



RPNG

Visual-Inertial State Estimation

Guoquan (Paul) Huang

Mechanical Engineering
Electrical and Computer Engineering
Computer and Information Sciences
University of Delaware

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Motivating Example: AR/VR



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And more...

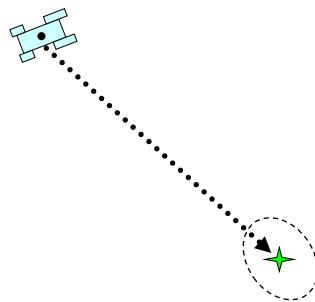
- Camera is an ideal aiding-source for *inertial navigation* in GPS-denied environments:
 - Light-weight, low-cost, but providing rich info.
- Numerous potential applications:
 - Autonomous driving
 - Emergence response (e.g., in Fukushima)
 - Situational awareness
 - Visually impaired navigation



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Simultaneous Localization and Mapping (SLAM)

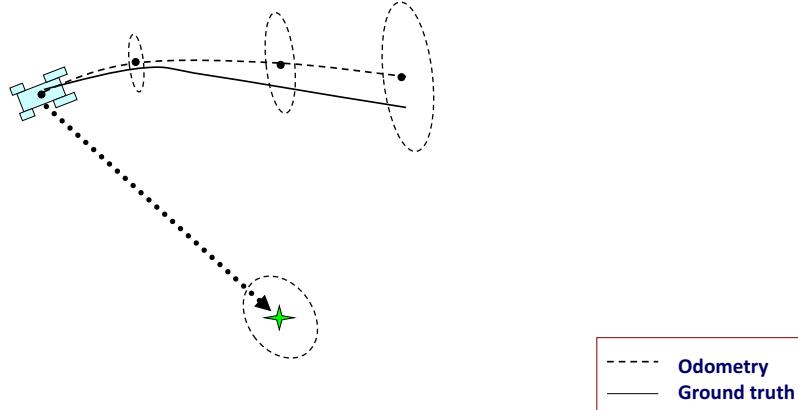
- Detect landmarks (features) in the environment
- Jointly estimate landmark positions and robot pose



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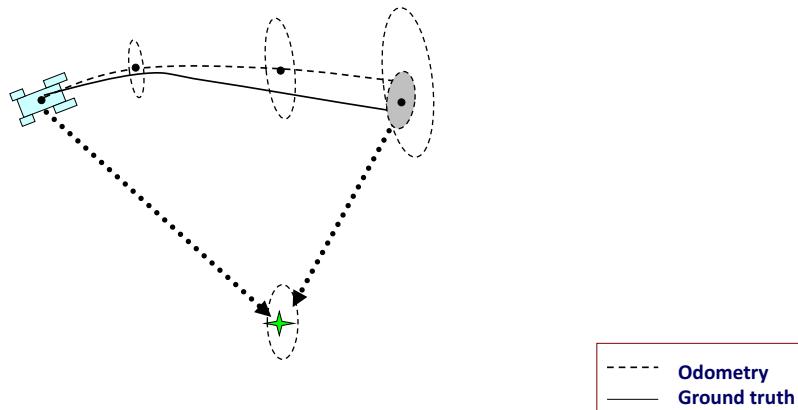
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Simultaneous Localization and Mapping (SLAM)

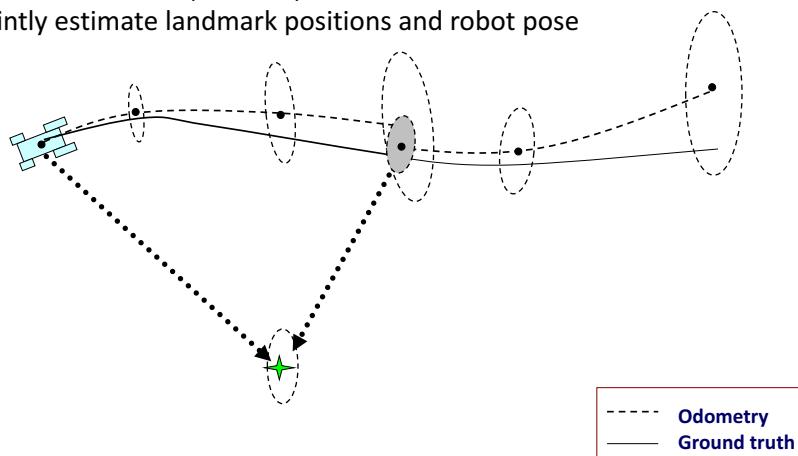
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Simultaneous Localization and Mapping (SLAM)

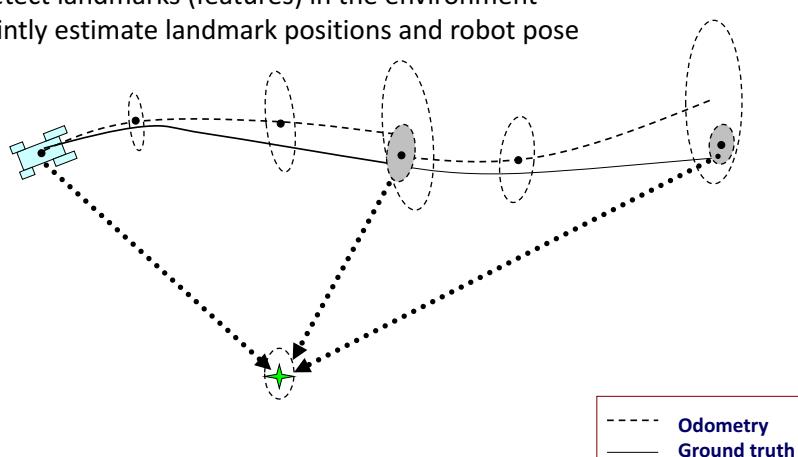
- Detect landmarks (features) in the environment
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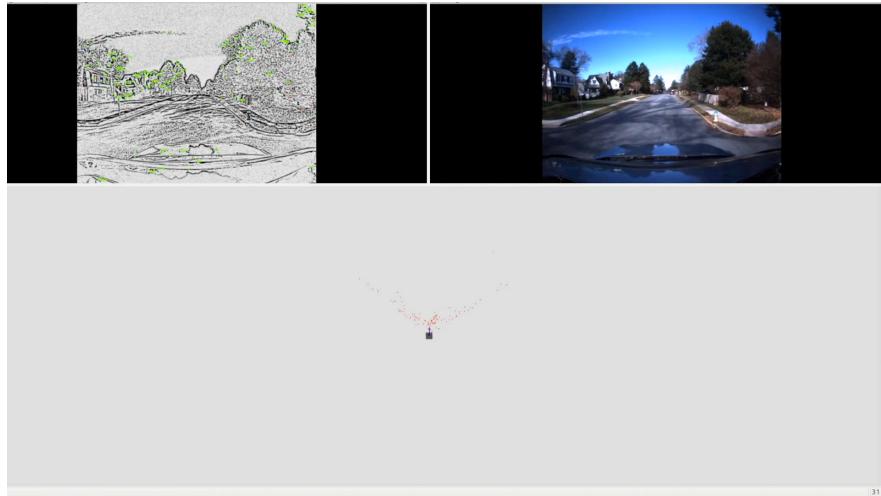
Simultaneous Localization and Mapping (SLAM)

- Detect landmarks (features) in the environment
- Jointly estimate landmark positions and robot pose



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SLAM in Action [IROS 18a]



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State Estimation

- Nonlinear estimation:
 - Nonlinear system: $\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{w}_k)$ $\mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q}_k)$
 - Nonlinear measurement: $\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k$ $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{R}_k)$
- Filtering: posterior pdf $p(\mathbf{x}_k | \mathbf{z}_{0:k})$
 - Linearized filters:
 - Extended Kalman Filter (EKF)
 - Unscented Kalman Filter (UKF)
 - Nonparametric method: Particle Filter (PF)
- Smoothing:
 - Sliding Window Filter (SWF): $p(\mathbf{x}_{k-N:k} | \mathbf{z}_{0:k})$
 - Batch Maximum A Posteriori (MAP) Estimator: $\hat{\mathbf{x}}_{0:k|k} = \arg \max p(\mathbf{x}_{0:k} | \mathbf{z}_{0:k})$

