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2018
MCM/ICM
Summary Sheet

Evaluation on Climate-based Fragile State Index

Summary

After '911' attacks, fragile states capture the attention of western countries rapidly, but most of related evaluation systems do not take climate into account. In this paper, we consider the impact of climate change on fragility and propose Climate-based Fragile State Index (CFSI).

First, primary indicators are set as: Politics, Society, Economy and Climate with 16 secondary indicators. Entropy-weight method is used to determine the weights of indicators based on data about 145 countries. A gravity-like model is designed to quantify the mutual impact among indicators. Weighted value is mapped to vector in 4-D space. The length of normalized vector projection on standard vector is defined as V_{CFSI}, the angle of vector repesents the potential of state's fragility. K-means algorithm is used to cluster 145 countries into 4 categories, and V_{CFSI} of 4 cluster centers are set as the threshold for 5 different fragility levels.

For clear visualization, colormap and projection are used to describe the impact of indicators change, especially climate-related ones. Based on the simulation of the fragility in South Sudan under different climate conditions, pratical proposals are designed to defend climate risks. The fragility of Egypt from 2018 to 2021 is predicted by Auto Regressive and Denial testing. Based on this prediction model, we discuss how climate changes lead to higher fragility. The result shows Egyptian fragility reached 'Extremely Fragile' and 'Vulnerable' in 2015 and 2017 respectively, and it probably stablizes in the near future.

Further, State Driven Intervention Model is built based on Bi-level Programming. The goal of upper level is to minimize fragility including climate risks, and the goals of two lower levels are to maximize economic development and to minimize social individuals's adverse influence. Suggestions of interventions and total cost are provided based on the distribution of optimal solution in two levels.

Finally, performance of our model is analysed on different sizes of 'state'. City modeling suffers from insufficient data, specific features in different cities. Continent modeling suffers from few indicators. Further, sugguestions aimed at the aforementioned problems are provided as our future work.

Contents

1 Introduction

After 'September 11 attacks', anti-terrorism becomes a hot spot issue. Increasing countries pay attention to the most fragile countries which bring lots of threats to international peace and security. A comprehensive assessment of the vulnerable countries has been widely studied.

There are many definitions of a fragile state. 'State' refers to a sovereign state or country and a fragile state is one where the state government is not able to, or chooses not to, provide the basic essentials to its people. For example, Canadian Institute of Financial Planning (CIFP) proposed the definition of fragile state [1]. So far, many assessment systems have been put forward to rank the fragility of states, such as Country Policy and Institutional Assessment (CPIA) proppposed by World Bank [2], CIFP proposed by Carleton University in Canada [3], and Center for Systemic Peace (CSP) proposed by George Mason University [4].

However, ther are many weaknesses in the current evaluation system. Most of the systems do not take climate as their struture index. Moreover, most systems simply add up the scores of all the indexes with the same weight. To solve the problems above, we propose our evaluating system to measure the fragility of states more reasonably.

Our paper is organized as follows. Section 2 clarifies the assumptions. Section 3 proposes our evaluating system to measure the fragility of state. Section 4 and section 5 take South Sudan and Egypt as examples to analyzes the impact of climate change, making predictions of fragility as well. Section 6 proposes the state driven interventions and section 7 discusses whether our model could be applied to cities and continents.

2 Assumptions

Our models are based on the following assumptions which are properly justified.

- **The impacts between climate and politics, society, economy are bidirectional.** Climate and politics, society, economy affect each other, so all indicators change in different levels based on their charactistics when something disturbs a state.

- **Imapact between the secondary indicators under different primary indicators is indirect.** The change of the secondary indicators leads to the change of the primary indicators directly and then change all the other secondary indicators through the interaction between the primary indicators.

- **Short-term accurate prediction is better than long-term inaccurate prediction.** For the prediction problem , long-term prediction models are often less accurate due to the unpredictability of climate change and the volatility of policy changes and so on. Therefore we establish short-term forecasting models by observing data relationships we find in model.

- **The primary goal of state is to reduce the fragility of state.** The goal of state is hierarchical. State needs to take the goals of climatic risks, economic development and citizens' standard of living into account, while the primary goal is to reduce fragility.

3 Evaluating the Fragility of State

3.1 Climate-based Fragile State Index

3.1.1 Selecting Indicators by PCA

In order to measure a country's fragility, our first task is to select indicators properly. It is essential to control the number of indicators. Because a few indicators cannot evaluate the fragility conprehensively and too many indicators will increase the difficulty of collecting real data.

We compare lots of current evaluating systems and finally decide to construct our own model based on a well-recoginized one. Fragile States Index (FSI) is an annual report published by the Fund for Peace in America, which has been widely used [5]. We reserve the categories of all the indicators of FSI, using them in our model [6].

- **Politics**: security apparatus, external intervention, factionalized elites, state legitimacy, public services, human right.
- **Economy**: economic decline, uneven economic development.
- **Politics**: group grievance, human flight, demographic pressures, displacement of refugees.

However, the system does not consider the impact of climate change on a country's fragility. So we introduce another primary indicator: Climate.

To set specific indicators for Climate, we draw lessons from the 2018 Environment Performance Index (EPI). Principal component analysis (PCA) is adopted to reduce the number of them. The essence of PCA is to find some orthogonal directions so that the variance of the data is maximum in these directions. We successfully pick up 4 indicators: Greenhouse Gases Emmision (GE), Water Resourses (WR) , Biodiversity and Habit (BH) and Forest Area (FA).

3.1.2 Determining the Weights by EWM

We have already selected 16 secondary indicators based on 4 primary indicators: Politics, Society, Economy and Climate. Then we choose a list of countries to evaluate.

We use two data sets: FSI by Fund for Peace [1] and 2018 Environment Performance Index (EPI)[2] aforementioned. FSI provides a list of 177 countries with the values of corresponding indicators and EPI provides the environment information of 181 countries. We select the shared countries and delete those lacking sufficient values. Finally, we get a list of 145 countries with all the values of 16 indicators.

[1] http://fundforpeace.org/fsi/data/

[2] https://epi.envirocenter.yale.edu/

Since these indiactors vary in dimensions, they cannot compare with each other directly. So we normalize the data to ensure all the data range from 0 to 1. Analyzing all the indicators, we find the larger value of an indicator, the higher value of CFSI, and the state will be more fragile.

Next we use the entropy-weight method (EWM) to determine the weights of secondary indicators. Suppose there are m secondary indicators under one primaty indicator. After getting the normalized values of each indicator, we construct a normalized matrix R_{nm}. Based on the entropy-weight method, we get the entropy value of the j th indicator.

$$H_j = -k \sum_{i=1}^{n} f_{ij} \ln f_{ij}, \quad j = 1, 2, 3, ..., m \tag{1}$$

The weight of the j th indicator is

$$w_j = \frac{1 - H_j}{m - \sum_{j=1}^{m} H_j} \quad j = 1, 2, 3, ..., m \tag{2}$$

where $k = \frac{1}{\ln n}$, $f_{ij} = \frac{R_{ij}}{\sum_{i=1}^{n} R_{ij}}$.

In this way, we have determined the weights of secondary indicators as Figure 1 shows.

