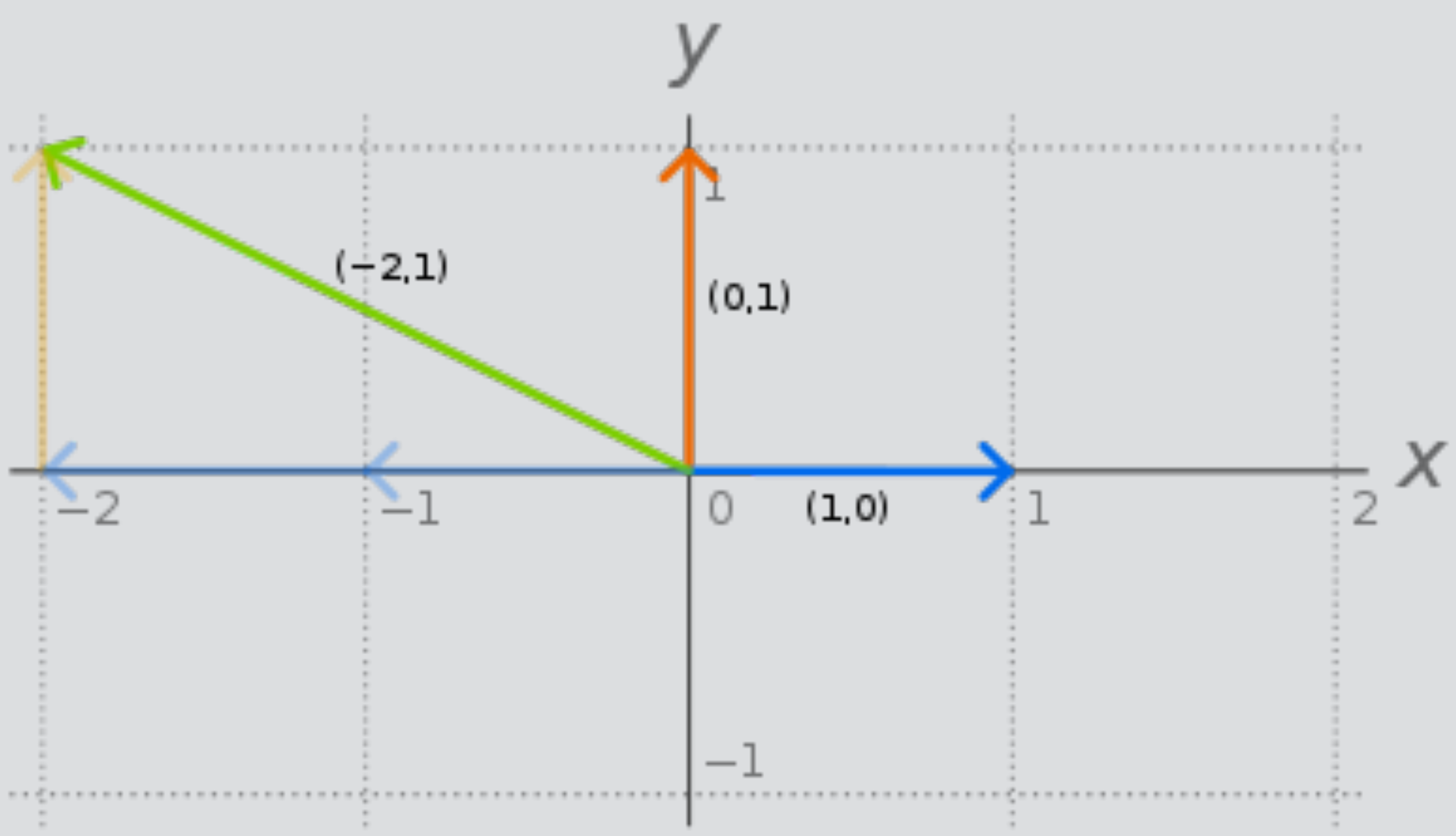
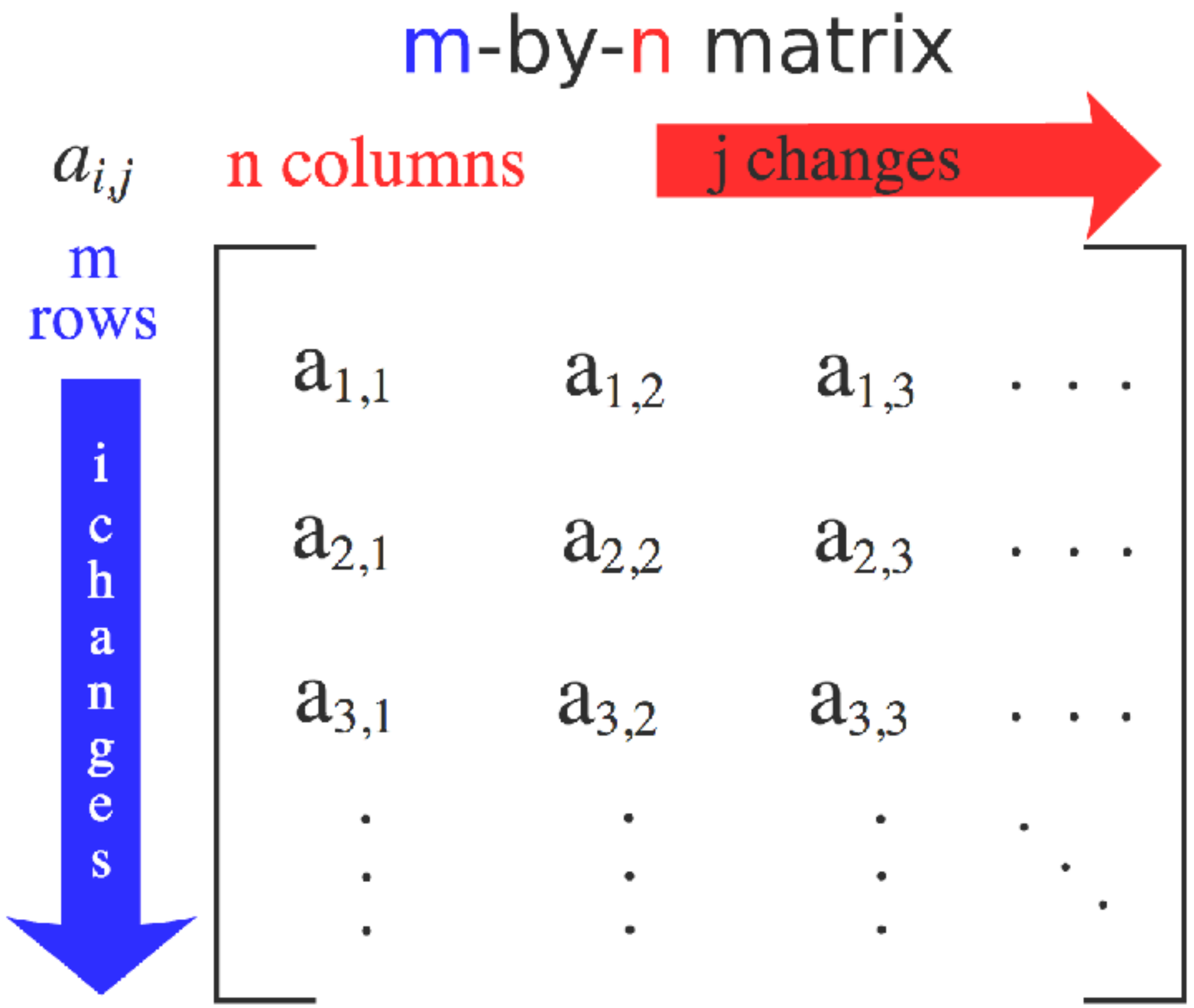
Linear Algebra Foundations Part 1



