Benchmarks of the CFCE Basis Reachability Graph

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Abstract

In this note, we report the comparison of the CFCE basis reachability graph (CFCE-BRG) and the reachability graph (RG) by means of four parameterized Petri net benchmarks taken from the literature. All algorithms are coded in MATLAB[1] (R2017a version), and all benchmarks are tested based on a laptop with Intel i7-5500U 2.40 GHz processor and 8 GB RAM.

Benchmark I

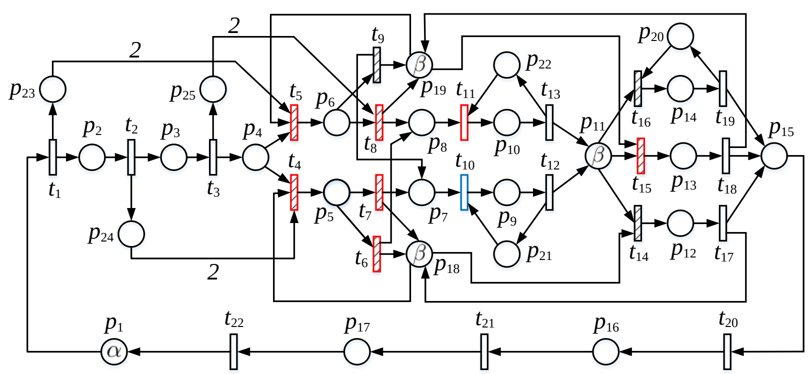

Figure 1: Benchmark I.

Table 1: Analysis of the RG and CFCE-BRG for Benchmark I.

Run	α	β	$\lvert R(N, M_0)\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}}\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}}\rvert/\lvert R(N, M_0)\rvert$	Time ratio
1	5	3	157304	o.t.[a]	4861	52	3.0%	<1.0%
2	6	4	1141871	o.t.	19042	834	1.6%	<1.0%
3	7	5	6325283	o.t.	55786	7556	0.9%	<1.0%
4	8	6	29540437	o.t.	152439	o.t.	0.5%	-
5	9	7	-	o.m.[b]	-	o.t.	-	-
6	10	8	-	o.m.	-	o.t.	-	-
7	12	8	-	o.m.	-	o.t.	-	-

[a] o.t. represents *overtime*, i.e., the program does not terminate within 43200 seconds (12 hours).
[b] o.m. represents *out of memory*, i.e., no additional memory can be allocated for use by the program.

We first test a hospital emergency service system (selected from [4] with a slight modification) modelled by a parameterized marked net $\langle N, M_0\rangle$ in Fig. 1. This example demonstrates the medical service process

[1] The source code can be requested by contacting the authors through email (gcmorninggc@gmail.com).

from patients' arrival to departure after treatment. Physically, place p_1 represents the count of patients in the emergency department. The Petri net N consists of 25 places and 22 transitions.

Consider $M_0 = \alpha p_1 + \beta p_{11} + \beta p_{18} + \beta p_{19}$ and $T_c = \{t_4, t_5, t_6, t_7, t_8, t_{11}, t_{15}\}$ (boxed in red). From the structure N, we have $T_{conf} = \{t_4, t_5, t_6, t_7, t_8, t_9, t_{14}, t_{15}, t_{16}\}$ (in shadow). Also, let t_{10} (boxed in blue) be an explicit transition to guarantee the acyclicity of the T_I-induced subnet. We have a basis partition $\pi = (T_E, T_I)$ for building the CFCE-BRG where $T_E = T_{conf} \cup T_c \cup \{t_{10}\}$ and $T_I = T \setminus T_E$.

For different values of parameters α and β, the number of basis markings $|\mathcal{M}_\mathcal{B}|$ and all reachable markings $|R(N, M_0)|$, as well as their computing times and corresponding ratios, are listed in Table 1. Through the test of Benchmark I, we can see that RG cannot be computed within the time limit in runs 1−4 and cannot be obtained with a RAM of 8 GB in the rest of the cases. However, the CFCE-BRG can be computed and is much smaller than the corresponding RG size-wise.

