Simulations of the Min-max Basis Reachability Graph

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Abstract

In this note, we report the simulation results of three benchmarks based on the min-max basis reachability graph (min-max-BRG). All tests are executed based on a laptop with Intel i7-5500U 2.40 GHz processor and 8 GB RAM.

Benchmark I

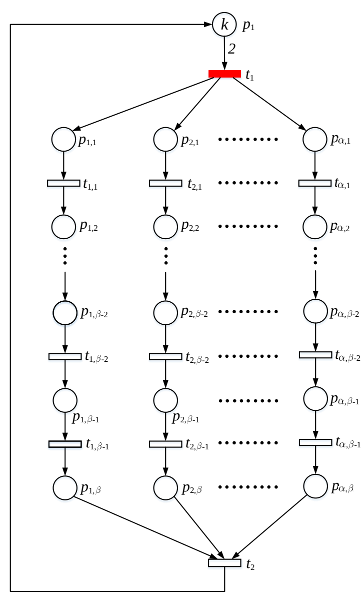

Figure 1: Benchmark I

As shown in Fig. 1, the first benchmark is a parameterized net system taken from [2] which represents a manufacturing system that contains a number of parallel production lines. However, some minor adjustments are made, i.e., the weight from p_1 to t_1 is increased to 2 and a series of transitions between places $p_{1,\beta-1}$ to $p_{\alpha,\beta-1}$ are removed. There are three parameters, i.e., k, α and β. With the change of the two parameters α and β, the scale of the system changes correspondingly. k indicates the initial resource quantity, while α and β represent the number of parallel lines and the length of each production line, respectively. Let $T_E = \{t_1\}$ (marked in red). For different values of k, α and β, the number of min-max basis markings $\mathcal{M}_{\mathcal{B}_\mathcal{M}}$ and all reachable markings $M \in R(N, M_0)$, as well as their computing times are listed in Table 1. Through all the testings, it can be concluded that the computation efficiency of obtaining the min-max-BRG is outperformed that of the reachability graph (RG).

Table 1: Analysis of the RG and min-max-BRG for the net in Fig. 1 with $T_E = \{t_1\}$.

Run	k	α	β	$\lvert R(N, M_0)\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}_\mathcal{M}}\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}_\mathcal{M}}\rvert/\lvert R(N, M_0)\rvert$	Time ratio
1	5	4	3	2921	21	7	0.03	0.2%	0.1%
2	6	4	3	14299	532	10	0.3	$<$ 0.1%	$<$ 0.1%
3	7	4	3	-	o.t.	13	0.9	-	-
4	8	4	3	-	o.t.	17	16	-	-
5	9	4	3	-	o.t.	21	48	-	-
6	10	4	3	-	o.t.	26	515	-	-
7	11	4	3	-	o.t.	31	1400	-	-
8	5	4	4	21029	1168	7	0.15	$<$0.1%	$<$0.1%
9	6	4	4	-	o.t.	10	14	-	-
10	7	4	4	-	o.t.	13	42	-	-
11	8	4	4	-	o.t.	17	4476	-	-

* The computing time is denoted by *overtime* (o.t.) if the program does not terminate within 36,000 seconds (10 hours).