Minimum Mean Square Error (MMSE) Estimator:
 $\hat{\mathbf{x}}_{k|k} = \mathbb{E}(\mathbf{x}_k | \mathbf{z}_{0:k})$

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SLAM: A State Estimation Problem

- State vector (1 landmark):

$$\mathbf{x}_k = [\mathbf{x}_{R_k}^T \quad \mathbf{p}_L^T]^T = [\mathbf{p}_{R_k}^T \quad \phi_k \quad \mathbf{p}_L^T]^T \in \mathcal{R}^{5 \times 1}$$

- Motion model:

$$\begin{aligned} \mathbf{p}_{R_{k+1}} &= \mathbf{p}_{R_k} + \mathbf{C}(\phi_k) {}^{R_k} \delta \mathbf{p}_k \\ \phi_{k+1} &= \phi_k + \delta \phi_k \\ \mathbf{p}_{L_{k+1}} &= \mathbf{p}_{L_k} \quad \mathbf{u}_k = \{{}^{R_k} \delta \mathbf{p}_k, \delta \phi_k\} = \text{odometry} \end{aligned}$$

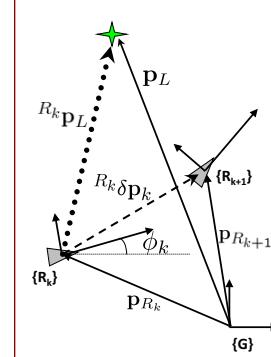
2x2 rotation matrix

$$\Rightarrow \mathbf{x}_{k+1} := f(\mathbf{x}_k, \mathbf{u}_k + \mathbf{w}_k)$$

- Measurement model:

$$\mathbf{z}_k = h({}^{R_k} \mathbf{p}_L) + \mathbf{v}_k = h(\mathbf{x}_k) + \mathbf{v}_k$$

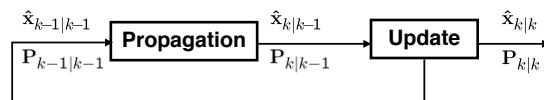
$${}^{R_k} \mathbf{p}_L = \mathbf{C}^T(\phi_k) (\mathbf{p}_L - \mathbf{p}_{R_k})$$



This is a state estimation problem and many different estimators available!

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Extended Kalman Filter (EKF)



$\hat{\mathbf{x}}_{i|j}, \mathbf{P}_{i|j}$: state and covariance estimates at time i given measurements up to time j

- EKF Propagation:

$$\begin{aligned} \hat{\mathbf{x}}_{k|k-1} &= f(\hat{\mathbf{x}}_{k-1|k-1}) \\ \mathbf{P}_{k|k-1} &= \Phi_{k-1} \mathbf{P}_{k-1|k-1} \Phi_{k-1}^T + \mathbf{G}_{k-1} \mathbf{Q}_{k-1} \mathbf{G}_{k-1}^T \end{aligned}$$

- EKF Update:

$$\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{S}_k^{-1}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k (\mathbf{z}_k - h(\hat{\mathbf{x}}_{k|k-1}))$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T$$

$$\begin{aligned} \Phi_{k-1} &= \nabla_{\mathbf{x}_{k-1}} f \Big|_{\{\hat{\mathbf{x}}_{k-1|k-1}, 0\}} \\ \mathbf{G}_{k-1} &= \nabla_{\mathbf{w}_{k-1}} f \Big|_{\{\hat{\mathbf{x}}_{k-1|k-1}, 0\}} \end{aligned}$$

$$\mathbf{H}_k = \nabla_{\mathbf{x}_k} h \Big|_{\hat{\mathbf{x}}_{k|k-1}}$$

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Observability Analysis

- Observability:
 - Given inputs/outputs of a dynamical system, to determine initial state.
 - For SLAM system, inputs typically include proprioceptive measurements (e.g., IMU, odometer), and outputs are exteroceptive measurements (e.g., LiDAR, camera).
 - Provides system insights about which state variables are *possible* to estimate and which not.
- Nonlinear observability analysis:
 - Lie derivative-based method [Hermann and Krener 1977]
- Linear observability analysis:
 - Observability matrix formed w/ system and measurement Jacobians:

$$\mathbf{M} = \begin{bmatrix} \mathbf{H}_{k_o} \\ \mathbf{H}_{k_o+1} \Phi_{k_o} \\ \vdots \\ \mathbf{H}_{k_o+m} \Phi_{k_o+m-1} \cdots \Phi_{k_o} \end{bmatrix}$$

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Observability-Constrained (OC)-EKF SLAM [IJRR 10]

- Sydney Victoria Park:
 - 4-wheeled vehicle with GPS, odometer, and laser
 - 4 km trajectory (**26 minutes**): 6969 robot poses and 151 landmarks
 - Real-time performance: OC-EKFs vs. iSAM (state of the art) [Kaess et al. 08]

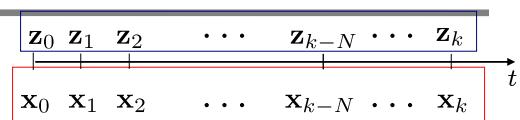


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Maximum A Posteriori (MAP) Estimator

- Maximize the posterior pdf

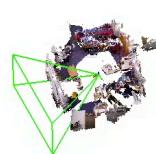
$$\hat{\mathbf{x}}_{0:K|0:K} = \arg \max p(\mathbf{x}_{0:K}|\mathbf{z}_{0:K})$$



$$\begin{aligned}\hat{\mathbf{x}}_{0:K|0:K} = \arg \min & \left(\sum_{k=0}^K \sum_{m=1}^{M_k} \frac{1}{2} \|\mathbf{z}_{km} - h(\mathbf{x}_k)\|_{\mathbf{R}_{km}}^2 \right. \\ & \left. + \sum_{k=0}^{K-1} \frac{1}{2} \|\mathbf{x}_{k+1} - f(\mathbf{x}_k, \mathbf{u}_k)\|_{\mathbf{Q}_k}^2 + \frac{1}{2} \|\mathbf{x}_0 - \mathbf{x}_{init}\|_{\mathbf{P}_0}^2 \right)\end{aligned}$$

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Graph SLAM [ECMR 13a]

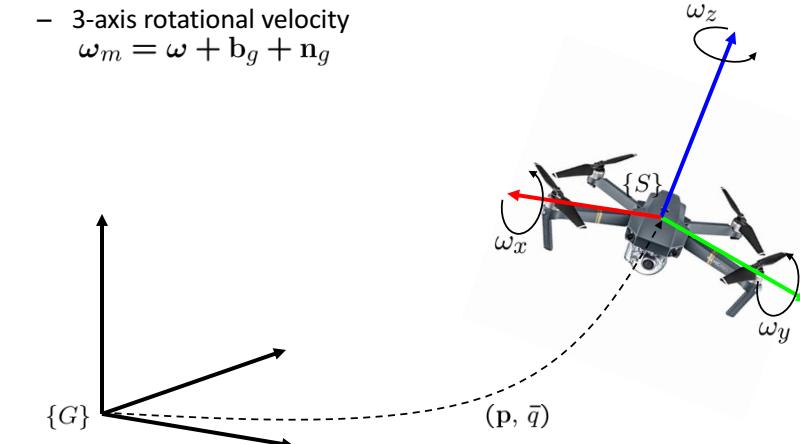


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Visual-Inertial Navigation System (VINS)

