Determining high-accuracy random phase-only masks for complex local modulation of arbitrary light fields

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We present an iterative algorithm that locally resolves the beam shaping problem through determining high-accuracy random phase-only masks. The algorithm is based on the discrimination of the target field spatial frequencies and a relation between the intensities of the incident and target fields, which is determined from a simple physical model. Considering an extensive set of arbitrary complex input fields, particularly those having random amplitude and phase values, the chosen variable relation between the maximal values of the intensities of the incident and target fields allows the user to find random phase-only masks with adjustable accuracy. Due to the intrinsic high-accuracy of the calculated random phase-only masks for incident random complex light fields, the implementation of an improved double image encryption method is direct.

The simultaneous shaping of amplitude and phase of a light beam circumscribed at a given spatial zone and obtained from a phase-only mask generates essential uses promoting laser applications in many technological fields. Optical trapping [1], laser micro-machine [2, 3], and material manufacturing [4] are among some of the most well-known. To enable a practical and efficient beam shaping technique implementation, the use of diffractive optics and iterative algorithms for phase-only masks calculation is a conventional approach (see [5–7] and references therein). Generally, light fields incident on the diffractive element exhibit low spatial complexity (i.e. plane wavefronts and Gaussian beams). For these input conditions, it is possible to obtain good quality output shaped beams. However, when the incident light beams are random or present considerable spatial complexity, the beam shaping process has not been addressed. Furthermore, the documented iterative algorithms do not show a systematic procedure derived from a physical model that assures the best accuracy found for a given incident light beam. In this work, we present an iterative algorithm that locally resolves the beam shaping problem, even for random input complex light fields, and with high accuracy.

The proposed algorithm forces the determined phase-only mask values to follow a random statistic while verifying a uniform distribution law. Thus, when the incident light field possesses random attributes in its amplitude and phase values, it can be shown that the obtained phase mask encrypts the target complex field. Due to the high-accuracy obtained by using the presented method, the complex values of the target field can be previously noiselet transform [8] or similar and then, the calculated phase-only mask values reproduce the noiselet transform coefficients for a given random input light field successfully. Similarly, several encryption methods based on chaos can be additionally implemented [9]. This point of view is related to the common double random phase encoding technique, but simultaneously considering target images corresponding to the amplitude and phase of the complex target field [10]. The complexity of the employed encryption optical setup is reduced, and no optical decryption implementation is necessary. For the interested readers, a review of iterative phase retrieval for measurement and encryption should be consulted in Ref. [11]. In what follows, we present an algorithm for random phase-only mask calculation from a practical point of view and discuss several methodological issues for easy understanding of the readers.

Figure 1 shows the employed optical scheme, which describes the main steps involved in the random phase-only mask calculation by using a phase retrieval approach. The random phase-only mask $\tau(x,y) = \exp[i\phi_\tau(x,y)]$, with $\phi_\tau(x,y)$ in the interval $(-\pi,\pi]$ and following a uniform distribution, modulates an incident light beam $E_i(x,y) = A_i(x,y)\exp[i\phi_i(x,y)]$ for arbitrary values of amplitude $A_i(x,y)$ and phase $\phi_i(x,y)$, where (x,y) denotes the spatial coordinates. This modulation is realized using a lens L of focal length f at the Fourier plane to obtain the output complex light beam $E_o(x,y)$. It is convenient to divide the Fourier plane in two spatial domains. A target domain, locally defined as $t(x,y)$, and the rest as $b(x,y)$ (see domains $t(x,y)$ and $b(x,y)$ in Fig. 1 for clarification). For the sake of clarity, we omitted the coordinates (x,y) and defined the domain $Ones$ as a matrix composed by ones, which is decomposed in two disjoint subsets b and t verifying the identity $Ones = b + t$ by adding zeros in b and t where necessary. Thus, the output light beam verifies the identity $E_o = E_o b + E_o t$. By adopting the target beam named as $E_t = E_o t = A_t \exp(i\phi_t)$, we remark that the user selects the target amplitude A_t and phase ϕ_t, $E_b = E_o b$ defines a light beam outside of the target area t, namely $b = Ones - t$. From Fig. 1, we obtain the following identity

