Bayesian Inference using Sequential Monte-Carlo Algorithms for Dynamical Systems Models

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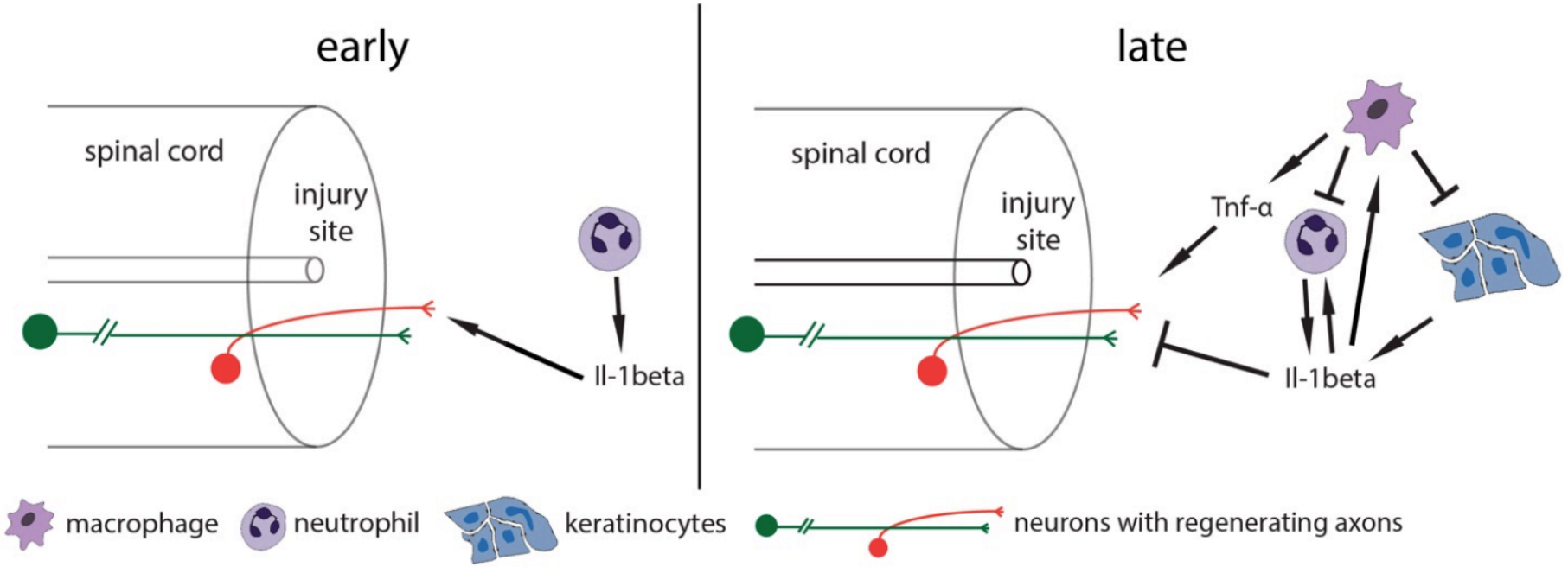
Background

- Zebrafish spinal cord repair
- Hypothesis: dynamically control by immune cells and molecules



Background

- Dynamic systems can be modelled as ODEs
- We wish to find the best parameters given the observed data

- Method: Bayesian inference

$$posterior \propto likelihood \times prior$$
$$p(\theta|D) \propto l(\theta|D) \times p(\theta)$$

- Find the best parameter values by measuring the posterior distribution of parameters θ

