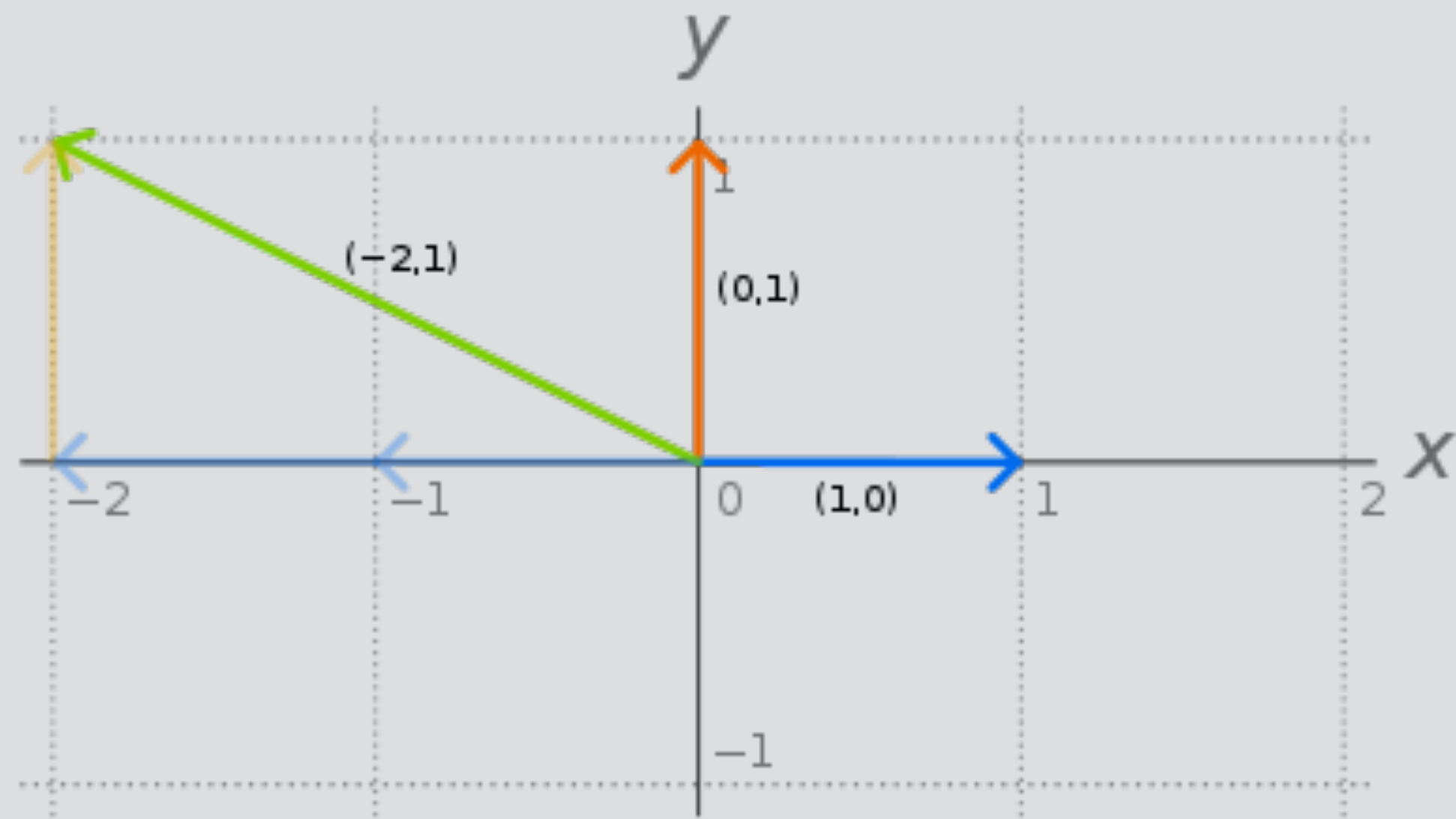


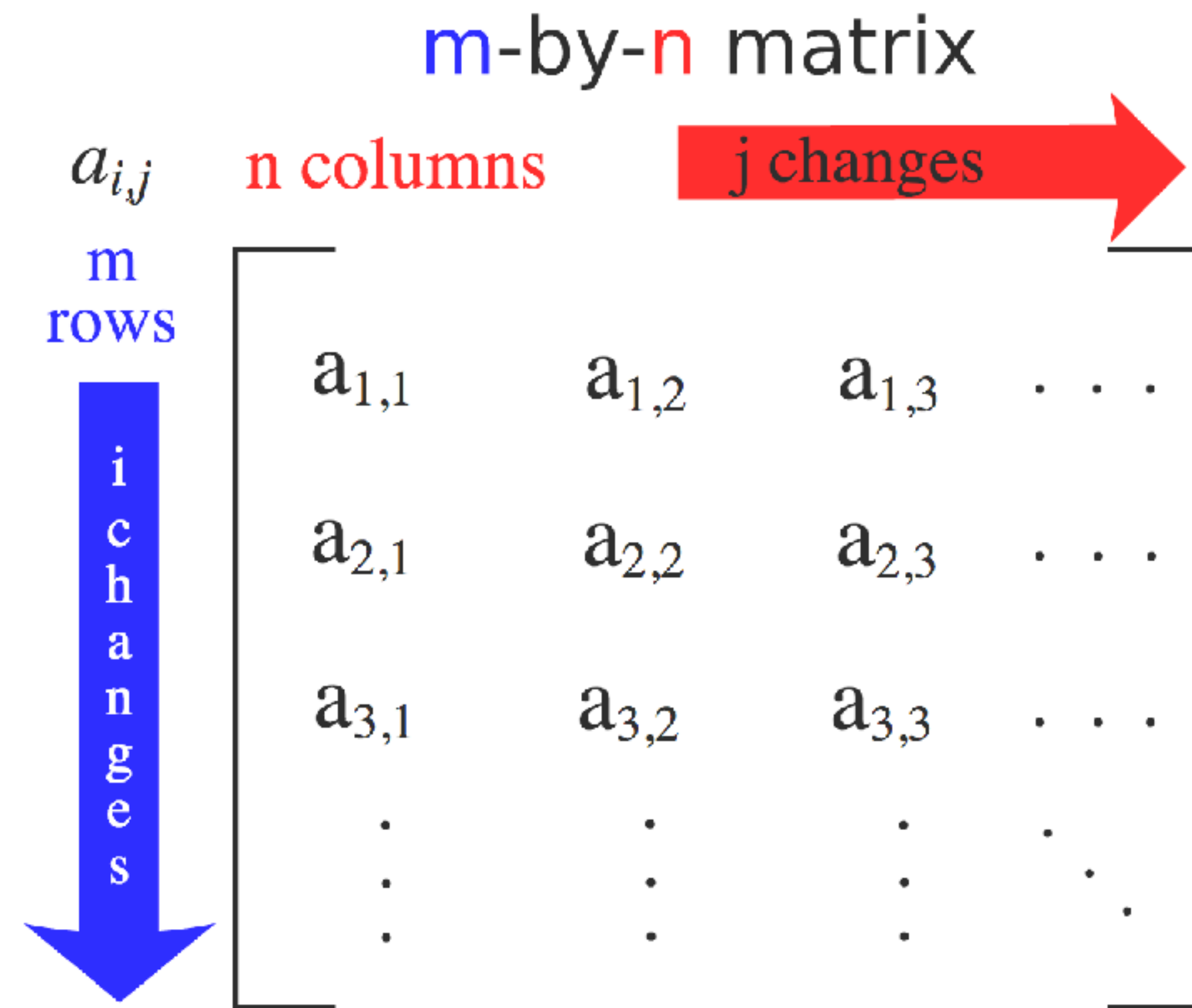
Linear Algebra Foundations Part 1



CS 3113

Matrix refresher.

A matrix.



A 2x3 matrix.

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 3 & 2 \end{bmatrix}$$

A 3x3 matrix.

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 3 & 2 \\ 3 & 4 & 2 \end{bmatrix}$$

Matrix operations.

Matrix addition.

