CS323 Written Assignment4 Sample Answer

Exercise 1

(0) $E' \to E$

(1) $E \to TX$

(2) $X \to +E$

(3) $X \to \epsilon$

(4) $T \to FY$

(5) $Y \to T$

(6) $Y \to \epsilon$

(7) $F \to PZ$

(8) $Z \to *Z$

(9) $Z \to \epsilon$

(10) $P \to (E)$

(11) $P \to a$

(12) $P \to b$

FIRST(E) = { (, a, b }, FOLLOW(E) = {), $ }

FIRST(X) = { +, ε }, FOLLOW(X) = {), $ }

FIRST(T) = { (, a, b }, FOLLOW(T) = { +,), $ }

FIRST(Y) = { (, a, b, ε }, FOLLOW(Y) = { +,), $ }

FIRST(F) = { (, a, b }, FOLLOW(F) = { (, a, b, +,), $ }

FIRST(Z) = { *, ε }, FOLLOW(Z) = { (, a, b, +,), $ }

FIRST(P) = { (, a, b }, FOLLOW(P) = { (, a, b, +,), *, $ }

$I_0 = \{[E' \to \cdot E], [E \to \cdot TX], [T \to \cdot FY], [F \to \cdot PZ], [P \to \cdot(E)], [P \to \cdot a], [P \to \cdot b]\}$

$I_1 = GOTO(I_0, E) = \{E' \to E\cdot\}$

ACTION(1, $) = accept

$I_2 = GOTO(I_0, T) = \{[E \to T \cdot X], [X \to \cdot + E], [X \to \cdot]\}$

ACTION(2,)) = reduce by $X \to \epsilon$

ACTION(2, $) = reduce by $X \to \epsilon$

$I_3 = GOTO(I_0, F) = \{[T \to F \cdot Y], [Y \to \cdot T], [Y \to \cdot], [T \to \cdot FY], [F \to \cdot PZ], [P \to \cdot(E)], [P \to \cdot a], [P \to \cdot b]\}$

ACTION(3, +) = reduce by $Y \to \epsilon$

ACTION(3,)) = reduce by $Y \to \epsilon$

ACTION(3, $) = reduce by $Y \to \epsilon$

$I_4 = GOTO(I_0, P) = \{[F \to P \cdot Z], [Z \to \cdot * Z], [Z \to \cdot]\}$

ACTION(4, () = reduce by $Z \to \epsilon$

ACTION(4, a) = reduce by $Z \to \epsilon$

ACTION(4, b) = reduce by $Z \to \epsilon$

ACTION(4, +) = reduce by $Z \to \epsilon$

ACTION(4,)) = reduce by $Z \to \epsilon$

ACTION(4, $) = reduce by $Z \to \epsilon$

$I_5 = GOTO(I_0, () = \{[P \to (\cdot E)], [E \to \cdot TX], [T \to \cdot FY], [F \to \cdot PZ], [P \to \cdot (E)], [P \to \cdot a], [P \to \cdot b]\}$

ACTION(0, () = shift to 5

$I_6 = GOTO(I_0, a) = \{[P \to a\cdot]\}$

ACTION(0, a) = shift to 6

ACTION(6, () = reduce by $P \to a$

ACTION(6, a) = reduce by $P \to a$

ACTION(6, b) = reduce by $P \to a$

ACTION(6, +) = reduce by $P \to a$

ACTION(6,)) = reduce by $P \to a$

ACTION(6, *) = reduce by $P \to a$

ACTION(6, $) = reduce by $P \to a$

$I_7 = GOTO(I_0, b) = \{[P \to b\cdot]\}$

ACTION(0, b) = shift to 7

ACTION(7, () = reduce by $P \to b$

ACTION(7, a) = reduce by $P \to b$

ACTION(7, b) = reduce by $P \to b$

ACTION(7, +) = reduce by $P \to b$

ACTION(7,)) = reduce by $P \to b$

ACTION(7, *) = reduce by $P \to b$

ACTION(7, $) = reduce by $P \to b$

$I_8 = GOTO(I_2, X) = \{[E \to TX\cdot]\}$

ACTION(8,)) = reduce by $E \to TX$

ACTION(8, $) = reduce by $E \to TX$

$I_9 = GOTO(I_2, +) = \{[X \to + \cdot E], [E \to \cdot TX], [T \to \cdot FY], [F \to \cdot PZ], [P \to \cdot (E)], [P \to \cdot a], [P \to \cdot b]\}$

