南方科技大学
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Chapter 2: Context-Free Grammars & Syntax Analysis

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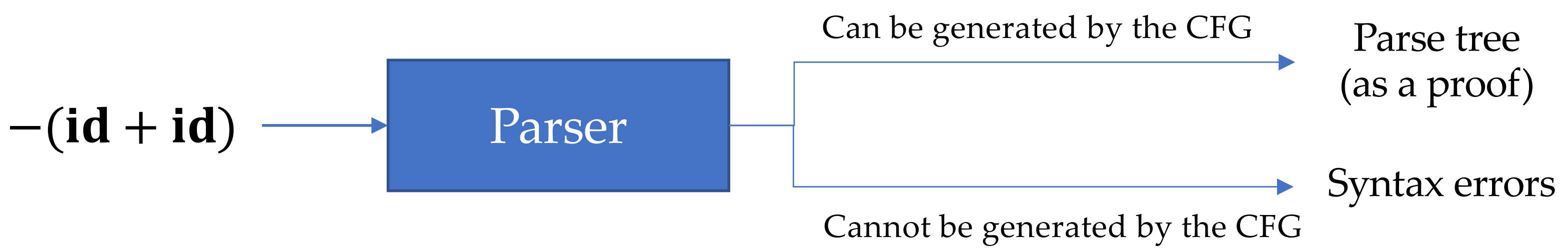
Outline

- Introduction: Syntax and Parsers
- Context-Free Grammars
- Top-Down Parsing Techniques
- Bottom-Up Parsing Techniques

Parsing Overview

- During program compilation, the syntax analyzer (a.k.a. parser) checks whether the string of token names produced by the lexer can be generated by the grammar for the source language
  - That is, if we can find a parse tree whose frontier is equal to the string, then the parser can declare "success"



CFG: $E \rightarrow - E \mid E + E \mid E * E \mid (E) \mid \mathbf{id}$

Top-Down Parsing

- **Problem definition:** Constructing a parse tree for the input string, starting from the root and creating the nodes of the parse tree in preorder (depth-first)

- **Two basic actions in top-down parsing:**
  - **Predict:** Determine the production to be applied for the leftmost nonterminal* (of the current parse tree's frontier)
  - **Match:** Match the leading terminals in the chosen production's body with the input string

* So that the sentential forms may contain leading terminals to match with the prefix of the input string

Top-Down Parsing Example

- **Grammar**

$$E \rightarrow TE'$$
$$E' \rightarrow +TE' \mid \epsilon$$
$$T \rightarrow FT'$$
$$T' \rightarrow * FT' \mid \epsilon$$
$$F \rightarrow (E) \mid id$$

- **Input string**

$$\mathbf{id} + \mathbf{id} * \mathbf{id}$$

Is the input string a sentence of the grammar?

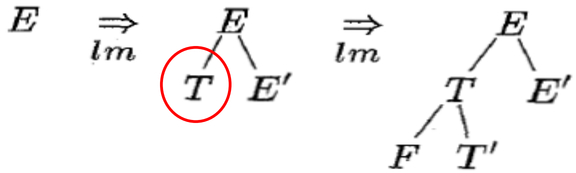
- **Grammar:** $E \rightarrow TE'$ $E' \rightarrow +TE' \mid \epsilon$ $T \rightarrow FT'$ $T' \rightarrow * FT' \mid \epsilon$ $F \rightarrow (E) \mid id$
- **Input string:** $\mathbf{id} + \mathbf{id} * \mathbf{id}$

E

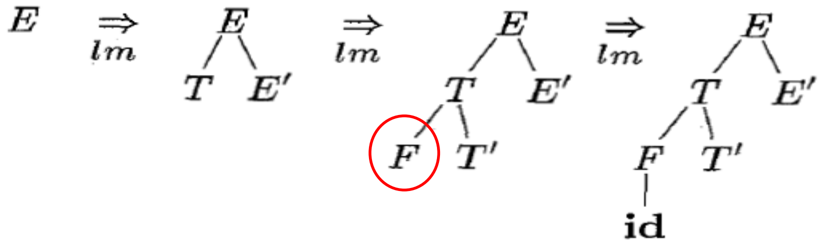
- **Grammar:** $E \rightarrow TE'$ $\quad E' \rightarrow +TE' \mid \epsilon \quad T \rightarrow FT' \quad T' \rightarrow * FT' \mid \epsilon \quad F \rightarrow (E) \mid id$
- **Input string:** **id + id * id** **The sentential form after rewrite:** TE'

$E \underset{lm}{\Rightarrow}$

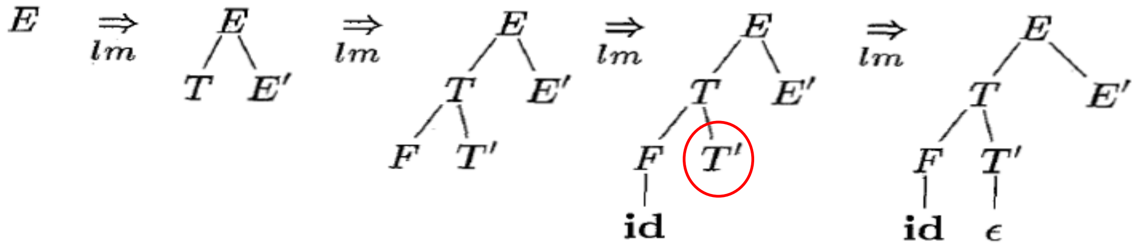
- **Grammar:** $E \to TE'$ $E' \to +TE' \mid \epsilon$ $T \to FT'$ $T' \to * FT' \mid \epsilon$ $F \to (E) \mid id$
- **Input string:** $\mathbf{id} + \mathbf{id} * \mathbf{id}$ **The sentential form after rewrite:** $FT'E'$



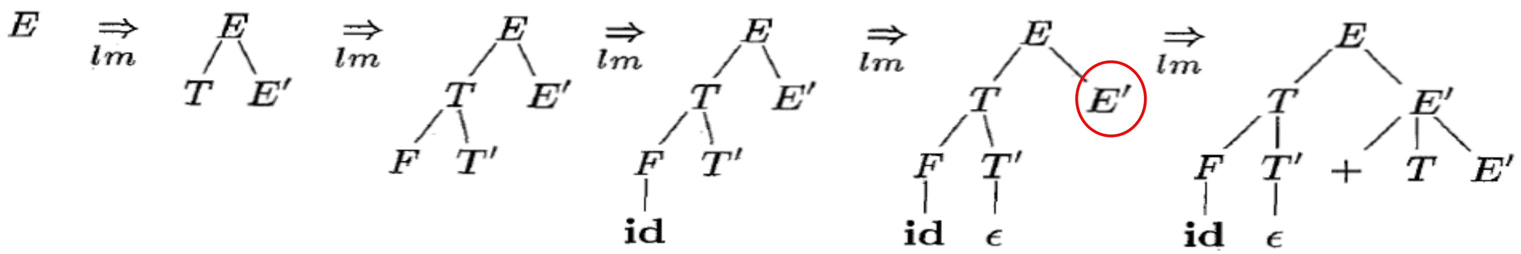
- **Grammar:** $E \to TE'$ $\quad E' \to +TE' \mid \epsilon$ $\quad T \to FT'$ $\quad T' \to * FT' \mid \epsilon$ $\quad F \to (E) \mid id$
- **Input string:** **id + id * id** **The sentential form after rewrite:** **id**$T'E'$



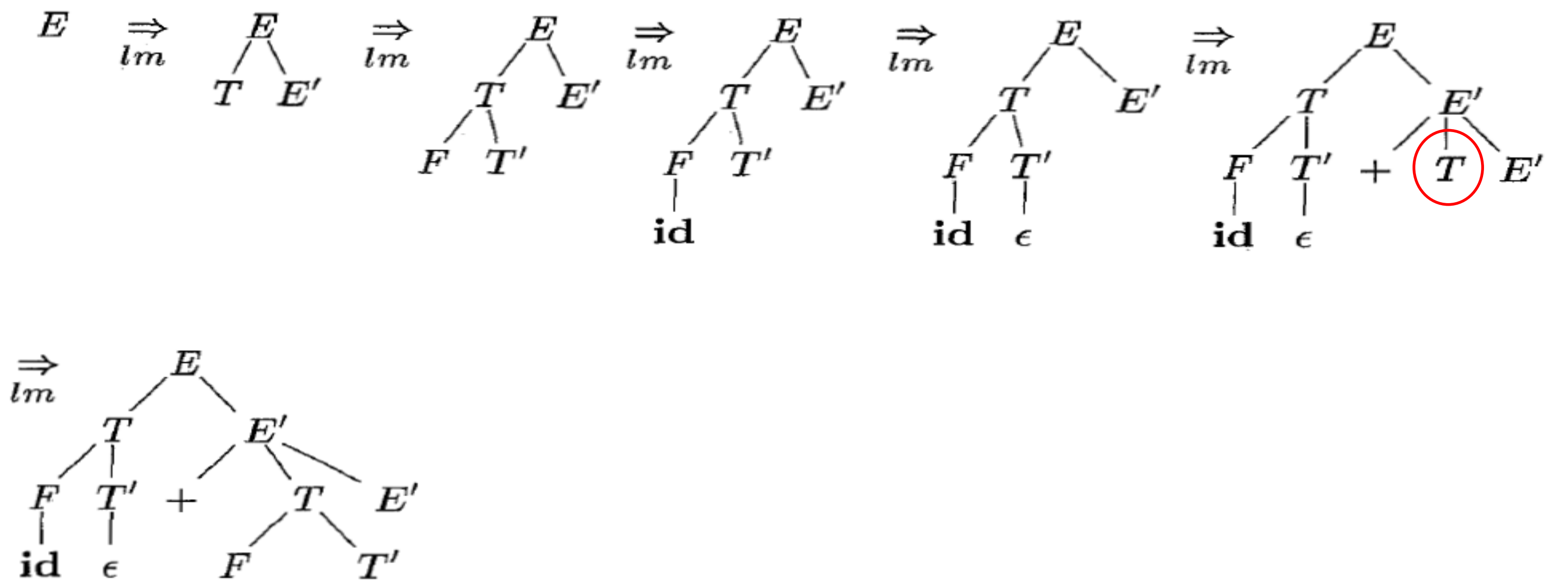
- **Grammar:** $E \rightarrow TE' \quad E' \rightarrow +TE' \mid \epsilon \quad T \rightarrow FT' \quad T' \rightarrow * FT' \mid \epsilon \quad F \rightarrow (E) \mid id$
- **Input string: id + id * id** **The sentential form after rewrite: id**E'



