# Geometric Adaptive Control of Attitude Dynamics on SO(3) with State Inequality Constraints

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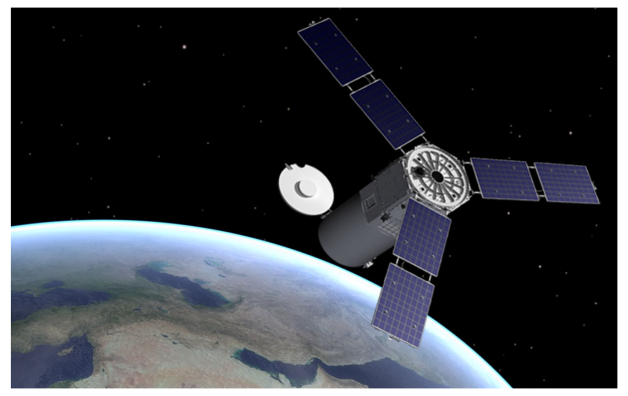
## Background and Motivation

- Autonomous control of vehicles is critical for missions
  - Typical operations require extensive planning and human interaction
  - Vehicles must operate safely in hazardous environments
  - Applicable to under-water, aerial, and spacecraft scenarios
- Key technology for autonomy is large angle reorientations in the presence obstacles
  - Spacecraft have sensitive payloads e.g. optical sensors
  - Reorient while not pointing in dangerous directions e.g. Sun, Moon

Hubble



Agile S/C

- Problem: reorient a vehicle while avoiding certain directions
  - Sensor exclusion zone around the Sun
  - UAVs manuevering in restricted and congested locations
  - Laser emitters on industrial robots

## Spacecraft Orientation

- Rigid body attitude dynamics has been extensively studied
- Configuration manifold is curved and nonlinear
  - Dynamics evolve on the Special Orthogonal Group: SO(3)
  - Unique properties: cannot be represented as a linear vector space
- Previous work is based on reduced attitude representations
  - Euler angles: 24 possible combinations which suffer singularities
  - Quaternions: no singularities but double cover SO(3)
- Geometric control: the development of control systems for systems evolving on nonlinear manifolds
  - Many systems cannot be defined correctly on Euclidean spaces
  - Innovative techniques avoid ambiguities and local coordinates and exactly describe the evolution of the system

## Attitude Dynamics

- Spacecraft is modeled as a rigid body rotating about its center of mass described by the Special Orthogonal Group

$$\mathrm{SO}(3) = \left\{ R \in \mathbb{R}^{3\times 3} \,|\, R^T R = I, \det R = 1 \right\}$$

- Euler's equations of motion govern the dynamics of a rigid body

$$J\dot{\Omega} + \Omega \times J\Omega = u + W(R,\Omega)\Delta,$$
$$\dot{R} = R\hat{\Omega},$$

- $R \in \mathrm{SO}(3)$ defines the orientation of the spacecraft with respect to an inertial reference frame
- $W(R,\Omega)\Delta$ models a wide range of external disturbances
  - Solar radiation pressure (SRP)
  - Gravity gradient moment
  - Air turbulence and gusts
  - Unknown mass distribution

## Adaptive Attitude Control with Collision Avoidance

- Constraint is defined in terms of unit-vectors on the two-sphere:

$$\mathrm{S}^2 = \left\{ q \in \mathbb{R}^3 \,|\, \|q\| = 1 \right\}$$

- We wish to avoid pointing spacecraft in a particular direction
  - Sensitive optical sensor - $r \in \mathrm{S}^2$ defines the sensor direction
  - Constraint direction - $v \in \mathrm{S}^2$ defines direction to distant object
- Hard cone constraint - strictly avoid pointing sensor towards the celestial object

$$r^T R^T v \leq \cos\theta$$

- Smooth, positive definite function which measures the error between current and desired configuration
- Error function is the product of two terms