Benchmark II

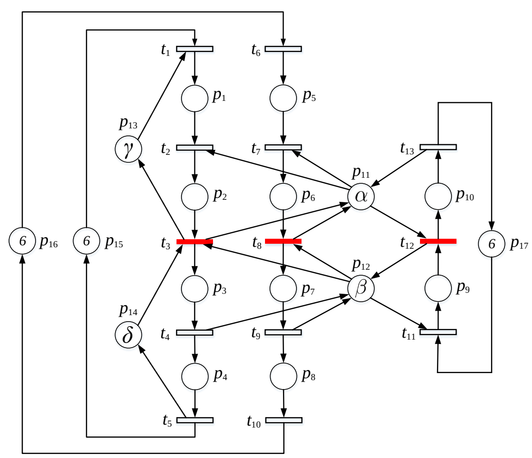

Figure 2: Benchmark II

Modified from a Petri net in [3], a parameterized net system shown in Fig. 2 is adopted as the second benchmark. The four parameters α, β, γ, and δ represent the numbers of tokens in places p_{11}, p_{12}, p_{13} and p_{14}, respectively. This system contains 17 places and 13 transitions and its scale does not change with changes in parameters. With $T_E = \{t_3, t_8, t_{12}\}$ (marked in red) and $T_I = \{t_1, t_2, t_4, t_5, t_6, t_7, t_9, t_{10}, t_{11}, t_{13}\}$, the analysis of min-max-BRG in comparison with the corresponding RG is illustrated in Table 2.

Table 2: Analysis of the RG and min-max-BRG for the net in Fig. 2 with $T_E = \{t_3, t_8, t_{12}\}$.

Run	α	β	γ	δ	$\lvert R(N, M_0)\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}_\mathcal{M}}\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}_\mathcal{M}}\rvert/\lvert R(N, M_0)\rvert$	Time ratio
1	2	2	1	1	4449	86	200	23	4%	26%
2	2	2	2	1	7523	253	312	65	4%	25%
3	2	2	2	2	12601	715	385	177	3%	24%
4	3	2	2	2	21026	2109	623	961	2.9%	45%
5	3	3	3	3	-	o.t.	1590	16580	-	-
6	3	4	3	3	-	o.t.	2369	57133	-	-
7	4	3	3	3	-	o.t.	2301	o.t.	-	-

* The computing time is denoted by *overtime* (o.t.) if the program does not terminate within 57,600 seconds (16 hours).

For different values of the parameters α, β, γ and δ, we report in different columns the sizes of the RG ($|R(N, M_0)|$) and the min-max-BRG ($|\mathcal{M}_{\mathcal{B}_\mathcal{M}}|$) as well as the time required to compute them. The ratio of $|\mathcal{M}_{\mathcal{B}_\mathcal{M}}|$ to $|R(N, M_0)|$ is also demonstrated.
With the increase of the four parameters α, β, γ, and δ, one can see that by selecting basis partition appropriately, the size of the min-max-BRG and the time required to construct it in the considered net are both smaller than that of the corresponding RG. Meanwhile, the ratios of $|\mathcal{M}_{\mathcal{B}_\mathcal{M}}|$ to $|R(N, M_0)|$decrease significantly.

Benchmark III

As shown in Fig. 3, the third benchmark is selected from [1] while we make some minor adjustments, i.e., two self-looped arcs $t_{33} \leftrightarrow p_6$ and $t_{36} \leftrightarrow p_{21}$ and one directed arcs $t_{39} \rightarrow p_{46}$ are removed. This system contains 46 places and 39 transitions and its scale does not change with changes in parameters. The initial marking shown in the figure is parameterized as:

$$M_0 = [\alpha\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \beta\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 8\ 1\ 0\ 0\ 0\ 0\ 0]^{\mathrm{T}}.$$

Let $T_E = \{t_1,\ t_3,\ t_7,\ t_9,\ t_{14},\ t_{20},\ t_{22},\ t_{24},\ t_{29},\ t_{32},\ t_{35}\}$ (marked in red). For different values of α and β, the number of min-max basis markings $\mathcal{M}_{\mathcal{B}_\mathcal{M}}$ and all reachable markings $M \in R(N,\ M_0)$, as well as their computing times are listed in Table 3. Through all the testings, it can be concluded that the computation efficiency of obtaining the min-max-BRG is outperformed that of the RG.

Table 3: Analysis of the RG and min-max-BRG for the net in Fig. 3 with
$T_E = \{t_1, t_3, t_7, t_9, t_{14}, t_{20}, t_{22}, t_{24}, t_{29}, t_{32}, t_{35}\}$.