Benchmark II

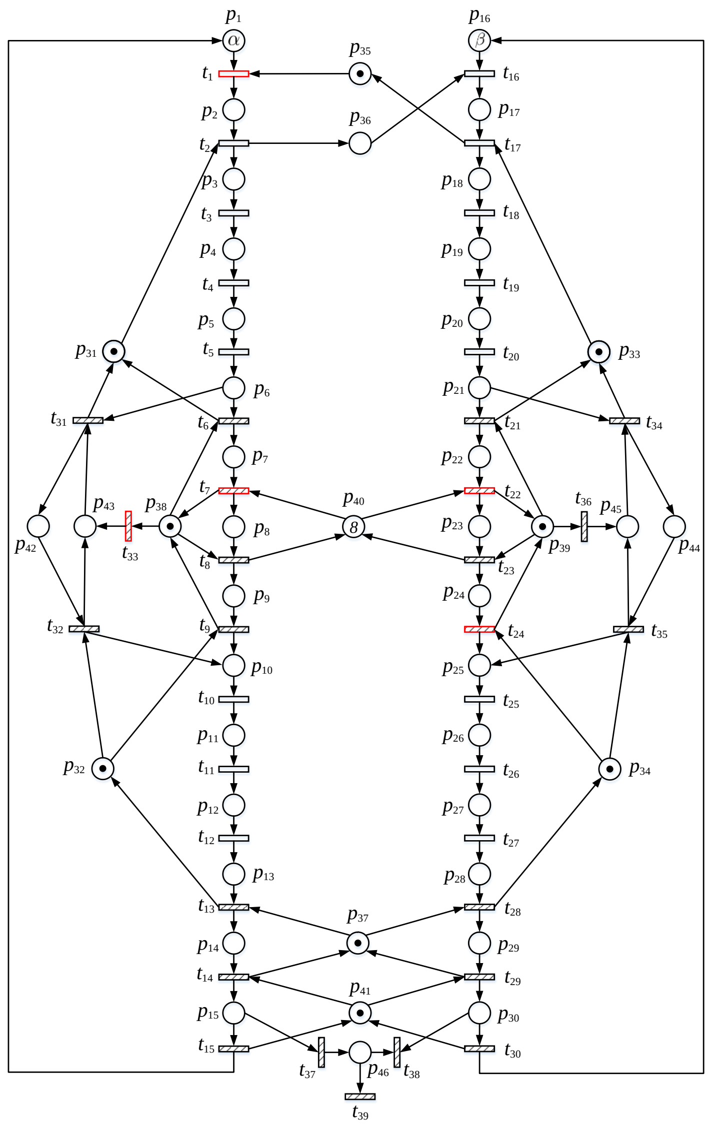

Figure 2: Benchmark II.

Table 2: Analysis of the RG and CFCE-BRG for Benchmark II.

Run	α	β	$\lvert R(N, M_0)\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}}\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}}\rvert/\lvert R(N, M_0)\rvert$	Time ratio
1	1	1	1966	10	604	2	30.7%	20%
2	1	2	12577	277	2145	11	17.0%	4.0%
3	2	2	76808	12378	7718	105	10.0%	0.8%
4	2	3	-	o.t.	16438	470	-	-
5	2	4	-	o.t.	26648	1248	-	-
6	3	3	-	o.t.	37118	2492	-	-
7	3	4	-	o.t.	59315	6449	-	-
8	4	4	-	o.t.	101420	19491	-	-
9	5	5	-	o.t.	201762	o.t.	-	-

The second benchmark is a parameterized Petri net in Fig. 4 (a slightly modified version of the Petri net in Fig. 5 in [2]) describing a manufacturing process in practice. This Petri net consists of 46 places and 39 transitions, where the initial marking M_0 is parameterized as $M_0 = \alpha p_1 + \beta p_{16} + p_{31} + p_{32} + p_{33} + p_{34} + p_{35} + p_{37} + p_{38} + p_{39} + 8p_{40} + p_{41}$. Let $T_c = \{t_1, t_7, t_{22}, t_{24}, t_{33}\}$ (boxed in red).

