ASSIGNMENT 7

PSTAT 160A – W19
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Release date: **Monday, February 25**
Due date: **Thursday, March 7, before your lecture in class**

The first part of this assignment sheet contains exercises for the sections. The second part consists of homework problems which have to be submitted on the due date.

Instructions for the homework: Please write your **full name**, **perm number**, your **TAs name**, and your **section's time & date** legibly on the front page of your homework sheet. Make sure that all your sheets are stapled together. Solve all homework problems. Your reasoning has to be comprehensible and complete. To receive full credit sufficient explanations need to be provided. **Two** randomly chosen questions will be graded.

Exercises for the Sections and Self-Study

1. You are given the following transition matrix for a Markov chain with state space $\mathcal{S} = \{1, 2\}$.

$$P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

   (a) Does the Markov chain have stationary distributions? If so, determine all of them. If not, explain why.

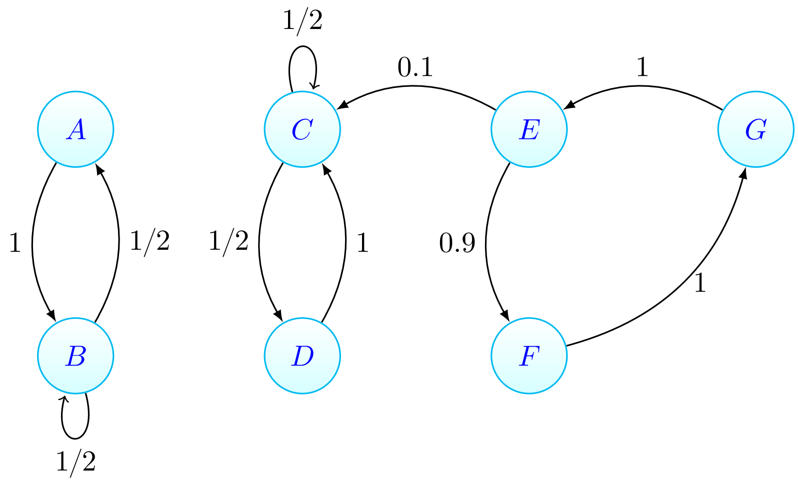
   (b) Does the Markov chain have a limiting distribution? If so, determine it. If not, explain why.

2. Let $(X_n)_{n\geqslant 0}$ be a Markov chain with state space $\mathcal{S} = \{1, 2, 3\}$ and transition probability matrix

$$P = \begin{pmatrix} 1/4 & 0 & 3/4 \\ 0 & 1/2 & 1/2 \\ 2/3 & 1/3 & 0 \end{pmatrix}.$$

   Compute the stationary distribution π.

3. A Markov chain $X_0, X_1, X_2, \ldots$ has the following transition graph:



   (a) Provide the transition matrix for the Markov chain.

   (b) Is there a limiting distribution? If so, determine it.

   (c) Determine the set of stationary distributions if any exist. If not, explain why.

4. Consider again the Markov chain from Exercise 3.

   (a) Determine its transient and recurrent states.

   (b) Determine its communication classes.

Homework Problems

1. (10 points) Let $(X_n)_{n\geqslant 0}$ be a Markov chain with state space $S = \{1, 2, 3\}$ and transition probability matrix

$$P = \begin{pmatrix} 0.4 & 0 & 0.6 \\ 0.3 & 0.1 & 0.6 \\ 0.2 & 0.2 & 0.6 \end{pmatrix}.$$

   Compute the stationary distribution π.

2. (10 points) Let $(X_n)_{n\geqslant 0}$ denote a Markov chain with state space $\mathcal{S} = \{1, 2, 3, 4\}$ and transition probability matrix

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{5} & \frac{4}{5} \\ 0 & p & 1-p & 0 \end{pmatrix}$$

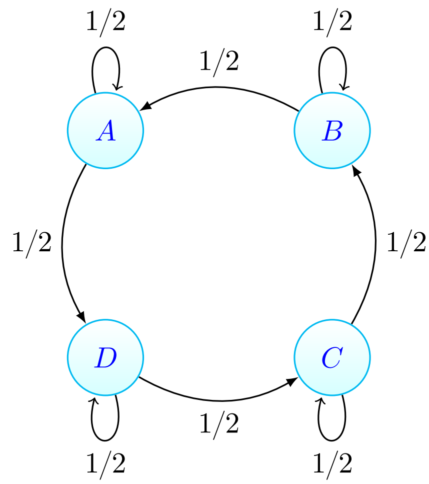
   where $p \in [0, 1]$ is some arbitrary probability.

(a) Provide the transition graph.

(b) Determine all communication classes. Which communication classes are closed?
Hint: You need to do a case-by-case analysis depending on the value of p.

(c) Fix $p = 0$. The initial distribution is given by $\alpha^T = (0, 0, \frac{5}{9}, \frac{4}{9})$. Compute the distribution of X_1. What is the distribution of X_{103}?

3. A Markov chain $X_0, X_1, X_2, \dots$ has the following transition graph:



(a) Provide the transition matrix for the Markov chain.

(b) Classify all states (recurrent/transient).

(c) Find the communication classes. Is the chain irreducible?

(d) Find the stationary distribution.

(e) What do you know about the limiting distribution?

4. (10 points) Consider a Markov chain on the state space $\mathcal{S} = \{1, 2, 3, 4, \dots\}$ with the following transition matrix:

$$P = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 & 0 & \cdots \\ 2/3 & 0 & 1/3 & 0 & 0 & \cdots \\ 3/4 & 0 & 0 & 1/4 & 0 & \cdots \\ 4/5 & 0 & 0 & 0 & 1/5 & \cdots \\ 5/6 & 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

That is, $P_{ij} = i/(i+1)$ if $j = 1$, $P_{ij} = 1/(i+1)$ if $j = i+1$, and $P_{ij} = 0$ otherwise.

(a) Classify all states of the Markov chain (transient, recurrent)

(b) Determine if the Markov chain is irreducible.