Packet Error Probability of Multi-carrier CDMA System in Fast/Slow Correlated Fading Plus Interference Channel

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Abstract. The probability of packet error for Multi-Carrier CDMA system with maximum ratio combining diversity is evaluated and compared in the slow and fast correlated Nakagami fading plus multiple access interference channel. From the results, the probability of packet error in fast fading is higher than that in slow fading, and the difference of packet error probability between the fast fading and slow fading diminishes with Nakagami fading parameter approaching infinity. When the error correction coding technique is used, it is observed that coding technique for Multi-Carrier CDMA system is more effective in the fast fading case than that in the slow fading case.

1 Introduction

Many urban and vehicular communication systems are subject to fading and co-channel interference caused by multipath propagation due to reflections, refractions and scattering by buildings and other large structures. Thus, the received signal is a sum of different signals that arrive via different propagation paths [1].

Several statistical models have been used in the literature to describe the fading envelope of the received signal [2]. A more versatile statistical model is Nakagami's m-distribution [3],[4], which can model a variety of fading environments including those modeled by the Rayleigh and one-sided Gaussian distributions. Also, the log-normal and Rician distributions may be closely approximated by the Nakagami distribution in some ranges of mean signal values. The Nakagami distribution is more flexible and more accurately fit experimental data for many physical propagation channels than the log-normal and Rician distributions. For this reason, there is contin-

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ued interest in modeling a variety of propagation channels with the Nakagami distribution.

Diversity reception technique is used extensively in radio channels to reduce the effect of fading on system performance, including both fixed terminals and mobile communication systems [5]. For example, diversity in land mobile radio can be used also in mobile terminals, on cars, and even in hand held portable radios. In order to obtain the diversity gain, there must be a sufficient degree of statistical independence in the fading of the received signal in each of the diversity branches. The assumption of statistical independence between the diversity channels is valid only if they are sufficiently separated. However, there are other cases of practical interest where the assumption of statistical independence is not valid. The effect of correlated fading on the performance of a diversity reception system has received a lot of attention in the literature [4],[6].

2 Analysis Model of Multi-carrier CDMA System

2.1 Multi-carrier CDMA System Model

This section is concerned with the calculation of the error probability of the Multi-Carrier CDMA system in a fading channel that is modeled by a discrete set of Nakagami faded paths. The block diagram of Multi-Carrier CDMA transmitter of the k-th user is shown in figure 1, where $a^{(k)}$ and $C_i^{(k)}$ denote the information symbol and the i-th spreading code of length M_C of the k-th user, respectively. By using M_C orthogonal codes, the maximum number of users is equal to M_C .

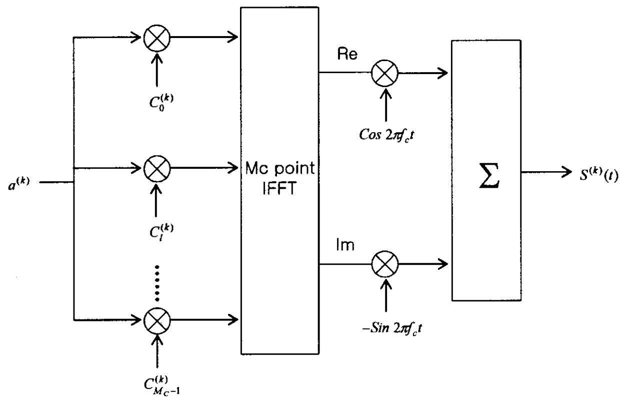


Fig. 1. Multi-Carrier CDMA transmitter for the k-th user in reverse link.

The transmitted signal of the k-th user is in general shaped in the time domain by a window function $w(t)$ to minimize excessive out-of-band emissions. The quadrature sinusoidal carriers are then modulated by the real and imaginary part of the baseband signals generated from IFFT respectively, while the former component at the *j*-th symbol instant can be described as

$$s_{I,j}^{(k)}(t)=\sqrt{\frac{2P}{M_C}}a_j^{(k)}\sum_{m=1}^{M_c}c_m^{(k)}\cos\left(\frac{2\pi mt}{T_s}\right)w(t-jT_s) \tag{1}$$

where $\{a_j^{(k)}\}$ is the bi-phase information sequence and $\{c_m^{(k)}\}$ is the spreading sequence over the alphabet {+1, -1}. It is assumed that the spreading gain M_C is the large and equals the number of carriers present. Also, the fundamental carrier frequency f_0, equals the symbol rate $1/T_S$ and the transmitted signal power P of every user is assumed to be same. Then, the *j*-th transmitted symbol $s_j^{(k)}$ of the *k*-th user with modulator phase shift θ_k is

