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**Paper**

# Mathematical Model of Bessel Beamformer with Automatic Gain Control for Smart Antenna Array System in Rayleigh Fading Channel

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This paper analyzes the mathematical model of a Bessel beamformer with automatic gain control (AGC) in a multipath scenario using a digital modulation technique and makes comparison with the one without AGC. The Bessel beamformer with AGC is designated here as the modified Bessel beamformer. The desired useful and interfering signals operate with the same carrier frequency and Doppler shifts but in different directions. Based on simulation results, this modified Bessel beamformer is shown to provide an optimum solution and to accommodate more users in real-time base stations of mobile communication system when implemented on smart antenna array systems. © 2014 Institute of Electrical Engineers of Japan. Published by John Wiley & Sons, Inc.

**Keywords:** smart antenna array system, modified Bessel beamformer, automatic gain control

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## 1. Introduction

A Bessel beamformer is used for forming a beam [1] for constant step sizes ($\mu$) ranging from zero to 1, i.e. $0 < \mu < (1/\lambda_{max})$ where $\lambda_{max}$ is the largest eigenvalue of the autocorrelation matrix $\mathbf{R}$ that is used to regulate the speed of adaptation, rate of convergence, and stability. The condition imposed on the step size bounds the beamformer to operate within 0–1. This beamformer is based on Bessel functions which are usually known as cylinder functions or Fourier–Bessel (FB) functions. An FB function generates an FB series that converges absolutely [2]. In Ref. [3], a theorem based on the $n$th order FB series expansion using Bessel function is developed which can be employed in diverse areas of signal processing, information representation, and communication systems. Coding of speech signals using Bessel functions as orthogonal signals in the FB expansion has been explored in Ref. [4], and it is found that speech quality and the bit rate increase when a large number of FB coefficients are used. In Ref. [5], a subcarrier weighting technique to suppress the out-of-band radiation of orthogonal frequency division multiplexing (OFDM) signals is proposed, and it is found that the proposed scheme improves the system performance in terms of bit error rate (BER) and gain. The performance of the continuous phase modulation (CPM) based OFDM (CPM-OFDM) system is analyzed in Ref. [6] and a CPM-based single-carrier frequency domain equalization (CPM-SC-FDE) structure for broadband wireless communication systems is proposed. It is shown that the proposed system has a better performance, better utilization of the channel frequency diversity, and better bandwidth efficiency than the CPM-OFDM system. In Ref. [7], a new beamforming strategy is proposed for multiuser systems with $N$ transmit antennas at the transmitter, and it is proved that the proposed scheme offers a remarkable improvement over the classical spatial division multiple accesses and achieves the same data rates as spatial multiplexing for all users but with significantly superior performance/diversity gain. The use of spherical Bessel functions for pattern synthesis of linear antennas is discussed in Ref. [8], which leads to antenna current distribution according to the Legendre polynomials of the first kind as they are easy to compute numerically. In Ref. [9], a smart antenna system based on direction-of-arrival estimation and adaptive beamforming (ABF) is designed, and in Ref. [10] an investigation is carried out to facilitate the minimum variance distortionless response (MVDR) beamforming technique for a cylindrical conformal array. The downlink performance analysis of a multi-input–multi-output (MIMO) system that combines ABF and spatial multiplexing (SM) procedures is detailed in Ref. [11]. Both prototypical experiments and simulations demonstrate that the ABF-MIMO OFDMA system improves the required signal-to-noise ratio (SNR) compared to the conventional MIMO OFDMA system. A novel approach by adjusting the step size of the LMS algorithm using the channel output autocorrelation (COA) has been proposed for application to unknown channel estimation or equalization in low SNR in Ref. [12]. It is shown that the proposed variable step size least mean square (LMS) (VSS-LMS) algorithm has considerably better performance than conventional LMS, recursive least squares (RLS), normalized LMS (N-LMS), and the other VSS-LMS algorithms.

In this paper, we propose a modified Bessel beamformer based on variable step size which depends on the signal array vector and simulated in Rayleigh fading channel model using a digital modulation technique. This modified Bessel beamformer is used for automatic adjustment of the tap weights of the filter according to the computed error estimation. Therefore, we call it a Bessel beamformer with automatic gain control (AGC). The step size for each iteration can be changed using the knowledge of the autocorrelation matrix.

The next section describes the system model. Section 3 covers the mathematical model for the modified Bessel beamformer. Section 4 presents the simulation results. Discussion and comments are presented in Section 5. Finally, Section 6 concludes the paper.

