Introduction:
In this lab, the 1D finite volume method was implemented to solve the advection equation, with the periodic boundary conditions. After successfully implemented the method, the stability and convergence of this method was discussed separately.

Methodology:
Firstly, a c++ solver of finite volume method has been implemented which takes the initial value, time step size, grid size and number of grids as parameters. The solver was then used to calculate the given problem with n fixed to be 40 and the value C equals to 0.4, 0.8, 1.0, 1.2 separately, and the TV value for each cases were calculated as a function over time steps. Finally, to calculate the convergence rate, the time step size t was fixed to be 0.005(C<1 has always been satisfied) and the cases when n=80, 160, 320, 640 have been solved. The error was calculated by comparing to the exact solution, and the convergence rate was calculated. The plotting was done using a matlab program finitevolplot.m.

Result:
1. The stability of this method with different C:
1.1 when n=40, C=0.4 the solution together with the TV value was plotted in Figure 1. The region with positive value is gradually moving to the left and the region expands as time goes on.

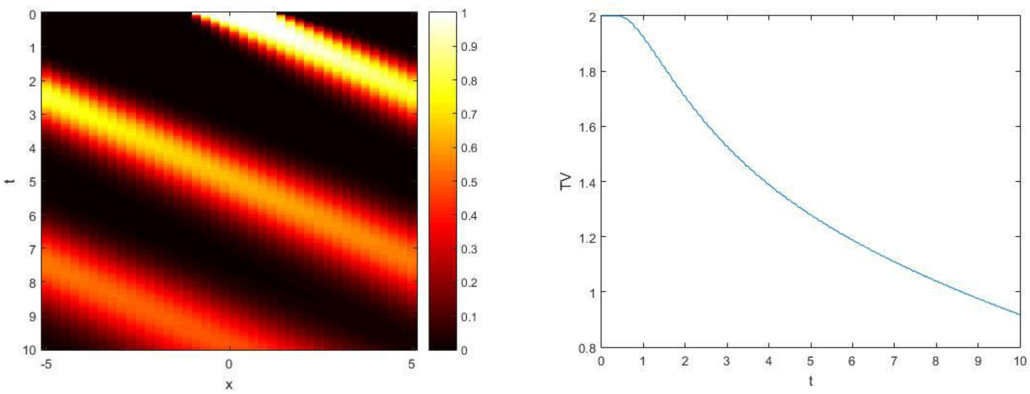

Figure 1: the solution(left figure) and TV value(right figure) when n=40, C=0.4.

1.2 when n=40, C=0.8 the solution together with the TV value was plotted in Figure 2. The region with positive value is gradually moving to the left and the region expands as time goes on, but not as significant as in 1.1.

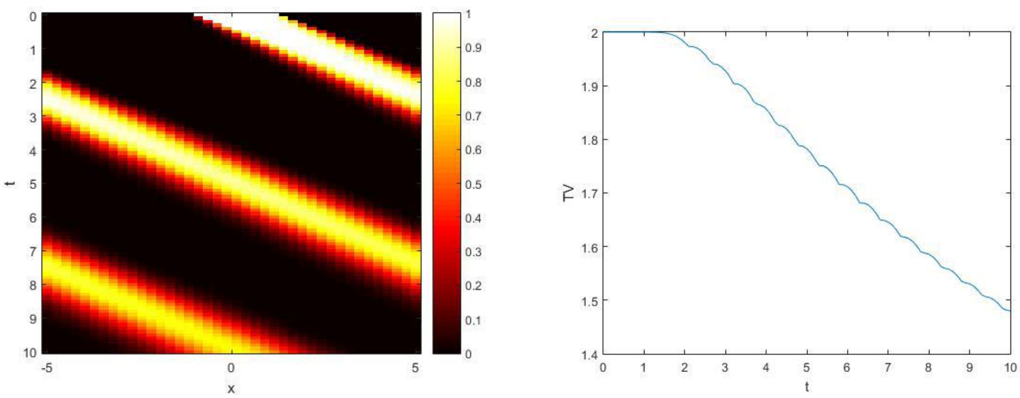

Figure 2: the solution(left figure) and TV value(right figure) when n=40, C=0.8.

1.3 when n=40, C=1 the solution together with the TV value was plotted in Figure 3. The region with positive value is gradually moving to the left and the width and value of the region keeps the same.