CS 3113

Matrix refresher.

A matrix.



m-by-n matrix
$a_{i,j}$
n columns
j changes
m rows
i changes
$a_{1,1}$ $a_{1,2}$ $a_{1,3}$ . . .
$a_{2,1}$ $a_{2,2}$ $a_{2,3}$ . . .
$a_{3,1}$ $a_{3,2}$ $a_{3,3}$ . . .

A 2x3 matrix.

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 3 & 2 \end{bmatrix}$$

A 3x3 matrix.

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 3 & 2 \\ 3 & 4 & 2 \end{bmatrix}$$

Matrix operations.

Matrix addition.

To add two matrices, add their corresponding entries.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} + \begin{bmatrix} J & K & L \\ M & N & O \\ P & Q & R \end{bmatrix} = \begin{bmatrix} A+J & B+K & C+L \\ D+M & E+N & F+O \\ G+P & H+Q & I+R \end{bmatrix}$$

Matrix subtraction.

To subtract two matrices, subtract their corresponding entries.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} - \begin{bmatrix} J & K & L \\ M & N & O \\ P & Q & R \end{bmatrix} = \begin{bmatrix} A-J & B-K & C-L \\ D-M & E-N & F-O \\ G-P & H-Q & I-R \end{bmatrix}$$

Matrix addition and subtraction can only happen
with matrices that are the same size!

