STA219: Probability and Statistics for Engineering

Assignment 4

Note: The assignment can be answered in Chinese or English, either is fine. Please provide derivation and computation details, not just the final answer. Please submit a PDF file on BB.

1. (10 points) Suppose X and Y are independent discrete random variables, and the joint PMF of (X, Y) is as follows:

X\Y	y_1	y_2	y_3
x_1	a	$1/9$	c
x_2	$1/9$	b	$1/3$

Please calculate the values of a, b, c.

2. (10 points) Let X and Y be independent Poisson random variables with parameter λ, i.e., $X \sim \text{Poisson}(\lambda)$, $Y \sim \text{Poisson}(\lambda)$. Let $U = 2X + Y$, $V = 2X - Y$. Please calculate the correlation coefficient between U and V.

3. (10 points) Suppose the joint PDF of random vector (X, Y) is

$$f(x, y) = \begin{cases} 1, & |y| < x, 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

(1) Please calculate $\mathrm{E}(X)$, $\mathrm{E}(Y)$, $\mathrm{Cov}(X, Y)$. (5 points)

(2) Is X and Y independent? (5 points)

4. (10 points) Suppose the joint PDF of random vector (X, Y) is

$$f(x, y) = \begin{cases} x + y, & 0 < x, y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

(1) When $0 < y < 1$, derive $\mathrm{E}(X|Y = y)$. (5 points)

(2) Based on (1), please calculate $\mathrm{E}(X)$ using the Law of Total Expectation. (5 points)

5. (10 points) Let X and Y be independent Uniform random variables, i.e., $X \sim \mathrm{U}(0,1)$, $Y \sim \mathrm{U}(0,1)$. What is the PDF of $T = X + Y$?

6. (10 points) Let X_1, X_2 and X_3 represent the time (in minutes) necessary to perform three successive repair tasks at a service facility. They are independent, normal random variables

with expected values 45, 50 and 75, and variances 10, 12 and 14, respectively. What is the probability that the service facility can finish all three tasks within 3 hours (that is, 180 minutes)?

7. (10 points) There are 40 light bulbs, and the lifespan of each light bulb follows an exponential distribution with an average lifespan of 25 days. Suppose that we use one light bulb at a time and replace it immediately with a new bulb once the previous one breaks. Please find the probability that these bulbs can be used for a total of more than 900 days.

8. (10 points) A large hotel has a total of 500 rooms, and each room has one air conditioner with a power rating of 2 kW. Suppose the occupancy rate is 80%, which means that each room has an 80% probability of being occupied, independently of other rooms. How many kW of power are needed to ensure a 99% probability of having enough power for the air conditioners?

9. (10 points) X and Y are independent random variables, and $X \sim Exp(\lambda)$, $Y \sim Exp(\mu)$. Define random variable Z as follows:

$$Z = \begin{cases} 1, & if\ X \leq Y, \\ 0, & if\ X > Y. \end{cases}$$

Derive the PMF of Z.

10. (10 points) Suppose X and Y are independent, identically distributed random variables, and follow geometric distribution, which is $P(X = k) = (1-p)^{k-1}p,\ k = 1,2, \ldots$. Derive the PMF of $Z = \max(X, Y)$.