ACTION(2, +) = shift to 9

$I_{10} = GOTO(I_3, Y) = \{[T \to FY\cdot]\}$

ACTION(10, +) = reduce by $T \to FY$

ACTION(10,)) = reduce by $T \to FY$

ACTION(10, $) = reduce by $T \to FY$

$I_{11} = GOTO(I_3, T) = \{[Y \to T \cdot]\}$

ACTION(11, +) = reduce by $Y \to T$

ACTION(11,)) = reduce by $Y \to T$

ACTION(11, $) = reduce by $Y \to T$

$GOTO(I_3, F) = I_3$

$GOTO(I_3, P) = I_4$

$GOTO(I_3, () = I_5$

$GOTO(I_3, a) = I_6$

$GOTO(I_3, b) = I_7$

$I_{12} = GOTO(I_4, Z) = \{[F \to PZ \cdot]\}$

ACTION(12, () = reduce by $F \to PZ$

ACTION(12, a) = reduce by $F \to PZ$

ACTION(12, b) = reduce by $F \to PZ$

ACTION(12, +) = reduce by $F \to PZ$

ACTION(12,)) = reduce by $F \to PZ$

ACTION(12, $) = reduce by $F \to PZ$

$I_{13} = GOTO(I_4, *) = \{[Z \to * \cdot Z], [Z \to \cdot * Z], [Z \to \cdot]\}$

ACTION(4, *) = shift to 13

ACTION(13, () = reduce by $Z \to \epsilon$

ACTION(13, a) = reduce by $Z \to \epsilon$

ACTION(13, b) = reduce by $Z \to \epsilon$

ACTION(13, +) = reduce by $Z \to \epsilon$

ACTION(13,)) = reduce by $Z \to \epsilon$

ACTION(13, $) = reduce by $Z \to \epsilon$

$I_{14} = GOTO(I_5, E) = \{[P \to (E \cdot)]\}$

$GOTO(I_5, T) = I_2$

$GOTO(I_5, F) = I_3$

$GOTO(I_5, P) = I_4$

$GOTO(I_5, () = I_5$

$GOTO(I_5, a) = I_6$

$GOTO(I_5, b) = I_7$

ACTION(9, () = shift to 5

ACTION(9, a) = shift to 6

ACTION(9, b) = shift to 7

$GOTO(I_9, F) = I_3$

$GOTO(I_9, P) = I_4$

$GOTO(I_9, T) = I_2$

$I_{15} = GOTO(I_9, E) = \{[X \to +E\cdot]\}$

ACTION(15,)) = reduce by $X \to +E$

ACTION(15, $) = reduce by $X \to +E$

$I_{16} = GOTO(I_{13}, Z) = \{[Z \to *Z\cdot]\}$

ACTION(16, () = reduce by $Z \to *Z$

ACTION(16, a) = reduce by $Z \to *Z$

ACTION(16, b) = reduce by $Z \to *Z$

ACTION(16, +) = reduce by $Z \to *Z$

ACTION(16,)) = reduce by $Z \to *Z$

ACTION(16, $) = reduce by $Z \to *Z$

ACTION(13, *) = shift to 13

$I_{17} = GOTO(I_{14},)) = \{[P \to (E)\cdot]\}$

ACTION(17, () = reduce by $P \to (E)$

ACTION(17, a) = reduce by $P \to (E)$

ACTION(17, b) = reduce by $P \to (E)$

ACTION(17, +) = reduce by $P \to (E)$

ACTION(17,)) = reduce by $P \to (E)$

ACTION(17, *) = reduce by $P \to (E)$

ACTION(17, $) = reduce by $P \to (E)$

ACTION(14,)) = shift to 17

SLR table:

