Department of Electrical Engineering
University of Moratuwa
B.Sc. Eng. Semester 3

EE 2210 – Circuits and Fields

Assignment: Passive Filter Design

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Circuits and Fields Assignment

① Woofer low pass = (−12dB/oct) −
Tweeter high pass = (−12dB/oct)
Midrange band pass = (24 dB/oct)

As butterworth has 6n dB/oct or 20ndB/dec ;

Woofer → 2nd order (12/6 = 2)
Tweeter → 2nd order (12/6 = 2)
Midrange → 8th order (24/6 = 4 + 4 = 8)

② $\Delta dB = \text{Slope} \times \log_2\left(\frac{f}{f_0}\right)$ } (as 2X of 'f' shows ΔdB difference)

∴ for woofer ;

$-3-(-6) = (-12) \times \log_2\left(\frac{f}{100}\right)$

$f_c = 84.09 \text{ Hz} \approx \boxed{84 \text{ Hz}}$

∴ at woofer −3dB → $\boxed{84 \text{ Hz} = f_c}$

for tweater ;

$-3-(-6) = 12 \times \log_2\left(\frac{f_c}{2000}\right)$

$f_c = 2378.4 \text{ Hz}$

at tweater, −3dB f_c → $\boxed{2378 \text{ Hz}}$

at Midrange ;

$f_{c_1} = 2^{+1/8} \times 100 = \boxed{109 \text{ Hz}}$

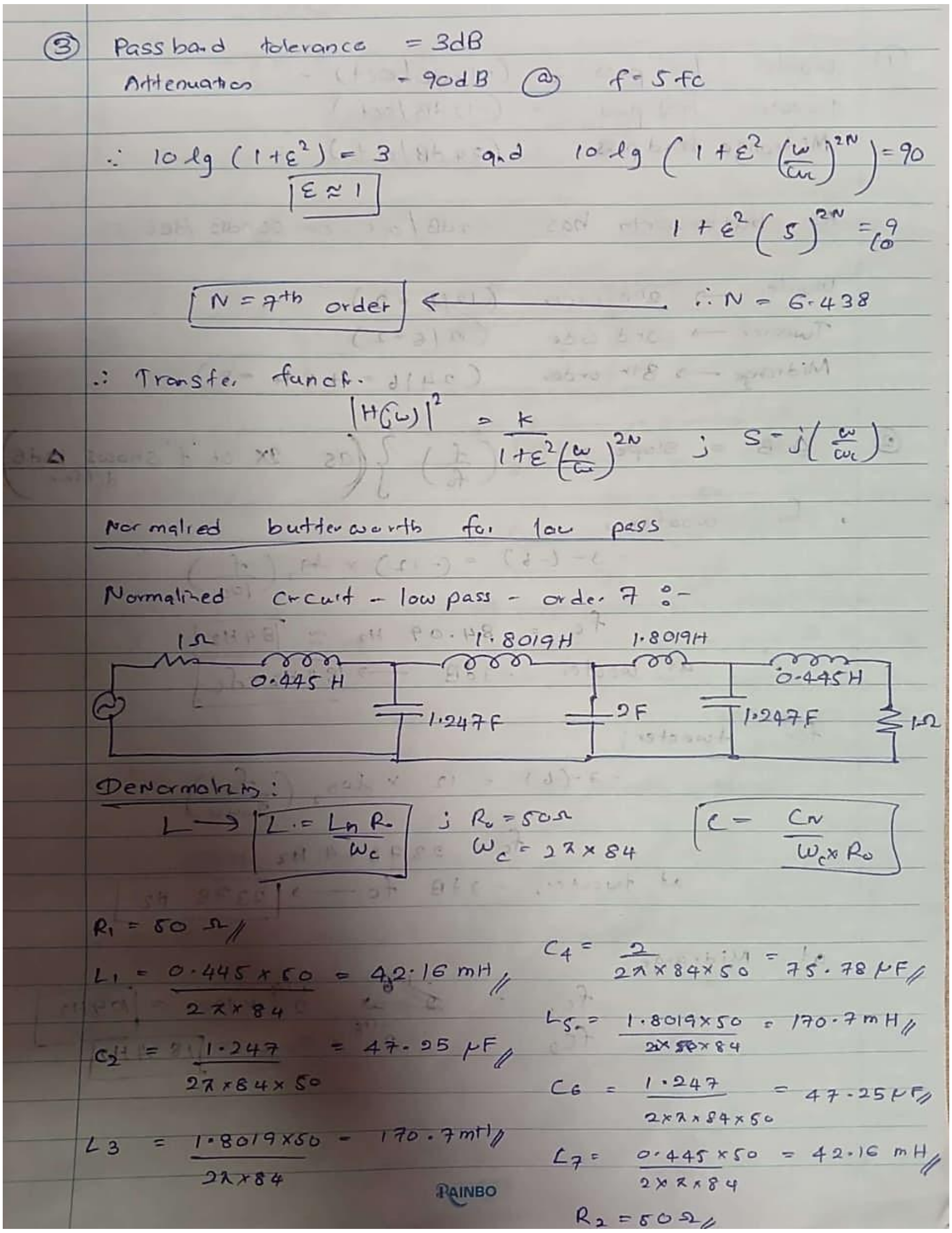
$f_{c_2} = 2^{-1/8} \times 2000 = \boxed{1834 \text{ Hz}}$

(The roll-off rate is determined by the filter's order, n, and is expressed as 20n dB per decade or 6n dB per octave)

2. Therefore, from the above plot and calculations;

Woofer (Low Pass Filter) Cut off frequency : 84 Hz
Tweater (High Pass Filter) Cut off frequency : 2378 Hz
Midrange (Band Pass Filter) Cut off frequency : 109 – 1834 Hz

3. Filter Calculations:



③ Pass band tolerance = 3dB
Attenuation = 90dB @ f = 5fc

$\therefore 10 \lg (1+\varepsilon^2) = 3$ and $10 \lg \left(1+\varepsilon^2 \left(\frac{\omega}{\omega_c}\right)^{2N}\right) = 90$

$\boxed{\varepsilon \approx 1}$

$1+\varepsilon^2 (5)^{2N} = 10^9$

$\boxed{N = 7^{th} \text{ order}} \longleftarrow \therefore N = 6.438$

$\therefore$ Transfer funct.

$$|H(\omega)|^2 = \frac{k}{1+\varepsilon^2\left(\frac{\omega}{\omega_c}\right)^{2N}} \; ; \; S = j\left(\frac{\omega}{\omega_c}\right)$$

Normalized butterworth for low pass

Normalized circuit - low pass - order 7 :-

Denormalizing:

$L \longrightarrow \boxed{L = \frac{L_n R_o}{W_c}}$; $R_o = 50\Omega$, $\omega_c = 2\pi \times 84$ $\boxed{C = \frac{C_N}{W_c \times R_o}}$