$$s_j^{(k)}(t)=s_{I,j}^{(k)}(t)\cos(2\pi f_c t+\theta_k)-s_{Q,j}^{(k)}(t)\sin(2\pi f_c t+\theta_k) \tag{2}$$

where f_c to f_c+M_C/T_S is the desired spectrum in used and $s_{Q,j}^{(k)}(t)$ equals (1) with $\cos(2\pi mt/T_s)$ replaced by $\sin(2\pi mt/T_s)$. For the sake of clarity, we let $u_k(t)=\sum_{m=1}^{M_C}c_m^{(k)}\cos(2\pi mt/T_S)$, $v_k(t)=\sum_{n=1}^{M_C}c_n^{(k)}\sin(2\pi nt/T_S)$ in the analysis of the following sections.

For simplicity, we assume *w(t)* in Eq. (1) equals $p_{T_S}(t)$. Where $p_{T_S}(t)$ is the pulse waveform of each symbol, given as

$$p_{T_S}(t)=\begin{cases}1, & 0\le t<T_S\\ 0, & otherwise\end{cases} \tag{3}$$

where T_S is a symbol period.

For the quadrature modulator and the receiver, the transmitted data sequence, $\{a_j^{(1)}\}$ is coherently detected and multipath fading is not presented. To simplify our argument, neither the guard interval nor the influence of inter-symbol interference (ISI) of each user will be taken into account.

With a perfect power control, the *j*-th transmitted symbol at each receiver is given by

$$r_j(t)=\sum_{k=1}^{U}\left\{s_{I,j}^{(k)}(t-\tau_k)\cos(2\pi f_c t+\phi_k)-s_{Q,j}^{(k)}(t-\tau_k)\sin(2\pi f_c t+\phi_k)\right\}+N(t) \tag{4}$$

where $N(t)$ is white Gaussian noise and $\phi_k=\theta_k-2\pi f_c\tau_k$. The delay and modified phase offset $\{a_j^{(k)}\}$ of every transmitted signal are assumed to be uniformly distributed on the interval $[0, T_S]$ and $[0, 2\pi]$ respectively. It is further assumed that the parameters τ_1 and ϕ_1 equal zero.

Multi-Carrier CDMA signal consists of sum of M_C waveform. As M_C increases, its waveform will have a Gaussian distribution. That is, for asynchronous Multi-Carrier CDMA system, Gaussian approximation is then used to estimate the probability of error in the Multi-Carrier CDMA system.

In the analysis, random signature sequences are assumed in operation, and for asynchronous Multi-Carrier CDMA system, interference from other users can be well approximated as a Gaussian noise with zero mean and the average value of the *k*-th multiple access user's variance in the Multi-Carrier CDMA system, $E\left[Var\left\{I^{(1,k)}\right\}\right]$ is as follow [7].

$$E\left[Var\left\{I^{(1,k)}\right\}\right] = \frac{PT_S^2}{4\pi^2 M_C}\left\{\left[\sum_{n=1}^{M_C}\sum_{i=1,i\neq n}^{M_C}\frac{3n^2+i^2}{(n^2-i^2)^2}\right]+\frac{2\pi^2}{3}\right\} \tag{5}$$

By assuming the interference among all other users are independent, the total interference power would simply be the summation of every $Var\left\{I^{(1,k)}\right\}$. The equivalent signal-to-noise and interference power ratio γ_{eq} at the receiver is then approximated as follow.

$$\gamma_{eq} = \frac{M_C PT_S^2/2}{\frac{(U-1)PT_S^2}{4\pi^2 M_C}\left\{\left[\sum_{n=1}^{M_C}\sum_{i=1,i\neq n}^{M_C}\frac{3n^2+i^2}{(n^2-i^2)^2}\right]+\frac{2\pi^2}{3}\right\}+\frac{M_C N_0 T_S}{4}} \tag{6}$$

where E_b is transmitted signal energy in one symbol period, U is the number of multiple access users, and N_0 is the one-side noise spectral density.

2.2 Nakagami Fading Channel Model

The Multi-Carrier CDMA signals of *k*-th user are transmitted to channel, and then they are distorted by the Nakagami fading. A Nakagami characterizes channels with different fading depths through a parameter (m) called amount of fading. The received signal envelope, R is a random variable with a Nakagami probability density function (pdf) [2] i.e.

$$f_\Omega(R) = \frac{2m^m R^{2m-1}}{G(m)\Omega^m}\exp\left(-\frac{mR^2}{\Omega}\right) \tag{7}$$

where $G(\cdot)$ is the Gamma function, $\Omega/2=\overline{R^2}/2$ is the mean power of the faded signal, and m is Nakagami fading parameter ($m=\Omega^2/\left(\overline{R^2-\Omega}\right)\geq 1/2$).