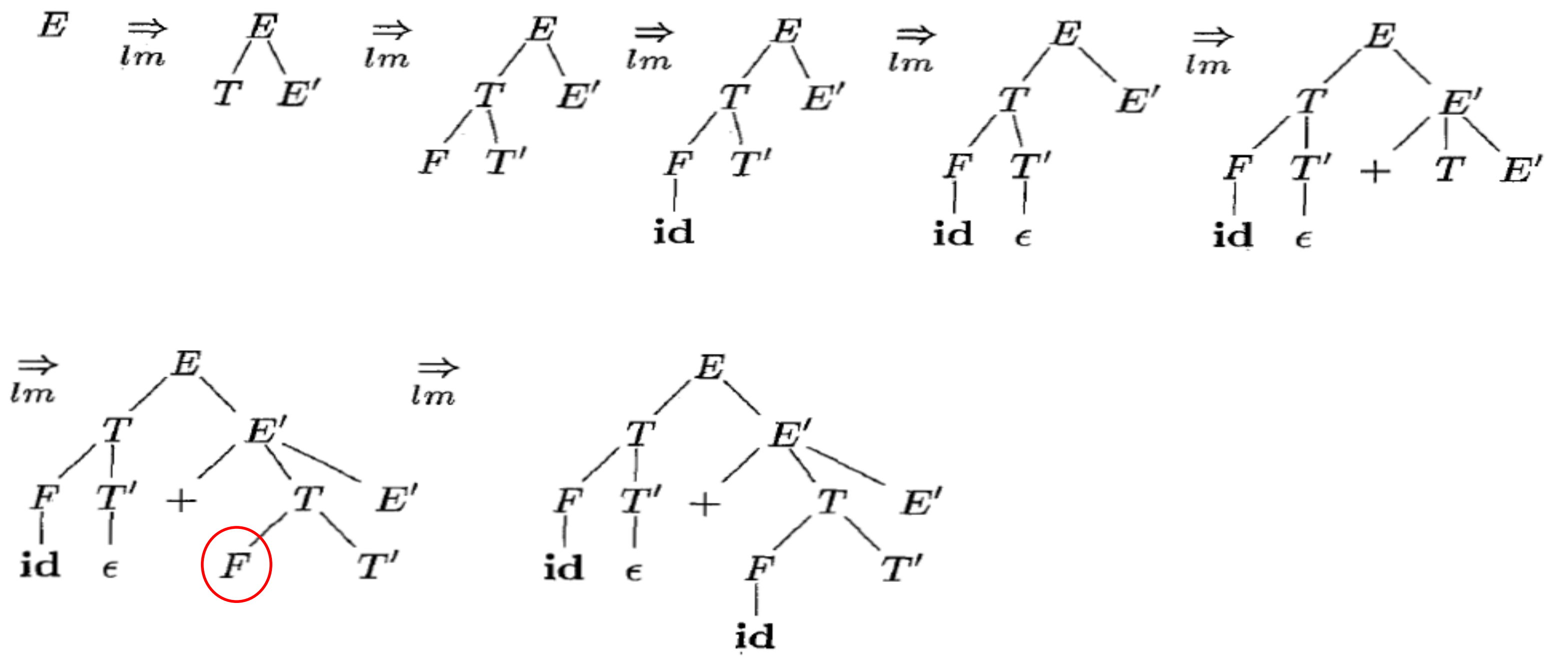
- **Grammar:** $E \rightarrow TE'$ $\quad E' \rightarrow +TE' \mid \epsilon$ $\quad T \rightarrow FT'$ $\quad T' \rightarrow * FT' \mid \epsilon$ $\quad F \rightarrow (E) \mid id$
- **Input string: id + id * id** **The sentential form after rewrite: id** $+TE'$



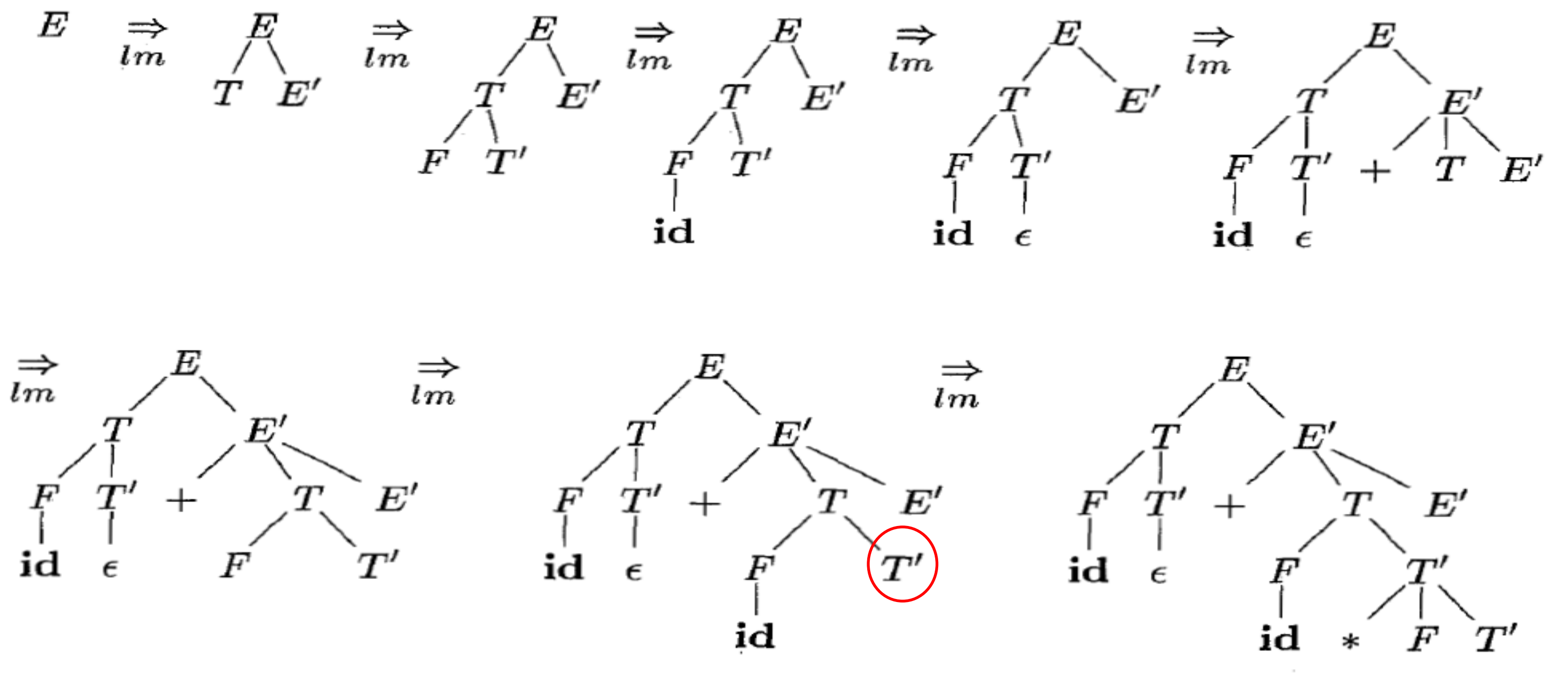
- **Grammar:** $E \to TE'$ $\quad E' \to +TE' \mid \epsilon$ $\quad \underline{T \to FT'}$ $\quad T' \to * FT' \mid \epsilon$ $\quad F \to (E) \mid id$
- **Input string: id + id $*$ id** **The sentential form after rewrite: id +**$FT'E'$



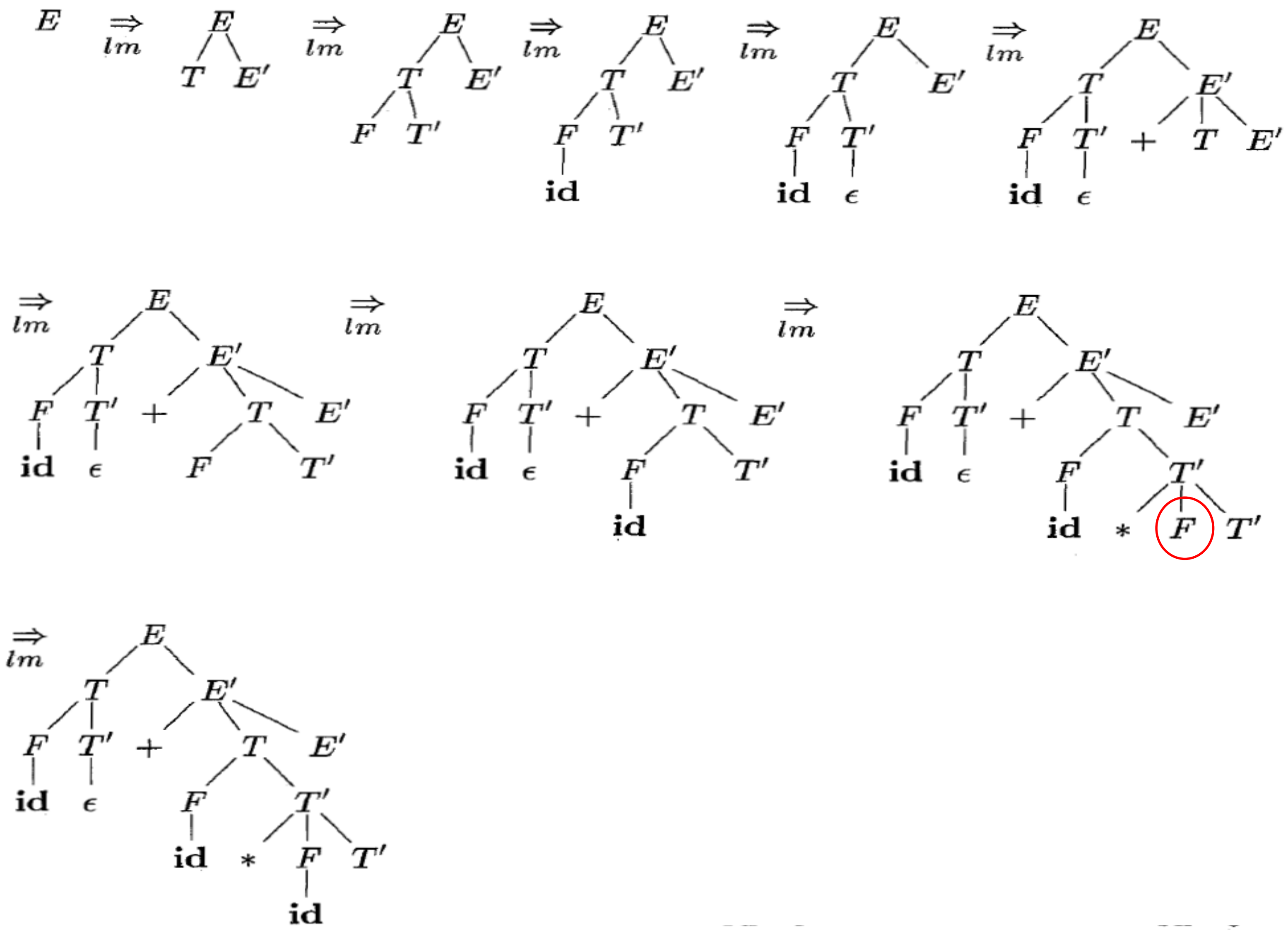
- **Grammar:** $E \rightarrow TE' \quad E' \rightarrow +TE' \mid \epsilon \quad T \rightarrow FT' \quad T' \rightarrow * FT' \mid \epsilon \quad F \rightarrow (E) \mid id$
- **Input string:** $\mathbf{id} + \mathbf{id} * \mathbf{id}$ **The sentential form after rewrite:** $\mathbf{id} + \mathbf{id}T'E'$



