Question 1

Rank 1 approximation is:

```
 1.8098    2.0608    2.3119
 4.4186    5.0314    5.6443
 7.0273    8.0021    8.9768
 9.6361   10.9727   12.3093
```
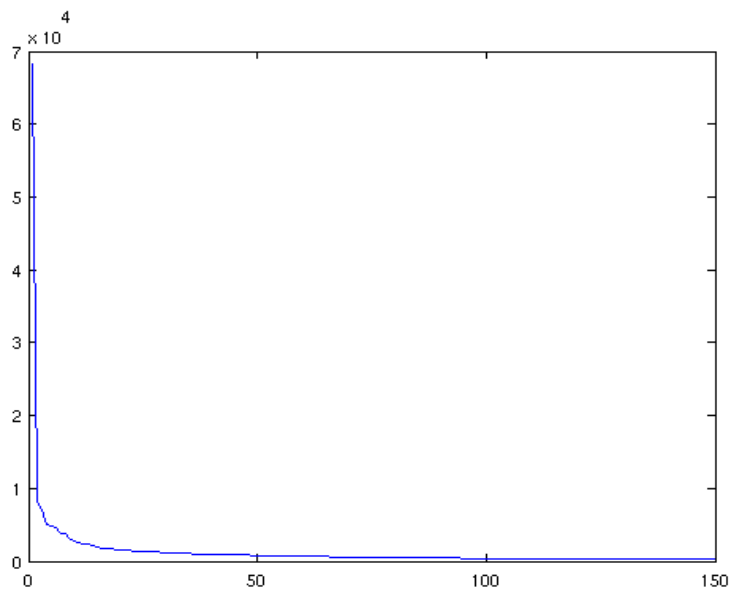
Relative Error calculated by method 1

  0.0506

Relative Error calculated by method 2

  0.0506

Question 2
a)

From the graph above we can see that the singular value drop down dramatically from about 6.8×10^4 (rank 1 singular value) down to 0(from about rank 100 singular value). This result implies that the terms which can have significant impact on picture's quality are the top ranked terms. Continuously increase the terms will make the increasing of picture's quality less and less significant, especially after about 100 terms(when the singular values are approximate to zero).

b)

The first row are rank 8, 16, 32 approximation(from left to right).
The second row are rank 64, 128, original picture(from left to right).

The picture with rank 8 approximation seems to have very low quality. Compared to that, the picture with rank 16 approximation is much better, though still a litter ambiguous. The quality of the rest of pictures are good to me, I can hardly tell the difference.

c) The rank is 361
The picture is:

Suppose the rank of matrix is n, If the rank of approximation is smaller than 2n, this kind of compression can save some space, because the rank k approximation only need to store first k columns form matrix U, first k columns of matrix V and k singular values. However, if the rank of approximation is greater than 2n, the storage space will be no less than the original picture, it is better to use original picture directly, instead approximation by this method.
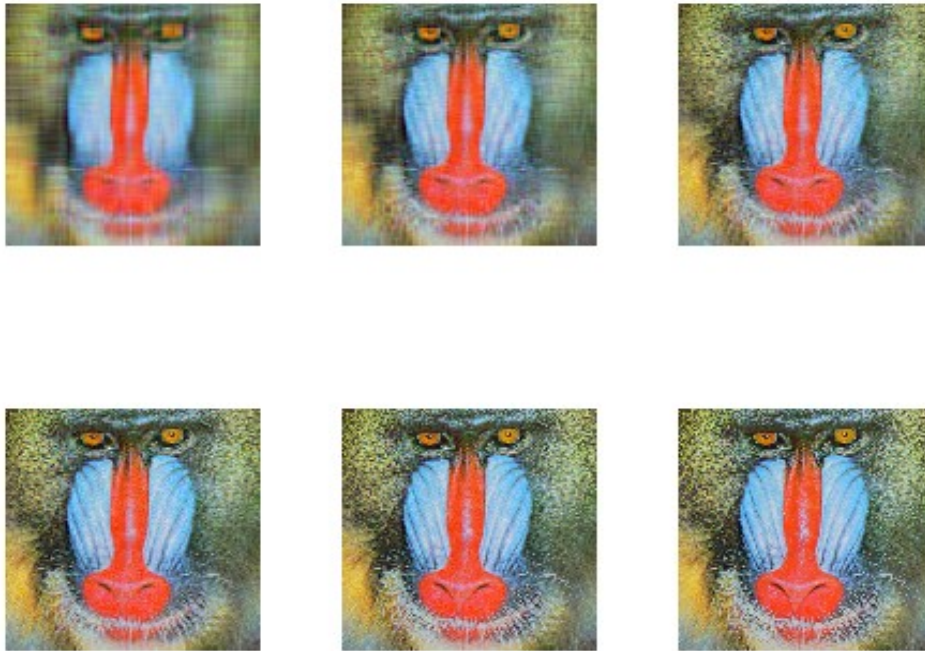
Question 3
a)
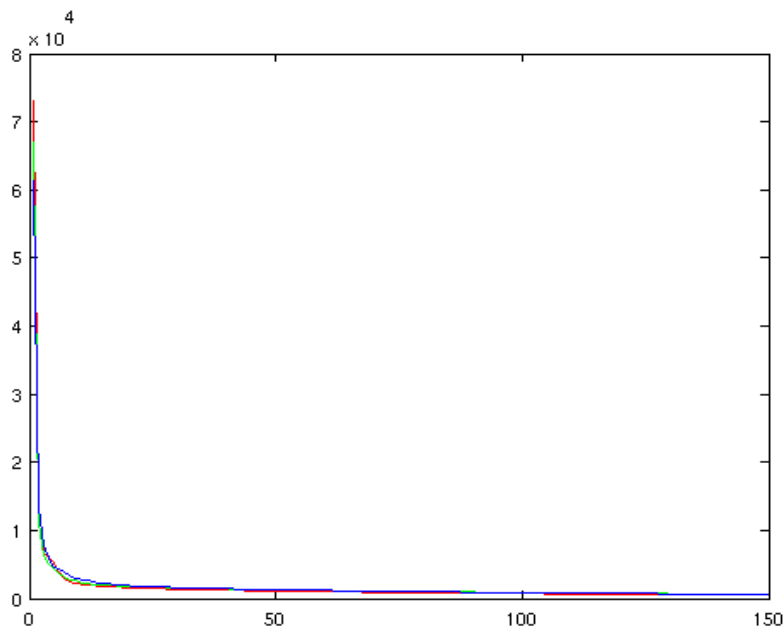
The red, green, blue lines are represent each of the three colors.
We can see that the distribution of the singular value for each of the color are very even, so the three curves are over lapped for the most part. As the same to the gray picture in question 2, the singular value drop down dramatically from about 7.3×10^4 (rank 1 singular value) down to 0(from about rank 120 singular value). This result implies that the terms which can have significant impact on picture's quality are the top ranked terms. Continuously increase the terms will make the increasing of picture's quality less and less significant.