$$E_i \tau = iFFT(E_b + E_t), \qquad \textbf{(1)}$$

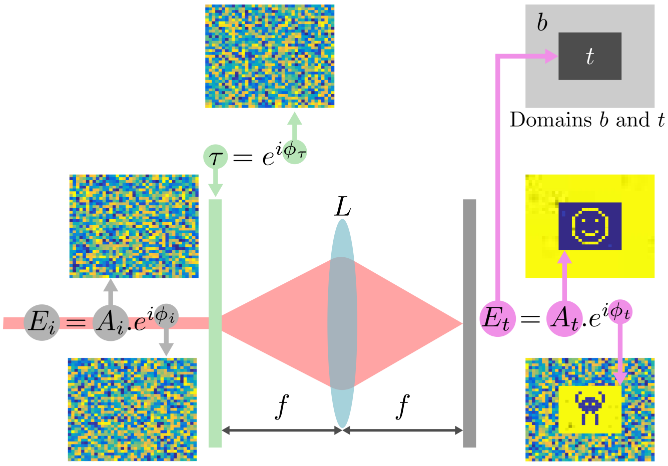

Fig. 1. Optical setup for beam shaping and encryption. $E_i(x,y) = A_i(x,y)\exp[i\phi_i(x,y)]$ is an incident light beam with arbitrary values of amplitude $A_i(x,y)$ and phase $\phi_i(x,y)$. $\tau(x,y) = \exp[i\phi_\tau(x,y)]$ is the calculated random phase-only mask. The target beam named as $E_t = A_t\exp[i\phi_t]$ is elected by the user that is reproduced within the target domain t by the lens L.

where $iFFT$ denotes the inverse Fast Fourier Transform (FFT). By renaming $B \equiv iFFT(E_b)$ and $T \equiv iFFT(E_t)$ in Eq. (1), the following condition about the amplitudes is verified $A_i^2 = |B|^2 + |T|^2 + (B^*T + BT^*)$, where $(^*)$ is the complex conjugation. If $B \equiv B_0\exp(i\phi)$ and $c \equiv T\exp(-i\phi) + T^*\exp(i\phi)$, then the amplitude B_0 is completely determined by the expression [12]

$$B_0 = \frac{1}{2}[-c \pm \sqrt{c^2 - 4(|T|^2 - A_i^2)}]. \qquad \textbf{(2)}$$

Given that the field B is a function of ϕ, we propose the iterative algorithm named as Algorithm 1, which recovers ϕ and then, in a direct way, determines the phase-only mask τ. In Ref. [12], a diagonal phase modulation of the form $\phi(x,y) = \alpha x + \beta y$ with coefficients α and β defined by an external operator was used, and an analytic solution for the phase introduced by a phase-only spatial light modulator (SLM) to generate far-field phase and amplitude distribution within a domain of interest was developed. It is important to remark that the approach described in Ref. [12] cannot determine phase masks without the intervention of an external operator by selecting and testing carriers ϕ. There is not any systematic approach based on a physical model to proceed in this selection, which is a tricky subject in applications that contain a wide variety of input fields, including those with random complex values. Besides, the application of these selected carriers ϕ competes with the elimination of the zero order effect in the performance obtained by using spatial light modulators. This drawback is eliminated by adopting a random phase-only mask approach.