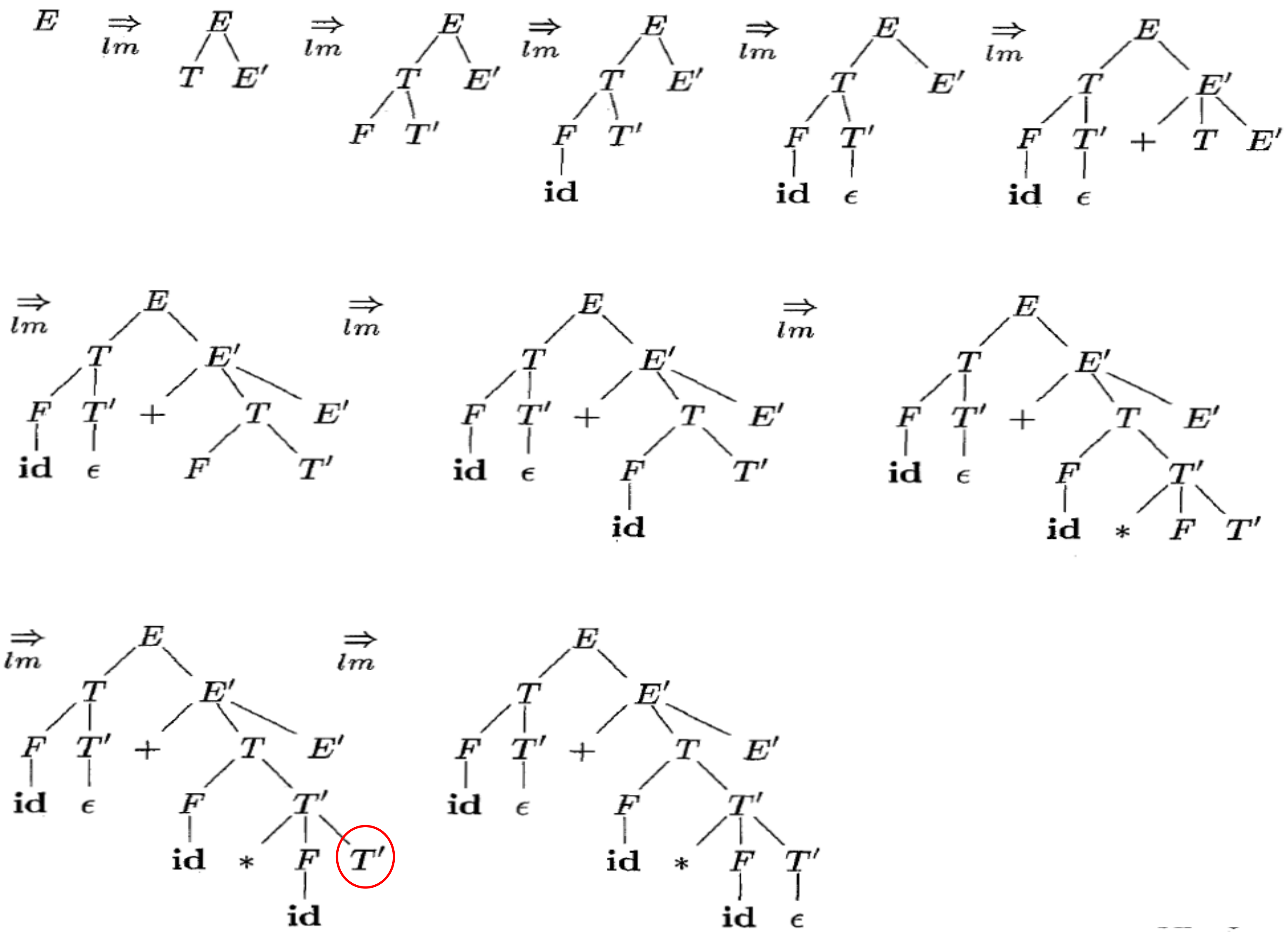
- **Grammar:** $E \rightarrow TE'$ $\quad E' \rightarrow +TE' \mid \epsilon$ $\quad T \rightarrow FT'$ $\quad T' \rightarrow * FT' \mid \epsilon$ $\quad F \rightarrow (E) \mid id$
- **Input string: id + id * id** **The sentential form after rewrite: id + id** $* FT'E'$



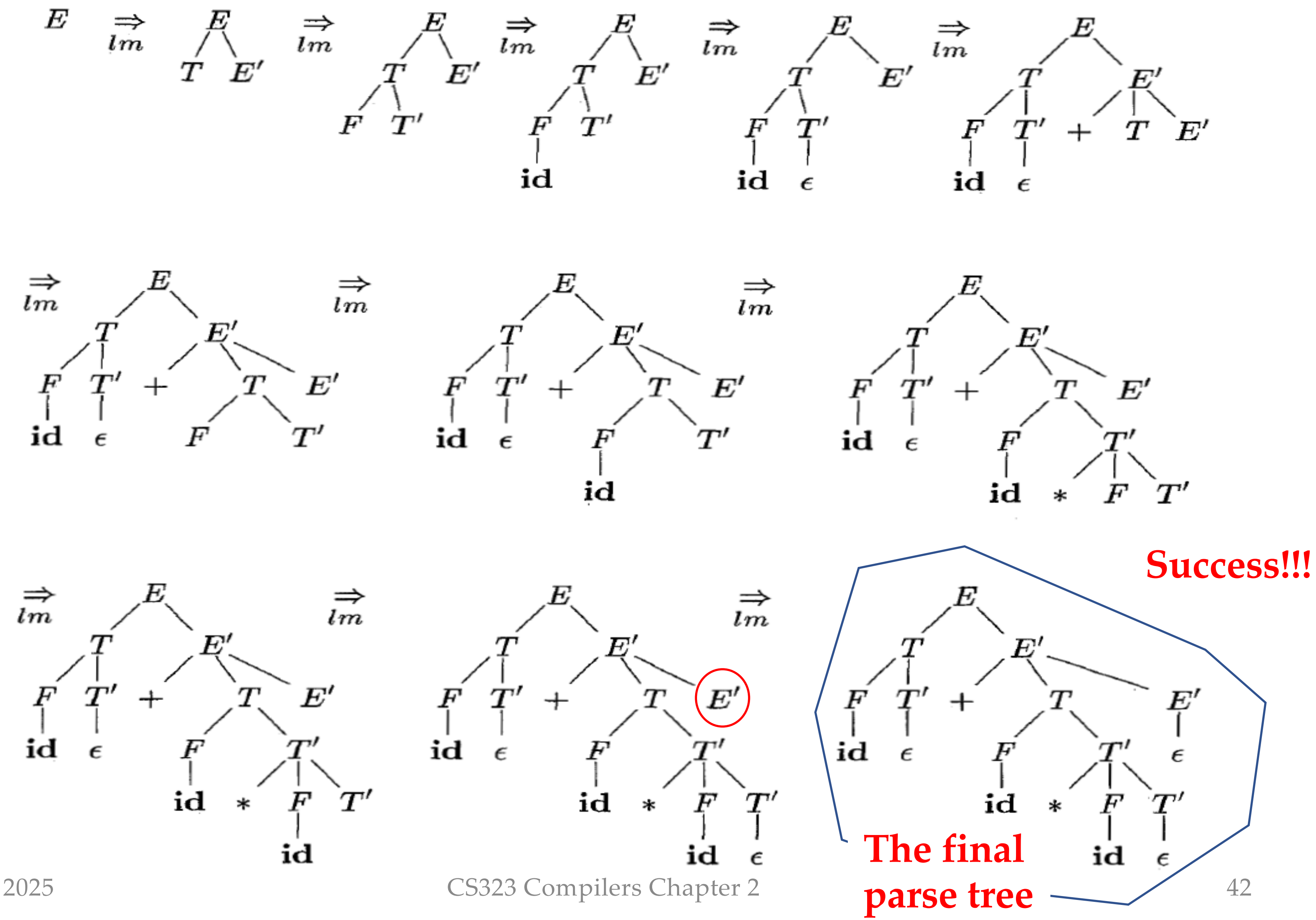
- **Grammar:** $E \to TE' \quad E' \to +TE' \mid \epsilon \quad T \to FT' \quad T' \to * FT' \mid \epsilon \quad F \to (E) \mid id$
- **Input string: id + id * id** **The sentential form after rewrite: id + id * id** $T'E'$



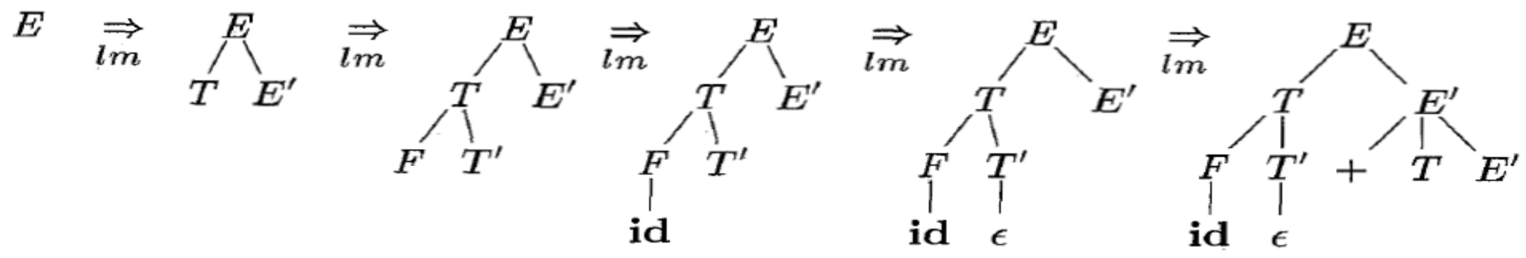
- **Grammar:** $E \rightarrow TE'$ $\quad E' \rightarrow +TE' \mid \epsilon$ $\quad T \rightarrow FT'$ $\quad T' \rightarrow * FT' \mid \epsilon$ $\quad F \rightarrow (E) \mid id$
- **Input string: id + id ∗ id** **The sentential form after rewrite: id + id ∗ id** E'



- **Grammar:** $E \rightarrow TE'$ $\quad E' \rightarrow +TE' \mid \epsilon$ $\quad T \rightarrow FT'$ $\quad T' \rightarrow * FT' \mid \epsilon$ $\quad F \rightarrow (E) \mid id$
- **Input string: id + id * id** **The sentential form after rewrite: id + id * id**

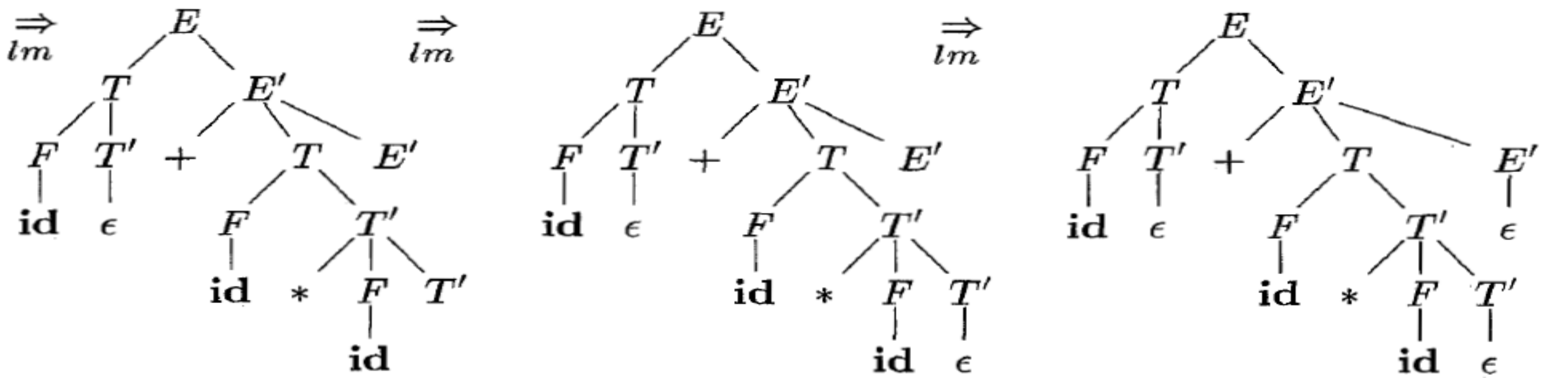


- **Grammar:** $E \to TE' \quad E' \to +TE' \mid \epsilon \quad T \to FT' \quad T' \to * FT' \mid \epsilon \quad F \to (E) \mid id$
- **Input string:** $\mathbf{id} + \mathbf{id} * \mathbf{id}$