Transpose of a matrix.

Transpose of a matrix is a matrix whose columns are the rows of the original matrix (and its rows are the columns).

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \qquad \begin{bmatrix} A & D \\ B & E \\ C & F \end{bmatrix}$$

$$M \qquad\qquad M^T$$

Matrix/scalar multiplication.

Multiply each entry of the matrix by the scalar.

$$S \times \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} = \begin{bmatrix} S \times A & S \times B & S \times C \\ S \times D & S \times E & S \times F \\ S \times G & S \times H & S \times I \end{bmatrix}$$

Matrix/matrix multiplication.

You can only multiply two matrices
if the number of columns of the first matrix
equals the number of rows of the second.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} = \quad ?$$

It results in a matrix that is number of rows of first matrix by number of columns of second matrix.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

For each row, find dot product with each column.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} =$$

$$\begin{bmatrix} A \times J + B \times M + C \times P & \\ & \end{bmatrix}$$

For each row, find dot product with each column.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} =$$

$$\begin{bmatrix} AxJ + BxM + CxP & AxK + BxN + CxQ \\ & \end{bmatrix}$$

For each row, find dot product with each column.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} = \begin{bmatrix} AxJ + BxM + CxP & AxK + BxN + CxQ \\ DxJ + ExM + FxP & \end{bmatrix}$$

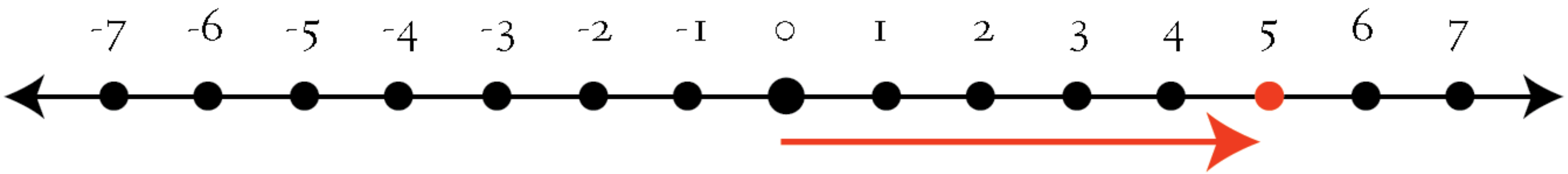
For each row, find dot product with each column.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} =$$

$$\begin{bmatrix} AxJ + BxM + CxP & AxK + BxN + CxQ \\ DxJ + ExM + FxP & DxK + ExN + FxQ \end{bmatrix}$$