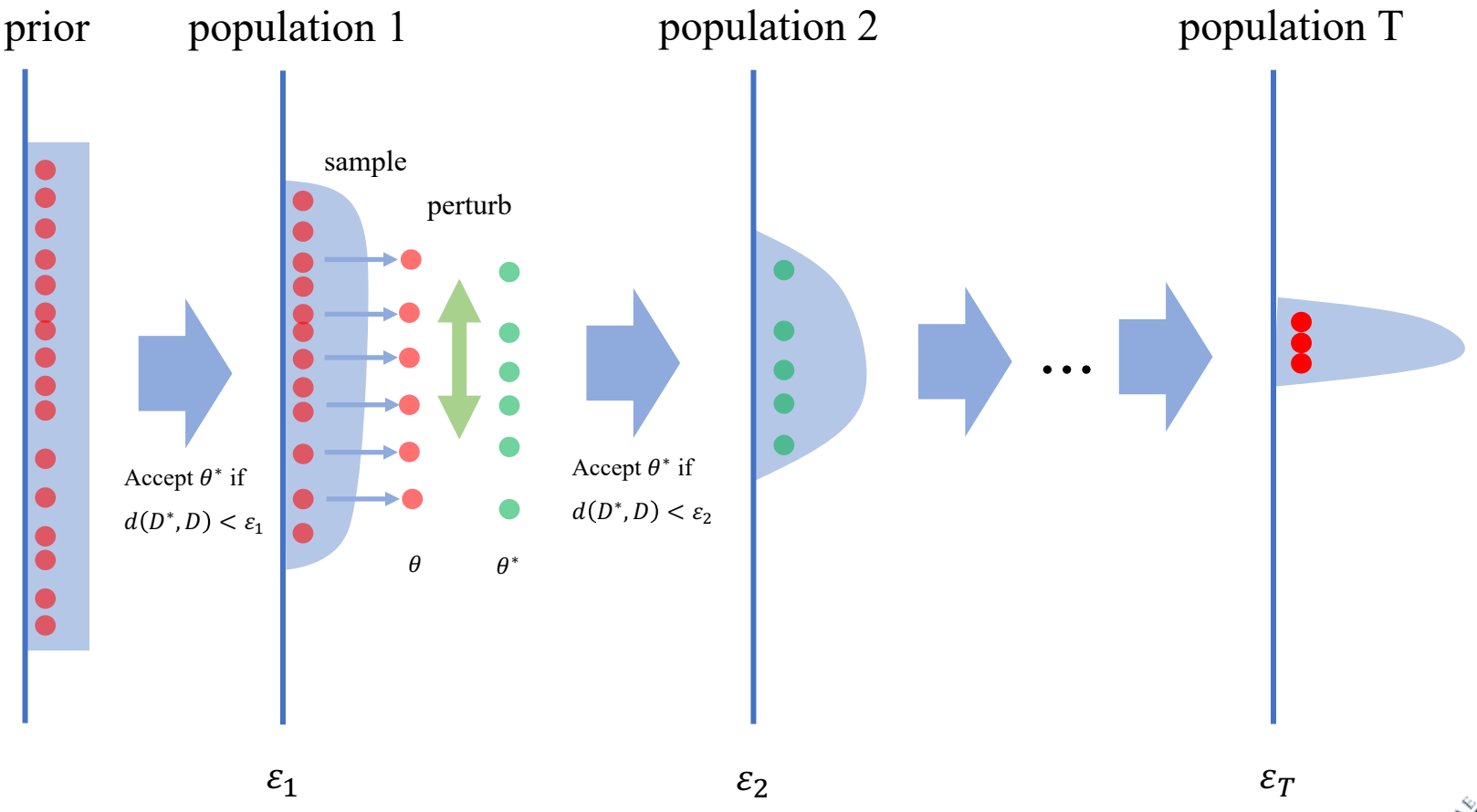
Background

- Hard to write down an expression for likelihood
  → likelihood-free numerical approximation

- Approximate Bayesian Computation (ABC)

Background

- ABC-SMC (Sequential Monte-Carlo)



Models

$$\frac{dN}{dt} = \lambda_N + \kappa_{N\beta}\beta - \mu_N N - \nu_{N\Phi} N\Phi$$
$$\frac{d\Phi}{dt} = \lambda_\Phi + \kappa_{\Phi\beta}\beta - \mu_\Phi \Phi$$
$$\frac{d\beta}{dt} = \frac{s_{\beta N} N}{1 + i_{\beta\Phi}\Phi} - \mu_\beta \beta$$
$$\frac{d\alpha}{dt} = s_{\alpha\Phi}\Phi - \mu_\alpha \alpha$$