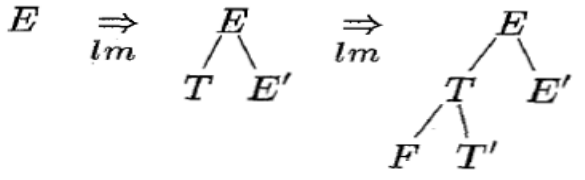
We can make two observations from the example:

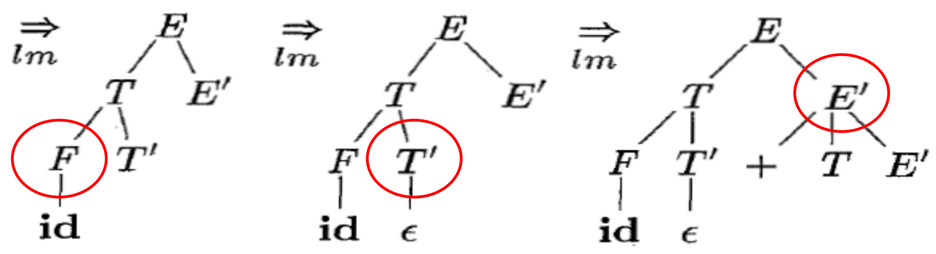
- Top-down parsing is equivalent to finding a leftmost derivation.
- At each step, the frontier of the tree is a left-sentential form.



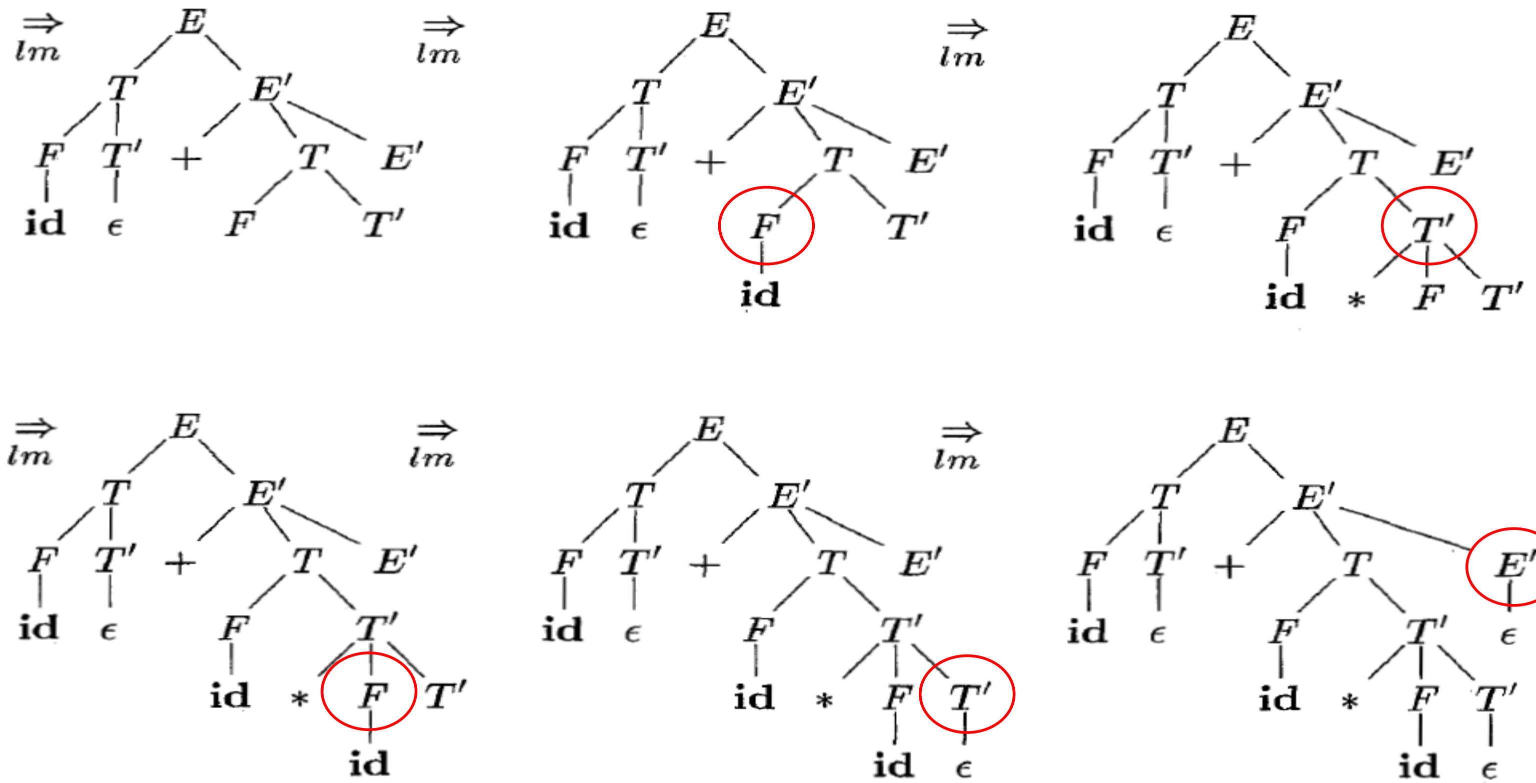
Key decision in top-down parsing: Which production to apply at each step?

Grammar: $E \to TE'$ $\quad E' \to +TE' \mid \epsilon$ $\quad T \to FT'$ $\quad T' \to * FT' \mid \epsilon$ $\quad F \to (E) \mid id$





F has two productions, why do we choose the second one?



Outline

- Introduction: Syntax and Parsers
- Context-Free Grammars
- Top-Down Parsing Techniques
  - Recursive-descent parsing
  - Non-recursive predictive parsing (Lab)
- Bottom-Up Parsing Techniques

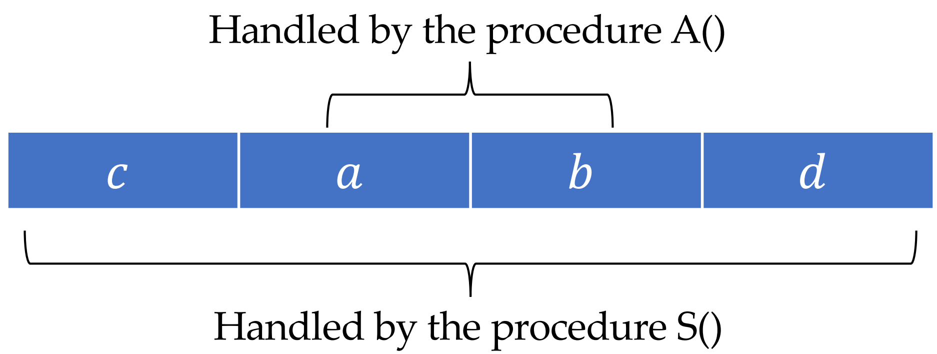
Recursive-Descent Parsing (递归下降的语法分析)

- A recursive-descent parsing program has a set of procedures, one for each nonterminal
  - The procedure for a nonterminal deals with a substring of the input
- Execution begins with the procedure for the start symbol
  - Announce success if the procedure scans the entire input (the start symbol derives the whole input via applying a series of productions)

CFG:

$$S \rightarrow cAd$$
$$A \rightarrow ab$$

Input string:



A Typical Procedure for A Nonterminal

```
void A() {
    Choose an A-production, A → X1 X2 ··· Xk;
    for ( i = 1 to k ) {
        if ( Xi is a nonterminal )
            call procedure Xi();
        else if ( Xi equals the current input symbol a )
            advance the input to the next symbol;
        else /* an error has occurred */;
    }
}
```

Predict

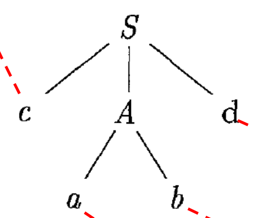
Match

CFG:

$S \rightarrow cAd$
$A \rightarrow ab$

Parsing input:

c	*a*	*b*	*d*



call S() → match "c"

→ call A() → match "ab" → A() return

→ match "d" → S() return

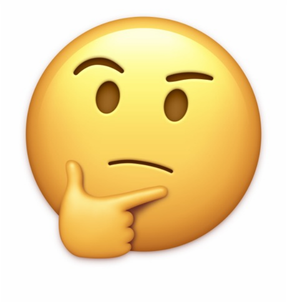
Backtracking (回溯)

```
   void A() {
1)     Choose an A-production, A → X1 X2 ··· Xk;
2)     for ( i = 1 to k ) {
3)         if ( Xi is a nonterminal )
4)             call procedure Xi();
5)         else if ( Xi equals the current input symbol a )
6)             advance the input to the next symbol;
7)         else /* an error has occurred */;
       }
   }
```

If there is a failure at line 7, does this mean that there must be syntax errors?

Backtracking (回溯)

```
   void A() {
1)     Choose an A-production, A → X_1 X_2 ··· X_k;
2)     for ( i = 1 to k ) {
3)         if ( X_i is a nonterminal )
4)             call procedure X_i();
5)         else if ( X_i equals the current input symbol a )
6)             advance the input to the next symbol;
7)         else /* an error has occurred */;
       }
   }
```



The failure might be caused by a wrong choice of *A*-production at line 1 !!!

Backtracking (回溯)

- General recursive-descent parsing may require repeated scans over the input (backtracking)
- To allow backtracking, we need to modify the algorithm

void $A()$ {
    Choose an A-production, $A \rightarrow X_1 X_2 \cdots X_k$;
    for ($i = 1$ to k) {
        if (X_i is a nonterminal)
            call procedure $X_i()$;
        else if (X_i equals the current input symbol a)
            advance the input to the next symbol;
        else /* an error has occurred */;
    }
}

Instead of exploring one *A*-production, we must try each possible production in some order.