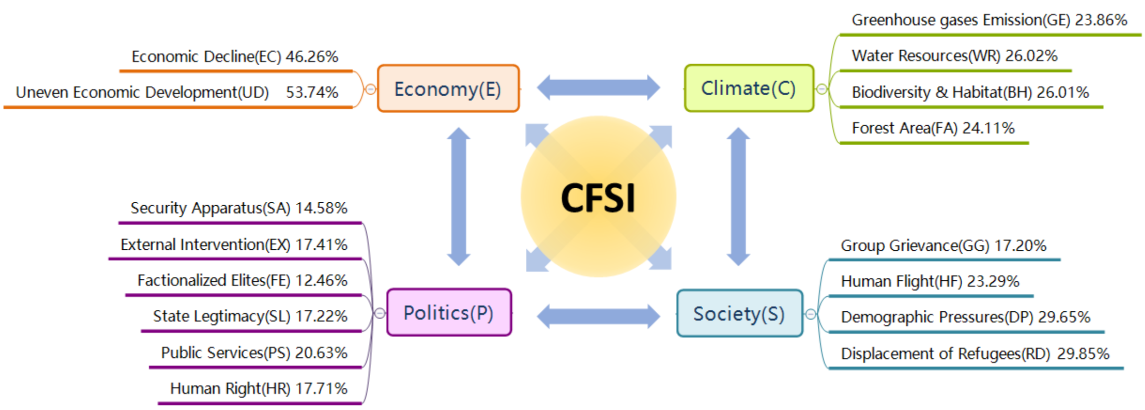


Figure 1: Evaluation system of CFSI

In Figure 1, we define the Climate-based Fragile State Index (CFSI), which has 4 primary indicators and 16 secondary indicators.

3.1.3 Influences Among Primary Indicators

In reality, there are complex influences among indicators. For example, climate shocks and environmental stress will aggravate the weak governance of a state. On the other hand, a stable governance may make more effort to improve the environment. The influences between them are mutual and complex.

The influences between two indicators depend on their values individually. For example, as a fragile country, politics, society, economy and climate have a tight connction. Due to the state's vulnerable structure, the change of a primary indiacator will result in the changes of others easily. By contrast, a stable country has a stronger ability to defend the sudden changes of one indicator. Politics, society, economy and climate develop in a more steady and independent mode which influenced by the other indicators less. So when the values of primary indicators are large and become more close to a fragile one, the influences between them will be greater.

Let z_i and z_j be the values of two primary indicators. Since ther are numerous factors hindering the transmission of influences, we denote the influence resistence as R_{ij} which is dependent on the level of national communication, the efficiency of policy implementation and the capacity of traffic flow, etc. So when the value of R_{ij} becomes smaller, the influences between indicators tend to be more significant.

Based on the analysis above, we utilize the gravity model to measure the influences between primary indicators. We denote the influence index between i and j as z_{ij}, which can be calculated by

$$z_{ij} = K\frac{z_i z_j}{R_{ij}} \tag{3}$$

where K is the coefficient to adjust the value varying from 0 to 1, and $z_{ij} = z_{ji}$ according to our assumption.

For a certain country, we denote its primaty index as a vector $\mathbf{X} = (X_1, X_2, X_3, X_4)$. It represents the values of Politics, Society, Economy and Climate respectively. After a period of time, the country's situations will change. This can be reflected by the changes of each secondary indicator, thus changing their weighted sum. So the values of primary indicators will change directly. We denote these direct changes as $\Delta X_1, \Delta X_2, \Delta X_3, \Delta X_4$, respectively.

For the i th indicator, it is influenced by other indiacators, and the influence from the j th indicator is $z_{ij}\Delta X_j (i \neq j)$. Since the value of primary indicator is the sum of the direct changes and indirect changes, we get an improved formula of the primary indicator.

$$\mathbf{X} = \begin{pmatrix} X_1 + \Delta X_1 + z_{12}\Delta X_2 + z_{13}\Delta X_3 + z_{14}\Delta X_4 \\ X_2 + \Delta X_2 + z_{21}\Delta X_1 + z_{23}\Delta X_3 + z_{24}\Delta X_4 \\ X_3 + \Delta X_3 + z_{31}\Delta X_1 + z_{32}\Delta X_2 + z_{34}\Delta X_4 \\ X_4 + \Delta X_4 + z_{41}\Delta X_1 + z_{42}\Delta X_2 + z_{43}\Delta X_3 \end{pmatrix} \tag{4}$$

The formula above improves our model by considering the influences between primary indicators: Politics, Society, Economy and Climate. In our case, we only investigate the impact of climate change. Then we have

$$\Delta X_1 = \Delta X_2 = \Delta X_3 = 0. \tag{5}$$

Thus, (4) can be simplified as

$$\mathbf{X}' = \begin{pmatrix} X_1 + z_{14}\Delta X_4 \\ X_2 + z_{24}\Delta X_4 \\ X_3 + z_{34}\Delta X_4 \\ X_4 + \Delta X_4 \end{pmatrix} \tag{6}$$

Each country represents a four-dimensional vector. After normalizing the vector, we set a standard vector (1,1,1,1) to represent the worst situation for a fragile state. We project each vector to the standard vector (1,1,1,1). Since the projected length ranges from 0 to 2, so we multiply the projected length by 1/2 for normalization. The normalized value is defined as the value of Climate-based Fragile State Index (CFSI).

$$V_{CFSI} = \frac{(X_1, X_2, X_3, X_4) \cdot (1, 1, 1, 1)}{2 \parallel (1, 1, 1, 1) \parallel} \tag{7}$$

In Figure 2, we present the top 10 most fragile states of CFSI . For each country, we plot a three-dimensional vector representing the normalized value of three primary indicators: Politics, Society and Economy. Then we reflect the fourth indicator by the magnitude of Climate besides.

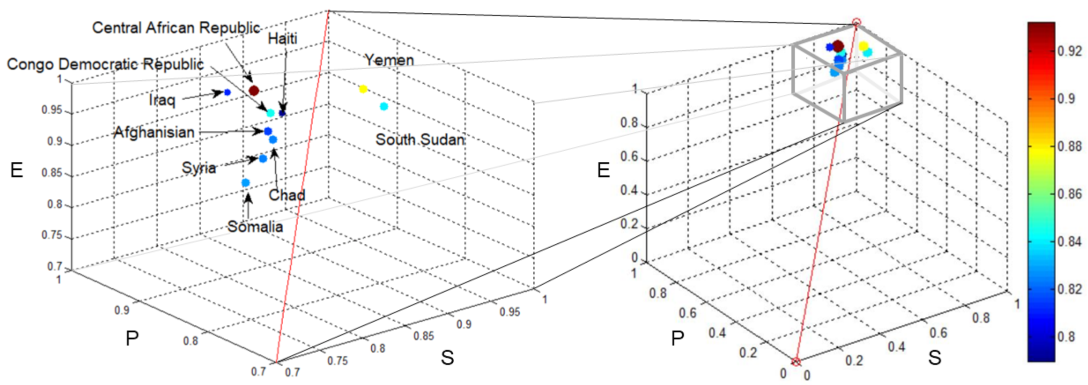


Figure 2: Top 10 most fragile states of CFSI

From the figure above, the endpoints of vectors are shown in different colors representing the values of climate. A warmer color indicates a more fragile situaiton of climate. The red line pointing from the origin to the vertex (1,1,1) represents the standard vector.

On one hand, we project the vector to the standard one and a longer projected length represents a more fragile state. On the other hand, the angle between the vector and the standard one indicates the potential for a state to become more fragile. When the angle between its vector and the standard one is realtively small, the fragile levels of different indicators are more close, triggering a higher potential of worsening its situation.

3.2 Impact of Climate Based on K-means Algorithm

From the sections above, we evaluate the fragility of a state based on our CFSI model. Furthermore, we aim to identify how climate change influence the fragility of a country in a more specific way.

As mentioned above, the influence index between the indicator i and j is z_{ij}. So the influence indexes of Climate on Politics, Society and Economy are denoted as a three-dimensional vector (z_{14}, z_{24}, z_{34}). Then we adopt the K-means Algorithm (KA) to claasify the impact of climate into three types.