- Estimate 6 DOF pose: position \mathbf{p} & orientation $\bar{\mathbf{q}}$
- Inertial Measurement Unit (IMU)
 - 3-axis rotational velocity

$$\omega_m = \omega + \mathbf{b}_g + \mathbf{n}_g$$



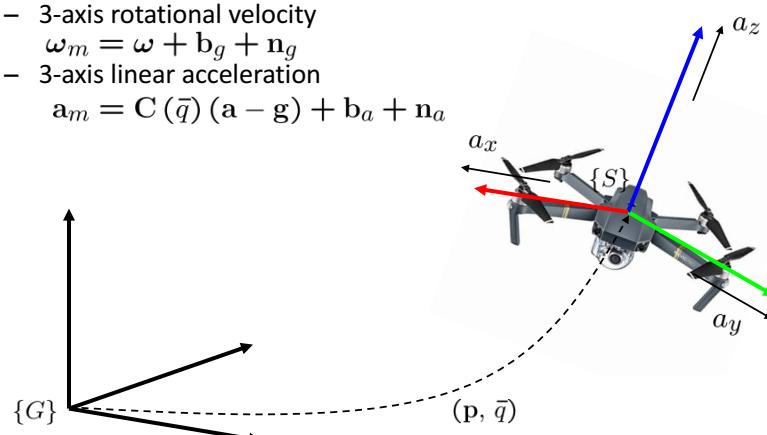
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Visual-Inertial Navigation System (VINS)

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- Inertial Measurement Unit (IMU)
 - 3-axis rotational velocity

$$\omega_m = \omega + \mathbf{b}_g + \mathbf{n}_g$$
 - 3-axis linear acceleration

$$\mathbf{a}_m = \mathbf{C}(\bar{\mathbf{q}})(\mathbf{a} - \mathbf{g}) + \mathbf{b}_a + \mathbf{n}_a$$

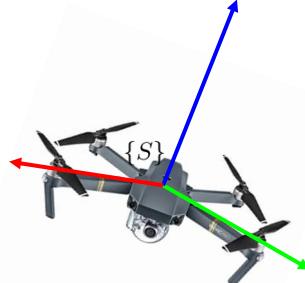


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Visual-Inertial Navigation System (VINS)

- Estimate 6 DOF pose: position \mathbf{p} & orientation $\bar{\mathbf{q}}$
- Inertial Measurement Unit (IMU)
 - 3-axis rotational velocity
 $\omega_m = \omega + \mathbf{b}_g + \mathbf{n}_g$
 - 3-axis linear acceleration
 $\mathbf{a}_m = \mathbf{C}(\bar{\mathbf{q}})(\mathbf{a} - \mathbf{g}) + \mathbf{b}_a + \mathbf{n}_a$
- Kinematic model of motion

$$\begin{aligned}\dot{\bar{\mathbf{q}}} &= \frac{1}{2} \Xi(\bar{\mathbf{q}}) \boldsymbol{\omega} \\ \dot{\mathbf{p}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= \mathbf{a} \\ \dot{\mathbf{b}} &= \mathbf{w}, \quad \mathbf{w} \sim \mathcal{N}(0, \mathbf{Q})\end{aligned}$$

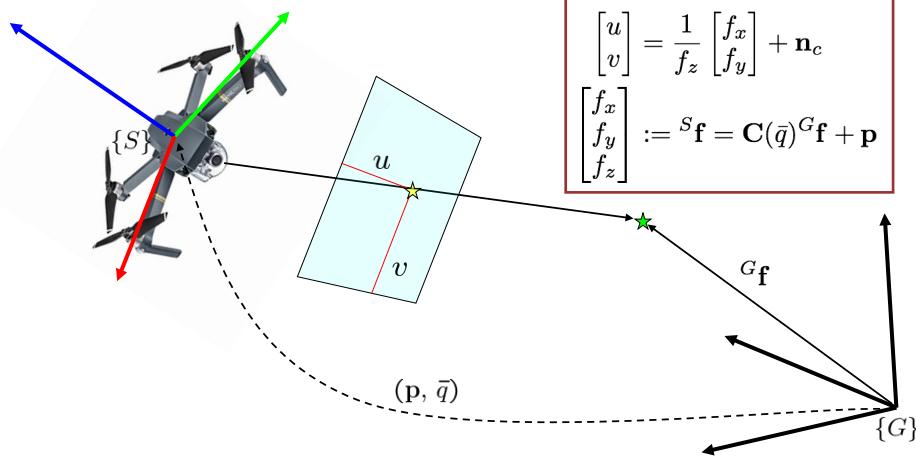


*However, integration of noise & bias causes **large drift** in the pose estimate!*

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Visual-Inertial Navigation System (VINS)

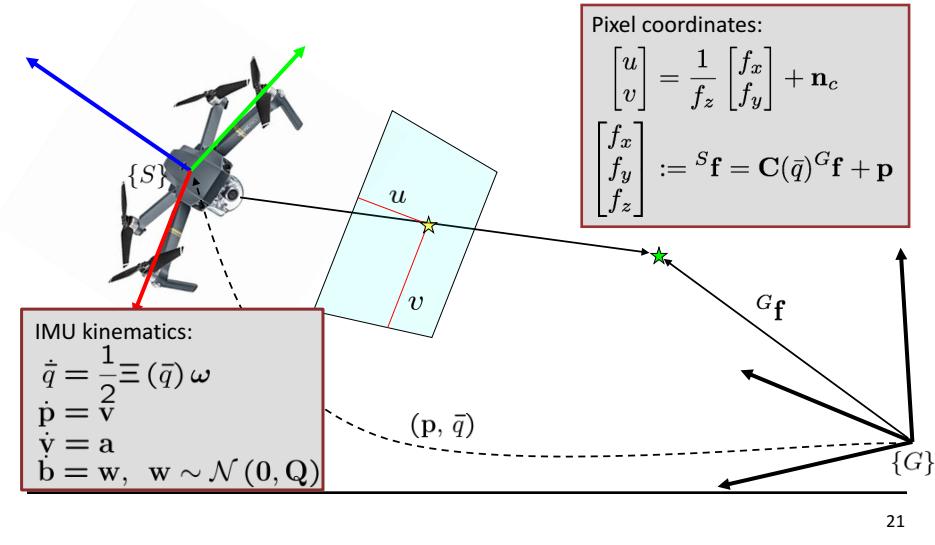
- Use IMU & camera to estimate $\bar{\mathbf{q}}$, \mathbf{p} , \mathbf{v} , \mathbf{b}
- Camera measurement model:



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Visual-Inertial Navigation System (VINS)

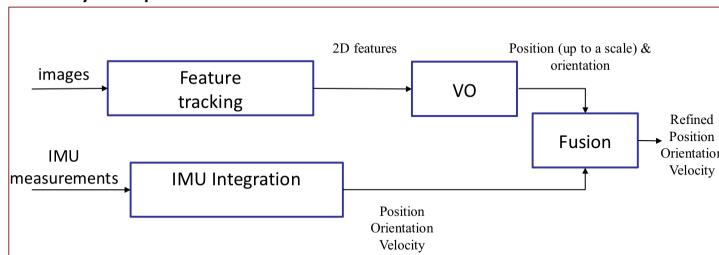
- Use IMU & camera to estimate \bar{q} , \mathbf{p} , \mathbf{v} , \mathbf{b}



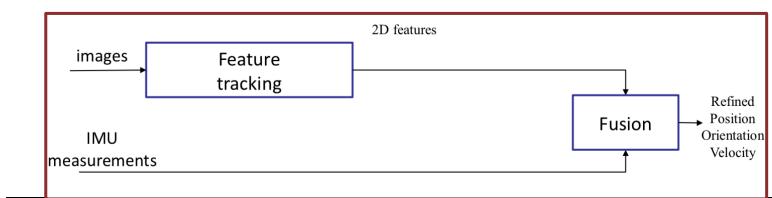
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Loosely vs. Tightly Coupled Estimation

- Loosely coupled estimator:



- Tightly coupled estimator:



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Filtering: MSCKF [Mourikis 2007]

- Key idea of Multi-State Constraints Kalman Filter (MSCKF):
 - Estimate a sliding window of IMU states without keeping features in the state vector, while tracking visual features in the window to provide motion constraints that are used for EKF update.