Model 1

Model 2
exponential decaying λ_N
$\lambda_N e^{-at}$

Model 3
remove $i_{\beta\Phi}$

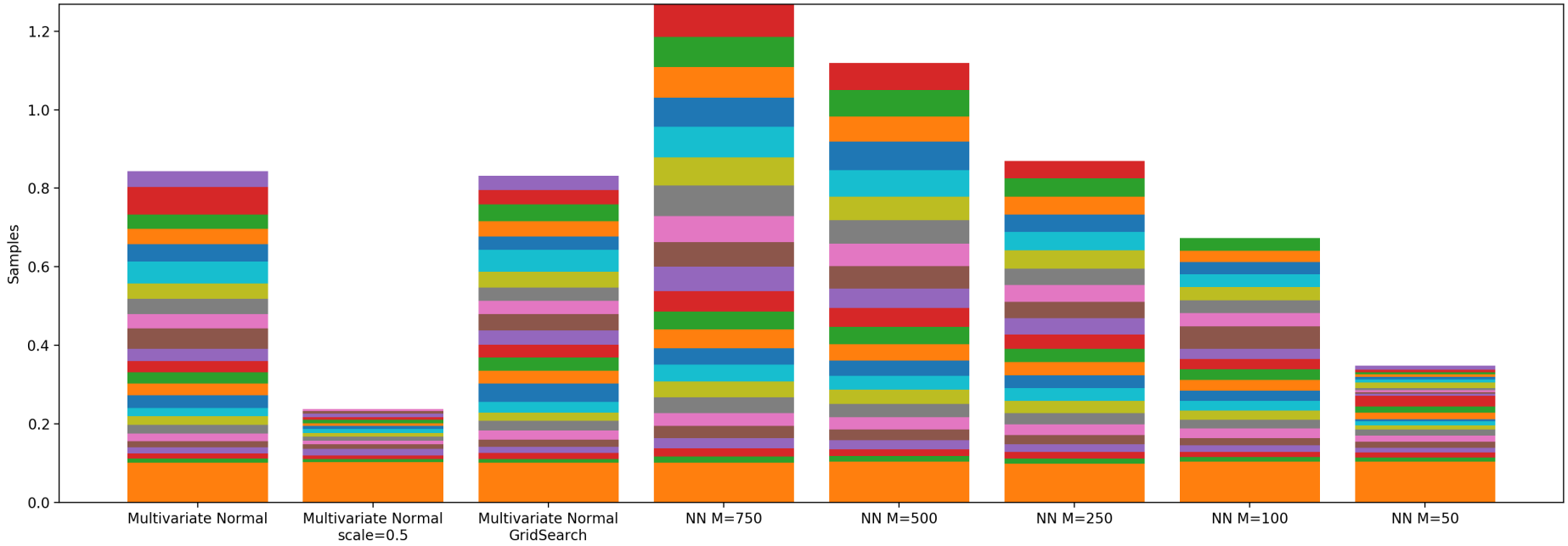
Implementations

- pyABC (modified) in Python

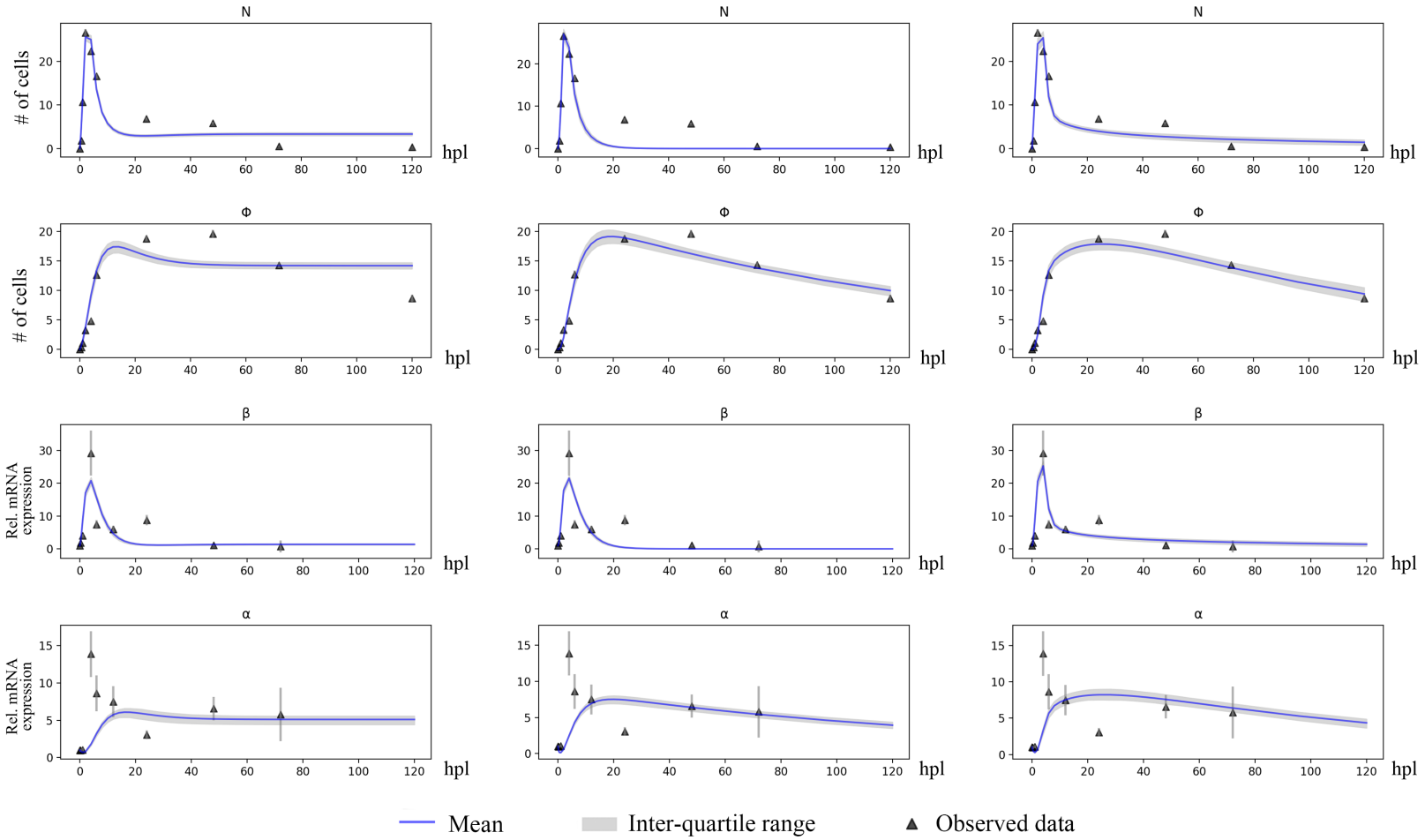
- Experiments
  - implementation options, algorithm hyperparameters
  - parameter inference
  - model comparison

Implementations

- Implementation options, algorithm hyperparameters
  - use synthetic data with know parameter values
  - compare efficiency and their influence on the goodness of fit
  - suggest proper setting for later parameter inference