In the K-means algorithm, we denote the three-dimensional vector for the k th country as Z_k and all the vectors compose the data set $Z = \{Z_1, Z_2, ..., Z_{145}\}$. Let μ_c be the cluster center of the partitions $(c = 1, 2, 3)$. The sum of squares for the distances from the cluster center is denoted as $J(c, \mu)$. The goal of KA can be determined by

$$\min\ J(c,\mu) = \sum_{Z_i \in Z} \parallel Z_i - \mu_c \parallel^2 \tag{8}$$

For the 145 countries, we plot the three-dimensional vectors (z_{14}, z_{24}, z_{34}). Figure 3 reflects the results of clustering. Different partitions are presented in different colours and the three big dots are the cluster centers. The influences of Climate on Politics, Society and Economy are shown in the x-axis, y-axis and z-axis respectively.

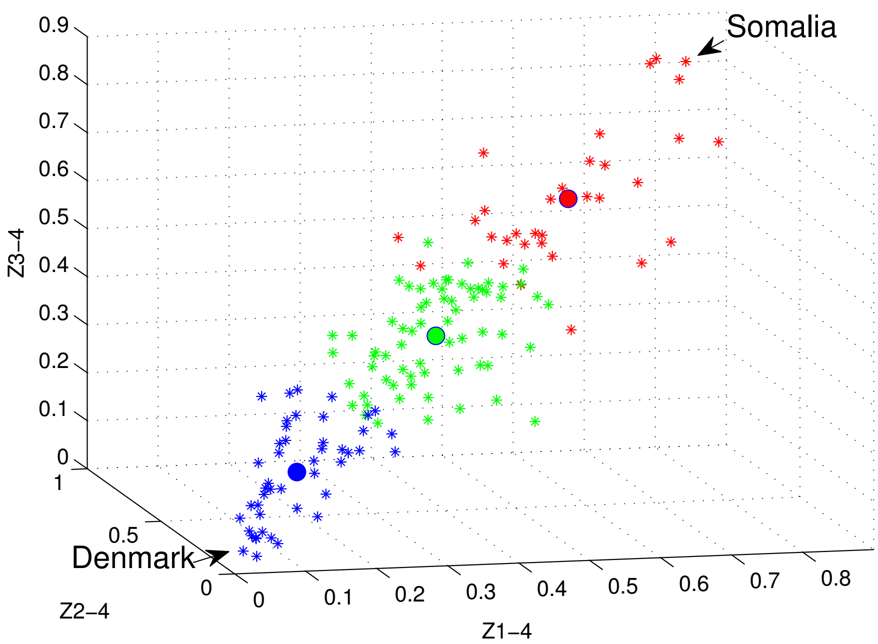


Figure 3: Influences of climate on other indicators

From the figure above, the impacts of climate are classified into three types: slight, moderate, significant. The blue dots represent the countries where climate has slight influence on other indicators. And Denmark is an extreme example near (0,0,0). The red dots represent the countries where climate has significant influence on other indicators. Somalia is an extreme example near (1,1,1). That is because Somalia ranks second in the top 10 list of FSI. It is so fragile that climate could exert tremendous influence on it. Between the blue and red dots are the green ones, representing the moderate influence of climate.

Besides, we find the ends of vectors mainly distribute near the vector (1,1,1). It means the influences of climate on other indicators are approximately similar.

3.3 Setting Standards for CFSI

We succssfully develop a method to determine the fragility of a state, however, our fundamental task is to reduce its fragility. In order to monitor the changes of CFSI, we decide to set standards to measure them more clearly.

Our purpose is to identify when a state is fragile, vulnerable, or stable. We calculate the statistical distribution of CFSI as Figure 4 shows. The values of CFSI do not obey the uniform distribution. Hence it is not a wise choice to divide CFSI form 0 to 1 evenly. In order to set an objective standard, we adopt the K-means Algorithm again.

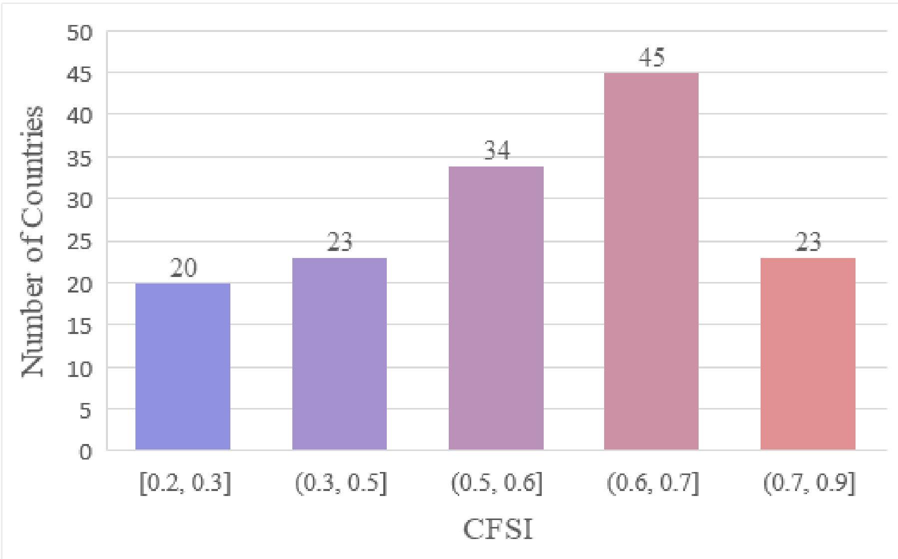


Figure 4: Distribution of CFSI from 145 countries

From our analysis aforementioned, the values of primary indicators are reflected in a vector (X_1, X_2, X_3, X_4). Using 145 countries as a dataset, we classify all the vectors into 4 partitions. We use the CFSI of four cluster centers as the thresholds, thus dividing the values of CFSI into 5 levels. As Figure 5 shows, we set 5 standards to describe CFSI: stable, normal, vulnerable, fragile and extremely fragile.

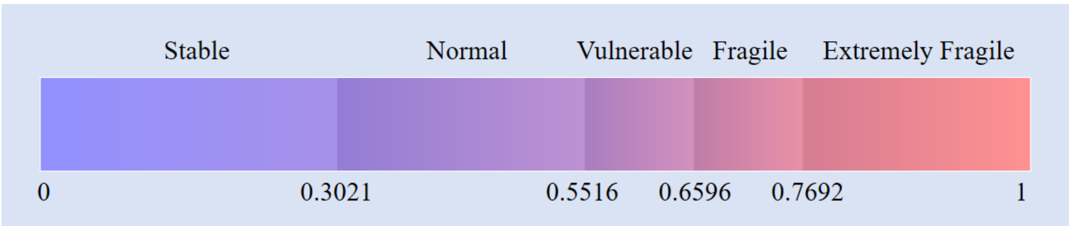


Figure 5: Standards of CFSI

The standards we set accord with the distribution of CFSI in Figure 4. The level of fragile countries lasts the shortest length from 0.6598 to 0.7692. That is because most countries have the values of CFSI varying from 0.6 to 0.7. So our classification is proved to be reasonable.

4 Impact of Climate Change in South Sudan

Based on the data set of 145 countries, we generally concluded the influence of climate on other indicators. Further, to make our findings more specific, we decide to select a certain country and investigate the impact of climate change in reality. So we select South Sudan from the top 10 most fragile states determined by FSI.

4.1 Analyzing the Impact of Climate Change

First we get the four-dimensional vector of South Sudan, which represents the values of four indicators: Politics (P), Society (S), Economy (E) and Climate (C). To analyze the impact of climate change, each time we only analyze the relationship between C and another indicator, remaining the left two indicators unchanged. So Figure 6 plots three subfigures to reflect the relationship between Climate and other indicators respectively.