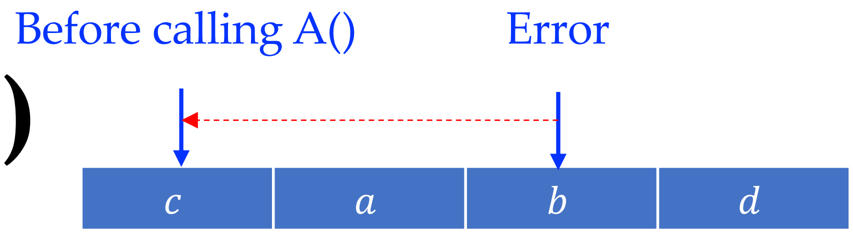
Backtracking (回溯)

- General recursive-descent parsing may require repeated scans over the input (backtracking)
- To allow backtracking, we need to modify the algorithm

```
    void A() {
1)      Choose an A-production, A → X1 X2 ··· Xk;
2)      for ( i = 1 to k ) {
3)          if ( Xi is a nonterminal )
4)              call procedure Xi();
5)          else if ( Xi equals the current input symbol a )
6)              advance the input to the next symbol;
7)          else /* an error has occurred */;
        }
    }
```

When there is a failure at line 7, return to line 1 and try another *A*-production.

Backtracking (回溯)



- General recursive-descent parsing may require repeated scans over the input (backtracking)
- To allow backtracking, we need to modify the algorithm

```
void A() {
        Choose an A-production, A → X1 X2 ··· Xk;
        for ( i = 1 to k ) {
                if ( Xi is a nonterminal )
                        call procedure Xi();
                else if ( Xi equals the current input symbol a )
                        advance the input to the next symbol;
                else /* an error has occurred */;
        }
}
```

In order to try another *A*-production, we must reset the input pointer that points to the next symbol to scan (failed trials consume symbols)

Backtracking Example

- Grammar: $S \rightarrow cAd \quad A \rightarrow ab \mid \boxed{a}$ One more production for A
- Input string: cad

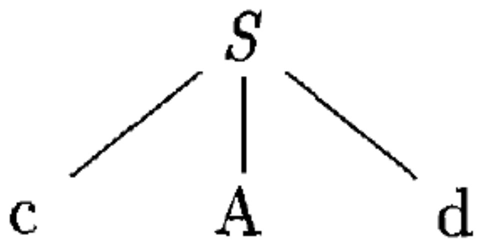
c	a	d

$\uparrow$ input pointer

Backtracking Example

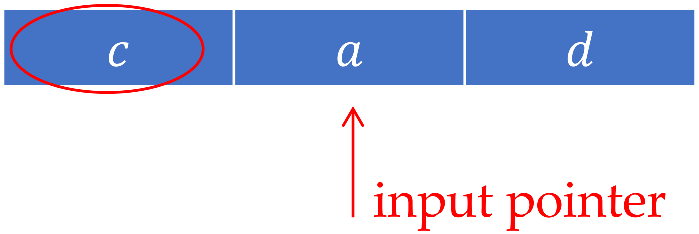
- Grammar: $S \rightarrow cAd \quad A \rightarrow ab \mid a$
- Input string: cad

  – S has only one production, apply it

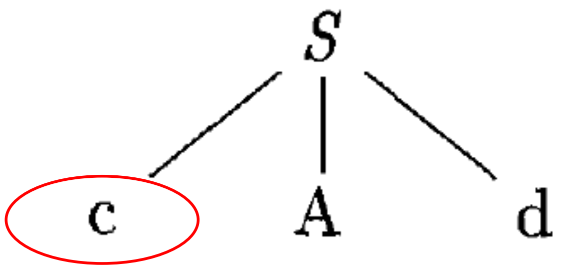


Backtracking Example

- Grammar: $S \rightarrow cAd \quad A \rightarrow ab \mid a$
- Input string: *cad*



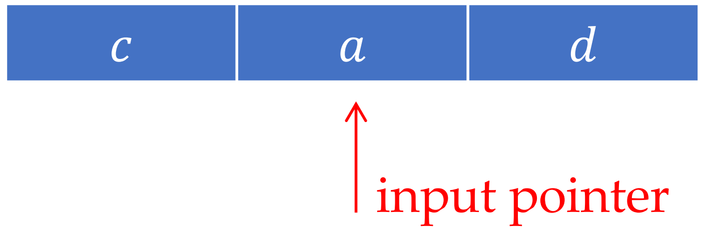
- The leftmost leaf matches *c* in input
- Advance input pointer

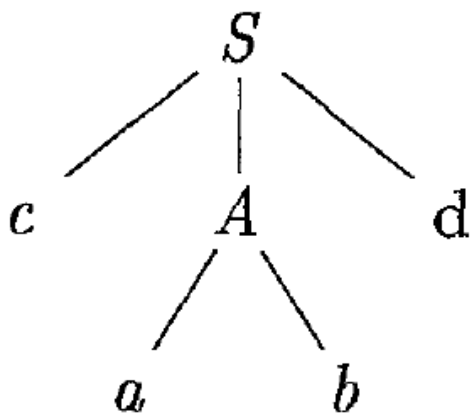


Backtracking Example

- Grammar: $S \rightarrow cAd \quad A \rightarrow ab \mid a$
- Input string: cad

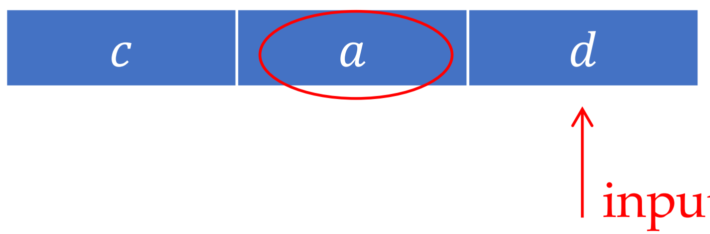
  – Expand A using the first production





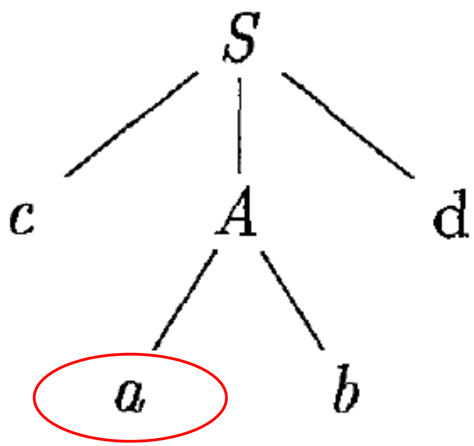
Backtracking Example

- Grammar: $S \rightarrow cAd \quad A \rightarrow ab \mid a$
- Input string: cad



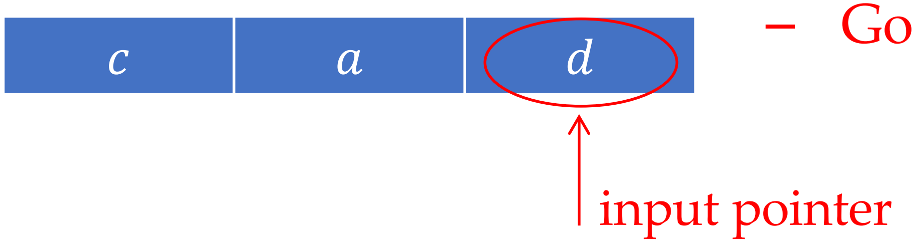
input pointer

- Leftmost leaf matches a in input
- Advance input pointer

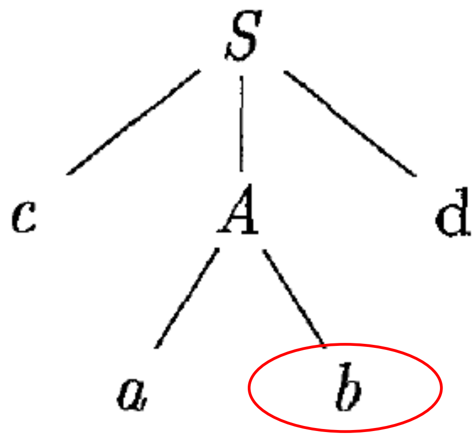


Backtracking Example

- Grammar: $S \rightarrow cAd \quad A \rightarrow ab \mid a$
- Input string: *cad*

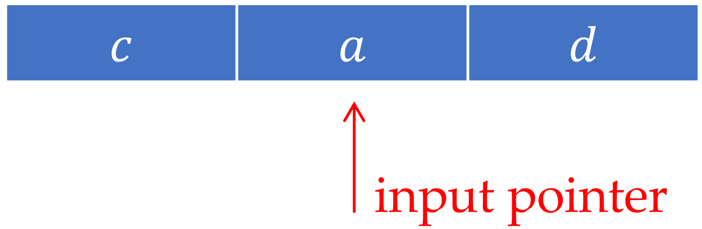


- Symbol mismatch
- Go back to try another *A*-production

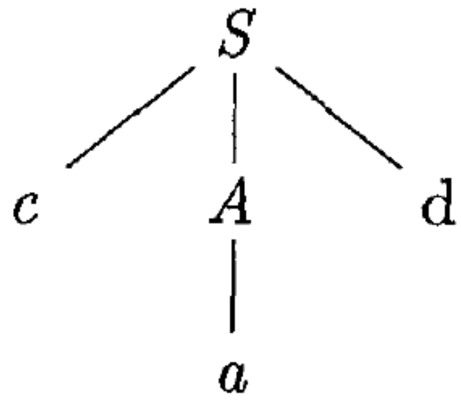


Backtracking Example

- Grammar: $S \rightarrow cAd \quad A \rightarrow ab \mid a$
- Input string: *cad*

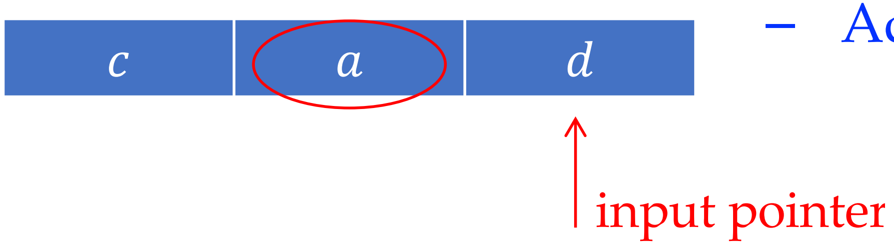


– Reset input pointer
– Expand *A* using its second production

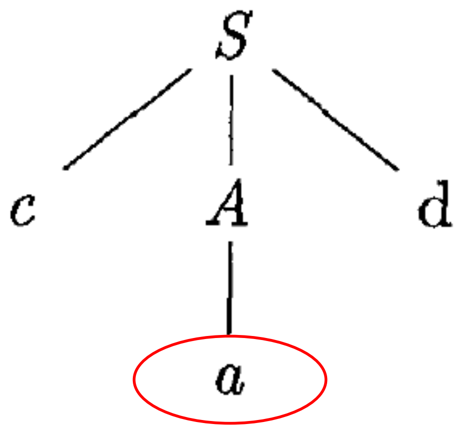


Backtracking Example

- Grammar: $S \rightarrow cAd \quad A \rightarrow ab \mid a$
- Input string: cad



– Leftmost leaf matches a in input
– Advance input pointer



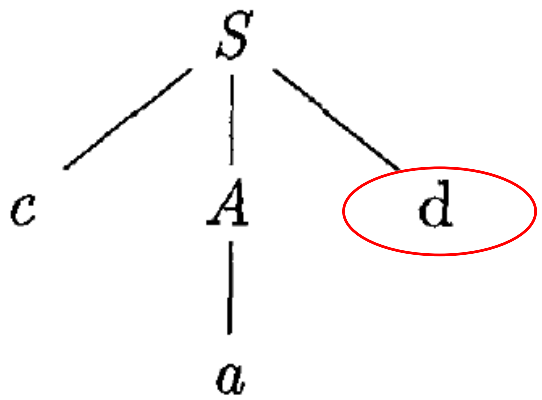
Backtracking Example

- Grammar: $S \rightarrow cAd \quad A \rightarrow ab \mid a$
- Input string: *cad*

– The last leaf node matches *d* in input

– Announce success!

