# Supplementary Materials

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This document provides supplementary materials for the paper entitled "Dynamic Security Assessment of Small-Signal Stability for Power Grids using Windowed Online Gaussian Process" by Chao Zhai, Hung D. Nguyen and Xiaofeng Zong. It includes the following four parts: GP regression and Lyapunov function, solutions to the issue of fast dynamics, information flow of online DSA scheme, and the identification of quasi-equilibrium.

## 1 GP Regression and Lyapunov Function

Essentially, a Gaussian process (GP) model is a probability distribution over possible functions that fit a set of sampling points. Since we have the probability distribution over all possible functions, it is feasible to calculate the means and the variances to indicate confidence level of predictions. By using the training set composed of sampling points and their values of unknown Lyapunov function, the GP regression allows to learn the unknown Lyapunov function $V(\mathbf{x})$, and it is able to provide the quantitative uncertainty assessment on the predictions. Figure 1 presents the estimation of Lyapunov function $V(\mathbf{x})$ with the training set of stable sampling points (i.e., blue crosses) by treating $V(\mathbf{x})$ as a GP. Thanks to the converse Lyapunov theorem, we are able to compute the value of Lyapunov function using the stable state trajectories of the concerned dynamical system without constructing an analytic Lyapunov function. Figure 2 illustrates three sampling points and their respective state trajectories converging to a stable equilibrium, which implies that the three sampling points are within the region of attraction of dynamical system. According to converse Lyapunov theorem [1], the value of Lyapunov function at the sampling

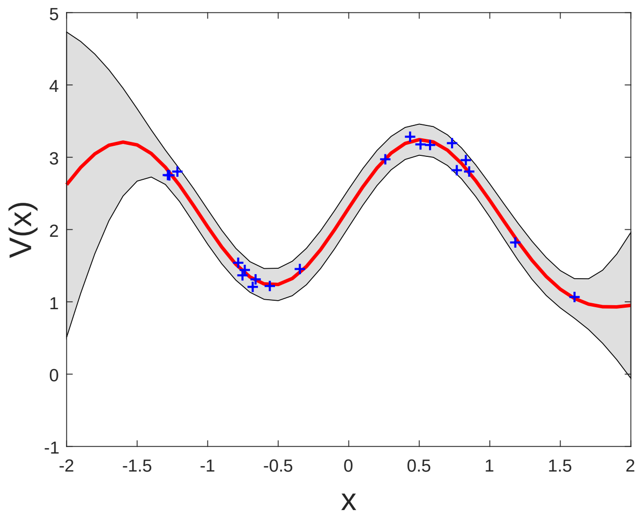


Figure 1: Gaussian process for estimating the value of Lyapunov function. The blue crosses denote the sampling points to train the GP model. The red curve refers to mean value of estimated Lyapunov function, and the gray region represents its standard deviation with the confidence level of 95%.

point $\mathbf{x}$ can be computed by

$$V(\mathbf{x}) = \int_0^{+\infty} \|\phi(\mathbf{x},t)\|^2 dt,$$

where $\phi(\mathbf{x},t)$ denotes the state trajectory of dynamical system originating from the sampling point $\mathbf{x}$.

# 2 Solutions to the Issue of Fast Dynamics

The fast dynamics of power electronic devices in distribution systems or microgrids may result in frequent fluctuations of power system states (e.g., voltage amplitude, phase angle, frequency), which makes it difficult to collect the stable state trajectories of power grids for the DSA. To solve the above issue, a threshold of disturbance magnitude can be introduced to filter out trivial state trajectories. Figure 3 illustrates the filtering mechanism of trivial state trajectories caused by power electronic devices in power systems. The red dot denotes a stable equilibrium point, and the dashed blue circle refers to the threshold of disturbance magnitude, which is quantified by the distance to the equilibrium. Two stable state trajectories starting at $A$ and $B$ are selected

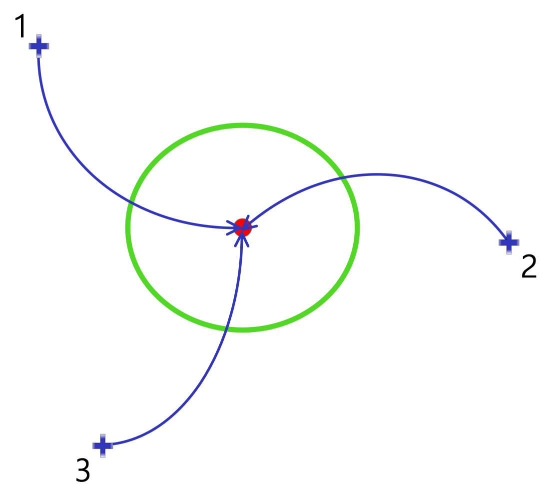


Figure 2: Sampling points and stable state trajectories of a dynamical system. The red dot denotes a stable equilibrium and blue crosses refer to the sampling points, which converge to the stable equilibrium along their respective trajectories. The green circle indicates the certified region of attraction for the dynamical system.

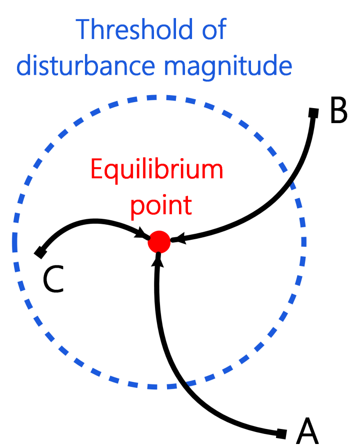


Figure 3: Filtering of trivial state trajectories according to the threshold of disturbance magnitude.

