Simulations of the Minimax Basis Reachability Graph

Chao Gu [a,c], Ziyue Ma [a], Zhiwu Li [a,b], Alessandro Giua [c]

[a] *SEME, Xidian University, Xian, China*

[b] *ISE, Macau University of Science and Technology, Taipa, Macau*

[c] *DIEE, University of Cagliari, Cagliari, Italy*

Abstract

In this note we report the simulation results of three benchmarks based on the minimax basis reachability graph (minimax-BRG). All tests are executed based on a laptop with Intel i7-5500U 2.40 GHz processor and 8 GB RAM.

1 Benchmark I

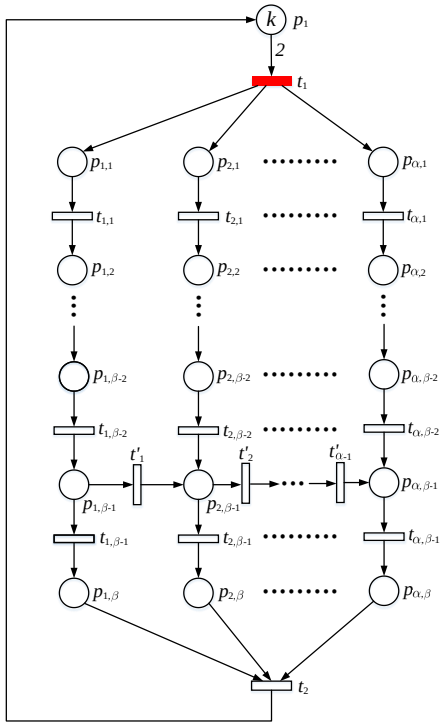

Fig. 1: Benchmark I

Email addresses: cgu1992@stu.xidian.edu.cn (Chao Gu), maziyue@gmail.com (Ziyue Ma), zhwli@xidian.edu.cn (Zhiwu Li), giua@unica.it (Alessandro Giua).

As shown in Fig. 1, the first benchmark is a parameterized net system taken from [2] which represents a manufacturing system that contains a number of parallel production lines. However, a minor adjustment is made, i.e., the weight from p_1 to t_1 is increased to 2. There are three parameters, i.e., k, α and β. With the change of the two parameters α and β, the scale of the system changes correspondingly. k indicates the initial resource quantity, while α and β represent the number of parallel lines and the length of each production line, respectively. Let $T_E = \{t_1\}$ (marked in red). For different values of k, α and β, the number of minimax basis markings $\mathcal{M}_{\mathcal{B}_\mathcal{M}}$ and all reachable markings $M \in R(N, M_0)$, as well as their computing times are listed in Table 1.

Table 1: Analysis of the RG and minimax-BRG for the net in Fig. 1 with $T_E = \{t_1\}$.

Run	k	α	β	$\lvert R(N,M_0)\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}_\mathcal{M}}\rvert$	Time (s)	$\lvert\mathcal{M}_{\mathcal{B}_\mathcal{M}}\rvert/\lvert R(N,M_0)\rvert$	Time ratio
1	5	4	3	10705	387	21	0.16	0.2%	<0.1%
2	6	4	4	72070	20447	52	5.2	<0.1%	<0.1%
3	7	4	3	-	o.t.	88	18	-	-
4	8	4	3	-	o.t.	140	835	-	-
5	9	4	3	-	o.t.	239	3127	-	-
6	10	4	3	-	o.t.	-	o.t.	-	-
7	5	5	3	30961	3400	21	0.3	<0.1%	<0.1%
8	6	5	3	-	o.t.	52	38	-	-
9	7	5	3	-	o.t.	88	155	-	-
10	8	5	3	-	o.t.	-	o.m.	-	-

* The computing time is denoted by *overtime* (o.t.) if the program does not terminate within 36,000 seconds (10 hours) and *out of memory* (o.m.) if the program runs out of memory.

