

Data Structures Using C++ 2E

Chapter 7
Stacks

Objectives

- Learn about stacks
- Examine various stack operations
- Learn how to implement a stack as an array
- Learn how to implement a stack as a linked list
- Discover stack applications
- Learn how to use a stack to remove recursion

Stacks

- Data structure
 - Elements added, removed from one end only
 - Last In First Out (LIFO)

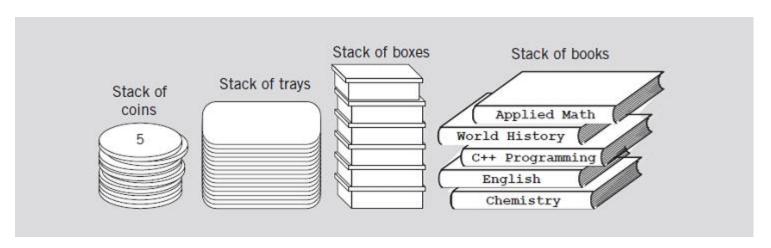


FIGURE 7-1 Various examples of stacks

- push operation
 - Add element onto the stack
- top operation
 - Retrieve top element of the stack, w/o remove top element
- pop operation
 - Remove top element from the stack

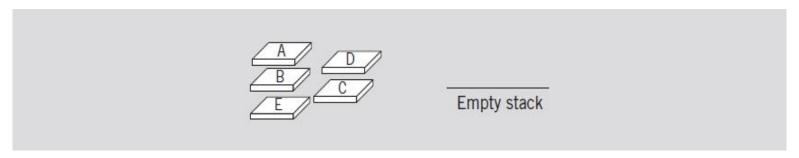


FIGURE 7-2 Empty stack

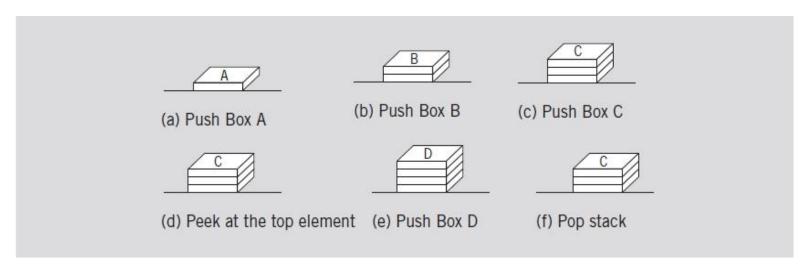


FIGURE 7-3 Stack operations

- Stack element removal
 - Occurs only if something is in the stack
- Stack element added only if room available
- isFullStack operation
 - Checks for full stack
- isEmptyStack operation
 - Checks for empty stack
- initializeStack operation
 - Initializes stack to an empty state

- Review code on page 398
 - Illustrates class specifying basic stack operations

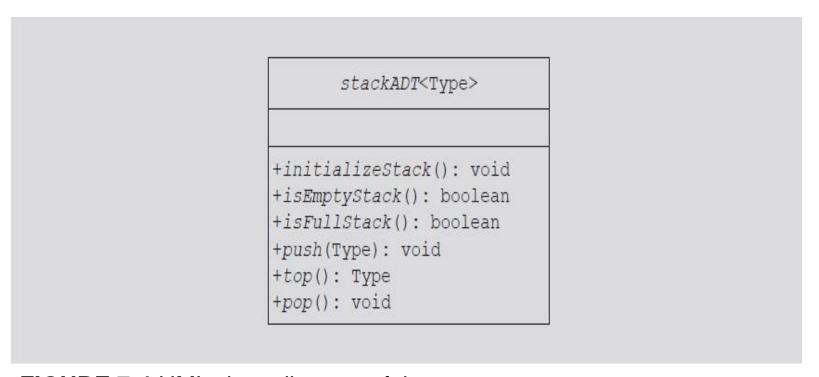


FIGURE 7-4 UML class diagram of the class stackADT

```
template <class Type>
class stackADT
public:
   virtual void initializeStack() = 0;
       //Method to initialize the stack to an empty state.
       //Postcondition: Stack is empty.
   virtual bool isEmptyStack() const = 0;
      //Function to determine whether the stack is empty.
      //Postcondition: Returns true if the stack is empty,
            otherwise returns false.
   virtual bool isFullStack() const = 0;
     //Function to determine whether the stack is full.
     //Postcondition: Returns true if the stack is full,
           otherwise returns false.
   virtual void push(const Type& newItem) = 0;
     //Function to add newItem to the stack.
     //Precondition: The stack exists and is not full.
     //Postcondition: The stack is changed and newItem is added
     // to the top of the stack.
   virtual Type top() const = 0;
     //Function to return the top element of the stack.
     //Precondition: The stack exists and is not empty.
     //Postcondition: If the stack is empty, the program
           terminates; otherwise, the top element of the stack
     //
           is returned.
   virtual\ void\ pop() = 0;
     //Function to remove the top element of the stack.
     //Precondition: The stack exists and is not empty.
     //Postcondition: The stack is changed and the top element
           is removed from the stack.
     //
```