Scanned entire input



The Problem of Left Recursion

Suppose there is only one A-production, ***A* → *Aα* ...**

```
void A() {
    Choose an A-production, [A → X1 X2 ··· Xk];
    for ( i = 1 to k ) {
        if ( Xi is a nonterminal )
            call procedure Xi();
        else if ( Xi equals the current input symbol a )
            advance the input to the next symbol;
        else /* an error has occurred */;
    }
}
```

Recursively rewriting *A* without matching any terminals

If there is left recursion in a CFG, a recursive-descent parser may go into an infinite loop! Revise the CFG before parsing!!!

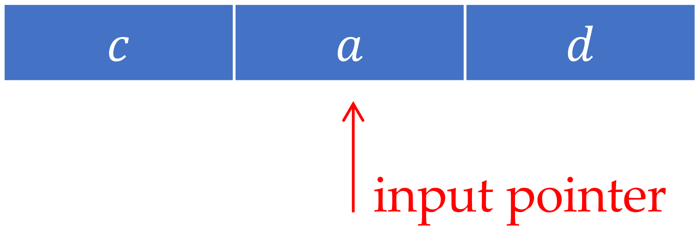
Can We Avoid Backtracking?

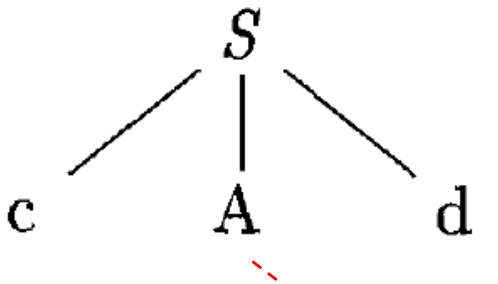
```
    void A() {
1)      Choose an A-production, A → X1 X2 ··· Xk;
2)      for ( i = 1 to k ) {
3)          if ( Xi is a nonterminal )
4)              call procedure Xi();
5)          else if ( Xi equals the current input symbol a )
6)              advance the input to the next symbol;
7)          else /* an error has occurred */;
        }
    }
```

Key problem: At line 1, we make ***random choices*** (brute force search)

Can We Avoid Backtracking?

- Grammar: $S \rightarrow cAd \quad A \rightarrow c \mid a$
- Input string: cad





When rewriting A, is it a good idea to choose $A \rightarrow c$?

No! If we look ahead, the next char in the input is a.
$A \rightarrow c$ is obviously a bad choice!!!

Looking Ahead Helps!

- Suppose the input string is $\boldsymbol{x}\boldsymbol{a}\ldots$
- Suppose the current sentential form is $\boldsymbol{x}\boldsymbol{A}\boldsymbol{\beta}$
  - $\boldsymbol{A}$ is a non-terminal; $\boldsymbol{\beta}$ may contain both terminals and non-terminals

If we know the following fact for the productions $\boldsymbol{A \rightarrow \alpha \mid \gamma}$:

- $a \in FIRST(\alpha)$: α derives strings that begin with a
- $a \notin FIRST(\gamma)$: γ derives strings that do not begin with a

*$FIRST(\alpha)$ denotes the set of beginning terminals of strings derived from α

After matching x, which production should we choose to rewrite A?

$\boldsymbol{A \rightarrow \alpha}$

Computing FIRST

- **FIRST(X)**, where X is a grammar symbol
  - If X is a terminal, then FIRST(X) = {X}
  - If X is a nonterminal and $X \rightarrow \epsilon$, then add ϵ to FIRST(X)
  - If X is a nonterminal and $X \rightarrow Y_1 Y_2 \ldots Y_k$ ($k \geq 1$) is a production
    - If for some i, a is in FIRST(Y_i) and ϵ is in all of FIRST(Y_1), …, FIRST(Y_{i-1}), then add a to FIRST(X)
    - If ϵ is in all of FIRST(Y_1), …, FIRST(Y_k), then add ϵ to FIRST(X)

Computing FIRST Cont.

- **FIRST($X_1 X_2 \dots X_n$)**, where $X_1 X_2 \dots X_n$ is a string of grammar symbols
  - Add all non-ϵ symbols of FIRST(X_1) to FIRST($X_1 X_2 \dots X_n$)
  - If ϵ is in FIRST(X_1), add non-ϵ symbols of FIRST(X_2) to FIRST($X_1 X_2 \dots X_n$)
  - If ϵ is in FIRST(X_1) and FIRST(X_2), add non-ϵ symbols of FIRST(X_3) to FIRST($X_1 X_2 \dots X_n$)
  - …
  - If ϵ is in FIRST(X_i) for all i, add ϵ to FIRST($X_1 X_2 \dots X_n$)

FIRST Example

- **Grammar**
  - $E \to TE'$ $\quad$ $E' \to +TE' \mid \epsilon$
  - $T \to FT'$ $\quad$ $T' \to * FT' \mid \epsilon$ $\quad$ $F \to (E) \mid \mathbf{id}$
- **FIRST sets**
  - $\text{FIRST}(F) = \{(, \mathbf{id}\}$
  - $\text{FIRST}(T) = \text{FIRST}(F) = \{(, \mathbf{id}\}$
  - $\text{FIRST}(E) = \text{FIRST}(T) = \{(, \mathbf{id}\}$
  - $\text{FIRST}(E') = \{+, \epsilon\}$ $\quad$ $\text{FIRST}(T') = \{*, \epsilon\}$
  - $\text{FIRST}(TE') = \text{FIRST}(T) = \{(, \mathbf{id}\}$
  - …

Strings derived from F or T must start with (or **id**

Looking Ahead Helps Cont.

- Suppose the input string is $\boldsymbol{x}\boldsymbol{a}\ldots$
- Suppose the current sentential form is $\boldsymbol{xA\beta}$
  - $\boldsymbol{A}$ is a non-terminal; $\boldsymbol{\beta}$ may contain both terminals and non-terminals

If we know that for the production $\boldsymbol{A \to \alpha}$, $\epsilon \in FIRST(\alpha)$, can we choose the production to rewrite A?

*$\epsilon \in FIRST(\alpha)$ means that rewriting A to α may result in an empty string (recall when we add ϵ to the $FIRST$ set)

If A can be followed by a in some sentential forms ($\boldsymbol{a \in FOLLOW(A)}$), it might# be a good idea to choose $\boldsymbol{A \to \alpha}$ to rewrite A.

"a belonging to the $FOLLOW$ set of A" is a necessary condition of a correct choice. If A can never be followed by a in any valid sentential forms, then the choice is definitely wrong.

Computing FOLLOW

- **Computing FOLLOW set for all nonterminals**
  - Add \$ in FOLLOW(S), where S is the start symbol and \$ is the input right endmarker
  - Apply the rules below, until all FOLLOW sets do not change
    1. If there is a production $A \to \alpha B \beta$, then everything in FIRST(β) except ϵ is in FOLLOW(B)
    2. If there is a production $A \to \alpha B$ (or $A \to \alpha B \beta$ and FIRST(β) contains ϵ) then everything in FOLLOW(A) is in FOLLOW(B)

By definition, ϵ will not appear in any FOLLOW set

FOLLOW Example

- **Grammar**
  - $E \rightarrow TE'$ $E' \rightarrow +TE' \mid \epsilon$
  - $T \rightarrow FT'$ $T' \rightarrow * FT' \mid \epsilon$ $F \rightarrow (E) \mid \mathbf{id}$
- **FOLLOW sets**
  - $\boxed{\text{FOLLOW}(E) = \{\$,)\}}$
  - $\text{FOLLOW}(E') = \{\$,)\}$
  - $\text{FOLLOW}(T) = \{+, \$,)\}$
  - $\text{FOLLOW}(T') = \{+, \$,)\}$
  - $\text{FOLLOW}(F) = \{*, +, \$,)\}$

- \$ is always in FOLLOW(E)
- Everything in FIRST()) except ϵ is in FOLLOW(E)

FOLLOW Example

- **Grammar**
  - $E \rightarrow TE'$ $\quad$ $E' \rightarrow +TE' \mid \epsilon$
  - $T \rightarrow FT'$ $\quad$ $T' \rightarrow * FT' \mid \epsilon$ $\quad$ $F \rightarrow (E) \mid \mathbf{id}$
- **FOLLOW sets**
  - $\text{FOLLOW}(E) = \{\$,)\}$
  - $\text{FOLLOW}(E') = \{\$,)\}$
  - $\boxed{\text{FOLLOW}(T) = \{+, \$,)\}}$
  - $\text{FOLLOW}(T') = \{+, \$,)\}$
  - $\text{FOLLOW}(F) = \{*, +, \$,)\}$

- Everything in FIRST(E') except ϵ is in FOLLOW(T)
- Since $E' \rightarrow \epsilon$, everything in FOLLOW(E) and FOLLOW(E') is in FOLLOW(T)

A Quick Summary

Why Do We Compute FIRST & FOLLOW?

- For a production $head \rightarrow body$, when we are trying to rewrite $head$, if we know $FIRST(body)$, that is, what terminals can strings derived from body start with, we can decide whether to choose $head \rightarrow body$ by looking at the next input symbol.
  - If the next input symbol is in $FIRST(body)$, the production may be a good choice.

- For a production $head \rightarrow \epsilon$ (or $head$ can derive ϵ in some steps), when we are trying to rewrite $head$, if we know $FOLLOW(head)$, that is, what terminals can follow $head$ in valid sentential forms, we can decide whether to choose $head \rightarrow \epsilon$ by looking at the next input symbol.
  - If the next input symbol is in $FOLLOW(head)$, the production may be a good choice.

LL(1) Grammars

- Recursive-descent parsers needing no backtracking can be constructed for a class of grammars called **LL(1)**
  - 1st L: scanning the input from left to right
  - 2nd L: producing a leftmost derivation (top-down parsing)
  - 1: using one input symbol of lookahead at each step to make parsing decision

LL(1) Grammars Cont.

A grammar G is LL(1) if and only if for any two distinct productions $A \rightarrow \alpha \mid \beta$, the following conditions hold:

1. There is no terminal a such that α and β derive strings beginning with a
2. At most one of α and β can derive the empty string
3. If $\beta \overset{*}{\Rightarrow} \epsilon$, then α does not derive any string beginning with a terminal in $\mathrm{FOLLOW}(A)$ and vice versa

*** The three conditions essentially rule out the possibility of applying both productions so that there is a unique choice of production at each "predict" step by looking at the next input symbol**

More formally:

1. $\mathrm{FIRST}(\alpha) \cap \mathrm{FIRST}(\beta) = \emptyset$ (conditions 1-2 above)
2. If $\epsilon \in \mathrm{FIRST}(\beta)$, then $\mathrm{FIRST}(\alpha) \cap \mathrm{FOLLOW}(A) = \emptyset$ and vice versa

LL(1) Grammars Cont.