In a Nakagami fading channel, the magnitude of the received signal is characterized by the Nakagami distribution. However, it is more convenient to use the square of the received signal envelope which is proportional to the signal power ($\gamma=R^2/2N_0$). The desired signal-to-noise power ratio γ is then gamma distributed with probability density given by [3]

$$f_\Gamma(\gamma)=\left(\frac{m}{\Gamma}\right)^m\frac{\gamma^{m-1}}{G(m)}\exp\left(-\frac{m\gamma}{\Gamma}\right),\qquad \gamma>0 \tag{8}$$

where $m\geq 1/2$ and Γ is the average signal-to-noise power ratio. The constant m is called the Nakagami fading parameter. m=1 and m=∞ correspond to the Rayleigh fading and the nonfading case, respectively.

2.3 Diversity Reception of Correlated Fading Signal

Next, we consider the reception of Multi-Carrier CDMA signal in correlated fading channel. For convenience we assume that the correlation between the fading signals is known and completely characterized by a single parameter ρ. Experimental data on correlation of the fading signals are usually given in terms of envelope correlation because of ease of measurement. Given the envelope correlation, however, the power correlation can be found, and vice versa. In fact, for many practical purposes, both correlations can be taken to be approximately equal [4].

In this section, we consider the constant correlation model which characterize the correlation between the fading signals in each of the diversity branches is presented as $\rho_{ij}=\rho$, where $i,j=1,2,\cdots,M_B$ and 0 < ρ < 1. Such a constant correlation model may approximate closely placed diversity antennas. For the above correlation model, Aalo [4] has obtained the probability density function for γ from its characteristic function. It is given by

$$f_{C\Gamma}(\gamma)=\frac{\left(\frac{\gamma m}{\Gamma}\right)^{M_B m-1}\exp\left(\frac{-\gamma m}{\Gamma(1-\rho)}\right)}{(\Gamma/m)(1-\rho)^{m(M_B-1)}(1-\rho+M_B\rho)^m G(M_B m)} \cdot {}_1F_1\left(m,M_B m;\frac{M_B m\rho\gamma}{\Gamma(1-\rho)(1-\rho+M_B\rho)}\right) \tag{9}$$

where ${}_1F_1(\cdot,\cdot;\cdot)$ is the confluent hypergeometric function. For ρ =0, Eq. (9) reduces to the probability density of the sum of M_B independent random variables, each of which has the density given by Eq. (8).

3 Performance Evaluation of Multi-carrier CDMA System

3.1 Probability of Packet Error for Multi-carrier CDMA System

In the analysis of data transmission with steady-signal reception, the probability of bit error plays a fundamental role in describing the performance of a digital communication system. Digital systems employing error detection or error correction coding are generally based on the transmission of blocks of L_P sequential bits, where each block of L_P bits may be a complete message or a submessage element, such as a character.

The performance of Multi-Carrier CDMA system will then depend upon the probability of occurrence of various numbers of errors in a block of L_P bits. For Multi-Carrier CDMA modulation and steady-signal reception in white Gaussian noise, the errors in a block will be binomially distributed, i.e., the probability of exactly t errors in a block of L_P bits is given by

$$P_t = \binom{L_P}{t} p^t (1-p)^{L_p - t} \tag{10}$$

where channel error rate, $p = P_e(\gamma)$ for a given signal to noise power ratio γ in AWGN and fading channel.

For the modulation schemes to which the binomial distribution applies, the probability of more than M errors within a block of L_P bits, which we define as the probability of packet error is

$$P(M, L_P) = \sum_{t=M+1}^{L_P} P_t \tag{11}$$

In a communication system that transmits data in blocks of L_P bits, the probability $P(M, L_P)$ of more than M bit errors in a block is an important quantity. If an error-correction code capable of correcting up to M errors in each block of L_P bits is employed, system performance is governed by the probability of more than M errors in a block. If a simple automatic repeat request (ARQ) scheme is used, the throughput can be determined from $P(0, L_P)$. On the other hand, if the use of forward error correction is to be investigated, then the knowledge of $P(M, L_P)$ is required. For AWGN channel in which the bit errors are independently and identically distributed, $P_g(M, L_P)$ can be readily calculated from the bit error probability $P_{eg}(\gamma)$, namely,

$$P_g(M, L_P) = \sum_{t=M+1}^{L_P} \binom{L_P}{t} P^t{}_{eg}(\gamma)(1 - P_{eg}(\gamma))^{L_P - t} \tag{12}$$