To add two matrices, add their corresponding entries.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} + \begin{bmatrix} J & K & L \\ M & N & O \\ P & Q & R \end{bmatrix} = \begin{bmatrix} A+J & B+K & C+L \\ D+M & E+N & F+O \\ G+P & H+Q & I+R \end{bmatrix}$$

Matrix subtraction.

To subtract two matrices, subtract their corresponding entries.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} - \begin{bmatrix} J & K & L \\ M & N & O \\ P & Q & R \end{bmatrix} = \begin{bmatrix} A-J & B-K & C-L \\ D-M & E-N & F-O \\ G-P & H-Q & I-R \end{bmatrix}$$

**Matrix addition and subtraction can only
happen
with matrices that are the same size!**

Transpose of a matrix.

Transpose of a matrix is a matrix whose columns are the rows of the original matrix (and its rows are the columns).

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix}$$

M

$$\begin{bmatrix} A & D \\ B & E \\ C & F \end{bmatrix}$$

M^T

Matrix/scalar multiplication.

Multiply each entry of the matrix by the scalar.

$$S \times \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} = \begin{bmatrix} S \times A & S \times B & S \times C \\ S \times D & S \times E & S \times F \\ S \times G & S \times H & S \times I \end{bmatrix}$$

Matrix/matrix multiplication.

You can only multiply two matrices
if the number of columns of the first matrix
equals the number of rows of the second.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} = ?$$

It results in a matrix that is number of rows of first matrix by number of columns of second matrix.

$$\begin{bmatrix} \textcolor{red}{A} & \textcolor{red}{B} & \textcolor{red}{C} \\ \textcolor{red}{D} & \textcolor{red}{E} & \textcolor{red}{F} \end{bmatrix} \begin{bmatrix} \textcolor{blue}{J} & \textcolor{blue}{K} \\ \textcolor{blue}{M} & \textcolor{blue}{N} \\ \textcolor{blue}{P} & \textcolor{blue}{Q} \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

For each row, find dot product with each column.

The diagram illustrates the dot product of a row and a column. On the left, a 2x3 matrix is shown with elements A, B, C in the first row and D, E, F in the second row. A red arrow points from the first row to a red box containing A, B, and C. On the right, a 3x2 matrix is shown with elements J, M, P in the first column and K, N, Q in the second column. A red arrow points from the first column to a red box containing J, M, and P. An equals sign follows the matrices.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} =$$

The diagram shows the calculation of the dot product for the first row and first column. A large red bracket on the left contains the expression $A \times J + B \times M + C \times P$. A large red bracket on the right is also shown.

$$\left[A \times J + B \times M + C \times P \right]$$

For each row, find dot product with each column.

The diagram illustrates the dot product of a row and a column. On the left, a 2x3 matrix is shown with elements A, B, C in the first row and D, E, F in the second row. A red arrow points to the first row, which is highlighted with a red background. On the right, a 3x2 matrix is shown with elements J, M, P in the first column and K, N, Q in the second column. A red arrow points to the second column, which is highlighted with a red background. An equals sign follows the matrices.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} =$$

The diagram shows the result of the dot product as a row vector. The first element is the dot product of the first row and first column, calculated as A times J plus B times M plus C times P. The second element is the dot product of the first row and second column, calculated as A times K plus B times N plus C times Q. The entire result is enclosed in large square brackets.

$$\begin{bmatrix} A \times J + B \times M + C \times P & A \times K + B \times N + C \times Q \end{bmatrix}$$

For each row, find dot product with each column.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} =$$

$$\begin{bmatrix} A \times J + B \times M + C \times P & A \times K + B \times N + C \times Q \\ D \times J + E \times M + F \times P & \end{bmatrix}$$

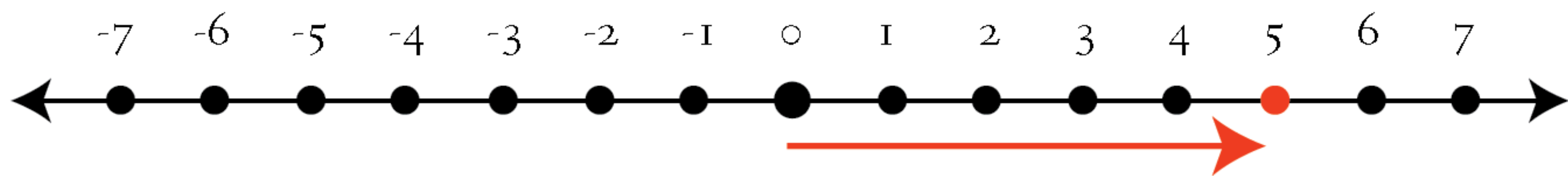
For each row, find dot product with each column.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} =$$

$$\begin{bmatrix} A \times J + B \times M + C \times P & A \times K + B \times N + C \times Q \\ D \times J + E \times M + F \times P & D \times K + E \times N + F \times Q \end{bmatrix}$$

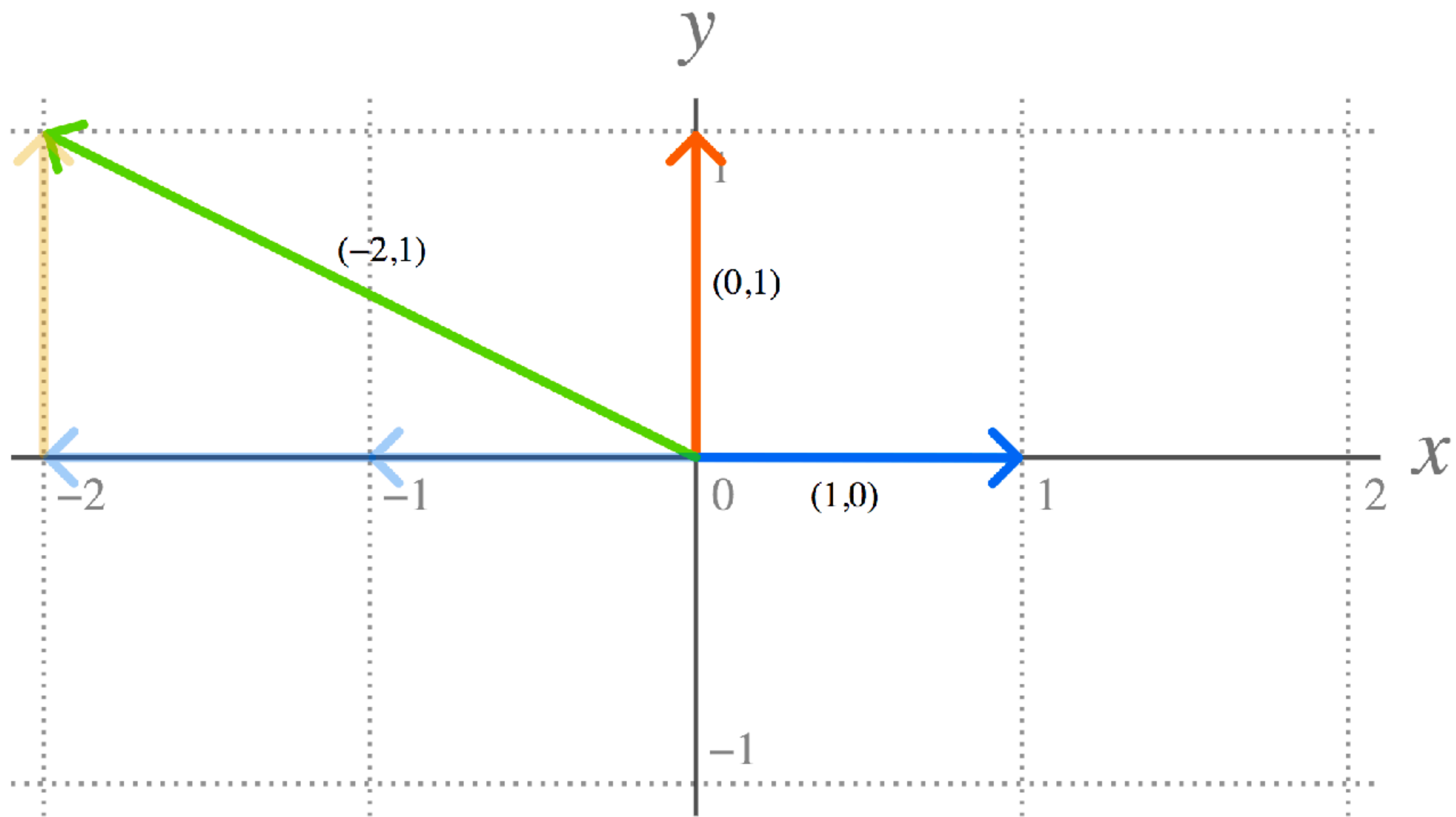
● Where am I?