STATE	ACTION							GOTO						
	+	*	(	)	a	b	$	E	X	T	Y	F	Z	P
0			s5		s6	s7		1		2		3		4
1							acc							
2	s9			r3			r3		8					
3	r6		s5	r6	s6	s7	r6			11	10	3		4
4	r9	s13	r9	r9	r9	r9	r9						12	
5			s5		s6	s7		14		2		3		4
6	r11	r11	r11	r11	r11	r11	r11							
7	r12	r12	r12	r12	r12	r12	r12							
8				r1			r1							
9			s5		s6	s7		15		2		3		4
10	r4			r4			r4							
11	r5			r5			r5							
12	r7		r7	r7	r7	r7	r7							
13	r9	s13	r9	r9	r9	r9	r9						16	
14				s17										
15				r2			r2							
16	r8		r8	r8	r8	r8	r8							
17	r10	r10	r10	r10	r10	r10	r10							

Yes. The grammar is $SLR(1)$.

The parsing steps for $(a*+b)+b$:

STEP	STACK	SYMBOLS	INPUT	ACTION
1	0	$	(a*+b)+b$	shift to 5
2	0 5	$(	a*+b)+b$	shift to 6
3	0 5 6	$(a	*+b)+b$	reduce by $P \to a$
4	0 5 4	$(P	*+b)+b$	shift to 13
5	0 5 4 13	$(P*	+b)+b$	reduce by $Z \to \epsilon$
6	0 5 4 13 16	$(P*Z	+b)+b$	reduce by $Z \to *Z$
7	0 5 4 12	$(PZ	+b)+b$	reduce by $F \to PZ$
8	0 5 3	$(F	+b)+b$	reduce by $Y \to \epsilon$
9	0 5 3 10	$(FY	+b)+b$	reduce by $T \to FY$
10	0 5 2	$(T	+b)+b$	shift to 9
11	0 5 2 9	$(T+	b)+b$	shift to 7
12	0 5 2 9 7	$(T+P	)+b$	reduce by $P \to b$
13	0 5 2 9 4	$(T+P	)+b$	reduce by $Z \to \epsilon$
14	0 5 2 9 4 12	$(T+PZ	)+b$	reduce by $F \to PZ$

15	0 5 2 9 3	$(T+F	)+b$	reduce by $Y \to \epsilon$
16	0 5 2 9 3 10	$(T+FY	)+b$	reduce by $T \to FY$
17	0 5 2 9 2	$(T+T	)+b$	reduce by $X \to \epsilon$
18	0 5 2 9 2 8	$(T+TX	)+b$	reduce by $E \to TX$
19	0 5 2 9 15	$(T+E	)+b$	reduce by $X \to +E$
20	0 5 2 8	$(TX	)+b$	reduce by $E \to TX$
21	0 5 14	$(E	)+b$	shift to 17
22	0 5 14 17	$(E)	+b$	reduce by $P \to (E)$
23	0 4	$P	+b$	reduce by $Z \to \epsilon$
24	0 4 12	$PZ	+b$	reduce by $F \to PZ$
25	0 3	$F	+b$	reduce by $Y \to \epsilon$
26	0 3 10	$FY	+b$	reduce by $T \to FY$
27	0 2	$T	+b$	shift to 9
28	0 2 9	$T+	b$	shift to 7
29	0 2 9 7	$T+b	$	reduce by $P \to b$
30	0 2 9 4	$T+P	$	reduce by $Z \to \epsilon$
31	0 2 9 4 12	$T+PZ	$	reduce by $F \to PZ$
32	0 2 9 3	$T+F	$	reduce by $Y \to \epsilon$
33	0 2 9 3 10	$T+FY	$	reduce by $T \to FY$
34	0 2 9 2	$T+T	$	reduce by $X \to \epsilon$
35	0 2 9 2 8	$T+TX	$	reduce by $E \to TX$
36	0 2 9 15	$T+E	$	reduce by $X \to +E$
37	0 2 8	$TX	$	reduce by $E \to TX$
38	0 1	$E	$	accept