};

Implementation of Stacks as Arrays

- (fixed size) array is allocated by constructor
- First stack element
 - Put in first array slot
- Second stack element
 - Put in second array slot, and so on
- Top of stack
 - Index of last element added to stack
- Stack element accessed only through the top
 - Problem: array is a random access data structure
 - Solution: use another variable (stackTop)
 - Keeps track of the top position of the array

Implementation of Stacks as Arrays (cont'd.)

- Review code on page 400
 - Illustrates basic operations on a stack as an array

```
stackType<Type>
-maxStackSize: int
-stackTop: int
-*list: Type
+operator=(const stackType<Type>&): const stackType<Type>&
+initializeStack(): void
+isEmptyStack() const: bool
+isFullStack() const: bool
+push (const Type&): void
+top() const: Type
+pop(): void
-copyStack(const stackType<Type>&): void
+stackType(int = 100)
+stackType(const stackType<Type>&)
+~stackType()
```

FIGURE 7-5 UML class diagram of the class stackType

Implementation of Stacks as Arrays (cont'd.)

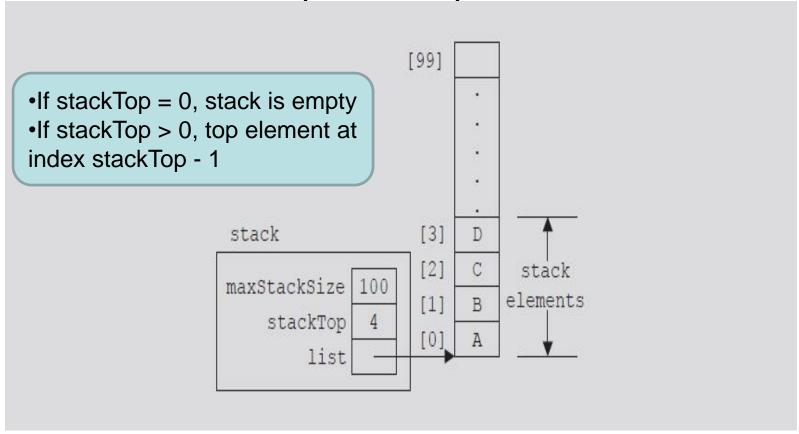


FIGURE 7-6 Example of a stack

Initialize Stack

- Value of stackTop if stack empty
 - Set stackTop to zero to initialize the stack
- Definition of function initializeStack

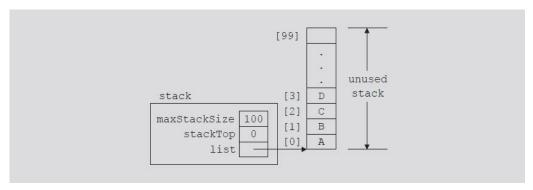


FIGURE 7-7 Empty stack

```
template <class Type>
void stackType<Type>::initializeStack()
{
    stackTop = 0;
}//end initializeStack
```

Empty Stack

- Value of stackTop indicates if stack empty
 - If stackTop = zero: stack empty
 - Otherwise: stack not empty
- Definition of function is EmptyStack

```
template <class Type>
bool stackType<Type>::isEmptyStack() const
{
    return(stackTop == 0);
}//end isEmptyStack
```

Full Stack

- Stack full
 - If stackTop is equal to maxStackSize
- Definition of function isFullStack

```
template <class Type>
bool stackType<Type>::isFullStack() const
{
    return(stackTop == maxStackSize);
} //end isFullStack
```

Push

- Two-step process
 - Store newItem in array component indicated by stackTop
 - Increment stackTop

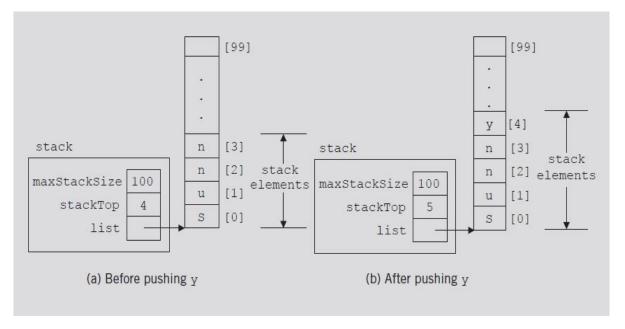


FIGURE 7-8 Stack before and after the push operation
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Push (cont'd.)