- State vector: $\mathbf{x} = \begin{bmatrix} {}^I\bar{q}_G^\top, \mathbf{b}_g^\top, {}^G\mathbf{v}_I^\top, \mathbf{b}_a^\top, {}^G\mathbf{p}_I^\top, \mathbf{x}_c^\top \end{bmatrix}^\top$

- IMU propagation:

$$\left. \begin{array}{l} \text{Attitude} \quad {}^I\dot{\bar{q}}_G(t) = \frac{1}{2}\Omega(\omega(t)){}^I\bar{q}_G(t) \\ \text{Velocity-Position} \quad {}^G\dot{\mathbf{p}}_I(t) = {}^G\mathbf{v}_I(t), \quad {}^G\dot{\mathbf{v}}_I(t) = {}^G\mathbf{a}(t) \\ \text{Gyroscope} \quad \omega_m = \omega + \mathbf{b}_g + \mathbf{n}_g \\ \text{Accelerometer} \quad \mathbf{a}_m = \mathbf{C}({}^I\bar{q}_G)({}^G\mathbf{a} - {}^G\mathbf{g}) + \mathbf{b}_a + \mathbf{n}_a \\ \text{Biases} \quad \dot{\mathbf{b}}_g(t) = \mathbf{n}_{wg}(t), \quad \dot{\mathbf{b}}_a(t) = \mathbf{n}_{wa}(t) \end{array} \right\} \Rightarrow$$

$\dot{\hat{\mathbf{x}}} = f(\hat{\mathbf{x}}) \quad \dot{\mathbf{P}} = \mathbf{FP} + \mathbf{PF}^T + \mathbf{Q} \quad \mathbf{F} = \nabla_{\mathbf{x}}f$

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MSCKF Update

- Observation of feature j from camera pose i :

$$\mathbf{z}_i^{(j)} = \frac{1}{C_i Z_j} \begin{bmatrix} C_i X_j \\ C_i Y_j \\ C_i Z_j \end{bmatrix} + \mathbf{n}_i^{(j)}$$

Measurement noise

$$C_i \mathbf{p}_j = \begin{bmatrix} C_i X_j \\ C_i Y_j \\ C_i Z_j \end{bmatrix} = {}^G_C \mathbf{C}({}^G \mathbf{p}_{\ell_j} - {}^G \mathbf{p}_{C_i})$$

Rotation between world and camera frames Feature position Camera position

- Measurement residual:

$$\begin{aligned} \mathbf{r}_i^{(j)} &\simeq \mathbf{H}_{OF_i}^{(j)} \tilde{\mathbf{X}} + \mathbf{H}_{f_i}^{(j)} \mathbf{G} \tilde{\mathbf{p}}_{\ell_j} + \mathbf{n}_i^{(j)} \Rightarrow (\text{stack all residuals of feature } j) \\ \mathbf{r}^{(j)} &\simeq \mathbf{H}_{OF}^{(j)} \tilde{\mathbf{X}} + \mathbf{H}_f^{(j)} \mathbf{G} \tilde{\mathbf{p}}_{\ell_j} + \mathbf{n}^{(j)} \end{aligned}$$

State Error Feature position error

- Project onto **left nullspace** of $\mathbf{H}_f^{(j)}$ to remove effect of feature position error

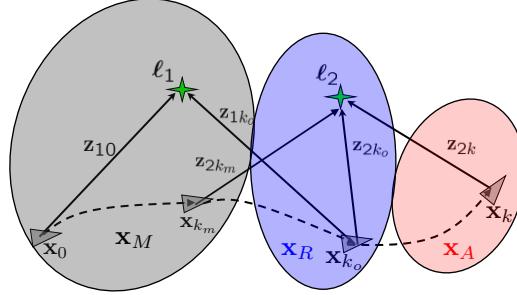
$$\mathbf{r}_o^{(j)} = \mathbf{U}^T \mathbf{r}^{(j)} = \mathbf{H}_o^{(j)} \tilde{\mathbf{X}} + \mathbf{n}_o^{(j)}$$

- Perform EKF update

- Used in Google ARCore (and will be in next Oculus devices)

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Smoothing: Sliding Window BA



- Formulate as a MAP (or nonlinear least squares) optimization:

$$\min_{\{\mathbf{x}_i, \ell_j\}} \sum_{i,j} \|\mathbf{z}_{ij} - h(\mathbf{x}_i, \ell_j)\|_{\mathbf{R}_{ij}}^2 + \sum_k \|\mathbf{x}_k - f(\mathbf{x}_{k-1}, \mathbf{u}_m)\|_{\mathbf{Q}_k}^2$$

Reprojection residuals
IMU residuals

- Visual tracking [Lucas 1981], IMU preintegration [IJRR 19a], marginalization [IROS 11]
- Iterative solvers such as G-N and L-M
- Examples: OKVIS [Leutenegger et al. 2014], VINS-Mono [Qin et al. 2018]

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VINS Review [ICRA 19]

2019 International Conference on Robotics and Automation (ICRA)
Palais des congrès de Montréal, Montréal, Canada, May 20-24, 2019

Visual-Inertial Navigation: A Concise Review

Guoquan Huang

Abstract—As inertial and visual sensors are becoming ubiquitous, visual-inertial navigation systems (VINS) have prevailed in a wide range of applications from mobile augmented reality to aerial navigation to autonomous driving, in part because of the complementary sensing capabilities and the decreasing costs and size of the sensors. In this paper, we survey thoroughly the research efforts taken in this field and strive to provide a concise but complete review of the related work – which is unfortunately missing in the literature while being greatly demanded by researchers and engineers – in the hope to accelerate the VINS research and beyond in our society as a whole.

I. INTRODUCTION

Over the years, inertial navigation systems (INS) [1, 2] have been widely used for estimating the 6DOF poses (positions and orientations) of sensing platforms (e.g., autonomous vehicles), in particular, in GPS-denied environments such as underwater, indoor, in the urban canyon, and on other planets. Most INS rely on a 6-axis inertial measurement unit (IMU) that measures the local linear acceleration and angular velocity of the platform to which it is rigidly connected. With the recent advancements of hardware design and manufacturing, low-cost light-weight micro-electro-mechanical (MEMS) IMUs have become ubiq-

uitous. As inertial and visual sensors are becoming ubiquitous, visual-inertial navigation systems (VINS) have prevailed in a wide range of applications from mobile augmented reality to aerial navigation to autonomous driving, in part because of the complementary sensing capabilities and the decreasing costs and size of the sensors. In this paper, we survey thoroughly the research efforts taken in this field and strive to provide a concise but complete review of the related work – which is unfortunately missing in the literature while being greatly demanded by researchers and engineers – in the hope to accelerate the VINS research and beyond in our society as a whole.

As evident, VINS technologies are emerging, largely due to the demanding mobile perception/navigation applications, which has given rise to a rich body of literature in this area. However, to the best of our knowledge, there is no contemporary literature review of VINS, although there are recent surveys broadly about SLAM [16, 36] while not specializing on VINS. This has made difficult for researchers and engineers in both academia and industry, to effectively find and understand the most important related work to their interests, which we have experienced over the years when we are working on this problem. For this reason, we are striving to bridge this gap by: (i) offering a concise (due to space limitation but complete) review of VINS with

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VINS Workshop @ IROS 2019

Visual-Inertial Navigation: Challenges and Applications

IROS 2019 Full-day Workshop: November 8, 2019, Macau, China



Updates

- (7/3) We are pleased to announce that [Lord Sensing](#) will sponsor the [Lord Best Paper Award](#) (with MicroStrain IMUs as prize)
- (7/2) Please prepare papers following the IROS template available in [LaTeX](#) and [MS Word](#).
- (7/1) We are pleased to announce that there will be a [Special Issue in the IET Cyber-systems and Robotics](#), which will invite some of best papers presented at this workshop.