$R_1 = 50\ \Omega$

$L_1 = \frac{0.445 \times 50}{2\pi \times 84} = 42.16\ mH$

$C_2 = \frac{1.247}{2\pi \times 84 \times 50} = 47.25\ \mu F$

$L_3 = \frac{1.8019 \times 50}{2\pi \times 84} = 170.7\ mH$

$C_4 = \frac{2}{2\pi \times 84 \times 50} = 75.78\ \mu F$

$L_5 = \frac{1.8019 \times 50}{2\pi \times 84} = 170.7\ mH$

$C_6 = \frac{1.247}{2\times\pi\times 84\times 50} = 47.25\ \mu F$

$L_7 = \frac{0.445 \times 50}{2\times\pi\times 84} = 42.16\ mH$

$R_2 = 50\Omega$

high pass transfer function:

$$|H(j\omega)|^2 = \frac{k}{1+\varepsilon^2\left(\frac{\omega_c}{\omega}\right)^{2N}} \quad ; \quad s = j\left(\frac{\omega_c}{\omega}\right)$$

$$= \frac{s^7}{s^7 + 4.494\omega^2 s^6 + 10.098\omega^2 s^5 + \dots}$$

$$(\omega = 2\pi \times 2378)$$

$$Z(s) = \frac{1+\gamma(s)}{1-\gamma(s)} \rightarrow \gamma(s) = \frac{s^7}{\left(s^7 + 4.493 s^6 + 10.397 s^5 + 14.591 s^4 + 14.597 s^3 + 10.697 s^2 + 4.493 s + 1\right)}$$

Transfer funct of Band pass

$$H(m) = H_L \times H_H$$

$$= \left[\frac{k}{1+\varepsilon^2\left(\frac{\omega_1}{\omega_c}\right)^{2N}}\right]\left[\frac{k}{1+\varepsilon^2\left(\frac{\omega_c}{\omega_2}\right)^{2N}}\right], \quad \omega =$$

or:

$$= \frac{\omega_1^7}{\left(s^7 + 4.494\omega_1 s^6 + \dots\right)} \times \left(\frac{s^7}{s^7 + 4.494\omega_2^2 s^6 + \dots}\right)$$

Transfer Functions for low pass filter and high pass filter:

Low pass transfer:

$$|H(j\omega)|^2 = \frac{k}{1+\varepsilon^2\left(\frac{\omega}{\omega_c}\right)^{2N}} \quad ; \quad s = j\left(\frac{\omega}{\omega_c}\right)$$

$$= \frac{1}{1+\left(\frac{s}{j}\right)^{2\times 7}} = \frac{1}{1-s^{14}}$$

$$\Gamma(s)\Gamma(-s) = 1 - \frac{4R_iR_L}{(R_i+R_L)^2}|H(s)|^2$$

$$= 1 - \frac{1}{1-s^{14}} = \frac{-s^{14}}{1-s^{14}} \quad \therefore (-s^7)(1+s^7)$$

$$\therefore \Gamma(s)\Gamma(-s) = \frac{s^7 \cdot -s^7}{(1+s^7)(1-s^7)}$$

$$\Gamma(s) = \frac{s^7}{1+s^7}$$

$1-s^7-s^7$ $\quad \frac{s^7}{1+s^7}+1$

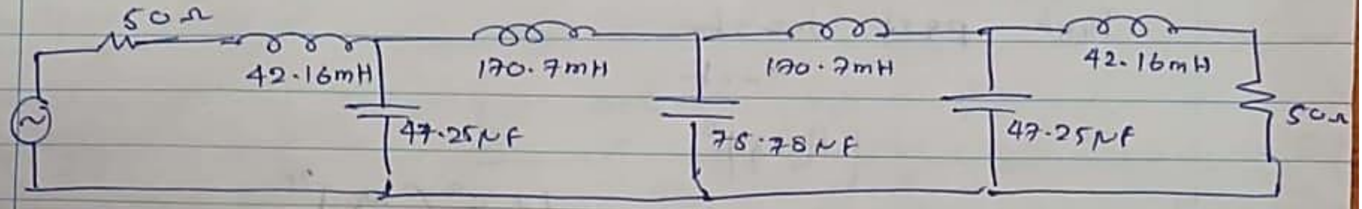
$$Z(s) = \frac{1+\Gamma(s)}{1-\Gamma(s)} = \frac{1+2s^7}{1}$$

$$\frac{1+\frac{s^7}{s^7+1}}{1-\frac{s^7}{s^7+1}} \quad \frac{2s^7+1}{1}$$

for high pass: $= \frac{k}{1+\varepsilon^2\left(\frac{j}{s}\right)^{14}} = \frac{s^{14}}{s^{14}-1} = \frac{s^7 \cdot s^7}{(s^7-1)(s^7+1)}$

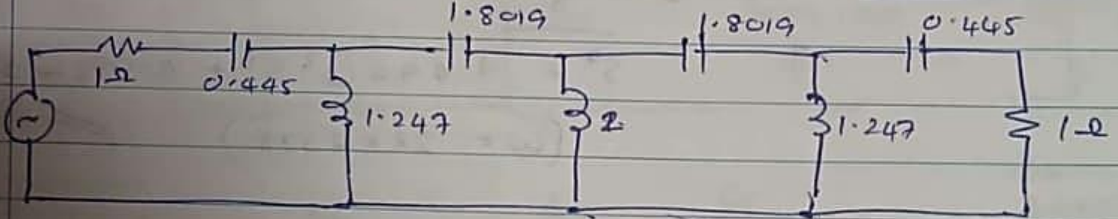
$$Y_s Y(-s) = \frac{-1}{s^{14}-1} \Rightarrow y(s) = \frac{1}{s^7+1} \quad ; \quad Z(s) = \frac{1+Y(s)}{1-Y(s)}$$

∴ Low pass filter circuit



High Pass Normalized Circuit

$$\boxed{\omega = 2\times\pi\times 2378}$$



Denormalizing

$L \longrightarrow \frac{1}{\omega L R}$ $\quad C \longrightarrow \frac{R}{\omega C}$

$R_1 = 50\ \Omega$

$C_1 = \frac{1}{2\times\pi\times 2378\times 0.445\times 50} = 3\mu F$

$L_2 = \frac{50}{2\pi\times 2378\times 1.247} = 2.68\ mH$

$C_3 = \frac{1}{2\times\pi\times 2378\times 1.8019\times 50} = 0.74\mu F$

$C_7 = \frac{1}{2\pi\times 2378\times 0.445\times 50} = 3\ \mu F$

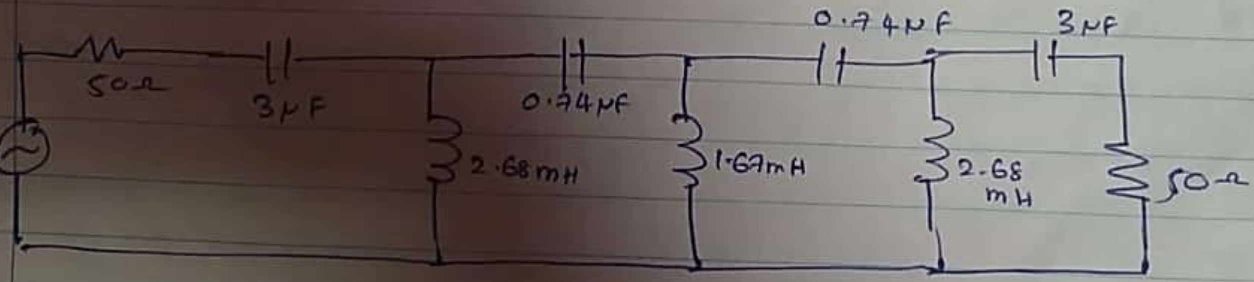
$L_4 = \frac{50}{2\times\pi\times 2378\times 2} = 1.67\ mH$