Definition of push operation

```
template <class Type>
void stackType<Type>::push(const Type& newItem)
{
    if (!isFullStack())
    {
        list[stackTop] = newItem; //add newItem at the top
            stackTop++; //increment stackTop
    }
    else
        cout << "Cannot add to a full stack." << endl;
}//end push</pre>
```

Return the Top Element

Definition of top operation

Pop

- Remove (pop) element from stack
 - Decrement stackTop by one

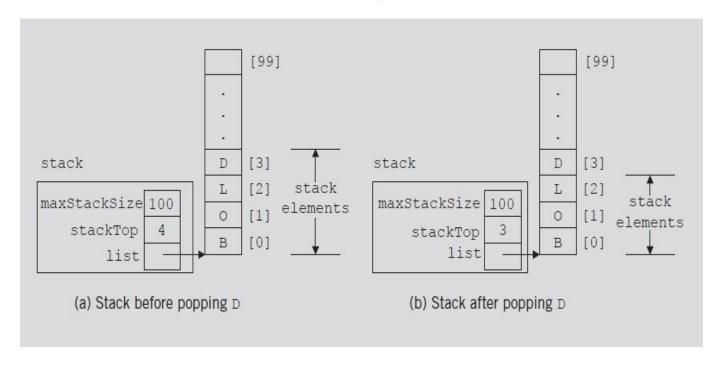


FIGURE 7-9 Stack before and after the pop operation

Pop (cont'd.)

- Definition of pop operation
- Underflow
 - Removing an item from an empty stack
 - Check within pop operation (see below)
 - Check before calling function pop

Copy Stack

Definition of function copyStack

```
O(n)
```

Constructor and Destructor

```
template <class Type>
       stackType<Type>::stackType(int stackSize)
           if (stackSize <= 0)
               cout << "Size of the array to hold the stack must "
                    << "be positive." << endl;
               cout << "Creating an array of size 100." << endl;</pre>
               maxStackSize = 100;
           else
               maxStackSize = stackSize; //set the stack size to
                                            //the value specified by
                                            //the parameter stackSize
           stackTop = 0;
                                            //set stackTop to 0
           list = new Type[maxStackSize]; //create the array to
                                            //hold the stack elements
       }//end constructor
       template <class Type>
       stackType<Type>::~stackType() //destructor
           delete [] list; //deallocate the memory occupied
                            //by the array
       }//end destructor
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```

Copy Constructor

Definition of the copy constructor

Overloading the Assignment Operator (=)

- Classes with pointer member variables
 - Assignment operator must be explicitly overloaded
 - Why?
- Function definition to overload assignment operator for class stackType

O(n)

Stack Header File

- myStack.h
 - Header file name containing class stackType definition

```
//Header file: myStack.h
#ifndef H StackType
#define H StackType
#include <iostream>
#include <cassert>
#include "stackADT.h"
using namespace std;
//Place the definition of the class template stackType, as given
//previously in this chapter, here.
//Place the definitions of the member functions as discussed here.
#endif
```

Stack Header File (cont'd.)

- Stack operations analysis
 - Similar to class arrayListType operations

TABLE 7-1 Time complexity of the operations of the class stackType on a stack with *n* elements

Function	Time complexity
isEmptyStack	0(1)
isFullStack	0(1)
initializeStack	O(1)
constructor	0(1)
top	0(1)
push	0(1)
pop	0(1)
copyStack	O(n)
destructor	0(1)
copy constructor	O(n)
Overloading the assignment operator	O(n)

Linked Implementation of Stacks

- Disadvantage of array (linear) stack representation
 - Fixed number of elements can be pushed onto stack
- Solution
 - Use pointer variables to dynamically allocate, deallocate memory
 - Use linked list to dynamically organize data
- Value of stackTop: array (linear) representation
 - Indicates number of elements in the stack
 - Gives index of the array
 - Value of stackTop 1
 - Points to top item in the stack
- Value of stackTop: linked representation
 - Locates top element in the stack
 - Gives address (memory location) of the top element of the stack

```
template <class Type>
struct nodeType
{
    Type info;
    nodeType<Type> *link;
};
```