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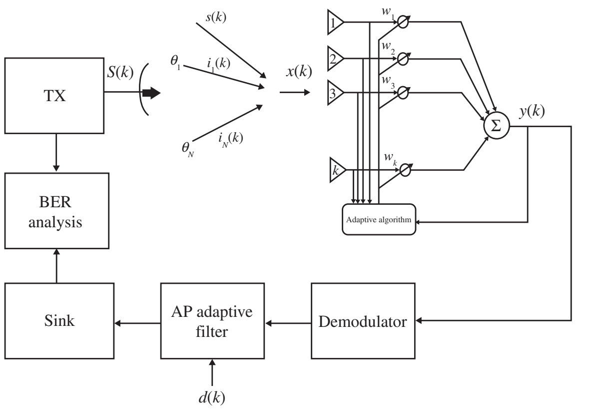


Fig. 1. Use of the modified Bessel beamformer in the Rayleigh fading channel model with the desired and interfering signals shown

## 2. System Model

A smart antenna array system consists of number of elements with uniform distance between two elements and equipped with a digital signal processor containing the ABF algorithm, i.e. the modified Bessel beamformer, as shown in Fig. 1. These ABF algorithms [13–15] are used to update the weights dynamically so that the mean-square error (MSE) is reduced and the SNR of the desired signal is optimized.

The proposed algorithm is based on variable step size and depends only on the signal array vector. This modified Bessel beamformer is used for automatic adjustment of the tap weights of the filter according to the computed error estimation. Therefore, the Bessel beamformer with AGC provides adaptive weight calculation efficiently. This is the modified version of our previous work [1] in which the self-adjustment property was missing and was based on an analog modulation scheme. In this case, the proposed algorithm is simulated in the Rayleigh fading channel model using a digital modulation technique. The modified Bessel beamformer adjusts itself automatically as the step size changes.

## 3. Mathematical Model

### 3.1. Modified Bessel Beamformer

The proposed algorithm is used for the automatic adjustment of the tap weights of the filter according to the computed error estimation and provides adaptive weight calculation efficiently. The algorithm is simulated in the Rayleigh fading channel environment and the step size changes for each iteration using the knowledge of the autocorrelation matrix, and therefore yields the set of optimum weights of the beamformer automatically. The AGC is derived from signal array vector as given by

$$\mu = \frac{1}{(2 * real(trace(\mathbf{R})))} \tag{1}$$

$$\mathbf{R} = [\mathbf{X_k} * (\mathbf{X_k})^T] \tag{2}$$

where $\mathbf{R}$ is the autocorrelation matrix describing correlation between various elements of signal array vector ($\mathbf{X_k}$), and $T$ denotes the transpose of signal array vector. The autocorrelation matrix provides the eigenvalues. The eigenvalues/vectors determine the quadratic MSE performance surface that regulates the speed of adaptation, rate of convergence, and stability. The signal array vector is written as

$$\mathbf{X_k} = [x_1, x_2, \ldots.., x_N]^T \tag{3}$$

Now consider a linear Bessel beamformer using multiple inputs at its array elements, as shown in Fig. 1; then its output will be

$$y_k = \mathbf{X}_k \widehat{\mathbf{W}}_k^T \tag{4}$$

where $\widehat{\mathbf{W}} = J_v(N)\mathbf{W}_k$ is the initial estimate weight vector which is equal to the dot product of the starting weight vector and the Bessel function, and $k$ is the iteration number.

Bessel function of the first kind is a mathematical function that generates an output array for each element of the input array [16]. Occasionally, Bessel functions are also known as FB functions [2]. It is important to note that initial estimate weight vector ideally has no impact on the end results [17]. Bessel function of the first kind is a highly convergent series [18] that helps the algorithm to converge efficiently and compute the array factor [18]. Bessel functions are eigenfunctions that are all mutually orthogonal. We use the orthogonality property for determining each of the coefficients that make the infinite series as a whole conform to the initial conditions. The infinite series is the solution of a time-dependent problem, involves a wave, and forms a basis for series expansion, similar to Fourier series. A Fourier series expresses a function in terms of its frequency components. In applying Fourier series to signal processing, the individual terms should be the ones that would be obtained if a narrow band-pass filter is applied to the signal. The eigenfunctions may have little physical significance, and are really just useful mathematical tools because of the property of orthogonality [4] [19–22]. Bessel beamformers employing Bessel functions have the ability to discriminate between the desired signal, noise, and other unwanted components using the principle of orthogonality. Because of the property that the desired useful and the interference components are orthogonal, we can achieve perfect recovery at the receiver.

