Lab 5 Report

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1 Introduction

In this lab, a statistical hypothesis testing was performed on the study of the distribution of the mean value of a binary dataset. The statistical power(1-β) was tested by numerical experiment and also theoretical approach, under different value of size for each of sample. The results are plotted and comparison was made between the experimental and theoretical result.

2 Methodology

100 samples was randomly picked form the dataset "survey1.dat" with different sample size n. The mean value of each of the samples($p*$) was calculated. To determine whether or not the sample should be rejected, I tested whether the p* meets the α threshold which is 0.05:
According to the question, we have

$$H_0 : p_0 = 0.5;\ H_a : p_0 < 0.5\ or\ p_0 > 0.5$$

because $\alpha = 0.05$, in order to reject null hypothesis of certain sample we need to have

$$Z < -0.025\ or\ Z > 0.025$$

so we have

$$(p * -\mu)/\sigma < -0.025\ or\ (p * -\mu)/\sigma > 0.025$$

where

$$\mu = p;\ \sigma = \sqrt{\frac{p(1-p)}{n}}$$

which is given by the question. Combining the equations above, we have the rejection criteria:

$$p* > 0.5 + \frac{0.98}{\sqrt{n}}\ or\ p* > 0.5 - \frac{0.98}{\sqrt{n}}$$

Based on this criteria above, I made the program to find the proportion of times the null hypothesis is rejected. The figure was plot using the data I got.
To calculate the theoretical power, based on the definition of power and the given conditions from the problem, I have:

$$1 - \beta = 1 - P(0.5 - \frac{0.98}{\sqrt{n}} < p* < 0.5 + \frac{0.98}{\sqrt{n}} | p = 0.55)$$

when $p = 0.55$, we have

$$\mu = p = 0.55;\quad \sigma = \sqrt{\frac{p(1-p)}{n}} = 0.4975/\sqrt{n}$$

Also

$$Z = (p * -\mu)/\sigma$$

so I have

$$1-\beta = 1-P(\frac{0.5-\frac{0.98}{\sqrt{n}}-0.55}{0.4975/\sqrt{n}} < Z < \frac{0.5+\frac{0.98}{\sqrt{n}}-0.55}{0.4975/\sqrt{n}})$$
$$= 1-P(\frac{-0.05\sqrt{n}-0.98}{0.4975} < Z < \frac{-0.05\sqrt{n}+0.98}{0.4975})$$
$$= 1-\int_{-0.1005\sqrt{n}-1.9698}^{-0.1005\sqrt{n}+1.9698} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}dx$$

which is the theoretical power of the experiment as a function of sample size.

3 Result

The numerical experiment to test the rejection rate with n=[50,100,250,500,1000] gives the result 0.11, 0.17, 0.39, 0.61, 0.86 separately. The result was plotted in Figure 1(blue line).
Using the formula

$$1-\beta = 1-\int_{-0.1005\sqrt{n}-1.9698}^{-0.1005\sqrt{n}+1.9698} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}dx$$

together with the online Z value table. I calculated the theoretical power for each value of n, the result is: 0.1077, 0.1688, 0.3519, 0.6093, 0.8865 for n=[50,100,250,500,1000] separately. The result was plotted in Figure 1(red line).

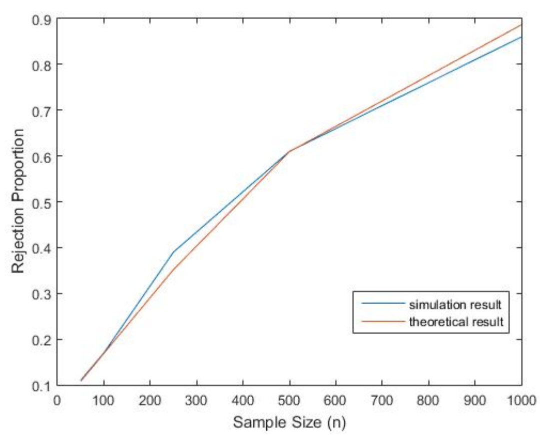

Figure 1: The rejection rate with different sample size obtained form numerical simulation(blue line) and the theoretical inference by calculating the statistical power(read line).

4 Discussion

From Figure 1 we can see that as the sample size increase, both the proportion of null hypothesis rejected by experiment and the theoretical power increase. Actually, this two group of values(data in blue line and read line) are very similar to each other, with the difference no more than ± 0.05 except form the data at n=250. The result is as expected since the real mean value of the data dataset is 0.55, which is exactly the given value of p when calculating the theoretical power. The difference between the experimental result and the theoretical power may caused by the limited amount of sample(only 100 samples was taken in numerical experiment) and drawing repeated samples.