1 Homework2

1. Let $\mathbf{X} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ where $\mathbf{X} \in \mathbb{R}^2$ and

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$$

where ρ is the correlation coefficient. Show that the pdf is given by

$$p(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} exp\Big(- \frac{1}{2(1-\rho^2)} \big(\frac{(x_1-\mu_1)^2}{\sigma_1^2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2} - 2\rho \frac{(x_1-\mu_1)}{\sigma_1} \frac{(x_2-\mu_2)}{\sigma_2} \big) \Big).$$

2. Consider a Bivariate Normal distribution with $\mu_1 = 0, \mu_2 = 2, \sigma_{11} = 2, \sigma_{22} = 1$ and $\rho_{12} = 0.5$.
(a) Write out the bivariate normal density.
(b) Write out the Mahalanobis distance as a function of x_1 and x_2.
(c) Determine and sketch the contour of the probability. (Please note the directions of the eigenvectors).

3. Let $\mathbf{X} \sim \mathcal{N}_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $\boldsymbol{\mu}^T = \{-3, 1, 4\}$ and

$$\Sigma = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Which of the following variables are independent?
(a) X_1 and X_2, (b) X_2 and X_3.