Bessel function of the first kind $J_v(N)$ is given by

$$J_v(N) = \left(\frac{N}{2}\right)^v \sum_{k=0}^{\infty} \frac{\left(-\frac{N^2}{4}\right)^k}{k!\Gamma(v+k+1)} \tag{5}$$

where $v$ denotes the order of the Bessel function of the first kind, which must be a real number. The number of elements in the array is represented by $N$, and $\Gamma$ is the gamma function.

As the signal array vector consists of the desired and other interfering signals [23], it can also be written as

$$\mathbf{X}_k = s_d(k)a(\theta_d) + \sum_{i=1}^{L} s_i(k)a(\theta_i) + n(k) \tag{6}$$

where $s_d$ and $s_i$ are the desired and interfering signals arriving at the array at angles $\theta_d$ and $\theta_i$, respectively, $L$ is the number of interfering signals, and $n$ is a white and zero-mean complex Gaussian noise at the array elements. $a(\theta_d)$ and $a(\theta_i)$ are the steering vectors for the desired and interfering signals, respectively. The steering vector is described as

$$a(\theta) = [1, e^{-j\varphi}, \ldots\ldots., e^{-j(N-1)\varphi}] \tag{7}$$

where $\varphi = (2\pi d/\lambda)\sin\theta$ is the phase shift observed at each sensor due to the angle of arrival (AOA) of the wave front, and we assume $d$ is the uniform distance between array elements. $\lambda = c/f$ where $f$ is in hertz. Therefore, the steering vector can be written as

$$a(\theta) = [1, e^{-j\frac{2\pi}{\lambda}d\sin(\theta)}, \ldots\ldots., e^{-j\frac{2\pi}{\lambda}d(N-1)\sin(\theta)}] \tag{8}$$

The error signal used for adjustment of the adaptive system by adjusting or optimizing the weight vector to minimize this error signal is given by

$$e_k = d_k - y_k \tag{9}$$

Putting value of $y_k$ in (9) and differentiating w.r.t. the weight $w$, we get

$$\frac{\partial e_k}{\partial w} = 0 - [\mathbf{X}_k J_v(N)(1) + 0] = -\mathbf{X}_k J_v(N) \tag{10}$$

Now, the gradient is obtained by differentiating the squared error by the receiver antenna weight, i.e. putting the value of (10) in the gradient estimate of the form, giving

$$\widehat{\nabla}_k = 2e_k \begin{bmatrix} \frac{\partial e_k}{\partial w_0} \\ \cdot \\ \cdot \\ \frac{\partial e_k}{\partial w_L} \end{bmatrix} = 2e_k(-\mathbf{X}_k J_v(N)) \tag{11}$$

From steepest decent method [20, Chap. 2, Eq. (2.35) and Chap. 4, Eq. (4.36)], [24], which is used for developing and analyzing a variety of adaptive algorithms, we have

$$\mathbf{W}_{k+1} = \mathbf{W}_k - \mu\nabla \tag{12}$$

Putting value of the gradient estimate (11) into (12), we get

$$\mathbf{W}_{k+1} = \mathbf{W}_k + 2\mu e_k J_v(N)\mathbf{X}_k \tag{13}$$

where μ is a variable step size that depends on signal array vector and can be considered one of the most effective variable step size algorithms. Therefore, it can be called an *automatic gain control* (AGC) as defined in (1). It is important to note that the weight vectors for the Bessel beamformer and the modified Bessel beamformer are same. However, the only difference between these two beamformers is in the step size (μ) measurement. The former is a constant step size algorithm, but the latter is a variable step size algorithm with the aim to make the signal power constant. In summary, the proposed beamformer performs the following steps:

*Step 1*: Obtain R in (2) by signal array vector.
*Step 2*: Get AGC in (1) for self-adjustment of the algorithm.
*Step 3*: Calculate the error signal used for optimizing the weight vector in (9).
*Step 4*: Calculate the robust adaptive beamformer weights in (13).
*Step 5*: Repeat the above steps in a closed loop to get the optimum results.