```
template <class Type>
class linkedStackType: public stackADT<Type>
public:
    const linkedStackType<Type>& operator=
                              (const linkedStackType<Type>&);
    bool isEmptyStack() const;
    bool isFullStack() const;
   void initializeStack();
   void push(const Type& newItem);
    Type top() const;
   void pop();
    linkedStackType();
    linkedStackType(const linkedStackType<Type>& otherStack);
    ~linkedStackType();
private:
    nodeType<Type> *stackTop; //pointer to the stack
    void copyStack(const linkedStackType<Type>& otherStack);
};
```

Linked Implementation of Stacks (cont'd.)

- Example 7-2
 - Stack: object of type linkedStackType

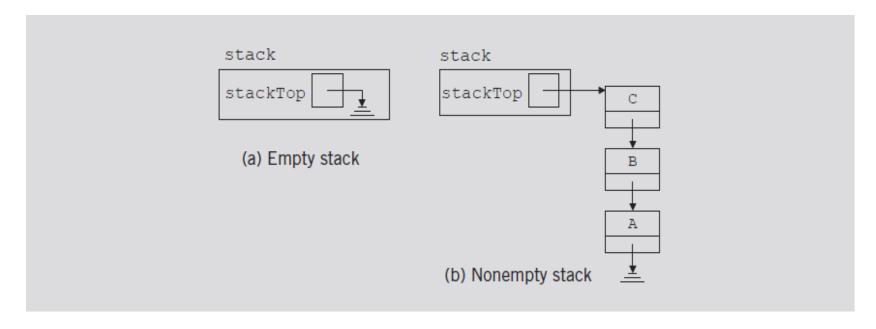


FIGURE 7-10 Empty and nonempty linked stacks

Default Constructor

- When stack object declared
 - Initializes stack to an empty state
 - Sets stackTop to NULL
- Definition of the default constructor

```
template <class Type>
linkedStackType<Type>::linkedStackType()
{
    stackTop = NULL;
}
```

Empty Stack and Full Stack

- Stack empty if stackTop is NULL
- Stack never full
 - Element memory allocated/deallocated dynamically
 - Function isFullStack always returns false value

```
template <class Type>
bool linkedStackType<Type>::isEmptyStack() const
{
    return(stackTop == NULL);
} //end isEmptyStack

template <class Type>
bool linkedStackType<Type>::isFullStack() const
{
    return false;
} //end isFullStack
```

Initialize Stack

- Reinitializes stack to an empty state
 - deallocate memory occupied by the stack elements,
 set stackTop to NULL
- Definition of the initializeStack function

```
O(n)
template <class Type>
void linkedStackType<Type>:: initializeStack()
   nodeType<Type> *temp; //pointer to delete the node
   while (stackTop != NULL) //while there are elements in
                              //the stack
        temp = stackTop;
                           //set temp to point to the
                            //current node
        stackTop = stackTop->link; //advance stackTop to the
                                    //next node
        delete temp;
                        //deallocate memory occupied by temp
  //end initializeStack
```

Push

- newElement added at the beginning of the linked list pointed to by stackTop
- Value of pointer stackTop updated

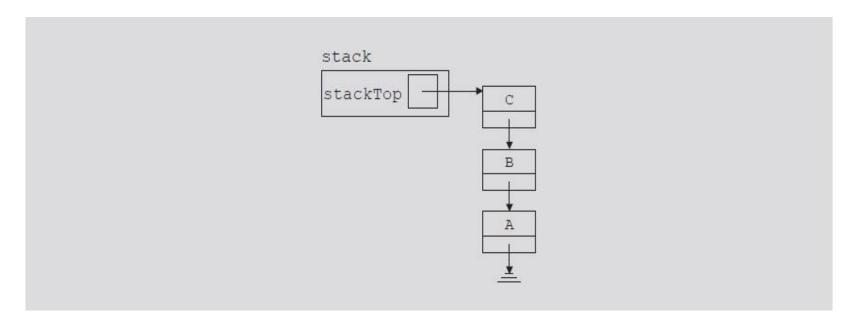


FIGURE 7-11 Stack before the push operation

Push (cont'd.)

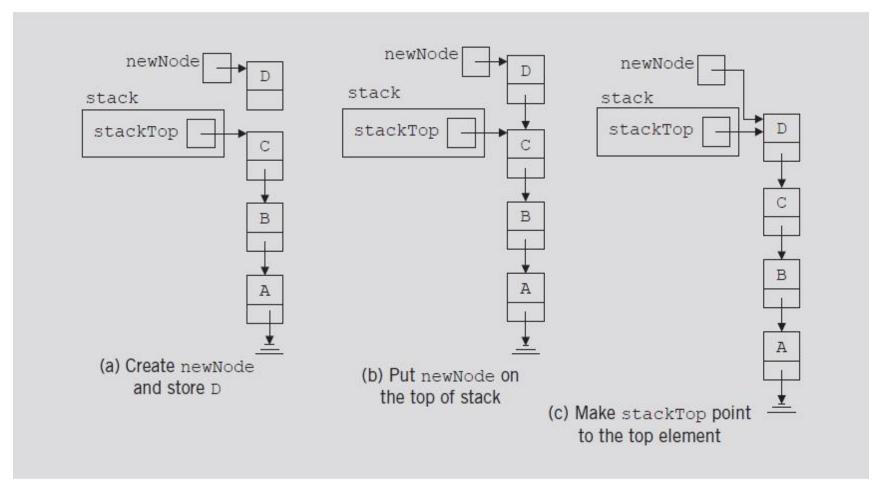


FIGURE 7-12 Push operation

Push (cont'd.)

Definition of the push function

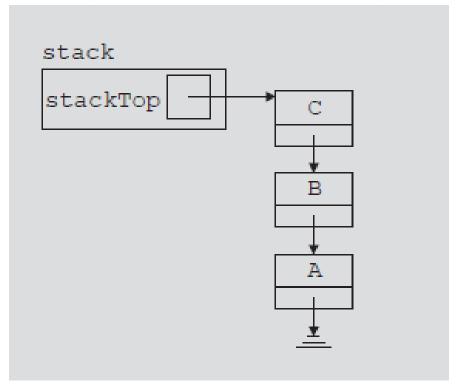
```
template <class Type>
void linkedStackType<Type>::push(const Type& newElement)
   nodeType<Type> *newNode; //pointer to create the new node
   newNode = new nodeType<Type>; //create the node
   newNode->info = newElement; //store newElement in the node
   newNode->link = stackTop; //insert newNode before stackTop
   //top node
} //end push
```

Return the Top Element

- Returns information of the node to which stackTop pointing
- Definition of the top function

Pop

- Removes top element of the stack
 - Node pointed to by stackTop removed
 - Value of pointer stackTop updated



Pop (cont'd.)

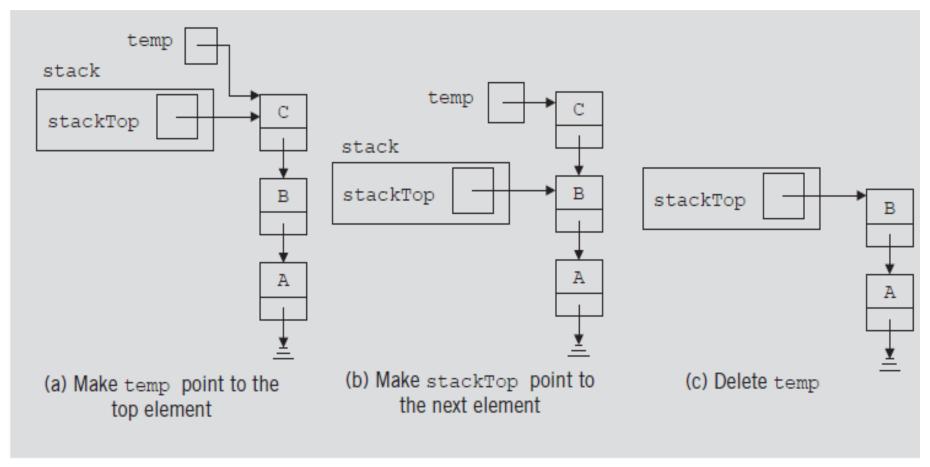


FIGURE 7-14 Pop operation

Pop (cont'd.)

Definition of the pop function