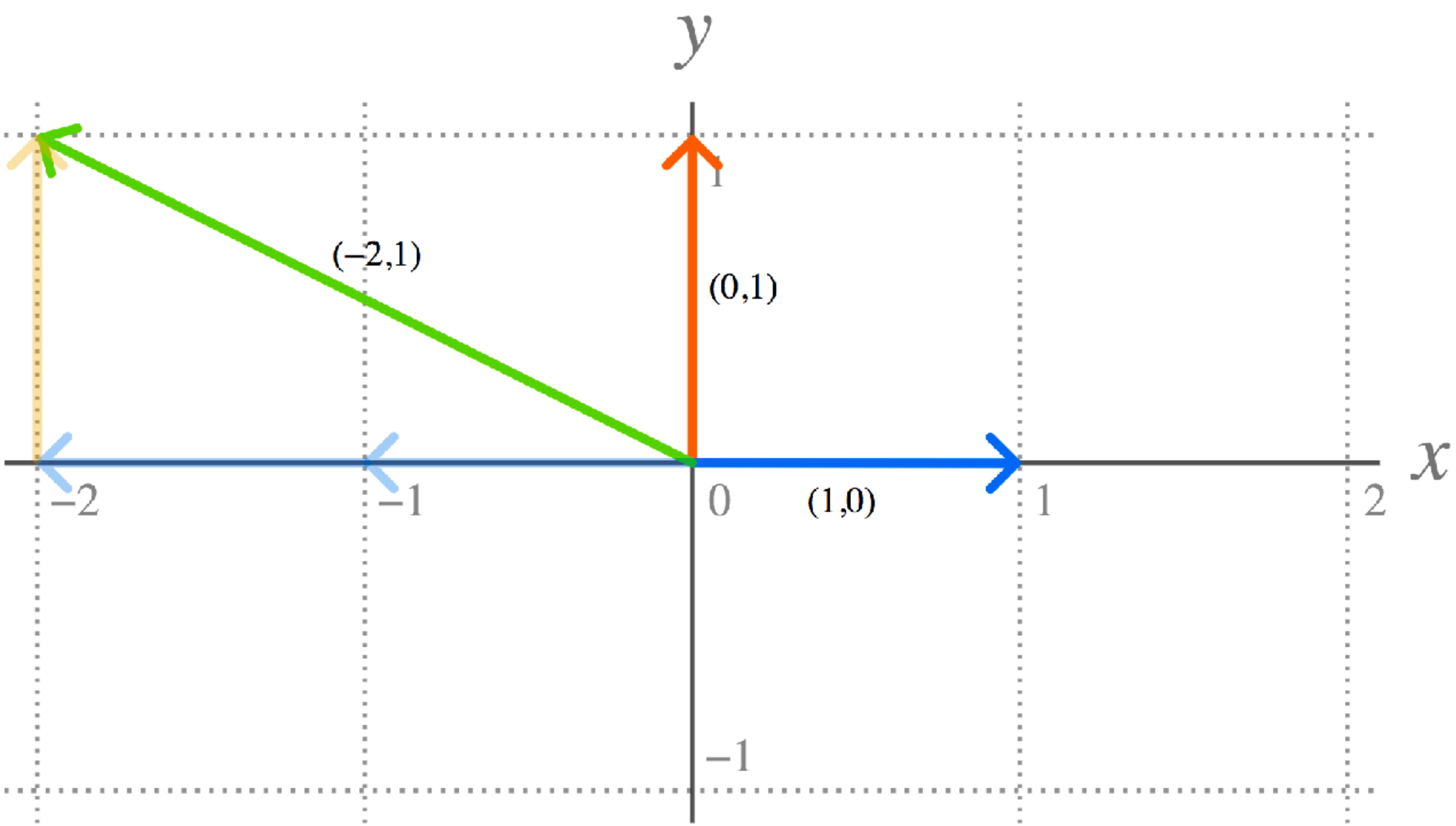
- Where am I?

