Introduction:
In this lab, a 1D heat equation was implemented using the forward Euler approach. In order to analyze the accuracy of this method, a manufactured solution was given. The data for that solution was calculated and the solver was used to solve the heat equation. The error and convergent rate was estimated using the manufactured solution.

Methodology:
The manufactured solution was given as: u(x,t)=sin(x)+1, based on these and together with the heat equation, we can get the right hand side data as: f(x,t)=sin(x).
Also the manufactured solution implies the initial condition:
u(x, t0)=sin(x)+1
and the boundary conditions:
u(x0, t)=u(xn,t)=1
Given this information, the PDE can be solved using the forward Euler approach on time triangulation. The simulation was done in space region of [0 2pi] and time region of [0 1]. After get the numerical solution, the average l2 error norm was calculated,and the convergence was analyzed. To calculate the convergence on space, time steps were finxed to be 1/10000 and the spacial grid number was set as 40, 80, 160, 320 separately. On the other hand, to calculate the convergence on space, the spatial grid number was fixed as 100 and the spacial grid number was set as 500, 1000, 2000, 4000 separately. The theoretical temporal convergence order is O(n) and the theoretical spatial convergence order is O(n^2).

Result:
1. test for convergence related to the spatial grid. Solutions for the PDE with time steps number to be 10000 and spacial grid number to be 40, 80, 160, 320 was plotted in following figure

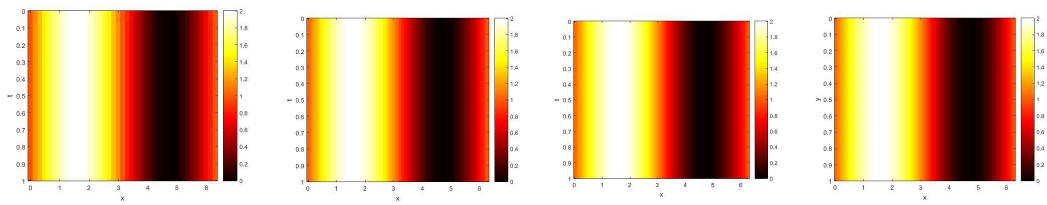

Figure 1: the solution for the PDE with time steps number to be 10000 and spacial grid number to be 40, 80, 160, 320(left to right) separately

The average l2 error norm was plotted in the following log-log plot:

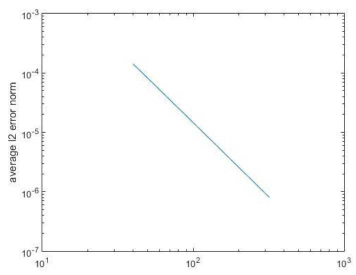

Figure 2: the average l2 error norm for different number of spatial grids 40, 80, 160, 320.
The convergence rate was calculated in the following table:

n	Average l2 error norm	Convergence rate
40	0.000141781	Na
80	2.53723e-05	2.4823
160	4.51306e-06	2.4911
320	8.00288e-07	2.4955

Table 1: The error and convergence rate with different value of n

2. test for convergence related to the temporal grid. Solutions for the PDE with spatial steps number to be 100 and temporal grid number to be 500, 1000, 2000, 4000 were plotted in following figure

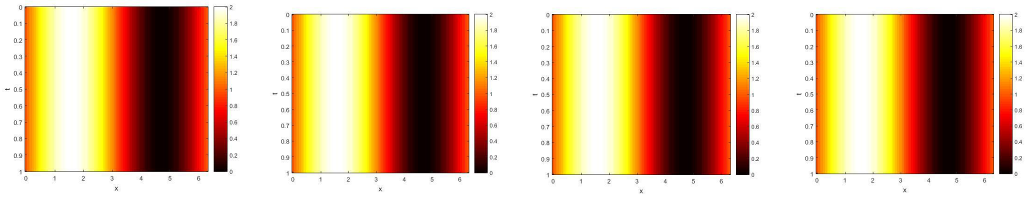

Figure 3: the solution for the PDE with spatial steps number to be 100 and time grid number to be 500, 1000, 2000, 4000 (left to right) separately.

The average l2 error norm was plotted in the following log-log plot:

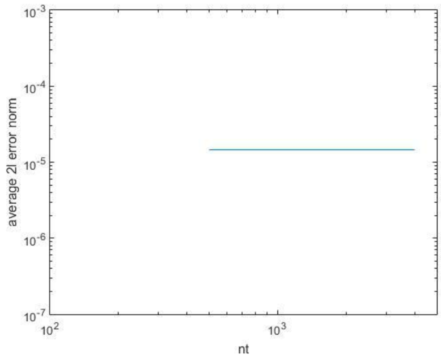

Figure4: the average l2 error norm for different number of temporal grids 500, 1000, 2000, 4000.

The convergence rate was calculated in the following table:

nt	Average l2 error norm	Convergence rate
500	1.45679e-05	Na
1000	1.45637e-05	4.1600e-04
2000	1.45616e-05	2.0804e-04
4000	1.45605e-05	1.0899e-04

Table 2: The error and convergence rate with different value of number of temporal grids

Conclusion:
From the solution of PDE in Figure 1 and 3, we can see that the result at the final time is basically the same as the initial condition, which is sin(x)+1. This phenomenon is consistent with the fact that the manufactured solution of this method is time dependent, thus indicate the correctly implementation of this 1D heat equation PDE solver.

For the spatial convergence rate, I get a convergence rate approximate to 2.5 as n increasing in size. Since the convergence rate of the theoretical spatial discretizationis 2, the simulation result is larger than expected. This difference can be cause by the error on temporal discretization, which also determines the error.

As to the temporal discretization, I get the convergence rate approximate to 0, despite the fact that the first order scheme was used. This phenomenon could be caused by the fact that the manufactured solution is time independent. The convergence of first order scheme can only be satisfied if the first derivative of underlying function exist. In this case, however, there are no first derivative at the direction of time axis, which lead to the error independent from the temporal step size.

In general, this manufactured solution can be used to justify the correctly implementation of the algorithm. But it is not suitable to be used on accuracy analysis. A time dependent manufactured solution is preferred which could give a better convergence rate estimation.