```
template <class Type>
void linkedStackType<Type>::pop()
   nodeType<Type> *temp; //pointer to deallocate memory
    if (stackTop != NULL)
       temp = stackTop; //set temp to point to the top node
        stackTop = stackTop->link; //advance stackTop to the
                                    //next node
       delete temp; //delete the top node
    else
       cout << "Cannot remove from an empty stack." << endl;
}//end pop
```

Copy Stack

- Makes an identical copy of a stack
- Definition similar to the definition of copyList for linked lists
- Definition of the copyStack function

```
template <class Type>
  void linkedStackType<Type>::copyStack
                        (const linkedStackType<Type>& otherStack)
   {
      nodeType<Type> *newNode, *current, *last;
                                                                       O(n)
       if (stackTop != NULL) //if stack is nonempty, make it empty
           initializeStack();
      if (otherStack.stackTop == NULL)
           stackTop = NULL;
      else
          current = otherStack.stackTop; //set current to point
                                      //to the stack to be copied
               //copy the stackTop element of the stack
           stackTop = new nodeType<Type>; //create the node
           stackTop->info = current->info; //copy the info
           stackTop->link = NULL; //set the link field to NULL
           last = stackTop;
                                  //set last to point to the node
           current = current->link; //set current to point to the
                                    //next_node
               //copy the remaining stack
          while (current != NULL)
              newNode = new nodeType<Type>;
               newNode->info = current->info;
              newNode->link = NULL;
               last->link = newNode;
               last = newNode;
               current = current->link;
Da
           }//end while
       }//end else
  } //end copyStack
```

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Constructors and Destructors

Definition of the functions to implement the copy constructor and the destructor

```
O(n)
    //copy constructor
template <class Type>
linkedStackType<Type>::linkedStackType(
                      const linkedStackType<Type>& otherStack)
    stackTop = NULL;
    copyStack(otherStack);
}//end copy constructor
    //destructor
                                                         O(n)
template <class Type>
linkedStackType<Type>::~linkedStackType()
    initializeStack();
}//end destructor
```

Overloading the Assignment Operator (=)

Definition of the functions to overload the assignment operator

