山东大学计算机科学与技术学院

机器学习与模式识别课程实验报告

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实验题目: Regularization

实验目的:

实现加入正则项的线性回归和逻辑回归。

硬件环境:

DELL 台式机

软件环境:

MATLAB R2018b

实验步骤与内容:

 Regularized Linear Regression 用 5 阶多项式进行拟合

$$h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4 + \theta_5 x^5$$

与非正则化线性回归相比, $J(\theta)$ 多了一个"惩罚项":

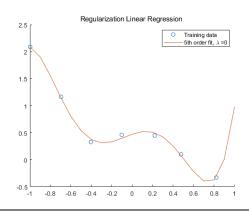
$$J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^{m} (h_{\theta}(x^{(i)} - y^{(i)})^{2} + \lambda \sum_{i=1}^{n} \theta_{j}^{2}) \right]$$

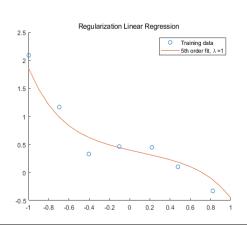
此时,

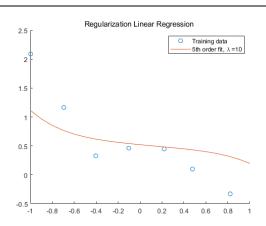
$$\theta = (x^T x + \lambda A)^{-1} x^T y$$

其中 A 为(n+1) 阶对角方阵,左上角为 0,其他对角元素为 1.

分别取 $\lambda = 0.1,10$, 得到拟合结果如下:

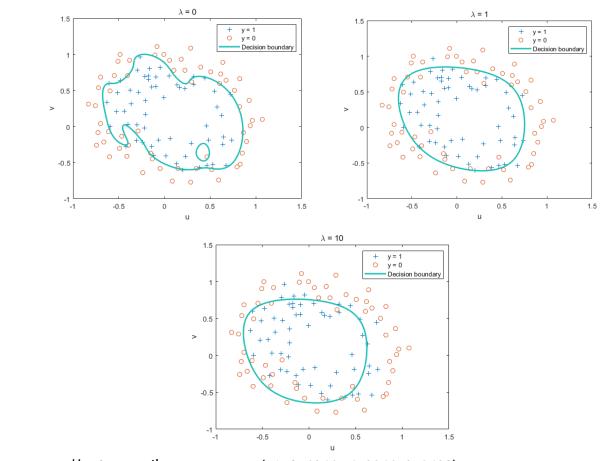






三次 theta 的 L2-norm 为 norm_theta=(8.1687,0.8098,0.5931)。

2. Regularized Logistic Regression 与正则化在线性回归中的应用步骤类似,在逻辑回归的基础上加入正则项迭代求 θ ,分别取 $\lambda=0,1,10$ 得到分类结果如下:



theta的L2-norm为norm_theta=(7172.6946, 4.0346, 0.8402).

结论分析与体会:

可见,当我们增加参数来使 $J^{(\theta)}$ 第一项的值减小时,正则化项会逐渐增大,从而平衡模型复杂度和拟合程度来避免过拟合问题。正则化项系数 λ 越大,最小化 $J^{(\theta)}$ 的结果更倾向于满足于正则项最小,从而 theta 的 L2-norm 也越小,并且拟合曲线对训练数据拟合得越不精确,如果 λ 过大,也可能导致欠拟合。

附录:程序源代码

```
%Experiment 5: Regularization
%Regularized Linear Regression
x = load('ex5Linx.dat');
y = load('ex5Liny.dat');
m = length(x);
n = 5; %拟合的多项式阶数
x1 = [ones(m, 1), x, x.^2, x.^3, x.^4, x.^5];
d = ones(n+1, 1);
D = diag(d);
D(1, 1) = 0;
lamda = [0, 1, 10];
t = (-1:0.1:1);
T = [ones(length(t), 1), t, t.^2, t.^3, t.^4, t.^5];
norm_theta = zeros(length(lamda), 1);
for i = 1:length(lamda)
   theta = (x1'*x1 + 1amda(i)*D) \x1'*y;
    y1 = T*theta;
    norm_theta(i) = norm(theta);
    figure;
    scatter(x, y);
    hold on
    plot(t, y1)
    legend('Training data', ['5th order fit, \lambda =' num2str(lamda(i))]);
    title('Regularization Linear Regression');
end
%Regularized Logistic Regression
```

```
x = load('ex5Logx.dat');
y = load('ex5Logy.dat');
pos = find(y==1);
neg = find(y==0);
plot(x(pos, 1), x(pos, 2), '+')
hold on
plot(x(neg, 1), x(neg, 2), 'o')
xlabel('u');
ylabel('v');
u = x(:, 1);
v = x(:, 2);
x = map_feature(u, v);
m = 117;
n = 28;
theta = zeros(28, 1);
g = inline('1.0./(1.0+exp(-z))');
lambda = 10;
d = ones(n, 1);
D = diag(d);
D(1, 1) = 0;
for i = 1:10
    grad = 1/m*((g(theta'*x')-y')*x)'+ lambda/m*theta;
    H = 1/m*x'*diag(g(theta'*x').*g(-theta'*x'))*x + lambda/m*D;
    theta = theta - H\grad
end
norm_theta = norm(theta);
u = 1inspace(-1, 1.5, 200);
v = 1inspace(-1, 1.5, 200);
z = zeros(length(u), length(v));
for i = 1:length(u)
    for j = 1: length(v)
        z(j, i) = map_feature(u(i), v(j))*theta;
    end
end
contour(u, v, z, [0,0], 'LineWidth', 2)
legend('y = 1', 'y = 0', 'Decision boundary');
title(['\lambda = ' num2str(lambda)]);
```