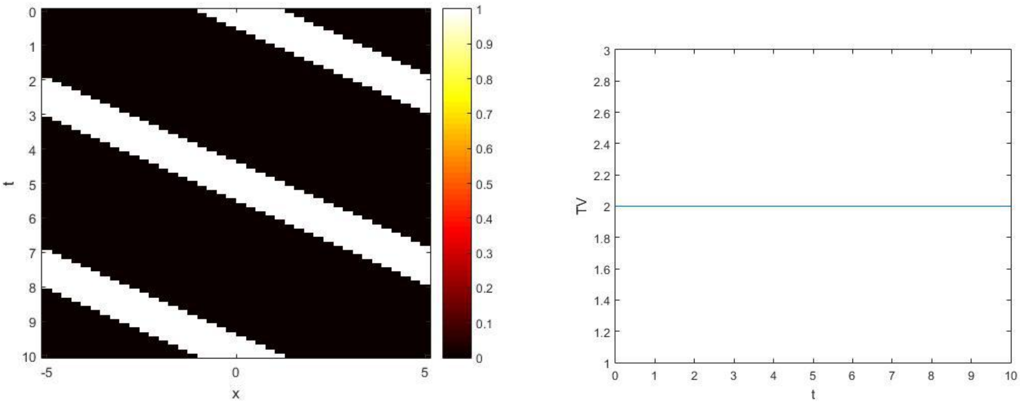

Figure 3: the solution(left figure) and TV value(right figure) when n=40, C=1;

1.4 when n=40, C=1.2 the solution together with the TV value was plotted in Figure 4. The method blows up.

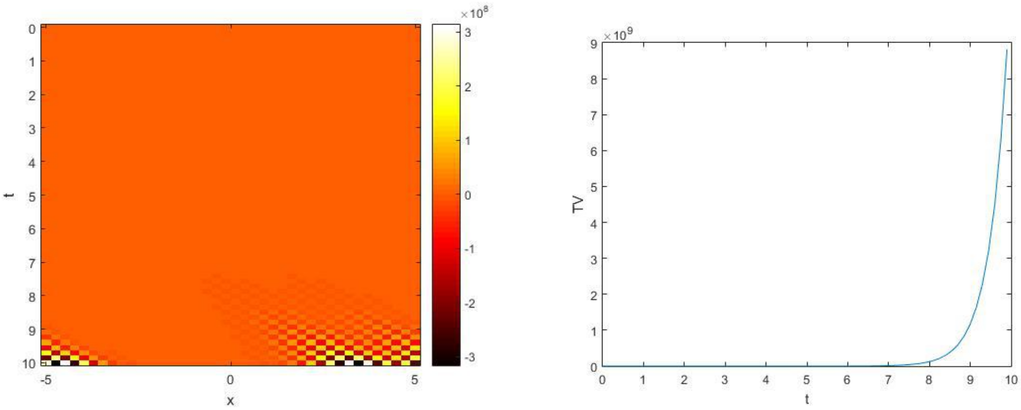

Figure 4: the solution(left figure) and TV value(right figure) when n=40, C=1.2;

2 The convergence of this method

The time step size t was fixed to be 0.005 and the cases when n=80, 160, 320, 640 have been solved. The average l2 error norm(scaled with n) was calculated by comparing to the exact solution, And plotted as a function of n in log-log plot(Figure 5):

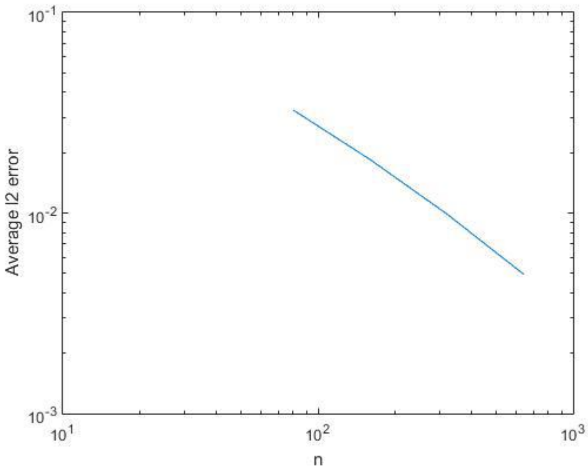

Figure 5: The Average l2 error norm as a function of grids number.

The error and convergence rate was reported in the following table:

n	Average l2 error norm	Convergence rate
80	0.032606	Na
160	0.018473	0.8197
320	0.0098879	0.9017
640	0.00494853	0.9987

Table 1: The error and convergence rate with different value of n.

Discussion:

In this lab, the stability and convergence of the finite volume method has been analyzed. From the part 1 of the result, we can see that when C is smaller than 1(C=0.4 , C=0.8), The method was stable. The region with positive value is gradually moving to the left and the region expands as time goes on. The function value of the positive region decreases, and the TV value also decreasing correspondingly as the time goes on. The expanding and function value change of positive region becomes less significant as the C value increase, until C reaches 1. When C=1, he expanding and function value change of positive region will not happen. When C is larger than 1(C=1.2), the method becomes unstable and blows up. We can draw the conclusion that the method is stable at $0<C<1$ and a larger value of C inside this region can reduce the expanding and function value change of the positive region.

As to the convergence, as the value of n doubled, the average l2 error norm decreases correspondingly. As the value of n increase, this decreasing rate becomes more significant and more closer to 50%. Correspondingly, convergence rate was more and more approximate to 1 as the value of n increased. This phenomenon indicates that this method has a first order convergence.