$C_5 = \frac{1}{2\pi\times 2378\times 1.8019\times 50} = 0.74\ \mu F$

$L_6 = \frac{50}{2\times\pi\times 2378\times 1.247} = 2.68\ mH$

$R_2 = 50\Omega$

high pass filter circuit



BandPass filter

$R_1 = R_L = 50\,\Omega$

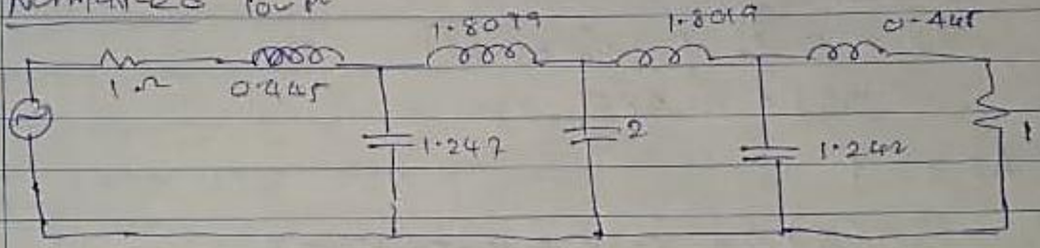
$f_1 = 109$ Hz $\longrightarrow \omega_{c_1} = 684.867$ rad/s

$f_2 = 1834$ $\qquad \omega_{c_2} = 11523.36$ rad/s

$BW = \omega_{c_2} - \omega_{c_1} = \underline{\underline{10838.5}}$ rad/s

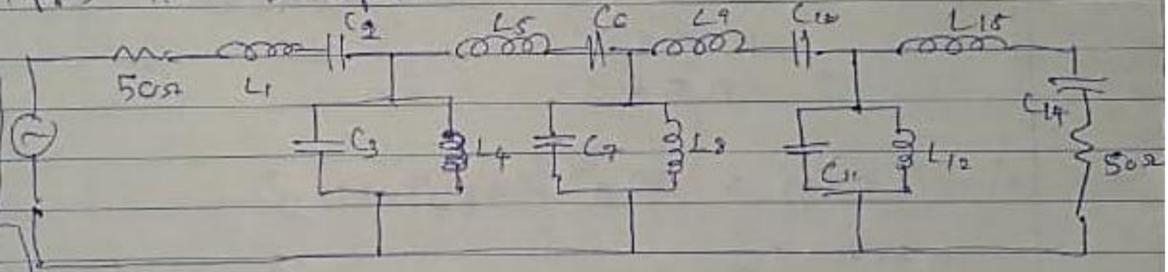
$\omega_0 = \sqrt{\omega_{c_1} \times \omega_{c_2}} = 2809.27$ //

Normalized low P



Appy for bandpass

series
$L \to LR/BW$
$C \to BW/\omega_0^2 R$
$C \to C/BW\,R$
$L \to BW/\omega^2 LR$



$L_1 = \dfrac{0.445 \times 50}{10838.5} = 2.05\,mH$ //

$C_6 = 15.2\,\mu F$ //

$C_{14} = 61\,\mu F$ //

$C_2 = \dfrac{10838.5}{(2809.27)^2 \times 0.445 \times 50} = 61\,\mu F$ //

$C_7 = 3.7\,\mu F$ //

$L_8 = 34.2\,mH$ //

$C_3 = \dfrac{1.242}{10838.5 \times 50} = 2.29\,\mu F$ /

$L_9 = 8.31\,mH$ //

$L_4 = \dfrac{10838.5 \times 50}{(2809.27^2) \times 1.242} = 53\,mH$ //

$C_{10} = 15.2\,\mu F$ //

$C_{11} = 2.2\,\mu F$ /

$L_5 = \dfrac{1.8019 \times 50}{10838.5} = 8.31\,mH$ //

$L_{12} = 55\,mH$ //

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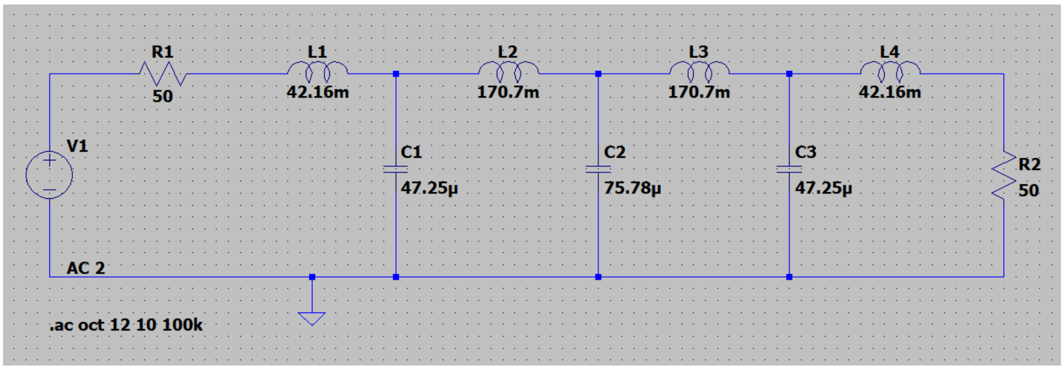
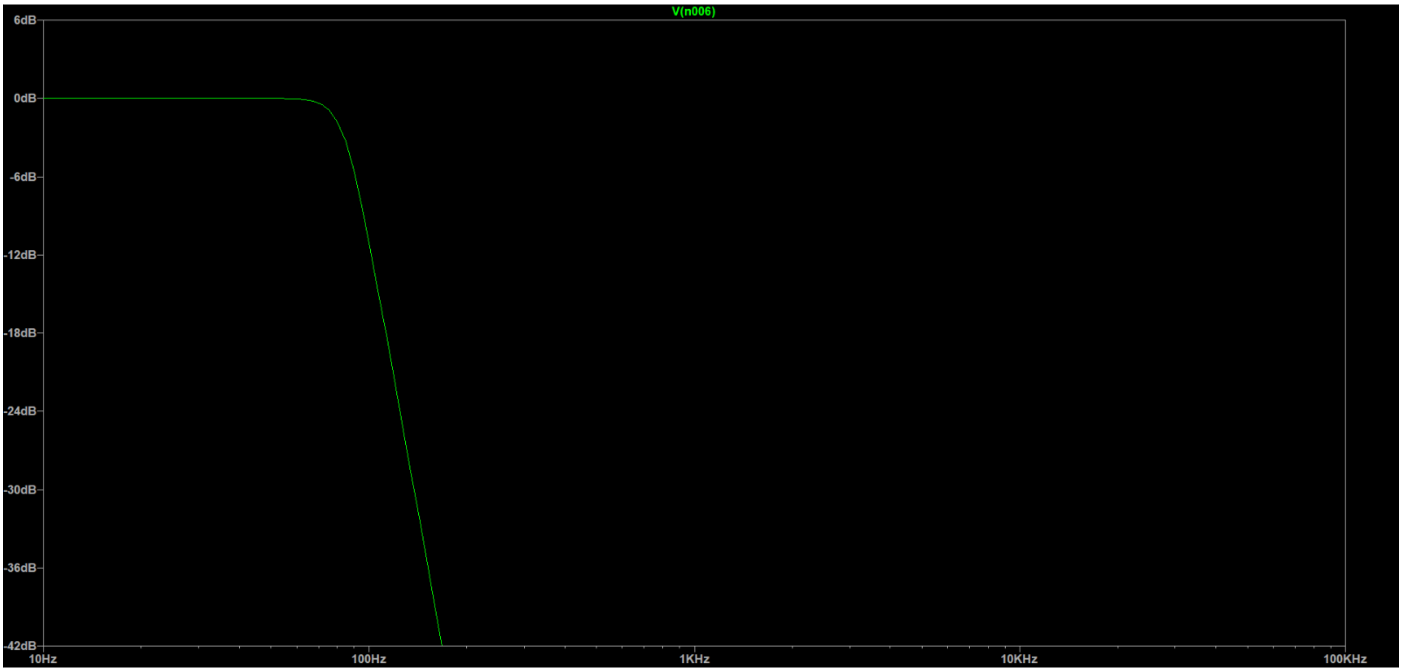
$L_{15} = 2.05\,mH$