- For LL(1) grammars, during recursive-descent parsing, the proper production to apply for a nonterminal can be selected by looking only at the current input symbol

Grammar: $stmt \rightarrow \underbrace{\textbf{if}(\textbf{expr})\ stmt\ \textbf{else}\ stmt}_{1} \mid \underbrace{\textbf{while}(\textbf{expr})\ stmt}_{2} \mid \underbrace{\textbf{a}}_{3}$

Parsing steps for input: **if(expr) while(expr) a else a**

1. Rewrite the start symbol *stmt* with ①: **if(expr)** *stmt* **else** *stmt*
2. Rewrite the leftmost *stmt* with ②: **if(expr) while(expr)** *stmt* **else** *stmt*
3. Rewrite the leftmost *stmt* with ③: **if(expr) while(expr) a else** *stmt*
4. Rewrite the leftmost *stmt* with ③: **if(expr) while(expr) a else a**

Parsing Table (预测分析表)

- We can build parsing tables for recursive-descent parsers (**LL parsers**)
- A predictive **parsing table** is a two-dimensional array that determines which production the parser should choose when it sees a nonterminal A and a symbol a on its input stream
- The parsing table of an LL(1) parser has no entries with multiple productions

NON-TERMINAL	INPUT SYMBOL					
	id	$+$	$*$	(	)	\$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \mathbf{id}$			$F \to (E)$		

Parsing Table Construction

The following algorithm can be applied to any CFG

- **Input:** Grammar G **Output:** Parsing table M
- **Method:**
  - For each production $A \to \alpha$ of G, do the following:
    - For each terminal a in $\text{FIRST}(\alpha)$, add $A \to \alpha$ to $M[A, a]$
    - If ϵ is in $\text{FIRST}(\alpha)$, then for each terminal b (including the right endmarker \$) in $\text{FOLLOW}(A)$, add $A \to \alpha$ to $M[A, b]$
  - Set all empty entries in the table to **error**

Fill the table entries so that when rewriting A, we know what production to choose by checking the next input symbol

Parsing Table Construction Example

- **Grammar**
  - $E \rightarrow TE'$ $\quad E' \rightarrow +TE' \mid \epsilon$
  - $T \rightarrow FT'$ $\quad T' \rightarrow * FT' \mid \epsilon$ $\quad F \rightarrow (E) \mid \mathbf{id}$
- **FIRST sets:** E, T, F: $\{(, \mathbf{id}\}$ $\quad E'$: $\{+, \epsilon\}$ $\quad T'$: $\{*, \epsilon\}$
- **FOLLOW sets:** E, E': $\{\$,)\}$ $\quad T, T'$: $\{+, \$,)\}$ $\quad F$: $\{*, +, \$,)\}$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	$
E	$\boxed{E \rightarrow TE'}$			$\boxed{E \rightarrow TE'}$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \mathbf{id}$			$F \rightarrow (E)$		

For $E \rightarrow TE'$:

FIRST(TE')

= FIRST(T)

= $\{(, \mathbf{id}\}$

Parsing Table Construction Example

- Grammar
  - $E \rightarrow TE'$ $E' \rightarrow +TE' \mid \epsilon$
  - $T \rightarrow FT'$ $T' \rightarrow * FT' \mid \epsilon$ $F \rightarrow (E) \mid \textbf{id}$
- **FIRST sets:** E, T, F: {(, **id**} E': $\{+, \epsilon\}$ T': $\{*, \epsilon\}$
- **FOLLOW sets:** E, E': {\$,)} T, T': {+, \$,)} F: {*, +, \$,)}

NON - TERMINAL	INPUT SYMBOL					
	id	$+$	$*$	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$\boxed{E' \rightarrow \epsilon}$	$\boxed{E' \rightarrow \epsilon}$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \textbf{id}$			$F \rightarrow (E)$		

For $E' \rightarrow \epsilon$:

ϵ in FIRST(ϵ)

FOLLOW(E')

= {\$,)}

Conflicts in Parsing Tables

- **Grammar:** $S \rightarrow \mathbf{i}E\mathbf{t}SS' \mid \mathbf{a}$ $\quad \boxed{S' \rightarrow \mathbf{e}S \mid \epsilon} \quad$ $E \rightarrow \mathbf{b}$
  - FIRST($\mathbf{e}S$) $=$ {**e**}, so we add $S' \rightarrow \mathbf{e}S$ to $M[S', \mathbf{e}]$
  - FOLLOW(S') $=$ {\$, **e**}, so we add $S' \rightarrow \epsilon$ to $M[S', \mathbf{e}]$

NON - TERMINAL	INPUT SYMBOL					
	a	b	e	i	t	\$
S	$S \rightarrow a$			$S \rightarrow iEtSS'$		
S'			$S' \rightarrow \epsilon$ $S' \rightarrow eS$			$S' \rightarrow \epsilon$
E		$E \rightarrow b$				

- LL(1) grammar is never ambiguous.
- This grammar is not LL(1). The language has no LL(1) grammar !!!

Recursive-Descent Parsing for LL(1) Grammars

void $A()$ {

1) Choose an A-production, $A \rightarrow X_1 X_2 \cdots X_k$;

2) **for** ($i = 1$ to k) {

3) **if** (X_i is a nonterminal)

4) call procedure $X_i()$;

5) **else if** (X_i equals the current input symbol a)

6) advance the input to the next symbol;

7) **else** /* an error has occurred */;

}

}

Replace line 1 with: Choose A-production according to the parsing table

- Assume input symbol is a, then the choice is the production in $M[A, a]$

Outline

- Introduction: Syntax and Parsers
- Context-Free Grammars
- Top-Down Parsing Techniques
  - Recursive-descent parsing
  - Non-recursive predictive parsing (Lab)
- Bottom-Up Parsing

Recall Recursive-Descent Parsing

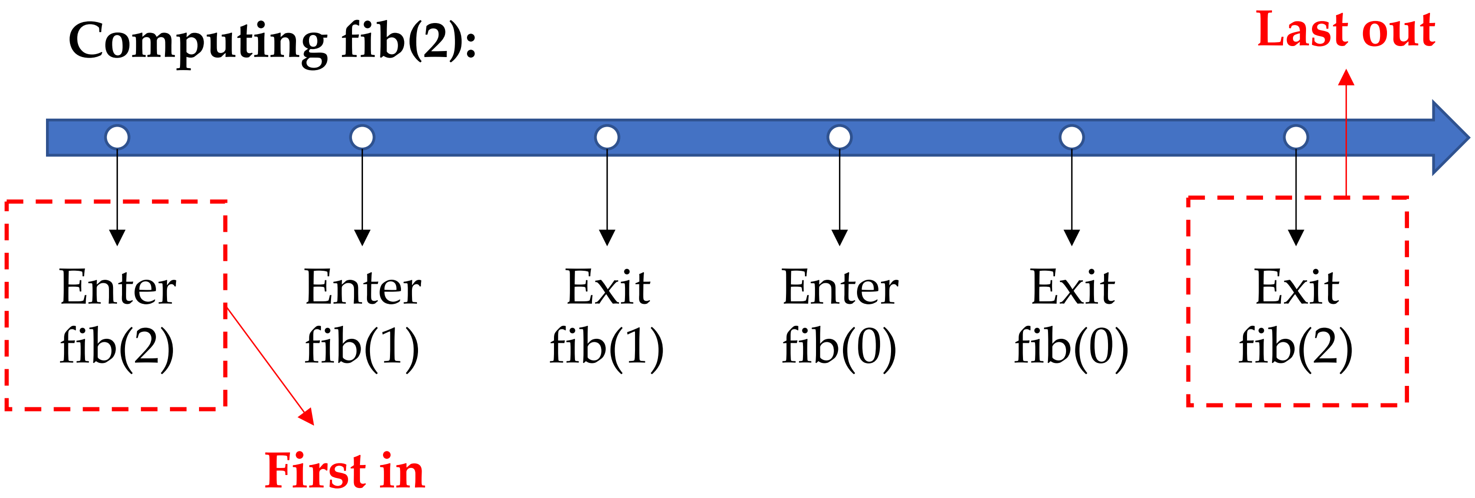
```
     void A() {
1)       Choose an A-production, A → X1 X2 ··· Xk;
2)       for ( i = 1 to k ) {
3)           if ( Xi is a nonterminal )
4)               call procedure Xi();
5)           else if ( Xi equals the current input symbol a )
6)               advance the input to the next symbol;
7)           else /* an error has occurred */;
         }
     }
```

Recursive-descent parsing has recursive calls.
Can we design a non-recursive parser?

How Is Recursion Handled?

```
int fib(int n) {
    if(n <= 1) return n;
    else {
        int a = fib(n-1) + fib(n-2);
        return a;
    }
}
```

Computing fib(2):



Non-Recursive Predictive Parsing

- A non-recursive predictive parser can be built by **explicitly maintaining a stack** (not implicitly via recursive calls)
  - **Input buffer** contains the string to be parsed, ending with $
  - **Stack** holds a sequence of grammar symbols with $ at the bottom. Initially, the stack contains only $ and the start symbol *S* on top of $

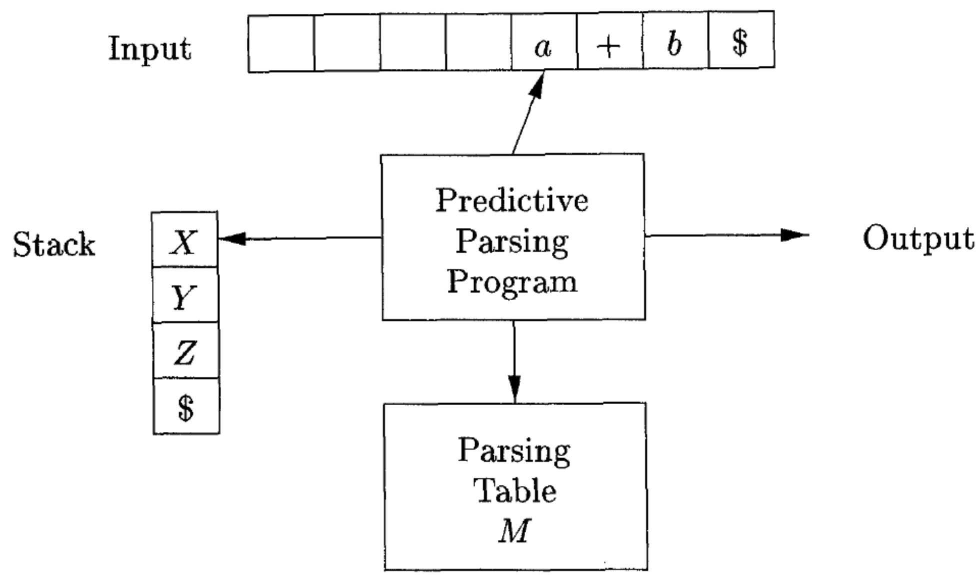
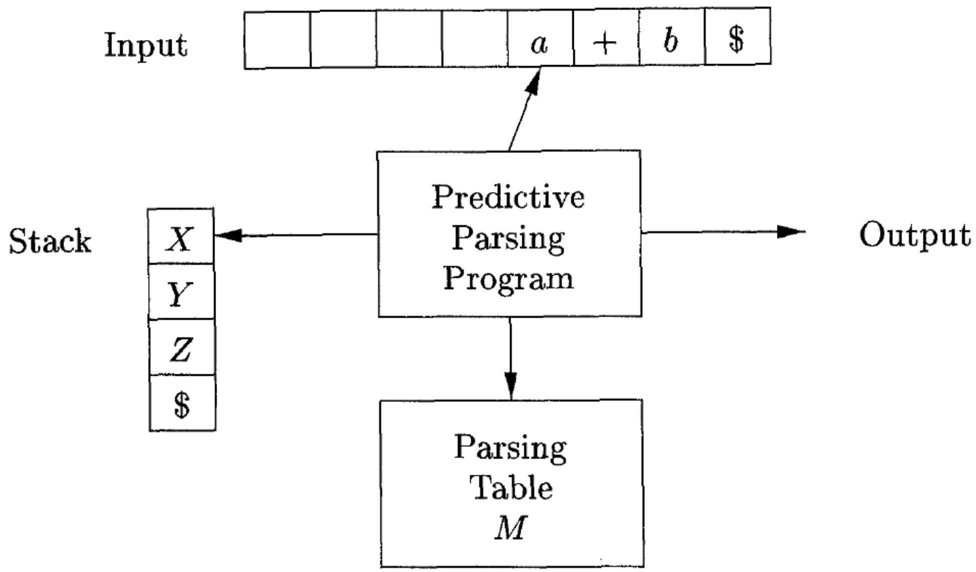


Table-Driven Predictive Parsing

- **Input:** A string ω and a parsing table M for grammar G
- **Output:** If ω is in $L(G)$, a leftmost derivation of ω (input buffer and stack are both empty); otherwise, an error indication



Initially, the input buffer contains $\omega\$$.