$$\Psi(R) = A(R)B(R)$$

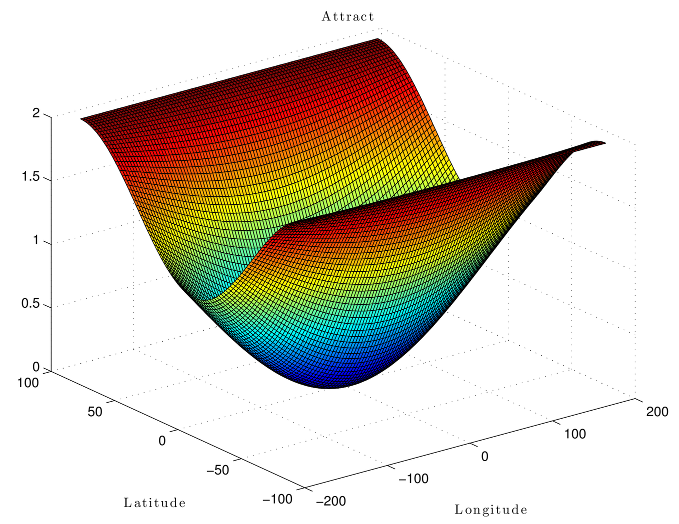
  - Attractive - drives system towards desired attitude

$$A(R) = \frac{1}{2}\mathrm{tr}\left[G\left(I - R_d^T R\right)\right]$$

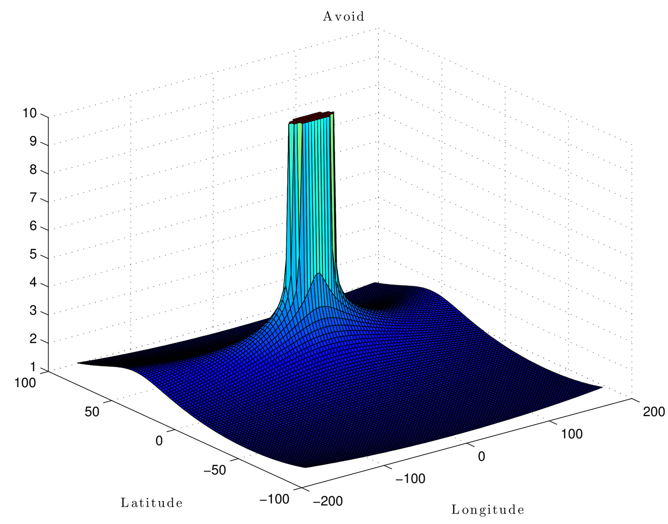
  - Repulsive - forces system away from constraint directions

$$B(R) = 1 - \frac{1}{\alpha}\ln\left(\frac{\cos\theta - r^T R^T v}{1+\cos\theta}\right)$$

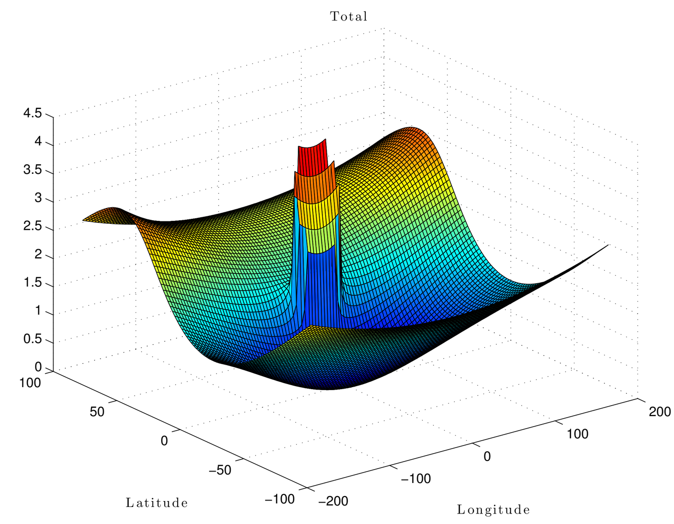
- Logarithmic barrier function causes the error to grow as $r^T R^T v \to \cos\theta$
  - $B(R) \to \infty$ as the constraint boundary is neared $r^T R^T v \to \cos\theta$
  - $B(R)$ has little impact on $\Psi$ when far from constraint as the logarithmic function quickly decays




Attractive $A(R)$




Repulsive $B(R)$




Configuration $\Psi$

- We can easily generalize this technique to an arbitrary number of constraints

$$\Psi = A\left[1 + \sum_i C_i\right] \quad \text{where } C_i = B - 1$$

- Lyapunov analysis is used to derive an adaptive control scheme which guarantees stability in the face of disturbances and obstacles

