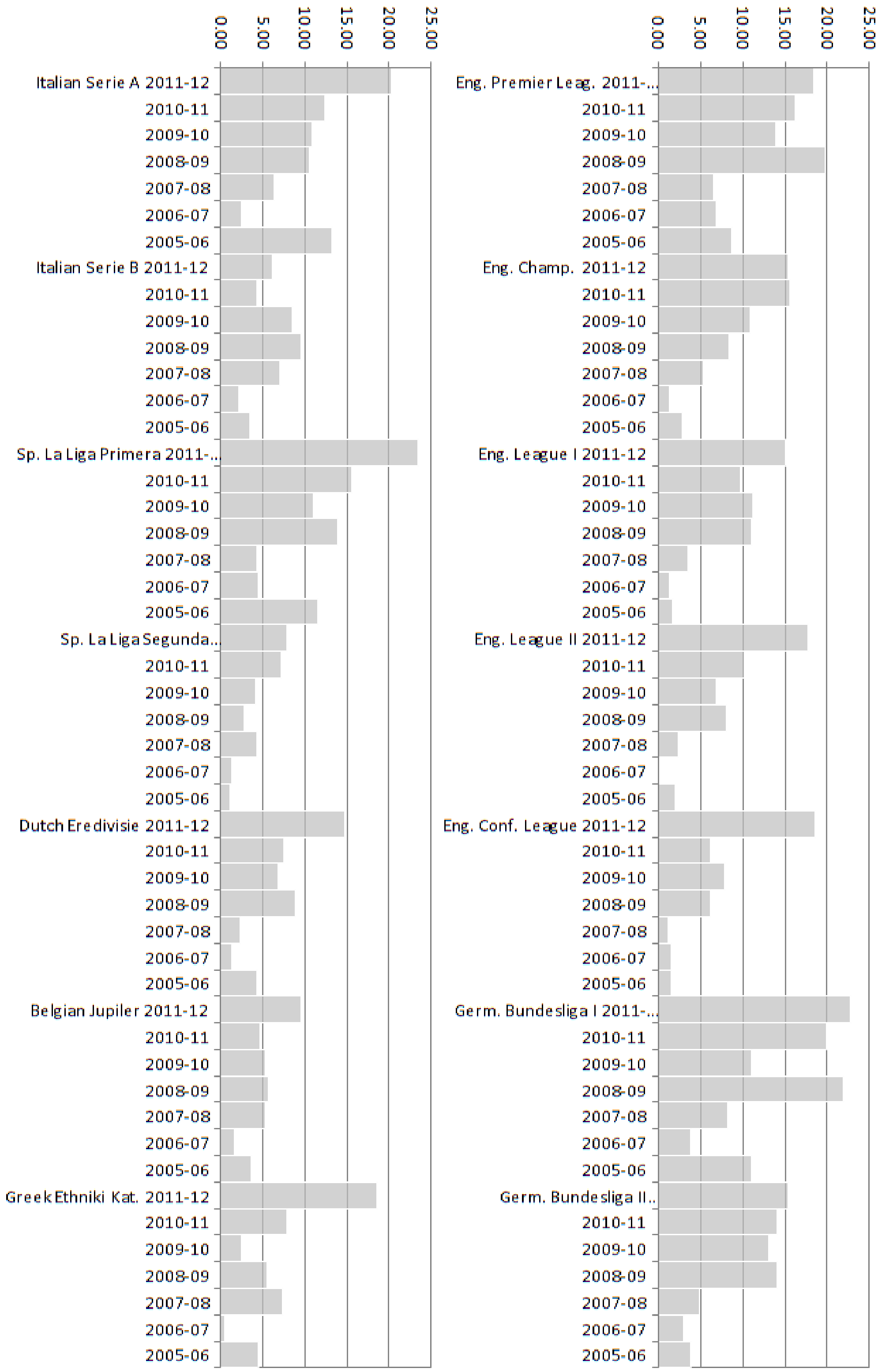

Figure A.3. Number of arbitrage instances discovered per 100 match instances for the specified league and season.