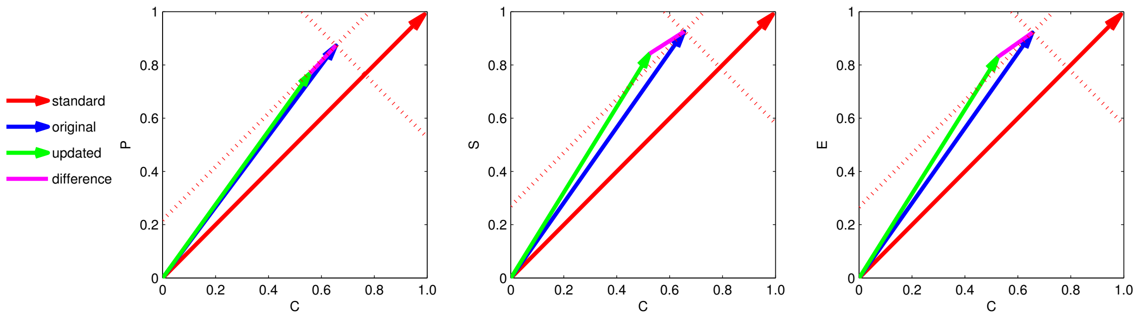


Figure 6: Impact of climate change by projection

Based on the figure above, we reduce the value of Climate to 80% of its original one, and the value of the other indicator will change as well, triggering the change of CFSI eventually. After projecting the vectors to the two-dimensional plane, the blue vector shows the original one and the green vector shows the updated one. The purple vector is the difference of them.

To measure the impact of climate change on CFSI, we also project the standard vector (1,1,1,1) to the two-dimensional plane as the red vector (1,1) in the figure. Since the purple vector is the difference between the original and updated ones, its projected length on the standard vector (red) represents the decreased value of fragility.

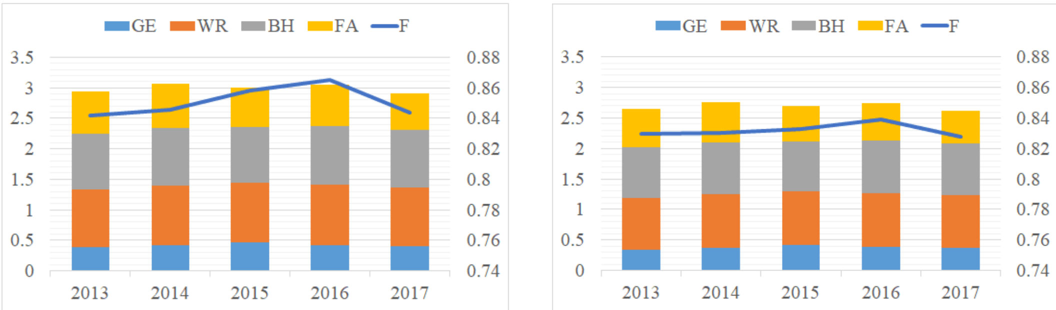
We first consider two extreme conditions. If the purple vector is perpendicular to the red one, its projected length on the red one is zero. That indicates the climate change contributes to the changes of another indicator, but the combination of their effects will not change the fragility of a state. On the contray, if the purple vector is parallel with the red one, the projected length on the red one will be relatively long, thus the fragility will change significantly.

Analyzing the figure carefully, we find climate is positively related to other indicators. When the value of Climate decreases, the values of other indicatoes will also decrease. Meanwhile, the projected length on the standard vector will decrease and the angle between them will increase. That indicates South Sudan will be more stable with the improvement of climate.

Moreover, when the climate changes, the change of CFSI is more influenced by the corresponding change of Society or Economy rather than Politics. This can be explained by our gravity model. For South Sudan, the values of Society and Economy are relatively high (0.9254 and 0.9215), while the value of Politics is relatively low (0.8728). So the influence index between Climate and Politics is relatively small in the gravity model. Meanwhile, our finding reflects the reality reasonably. In South Sudan, rain goes together with heat and the distribution of rainfall is not even. Climate change plays a crucial role in agricultural economy and social services.

4.2 Fragility of South Sudan under Different Climates

Climate change exerts significant influence on the fragility of Souhth Sudan, then we analyze how the fragility will change without these effects. Figure 7 shows the values of 4 climate indicators and CFSI from 2013 to 2017.



(a) original climate (b) more frienfly climate

Figure 7: Fragility of South Sudan under different climate

Subfigure (a) reflects the real situation. CFSI is increasing steadily from 2013 to 2016 due to the decrease of water resoures (WR). It is worthy to note that the larger value of WR indicates the worse situaion. Since South Sudan has the tropical grassland climate, the amount of rainfall changes dramatically. So the lack of rainfall increases the fragility of South Sudan.

To simulate the situation of Egypt without climate change, we then decrease the values of 4 climate indicators to 90%, representing a more friendly climate. As the subfigure (b) shows, CFSI decreases in 2016, further verifying the significant effect of climate change. So South Sudan will become more stable in a more friendly climate especially with more rainfall.

4.3 Ways to Reduce Fragility from Climate Change

We have already analyzed the impact of climate change in South Sudan, our following task is to study how to reduce the fragility considering the effect of climate change. As we mentioned before, CSFI is defined as the projected length of a four-dimensional vector on the standard one (1,1,1,1).

To reduce the fragility of South Sudan, we attempt to reduce the magnitude of $\mathbf{X}'$ as much as possible. In our case, we only consider the impact of climate change which is reflected by the influence index $z_{ij} = k\frac{z_i z_j}{R_{ij}}$ in (3). To reduce the value of z_{ij}, there are mainly 2 basic principles: reducing the numerator and increasing the denominator.

Concretely, we propose pratical suggestions for South Sudan from 3 perspectives.

- **Reduce the value of z_{24} and z_{34}.**
  Since South Sudan has diverse terrains such as basins, rivers, lakes, forests and marshes, gradient cultivation is a reasonable choice. Meanwhile South Sudan should make use of the laterite land and develop characteristic agriculture, cultivating suitable products such as rubber, coconut and pepper.

- **Reduce the value of z_{14}**
  The turbulent political situation greatly results in the unstable lifes of people. According to statistics, there are more than 1.9 million tramps in South Sudan and approximately 1.6 million people seeking shelter in neighbouring countries [7]. Therefore, the government is recommended to implement the peace agreement, maintain the stability of the state, and solve the humanitarian crisis.

- **Increase the value of R_{ij}**
  As mentioned before, the traffic resistence reflects the obstacles during the transmission of influences. So we could reduce the impact of climate change by increasing the influence resistence R_{ij}. By 2014, there have been 11 approved Internet Service Provider (ISP) in South Sudan. And we recommend South Sudan further improve the national mobile communications and increase the coverage of Internet and optical fiber.

In conclusion, adopting the sugesstions above, South Sudan can enhance its ability to defend climate shocks, thus reducing its level of fragility.

5 Predicting the Fragility of Egypt

From the sections above, we determine a country's fragility by the CFSI model and use South Sudan as a case study to further investigate the impact of climate change. Since the fundamental function of fragility evaluation is to monitor those fragile states and help them become more stable, our following work is to predict their trends based on the CFSI model.

5.1 Prediction Based on Auto Regressive Model

In the long term, politics, society, economy and climate change under certain regulations. But the short-term prediction is relatively hard. That is becasuse those indicators will be influenced by lots of complex factors such as sudden climate change, population mobility and policy adjustment.

There are many prediction methods and the typical ones are Grow Curve Prediction [8] and Exponential Smoothing [9]. However, their results of short-term prediction are not precise enough. So we consider to introduce the Auto Regressive model. Auto Regressive (AR) Prediction is especially suitable for the short-term prediction. It not only considers the dependency of fragility in the time series, but also considers the interference of random fluctuation. Above all, we decide to adopt the Auto Regressive (AR) model to predict the fragility of states.