Vectors

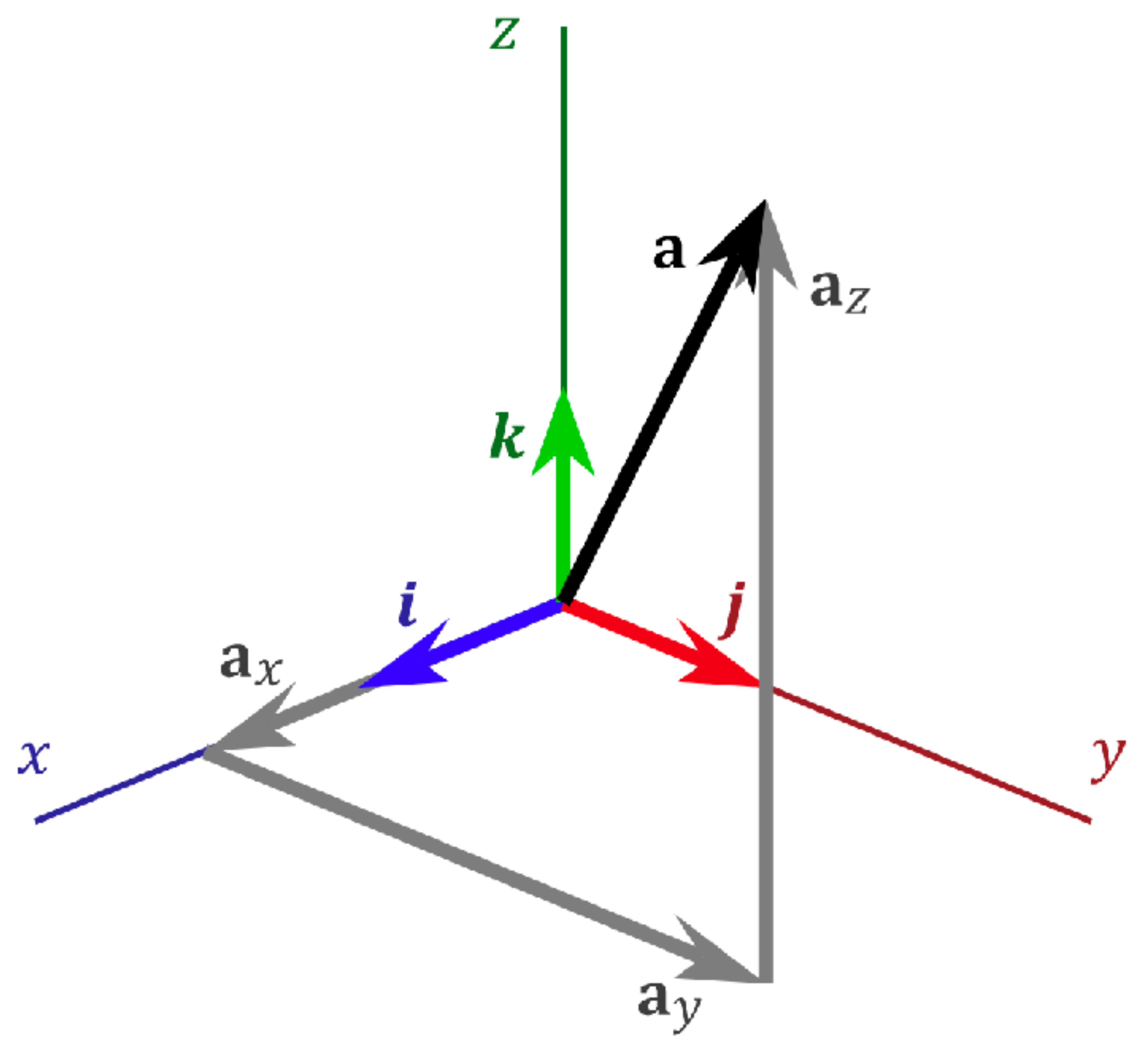

-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

Basis Vectors

Vectors in N dimensions


y
x
(−2,1)
(0,1)
(1,0)
−2
−1
0
1
2
1
−1

Basis vectors.



$$\vec{v} = \sum_i c_i \vec{b}_i .$$

$$\vec{v} = \sum_i c_i \vec{b}_i = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} .$$

