Introduction

In this lab, I implemented the Controlled Random Search method, which is a global minimization method. This method is based on the research of Price et al.(1977). The result of the method has been analyzed and compared with the BFGS method I implemented last lab.

Methodology

The CRS method is implemented as following: Firstly, choose certain number of points N (N=50 in this experiment) randomly and store them in an array A, which is the cloud. Then generate the new trial point based on the centroid of the randomly chosen points from A(discussed later). If the new trial point meets the constrains and the function values of this point is smaller than the point in array A which has the largest function value(point M), replace the M with the newly generated point. By continuously replacing the points in A with the points which have the smaller function values, the method can converged to the global minimum.

The coordinate of new trial point is generated by calculating P=2*G-R, where G is the centroid of randomly chosen points(n points) from A, and R is the another point chosen from A which represent the pole of the simplex.

This method was evaluated using the example 4 of the paper from Price (1977). The method was then compared with BFGS method implemented last week.

Result

When running the RCS program, the cloud finally converged to point x=1, y=1, with the function value to be 0. The minimum, average, and maximum of the clouds function values has been plotted as the following figure(Fig 1)

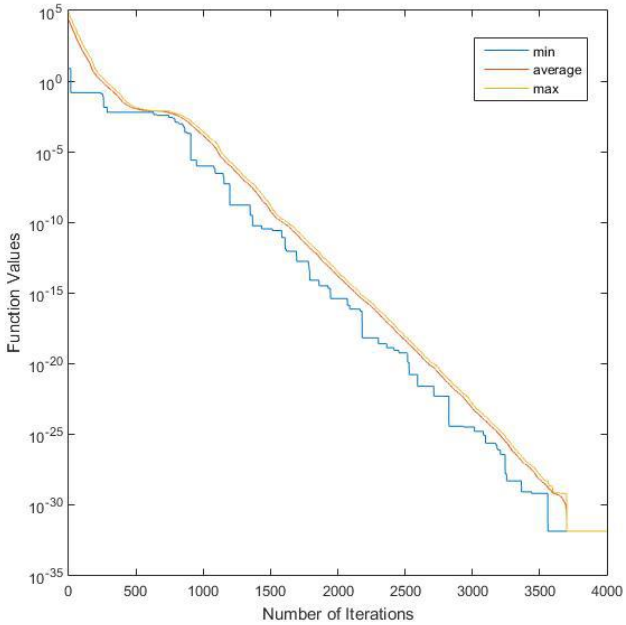


Figure 1: The minimum, average, and maximum value of the cloud in iterations from 1 to 4000

The cloud distribution of the initial stage, 1/3 of iterations, 2/3 of iterations and final stage have been plotted. We can see that the cloud converged to (1,1).

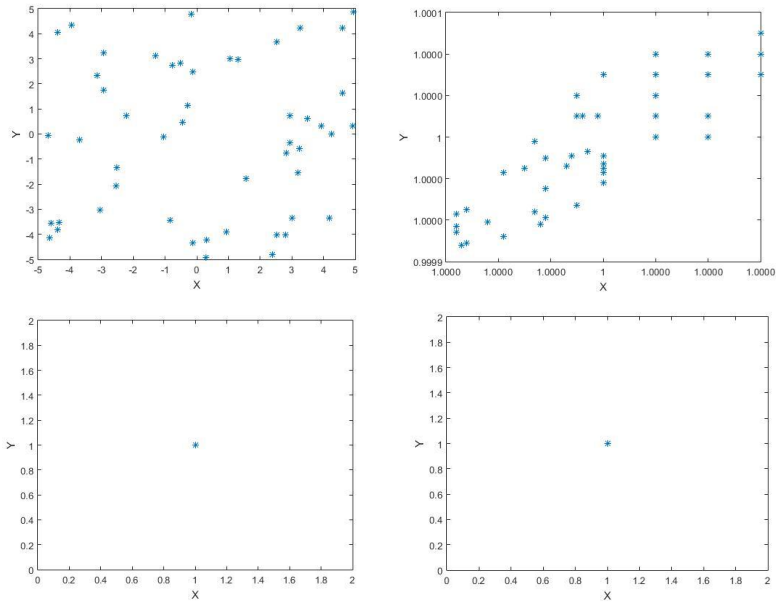


Figure 2: The distribution of the cloud. From left to right, up to down, the cloud distribution at the iteration 1, 1333, 2667 and 4000.

When running the BFGS program, the method was converged to x= 0.341, y= 0.116 with f(x,y)= 8.75777e-20 in 10 iterations, which is another global minimum.

Discussion:
In this method, I implemented the algorithm CRS, and tested it on the given example. From figure 1, we can see that the method converged to one of the global minimum with exponential convergence rate(since figure 1 is a semi-log plot ). The cloud also concentrated to x=1, y=1, which is the axis of the corresponding local minimum, as the number of iterations increase. This result suggested the successfully implementation of the CRSS method.

When compared with the result of paper, however, the method implemented in this lab sometimes cannot completely abandon the local minimum. When running the method 10 times, there is one time that this method was converged to the local minimum with x=-0.664, y=0.441, f(x,y)=0.0074. This phenomenon hasn't been reported in Price's paper.

The BFGS method will always converged to one of the global minimum with x= 0.341, y= 0.116, for the given initial point. This method is much efficient compared with CRS, it only need 10 iterations to converge. However, this method need the first order and second order derivative of f(x, y). Also it cannot grantee that the minimum found is global minimum, which minimum it converges to is also determined on the choose of start point.