The iterative Algorithm 1 is based on alternating projections. These recursive methods often fail to work or show partial results that are difficult to interpret. A reason to explain this difficulty is that there is no guarantee to find an algorithmic solution. This problem is not convex, and its solution depends on the initialization and the complex object signal. In practice, a strategy to follow is to combine prior information that increases the probability of convergence to the right solution. Practitioners currently resort to various ad hoc methods with adequate

Algorithm 1. Iterative algorithm for phase retrieval

1: **procedure** PR(E_i,E_o,*thresh*) ▷ Find τ for input arguments: E_i, E_o, *thresh*
2: $T \leftarrow iFFT[E_o t]$
3: $\phi \leftarrow \phi$ ▷ Initialize with arbitrary random ϕ
4: **while** $rms > thresh$ **do** ▷ Break if $rms \leq thresh$
5: $c \leftarrow T\exp(-i\phi) + conj(T)\exp(i\phi)$
6: $B_0 \leftarrow [-c + \sqrt{c^2 - 4(|T|^2 - A_i^2)}]/2$ ▷ Eq. 2
7: $B \leftarrow iFFT\{FFT[T + B_0\exp(i\phi)]c\}$
8: $\phi \leftarrow \arg(B)$
9: $\tau \leftarrow [T + B_0\exp(i\phi)]/E_i$ ▷ Hybrid condition
10: $rms \leftarrow \log\{\frac{1}{N^2}\sum[|E_t| - |FFT(E_i\tau)t|]^2\}$
11: **return** τ ▷ $\tau = \exp(i\phi_\tau)$

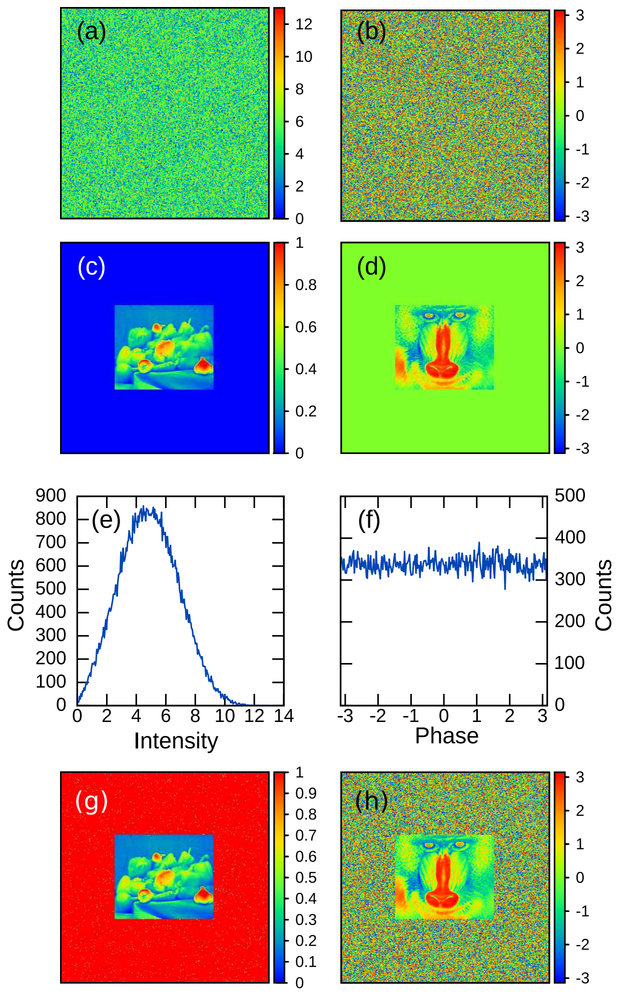

Fig. 2. Amplitude (a) and phase (b) of the incident light field E_i. Amplitude (c) and phase (d) of the target beam E_t. Amplitude (e) and phase (f) histograms corresponding to the incident field. Amplitude (g) and phase (h) of the recovered field $E_r = FFT(E_i\tau)$.

prior information [13]. For that reason, Algorithm 1 incorporates the numerical constraint defined by Eq. (2) specified in step 6 and employs a hybrid condition defined in step 9. Both resources successfully resolve the beam shaping problem as it is shown in what follows.