```
O(n)
::operator=
```

Overloading the Assignment Operator (=) (cont'd.)

TABLE 7-2 Time complexity of the operations of the class linkedStackType on a stack with *n* elements

Function	Time complexity
isEmptyStack	0(1)
isFullStack	0(1)
initializeStack	O(n)
constructor	0(1)
top	O(1)
push	O(1)
pop	O(1)
copyStack	O(n)
destructor	O(n)
copy constructor	O(n)
Overloading the assignment operator	O(n)

Stack as Derived from the class unorderedLinkedList

- Stack push function, list insertFirst function
 - Similar algorithms
 - initializeStack and initializeList,
 isEmptyList and isEmptyStack, etc.
- class linkedStackType can be derived from class linkedListType
 - class linkedListType: abstract class
- class linkedStackType can be derived from class unorderedLinkedListType
 - class unorderedLinkedListType: derived from class linkedListType

```
template<class Type>
class linkedStackType: public unorderedLinkedList<Type>
{
public:
    void initializeStack();
    bool isEmptyStack() const;
    bool isFullStack() const;
    void push(const Type& newItem);
    Type top() const;
    void pop();
};
template<class Type>
void linkedStackType<Type>::initializeStack()
    unorderedLinkedList<Type>::initializeList();
template<class Type>
bool linkedStackType<Type>::isEmptyStack() const
{
    return unorderedLinkedList<Type>::isEmptyList();
template<class Type>
bool linkedStackType<Type>::isFullStack() const
    return false;
}
template<class Type>
void linkedStackType<Type>::push(const Type& newElement)
    unorderedLinkedList<Type>::insertFirst(newElement);
template<class Type>
Type linkedStackType<Type>::top() const
    return unorderedLinkedList<Type>::front();
template<class Type>
void linkedStackType<Type>::pop()
{
    nodeType<Type> *temp;
    temp = first;
    first = first->link;
    delete temp:
```

Application of Stacks: Postfix Expressions Calculator

- Arithmetic notations
 - Infix notation: operator between operands
 - Prefix (Polish) notation: operator precedes operands
 - Postfix (Reverse Polish) notation: operator follows operands
- Stack use in compliers
 - Translate infix expressions into some form of postfix notation
 - Translate postfix expression into machine code

Infix, Prefix, and Postfix

Infix	Prefix	Postfix
A + B	+ A B	A B +
A * B + C	+ * A B C	A B * C +
A * (B + C)	* A + B C	A B C + *
A - (B - (C - D))	- A - B - C D	A B C D
A - B - C - D	ABCD	AB-C-D-

Prefix and Postfix: no parentheses

Infix and Postfix Expression Conversion

EXAMPLE 7-4

Infix expression

$$a + b$$

 $a + b * c$
 $a * b + c$
 $(a + b) * c$
 $(a - b) * (c + d)$
 $(a + b) * (c - d / e) + f$

Equivalent postfix expression

Postfix Algorithm

- Postfix expression can be evaluated using the following algorithm
 - Scan the expression from left to right
 - When an operator is found, back up to get the required number of (preceding) operands, perform the operation
 - Repeat until reaching the end of the expression

Application of Stacks: Postfix Expressions Calculator (cont'd.)

Postfix expression: 6 3 + 2 * =

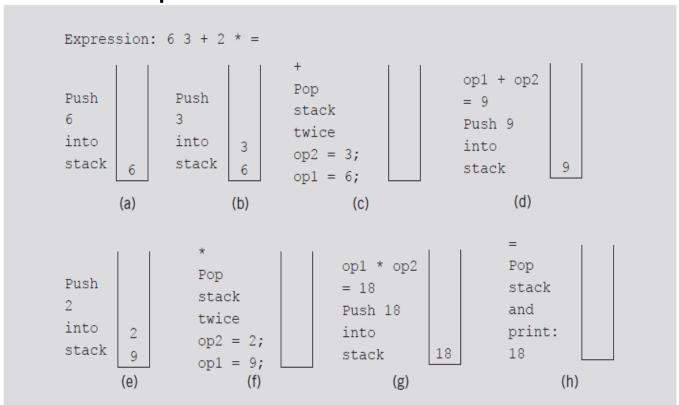


FIGURE 7-15 Evaluating the postfix expression: 6 3 + 2 * =

Postfix Algorithm w/ Stack

- When an operand (number) is encountered in an expression, it is pushed onto the stack.
- When we read a symbol other than a number, the following cases arise
 - 1. The symbol we read is one of +, -, *, /, or =.
 - a) If the symbol is +, -, *, or /, it is an operator and we must evaluate it. Because an operator requires two operands, the stack must have at least two elements; otherwise, the expression has an error
 - b) If the symbol is =, the expression ends and we must print the answer. The stack must contain exactly one element, which is the result; otherwise, it has an error.
 - 2. The symbol we read is something other than +, -, *, /, or =. In this case, the expression contains an illegal operator.

Postfix Expression

Consider the following expressions