Implementations – Parameter inference



Model 1 Model 2 Model 3

Implementations – Parameter inference

Model 3

$$\frac{d\alpha}{dt} = s_{\alpha\Phi}\Phi - \mu_{\alpha}\alpha$$

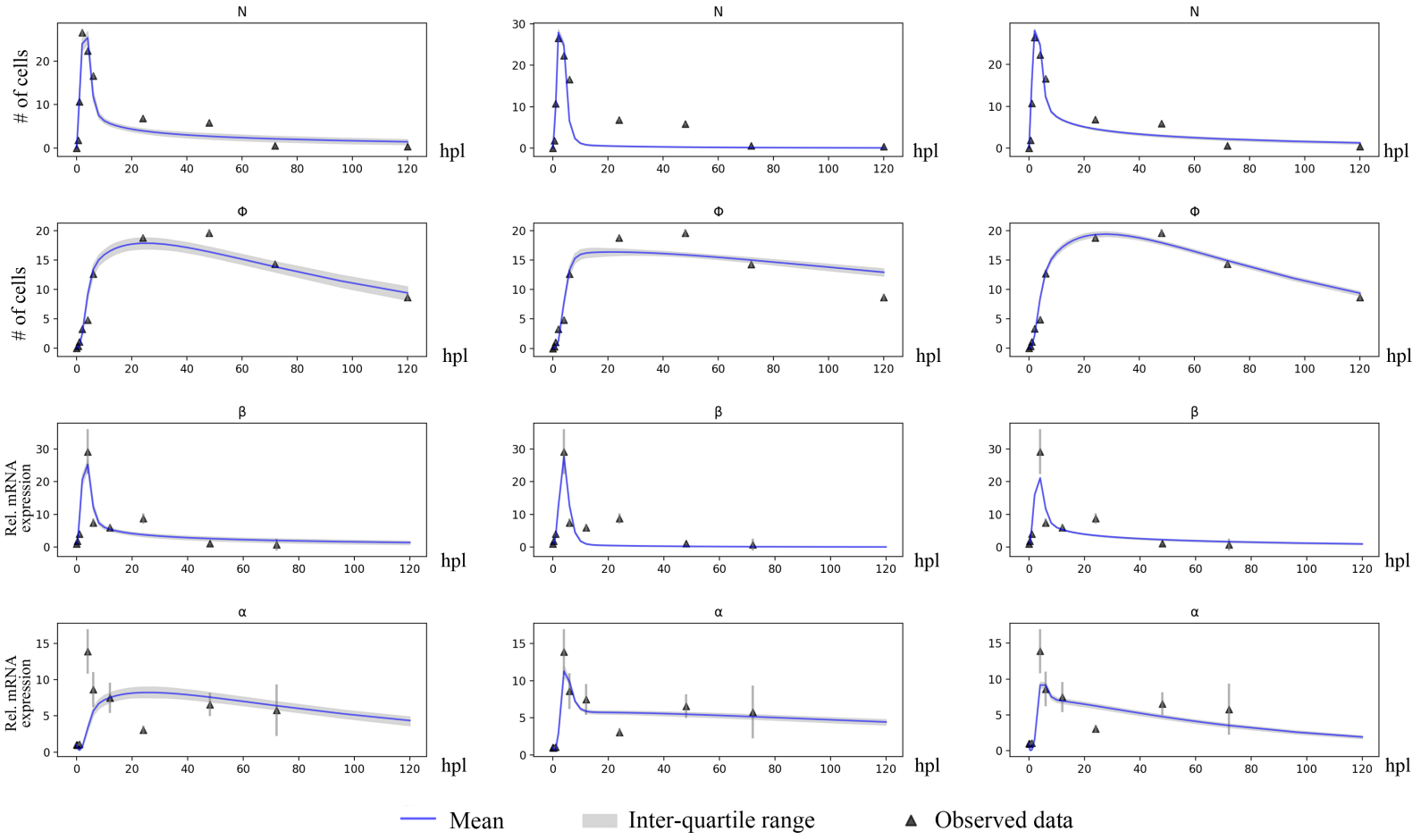
Model 4

$$\frac{d\alpha}{dt} = s_{\alpha\Phi}\Phi - \mu_{\alpha}\alpha + d_{\beta\alpha}\beta$$

Model 5

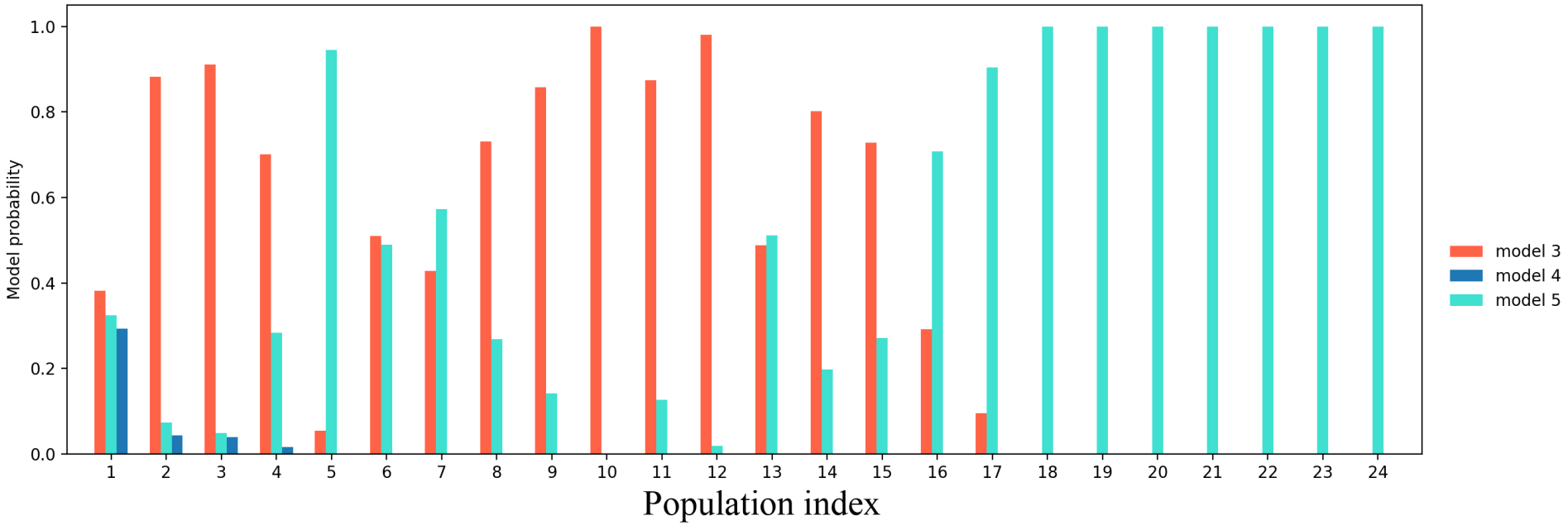
$$\frac{d\alpha}{dt} = \left(s_{\alpha\Phi} + f_{\beta\alpha}\beta\right)\Phi - \mu_{\alpha}\alpha$$