Run	α	β	$\lvert R(N, M_0)\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}_\mathcal{M}}\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}_\mathcal{M}}\rvert/\lvert R(N, M_0)\rvert$	Time ratio
1	1	1	1966	21	361	7	18%	33%
2	1	2	12577	537	1724	52	14%	9%
3	2	2	76808	20415	9294	498	12%	2%
4	2	3	-	o.t.	25031	3794	-	-
5	2	4	-	o.t.	46388	9872	-	-
6	3	3	-	o.t.	71753	31965	-	-
7	3	4	-	o.t.	130857	o.t.	-	-

* The computing time is denoted by *overtime* (o.t.) if the program does not terminate within 36,000 seconds (10 hours).

References

[1] M. P. Cabasino, A. Giua, M. Pocci, and C. Seatzu. Discrete event diagnosis using labeled Petri nets. an application to manufacturing systems. *Control Engineering Practice*, 19(9):989–1001, 2011.

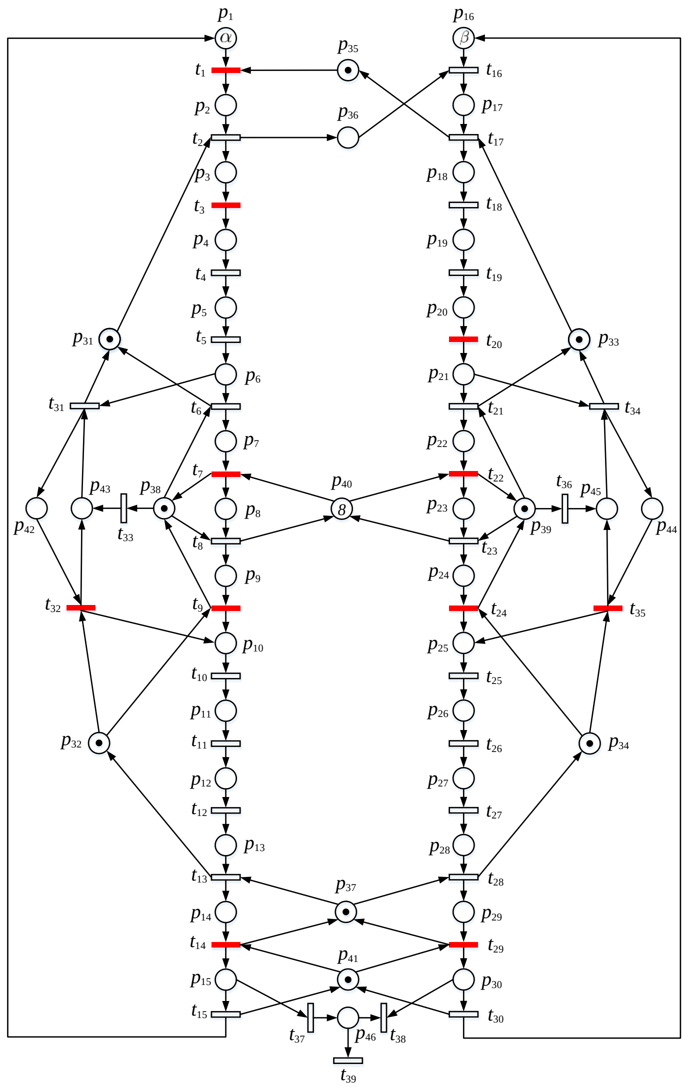

Figure 3: Benchmark III

[2] S. Lai, D. Nessi, M. P. Cabasino, A. Giua, and C. Seatzu. A comparison between two diagnostic tools based on automata and Petri nets. In *2008 9th International Workshop on Discrete Event Systems*, pages 144–149. IEEE, 2008.

[3] G. Y. Liu and Z. W. Li. General mixed integer programming-based liveness test for system of sequential systems with shared resources nets. *IET control theory & applications*, 4(12):2867–2878, 2010.