```
-76+3;6-=
-14+23*=
-1323+=
Invalid operator;
Insufficient operand for +
```

 To make the input easier to read, we assume the postfix expressions are in the following form:

The symbol # precedes each number in the expression

Application of Stacks: Postfix Expressions Calculator (cont'd.)

Main algorithm pseudocode

```
Read the first character
while not the end of input data
{
    a. initialize the stack
    b. process the expression
    c. output result
    d. get the next expression
}
```

- Broken into four functions for simplicity
 - Function evaluateExpression
 - Function evaluateOpr
 - Function discardExp
 - Function printResult

```
void evaluateExpression(ifstream& inpF, ofstream& outF,
                         stackType<double>& stack,
                         char& ch, bool& isExp0k)
£
    double num;
    outF << ch;
    while (ch != '-')
    £
        switch (ch)
        case '#':
            inpF >> num;
            outF << num << " ";
            if (!stack.isFullStack())
                stack.push(num);
            else
            £
                cout << "Stack overflow. "
                     << "Program terminates!" << endl;
                exit(0); //terminate the program
            7
            break:
        default:
            evaluateOpr(outF, stack, ch, isExpOk);
        }//end switch
        if (isExp0k) //if no error
        £
            inpF >> ch;
            outF << ch;
            if (ch != '#')
                outF << " ":
        }
        else
            discardExp(inpF, outF, ch);
    } //end while (!= '=')
} //end evaluateExpression
```

```
char& ch, bool& isExp0k)
{
   double op1, op2;
   if (stack.isEmptyStack())
   {
       out << " (Not enough operands)";
       isExp0k = false;
   }
                                                                     case '/':
   else
                                                                         if (op2 != 0)
   {
                                                                              stack.push(op1 / op2);
       op2 = stack.top();
                                                                         else
        stack.pop();
                                                                             out << " (Division by 0)";
       if (stack.isEmptyStack())
                                                                             isExp0k = false;
        {
            out << " (Not enough operands)";
                                                                         }
            isExp0k = false;
                                                                         break;
        }
       else
                                                                     default:
                                                                         out << " (Illegal operator)";
            op1 = stack.top();
                                                                         isExp0k = false;
            stack.pop();
                                                                     }//end switch
                                                                 } //end else
            switch (ch)
                                                            } //end else
                                                        } //end evaluateOpr
            case '+':
                stack.push(op1 + op2);
                break;
            case '-':
                stack.push(op1 - op2);
                break;
                                                                                                     55
            case '*':
                stack.push(op1 * op2);
                break;
```

void evaluateOpr(ofstream& out, stackType<double>& stack,

```
void discardExp(ifstream& in, ofstream& out, char& ch)
    while (ch != '=')
        in.get(ch);
        out << ch;
} //end discardExp
void printResult(ofstream& outF, stackType<double>& stack,
                 bool isExp0k)
{
    double result;
    if (isExp0k) //if no error, print the result
    {
        if (!stack.isEmptyStack())
            result = stack.top();
            stack.pop();
            if (stack.isEmptyStack())
                outF << result << endl;
            else
                outF << " (Error: Too many operands)" << endl;
        } //end if
        else
            outF << " (Error in the expression)" << endl;
    }
    else
        outF << " (Error in the expression)" << endl;
    outF << ".
         << endl << endl;
} //end printResult
```

```
int main()
    bool expression0k;
    char ch;
    stackType<double> stack(100);
    ifstream infile;
    ofstream outfile;
    infile.open("RpnData.txt");
    if (!infile)
        cout << "Cannot open the input file. "
             << "Program terminates!" << endl;
        return 1;
    }
    outfile.open("RpnOutput.txt");
    outfile << fixed << showpoint;
    outfile << setprecision(2);
    infile >> ch;
    while (infile)
        stack.initializeStack();
        expression0k = true;
        outfile << ch;
        evaluateExpression(infile, outfile, stack, ch,
                           expressionOk);
        printResult(outfile, stack, expression0k);
        infile >> ch; //begin processing the next expression
    } //end while
    infile.close();
    outfile.close();
    return 0:
```

fixed: floating-point values are written using fixed-point notation: the value is represented with exactly as many digits in the decimal part as specified by the *precision field* (<u>precision</u>) and with no exponent part.

showpoint: the decimal point is always written for floating point values inserted into the stream (even for those whose decimal part is zero).

Infix to Postfix Conversion

- 1. Initialize an empty stack of operators
- 2. While (no error && !end of expression)
 - a) Get next input "token" from infix expression
 - b) If token is ...
 - i. "(" : push onto stack
 - ii. ")" : pop and append stack elements until "(" occurs, do not display it
 - iii. operator
 - if (stack is empty or operator has higher priority than top of stack) push token onto stack

else

pop and append top of stack to postfix; repeat comparison of token with top of stack

- iv. Operand: append to postfix
- 3. When end of infix reached, pop and append stack items to postfix until empty
- Ex: A * B + C
- Ex: A * (B + C)

NOTE: (in stack

has lower priority

than operators

Print a Linked List in Reverse Order (Chapter 6)

 Function template to implement previous algorithm and then apply it to a list

Removing Recursion: Nonrecursive Algorithm to Print a Linked List Backward

- Stack
 - Used to design nonrecursive algorithm
 - Print a linked list backward
- Use linked implementation of stack