b)

The first row are rank 8, 16, 32 approximation(from left to right).
The second row are rank 64, 128, original picture(from left to right).

Similar to the picture in question 2, the picture with rank 8 approximation seems to have very low quality. Compared to that, the picture with rank 16 approximation is much better, though still not very clear. The quality of the rest of pictures are good to me, I can hardly tell the difference. Every color seems to be well mixed.

c) I would say rank 32 approximation is good enough to be eyeball norm. We can see form result in a) that after the rank 32, the singular values are approximate to zero, which means increasing the rank by adding the following terms to the approximation will not make a large difference to the picture.

d)
d-a)
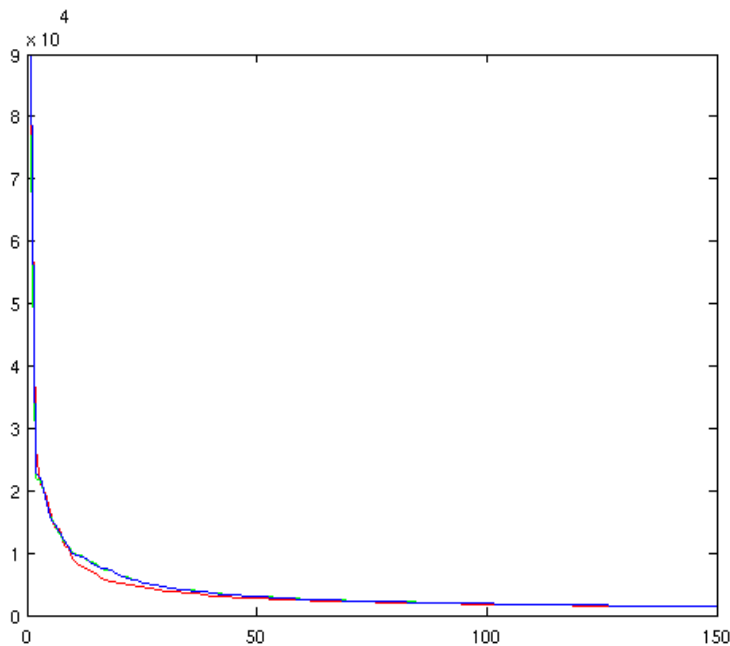
The red, green, blue lines are represent each of the three colors.
We can see that the distribution of the singular value for each of the color are very even, so the three curves are over lapped for the most part. As the same to the gray picture in question 2, the singular value drop down dramatically from about 9×10^4 (rank 1 singular value) down to 0.1. This result implies that the terms which can have significant impact on picture's quality are the top ranked terms. Continuously increase the terms will make the increasing of picture's quality less and less significant. However, compared to graph in a), the dropping of singular value is slower, which implies higher order of approximation is needed.

d-b)

The first row are rank 8, 16, 32 approximation(from left to right).
The second row are rank 64, 128, original picture(from left to right).

The first two picture(rank 8 and rank 16) is not clear, but the picture with rank 16 is much better than the first picture, The pictures which have rank 32 or higher rank of approximation are acceptable to me and there is no significance between them.

d-c)
I would say rank 32 approximation is good enough to be eyeball norm. similar to the conclusion in c), we can see form result in d-a) that after the rank 32, the singular values are approximate to zero, which means increasing the rank by adding the following terms to the approximation will not make a large difference to the picture.