Suppose that is necessary to obtain a random phase-only mask of spatial dimensions of $N_x = 270$ and $N_y = 320$ pixels, capable of reproducing a target field of $N^2 = 128 \times 128$ pixels containing images of peppers as its amplitude and a baboon as the phase for a given incident random complex light field as shown in Fig. 2. More precisely, Figs. 2(a) and 2(b) show the considered incident light beam of random amplitude A_i and phase ϕ_i, Figs. 2(e) and 2(f) illustrate the corresponding histograms of Figs. 2(a) and 2(b), and the original target images of peppers and the baboon are also shown in Figs. 2(c) and 2(d), respectively. For these input arguments, Figs. 2(g) and 2(h) successfully demonstrate the recovered amplitude of peppers $|E_r|$ and the baboon phase $\arg(E_r)$ images obtained by means of the expression $E_r = FFT\{E_i \exp[i \arg(\tau)]\}$ for the best threshold found by using Algorithm 1 and the calculated random phase-only mask. The obtained random phase-only mask with its corresponding histogram are depicted in Figs. 3(a) and 3(b), respectively. The RMS values calculated as $rms_a = \log_{10}[1/N^2 \sum_{N^2}(|E_t| - |E_r|)^2 t]$, and $rms_p = \log_{10}\{1/N^2 \sum_{N^2}[\arg(E_t) - \arg(E_r)]^2 t\}$, for the corresponding iterations of amplitudes and phases are shown in Figs. 3(c) and 3(d), respectively. The replication of the presented results in Figs. 2 and 3 can be carried out by the open source scripts available in Ref. [14]. Critical issues that can lead to misunderstanding of the presented approach are highlighted and discussed in what follows. These points are of importance given that it is common knowledge the development of iterative phase retrieval algorithms often gives relative accuracy with high dependence of the input fields, oversampling conditions, and convergence constraints to name a few.

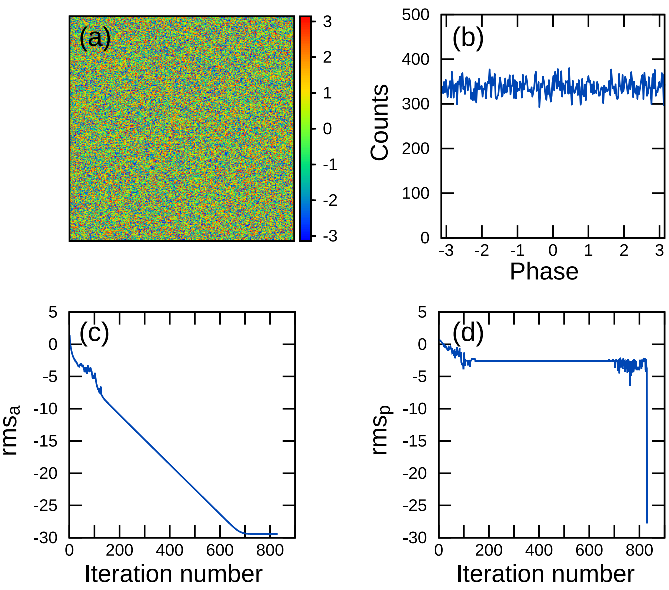

Fig. 3. Phase-only mask (a) obtained in Fig. 2 and its histogram (b). Evolution of the RMS values for the recovered amplitude (c) and phase (d) using Algorithm 1.

Let us start remarking the aim of the iterative algorithm implementation. For that, it is assumed that the oversampling conditions are satisfied [15]. The identity given by Eq. (1) shows that ϕ is a variable to be determined because the amplitude B_0 is presently fixed by Eq. (2). The readers should note that the information contained in ϕ substantially varies in each iteration with the information placed outside of the target region (i.e. b), which has no experimental interest. This point of view does not reflect the traditional approach used in the development of iterative phase retrieval algorithms. Furthermore, each arbitrary random initialization ϕ in step 3 of Algorithm 1 successfully resolves the beam shaping problem obtaining a different random phase-only mask τ.

