Lab 4 Report

Question 1:
The black line which pass though the noisy data(Figure 1) is the least-squares B-spline approximation to the data:

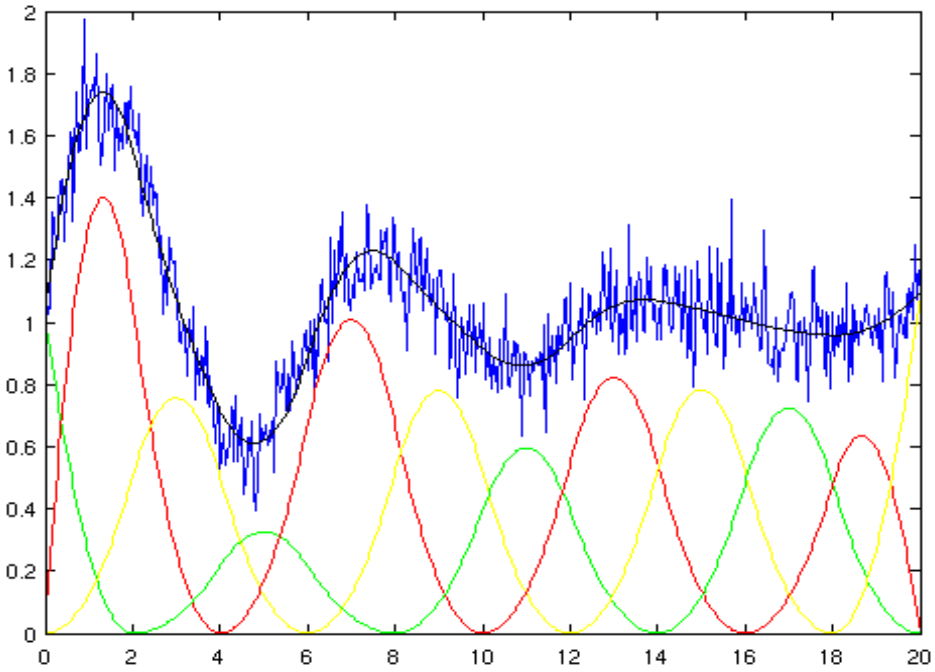

Figure1: blue line indicate the original noisy data, black line indicate the least-squares B-spline approximation of the data. The blue lines, red lines and yellow lines indicate the 12 basis functions.

Question 2:
The condition number of matrix A is 3.1662. Using Cholesky decomposition and QR algorithms to solve the LLS problems in question 1 and compare their speed and accuracy, I got the following result:

Cholesky decomposition:
Elapsed time = 0.015910
Least squared error is 5.171333

QR algorithm:
Elapsed time = 0.017252
Least squared error is 5.171333

It seem that the Cholesky decomposition is faster than QR algorithm and they have the same accuracy.

These two method was also tested using approximation of different numbers of bins (from 10 to 50) to see if the observation is consistent. The result was shown in Figure 2 and Figure 3, for the speed and accuracy separately

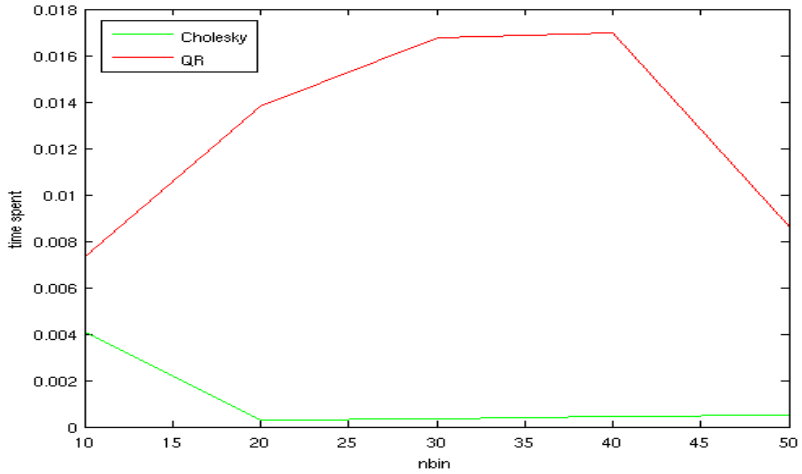

Figure 2: the number of bin related to the calculation time of Cholesky decomposition(green line) and QR algorithm(red line).