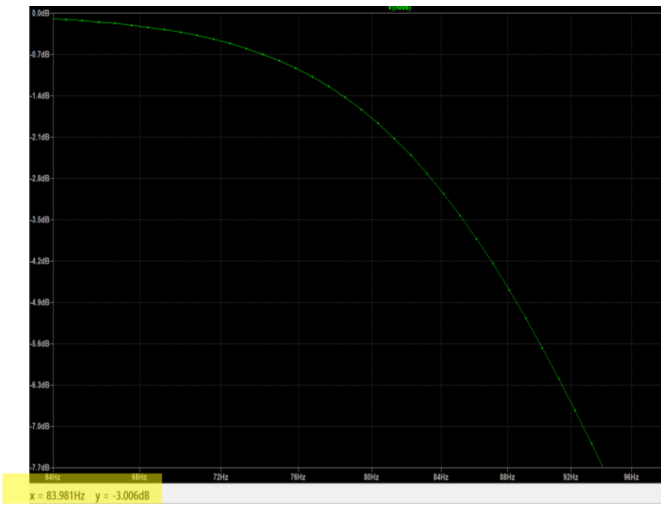
➢ LTspice Schematic: Low Pass Filter



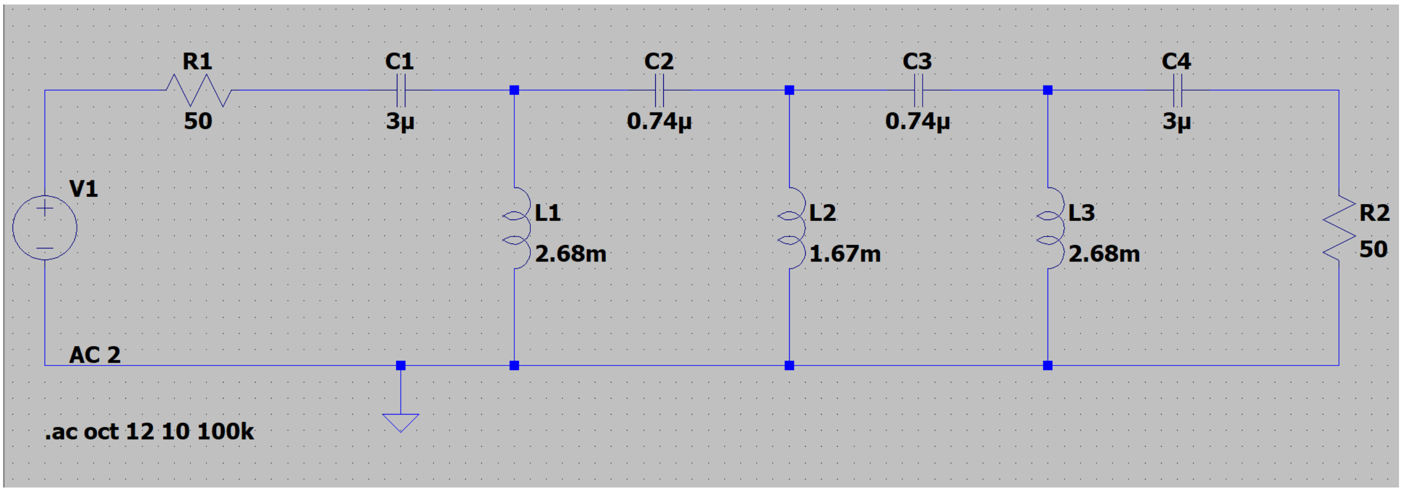
➢ LTspice Simulation:



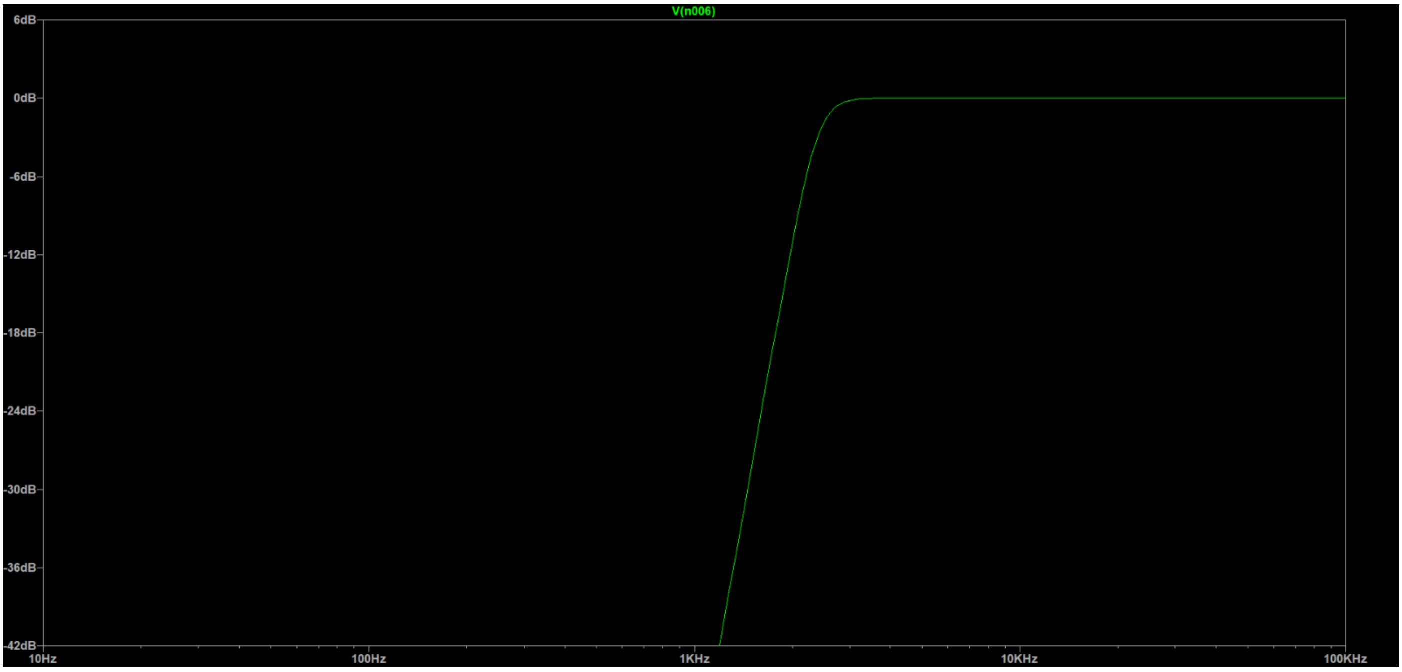
From this Cut off frequency (-3 dB) is 84 Hz.



