# A **Tutorial**: Computing QNMs for a Silver Grating in Air

## 1. Introduction

In this document, we present how, with **QNMEig,** computing QNMs for a 1D Ag grating. Since we provide all details about implementing **QNMEig** with COMSOL Multiphysics in the document*''QNMEig\_Sphere.pdf*", we do not repeat here the steps that are already documented in *''QNMEig\_Sphere.pdf*" and we only give the essential details. This document should be read in conjugation with the COMSOL model sheet ***''QNMEig\_1Dgrating.mph".*** This tutorial has been established in relation with Ref. [1].

## 2. Model Definition

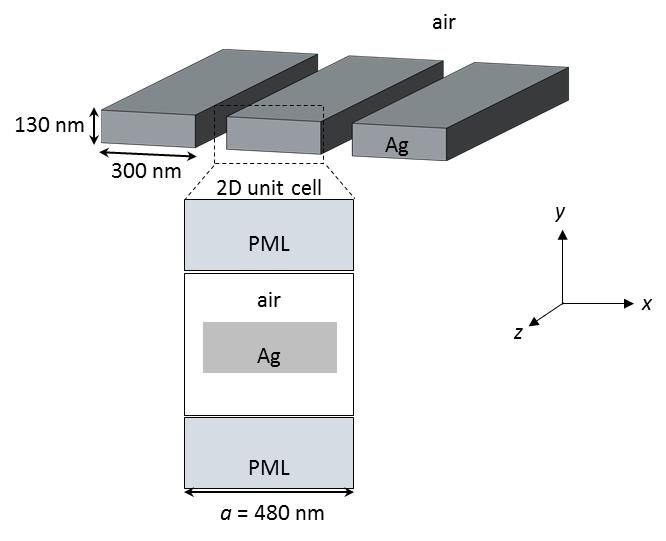


Figure 1. Geometry of a 1D Ag grating surrounded by air.

1 Ag has permittivity

, (1)

with , corresponding to nm in vacuum, , and .

2 We compute TM-polarized Bloch modes, with electric fields only having *x, y* components that are in the same plane as the 2D unit cell in Fig. 1. The magnetic field is parallel to the slits. Owing to the periodicity in the *x* direction, see Fig. 1, the electric fields of the Bloch modes take the form

, (2)

where is called the Bloch wavenumber, and it is a free parameter input by the user; is a periodical function satisfying , where *a* is the grating periodicity.

## 3. Modelling Instructions

Open **COMSOL Multiphysics**. From its **File** menu, choose **New**.

### NEW

In the **New** window, click **Model** **Wizard**.

### MODEL WIZARD

1 In the **Model** **Wizard** window, click **2D**.

2 We choose two physics modules: 1/ **Radio Frequency-**>**Electromagnetic** **Waves**, **Frequency** **Domain (emw)**; 2/ **Mathematics-**>**PDE** **Interfaces**, **Weak** **Form** **PDE**, with dependent variables named ***P1x***, ***P1y***, defined by

, (3)

in the Ag domain.

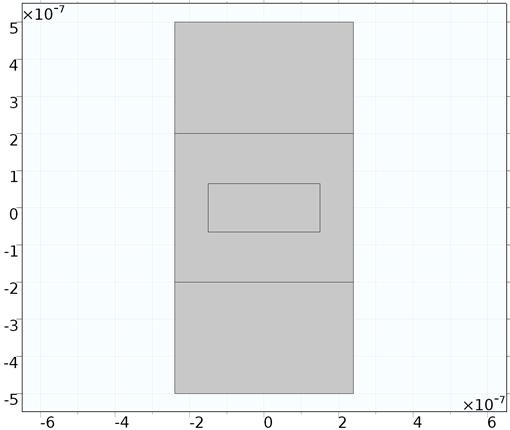
3 We choose the **Eigenfrequency** study.

### DEFINITIONS

All global parameters with their descriptions can be found in the COMSOL model sheet *''QNMEig\_1Dgrating.mph*". Here, we only emphasize important parameter, , the Bloch wavenumber, which is defined in Eq. (1) and is chosen to have a value *0.2\*pi/a* (*a* denotes the grating periodicity)in the *''QNMEig\_1Dgrating.mph*". This parameter is later used when imposing the periodic boundary condition for the grating.

### GEOMETRY

The geometry of the built 2D unit cell is given below.



### MATERIALS

We set relative permittivity of Ag with its non-dispersive permittivity, .