First, AR model requires us to identify whether the series are smooth by Daniel Testing. We denote the time series data as $a_t(t = 1, 2, ..., m)$ and set the significance level $\alpha = 0.05$. The Spearman correlation coefficient is denoted as q_s. The hypothesis test is:

$$H_0 : \rho_{xy} = 0, H_1 : \rho_{xy} \neq 0 \tag{9}$$

where ρ_{xy} is the overall correlation coefficent.

When (X, Y) is a bivariate normal population and H_0 is true, we then construct the statistic T.

$$T = \frac{q_{XY}\sqrt{n-2}}{\sqrt{1-q_{XY}^2}} \tag{10}$$

T obeys the t distribution $t(n-2)$. The degree of freedom is $n-2$.

When $| T |\leq t_{\alpha/2}(n-2)$, we accept H_0. Otherwise we refuse H_0. We denote the rank of a_t as $R_t = R(a_t)$. The Spearman correlation coefficient is

$$q_s = 1 - \frac{6}{n(n^2-1)}\sum_{i=1}^{n}(t - R_i^2) \tag{11}$$

Then we get the value of T.

$$T = \frac{q_s\sqrt{n-2}}{\sqrt{1-q_s^2}} \tag{12}$$

If $| T |\leq t_{\alpha/2}(n-2)$, the time series are smooth.

If $| T |> t_{\alpha/2}(n-2)$, the times series are not smooth. Then we need to construct smooth time series. For the time series $a_t(t = 1, 2, ..., m)$, we calculate the first order difference: $b_t = a_{t+1} - a_t$ and the time series $b_t(t = 1, 2, ..., m)$ are smooth.

Based on the smooth time series, we build the AR(2) model below.

$$y_t = c_1 y_{t-1} + c_2 y_{t-2} + \varepsilon_t \tag{13}$$

where c_1, c_2 are parameters and ε_t is the stationary white noise (stochastic disturbance).

5.2 Predicting the Impact of Climate Change on CFSI

Egypt is a representative country in North Africa. As one of the first countries to establish a social security system in that region, it provides a model for other surrounding countries to follow. Besides its climate represents the typical characteristics of Africa.

Hence we select Egypt for our case study and it is not in the top 10 list of FSI. We collect the values of 16 secondary indicators of Egypt from 2006 to 2017 by the data set of FSI and EPI mentioned before. Since EPI is published every two years, we assume the value of the missing year as the average between the two years nearby. Then we use two methods to predict the values of CFSI as Figure 8 shows.

The first method is shown in the upper part of the figure. The normalized data of Egypt are the values of 16 indicators from 2006 to 2017. Based on Daniel Testing, $| T |= 0.0885$ and $t_{\alpha/2}(n-2) = 2.2281$. So $| T |\leq t_{\alpha/2}(n-2)$, and the time series are proved

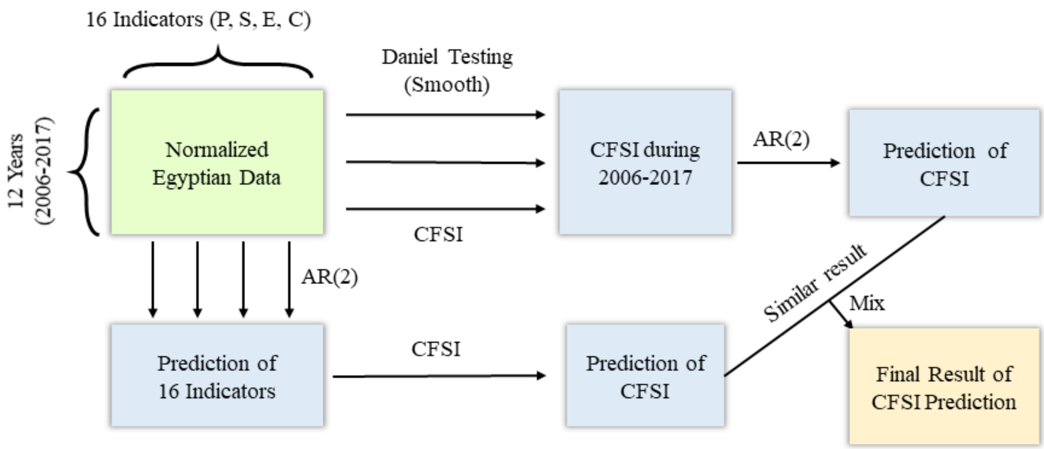


Figure 8: Prediction on the Values of CFSI

to be smooth. In our evaluation system, we determine the values of CFSI from 2006 to 2017. Then we use the AR(2) model to predict their values from 2017 to 2021. The result of AR(2) prediction is shown below.

$$V_{CFSI}(t) = 0.3033V_{CFSI}(t-1) - 0.5311V_{CFSI}(t-2) \tag{14}$$

The mean square error is 0.00046, thus indicating the high accuracy of our prediction.

The second method is shown in the lower part of the figure. We predict the values of secondary indicators by the AR(2) model. Then we calculate CFSI for each year based on our evaluation system.

Ultimately, the results of CFSI are similar in different methods, thus validating the stability of our model. So we take the average as the final result of CFSI. The predictions are shown in Figure 9.

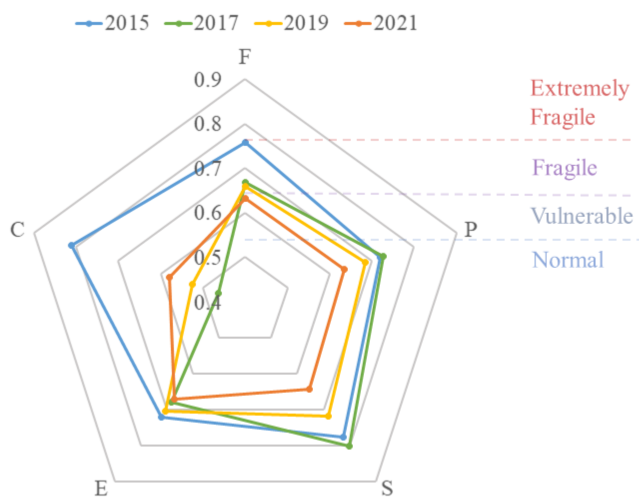


Figure 9: Values of CFSI and primary inicators in the given years

We find CFSI is decreasing and Egypt will gradually transform from a fragile country to a vulnerable one. As for the primary indicators, their values tend to decrease gradually except for Climate. From 2015 to 2017, the value of Climate drops greatly from 0.8 to 0.4, then it slowly rises to 0.6 in 2021. This may indicate the environment of Egypt will worsen in the future.

Specifically, to investigate the impact of climate change, we plot the trends of 4 indicators of Climate in Figure 10. The solid lines show the values of Greenhouse gases Emission (GE), Water Resources (WR), Biodiversity and Habit (BH) and Forest Area (FA). The dashed lines show their predicted values.