Implementations – Parameter inference



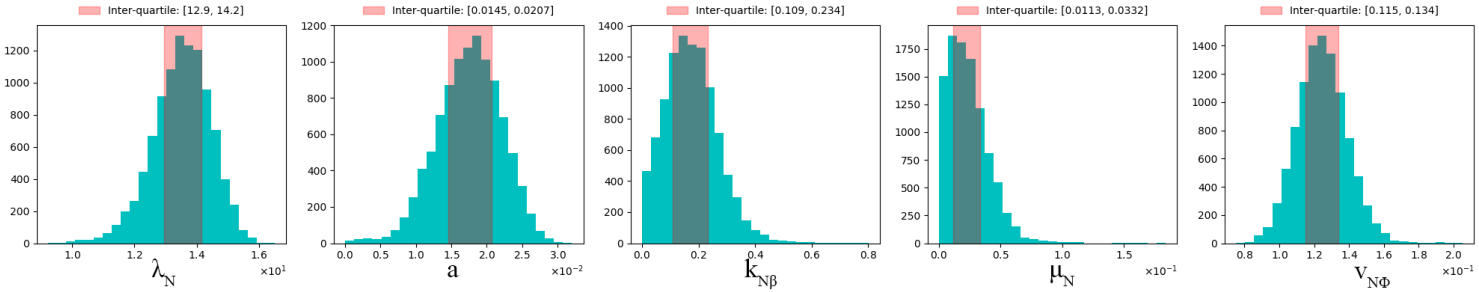
Model 3 Model 4 Model 5

Implementations – Parameter inference

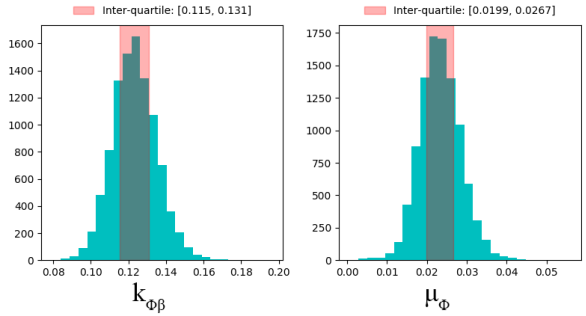


Implementations – Parameter inference

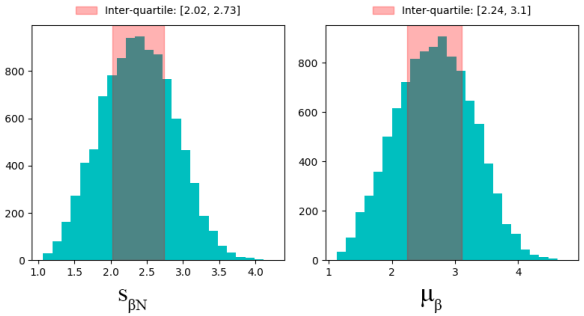
ODE 1



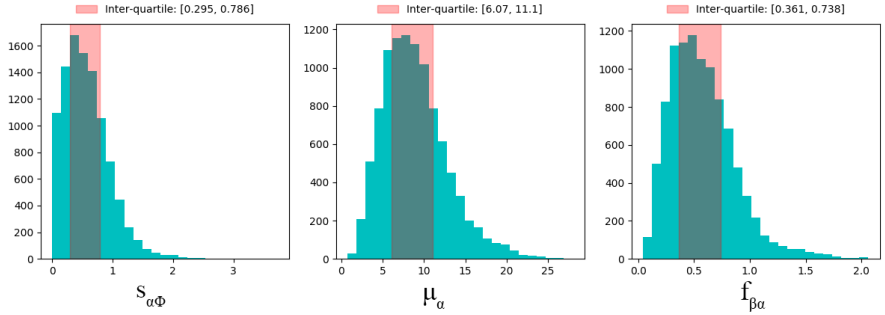
ODE 2



ODE 3