2 Benchmark II

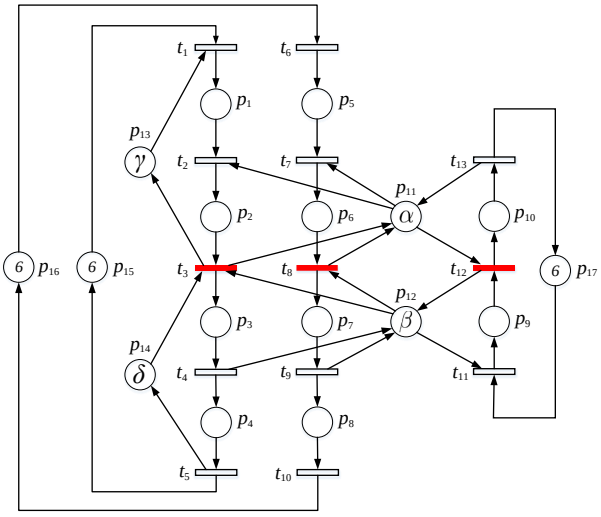

Fig. 2: Benchmark II

Selected from [3], a parameterized net system shown in Fig. 2 is the second benchmark. The four parameters α, β, γ, and δ represent the numbers of tokens in places p_{11}, p_{12}, p_{13} and p_{14}, respectively. This system contains 17 places and 13 transitions and its scale does not change with changes in parameters. With $T_E = \{t_3, t_8, t_{12}\}$ (marked in red) and $T_I = \{t_1, t_2, t_4, t_5, t_6, t_7, t_9, t_{10}, t_{11}, t_{13}\}$, the analysis of minimax-BRG in comparison with reachability graph is illustrated in Table 2.

Table 2: Analysis of the RG and minimax-BRG for the net in Fig. 1 with $T_E = \{t_3, t_8, t_{12}\}$.

Run	α	β	γ	δ	$\|R(N, M_0)\|$	Time (s)	$\|\mathcal{M}_{\mathcal{B}_\mathcal{M}}\|$	Time (s)	$\|\mathcal{M}_{\mathcal{B}_\mathcal{M}}\|/\|R(N, M_0)\|$	Time ratio
1	2	2	1	1	4449	86	200	23	4%	26%
2	2	2	2	1	7523	253	312	65	4%	25%
3	2	2	2	2	12601	715	385	177	3%	24%
4	3	2	2	2	21026	2109	623	961	2.9%	45%
5	3	3	3	3	-	o.t.	1590	16580	-	-
6	3	4	3	3	-	o.t.	2369	57133	-	-
7	4	3	3	3	-	o.t.	2301	o.t.	-	-

* The computing time is denoted by *overtime* (o.t.) if the program does not terminate within 57,600 seconds (16 hours).

For different values of the parameters α, β, γ and δ, we report in different columns the sizes of the reachability graph $|R(N, M_0)|$ and the minimax-BRG $|\mathcal{M}_{\mathcal{B}_\mathcal{M}}|$ as well as the time required to compute them. The ratio of $|\mathcal{M}_{\mathcal{B}_\mathcal{M}}|$ to $|R(N, M_0)|$ is also demonstrated.

With the increase of the four parameters α, β, γ, and δ, one can see that by selecting basis partition appropriately, the size of the minimax-BRG and the time required to construct it in the considered net are both smaller than that of the corresponding RG. Meanwhile, the ratios of $|\mathcal{M}_{\mathcal{B}_\mathcal{M}}|$ to $|R(N, M_0)|$ decrease significantly.

3 Benchmark III

As shown in Fig. 3, the third benchmark is selected from [1] while we make some minor adjustments, i.e., two self-looped arcs $t_{33} \leftrightarrow p_6$ and $t_{36} \leftrightarrow p_{21}$ and one directed arcs $t_{39} \rightarrow p_{46}$ are removed. This system contains 46 places and 39 transitions and its scale does not change with changes in parameters. The initial marking shown in the figure is parameterized as:

$$M_0 = [\alpha\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \beta\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0]^{\mathrm{T}}.$$

Let $T_E = \{t_1, t_3, t_7, t_9, t_{14}, t_{20}, t_{22}, t_{24}, t_{29}, t_{32}, t_{35}\}$ (marked in red). For different values of α and β, the number of minimax basis markings $\mathcal{M}_{\mathcal{B}_\mathcal{M}}$ and all reachable markings $M \in R(N, M_0)$, as well as their computing times are listed in Table 3.