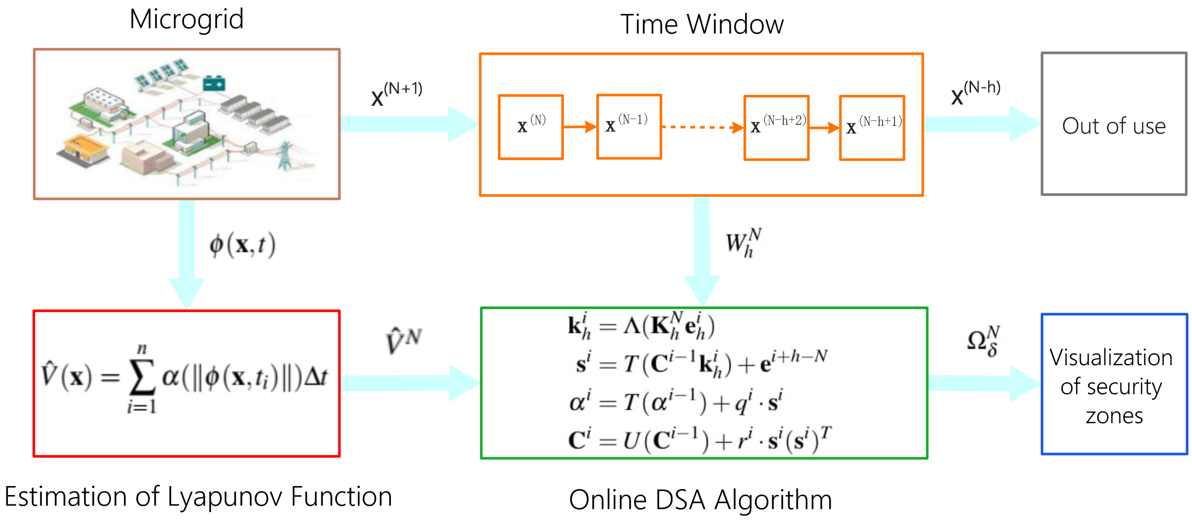


Figure 4: Illustration of online DSA scheme for a smart grid.

as the training data for the online DSA of power grids, because their corresponding disturbance magnitudes are larger than the threshold. In comparisons, the state trajectory starting at $C$ is filtered out as its disturbance magnitude is smaller than the threshold. In other words, all stable state trajectories starting in the dashed blue circle are filtered out, while those starting at the outside are collected for the online DSA. In this way, the sampling rate can be slowed down by adjusting the threshold of disturbance magnitude, so that the proposed online assessment approach is able to cope with the fast dynamics of power electronic devices.

# 3 Information Flow of Online DSA Scheme

Figure 4 illustrates the information flow and data processing for the online DSA. First of all, PMU data are collected from the smart grid. Then the sampling data are sent to a time window $W_h^N$ that can accommodate the latest $h$ sampling points, and the out-of-date sampling points are deleted to characterize the system evolution and reduce computational burdens. For each sampling point $\mathbf{x}^N$, the state trajectory $\phi(\mathbf{x}^N, t)$ is used to estimate the value of Lyapunov function $\hat{V}^N$. The sampling points in the time window are used to train the GP model and implement the online DSA scheme.

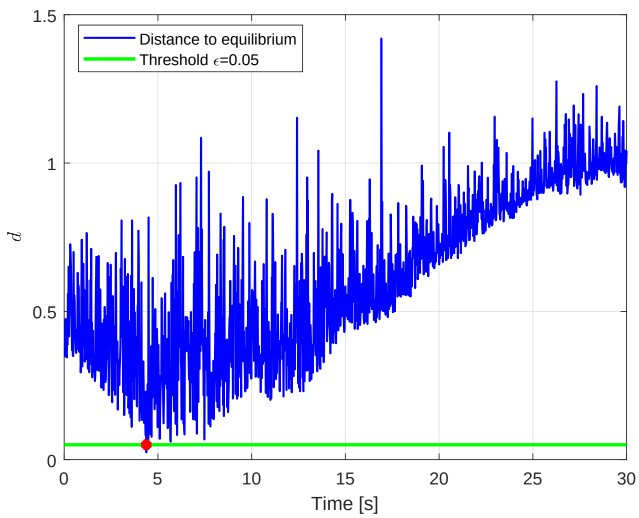


Figure 5: Identification of quasi-equilibrium of smart grid using PMU data.

# 4 Identification of Quasi-equilibrium

The identification of quasi-equilibrium is largely dependent on the threshold $\varepsilon$ and the time interval $I$. By setting the threshold $\varepsilon$ and solving $t^* = \inf\{t : d(t) \leq \varepsilon\}$, the quasi-equilibrium of smart grid can be determined as $(U(t^*), \theta(t^*), \omega(t^*))$. Figure 5 presents the time evolution of the variable $d(t)$ (blue time series) and the threshold (green line), and their cross point is defined as the quasi-equilibrium (red dot). By choosing $\varepsilon = 0.05$ and $I = [0, 30s]$, the quasi-equilibrium of smart grid can be identified as $(12.19\texttt{kV}, 2.46\texttt{rad}, 0.002\texttt{rad/s})$ with $t^* = 4.38$s.

# References


[1] H. Khalil, and J. Grizzle, Nonlinear Systems, Upper Saddle River, NJ: Prentice Hall, 2002.