Vectors

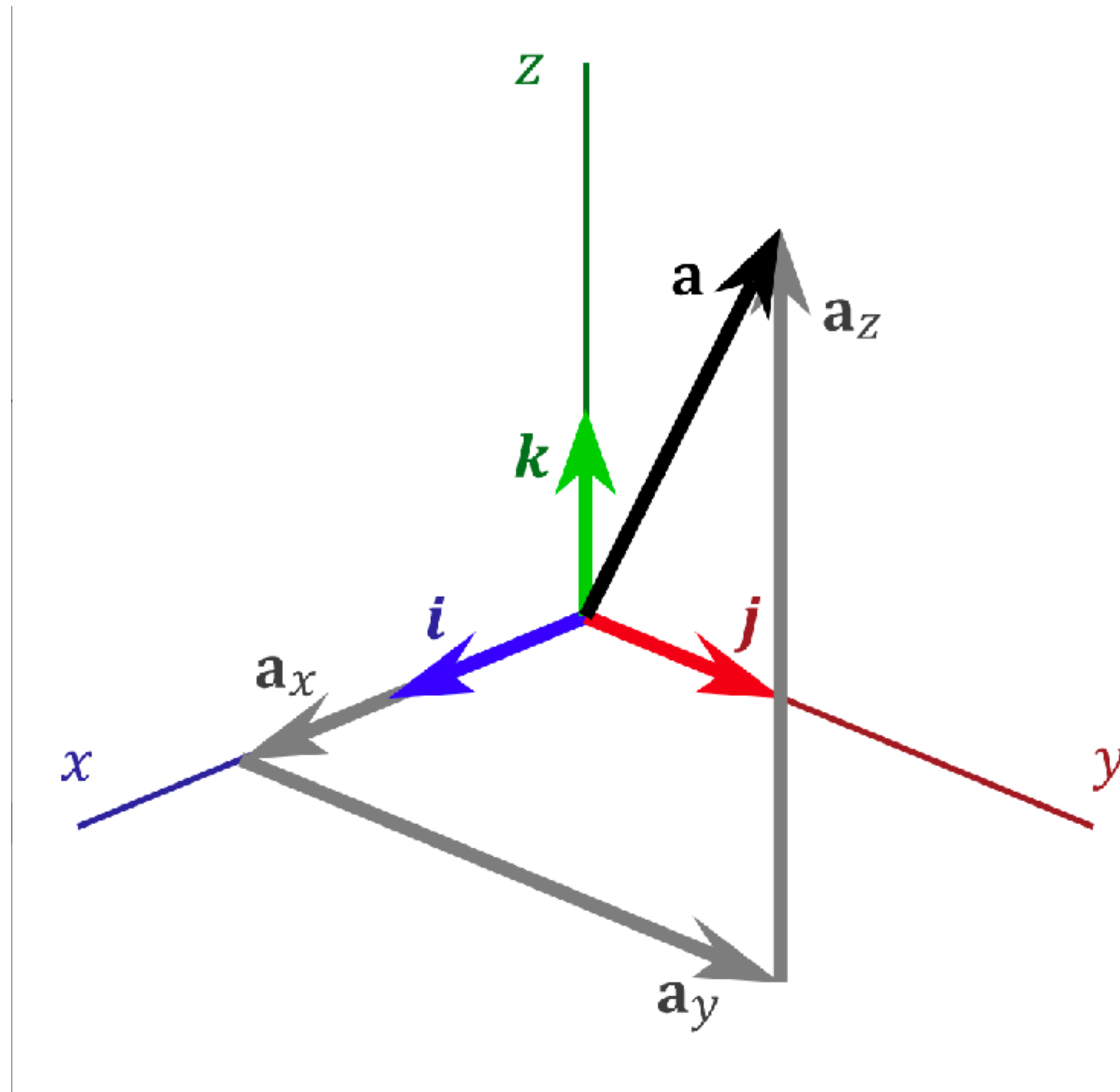


Basis Vectors

Vectors in N dimensions



Basis vectors.



$$\vec{v} = \sum_i c_i \vec{b}_i.$$

$$\vec{v} = \sum_i c_i \vec{b}_i = \left[\begin{array}{ccc} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{array} \right] \left[\begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} \right].$$

Basis vectors as matrices

Transformation matrices

Linear transformations

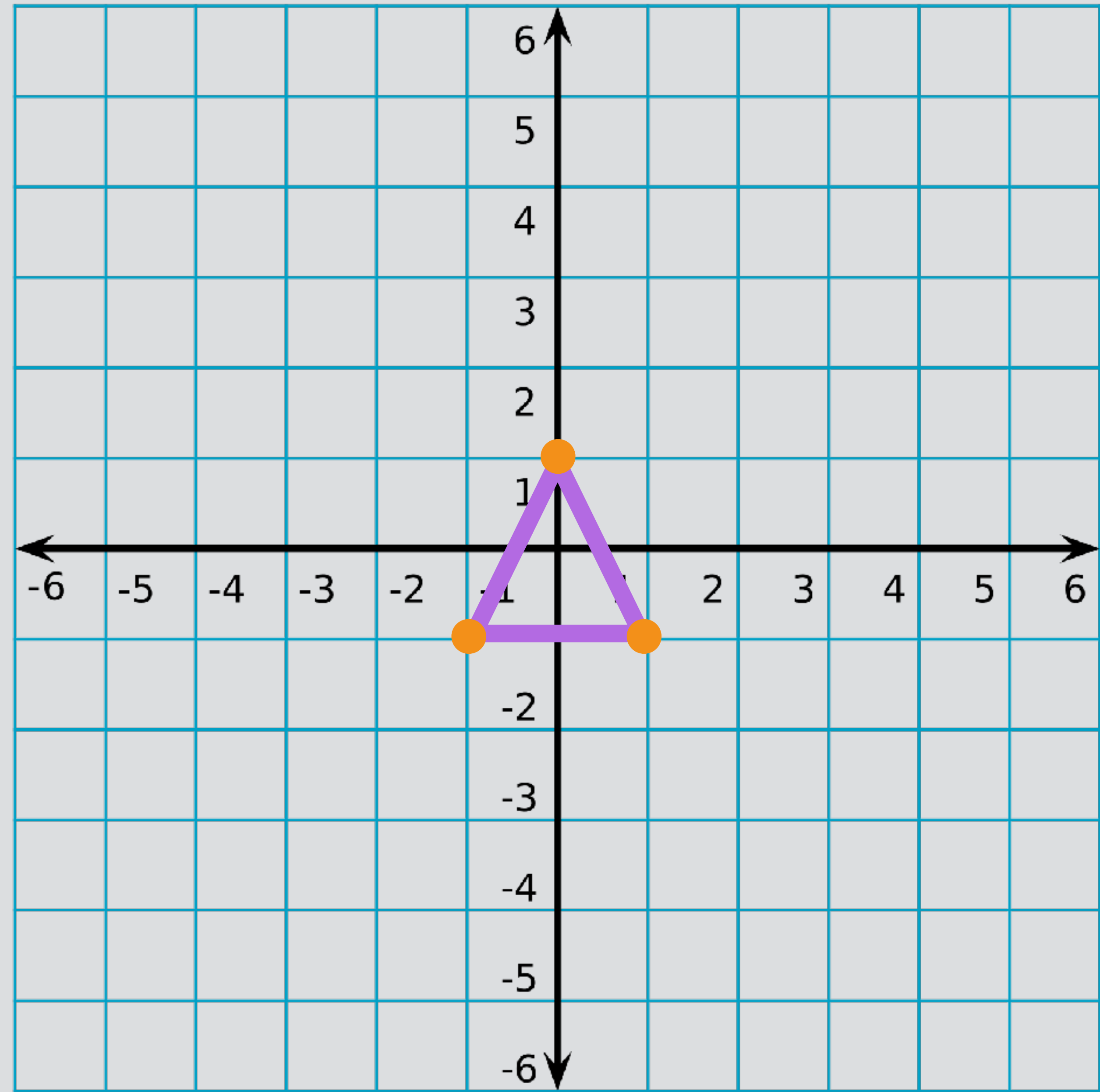
Scale

Scale

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} AX + BY \\ CX + DY \end{bmatrix}$$