However, for a fading channel as encountered in VHF or UHF mobile radio applications, no simple relationship exists between $P_g(M, L_P)$ and $P_{eg}(\gamma)$. One commonly used analytic model for the signal fluctuations assumes that they can be closely approximated by a Rayleigh distribution [8]-[11]. In the past, a number of authors [8],[9] have derived expression for $P_g(M, L_P)$ in the assumption that the fading is so slow that the received signal strength can be assumed constant over the duration of a block of L_P bits. We will refer to such a channel as a very slow fading channel. Unfortunately, these expressions have proved awkward to evaluate and ad hoc approximations for $P_g(0, L_P)$ have been suggested.

It is well known [7] that the standard Gaussian approximation formula of uncoded Multi-Carrier CDMA BPSK system is therefore given by

$$P_{eg}(\gamma) \approx Q(\sqrt{\gamma_{eq}}) = 0.5 \cdot erfc(\sqrt{0.5\gamma_{eq}}) \tag{13}$$

where γ_{eq} is the equivalent signal to noise plus interference power ratio at the base station receiver and *Q(x)* is the complementary error function. If we have a very slow fading channel in which the received average SNR varies according to a probability density function (pdf) $f_{CT}(\gamma)$, then the resulting probability of packet error in fading channel $P_f(M, L_P)$ can be written as

$$\begin{aligned} P_f(M,N) &= < P_f(M,N;\gamma) > \\ &= \int_0^{\infty} P_g(M,N) \cdot f_{c\Gamma}(\gamma) d\gamma \end{aligned} \tag{14}$$

where is a pdf of correlated Nakagami fading channel.

In the case of fast fading where the signal strength varies continuously such that two adjacent bits in a block are faded independently, we have

$$P_f(M, L_P) = \sum_{m=M+1}^{L_P} \binom{L_P}{t} P^t{}_{ef}(\gamma)(1 - P_{ef}(\gamma))^{L_P - t} \tag{15}$$

where $P_{ef}(\gamma)$ is the bit error probability under correlated Nakagami fading given by

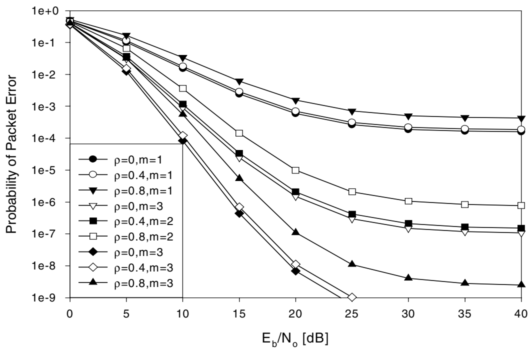
$$P_{ef}(\gamma) = \int P_{eg}(\gamma) \cdot f_{C\Gamma}(\gamma) d\gamma \tag{16}$$

In the absence of error-correction coding, the probability of packet error under fast fading channel is

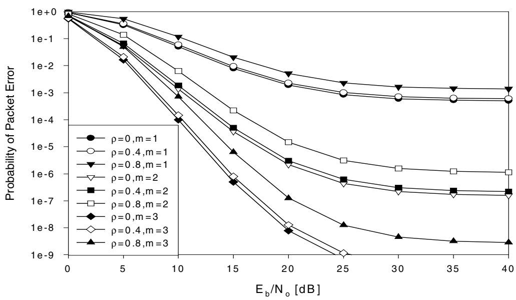
$$\begin{aligned} P_f(0, L_P) &= \sum_{t=1}^{L_P} \binom{L_P}{t} P^t{}_{ef}(\gamma)(1 - P_{ef}(\gamma))^{L_P - t} \\ &= 1 - (1 - P_{ef}(\gamma))^{L_P} \end{aligned} \tag{17}$$

4 Numerical Results and Discussion

In this paper, the probability of packet error for Multi-Carrier CDMA system are evaluated as a function of the Nakagami fading parameter (*m*), the number of Multi-Carrier (M_C), the number of multiple access users (*U*), correlation parameter (ρ), error correcting capability (*M*), the length of packet (L_P), and the number of branch (M_B). For the correlation parameter ρ =0, the probability density reduced the sum of M_B independent random variables, each of which has the uncorrelated probability density. In this section, to compare the performance of Multi-Carrier CDMA system, we select the error correcting capability at the same bits, and same correlation parameter.