Overview

As cameras and IMUs are becoming ubiquitous, visual-inertial navigation systems (VINS) estimation, hold great potentials in a wide range of applications from augmented reality to driving, in part because of the complementary sensing capabilities and the decreasing cost. Inertial navigation, alongside with SLAM, has witnessed tremendous progress in the past. However, visual-inertial systems remain poorly explored, greatly hindering the widespread deployment. For example, many VINS algorithms are yet not robust to high dynamics and poor lighting or long-term, large-scale operations, in particular, in life-critical scenarios; and yet they are understood to support high-level decision making. This workshop brings together researchers from both academia and industry, to share their insights and thoughts on the R&D of VINS forward the latest breakthroughs and cutting-edge research on visual-inertial navigation, technical challenges and future research directions for the community, and to identify new opportunities.

Call for Contributions

We welcome submissions of papers describing VINS-related work in progress, preliminary experiences. All submitted papers will be reviewed by at least two experts (see Program Committee).

Organizers

- [Guoquan \(Paul\) Huang](#), University of Delaware
- [Shaojie Shen](#), Hong Kong University of Science and Technology
- [Michael Kaess](#), CMU
- [Stergios Roumeliotis](#), University of Minnesota
- [John Leonard](#), MIT

Invited Speakers and Panelists (confirmed)

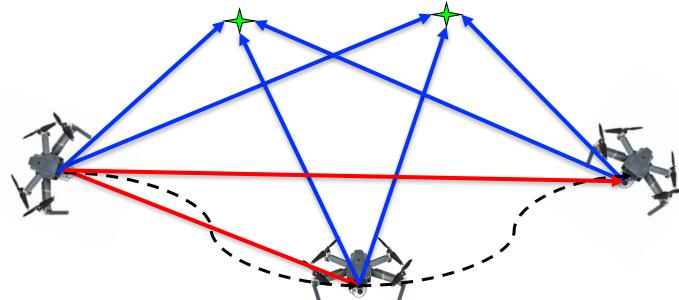
- [Luca Carlone](#), MIT
- [Jakob Engel](#), Facebook Oculus
- [Maurice Fallon](#), Oxford
- [Jonathan Kelly](#), University of Toronto
- [Giuseppe Loiacono](#), NYU
- [Laurent Kneip](#), Shanghai Tech
- [Davide Scaramuzza](#), University of Zurich
- [Guofeng Zhang](#), Zhejiang University
- [Ross Hartley](#), Amazon (University of Michigan)

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Visual-Inertial Estimation Research @ My Lab (RPNG)

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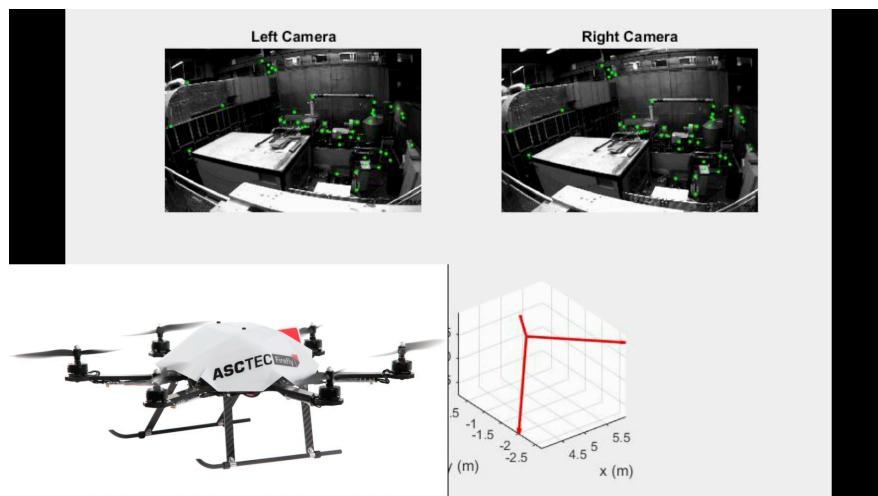
Optimal-State-Constraint (OSC)-EKF [ISRR 15; ARL 17]



- Key ideas:
 - Process a sliding window of images *only* to infer **optimal** state constraints btw. corresponding camera poses
 - Perform EKF propagation using IMU measurements, and EKF update using the inferred state constraints
 - As a result, *no* need to store features in the state vector, yielding *constant* complexity

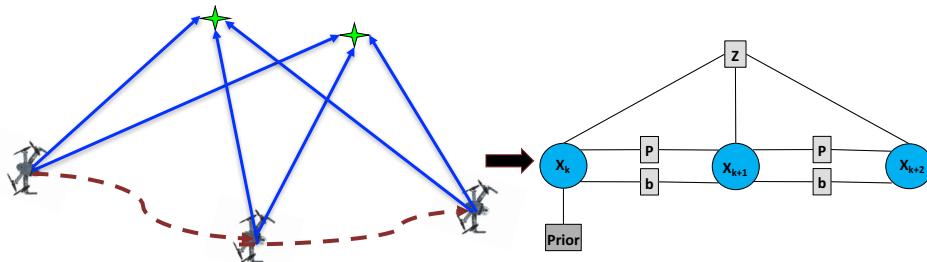
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Results: OSC-EKF VIO



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Closed-form Preintegration (CPI) for Graph-VINS [WAFR 16; IJRR 19a]

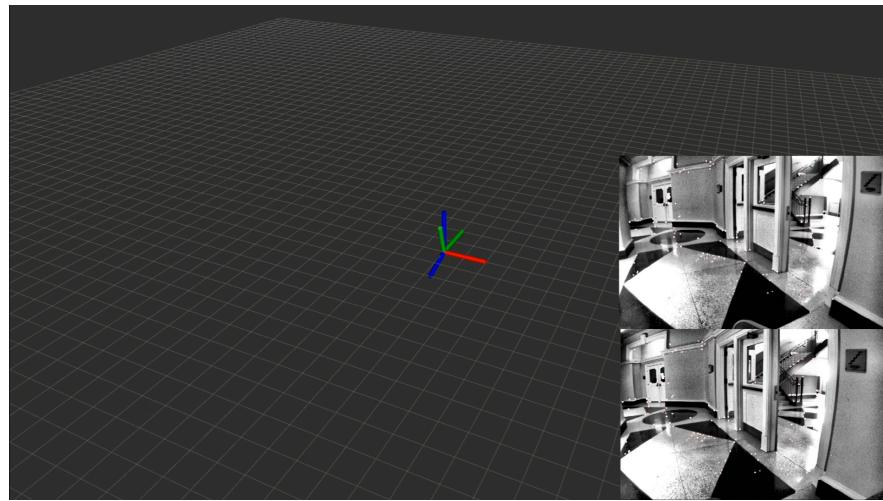


- Sliding-window BA to *tightly* fuse visual and inertial measurements
 - Analytical continuous-time IMU preintegration: (i) piecewise constant measurement, and (ii) piecewise constant local acceleration
 - Perform bundle adjustment (BA) to optimally estimate a sliding window of states and features detected
 - Marginalize out features to bound computational complexity