Exercise 2

(0) $S' \to S$

(1) $S \to 0A$

(2) $A \to S1A$

(3) $A \to \epsilon$

FIRST(S) = {0}, FOLLOW(S) = {1, $}

FIRST(A) = {0, ε}, FOLLOW(A) = {1, $}

SLR part:

$I_0 = \{[S' \to \cdot S], [S \to \cdot 0A]\}$

$I_1 = GOTO(I_0, S) = \{[S' \to S\cdot]\}$

ACTION(1, $) = accept

$I_2 = GOTO(I_0, 0) = \{[S \to 0 \cdot A], [A \to \cdot S1A], [A \to \cdot], [S \to \cdot 0A]\}$

ACTION(0, 0) = shift to 2

ACTION(2, 0) = shift to 2

ACTION(2, 1) = reduce by $A \to \epsilon$

ACTION(2, $) = reduce by $A \to \epsilon$

$I_3 = GOTO(I_2, A) = \{[S \to 0A\cdot]\}$

ACTION(3, 1) = reduce by $S \to 0A$

ACTION(3, $) = reduce by $S \to 0A$

$I_4 = GOTO(I_2, S) = \{[A \to S \cdot 1A]\}$

$I_5 = GOTO(I_4, 1) = \{[A \to S1 \cdot A], [A \to \cdot S1A], [A \to \cdot], [S \to \cdot 0A]\}$

ACTION(4, 1) = shift to 5

$GOTO(I_5, S) = I_4$

$GOTO(I_5, 0) = I_2$

ACTION(5, 0) = shift to 2

ACTION(5, 1) = reduce by $A \to \epsilon$

ACTION(5, $) = reduce by $A \to \epsilon$

$I_6 = GOTO(I_5, A) = \{[A \to S1A\cdot]\}$

ACTION(6, 1) = reduce by $A \to S1A$

ACTION(6, $) = reduce by $A \to S1A$

SLR table:

STATE	ACTION			GOTO	
	0	1	$	S	A
0	s2			1	
1			accept		
2	s2	r3	r3	4	3
3		r1	r1		
4		s5			
5	s2	r3	r3	4	6
6		r2	r2		

Yes. It is SLR(1).

CLR part:

$I_0 = \{[S' \to \cdot S, \$], [S \to \cdot 0A, \$]\}$

$I_1 = GOTO(I_0, S) = \{[S' \to S\cdot, \$]\}$

ACTION(1, $) = accept

$I_2 = GOTO(I_0, 0) = \{[S \to 0 \cdot A, \$], [A \to \cdot S1A, \$], [A \to \cdot, \$], [S \to \cdot 0A, 1]\}$

ACTION(0, 0) = shift to 2

ACTION(2, $) = reduce by $A \to \epsilon$

$I_3 = GOTO(I_2, A) = \{[S \to 0A\cdot, \$]\}$

ACTION(3, $) = reduce by $S \to 0A$

$I_4 = GOTO(I_2, S) = \{[A \to S \cdot 1A, \$]\}$

$I_5 = GOTO(I_2, 0) = \{[S \to 0 \cdot A, 1], [A \to \cdot S1A, 1], [A \to \cdot, 1], [S \to \cdot 0A, 1]\}$

ACTION(2, 0) = shift to 5

ACTION(5, 1) = reduce by $A \to \epsilon$

$I_6 = GOTO(I_4, 1) = \{[A \to S1 \cdot A, \$], [A \to \cdot S1A, \$], [A \to \cdot, \$], [S \to \cdot 0A, 1]\}$