## 4. Simulation Results

The simulations were designed with 8 bits per symbol in quadrature amplitude modulation (QAM) to analyze the properties of modified Bessel beamformer in the digital modulation technique. The multipath Rayleigh fading channel is taken for simulation purpose. To make the channel noisier, additive white Gaussian noise (AWGN) with SNR = 20 dB is added in order to evaluate the efficiency of the proposed beamformer in a rough scenario. The order of the Bessel function is 1 in this case.

**4.1. Message signal in discrete form** The message input signal consists of discrete time signals. The message signal shown in Fig. 2 can be converted into symbol to evaluate the performance of proposed beamformer in multipath Rayleigh fading model and compared with [1]. In both cases, when the message signal is passed through a multipath Rayleigh fading channel from the radio transmitter to the receiver, major paths result in the arrival of delayed versions of the signal at the receiver. In addition, the radio signal undergoes scattering on a local scale for each major path. Such local scattering is typically characterized by a large number of reflections by objects near the mobile. These unresolvable components combine at the receiver and give rise to the phenomenon known as *multipath fading*.

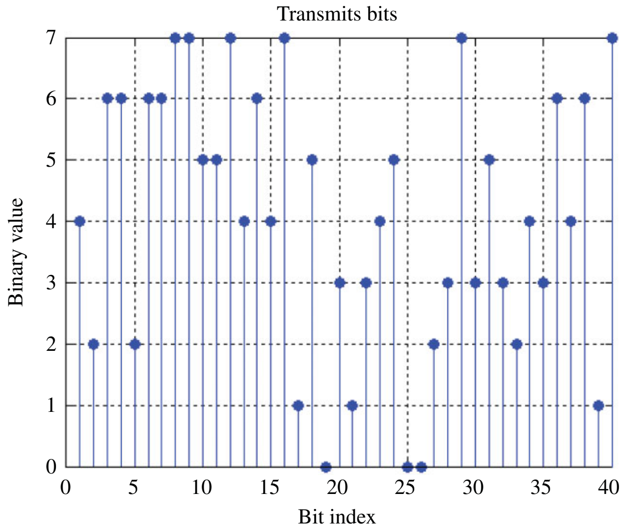


Fig. 2. Message signal in discrete form

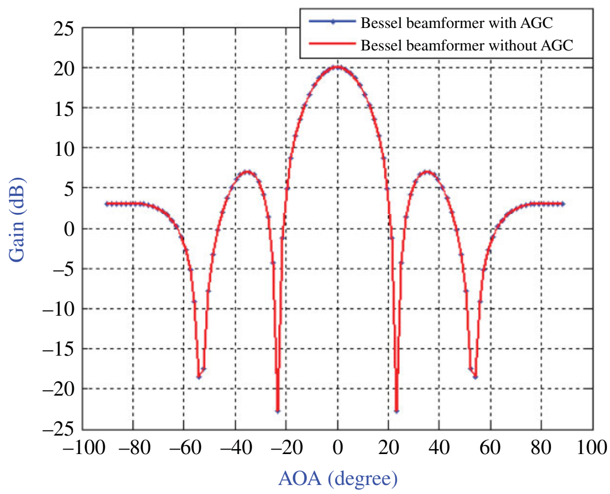


Fig. 3. Array factor plot for the Bessel beamformer with and without AGC with AOA for desired user of 0 degree

**4.2. Gain enhancement by smart antenna array system** The uniform linear array is taken for 16 elements as shown in Fig. 3 and the distance between two elements is maintained as $\lambda/2$ for both algorithms under study. One-hundred samples are taken for the simulation. The AOA for desired user is 0 degree. The interfering signals operate with same carrier frequency and Doppler shifts but in different directions with different path gains. The beam width is measured between the first two nulls of the array response function. The desired signal and the interferers are received by an array for various numbers of elements. It is observed that the array directivity increases with the number of elements, but at the same time the number of sidelobes also increases. The directivity of modified Bessel beamformer for 16 elements is observed to be 20.0018 dB, whereas that for the Bessel beamformer without AGC is found to be 19.9981 dB. Both beamformers steer the main beam toward the desired direction. The ratio between the powers of the main lobe and the first sidelobe is 13.0 dB. The null depth performance is found to be −23.0 dB. Subsequent data obtained from Figs 3 and 4 are given in the Table I, which indicates that the beam width increases when number of elements in the array decreases. Similarly, the gain of the smart antenna increases when number of elements in the array increases.