```
current = first;
                                                       //Line 1
                    while (current != NULL)
                                                       //Line 2
                                                       //Line 3
                         stack.push (current);
                                                       //Line 4
                         current = current->link;
                                                       //Line 5
                                                       //Line 6
                    while (!stack.isEmptyStack())
                                                         //Line 7
                                                         //Line 8
                        current = stack.top();
                        stack.pop();
                        cout << current->info << " ";
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                                                         //Line 12
```

delete current:

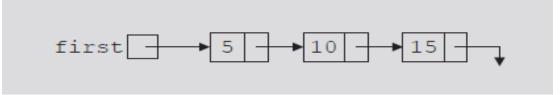


FIGURE 7-16 Linked list

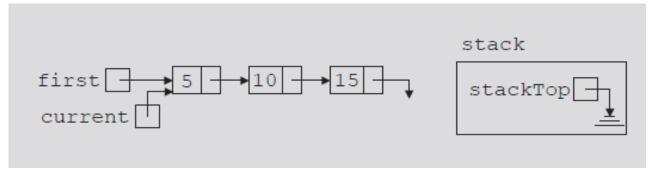


FIGURE 7-17 List after the statement current = first; executes

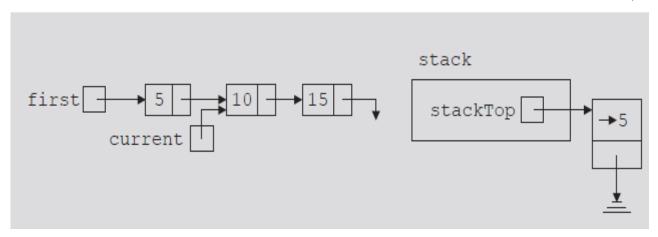


FIGURE 7-18 List and stack after the statements

stack.push(current); and current = current->link;
execute

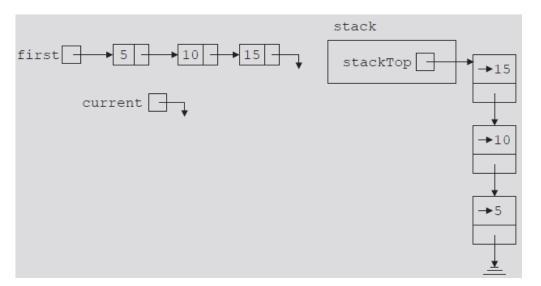


FIGURE 7-19 List and stack after the 1st while statement executes

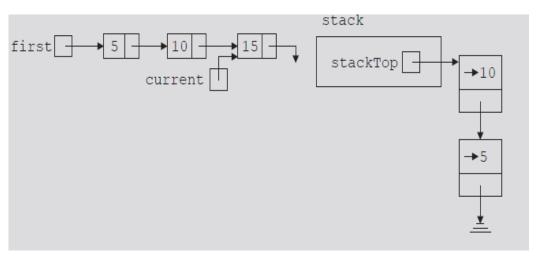


FIGURE 7-20 List and stack after the statements current =
stack.top(); and stack.pop(); execute

STL class stack

- Standard Template Library (STL) library class defining a stack
- Header file containing class stack definition

- stack

TABLE 7-3 Operations on a stack object

O peration	Effect
size	Returns the actual number of elements in the stack.
empty	Returns true if the stack is empty, and false otherwise.
push(item)	Inserts a copy of item into the stack.
top	Returns the top element of the stack, but does not remove the top element from the stack. This operation is implemented as a value-returning function.
pop	Removes the top element of the stack.

Summary

Stack

- Last In First Out (LIFO) data structure
- Implemented as array or linked list
- Arrays: limited number of elements
- Linked lists: allow dynamic element addition
- Stack use in compliers
 - Translate infix expressions into some form of postfix notation
 - Translate postfix expression into machine code
- Standard Template Library (STL)
 - Provides a class to implement a stack in a program

Self Exercises

• Programming Exercises: 1, 2, 3, 4, 7, 9