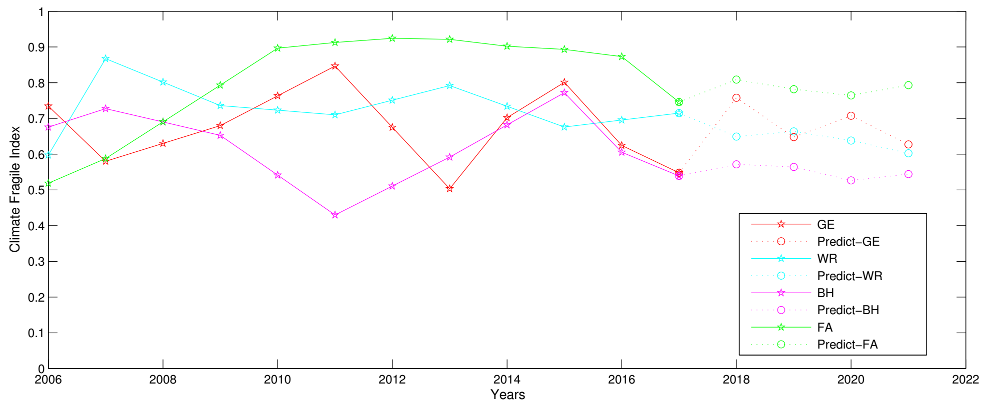


Figure 10: Trends of climate change in Egypt

From the figure above, there are mainly two types of trends. The values of Water Resources and Forest Area both rise steadily from 2006 and arrive to the peak in 2007 and 2014 respectively. Then they gradually drop regardless of some fluctuations. Finally their values remain stable near 0.7 and 0.5 after 2018. That indicates the situations of Water Resources and Forest Area both get improved after severe damage. Meanwhile, the values of Greenhouse Gasses Emission and Biodiversity and Habit keeps fluctuaing violently from 2006 to 2017, so the predicted ones also experience extreme fluctuations. This can be explained by its social unrest and conflicts, which increase the fragility of environment greatly.

Comparing Figure 9 and Figure 10, we could analyze how climate change influence the fragility of Egypt. From 2015 to 2017, the values of indicaters all decline sharply apart from a slight increase of water resources, hence the improved situation of climate may promote the stability of Egypt.

Furthermore, we could predict how climate change may influence the fragility of Egypt. From 2017 to 2018, the greenhouse gasses emission is predicted to increase dramatically, contributing to the climate warming. Thus the value of CFSI will rise from 0.665 to 0.671 according to our prediction. The increaing amount is about 1%.

From 2018 to 2021, the values of these 4 indicators are predicted to decline gradually and stablize eventually. So the improved situation of climate may promote the stability of Egypt.

5.3 Predicting the Tipping Point of Fragility

Since there are 4 levels of fragility: normal, vulnerable, fragile and extremely fragile, we define the tipping point as the time when the fragility of a state changes from one level to another.

We plot the values of CFSI from 2006 to 2021 in Figure 11. The solid lines represent the historical data and the dashed lines represent the predicted ones. We also mark four levels of fragility besides.

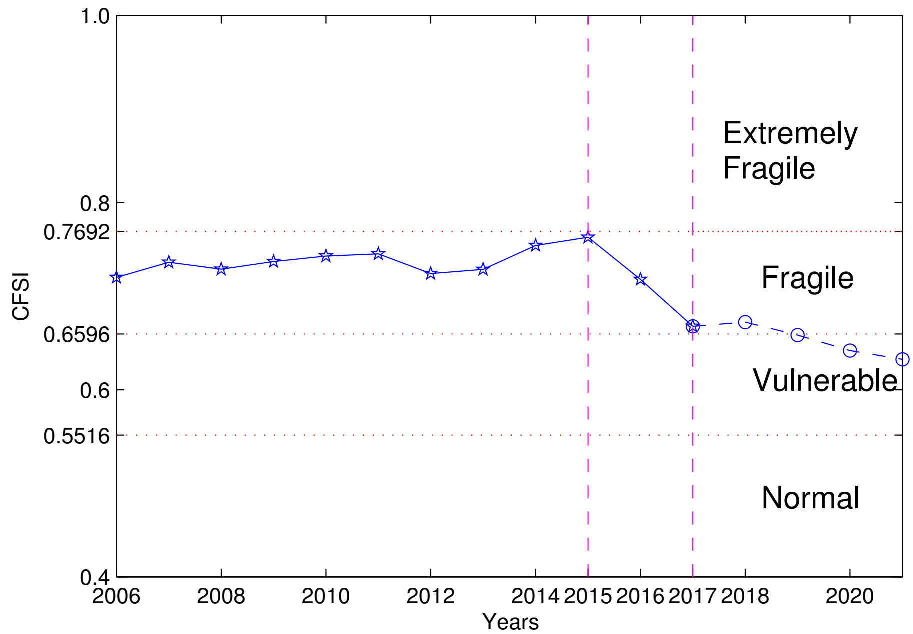


Figure 11: Variation trend of CFSI from 2006 to 2021 in Egypt

Generally speaking, Egypt remains as a fragile country. Since 2006, the value of CFSI climbs slowly with slight fluctuations and it reaches the peak point in 2015. Since that value is extremely close to the boundary between fragile and extremely fragile, we could say Egypt reaches its tipping point in 2015.

CFSI falls rapidly after 2015 and the prediction shows the declining rate will be slowed down after 2017. Egypt is going to transform from a fragile country to a vulnerable one.

In conclusion, Egypt reaches its tipping point in 2015 and 2017.

6 Intervention Model Based on Bilevel Programming

After evaluation and prediction of the fragility of a state, our fundamental task is to reduce V_{CFSI}, helping more countries to get over the stuggle of fragility. In this section, we will introduce the effect of human intervention.

6.1 Bilevel Programming

The fragility of a state is dependent on a great deal of factors. Facing the impact of climate change, we could not underestimate the effect of state driven interventions. How to balance the needs among country, enterprises and individuals is an objective programming problem essentially.

Multi-level programming [10] and multi-objective programming [11] are not suitable for the macro-regulation model, which has different levels of decision makers. Multi-level programming lacks the top decision maker and multi-objective programming is only used for one decision maker. In view of these considerations, we bulid a macro-regulation model based on the bilevel programming [12].

In the bilevel programming model, there are two levels of decision makers. The upper level guilds the behaviour of the lower one instead of the direct intervention. The lower level could make decisions freely based on the decisions of the upper level. This clearly reflects the relationship between government and individuals.

We use 3 entities in our model as Figure 12 shows.

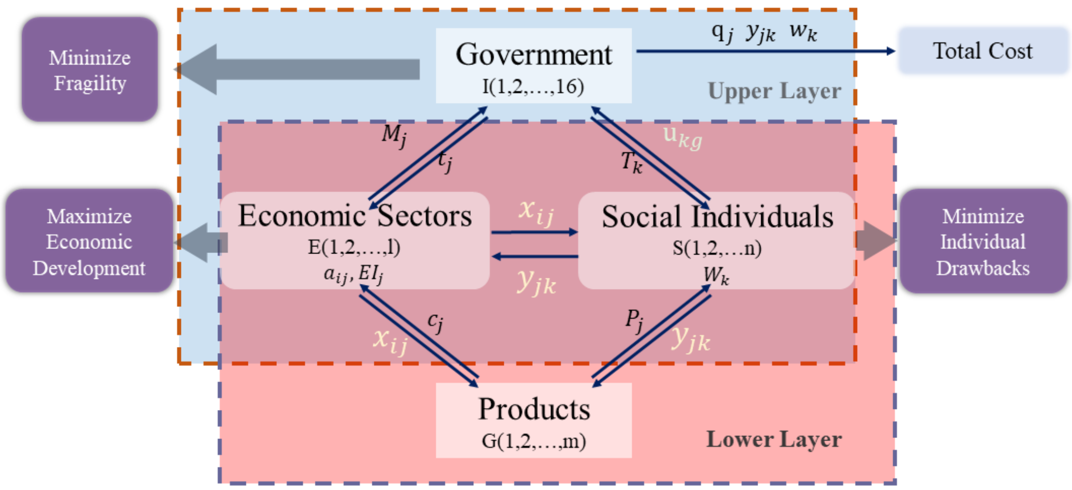


Figure 12: State driven interventions based on bilevel programming

- **Economic Sectors**
  For the economic sectors, their production activities will influence the economy, thus influencing the fragility of the state. Suppose there are l economic sectors producing m pieces of goods. We denote them as $E(1, 2, ..., l)$ and $G(1, 2, ..., m)$ respectively. For the i th sector, let the production capacity of the j th piece of goods as a_{ij} and the production efficiency be x_{ij}. For the j th piece of goods, let EI_j be its net import, P_j and C_j be its price and cost, and M_j be the upper limit of its quota in the market. The decision variable is the production efficiency x_{ij}.