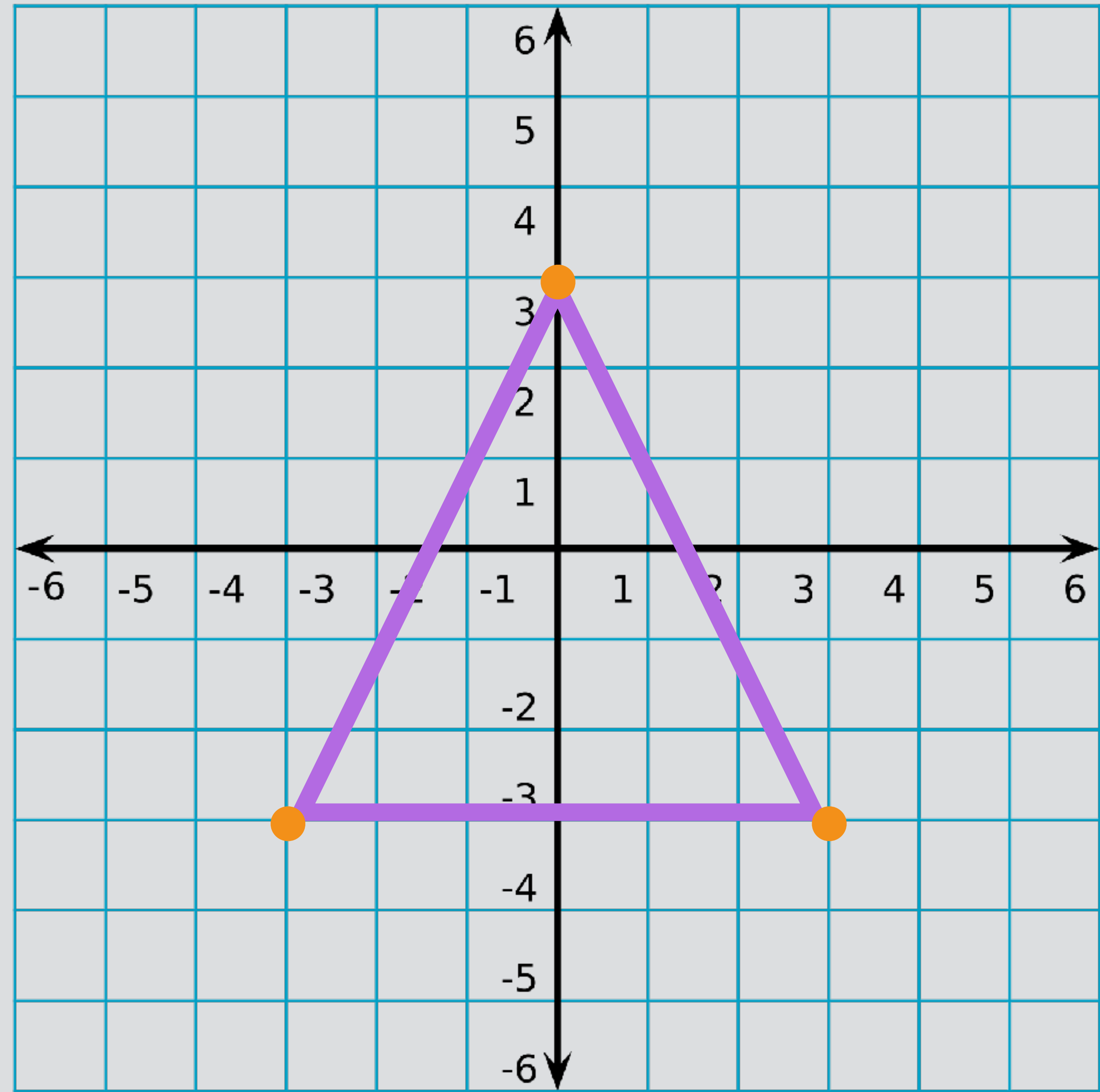
$$\begin{bmatrix} sx & 0 \\ 0 & sy \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} sxX + 0Y \\ 0X + syY \end{bmatrix}$$