### DEFINITIONS (Local)

In the **Model Builder Window->Component 1->Definition**, we define/add the following things:

1 PMLs, chosen to be *Cartesian-type* PMLs.

2 Two sets of variables, {***DP1x, DP1y***} that relate with {***P1x***, ***P1y*** } defined in the Ag domain

DP1x=epsilon0\_const\*(emw.epsilonrxx\*P1x+emw.epsilonrxy\*P1y),

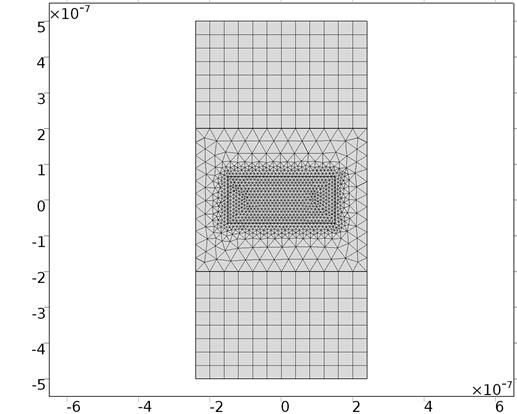
DP1y=epsilon0\_const\*(emw.epsilonryx\*P1x+emw.epsilonryy\*P1y) .

4 Two integration operators: **intAll,** integration over all domains including PML domains; **intMetal**, integration over the Ag domain.

5 One linear extrusion operator, **linext1**, which maps the fields, e.g., ***E***(*x,y*), to ***E***(*x,y*).

6 QNM normalization factor, **QN.** For a short description of the normalization of Bloch modes , we refer to Section 4 ''**Normalization of Bloch Modes"**.

### MESHES



### ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EWV)

1 We add one weak contribution in the Ag domain:

mu0\_const\*QNM\_omega^2\*(test(emw.Ex)\*DP1x+test(emw.Ey)\*DP1y) )\*pml1.detInvT

2 We impose the **Floquet periodicity** condition for the left and right boundaries of the unit cell. We specify the Bloch-wavevector components[Kx Ky] as

|  |  |
| --- | --- |
| Kx | kb |
| Ky | 0 |

### WEAK FORM PDE FOR AUXILIARY FIELDS P1x, P1y

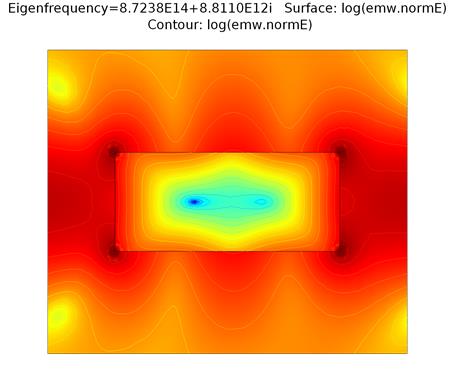
The input weak formulation is

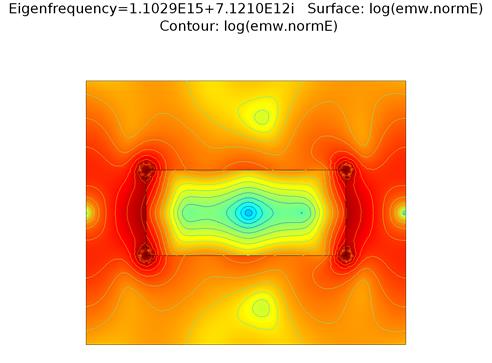
1/lambda\_N^2\*((test(P1x)\*P1x+test(P1y)\*P1y)\*(QNM\_omega^2-j\*gamma\_Ag\*QNM\_omega-omega0\_Ag^2)/omegap\_Ag^2+(test(P1x)\*emw.Ex+test(P1y)\*emw.Ey))

### STUDY 1

We ask the eigensolver to compute 80 QNMs around .

### RESULTS





## 4. Normalization of Bloch Modes

The normalization of a Bloch mode involve another Bloch mode with the opposite Bloch wavenumber and the same eigenfrequency, . Specifically, the normalization factor *QN* (see subsection *''****DEFINITIONS (Local)"*** in "***Modelling Instructions"***) is

, (4)

which, for non-dispersive , could be simplified to

. (5)

For our Ag grating that has inversion symmetry in the *x* direction, and are linked by a simple relation

, (6)

where the subscripts *''", ''", ""* specify the *x, y, z* components of the electric fields. Thus, knowing , is straightforwardly computed from Eq. (6). Technically, Eq. (6) is implemented by defining a linear extrusion operator, **linext1,** see subsection *''****DEFINITIONS (Local)"*** in "***Modelling Instructions"****;* that is **, , .**

## 5. References

[1] P. Lalanne, W. Yan, A. Gras, C. Sauvan, J.-P. Hugonin, M. Besbes, G. Demésy, M. D. Truong, B. Gralak, F. Zolla, A. Nicolet, F. Binkowski, L. Zschiedrich, S. Burger, J. Zimmerling, R. Remis, P. Urbach, H. T. Liu, and T. Weiss, "Quasinormal mode solvers for resonators with dispersive materials," J. Opt. Soc. Am. A **36**, 686-704 (2019).