

This is the part2 of the report, which is only for question 5-(i), the other part of questions are reported in the submitted hand-written document. The four method was plotted by running the code lab06.cpp and lab06plot.m

1.Forward Euler

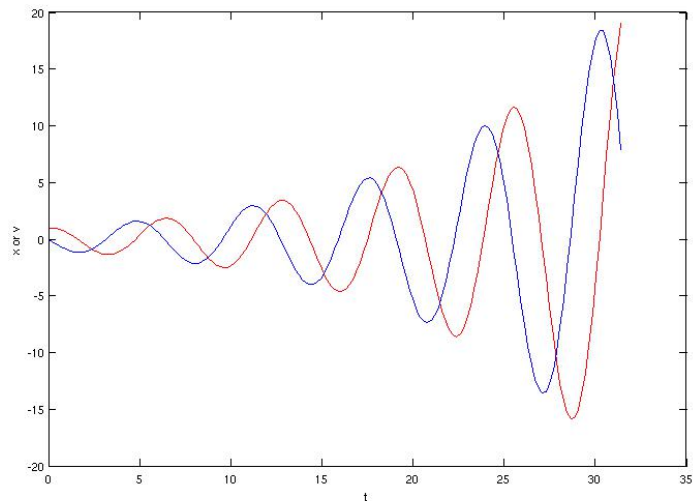


Figure 1: $X(t)$ (red line) and $v(t)$ (blue line)of FE method changing with time t .

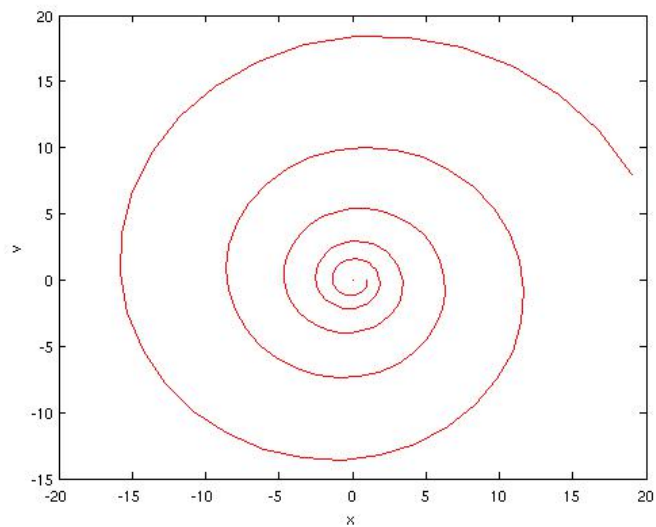


Figure 2:red line in the figure indicate a phase diagram x versus v of FE method.

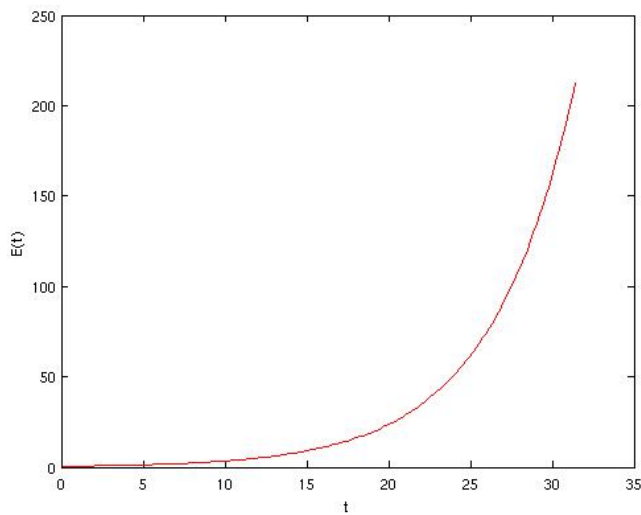


Figure 3: Total energy $E(t)$ (red line) changing with time in FE method.

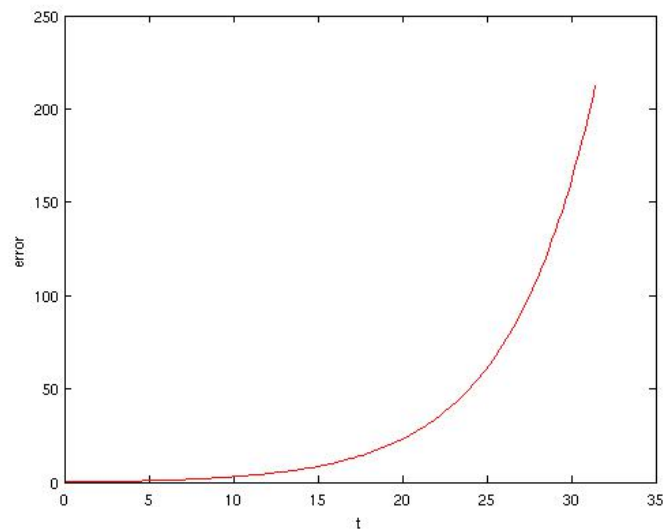


Figure 4: red line shows the error of FE method compared with exact solution

From figure 1-4 we can see that compared with the exact solution, the energy is increasing dramatically when the time becomes larger and larger. So the FE method is not stable.