Table A.1. Arbitrage opportunities discovered per football league and season.

League	Season	Match instances considered[7]	Bookmaking firms considered[8]	Arbitrage instances found	Average profit per match instance
English Premier League	2005-06	380	57.84	33	2.29%
	2006-07	380	44.70	26	2.63%
	2007-08	380	43.39	25	0.54%
	2008-09	380	40.16	75	0.53%
	2009-10	380	36.02	53	0.57%
	2010-11	380	35.73	62	0.55%
	2011-12	380	41.56	70	0.52%
English Champ.	2005-06	462	55.30	13	2.87%
	2006-07	462	41.39	6	2.67%
	2007-08	462	38.42	25	2.74%
	2008-09	462	37.58	39	0.51%
	2009-10	462	34.67	50	0.60%
	2010-11	166	41.33	26	0.41%
	2011-12	462	40.07	71	0.40%
English League I	2005-06	462	49.10	8	1.07%
	2006-07	461	37.86	6	0.21%
	2007-08	462	36.81	16	1.06%
	2008-09	462	34.38	51	0.83%
	2009-10	462	32.74	52	0.71%
	2010-11	462	34.33	45	0.55%
	2011-12	462	38.88	70	0.47%
English League II	2005-06	462	47.13	9	3.34%
	2006-07	462	37.22	1	4.05%
	2007-08	461	35.72	11	1.33%
	2008-09	462	34.80	37	0.91%
	2009-10	462	31.68	32	0.94%
	2010-11	462	33.50	47	0.63%
	2011-12	462	38.68	82	0.63%
English Conference League	2005-06	462	36.38	7	1.63%
	2006-07	461	31.17	7	3.45%
	2007-08	462	32.17	5	0.50%
	2008-09	455	31.55	29	0.81%
	2009-10	462	28.47	36	1.28%
	2010-11	462	32.01	29	0.69%
	2011-12	453	37.05	86	0.74%
German Bundesliga I	2005-06	306	56.49	34	0.92%
	2006-07	306	44.46	12	0.56%
	2007-08	306	41.93	25	0.62%
	2008-09	306	38.55	67	0.72%
	2009-10	306	34.74	34	0.54%
	2010-11	306	36.14	61	0.83%
	2011-12	306	41.56	70	0.68%
German Bundesliga II	2005-06	306	40.65	12	1.29%
	2006-07	303	30.75	9	2.36%
	2007-08	305	36.59	15	0.54%
	2008-09	306	30.81	43	0.71%
	2009-10	306	30.53	40	0.90%
	2010-11	306	32.46	43	0.97%
	2011-12	306	38.82	47	1.07%
	2005-06	380	55.74	50	1.62%

[7] The number of match instances considered for discovering arbitrage opportunities. For instance, if there were 10 missing entries in an EPL dataset (which consists of a total of 380 match instances over a single season), the reported number of instances considered would be 370.
[8] Average bookmaking firms considered per match instance.