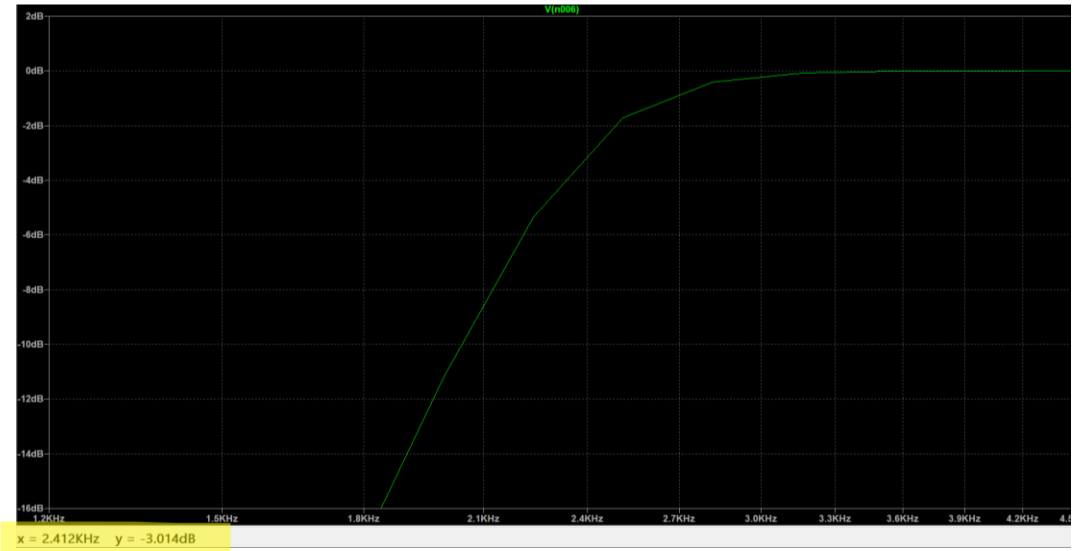
➤ LTspice schematic: High Pass Filter



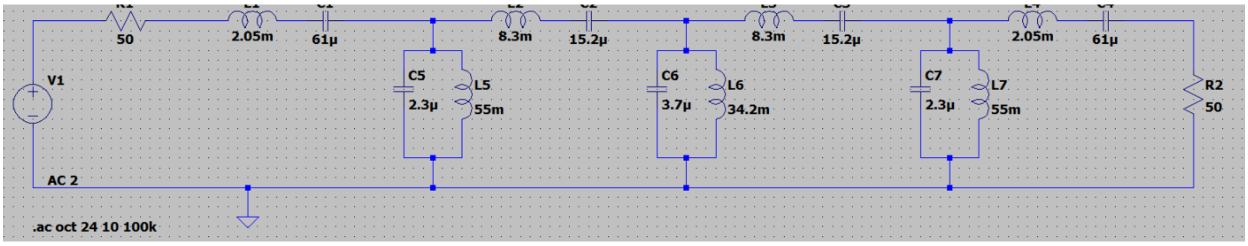
➤ LTspice simulation: High Pass Filter



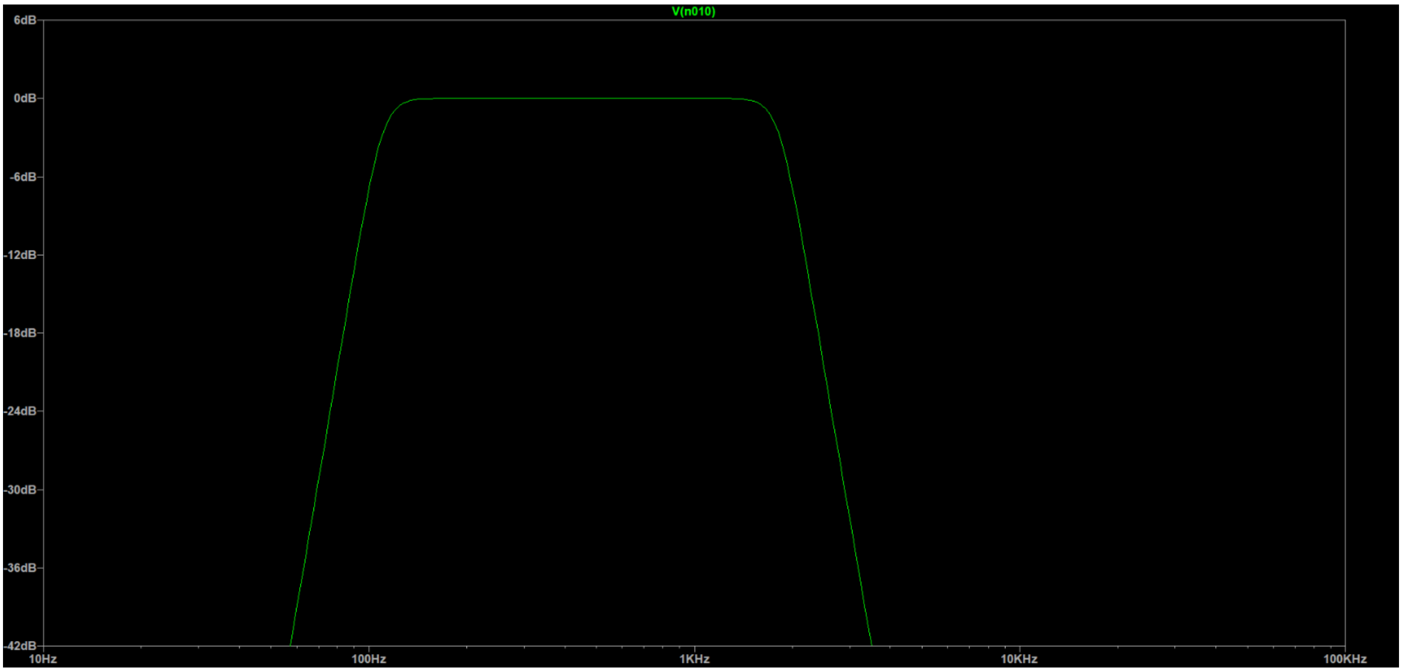
From this plot Cut off frequency is 2412 Hz.