2.Backward Euler

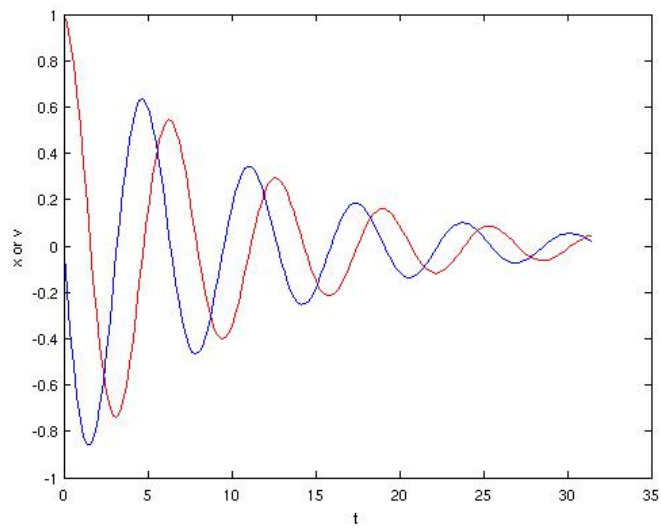


Figure 5: $X(t)$ (red line) and $v(t)$ (blue line)of BE method changing with time t .

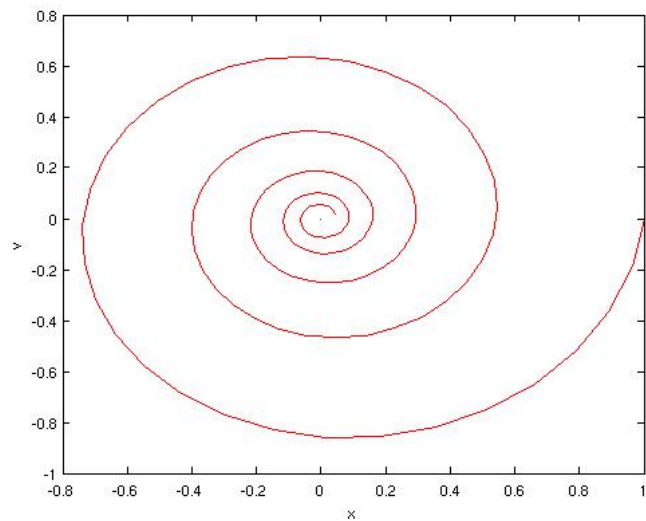


Figure 6:red line in the figure indicate a phase diagram x versus v of BE method.

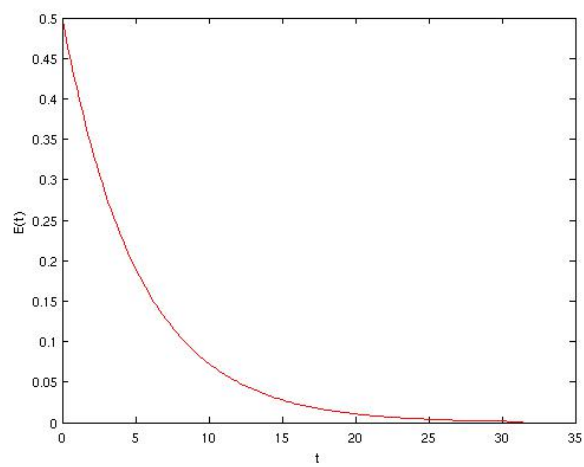


Figure 7: Total energy $E(t)$ (red line) changing with time in BE method.

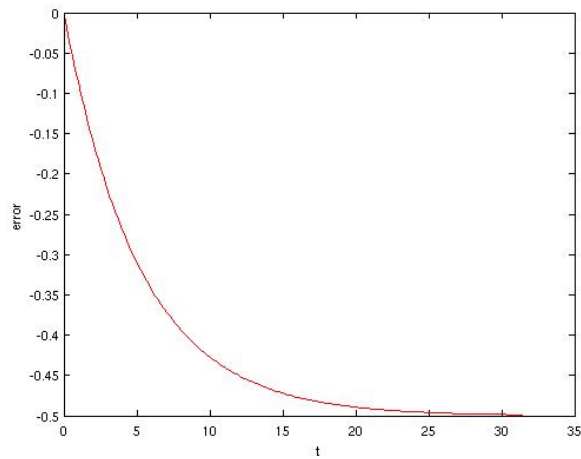


Figure8: red line shows the error of BE method compared with exact solution

From figure 5-8 we can see that compared with the exact solution, the energy is decreasing dramatically when the time becomes larger and larger. So the BE method is also not stable.