The relationship between the maximum amplitudes $|T|$ and A_i, named $\alpha \equiv \sup\{|T|\}/\sup\{A_i\}$, allows modifying the convergence velocity of the iterative algorithm and the obtained accuracy of the determined phase-only mask. This mechanism can be seen by developing Eq. (1) to the first order in $\alpha << 1$ and substituting B_0 with the value given by Eq. (2). Thus, $\tau = \exp i(\phi - \phi_i)[1 + i\alpha \sin(\phi_T - \phi)]$, where $\phi_T = \arg(T)$. In the limit of $\alpha << 1$, the amplitude of τ tends to the unity. Particularly, the determined phase mask in Fig. 3 exhibits a unitary mean value of amplitude with a standard deviation of $std = 2.05 \times 10^{-16}$ considering a value of the parameter $\alpha = 0.5$ (see maximum ratings in Figs. 2(a) and 2(c)). It can be seen that minor accuracy is reached by using few iterations for determined α values. High values of the parameter α can even generate tolerable deviations from the unity in the mask amplitude according to the convenience selected by the user. Fig. 4 resumes this claim. For the incidence conditions used in Fig. 2, Fig. 4 shows the amplitude and phase obtained by using 150 iterations, a relation $\alpha = 0.65$, and adopting a restricted resolution of only 8 bits for the phase mask values and 12 bits in the detector. The phase values ϕ_τ discretized in 8 bits were taken as $\phi_\tau = \arg(\tau)$ and, in this case, the calculated standard deviation for τ was of $std = 0.0084$. To quantify the performance of the obtained numerical results with low accuracy shown in Fig. 4, the spatial distortions of phase and amplitude between the original and the recovered images were evaluated utilizing the structural similarity (SSIM) index Q [16]. This index is in the values range $[-1, 1]$, where $Q = 1$ is satisfied for the exact result. By using a sliding window of 5×5 pixels, the obtained values were $Q = 0.89$ and $Q = 0.93$ for peppers and the baboon, respectively.

Noiselets are noise-like wavelet functions and give the worst compressibility behavior concerning other types of orthogonal wavelet analysis. In practice, this means that coefficients with large values which describe the substantial information of the analyzed data are not found. Therefore, the noiselet transform is a useful approach in encryption procedures. Figs. 5(a) and 5(b) show the amplitude and phase of the obtained noiselet coefficients by using a 2D fast noiselet transform [17] of the target complex field depicted in Fig. 2(c) and (d), respectively. With the same initial conditions specified in Fig. 2, for 150 iterations, $\alpha = 0.65$, with a level of discretization of 8 bits in the values of the calculated phase mask and 12 bits in the detector, by applying Algorithm 1 on the target field defined by these noiselet coefficients, Figs. 5(c) and 5(d) show the results of the amplitude and phase of the antitransform coefficients that recover peppers and the baboon images, respectively. The obtained values for the quality indexes were $Q = 0.66$ for peppers and $Q = 0.74$ for the baboon.

Another topic of interest is the implementation of SLM's as phase-only masks. In this case, by adopting an SLM format of $N_y = 1920$ by $N_x = 1080$ pixels and defining macro pixels of 6×4 pixels with a width of 1 inactive pixel in line, the phase mask determined in the conditions of Fig. 4 achieved the following results when it was identically transported to the SLM's format by using the macro pixel structure. Figs. 5(e) and 5(f) illustrate the amplitude and phase as seen by the detector, respec-

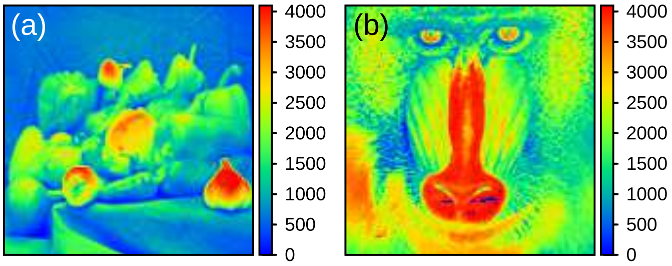

Fig. 4. (a) Amplitude (peppers) and (b) phase (baboon) obtained by using Algorithm 1 in the initial conditions of Fig. 2 for 150 iterations, $\alpha = 0.65$, with a level of discretization of 8 bits in the values of the phase mask and 12 bits in the detector.

tively. The reader should also note that the target field was displaced from the origin in the calculus ($x_{offset} = 72, y_{offset} = 97$ pixels). This procedure prevents the common zero order problem. Figs. 5(g) and 5(h) zoom the details of the obtained target beam with values of the quality indexes of $Q = 0.87$ and $Q = 0.86$ for peppers and the baboon, respectively.