ACTION(4, 1) = shift to 6

ACTION(6, $) = reduce by $A \to \epsilon$

$I_7 = GOTO(I_5, A) = \{[S \to 0A\cdot, 1]\}$

ACTION(7, 1) = reduce by $S \to 0A$

$I_8 = GOTO(I_5, S) = \{[A \to S \cdot 1A, 1]\}$

$GOTO(I_5, 0) = I_5$

$I_9 = GOTO(I_6, A) = \{[A \to S1A\cdot, \$]\}$

ACTION(9, $) = reduce by $A \to S1A$

$GOTO(I_6, S) = I_4$

$GOTO(I_6, 0) = I_5$

ACTION(6, 0) = shift to 5

$I_{10} = GOTO(I_8, 1) = \{[A \to S1 \cdot A, 1], [A \to \cdot S1A, 1], [A \to \cdot, 1], [S \to \cdot 0A, 1]\}$

ACTION(8, 1) = shift to 10

ACTION(10, 1) = reduce by $A \to \epsilon$

$I_{11} = GOTO(I_{10}, A) = \{[A \to S1A\cdot, 1]\}$

ACTION(11, 1) = reduce by $A \to S1A$

$GOTO(I_{10}, S) = I_8$

$GOTO(I_{10}, 0) = I_5$

ACTION(10, 0) = shift to 5

CLR table:

STATE	ACTION			GOTO	
	0	1	$	S	A
0	s2			1	
1			accept		
2	s5		r3	4	3
3			r1		
4		s6			
5	s5	r3		8	7
6	s5		r3	4	9
7		r1			
8		s10			
9			r2		
10	s5	r3		8	11
11		r2			

Yes. It is LR(1).

$LALR$ part:

We can merge I_2 and I_5 to I_{25} , I_3 and I_7 to I_{37}, I_4 and I_8 to I_{48}, I_6 and I_{10} to I_{610}, I_9 and I_{11} to I_{911}

$LALR$ table:

STATE	ACTION			GOTO	
	0	1	$	S	A
0	s25			1	
1			accept		
25	s25	r3	r3	48	37
37		r1	r1		
48		s610			
610	s25	r3	r3	48	911
911		r2	r2		

Yes. It is LALR(1).

The parsing steps for 000001111:

STEP	STACK	SYMBOLS	INPUT	ACTION
1	0	$	000001111$	shift to 25
2	0 25	$0	00001111$	shift to 25
3	0 25 25	$00	0001111$	shift to 25
4	0 25 25 25	$000	001111$	shift to 25
5	0 25 25 25 25	$0000	01111$	shift to 25
6	0 25 25 25 25 25	$00000	1111$	reduce by $A \to \epsilon$
7	0 25 25 25 25 25 37	$00000A	1111$	reduce by $S \to 0A$
8	0 25 25 25 25 48	$0000S	1111$	shift to 610
9	0 25 25 25 25 48 610	$0000S1	111$	reduce by $A \to \epsilon$
10	0 25 25 25 25 48 610 911	$0000S1A	111$	reduce by $A \to S1A$
11	0 25 25 25 25 37	$0000A	111$	reduce by $S \to 0A$
12	0 25 25 25 48	$000S	111$	shift to 610
13	0 25 25 25 48 610	$000S1	11$	reduce by $A \to \epsilon$
14	0 25 25 25 48 610 911	$000S1A	11$	reduce by $A \to S1A$
15	0 25 25 25 37	$000A	11$	reduce by $S \to 0A$
16	0 25 25 48	$00S	11$	shift to 610
17	0 25 25 48 610	$00S1	1$	reduce by $A \to \epsilon$
18	0 25 25 48 610 911	$00S1A	1$	reduce by $A \to S1A$
19	0 25 25 37	$00A	1$	reduce by $S \to 0A$
20	0 25 48	$0S	1$	shift to 610
21	0 25 48 610	$0S1	$	reduce by $A \to \epsilon$
22	0 25 48 610 911	$0S1A	$	reduce by $A \to S1A$
23	0 25 37	$0A	$	reduce by $S \to 0A$
24	0 1	$S	$	accept