- LTspice schematics: Band Pass Filter

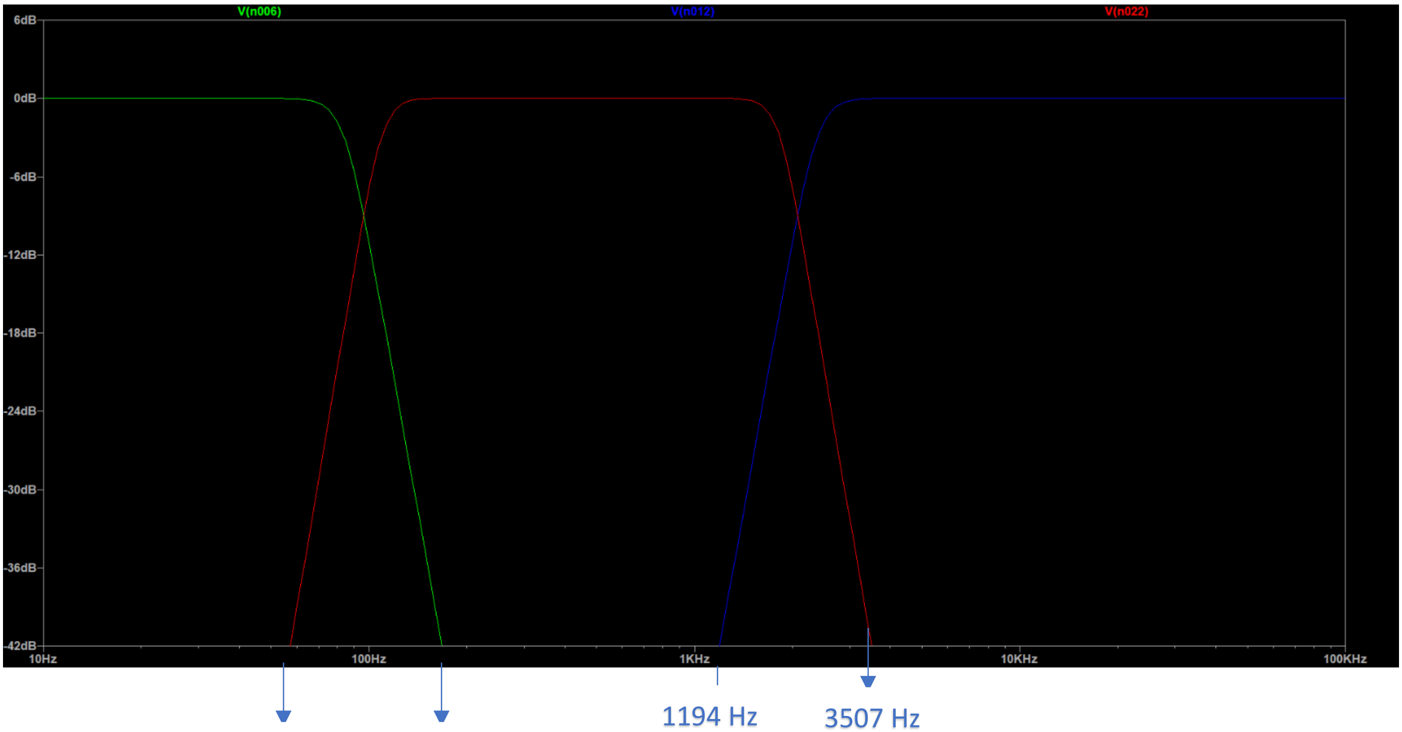


- LTspice Simulation:

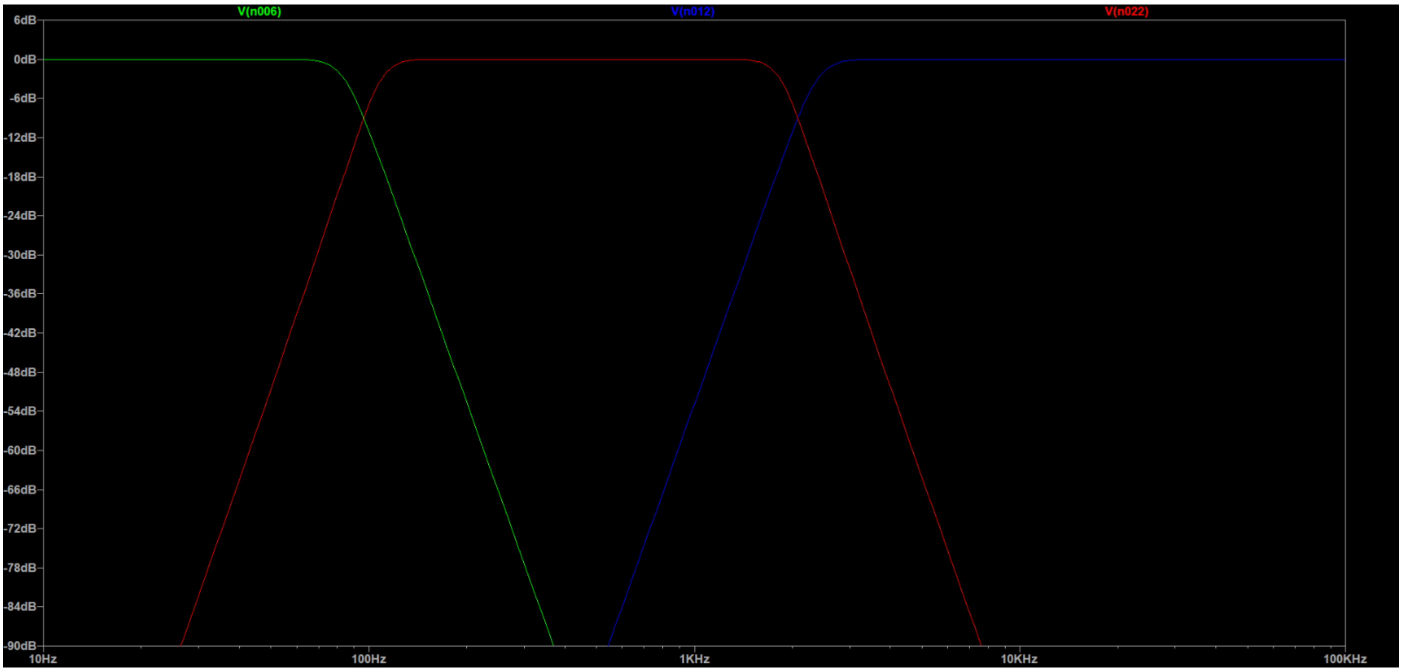


From this Pass Band frequencies are 109.3 Hz – 1832 Hz.

➢ Comparison:



ıre 1: Same dimensions as Data plot



In data plot the cross-over frequency of woofer and midranger is 100 Hz , -6 dB. The LTspice simulation gives cross over frequency of 96 Hz, -9dB.

In data plot upper cross over frequency is 2000 Hz, -6dB. LTspice simulation gives cross over frequency of 2078 Hz , -9 dB.

In transition band of woofer and midranger(Low end and low pass band of wooofer):

the frequency difference from plot	: 167 – 57 = 110 Hz
Data plot value for the transitional band	: 300 – 20 = 280 Hz
Therefore, reduction in region	: 280 – 110 = 170 Hz

In transition band of midranger and Tweeter. (High end and high pass filter)

frequency difference	: 3507 – 1194 = 2313 Hz
Data plot value for the transitional band	: 7000 – 600 = 6300 Hz
Therefore, reduction in region	: 6300 – 2313 = 3987 Hz

4.

Chebyshev type I

Passband $= 0.5\,dB$

Attenuation $= 90\,dB$ @ $f_c = f$

$\therefore y = 5$;

$0.5 = 10\lg(1+\varepsilon^2) \longrightarrow \varepsilon = 0.35$

~~$1+\varepsilon^2 T_n^2(y)$~~

$90 = 10\lg\left(1+\varepsilon^2 T_n^2(y)\right)$

$\therefore T_n(y) = 91287$

As $y > 1$:

$T_n(5) = \cosh(n \times \cosh^{-1} 5)$

$91287 = \cosh(n \times \cosh^{-1} 5)$

$n = 5.3$

$n = 6$

$\therefore T_6(y) = 32y^6 - 48y^4 + 18y^2 - 1$

n = even :

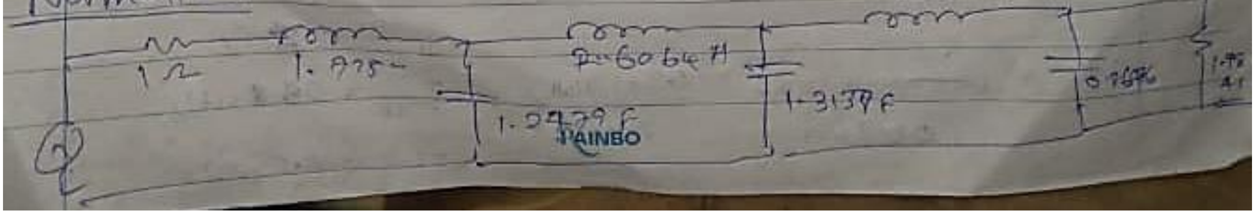
$k = \dfrac{(1+\varepsilon^2)\,4R_1R_2}{(R_1+R_2)^2} = k =$