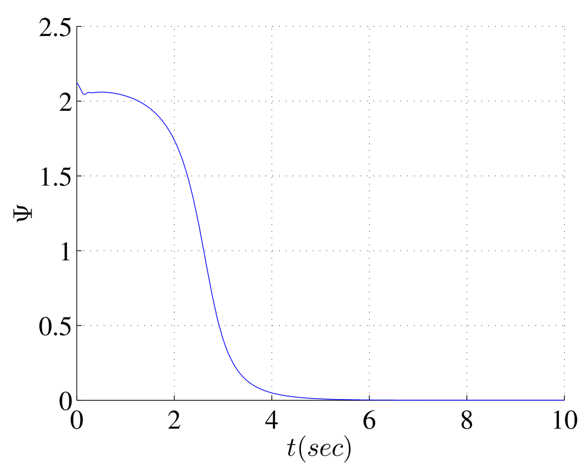
$$u = -k_R e_R - k_\Omega e_\Omega + \Omega \times J\Omega - W\bar{\Delta}$$
$$\dot{\bar{\Delta}} = k_\Delta W^T\left(e_\Omega + c e_R\right)$$

## Numerical Simulation

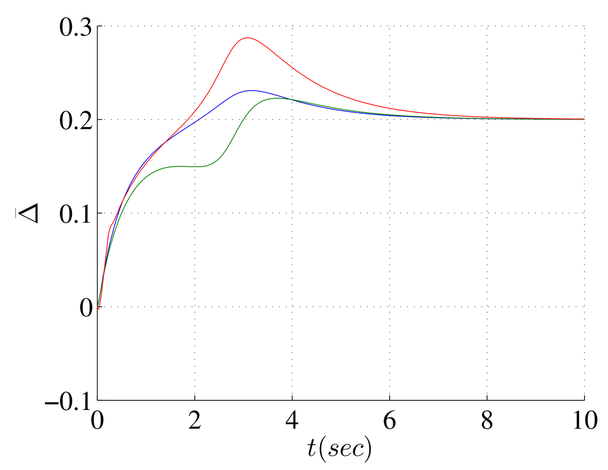
- Geometric Adaptive Controller is able to stabilize the rigid body while avoiding multiple constraints with a fixed but unknown external disturbance

$$\text{Initial: } R_0 = \exp\left(225^\circ \times \frac{\pi}{180}\hat{e}_3\right) \quad \text{Final: } R_d = I \quad \text{Disturbance: } \Delta = \left[0.2\ 0.2\ 0.2\right]^T$$

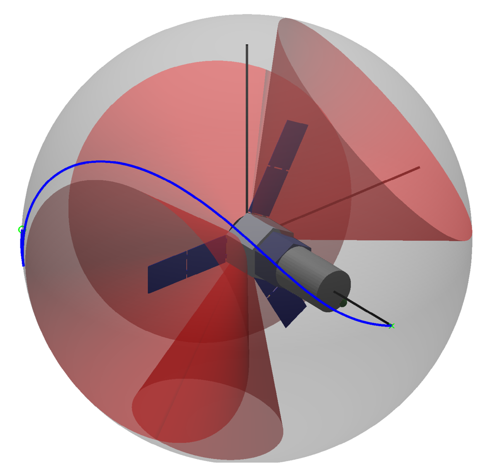
- The adaptive controller accurately accounts for the disturbance and ensures all constraints are satisfied