The start symbol S of G is on top of the stack, above $.

Example

$$E \to TE' \qquad E' \to +TE' \mid \epsilon$$

$$T \to FT' \qquad T' \to * FT' \mid \epsilon \qquad F \to (E) \mid \mathbf{id}$$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \mathbf{id}$			$F \to (E)$		

Input:

id + id * id

MATCHED	STACK	INPUT	ACTION
	$\underline{E}\$$	$\underline{\mathbf{id}} + \mathbf{id} * \mathbf{id}\$$	

Example

$E \rightarrow TE' \qquad E' \rightarrow +TE' \mid \epsilon$

$T \rightarrow FT' \qquad T' \rightarrow * FT' \mid \epsilon \qquad F \rightarrow (E) \mid \mathbf{id}$

Non - terminal	Input Symbol					
	id	$+$	$*$	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \mathbf{id}$			$F \rightarrow (E)$		

Input:

id + id * id

Matched	Stack	Input	Action
	$E\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	
	$TE'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	output $E \rightarrow TE'$

Example

$$E \rightarrow TE' \qquad E' \rightarrow +TE' \mid \epsilon$$

$$T \rightarrow FT' \qquad T' \rightarrow * FT' \mid \epsilon \qquad F \rightarrow (E) \mid \textbf{id}$$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \textbf{id}$			$F \rightarrow (E)$		

Input:

id + id * id

MATCHED	STACK	INPUT	ACTION
	$E\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	
	$TE'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	output $E \rightarrow TE'$
	$FT'E'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	output $T \rightarrow FT'$

Example

$E \rightarrow TE'$ $\quad E' \rightarrow +TE' \mid \epsilon$

$T \rightarrow FT'$ $\quad T' \rightarrow * FT' \mid \epsilon$ $\quad F \rightarrow (E) \mid \textbf{id}$

NON - TERMINAL	INPUT SYMBOL					
	id	$+$	$*$	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \textbf{id}$			$F \rightarrow (E)$		

Input:

id + id * id

MATCHED	STACK	INPUT	ACTION
	$E\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	
	$TE'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	output $E \rightarrow TE'$
	$FT'E'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	output $T \rightarrow FT'$
	$\textbf{id}\ T'E'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	output $F \rightarrow \textbf{id}$

Example

$$E \to TE' \qquad E' \to +TE' \mid \epsilon$$

$$T \to FT' \qquad T' \to * FT' \mid \epsilon \qquad F \to (E) \mid \mathbf{id}$$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \mathbf{id}$			$F \to (E)$		

Input:

id + id * id

MATCHED	STACK	INPUT	ACTION
	$E\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	
	$TE'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	output $E \to TE'$
	$FT'E'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	output $T \to FT'$
	$\mathbf{id}\ T'E'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	output $F \to \mathbf{id}$
id	$T'E'\$$	$+ \mathbf{id} * \mathbf{id}\$$	match **id**

Example

$$E \to TE' \qquad E' \to +TE' \mid \epsilon$$

$$T \to FT' \qquad T' \to * FT' \mid \epsilon \qquad F \to (E) \mid \mathbf{id}$$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \mathbf{id}$			$F \to (E)$		

Input:

id + id * id

MATCHED	STACK	INPUT	ACTION
	$E\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	
	$TE'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	output $E \to TE'$
	$FT'E'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	output $T \to FT'$
	$\mathbf{id}\ T'E'\$$	$\mathbf{id} + \mathbf{id} * \mathbf{id}\$$	output $F \to \mathbf{id}$
id	$T'E'\$$	$+ \mathbf{id} * \mathbf{id}\$$	match **id**
id	$E'\$$	$+ \mathbf{id} * \mathbf{id}\$$	output $T' \to \epsilon$

Example

$E \rightarrow TE' \qquad E' \rightarrow +TE' \mid \epsilon$

$T \rightarrow FT' \qquad T' \rightarrow * FT' \mid \epsilon \qquad F \rightarrow (E) \mid \textbf{id}$

NON - TERMINAL	INPUT SYMBOL					
	id	+	$*$	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \textbf{id}$			$F \rightarrow (E)$		

Input:

id + id $*$ id

MATCHED	STACK	INPUT	ACTION
	$E\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	
	$TE'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	output $E \rightarrow TE'$
	$FT'E'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	output $T \rightarrow FT'$
	$\textbf{id}\ T'E'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	output $F \rightarrow \textbf{id}$
id	$T'E'\$$	$+ \textbf{id} * \textbf{id}\$$	match **id**
id	$E'\$$	$+ \textbf{id} * \textbf{id}\$$	output $T' \rightarrow \epsilon$
id	$\underline{+}\ TE'\$$	$\underline{+}\ \textbf{id} * \textbf{id}\$$	output $E' \rightarrow +TE'$

Example

$E \rightarrow TE' \quad E' \rightarrow +TE' \mid \epsilon$

$T \rightarrow FT' \quad T' \rightarrow * FT' \mid \epsilon \quad F \rightarrow (E) \mid \textbf{id}$

NON - TERMINAL	INPUT SYMBOL					
	id	$+$	$*$	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow \textbf{id}$			$F \rightarrow (E)$		

Input:

$\textbf{id} + \textbf{id} * \textbf{id}$

MATCHED	STACK	INPUT	ACTION
	$E\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	
	$TE'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	output $E \rightarrow TE'$
	$FT'E'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	output $T \rightarrow FT'$
	$\textbf{id}\ T'E'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	output $F \rightarrow \textbf{id}$
id	$T'E'\$$	$+ \textbf{id} * \textbf{id}\$$	match **id**
id	$E'\$$	$+ \textbf{id} * \textbf{id}\$$	output $T' \rightarrow \epsilon$
id	$+ TE'\$$	$+ \textbf{id} * \textbf{id}\$$	output $E' \rightarrow + TE'$
id $+$	$TE'\$$	$\textbf{id} * \textbf{id}\$$	match $+$
…	…	…	…
$\textbf{id} + \textbf{id} * \textbf{id}$	\$	\$	output $E' \rightarrow \epsilon$

There are eight more steps before accepting.

The parser announce success when both stack and input are empty.

Example

$$E \to TE' \qquad E' \to +TE' \mid \epsilon$$

$$T \to FT' \qquad T' \to * FT' \mid \epsilon \qquad F \to (E) \mid \mathbf{id}$$

NON - TERMINAL	INPUT SYMBOL					
	id	+	*	(	)	\$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \to *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \to \mathbf{id}$			$F \to (E)$		

Input:

id + id * id

MATCHED	STACK	INPUT	ACTION
	$E\$$	$\mathbf{id}+\mathbf{id}*\mathbf{id}\$$	
	$TE'\$$	$\mathbf{id}+\mathbf{id}*\mathbf{id}\$$	output $E \to TE'$
	$FT'E'\$$	$\mathbf{id}+\mathbf{id}*\mathbf{id}\$$	output $T \to FT'$
	$\mathbf{id}\ T'E'\$$	$\mathbf{id}+\mathbf{id}*\mathbf{id}\$$	output $F \to \mathbf{id}$
id	$T'E'\$$	$+\mathbf{id}*\mathbf{id}\$$	match **id**
id	$E'\$$	$+\mathbf{id}*\mathbf{id}\$$	output $T' \to \epsilon$
id	$+TE'\$$	$+\mathbf{id}*\mathbf{id}\$$	output $E' \to +TE'$
id +	$TE'\$$	$\mathbf{id}*\mathbf{id}\$$	match +
...	...	...	...
$\mathbf{id}+\mathbf{id}*\mathbf{id}$	\$	\$	output $E' \to \epsilon$

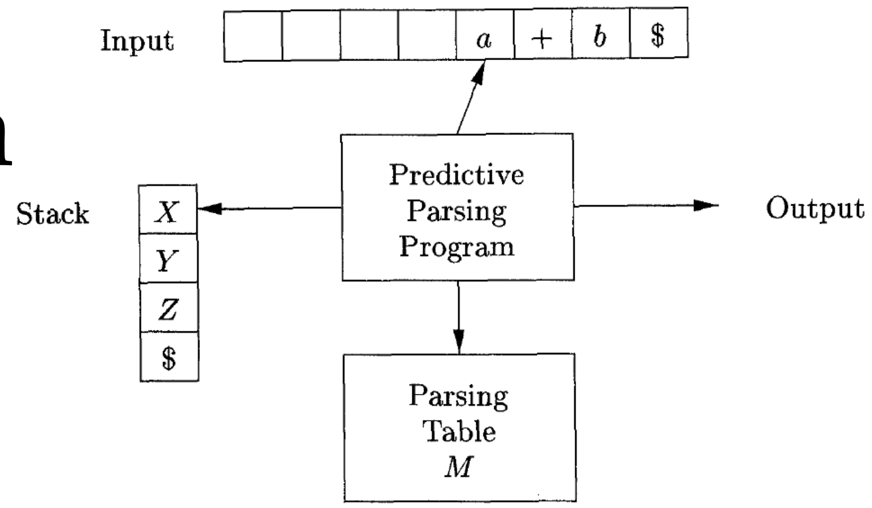
Leftmost derivation

Matched part
+
Stack content
(from top to bottom)
=
A left-sentential form

总是最左句型

SELF LEARNING E-MATERIALS

Parsing Algorithm




```
let a be the first symbol of ω;
let X be the top stack symbol;
while ( X ≠ $ ) { /* stack is not empty */
    if ( X = a ) pop the stack and let a be the next symbol of ω;
    else if ( X is a terminal) error();   /* X can only match a, cannot be another terminal */
    else if ( M[X, a] is an error entry ) error();
    else if ( M[X, a] = X → Y1Y2 ... Yk ) {
        output the production X → Y1Y2 ... Yk;
        pop the stack;
        push Yk, Yk−1, ..., Y1 onto the stack, with Y1 on top;  /* order is critical */
    }
    let X be the top stack symbol;
}
```