$k = (1+0.35^2) \times 1 = 1.1225$

$\therefore$ Transfer func $= 1.1225 / 1+\varepsilon^2(T_n^2(y))$

$= \dfrac{1.1225}{1+0.35^2(32y^6-48y^4+18y^2-1)}$

Normalized



Transfer function remaining part:

$$Y(s)\,Y(-s) = 1 - \frac{4R_1R_2}{(R_1+R_2)^2}\,|H(s)|^2$$

$$= 1 - 1 \times \left(\frac{1.1225}{(1+0.35^2)\left[32\left(\frac{s}{j}\right)^6 - 48\left(\frac{s}{j}\right)^4 + 18\left(\frac{s}{j}\right)^2 - 1\right]} \right)$$

Cherbyshev filter De normalizing:

De normalize

~~for passband tol = 0.5dB;~~

$W = 84\ Hz$

$L_1 = \frac{1.7254 \times 50}{2\pi \times \cancel{73}\ 84} = \cancel{188\ mH}$ $163.4\ mH$

$C_2 = \frac{1.2479}{2\pi \times \cancel{73}\ 84 \times 50} = \cancel{54.4\ \mu F}$ $47.3\ pF$

$L_3 = \frac{2.6064 \times 50}{2\pi \times \cancel{73}\ 84} = \cancel{284\ mH}$ $= 246.8\ mH$

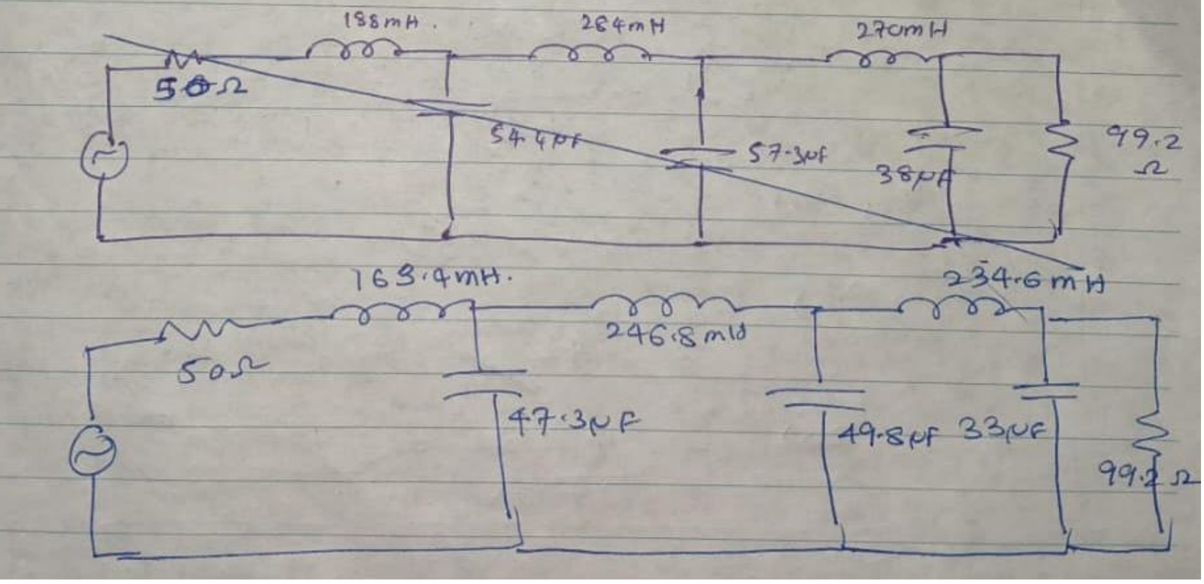
$C_4 = \frac{1.3137}{2\pi \times \cancel{73}\ 84 \times 50} = \cancel{57.3\ \mu F}$ $= 49.8\ \mu F$

$L_5 = \frac{2.4758 \times 50}{2\pi \times \cancel{73}\ 84} = \cancel{270\ mH}$ $234.6\ mH$

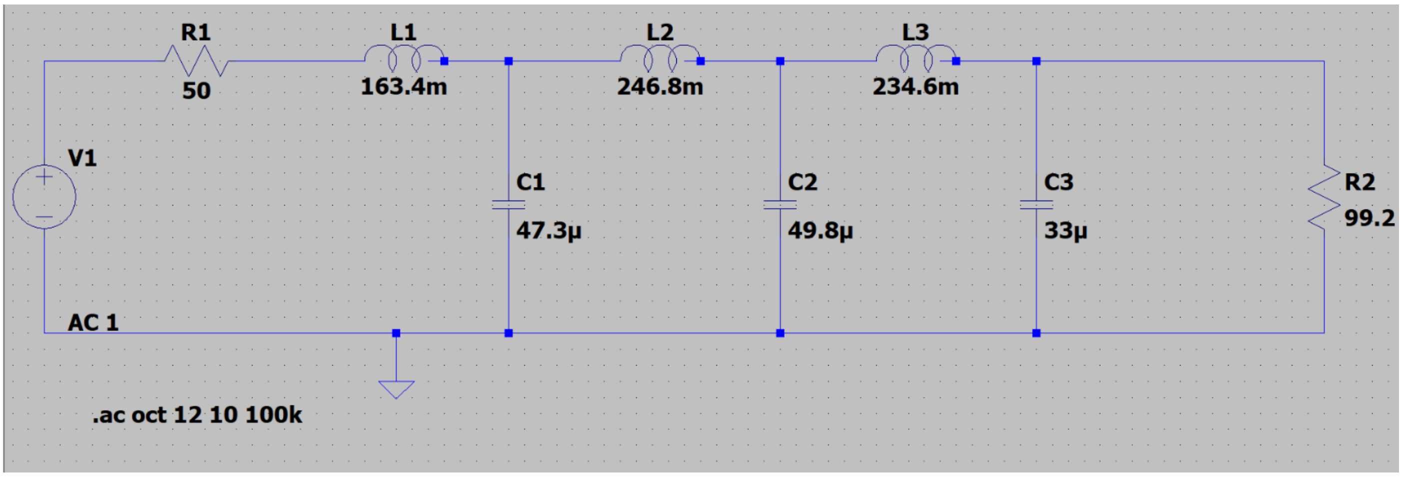
$C_6 = \frac{0.8696}{2\pi \times \cancel{73}\ 84 \times 50} = \cancel{38\ \mu F}$ $33\ \mu F$

