Introduction
In this lab, we used the Fourier series approximation to approximate different functions. Firstly, a numerical integration method was implemented and then used to calculate the coefficients of Fourier series. The coefficients were then used to plot the figure.

Methodology
Firstly, the Romberg integration method was implemented which used the underlying Trapezoid rule to successively calculate the integration until reaching the stopping criteria, with the level of refinement to be 8. The Romberg method was used to calculate the coefficients of the Fourier series, based on the given formula. The coefficients were calculated by the c++ programs(fs1.cpp, fs2.cpp, fs3.cpp) one for each of the given functions. The coefficients for specific value of n was then used to plot the simulated function using MATLAB.

Results
The Fourier approximation to the function f(x)=x(x+1) has been computed, for n=2, 4, 8, 16. The coefficients were then used to plot Figure 1, together with the original function.

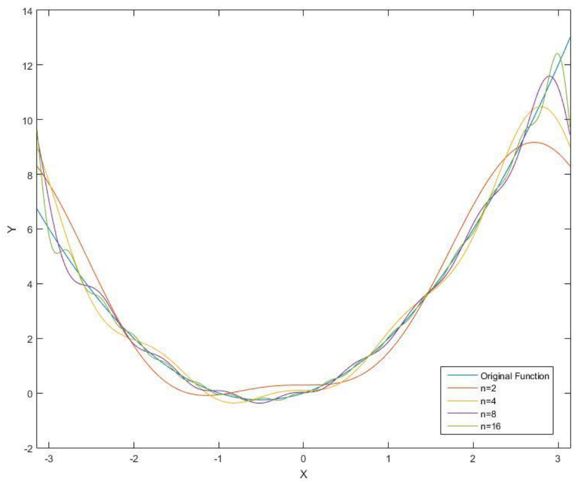

Figure 1: The function f(x)=x(x+1) together with its Fourier series approximation with n=2, 4, 8, 16.

The Fourier approximation to the function f(x)=x^3 has been computed, for n=2, 4, 8, 16. The coefficients were then used to plot Figure 2, together with the original function.

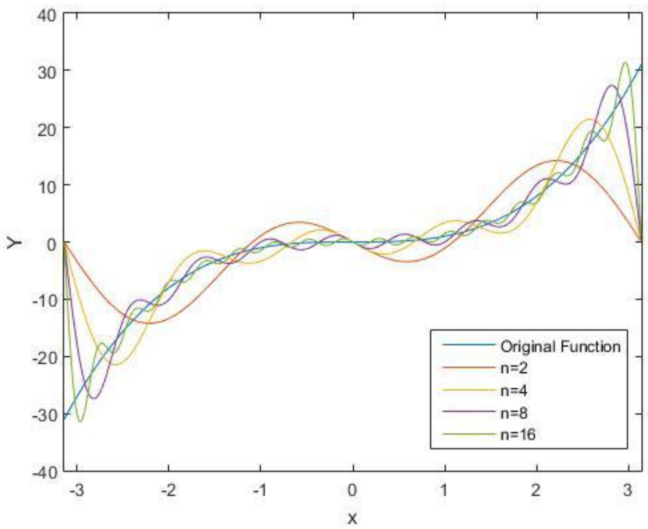

Figure2: The function f(x)=x^3 together with its Fourier series approximation with n=2, 4, 8, 16.

The Fourier approximation to the square wave function with A=10, P=10 has been computed, with 5, 10, 15, 20 terms (corresponding n are 4, 9, 14, 19, since the index start with 0). The coefficients were then used to plot Figure3, together with the original function.