y-axis



x-axis

y-axis

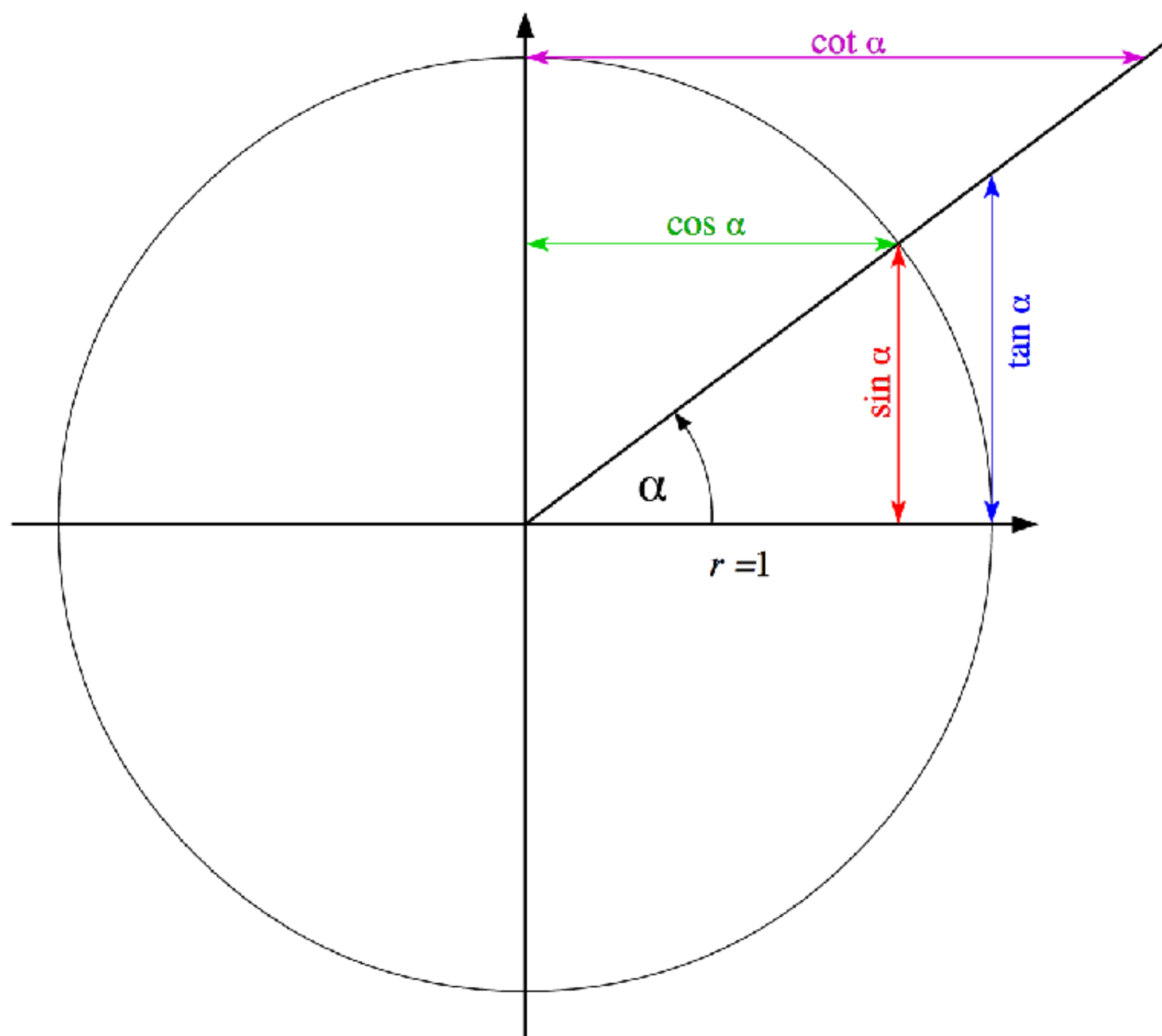


x-axis

Rotation

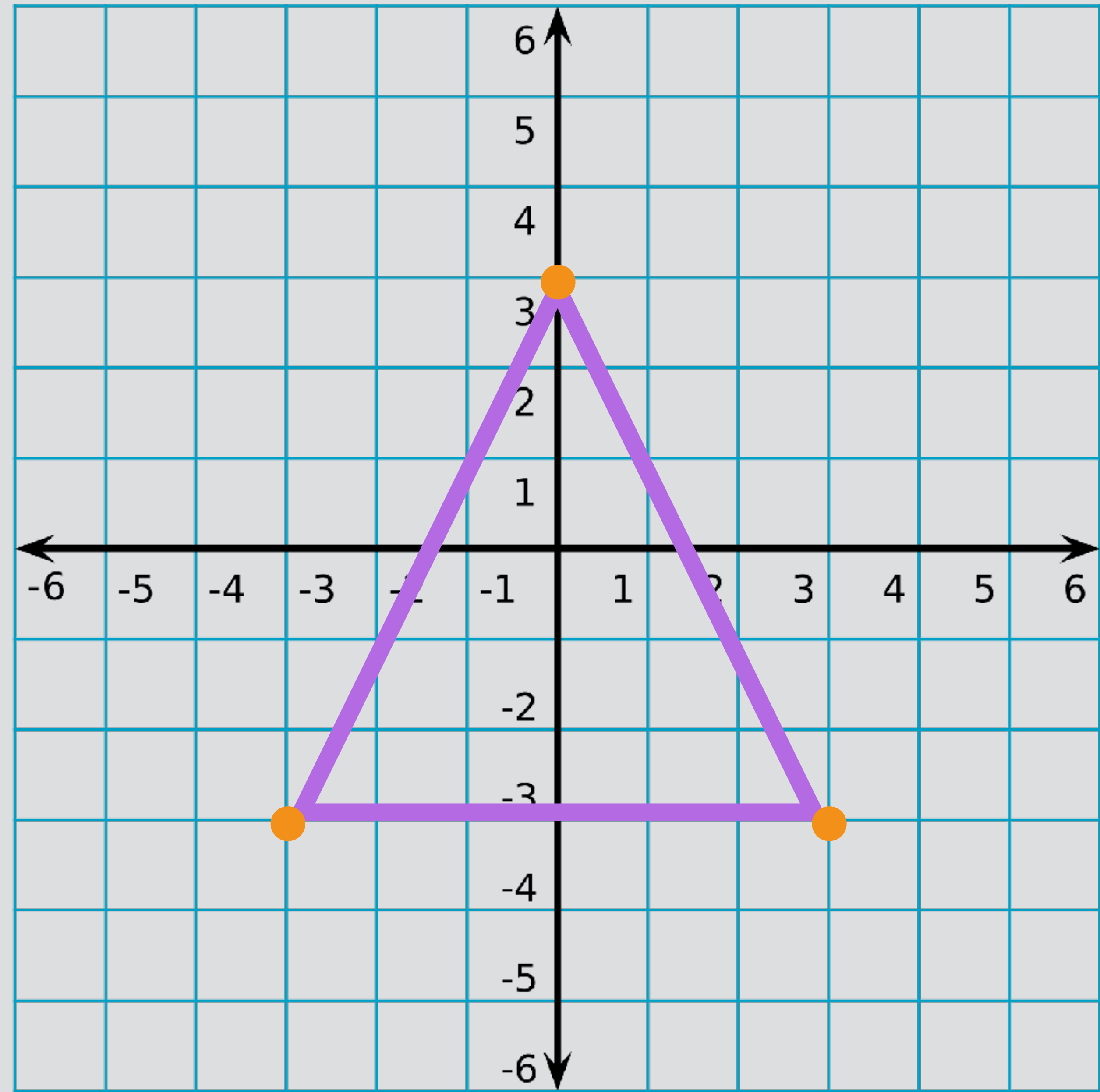
Rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\theta X + -\sin\theta Y \\ \sin\theta X + \cos\theta Y \end{bmatrix}$$

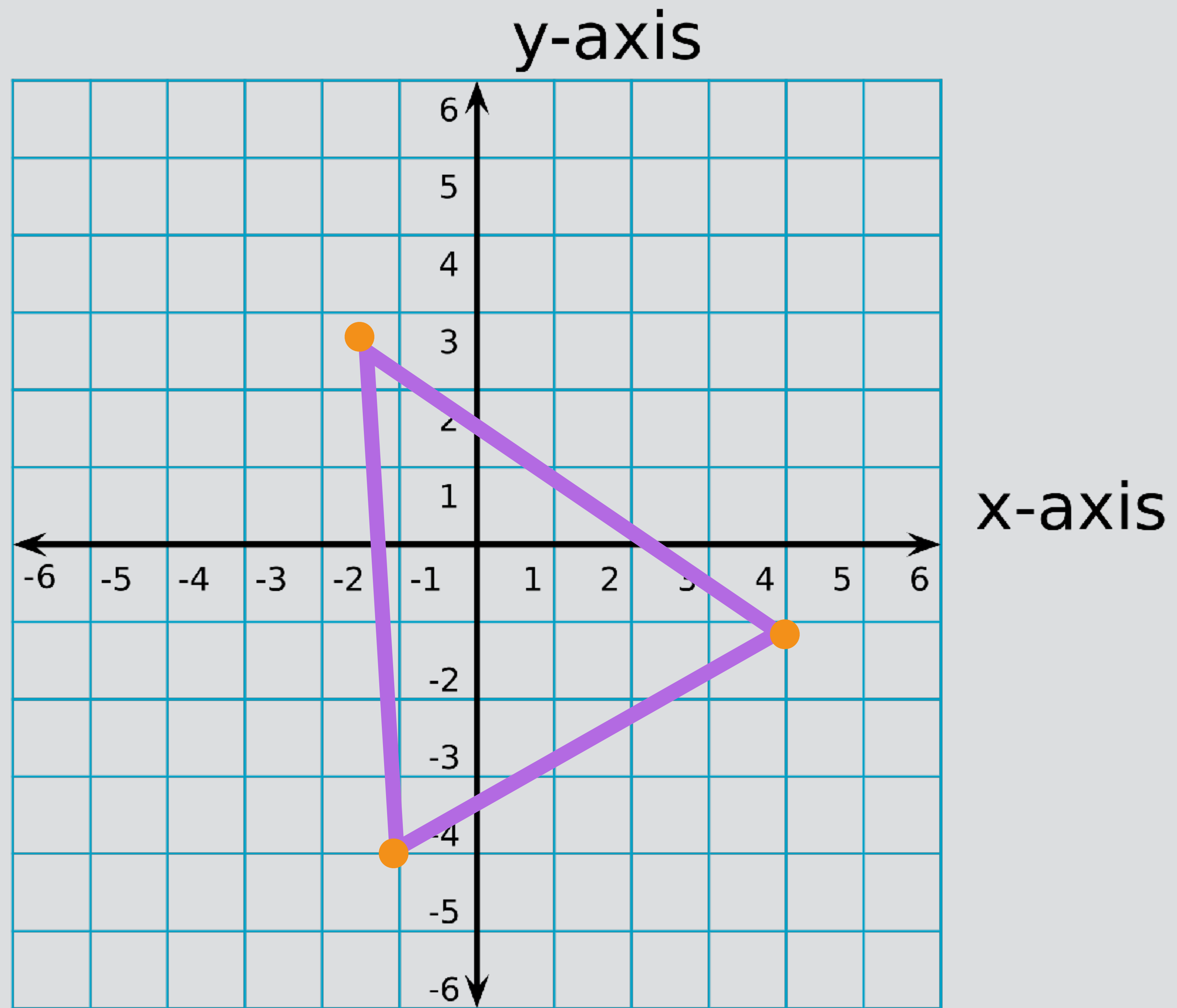


$$\begin{bmatrix} \cos \theta X + -\sin \theta Y \\ \sin \theta X + \cos \theta Y \end{bmatrix}$$

