ASSIGNMENT 5

PSTAT 160A – W19
Nils Detering

Release date: **Monday, February 4**
Due date: **Thursday, February 14, before your lecture in class**

The first part of this assignment sheet contains exercises for the sections. The second part consists of homework problems which have to be submitted on the due date.

Instructions for the homework: Please write your **full name**, **perm number**, your **TAs name**, and your **section's time & date** legibly on the front page of your homework sheet. Make sure that all your sheets are stapled together. Solve all homework problems. Your reasoning has to be comprehensible and complete. To receive full credit sufficient explanations need to be provided. **Two** randomly chosen questions will be graded.

Exercises for the Sections and Self-Study

1. "The power of commited minorities": Assume there is an election with two candidates A and B and 1 000 000 voters. 2000 of these voters are definitely voting for candidate A. The remaining 998 000 voters are indecisive and will independently of each other vote for one of the candidate with equal probability 1/2 (they might simply throw a fair coin). Determine approximately the probability p_A that candidate A wins the election.

2. Let $(S_n)_{n\geqslant 0}$ be a symmetric random walk starting at 2, i.e. $S_0 = 2$. Compute the following probabilities:

   - $\mathbb{P}(S_3 = 1)$,
   - $\mathbb{P}(S_1 = 1, S_3 = 3)$,
   - $\mathbb{P}(\{S_5 = 3\} \cup \{S_5 = 2\})$.

3. Use the reflection principle to find the number of paths for a simple random walk from $S_0 = 2$ to $S_{15} = 5$ that do **not** hit the line $y = 6$.

4. *Bertrand's Ballot Problem*: In an election candidate A receives 200 votes while candidate B only receives 100. Assume that the probability of getting a vote is identical (50% each) for A and B. What is the probability that A is always ahead throughout the count?

Homework Problems

1. (10 points) Let $(S_n)_{n\geqslant 0}$ be a simple random walk starting at $1 (S_0 = 1)$ and with $p = 0.4$ and $q = 1 - p = 0.6$. Compute the following probabilities:

   - $\mathbb{P}(S_2 = 1 | S_5 = 0)$,
   - $\mathbb{P}(S_5 = 0 | S_2 = 0)$,
   - $\mathbb{P}(M_{10} \geqslant 4, S_{10} \geqslant 4)$, where $M_{10} = \max_{0\leqslant i\leqslant 10} S_i$.

2. (10 points) Let$(S_n)_{n\geqslant 0}$ be a simple random walk starting at 0 with $p = 0.3$ and $q = 1-p = 0.7$. Compute the following probabilities:

   - $\mathbb{P}(S_2 = 0, S_4 = 0, S_5 = -1)$,
   - $\mathbb{P}(\{S_4 = 4\} \cup \{S_4 = -2\})$,
   - $\mathbb{P}(M_{17} \leqslant -5, S_7 = -5)$, where $M_{17} = \min_{0\leqslant i\leqslant 17} S_i$.

3. (10 points) Use the reflection principle to find the number of paths for a simple random walk from $S_0 = 2$ to $S_{10} = 6$ that hit the line $y = 0$

4. (10 points) Use the reflection principle to find the probability $\mathbb{P}(M_8 = 6)$, where $M_8 = \max_{0\leqslant i\leqslant 8} S_i$ and $(S_n)_{n\geqslant 0}$ is a simple symmetric random walk starting in 0 $(S_0 = 0)$.