Summarizing, we solve the local beam shaping problem by calculating a high accuracy random phase-only mask. This is performed by introducing an iterative algorithm, which is also offered to the optical community for reproducibility and testing purposes. By adjusting the relationship between the maximum amplitude ratings of the input and target fields, the user can restrain the convergence velocity of the algorithm and also the final accuracy of the computed random phase mask. High performance in the computed mask is found when the incident beams are selected with random characteristics in its amplitude and phase values. This issue favors the implementation of an improved double images encryption method obtaining an additional target phase image by using a simple optical setup. The proposed algorithm supports the addition of supplementary encryption processes like the use of the noiselet transform. New insights and a full evaluation of the proposed method will be presented in a future paper.

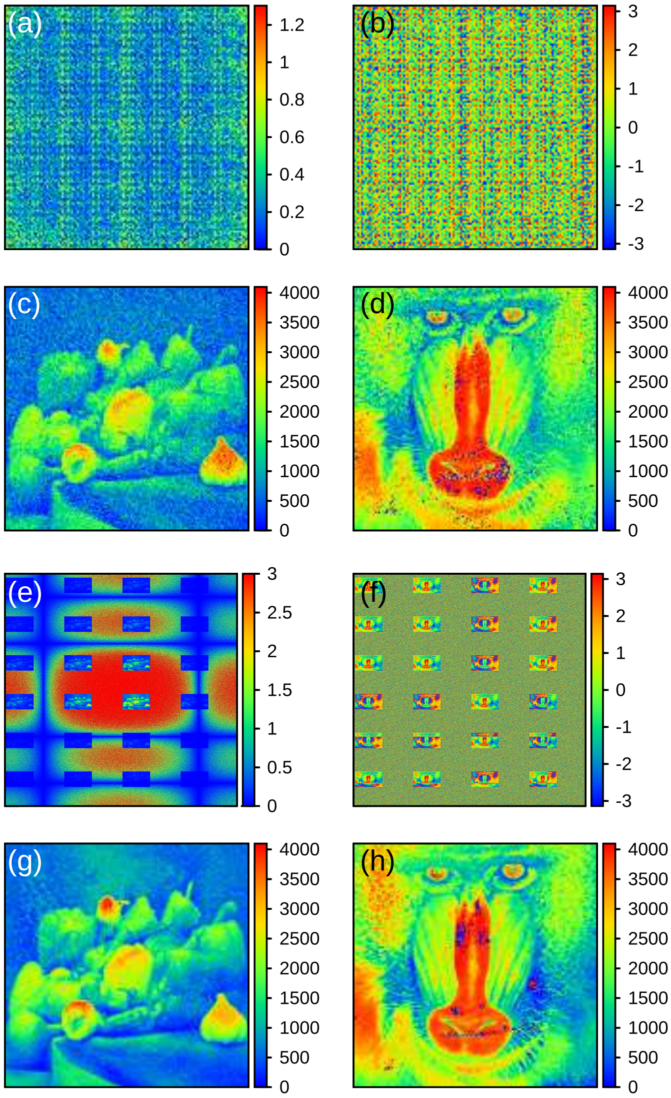

Fig. 5. Amplitude (a) and phase (b) of the noiselet transform coefficients of peppers (Fig. 2(c)) and the baboon (Fig. 2(d)) images, respectively. Amplitude (c) and phase (d) of the retrieved noiselet antitransform coefficients. Recovered amplitude (e) and phase (f) employing a SLM with a macropixel of 6×4 pixels by using Algorithm 1 in the same calculus conditions that Fig. 4 and avoiding the zero order effect. Amplitude (g) and phase (h) of the recovered field.

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