Table I. Performance analysis of both algorithms under study

| Bessel beamformer | Element spacing | AOA (degree) | Max. MSE | Gain (dB) | Beamwidth (degree) |
|---|---|---|---|---|---|
| With AGC | 0.5 | 0 | 2.925 | 20.0018 | 42 |
| Without AGC | 0.5 | 0 | 12.029 | 19.9981 | 42 |

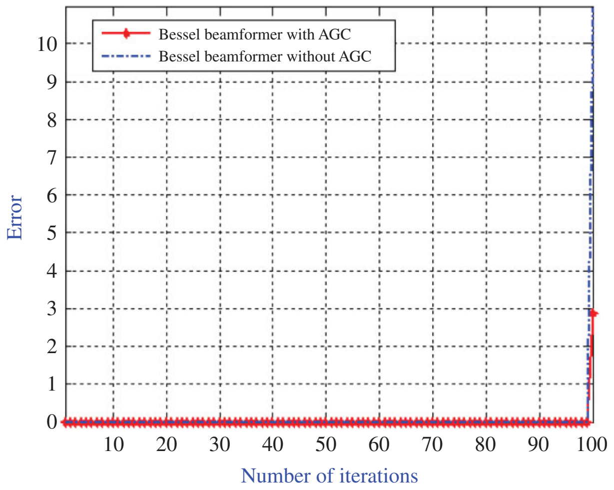


Fig. 4. Mean square error performance for the Bessel beamformer with and without AGC

### 4.3. Mean Square Error Performance

The minimum mean-square error (MMSE) describes the performance of the given system as shown in Fig. 4.

An adaptive beamformer like the modified Bessel beamformer or the Bessel beamformer without AGC combines the signals received by different elements of a smart antenna array to form a single output. This is achieved by minimizing the MSE between the desired output and the actual array output. The MMSE for the modified Bessel beamformer and the Bessel beamformer without AGC is shown in Fig. 4, which indicates that the modified Bessel beamformer has minimum MSE for various iterations as the modified Bessel beamformer adjusts itself automatically for higher throughput in terms of gain towards desired user. The MMSE for both beamformers is zero whereas the maximum MSE is found as 12.029 and 2.925 dB, respectively. Therefore the performance of the modified Bessel beamformer is better than that of the Bessel beamformer without AGC.

### 4.4. Receiver Performance

In both cases, i.e. in the proposed beamformer and [1], the receiver performance is judged by the recovery of the original desired signal. It utilizes an adaptive equalizer/filter known as the *affine projection adaptive filter*, which extracts desired signal i.e. streams of bits are filtered in order to remove higher frequency contents leaving only original data/signal as shown in Fig. 5. It is a true copy of transmitted message signal shown in Fig. 2.

### 4.5. Bit Error Rate Performance

BER is one of the important parameters to assess a communication system that transmits digital data from a transmitter to a receiver. BER curves display the required SNR, usually expressed in Eb/No, necessary to achieve a specified BER at the receiver. The comparison is made between the BER obtained through the semianalytic technique and the theoretical BER for the smart antenna array operating in the Rayleigh fading channel environment. The semianalytic technique is one of the ways to compute error rates. For certain types of systems, the semianalytic technique can produce results much