y-axis



x-axis



Homogeneous coordinates

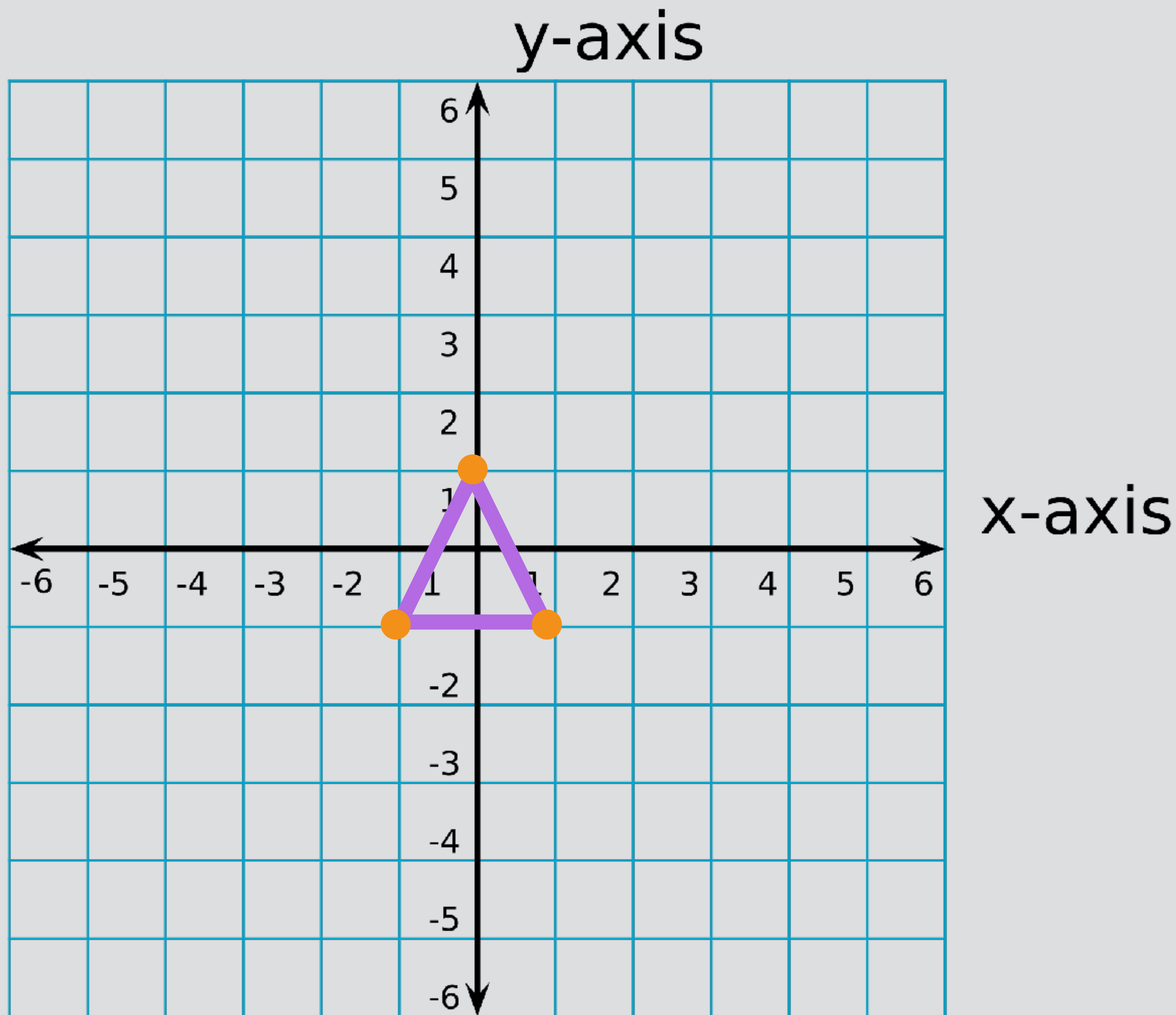
Affine transformations

An affine transformation matrix combines a linear transformation matrix with a translation using homogeneous coordinates.

Affine Identity

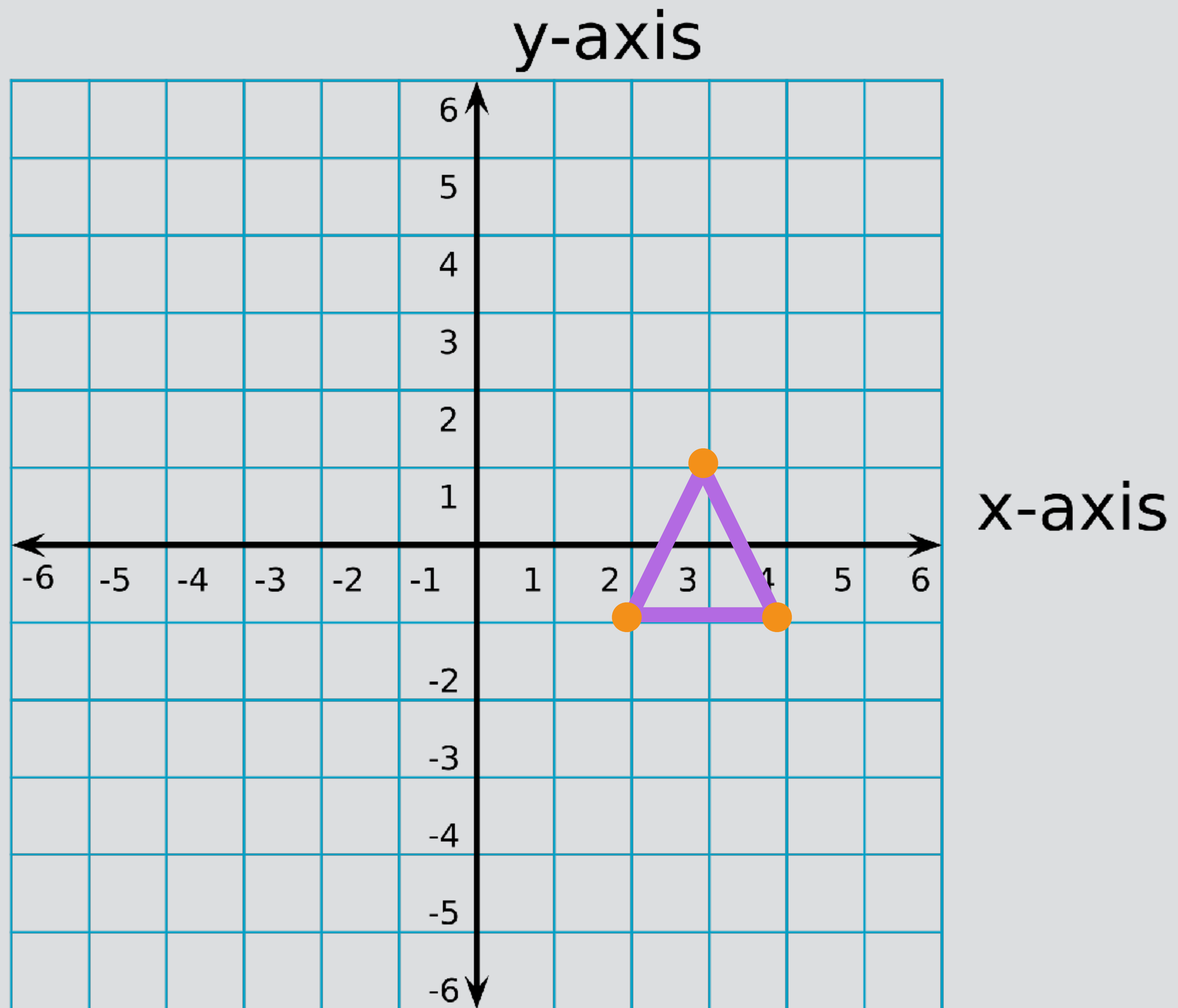
Linear part

$$\begin{bmatrix} \boxed{1} & \boxed{0} & \boxed{0} & 0 \\ \boxed{0} & \boxed{1} & \boxed{0} & 0 \\ \boxed{0} & \boxed{0} & \boxed{1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