Open source: <https://github.com/rpng/cpi>

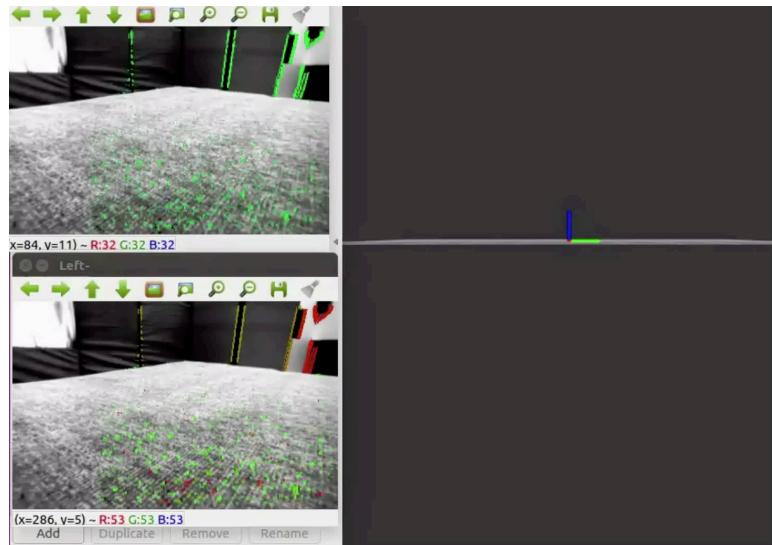
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Results: Graph-VINS [IJRR 19a]



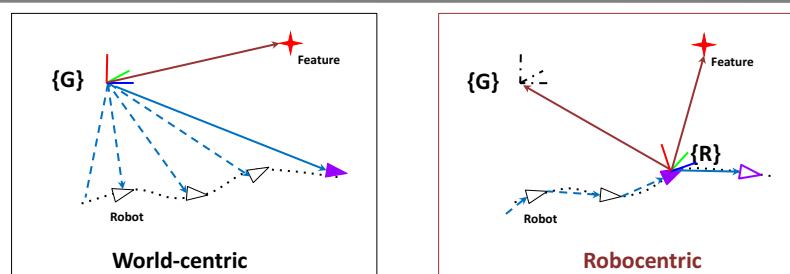
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Direct VINS [ICRA 17a, IJRR19a]



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Robocentric VIO [IROS 18a, IJRR19b]



- Robocentric VIO (R-VIO) within the MSCKF framework:
 - State includes local gravity and a sliding window of relative poses
 - Close-form IMU preintegration (for propagation)
 - Inverse depth-based measurement model (for update)
 - Composition is employed to shift robocentric frame after update

Open source: <https://github.com/rpng/r-vio>

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Results: R-VIO [IJRR 19b]

Robocentric Visual-Inertial Navigation

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Results: R-VIO [IJRR 19b]

Robocentric Visual-Inertial Odometry

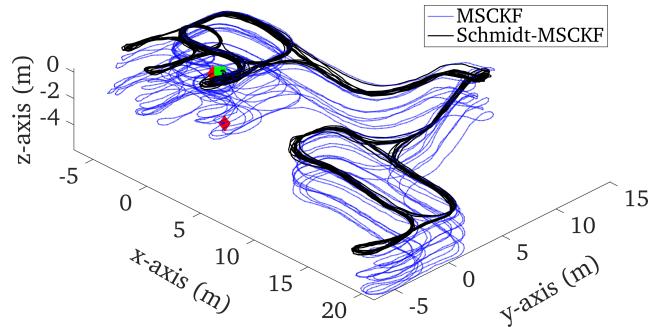
Zheng Huai and Guoquan Huang

Robot Perception and Navigation Group (RPNG)
University of Delaware

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Schmidt MSCKF for VINS w/ Loop Closures [ICRA 19c]

- A novel linear-complexity algorithm for VINS with loop closures:
 - Exploit Schmidt-KF for real-time consistent inclusion of old keyframes by only updating their cross-correlations
 - Leverage MSCKF nullspace-based marginalization, allowing for efficient processing measurements of keyframe-based loop-closures



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Results: Schmidt MSCKF [ICRA 19c]

A Linear-Complexity EKF for
Visual-Inertial Navigation with Loop Closures

Patrick Geneva, Kevin Eckenhoff,
and Guoquan Huang

RPNG, University of Delaware, USA

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Schmidt-EKF VI-SLAM [CVPR 19]



Active Feature Tracks:

Current image with feature tracks shown.

Features that reach max track length become SLAM features, while features that are lost are used as VIO feature update.

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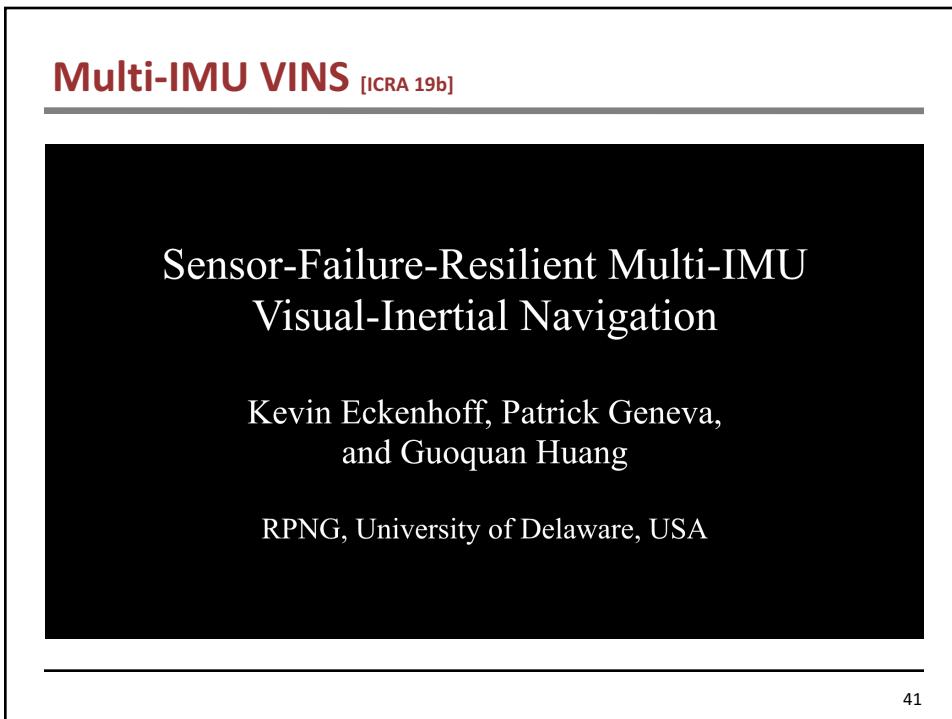
Multi-Camera VINS [ICRA 19a]

Multi-Camera Visual-Inertial Navigation
with Online Intrinsic and Extrinsic Calibration

Kevin Eckenhoff, Patrick Geneva,
Jesse Bloecker, and Guoquan Huang

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Observability Analysis and Representations [ICRA 19f; TRO 19]						
Model #	Point	Error states	Line	Error states	Plane	Error states
1: General Form	f_1, f_2, f_3, f_4	not minimal	$\mathbf{n}_l, \mathbf{v}_l$	$\delta\theta_l, \delta\phi_l$	$\pi_1, \pi_2, \pi_3, \pi_4$	not minimal
2: Geometric Form	\mathbf{b}_f, r_f	not minimal	$\mathbf{n}_e = \frac{\mathbf{n}_l}{\ \mathbf{n}_l\ }$ $\mathbf{v}_e = \frac{\mathbf{v}_l}{\ \mathbf{v}_l\ }$ $d_e = \frac{\ \mathbf{n}_l\ }{\ \mathbf{v}_l\ }$	not minimal	\mathbf{n}_π, d_π	not minimal
3: Spherical Form	θ_f, ϕ_f, r_f	$\tilde{\theta}_f, \tilde{\phi}_f, \tilde{r}_f$	$\theta_l, \phi_l, \alpha_l, d_l$	$\tilde{\theta}_l, \tilde{\phi}_l, \tilde{\alpha}_l, \tilde{d}_l$	$\theta_\pi, \phi_\pi, d_\pi$	$\tilde{\theta}_\pi, \tilde{\phi}_\pi, \tilde{d}_\pi$
4: Inverse Depth	$\theta_f, \phi_f, \lambda_f = \frac{1}{r_f}$	$\tilde{\theta}_f, \tilde{\phi}_f, \tilde{\lambda}_f$	$\theta_l, \phi_l, \alpha_l, \lambda_l = \frac{1}{d_l}$	$\tilde{\theta}_l, \tilde{\phi}_l, \tilde{\alpha}_l, \tilde{\lambda}_l$	$\theta_\pi, \phi_\pi, \lambda_\pi = \frac{1}{d_\pi}$	$\tilde{\theta}_\pi, \tilde{\phi}_\pi, \tilde{\lambda}_\pi$
5: Quaternion	$\bar{q}_f = \frac{1}{\sqrt{1+r_f^2}} \begin{bmatrix} \mathbf{b}_f \\ r_f \end{bmatrix}$	$\delta\theta_f$	\bar{q}_l, d_l	$\delta\theta_l, \tilde{d}_l$	$\bar{q}_\pi = \frac{1}{\sqrt{1+d_\pi^2}} \begin{bmatrix} \mathbf{n}_\pi \\ d_\pi \end{bmatrix}$	$\delta\theta_\pi$
6: Closest Point	$\mathbf{p}_f = r_f \mathbf{b}_f$	$\mathbf{p}_f = \bar{\mathbf{p}}_f + \tilde{\mathbf{p}}_f$	$\mathbf{p}_l = d_l \bar{q}_l$	$\mathbf{p}_l = \bar{\mathbf{p}}_l + \tilde{\mathbf{p}}_l$	$\mathbf{p}_\pi = d_\pi \mathbf{n}_\pi$	$\mathbf{p}_\pi = \bar{\mathbf{p}}_\pi + \tilde{\mathbf{p}}_\pi$