Basis vectors as matrices

Transformation matrices

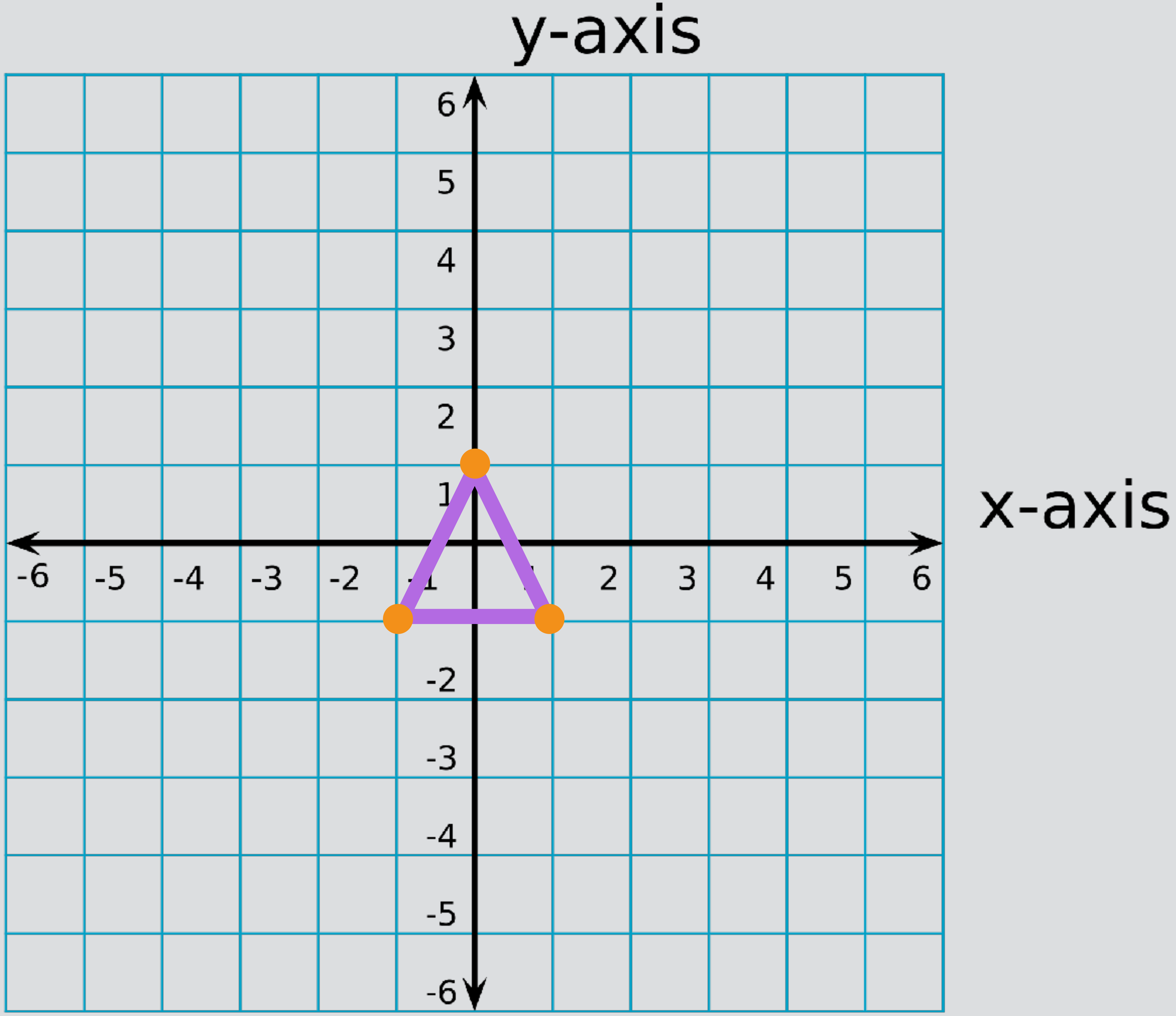
Linear transformations

Scale

Scale

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} AX + BY \\ CX + DY \end{bmatrix}$$

$$\begin{bmatrix} sx & 0 \\ 0 & sy \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} sxX + 0Y \\ 0X + syY \end{bmatrix}$$