- **Social individuals**
  Suppose there are n social individuals denoted as $S(1, 2, ..., n)$. For the k th individual, let its salary be W_k. Its consumption capacity for the j th piece of goods is y_{jk} which is its decision variable.

- **Government**
  Government adopts a series of policies to reduce its fragility. It balances the behavior of individuals and economic sectors by the tax rate. Suppose the individual income tax for the k th indivial is T_k and the tax rate for the j product is t_j.

Then we constrcut the State Driven Intervention model based on Bilevel Programming.

- **the Lower Level**
  For economic sectors, the goal is to maximize their profits. So the optimization model is

$$\max\{\Sigma_{j\in G}P_j(1-t_j)a_{ij}x_{ij} - \Sigma_{j\in G}c_j a_{ij}x_{ij}\} \tag{15}$$

$$s.t.\begin{cases} \Sigma_{j\in G}x_{ij}=1, & \forall i\in E, \quad (16)\\ P_j,\ t_j, a_{ij}, x_{ij}\geq 0, & \forall i\in E, j\in G. \quad (17)\end{cases}$$

where (16) is the constraint of production time and (17) is the nonnegativity constraint.

For social individuals, they have their influences on the climate. We denote the function $u_{kg}(y_{k_1},...,y_{k_m},GE,WR,BH,FA)$. The former m variables reflect their life demands and the last 4 variables represent their influences on the 4 indicators of climate aforementioned.

Suppose the goal of each social individual is to minimize their effects on worsening the fragility of the state, meanwhile satisfying their life demands. So the optimization model is

$$\min \Sigma_{g\in\{P,S,E,C\}}w_g u_{kg} \tag{18}$$

$$s.t.\begin{cases} \Sigma_{j\in G}P_j y_{kj}\leq W_k(1-T_k), & \forall k\in D, \quad (19)\\ P_j, y_{kj}, W_k, T_k\geq 0, & \forall j\in G, k\in D. \quad (20)\end{cases}$$

where (19) is the constraint of the life demands of individuals and (20) is the nonnegativity constraint.

- **the Upper Level**
  For the government, the goal is to minimize the fragility of the state. Denote the values of 16 secondary indicators as $I(1,2,...,16)$. So the optimization model is

$$\min \Sigma_{k\in D}\Sigma_{g\in\{P,S,E,C\}}w_g u_{kg} \tag{21}$$

$$s.t.\begin{cases} \Sigma_{i\in E}\Sigma_{j\in G}a_{ij}x_{ij}+\Sigma_{j\in G}EI_j=\Sigma_{j\in G}\Sigma_{k\in D}y_{kj}, & (22)\\ \Sigma_{i\in E}a_{ij}x_{ij}c_{ij}\leq M_j, & \forall j\in G, \quad (23)\\ \Sigma_{k\in D}W_k(1-T_k)\geq 0,, & (24)\\ a_{ij}, x_{ij}, EI_j, C_j, M_j, W_k, T_k\geq 0, & \forall i\in E, j\in G, k\in D (25)\end{cases}$$

where (22) is the constraint of balance between supply and demand, (23) is the capital constraint, (24) is the constraint of individuals' income and (25) is the non-negativity constraint.

In order to calculate the country's total expenditure on climate control, a new variable q is introduced as the cost factor for each commodity. q is determined by the ratio of the volume of the commodity to the total volume of the transaction. The formula for calculating expenses is

$$N = \Sigma_{k \in D} w_k \Sigma_{j \in G} q_j y_{jk} \tag{26}$$

where $q_k = \frac{M_j}{\Sigma_{j \in G} M_j}$ and N is the total expenditure of the country.

6.2 Solutions of Model and State Driven Interventions

Based on the Bilevel Programming model, we use the statistical data published by Egypt in 2015[3] and the information about Egypt in CIA[4]. After normalizing the data, we set the initial value x_{ij} and y_{kj} between 0 and 1 randomly. Then we determine the parameters with the opimal objective function.

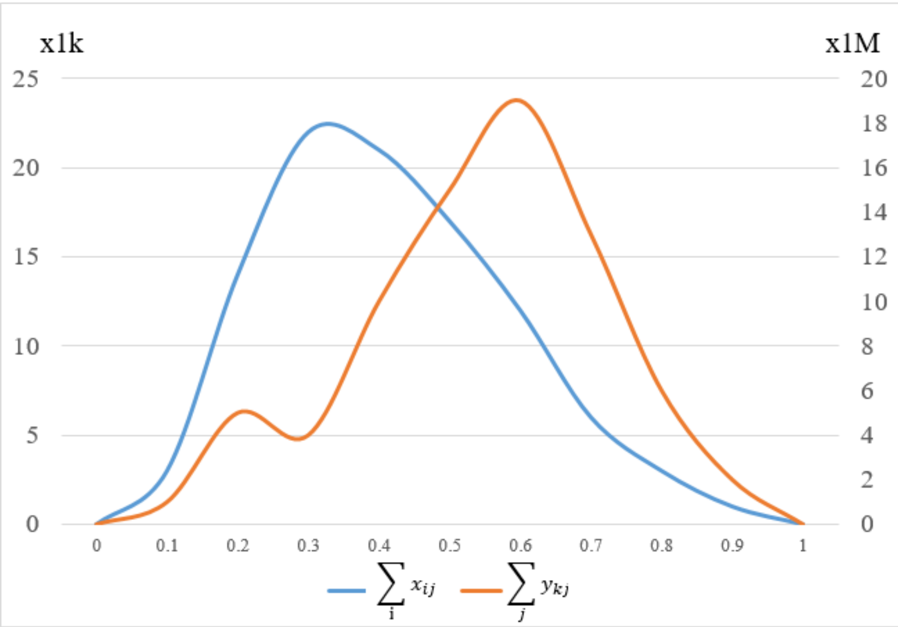


Figure 13: Distribution of optimal solutions in lower level

From Figure 13, we find the distribution of the optimal solution in lower level is unimodal. In a stable country, people's living standard Σy_{kj} and economic productivity Σx_{ij} tends to be 0.6 and 0.3 respectively. Based on it, we recommend the government improve macro-economic regulationuses through three variables below.

[3]https://www.cia.gov/library/publications/the-world-factbook/geos/eg.html
[4]sis.gov.eg/newvr/EgyptinFigures2015/EgyptinFigures/pages/englishLink.htm

- **Tax**: In terms of commodity tax, government should impose higher tax on the products which comsume too much energy and pollute the environment. It can raise the cost of goods and reduce the sales, thus reducing the resource consumption.

  In terms of personal income tax, the government may increase the tax of those people with high income, in order to reduce the consumption of luxury goods.

- **Import and export volumn**: For high energy consuming products, the government should encourage imports to reduce polluted emissions from local production.

- **Regulating market share**: Government should reduce the market share of those energy-consuming products and subsidize their alternatives.

Through the state driven interventions above, energy consumption and environmental pollution could be reduced, thus relieving global warming. This can promote a more friendly climate and decrease the fragility of country.

According to (26), the budget for controlling climate change is 43.74 million dollars and it accounts for 0.95% of the country's public expenditure budget, which is acceptable.

7 Evaluation of Model on Different Sizes of States

In order to measure the sensitivity of the model aforementioned, we evaluate the performance of model on different sizes of 'state'.