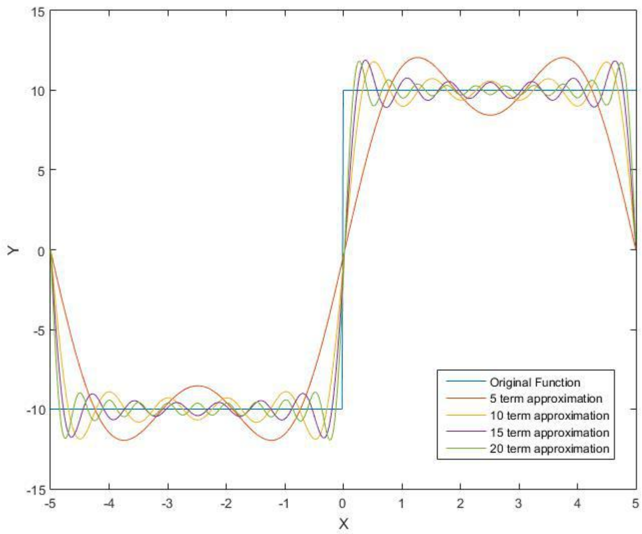

Figure3: The square wave function with A=10, P=10, together with its Fourier series approximation with 5, 10, 15, 20 terms (corresponding n are 4, 9, 14, 19, since the index start with 0).

Discussion

From Figure 1, we can see that as the number of n increase, the Fourier series approximation are more closer to the original functions (with smaller residuals), which indicate the smaller error. However, the errors at the boundary of the function, which is the area near -3 and 3, are more significant compared with other regions. This phenomenon is known as Gibbs phenomenon.

Figure 2 Implies the similar situation as Figure 1. We can see that as the number of n increase, the Fourier series approximation are more closer to the original functions , the error is also smaller . The Gibbs phenomenon for the second function is more significant than first function since the discontinuity in f(x)=x^3 is more significant is this region (if we consider the f(x) as periodic with symmetric boundary).

From Figure 3, we can see that as the term increase, the error becomes smaller and the approximation of the function is more close to the original function.

Interestingly, when expecting the coefficients of the Fourier series(Supplementary material), we can see that a0-an are zeros, only odd number of bi are non-zero values, which means the trigonometric of the approximation only contain the terms bn*sin(nx), with n to be odd number.

Supplementary material

Coefficients of the square wave function approximation:

Function 3:

The fourier coefficients for term= 5 are(in order of a0 a1-an b1-bn):

0 1.1616e-15 -1.32887e-15 -7.64296e-16 -5.30449e-16 12.7324 -2.00337e-15 4.24413 -5.59085e-16

The fourier coefficients for term= 10 are(in order of a0 a1-an b1-bn):
0 1.1616e-15 -1.32887e-15 -7.64296e-16 -5.30449e-16 5.88179e-16 1.03413e-15 1.2621e-15 -2.20858e-15 6.24359e-16 12.7324 -2.00337e-15 4.24413 -5.59085e-16 2.54648 1.67361e-15 1.81891 5.07463e-17 1.41478

The fourier coefficients for term= 15 are(in order of a0 a1-an b1-bn):
0 1.1616e-15 -1.32887e-15 -7.64296e-16 -5.30449e-16 5.88179e-16 1.03413e-15 1.2621e-15 -2.20858e-15 6.24359e-16 6.33243e-17 8.34597e-16 5.18581e-15 6.87072e-16 -3.36529e-15 12.7324 -2.00337e-15 4.24413 -5.59085e-16 2.54648 1.67361e-15 1.81891 5.07463e-17 1.41478 9.74078e-16 1.15765 -1.05838e-15 0.980031 -3.95423e-15

The fourier coefficients for term= 20 are(in order of a0 a1-an b1-bn):
0 1.1616e-15 -1.32887e-15 -7.64296e-16 -5.30449e-16 5.88179e-16 1.03413e-15 1.2621e-15 -2.20858e-15 6.24359e-16 6.33243e-17 8.34597e-16 5.18581e-15 6.87072e-16 -3.36529e-15 1.6038e-15 -7.06231e-15 4.18405e-15 1.3438e-15 4.89789e-15 12.7324 -2.00337e-15 4.24413 -5.59085e-16 2.54648 1.67361e-15 1.81891 5.07463e-17 1.41478 9.74078e-16 1.15765 -1.05838e-15 0.980031 -3.95423e-15 0.853132 8.57572e-17 0.739279 3.46511e-15 0.663118