Table 3: Analysis of the RG and minimax-BRG for the net in Fig. 3 with $T_E = \{t_1, t_3, t_7, t_9, t_{14}, t_{20}, t_{22}, t_{24}, t_{29}, t_{32}, t_{35}\}$.

Run	α	β	$\|R(N, M_0)\|$	Time (s)	$\|\mathcal{M}_{\mathcal{B}_\mathcal{M}}\|$	Time (s)	$\|\mathcal{M}_{\mathcal{B}_\mathcal{M}}\|/\|R(N, M_0)\|$	Time ratio
1	1	1	1966	19	361	5	18%	26%
2	1	2	12577	544	1724	42	14%	7%
3	2	2	76808	21304	9294	502	12%	2%
4	2	3	-	o.t.	25031	3520	-	-
5	2	4	-	o.t.	46388	11998	-	-
6	3	3	-	o.t.	71753	30584	-	-
7	3	4	-	o.t.	130857	o.t.	-	-

* The computing time is denoted by *overtime* (o.t.) if the program does not terminate within 36,000 seconds (10 hours).

References

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[2] S. Lai, D. Nessi, M. P. Cabasino, A. Giua, and C. Seatzu. A comparison between two diagnostic tools based on automata and Petri nets. In *2008 9th International Workshop on Discrete Event Systems*, pages 144–149. IEEE, 2008.

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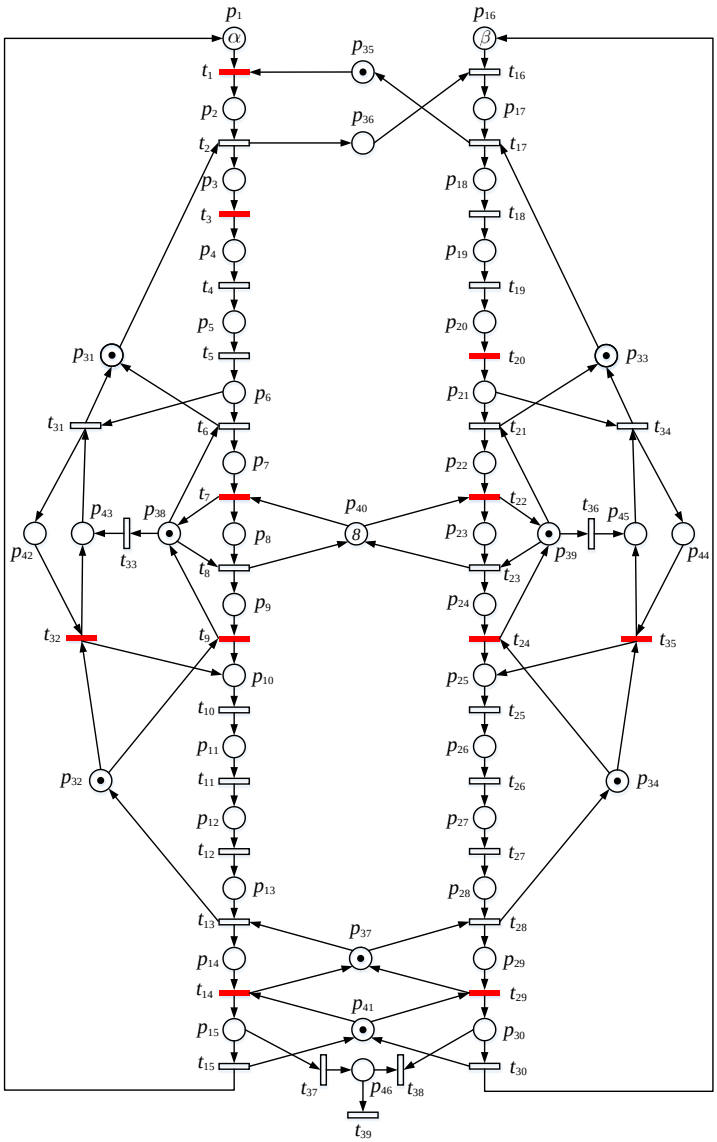

Fig. 3: Benchmark III