1. Please derive the dual problem for the following objective function.

$$\min_{\boldsymbol{w}} \frac{1}{2}\|\boldsymbol{w}\|^2 + C\sum_{i=1}^{n} \xi_i$$

$$s.t.\ y_i(\boldsymbol{w}^T\boldsymbol{x}_i + b) \geq 1 - \xi_i, i = 1,2, \dots, n$$

$$\xi_i \geq 0, i = 1,2, \dots, n$$

The answer should be:

$$\max_{\boldsymbol{\alpha}} \sum_{i=1}^{n} \alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i\alpha_j y_i y_j \boldsymbol{x}_i{}^{\boldsymbol{T}}\boldsymbol{x}_j$$

$$s.t.\ 0 \leq \alpha_i \leq C, i = 1, \dots, n$$

$$\sum_{i=1}^{n} \alpha_i y_i = 0$$

2. Please prove that for the following objective function,

$$\min_{\boldsymbol{w}} \frac{1}{2}\|\boldsymbol{w}\|^2 + C\sum_{i=1}^{n} \xi_i$$

$$s.t.\ y_i(\boldsymbol{w}^T \boldsymbol{x}_i + b) \geq 1 - \xi_i, i = 1,2,\dots,n$$

$$\xi_i \geq 0, i = 1,2,\dots,n$$

We have

$$\alpha_i = 0 \Rightarrow y_i(\boldsymbol{w}^T \boldsymbol{x}_i + b) \geq 1$$
$$\alpha_i = C \Rightarrow y_i\left(\boldsymbol{w}^T \boldsymbol{x}_i + b\right) \leq 1$$
$$0 < \alpha_i < C \Rightarrow y_i\left(\boldsymbol{w}^T \boldsymbol{x}_i + b\right) = 1$$