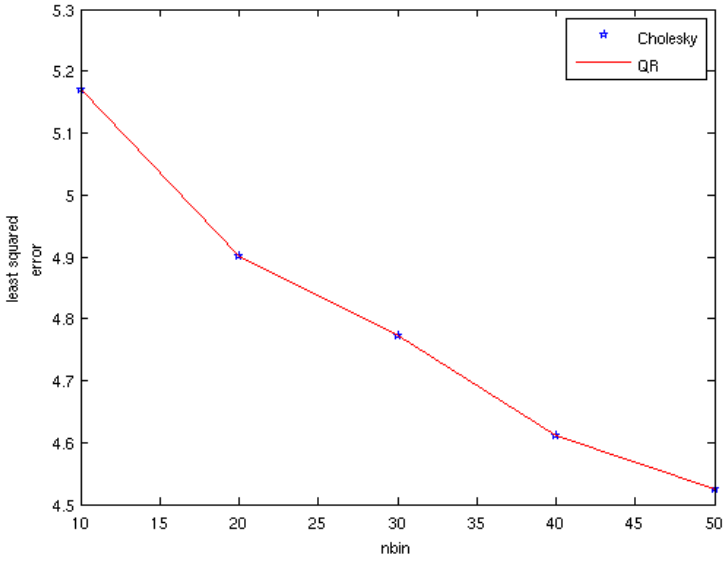

Figure3: the number of bins related to the accuracy of Cholesky decomposition(blue dots) and QR algorithm(red line).

We can see that the Cholesky decomposition is always faster compared with QR algorithm in calculating the linear least square problem here. The accuracy of this two method do not have significant difference since the errors of these two method are identical in this problem.

Question 3:
The relationship between the order p and least squared error is in Figure 4

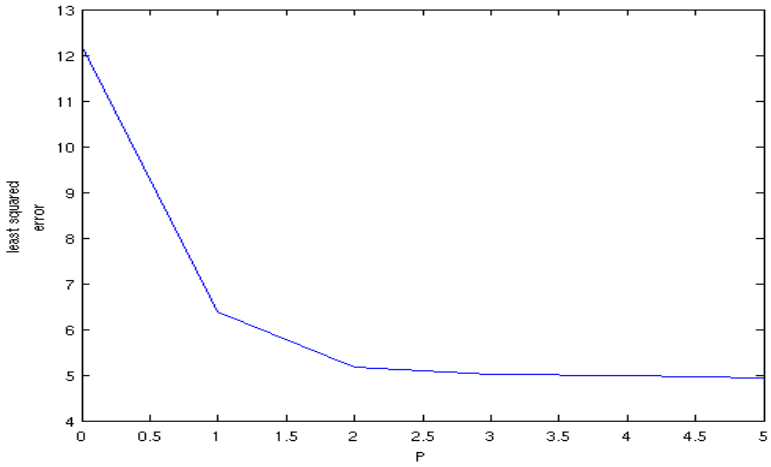

Figure 4: the relationship between the order p and least squared error of B-spline approximation

From figure 4 we can see that as the value of p become larger, the least squared error will become smaller. However, the changing of error become less significant as p become larger. The least squared error appears to be a logarithmic function of p.

Question 4:

The relationship between the number of bins and least squared error is in Figure 5

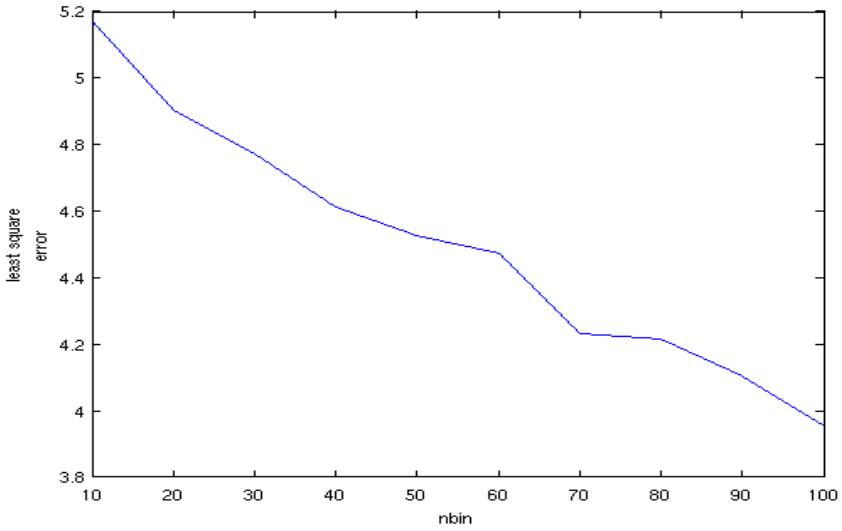

Figure 5: the relationship between the number of bins and least squared error of B-spline approximation

From figure 5 we can see that as the number of bins become larger, the least squared error will become smaller. As the number of bins become larger, the changing of least squared error seems consistent, although there is a fluctuation exist when nbin changed form 60 to 80 . In general, the least squared error appears to be a linear function of the number of bins.