3.Implicit Trapezoidal

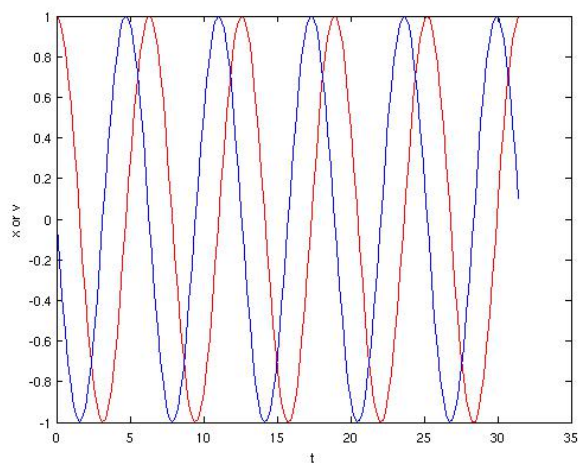


Figure 9: $X(t)$ (red line) and $v(t)$ (blue line)of IT method changing with time t .

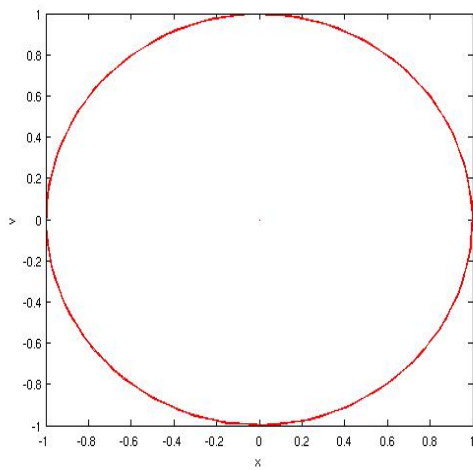


Figure 10:red line in the figure indicate a phase diagram x versus v of IT method.

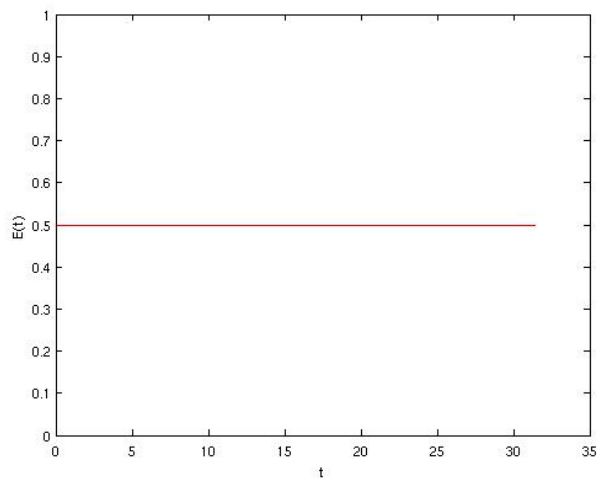


Figure 11: Total energy $E(t)$ (red line) changing with time in IT method.

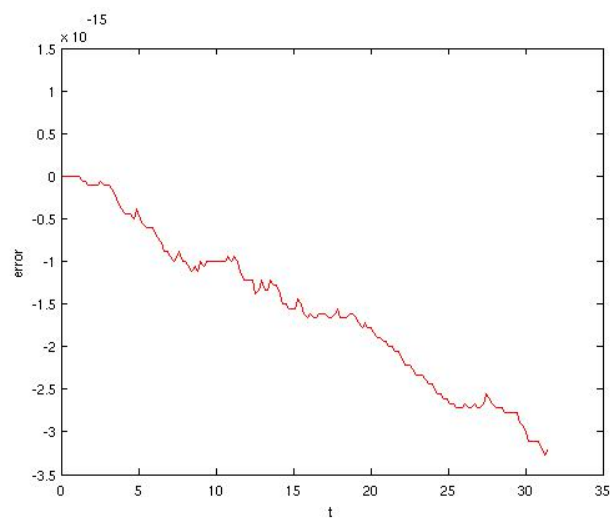


Figure 12: red line shows the error of IT method compared with exact solution

From figure 9-12 we can see that compared with the exact solution, the energy is much more stable compared with BE and FE. The energy is maintained very close to 0.5 as the time increasing, and the error is about 10^{-15} , which is very small. So the IT method is much more stable compared with BE and FE.