Features	Unobservable Directions
Single or multiple points	4
Non-parallel lines	
Planes with non-parallel intersections	
Point and line	
Point and plane	5
Single line non-parallel to planes	
Plane intersections non-parallel to lines	
Point, line and plane	
Single line	
Single line parallel to single plane	7
Two non-parallel planes	
Single plane	

- Unified representations for **points, lines and planes**: (i) quaternion, and (ii) closest point
- Aided INS with combination of all the geometrical features has 4 unobservable directions: global position and global yaw

Observability Analysis and Representations [ICRA 19f; TRO 19]

Aided Inertial Navigation: Unified Feature Representations and Observability Analysis

Yulin Yang and Guoquan Huang

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VINS w/ Point and Plane Features [ICRA 19e]

Tightly-Coupled Aided Inertial Navigation with Point and Plane Features

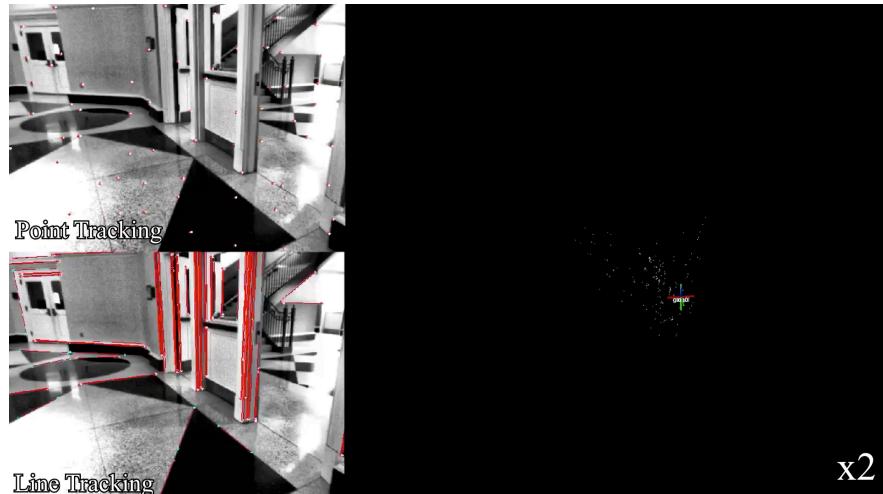
Yulin Yang, Patrick Geneva, Xingxing Zuo,*
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VIO w/ Point and Line Features [IROS 19b]



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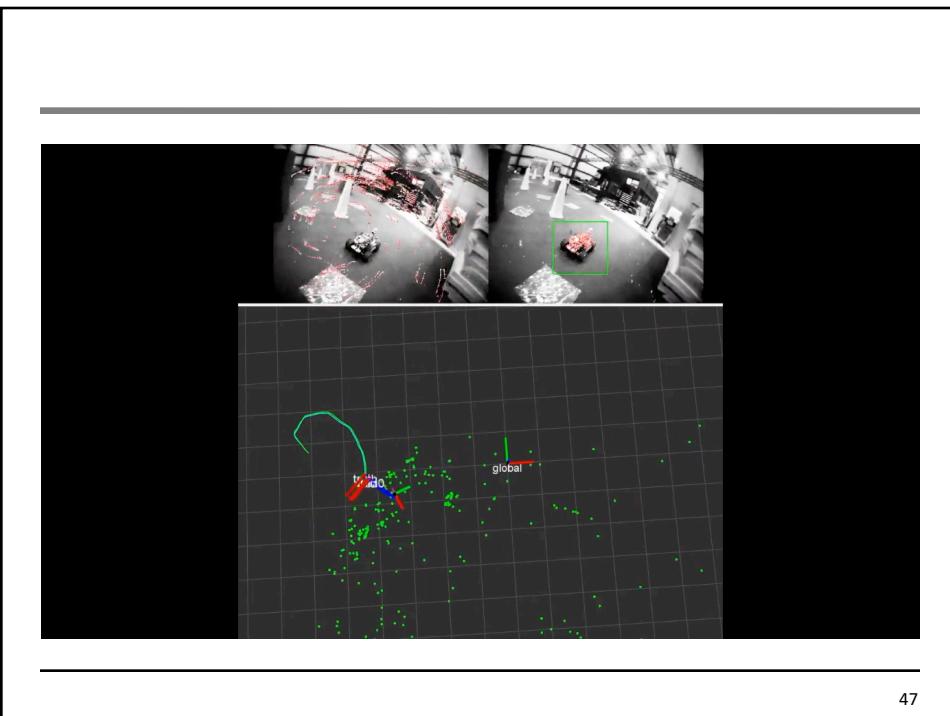
Visual-Inertial Localization & Moving Object Tracking [RAL 19a]

Tightly-Coupled Visual-Inertial Localization
and 3D Rigid-Body Target Tracking

Kevin Eckenhoff, Yulin Yang,
Patrick Geneva, and Guoquan Huang

RPNG, University of Delaware, USA

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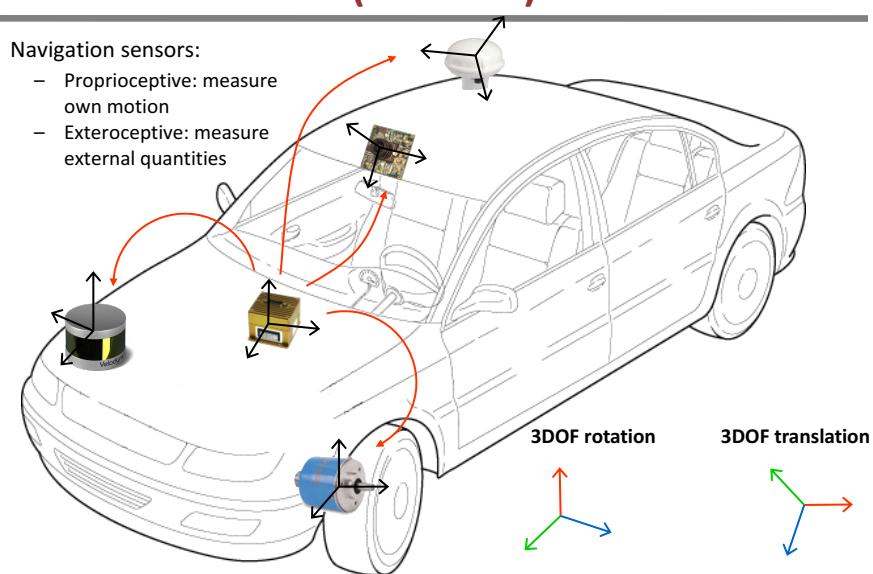