Italian Serie A	2006-07	380	42.16	9	0.70%
	2007-08	380	39.71	24	0.43%
	2008-09	379	37.72	40	1.09%
	2009-10	380	34.75	41	0.84%
	2010-11	380	35.24	47	0.67%
	2011-12	380	41.50	77	0.52%
Italian Serie B	2005-06	460	31.81	16	2.52%
	2006-07	462	30.2	10	1.30%
	2007-08	461	32.52	32	0.69%
	2008-09	461	29.94	44	1.07%
	2009-10	456	25.24	39	1.24%
	2010-11	462	27.81	20	0.91%
	2011-12	462	37.69	28	2.15%
Spanish La Liga Primera	2005-06	380	56.96	44	1.29%
	2006-07	380	44.15	17	0.49%
	2007-08	380	42.26	16	0.71%
	2008-09	380	37.28	53	0.66%
	2009-10	380	36.19	42	1.17%
	2010-11	380	36.15	59	0.79%
	2011-12	380	41.54	89	0.51%
Spanish La Liga Segunda	2005-06	462	32.75	5	1.40%
	2006-07	461	28.35	6	4.45%
	2007-08	462	33.71	20	0.94%
	2008-09	453	28.81	13	0.62%
	2009-10	462	30.19	19	0.82%
	2010-11	462	32.16	33	1.06%
	2011-12	462	38.80	36	0.74%
Dutch Eredivisie	2005-06	306	48.92	13	1.93%
	2006-07	306	39.42	4	1.01%
	2007-08	306	33.49	7	0.47%
	2008-09	306	34.84	27	1.98%
	2009-10	306	33.67	21	0.71%
	2010-11	306	34.28	23	0.43%
	2011-12	306	40.74	45	0.68%
Belgian Jupiler	2005-06	306	45.12	11	1.47%
	2006-07	306	36.32	5	0.24%
	2007-08	306	37.08	16	1.20%
	2008-09	306	33.41	17	1.40%
	2009-10	210	30.76	11	0.34%
	2010-11	240	31.24	11	0.65%
	2011-12	240	37.21	23	0.53%
Greek Ethniki Katigoria	2005-06	204	25.44	9	2.01%
	2006-07	200	24.93	1	0.22%
	2007-08	204	29.40	15	1.46%
	2008-09	204	29.13	11	1.42%
	2009-10	204	25.30	5	0.57%
	2010-11	204	28.45	16	0.83%
	2011-12	204	35.88	38	0.85%

APPENDIX B: ODDS BIASES

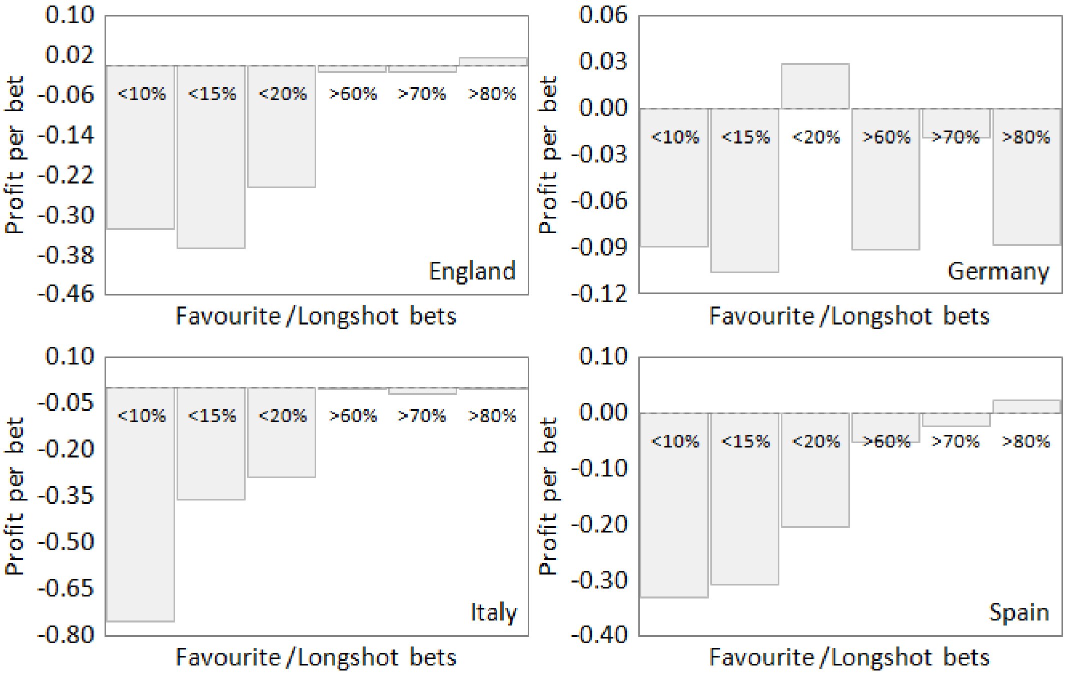

Figure B.1. Profit rates as generated by simulating £1 bets on match outcomes which satisfy the specified probabilistic favourite-longshot conditions, given all of the top division match instances over the seven football seasons (from 2005/06 to 2011/12) and as partitioned for England, Germany, Italy and Spain.