If 'state' represents a city: less authoritative available data could affect the accuracy of result. More importantly, cities usually have less diversity of climate features and unevenly distributed resource. It is less self-contained, which leads to greater mutual influence among fragile indicators. Specifically, Juba, the capital of South Sudan, has suitable climate conditions for agricultural cultivation and economic developemnt, but still rely on imports from northern Sudan severely, because of the lack of technology and turbulent political environment. Statistics shows 17% of the population in Juba can not have stable food provision [13]. Factors like these can hard be considered in our model due to the difference in different cities, thereby affecting the evaluation of CFSI and corresponding prediction and interventions.

If 'state' represents a continent: We only set 16 indicators besides 4 climate-related indicators totally to model, obviously it is not enough to measure the actual situation in a continent. For example, African continent has 5 kinds of climate including tropical desert climate, savannah climate, tropical rainforest climate, mediterranean climate and plateau mountain climate [14]. Too few indicators will result that model results deviate from the reality despite accurate dataset.

We provide suggestions for improvement below.

- City modeling should consider the particular characteristics of the city and the mutual influence between different indicators, also, we prefer to choose the dataset with complete data.

- In continent modeling, more relevant indicators should be considered such as climate type, distribution of light, latitude change.

8 Strengths and Weaknesses

8.1 Strengths

- By using four-dimensional vector to reflect the fragility of countries in CFSI model, the indicators are mapped to high-dimensional space. It can provide the information which can not be expressed in low-dimensional space such as angle and position. Further, by the method of colormap and projection from 4D space to 2D plane, we clearly visualize the relationship between the various indicators and the impact on the fragile indicators.
- We adopt K-means clustering to identify the degree of climate impact and countries's fragility. It investigates the distribution of data. Also, from the inspiration of gravity law, the method we design to measure the mutual influence coefficient of the indicators can be applied universally in weighing the mutual influence of nodes in bidirectional network.
- By combing AR time series analysis, Egypt's national and climatic conditions and the discipline we discover in CSFI model, we successfully predict how things go in next four years with high acurracy and provide specific advice for egypt.
- We employ bilevel programming to give state driven interventions, in which the government take economic development and social effect into account when making strategies to minimize climatic risks and fragility. Decision variables x_{ij}, y_{jk}, function u_{kg}) and constants interact with each other in four entities, so we can provide reasonable interventions in a particular state if data is available.

8.2 Weaknesses

- The method of mapping indicators to high dimension could be hard to visualize if dimension is too high.
- Auto regression can not maintain high prediction accuracy when comes to long-term prediction with limited data.
- Bilevel programming could encounter the problom of data restriction, especially in poor areas. Also, the model might have no optimal solution or have several local optimal solutions when the objective function is nonconvex and noncontinuous.

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Appendices

Here are the main programs we used in our model.

K-Means Algorithm:

```
% Clustering method by improved Kmeans

function [centroid, result] = Kmeanspp(data, k, iteration)
% choose an initial cluster centroid randomly from data
centroid = data(randperm(size(data,1),1),:);
% choose other centroids (a total number of k-1) through roulette method
for i = 2:k
    distance_matrix = zeros(size(data,1),i-1);
    for j = 1:size(distance_matrix,1)
        for k = 1:size(distance_matrix,2)
            distance_matrix(j,k) = sum((data(j,:)-centroid(k,:)) .^ 2);
        end
    end
    % choose next centroid according to distances between samples and
    % previous cluster centroids.
    index = Roulettemethod(distance_matrix);
    %Calculate the kth centroid by using the first k-1 centroids
    centroid(i,:) = data(index,:);
    clear distance_matrix;
end

% following steps are same to original k-means
result = zeros(size(data,1),1);
distance_matrix = zeros(size(data,1), k);
SSE = [];
flag = 0;

for i = 1:iteration
    previous_result = result;
    % calculate distance between each sample and each centroid
    for j = 1:size(distance_matrix,1)
        for k = 1:size(centroid,1)
            distance_matrix(j,k) = sqrt(sum((data(j,:)-centroid(k,:)) .^ 2));
        end
    end

    % assign each sample to the nearest controid
    [~,result] = min(distance_matrix,[],2);
    SSE(i) = 0;
    % recalculate centroid locations after assignment
    for j = 1:k
        centroid(j,:) = mean(data(result(:,1) == j,:));
        [m, ~] = size(data(result(:,1)==j,:));
        % SSE represents the sum of square errors
        SSE(i) = SSE(i) + sum(sqrt(sum( (repmat(centroid(j,:),m,1) - data(result(:,1)==j,:)).^2
    end
    % if classified results on all samples do not change over 5 iteration,
    % clustering process will stop
    if(result == previous_result)
        flag =flag +1;
        if flag == 5
            fprintf('Clustering over after %i iterations...\n',i);
```

```
            break;
        end
    end
end
colors = ['g','b','r','m','c','k'];
for i=1:k
    hold on
    plot3(centroid(i,1),centroid(i,2),centroid(i,3),'Marker','o','MarkerFaceColor',colors(i),'M
    hold on
    subdata = data(result(:,1)==i,:); % n numbers of point in ith class
    [n,~] = size(subdata);
    for j=1:n
        %'Color',[rand(),rand(),rand()]
        plot3(subdata(j,1),subdata(j,2),subdata(j,3),colors(i),'Marker','*','MarkerSize',5)
    end
end
figure;
plot(SSE,'-o')
title('SSE');

end
% grid on
% hold on
% xlabel('Z1-4');
% ylabel('Z2-4');
% zlabel('Z3-4');
```

Entropy Weight Method:

```
% Using entropy weight method to find the index weight
% R is the input matrix, weight vector weights

function weights = EntropyWeight(R)
% get the size of the matrix, rows for the number of objects
% cols forindex number
[rows,cols]=size(R);
h=1/log(rows);
F=zeros(rows,cols);
sumBycols=sum(R,1);
% calculate F
for i=1:rows
  for j=1:cols
      F(i,j)=R(i,j)./sumBycols(1,j);
  end
end

entropy=zeros(rows,cols);
% calculate entropy
for i=1:rows
  for j=1:cols
      if F(i,j)==0
          entropy(i,j)=0;
      else
          entropy(i,j)=log(F(i,j));
      end
  end
end

Hj=-h*(sum(F.*entropy,1)); % Calculate the entropy value Hj
```

```
weights=(1-Hj)/(cols-sum(Hj));
end
```

Auto Regression Prediction Model:

```
% Prediction method by AR2 and Daniel testing
function [fitting,pre_result,RE]=AR2_predict(input)

% input is row time series
%Find the rank of the original time series
R=tiedrank(input);
n=length(input);
T=1:n;

qs=1-6/(n*(n^2-1))*sum((T-R).^2);   % qs
T=qs*sqrt(n-2)/sqrt(1-qs^2) ;  % |T|
t_alpha=tinv(0.975,n-2);     % talpha/2
if  abs(T) > t_alpha
    fprintf( 'not smooth')
    % Daniel testing:time series are not smooth
    % Find the first difference of the original time series
    delta=diff(input);
    %Estimate the parameters of the model by using the AR(2) method
    fitting=ar(delta,2,'ls') ; % fitting struct
    hat=predict(fitting,[delta'; 0],2);
    pre_result=[input(1),input+hat(2)'];% prediction
    RE=abs((pre_result(1:end-1)-input)./input);  %Relative error

else
    fprintf(' smooth')
    % Daniel testing:time series are not smooth
    fitting=ar(input,2,'ls');
    hat = predict(fitting,[input';0],2);
    pre_result=hat';
    RE=abs((pre_result(1:end-1)-input)./input);
end
%
%
```