Configuration error $\Psi$



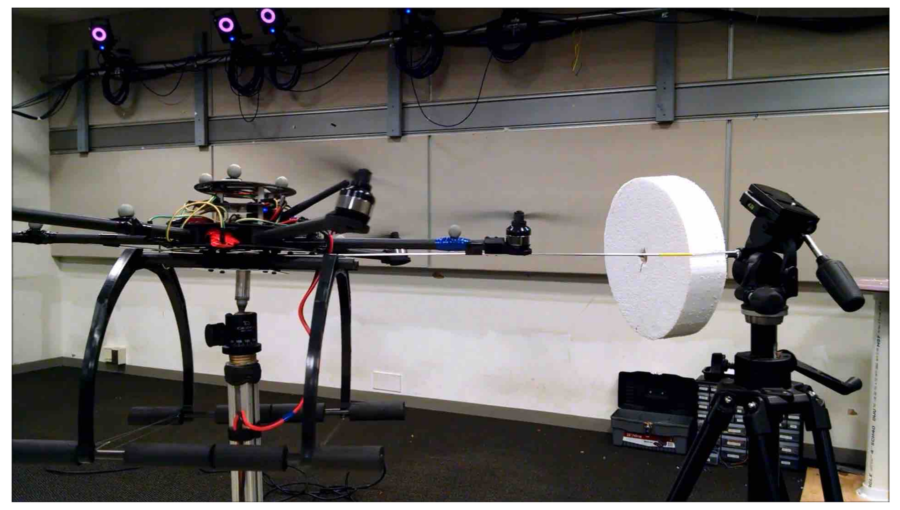

Disturbance estimate $\bar{\Delta}$



Attitude trajectory

## UAV Validation

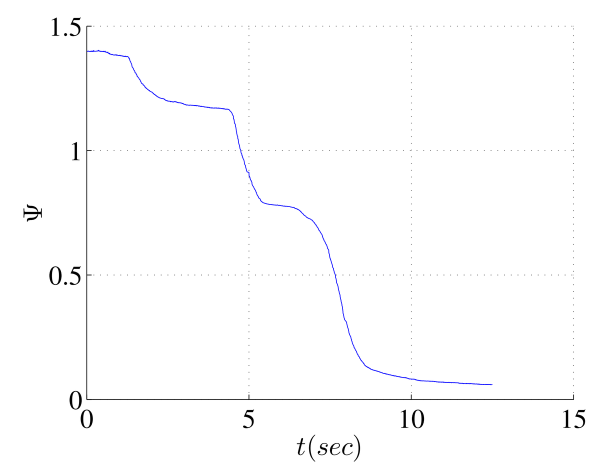
- Hexrotor UAV developed by the Flight Dynamics and Controls Laboratory
  - Three pairs of counter-rotating propellers
  - Attached to a spherical joint to emulate a fully actuated rigid body
  - Onboard computer module receives measurements from Vicon motion capture system and computes control input in real-time



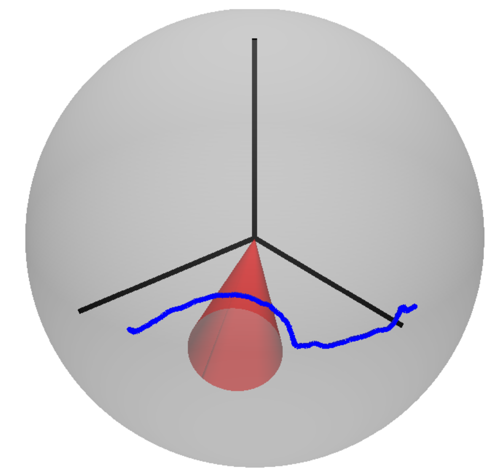
Attitude control testbed

- Hexrotor rotates about vertical axis while automatically avoiding the obstacle

$$\text{Initial: } R_0 = \exp\left(\frac{\pi}{2}\hat{e}_3\right) \quad \text{Final: } R_d = I$$




Configuration error $\Psi$



Attitude Trajectory

- Adaptive controller is robust to uncertainties and disturbances

## Conclusions

- Constrained geometric adaptive controller on SO(3)
  - Completely avoids singularities and ambiguities
  - Geometrically exact and conceptually simple attitude controller
  - Automatically satisfies multiple constraints without added complexity
- Obstacle avoidance computed in real-time with on-board software
  - Typical planning methods are only able to determine an obstacle-free path after multiple iterations and extensive computation
  - Large computation costs limit these methods to a priori calculation and make responsive control impossible
  - Randomized search algorithms can only offer a stochastic guarantee of convergence as the computation time increases
- Our control system is capable of handling any number of obstacles and offers a rigorous stability proof
  - Ideal for challenging scenarios with multiple obstacles or an environment which requires complex control
  - Computationally efficient and ideal for embedded systems with energy or computation limitations
  - Stability proof ensures manuvers always satisfy pointing constraints