$R_2 = 99.2\ \Omega$

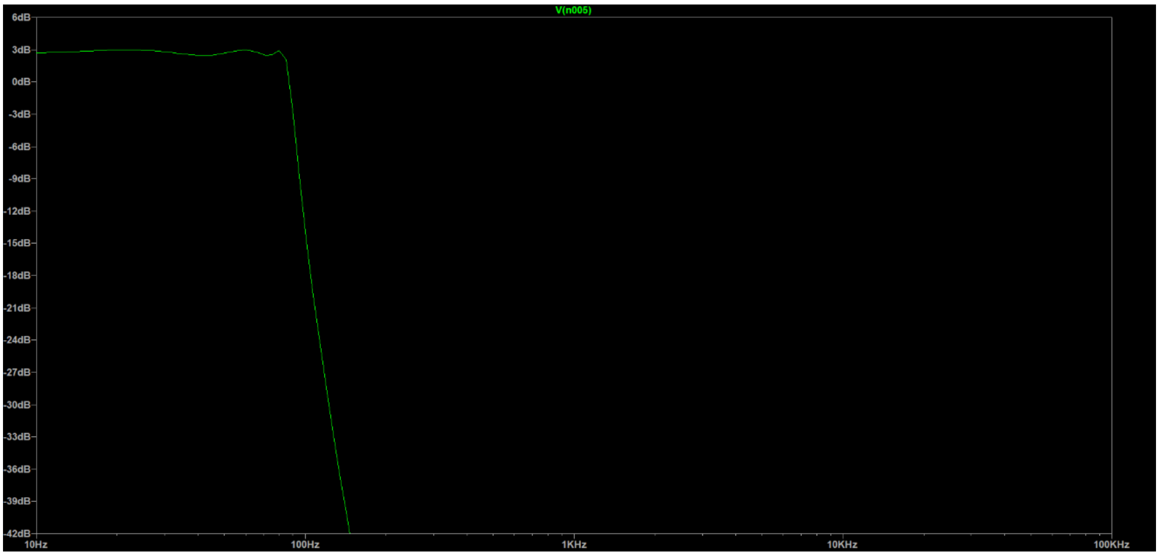
circuit for Cheby Type I filter



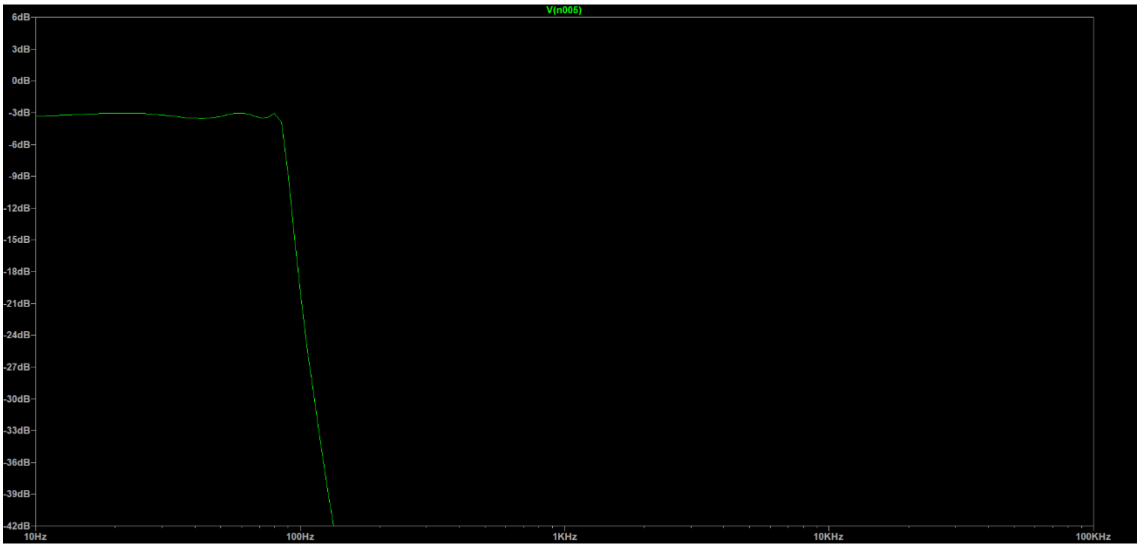
- LTspice schematics: Cherbyshev I filter



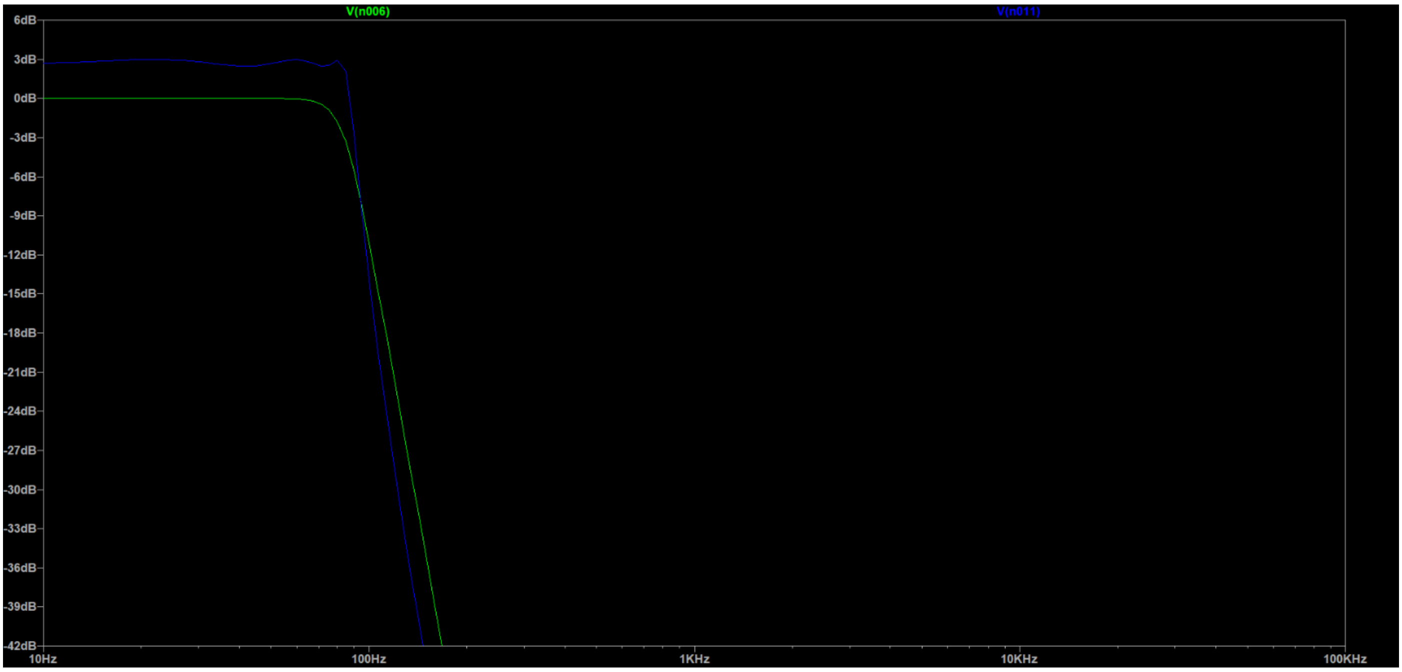
- LTspice simulation:
  Here using voltage amplitude 2 V.



Here using voltage amplitude 1 V.



- Comparing two filters:



4) c.

- Butterworth (Green curve) filter in has flat passband with no ripple visible. The response is maximally flat at passband. The transition region has a broader frequency range (not a steep roll - off).
- The Chebyshev type (Blue curve) I filter has noticeable ripples in passband and it's transition region is not broader as butterworth (steeper roll-off).

5) In the case of a Butterworth filter, an order of n=7 is required, while the same design specifications can be met Chebyshev filter of order n=6. This reduction in order minimizing circuit size and complexity as well as reducing overall cost.

Additionally, the Butterworth filter exhibits a relatively wide transition band. The Chebyshev filter, on the other hand, achieves a comparatively narrower transition band, improving performance in audio. (Steeper roll-off)

Hence, **the Chebyshev filter is more suitable option** for this design.