It can be inferred that $T_{conf} = \{t_6, t_7, t_8, t_9, t_{13}, t_{14}, t_{15}, t_{21}, t_{22}, t_{23}, t_{24}, t_{28}, t_{29}, t_{30}, t_{31}, t_{32}, t_{33}, t_{34}, t_{35}, t_{36}, t_{37}, t_{38}, t_{39}\}$ (marked in shadow). Since the subnet induced by all transitions $t \in T\setminus(T_{conf} \cup T_c)$ is acyclic, a CFCE basis partition $\pi = (T_E, T_I)$ can be obtained, i.e., $T_E = T_{conf} \cup T_c$ and $T_I = T \setminus T_E$. Thus, the corresponding CFCE-BRG $\mathcal{B}$ can be constructed based on $\pi = (T_E, T_I)$.

In Table 2, for different values of α and β, the number of basis markings in the CFCE-BRG, all reachable markings in the RG, and their computing times are listed in columns $1-7$. The corresponding ratios (size and time ratios) are reported in columns $8-9$. Through this benchmark tests, we conclude that the CFCE-BRG is much smaller in size and costs less computation time than the RG in all cases.

Benchmark III

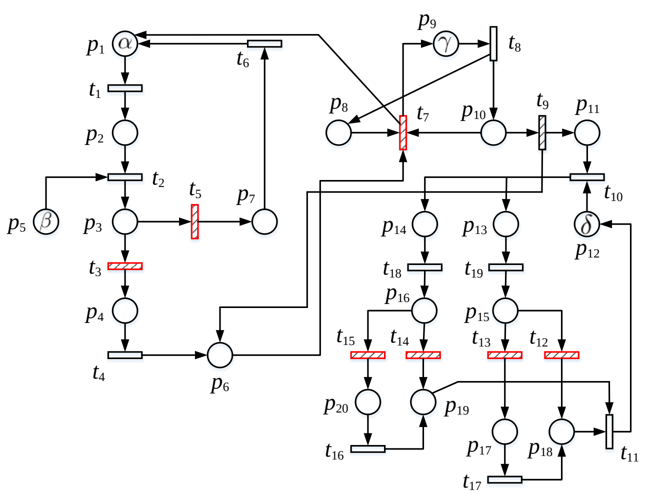

Figure 3: Benchmark III

As shown in Fig. 3, taken from [1] (with slight modifications), the third benchmark is a parameterized marked net $\langle N, M_0\rangle$ that characterizes an XML firewall security model. The initial marking is parameterized as $M_0 = \alpha p_1 + \beta p_5 + \gamma p_9 + \delta p_{12}$. Let $T_c = \{t_3, t_5, t_7, t_{12}, t_{13}, t_{14}, t_{15}\}$ (boxed in red) and $T_{uc} = T\setminus T_c$. It can be concluded that $T_{conf} = \{t_3, t_5, t_7, t_9, t_{12}, t_{13}, t_{14}, t_{15}\}$ (marked with shadow). Based on the CFCE condition, for the basis partition $\pi = (T_E, T_I)$, we set $T_E = \{t_3, t_5, t_7, t_9, t_{12}, t_{13}, t_{14}, t_{15}\}$ and $T_I = T\setminus T_E$.

With different parameters, the number of reachable markings and basis markings, along with the time

Table 3: Analysis of the RG and CFCE-BRG for Benchmark III.