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Sensor Calibration (Extrinsic)

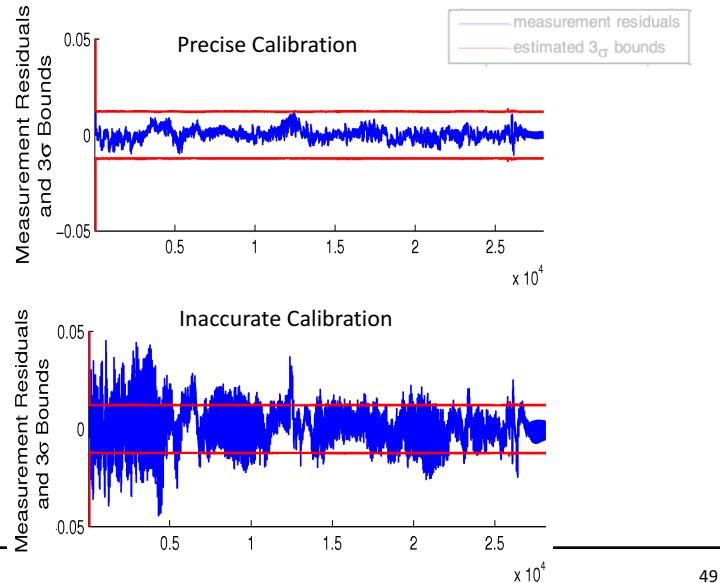
- Navigation sensors:

- Proprioceptive: measure own motion
- Exteroceptive: measure external quantities



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Significance of Sensor Calibration



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Degenerate Motions for Sensor Calibration [RAL 19b]

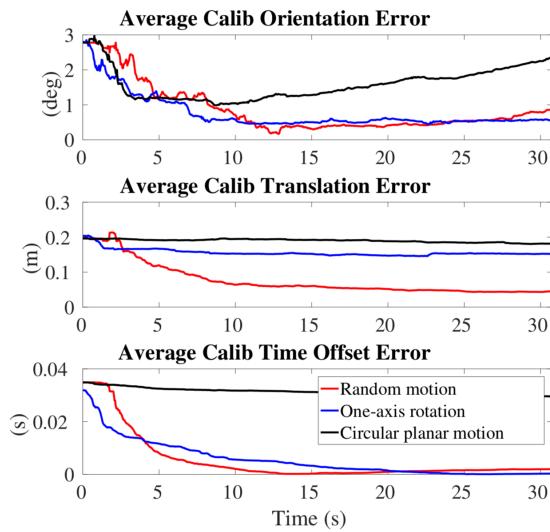
Degenerate Motion Analysis for Aided INS with Online Spatial and Temporal Sensor Calibration

Yulin Yang, Patrick Geneva,
Kevin Eckenhoff, and Guoquan Huang

RPNG, University of Delaware, USA

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Degenerate Motions for Sensor Calibration [RAL 19b]



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LiDAR-Inertial Plane SLAM [IROS 18b]

LIPS: LiDAR Inertial 3D Plane SLAM

Patrick Geneva, Kevin Eckenhoff,
Yulin Yang, and Guoquan Huang

RPNG, University of Delaware, USA

Open source: <https://github.com/rpng/lips>

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VINS w/ Prior LiDAR Map [RAL 19c]

Visual-Inertial Localization with Prior LiDAR Map Constraints

Xingxing Zuo*, Patrick Geneva, Yulin Yang,
Wenlong Ye*, Yong Liu* and Guoquan Huang

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LiDAR-Inertial-Visual Odometry [IROS 19c]

LIC-Fusion: LiDAR-Inertial-Camera Odometry

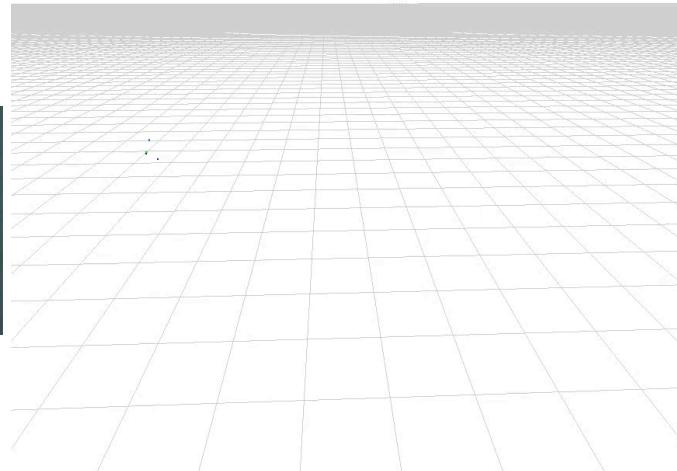
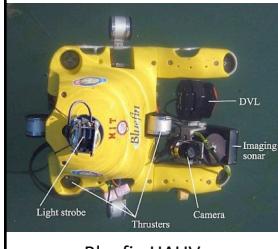
Xingxing Zuo*, Patrick Geneva, Woosik Lee, Yong Liu*, Guoquan Huang

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Underwater SLAM

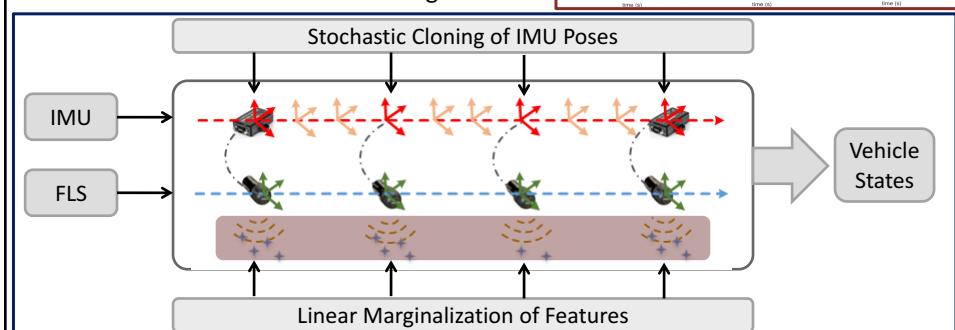
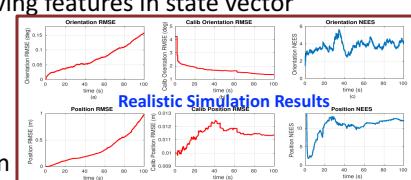


[Joint work with Kaess & Leonard]

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Acoustic-Inertial Navigation System (AINS) [ICRA 17b]

- MSCKF-based acoustic-inertial odometry:**
 - Utilize all measurement info w/o having features in state vector
- Key ideas:**
 - Stochastic cloning
 - EKF update of motion-only meas.
 - Online calibration
 - Linear initialization & marginalization



[Johannsson @ Teledyne shared BlueView MB2250 data]

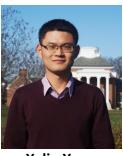
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Summary

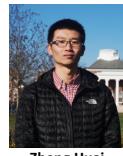
- Visual-inertial estimation is an enabling technology for mobile perception:
 - Current research: state estimation for SLAM/VINS
 - Future research: distributed estimation and perception
- My lab: Robot Perception and Navigation Group (RPNG)
<http://sites.udel.edu/robot> <https://github.com/rpng>



Kevin Eckenhoff



Yulin Yang



Zheng Huai



Patrick Geneva



James Maley



Woosik Lee



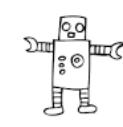
Jesse Bloecker



Nate Merrill

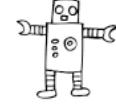


Xingxing Zuo (ZJU)



Chuchu Chen

We're hiring!



- Funded by NSF, DTRA, ARL, NASA, Huawei, Google, Bosch, JRE/MIT, etc.

Thank you!

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