Affine Translation

$$\begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

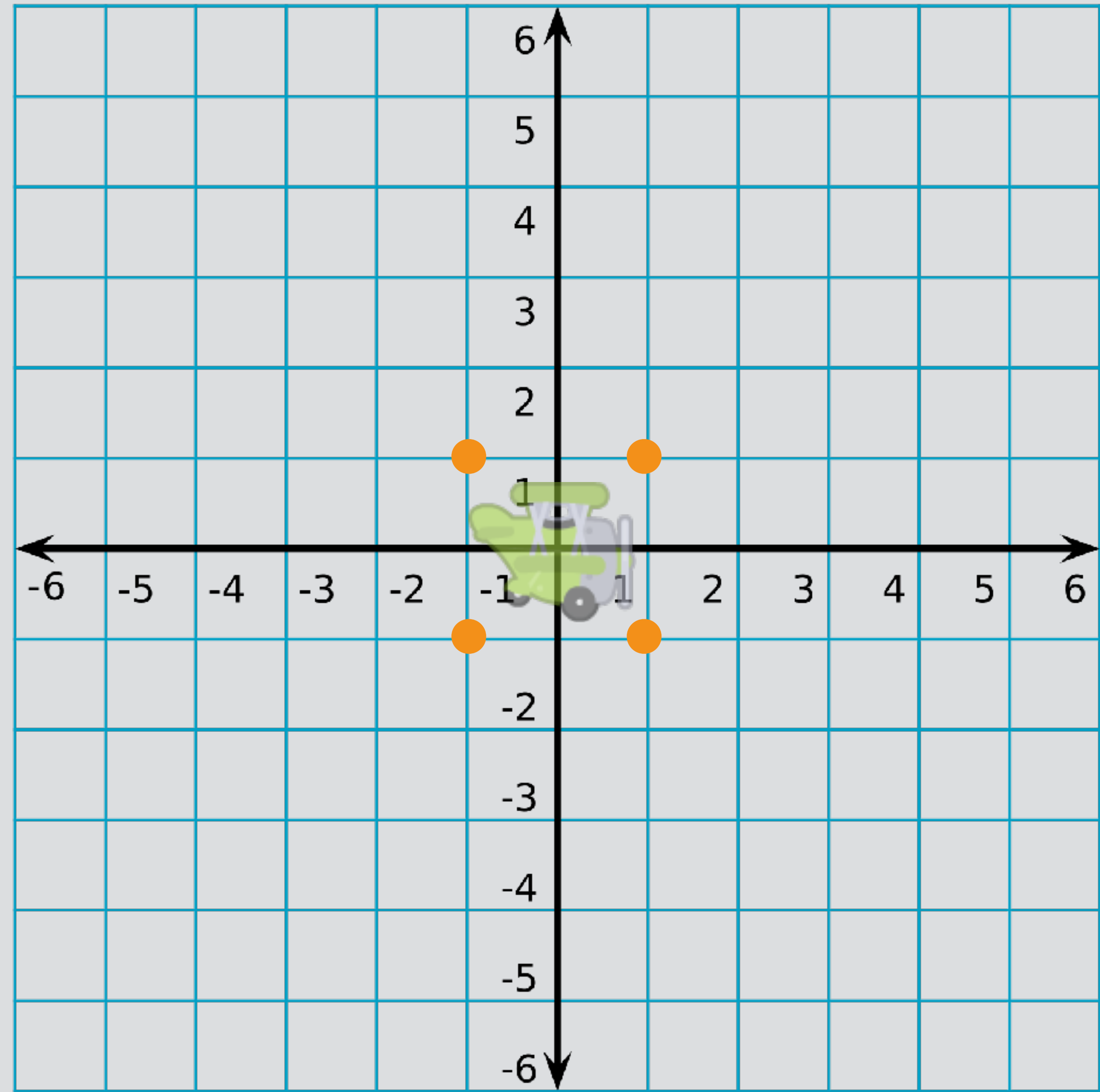


Multiplying transformation matrices

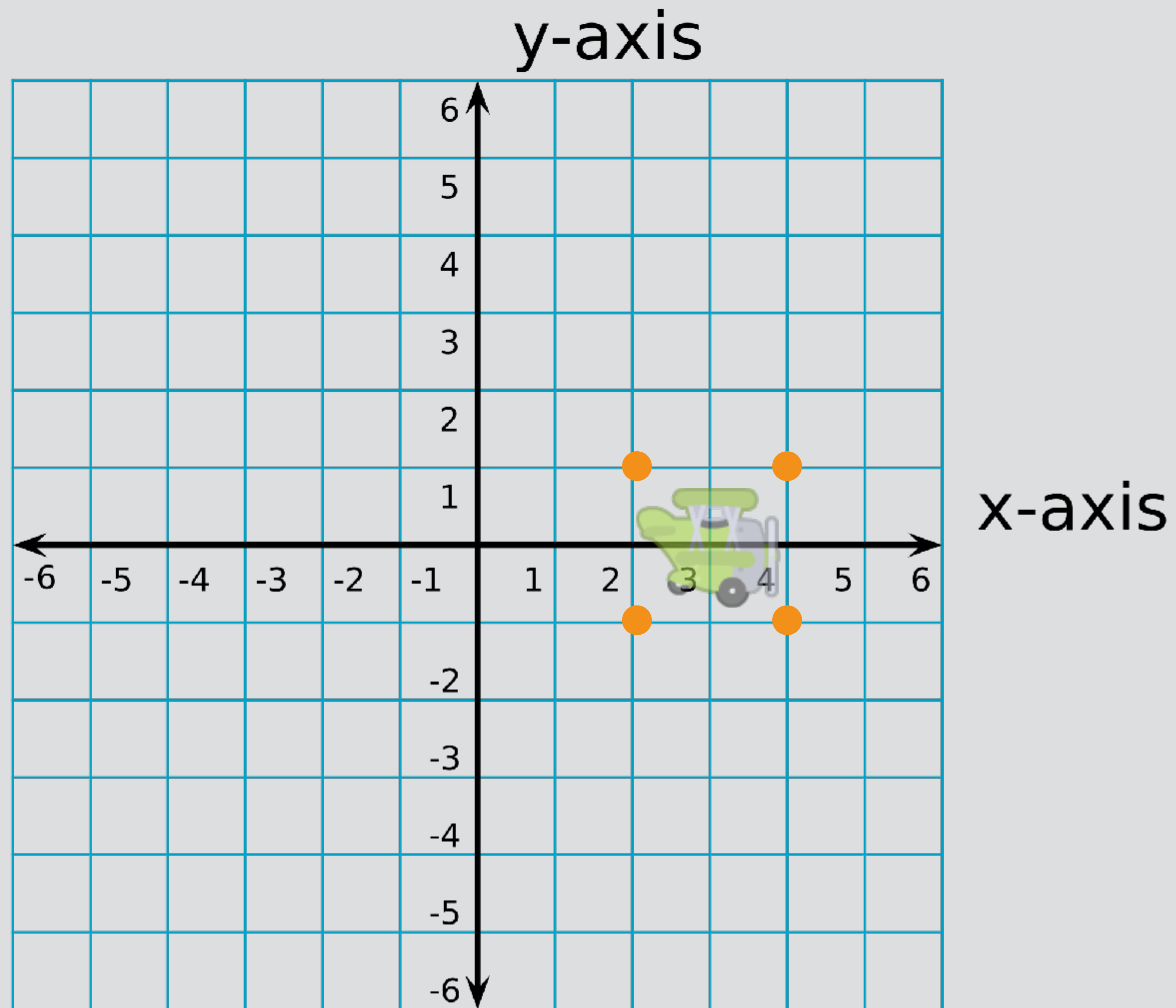
Matrix multiplication is non-commutative!