(a) *U=5,* M_B =2, M_C =127, L_P =100, M=0, ρ =0.4, slow fading



(b) *U=5,* M_B =2, M_C =127, L_P =100, M=0, ρ =0.4, fast fading

Fig. 2. Average y of pa cprobabilitket error for various E_b / N_0

Figure 2(a) and figure 2(b) show the effect of slow and fast fading according to the Nakagami fading parameter (m), the correlation parameter (ρ), and average E_b / N_0. Clearly, as the Nakagami fading parameter (m) increase, the packet error performance is getting better and the curves approach that corresponding to the case of non fading channel. Also, as the correlation parameter (ρ) decrease, the packet error performance is getting better and the curves approach that corresponding to the uncorrelated case. From the curves, it is noting that the performance of $\rho < 0.4$ can result in similar packet error performance of $\rho = 0$ in slow and fast fading channel.

Figure 3 shows the probability of packet error for various block length (N), fading parameters (m) and fading rate (slow/fast) in Nakagami fading channel. From the Fig. 3, it is observed that for a given value of m, the probability of packet error in fast fading is always higher than that in slow fading, especially when m is small (large amount of fading). The difference of packet error probability between the fast fading and slow fading diminishes as m approaches to infinity.

Figure 4(a) and figure 4(b) show the effects of error correction coding in slow and fast fading conditions respectively. Comparing the figure 4(a) and the figure 4(b), it is observed that error correction coding technique is more effective in the fast fading case than that in the slow fading case.

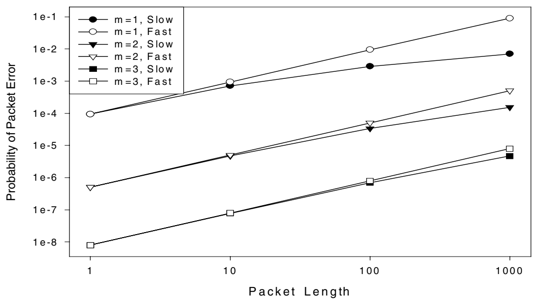
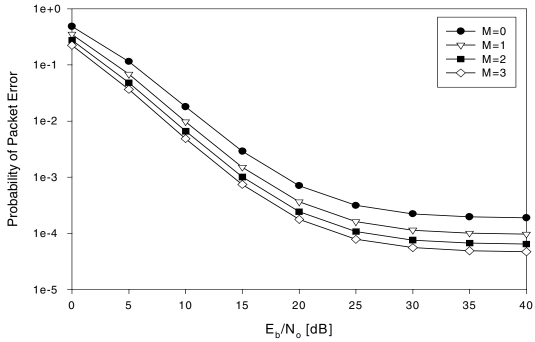


Fig. 3. Average probability of packet error for various block length (E_b / N_0 =15dB, U=5, M=0, M_C =127, ρ =0.4)

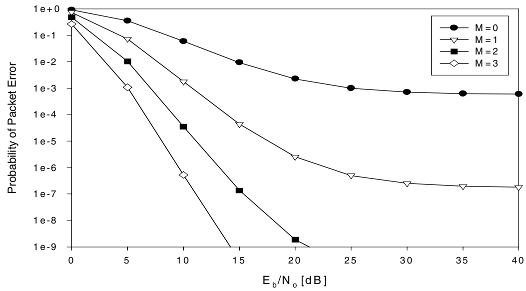
5 Conclusion

In this paper, we have considered the probability of packet error for the Multi-Carrier CDMA system in correlated Nakagami fading channel that includes the Rayleigh fading channel. As a technique for the performance improvement, MRC diversity and block coding techniques have been used, and the performance of Multi-Carrier CDMA system has been compared and analyzed in slow and fast correlated Nakagami fading plus multiple access interference environments.

From the results, the probability of packet error in fast fading is always higher than that in slow fading, and the difference of packet error probability between the fast fading and slow fading diminishes as m approaches to infinity. When the error correction coding technique is used, it is observed that error correction coding technique is more effective in the fast fading case than in the slow fading case.



(a) $U=5,\ M_B=2,\ M_C=127,\ L_P=100,\ \rho=0.4$, slow fading



(b) $U=5,\ M_B=2,\ M_C=127,\ L_P=100,\ \rho=0.4$, fast fading

Fig. 4. Average probability of packet error for various error correcting capability

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