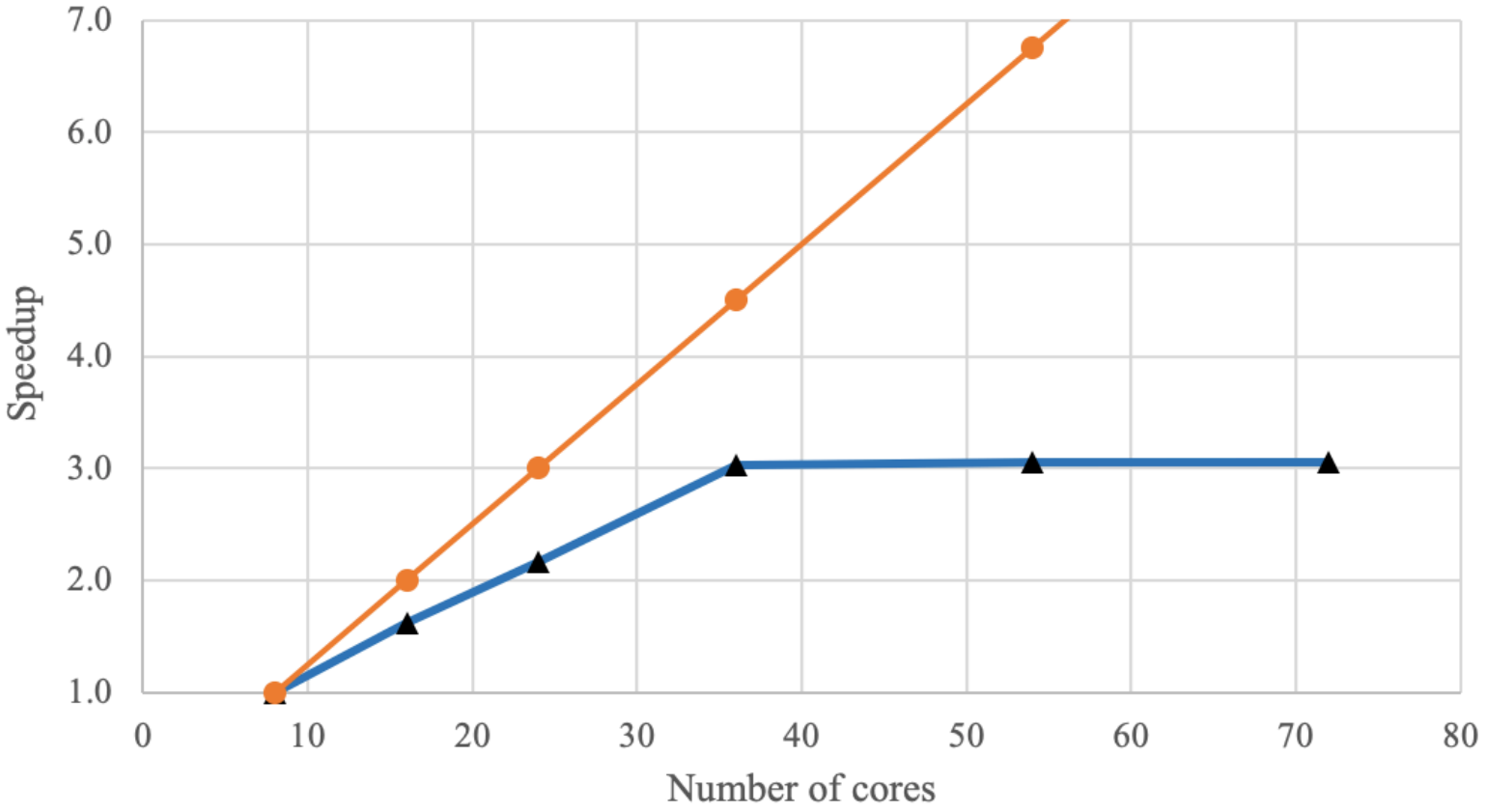


ODE 4

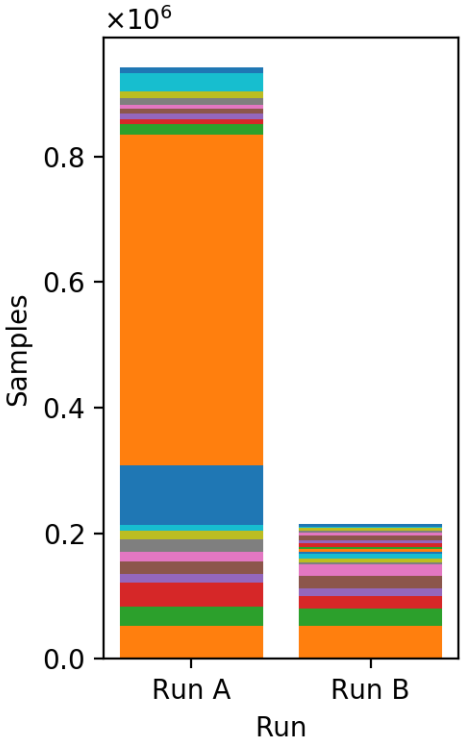


Model 5

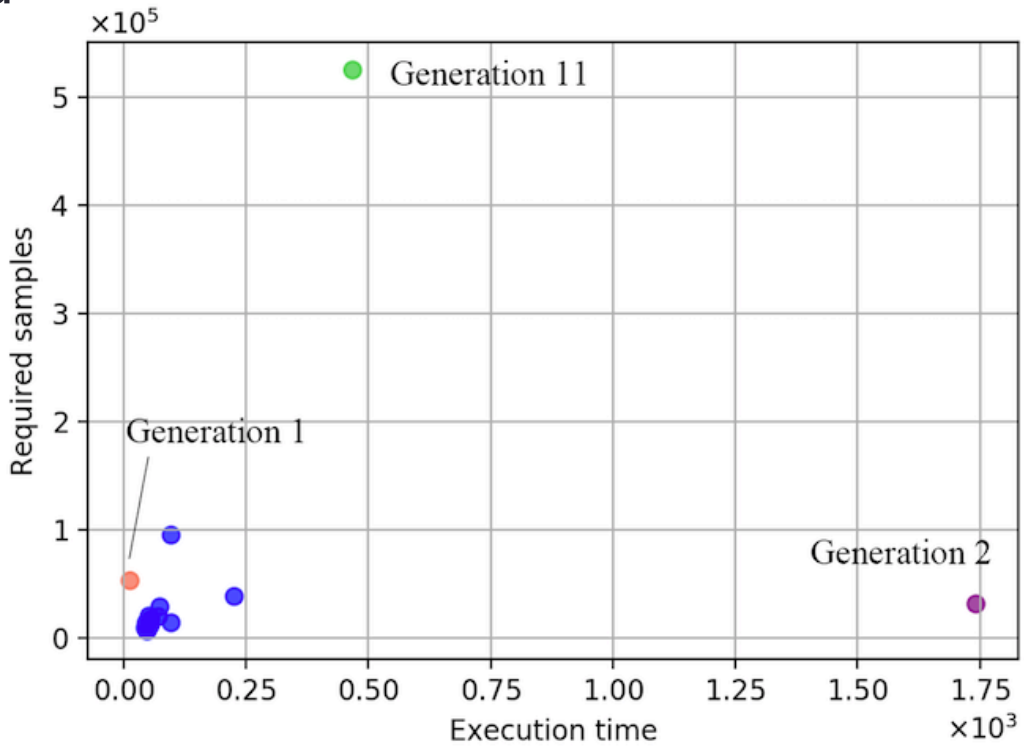
Performance



Performance



- High variance observed in required number of samples of different runs
  - related to local optima
- Execution time for each sample is not fixed



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Conclusions

- ABC SMC performed well in our parameter inference and model comparison task
  - with model 5 being the best model and its parameters being well-inferred
- Scaling-up performance is reasonable
  - However, further analysis was affected by the uncertainties

Thanks!

- Questions?