4. Leap-Frog

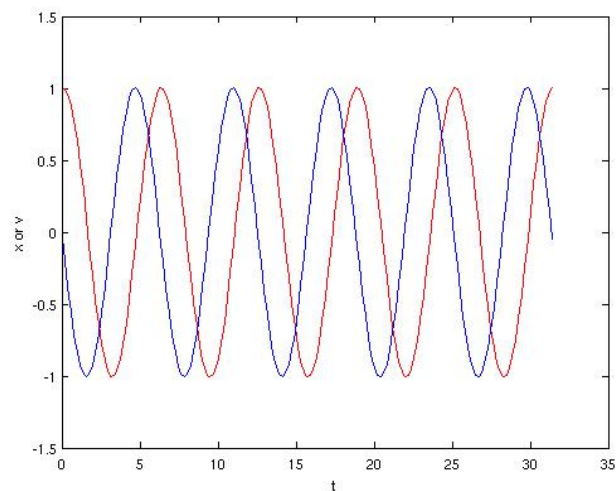


Figure 13: $x(t)$ (red line) and $v(t)$ (blue line) of LF method changing with time t .

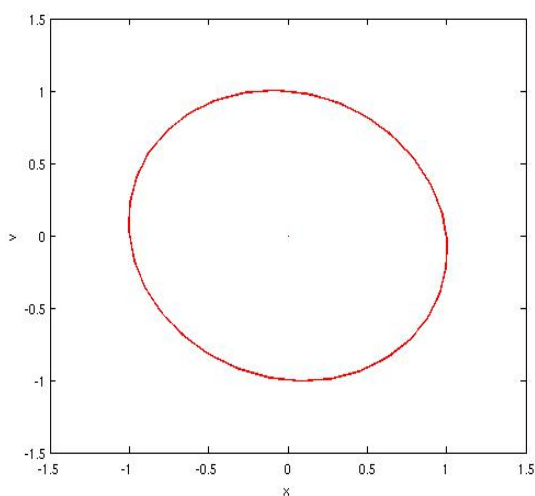


Figure 14: red line in the figure indicate a phase diagram x versus v of LF method.

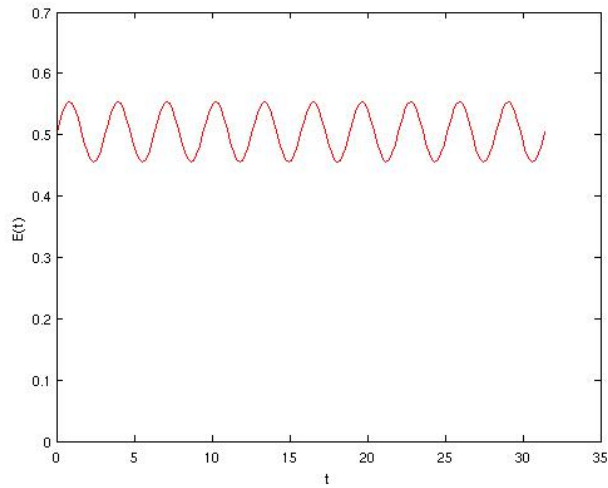


Figure 15: Total energy $E(t)$ (red line) changing with time in LF method.

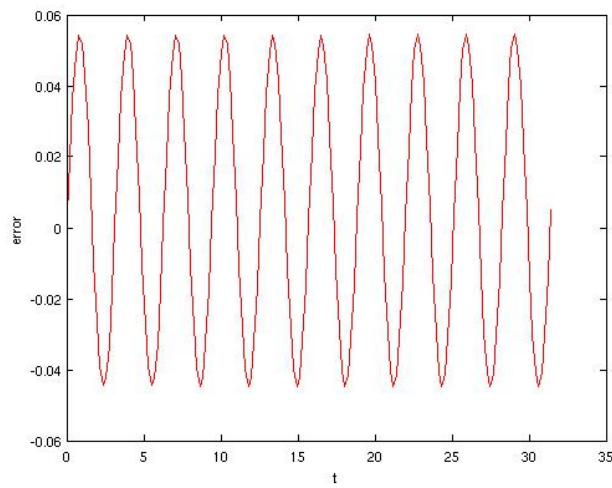


Figure 16: red line shows the error of LF method compared with exact solution

From figure 13-16 we can see that compared with the exact solution, the energy is fluctuating around the real energy, which is 0.5, as the time increasing. The method is still stable since the error is not increasing. However, compared with IT, this method is less accurate.