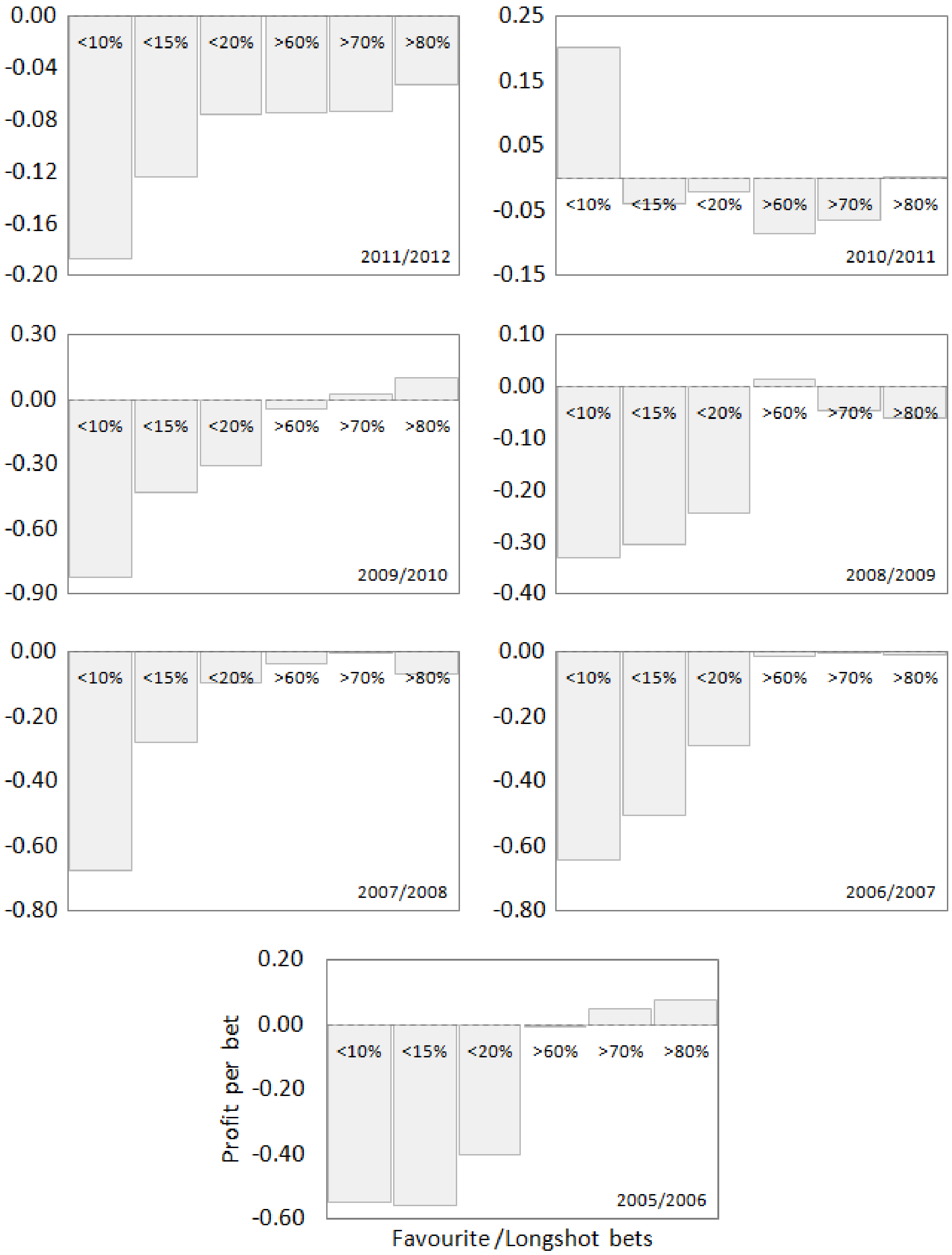

Figure B.2. Profit rates generated by simulating £1 bets on match outcomes which satisfy the specified probabilistic conditions, given all of the top division match instances of England, Germany, Italy and Spain, and as partitioned for each of the seven football seasons.

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