Run	α	β	γ	δ	$\lvert R(N, M_0)\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}}\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}}\rvert/\lvert R(N, M_0)\rvert$	Time Ratio
1	2	2	2	2	13272	406	910	1	6.8%	0.2%
2	3	3	3	3	1534512	o.t.	8540	122	0.6%	<0.2%
3	4	4	4	4	5051802	o.t.	49945	4937	1.0%	<0.2%
4	5	5	4	4	10293738	o.t.	79912	14553	0.8%	<0.2%
5	5	6	4	4	14827974	o.t.	117271	32791	0.8%	<0.2%
6	6	6	4	4	19274094	o.t.	119868	30857	0.6%	<0.2%
7	6	6	5	4	-	o.m.	285768	o.t.	-	-

consumptions, are respectively listed in Table 3. In Benchmark III, the data show that, as the scale of the system increases, the CFCE-BRG is much smaller in size and costs less time for computation than that of the RG in all cases. Meanwhile, the set of reachable markings cannot be computed within 12 hours after case 1 and cannot be obtained in case 7 due to insufficient memory.

Benchmark IV

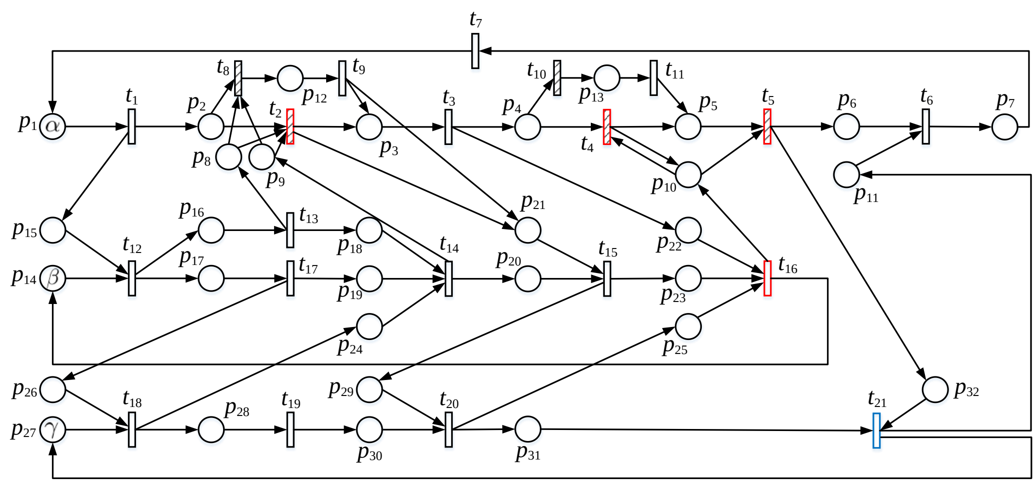

Figure 4: Benchmark IV.

Table 4: Analysis of the RG and CFCE-BRG for Benchmark IV.

Run	α	β	γ	$\lvert R(N, M_0)\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}}\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}}\rvert/\lvert R(N, M_0)\rvert$	Time Ratio
1	6	6	6	1838212	o.t.	3229	21	0.1%	<0.1%
2	10	10	10	-	o.m.	41832	4693	-	-
3	11	11	11	-	o.m.	70004	13575	-	-
4	12	12	12	-	o.m.	113034	43257	-	-
5	13	13	13	-	o.m.	176930	o.t.	-	-

Shown in Fig. 4, The fourth benchmark is a parameterized marked net $\langle N, M_0\rangle$ modelled following a radio block center system [3]. Let $T_c = \{t_2, t_4, t_5, t_{16}\}$ (boxed in red). It can be concluded that $T_{conf} = \{t_2, t_4, t_5, t_8, t_{10}\}$ (marked in shadow). Also, let t_{21} (boxed in blue) be an explicit transition to guarantee the acyclicity of the T_I-induced subnet. We conclude a CFCE basis partition $\pi = (T_E, T_I)$ where $T_E = T_{conf} \cup T_c \cup \{t_{21}\}$ and $T_I = T \setminus T_E$. For different values of parameters α, β and γ, the simulation results regarding the comparison of CFCE-BRG and RG, are all listed in Table 4. Through the test of Benchmark IV, we can see that RG cannot be computed within the time limit in run 1 and cannot be obtained with a RAM of 8 GB in the other cases. However, the CFCE-BRG can be computed in all cases and is much smaller than the corresponding RG size-wise.

References

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