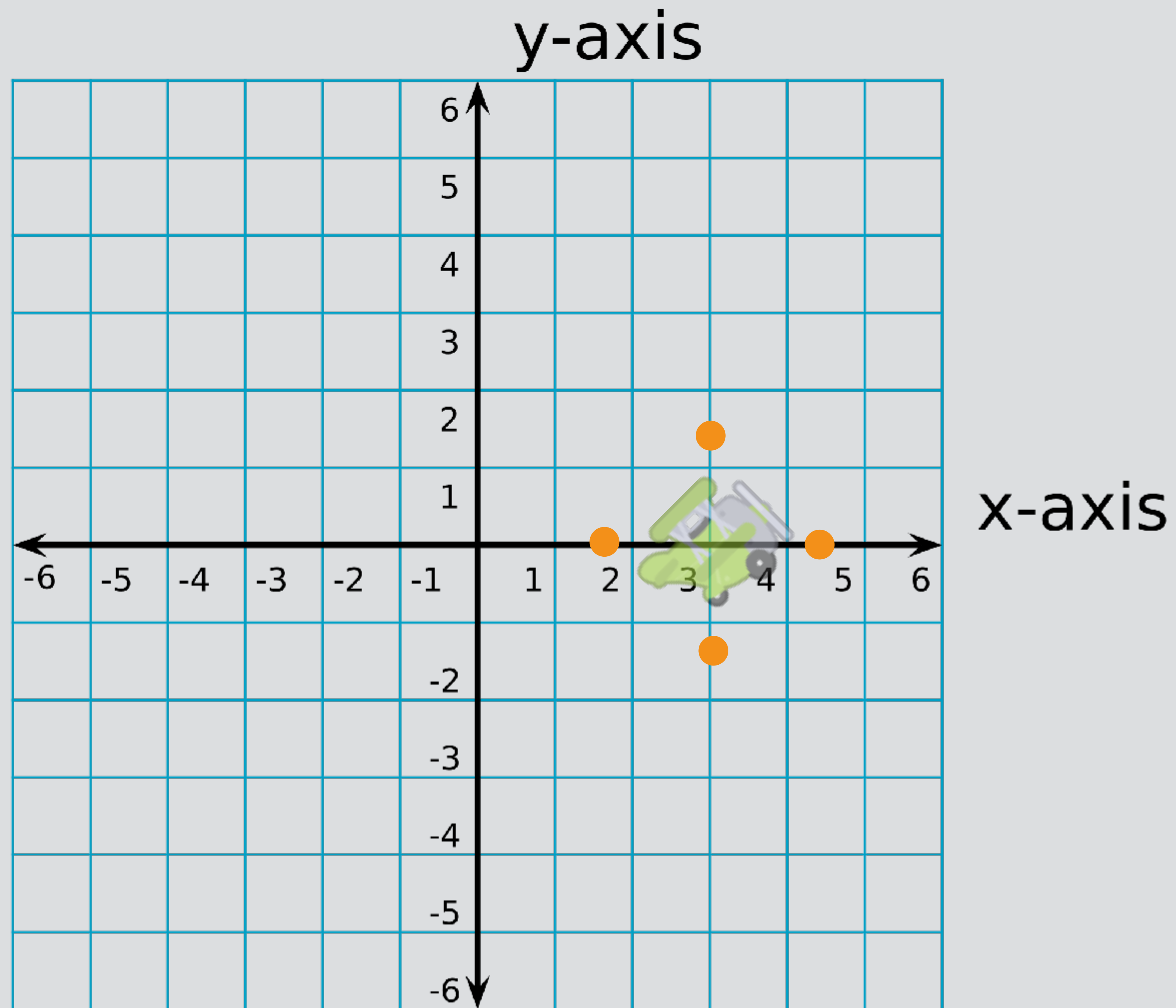
$$p = MR * MT * c$$

y-axis



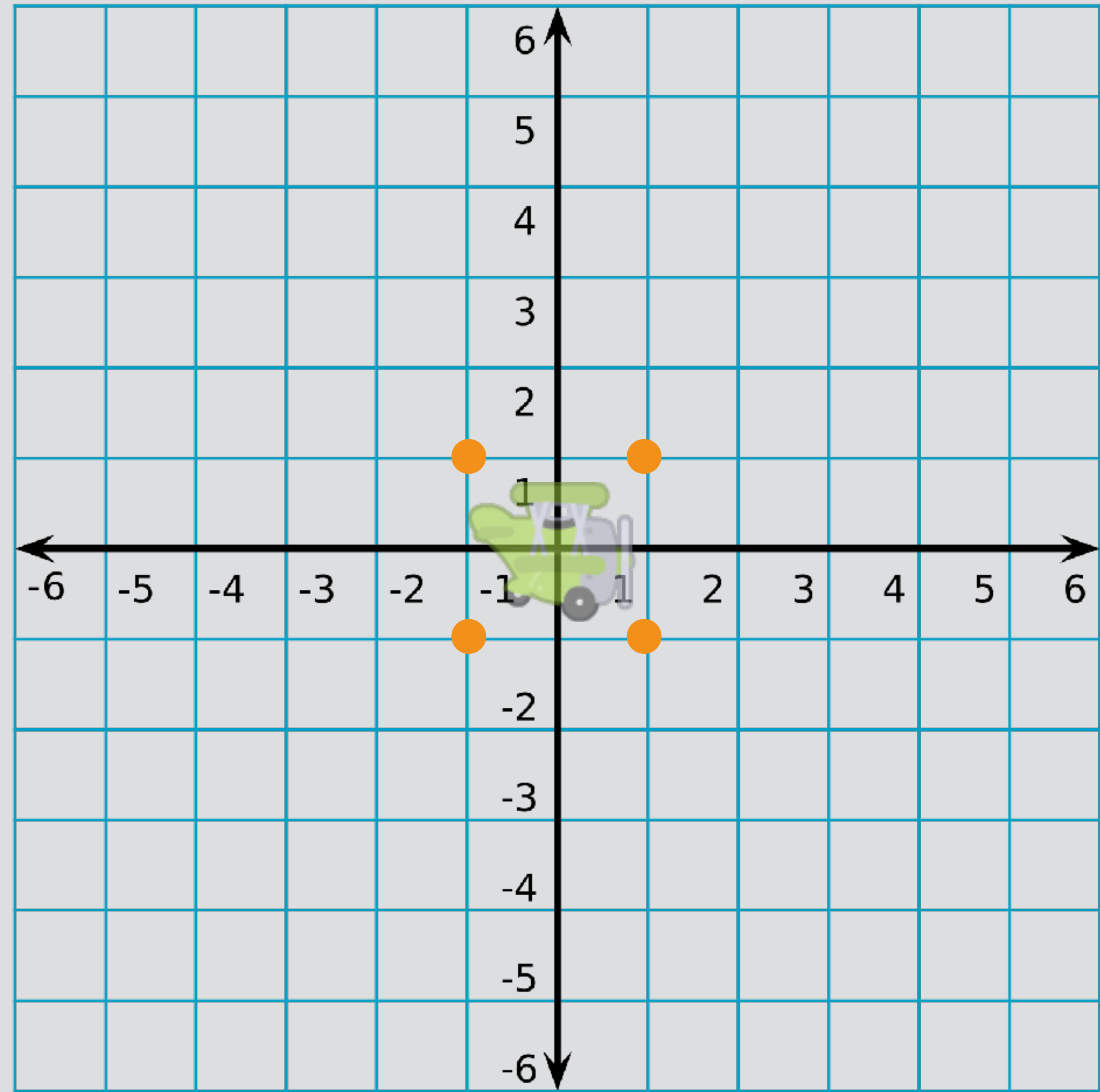
x-axis





$$p = MT * MR * c$$

y-axis



x-axis