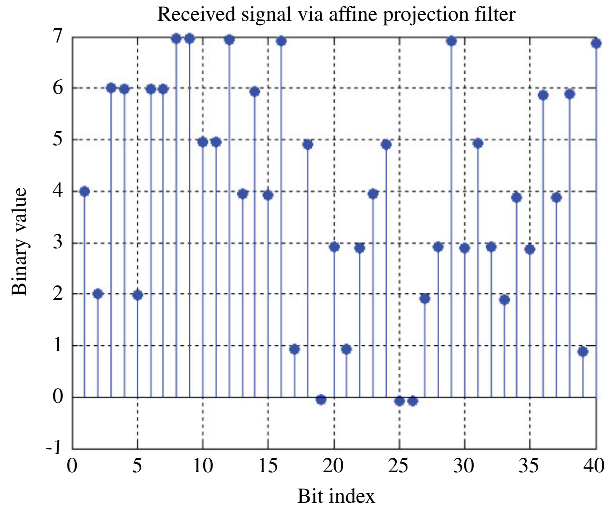


Fig. 5. Original signal recovered by receiver

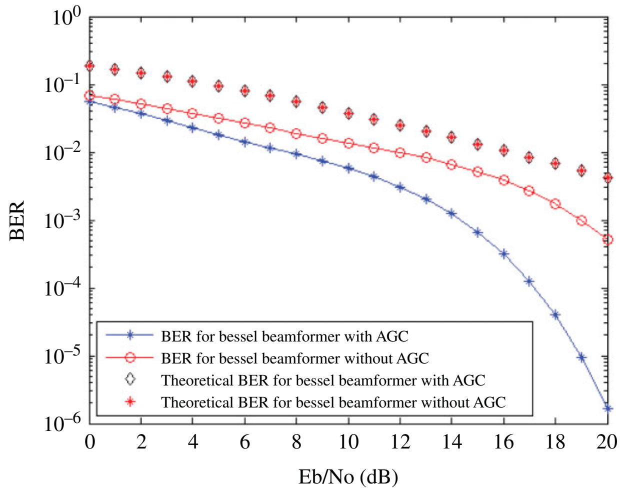


Fig. 6. Comparison between the calculated and theoretical BER

more quickly than a nonanalytic method that uses only simulated data. The semianalytic technique uses a combination of simulation and analysis to determine the error rate of a communication system, whereas theoretical data is useful for comparison. The plot shown in Fig. 6 shows that the BERs obtained using the semianalytic technique are less than the theoretical BER for both algorithms under study. However, the measured values of BER for the proposed beamformer are minimum as compared to published beamformer in Ref. [1]. The data extracted from Fig. 6 are given in the Table II for comparison. It can be seen that the proposed system is better than the one published in Ref. [1]. This is because of the self-adjustment property (that is AGC), which is not present in the case of [1]. Thus the proposed digital system under study is better for wireless communication. The BER performance curves shown in Fig. 6 show that the Bessel beamformer with AGC performs better than the Bessel beamformer without AGC.

## 5. Discussion and Comments

The findings of the simulation are as follows:

1. The modified Bessel beamformer has slightly more directive gain (20.0018 dB) as seen from Table I. However, this directive gain is achieved automatically with the help of AGC and adjusts itself automatically for higher throughput in terms of gain toward desired user, whereas in case of the Bessel

Table II. Effect of SNR on the bit error rate

| Parameters for comparison | | Bessel beamformer with AGC | Bessel beamformer without AGC |
|---|---|---|---|
| SNR (dB) | BER theory | Measured BER | Measured BER |
| 0 | 0.1888 | 0.0567 | 0.0686 |
| 2 | 0.1481 | 0.0370 | 0.0515 |
| 4 | 0.1113 | 0.0230 | 0.0377 |
| 6 | 0.0803 | 0.0143 | 0.0269 |
| 8 | 0.0559 | 0.0092 | 0.0190 |
| 10 | 0.0377 | 0.0057 | 0.0136 |
| 12 | 0.0249 | 0.0030 | 0.0098 |
| 14 | 0.0162 | 0.0012 | 0.0066 |
| 16 | 0.0104 | 0.0003 | 0.0038 |
| 18 | 0.0066 | 3.9539e-05 | 0.0017 |
| 20 | 0.0042 | 1.6457e-06 | 0.0005 |

beamformer [1] the requirement of adjustment of the tap weights is done by trial and error with a constant step size. Therefore the performance of modified Bessel beamformer is better than that of the Bessel beamformer.

2. Both the algorithms under study give −23.0 dB null depth performances as shown in Fig. 3.
3. The modified Bessel beamformer has a better capability to minimize the MSE (0 to 2.925 dB) because of the self-adjustment property, which is missing in case of the Bessel beamformer (0 to 12.029 dB).
4. The modified Bessel beamformer is more accurate and stable because of the AGC development. Therefore, the proposed algorithm is robust and with minimum degradation in performance.
5. The modified Bessel beamformer provides better stability and rate of convergence because of the AGC as compared to the Bessel beamformer reported in Ref. [1].
6. The modified Bessel beamformer saves power in a more efficient manner because of its directional gain and better stability.
7. The BER performance is optimum for the modified Bessel beamformer, as shown in Fig. 6 and Table II. The maximum and minimum BER for the modified Bessel beamformer is 0.0567 and $1.6457 \times 10^{-6}$ whereas the values are 0.0686 and 0.0005 for the Bessel beamformer reported in [1]. This is due to the self-adjustment property, which is missing in case of the Bessel beamformer reported in [1].

## 6. Conclusions

From the above facts and figures, it can be concluded that the performance of the modified Bessel beamformer is superior to that of the Bessel beamformer, and therefore can increase the capacity and quality of mobile communication systems when employed in smart antenna array system.

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