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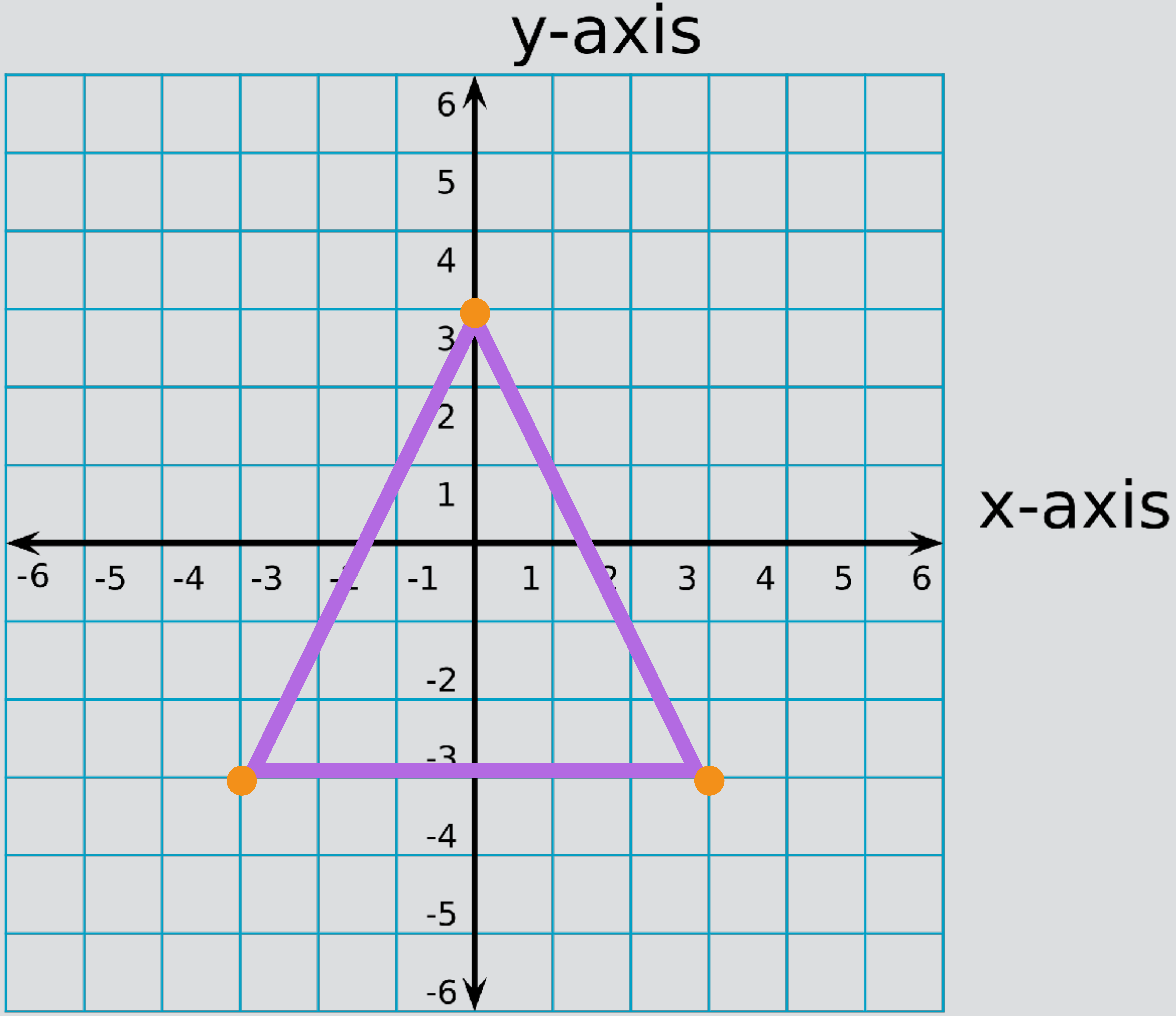
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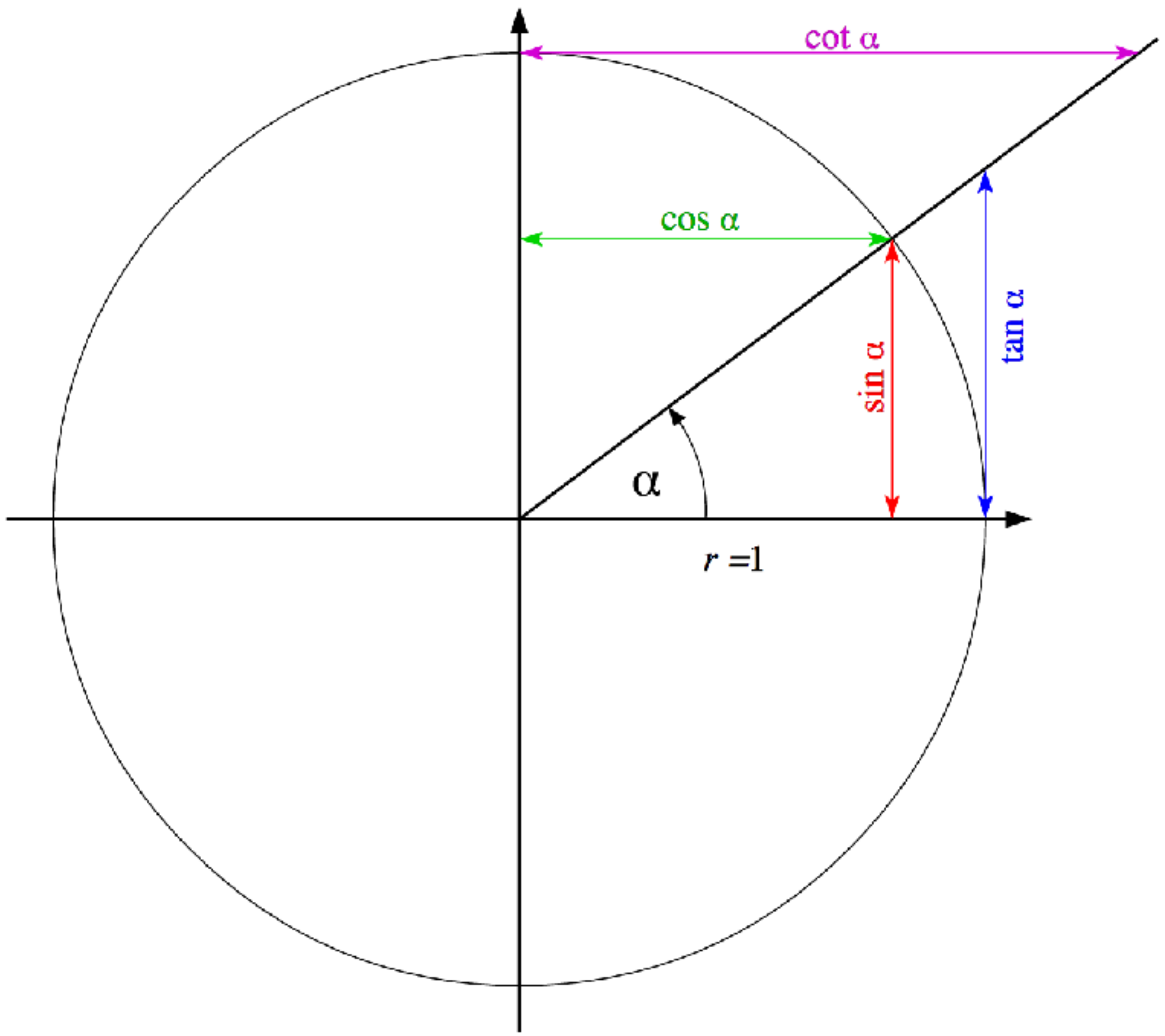
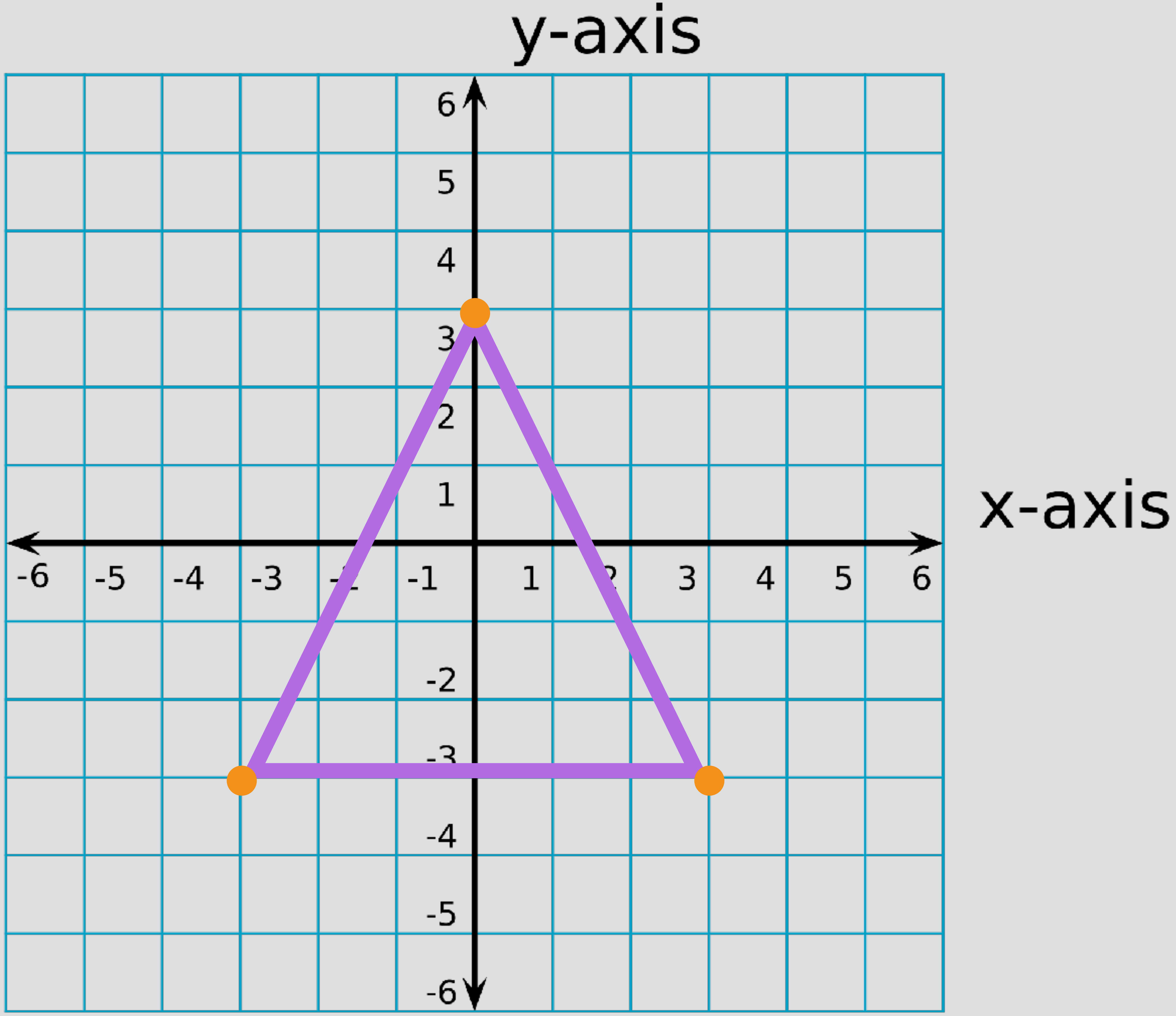
Rotation

Rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\theta X + -\sin\theta Y \\ \sin\theta X + \cos\theta Y \end{bmatrix}$$



$$\begin{bmatrix} \cos\theta X + -\sin\theta Y \\ \sin\theta X + \cos\theta Y \end{bmatrix}$$



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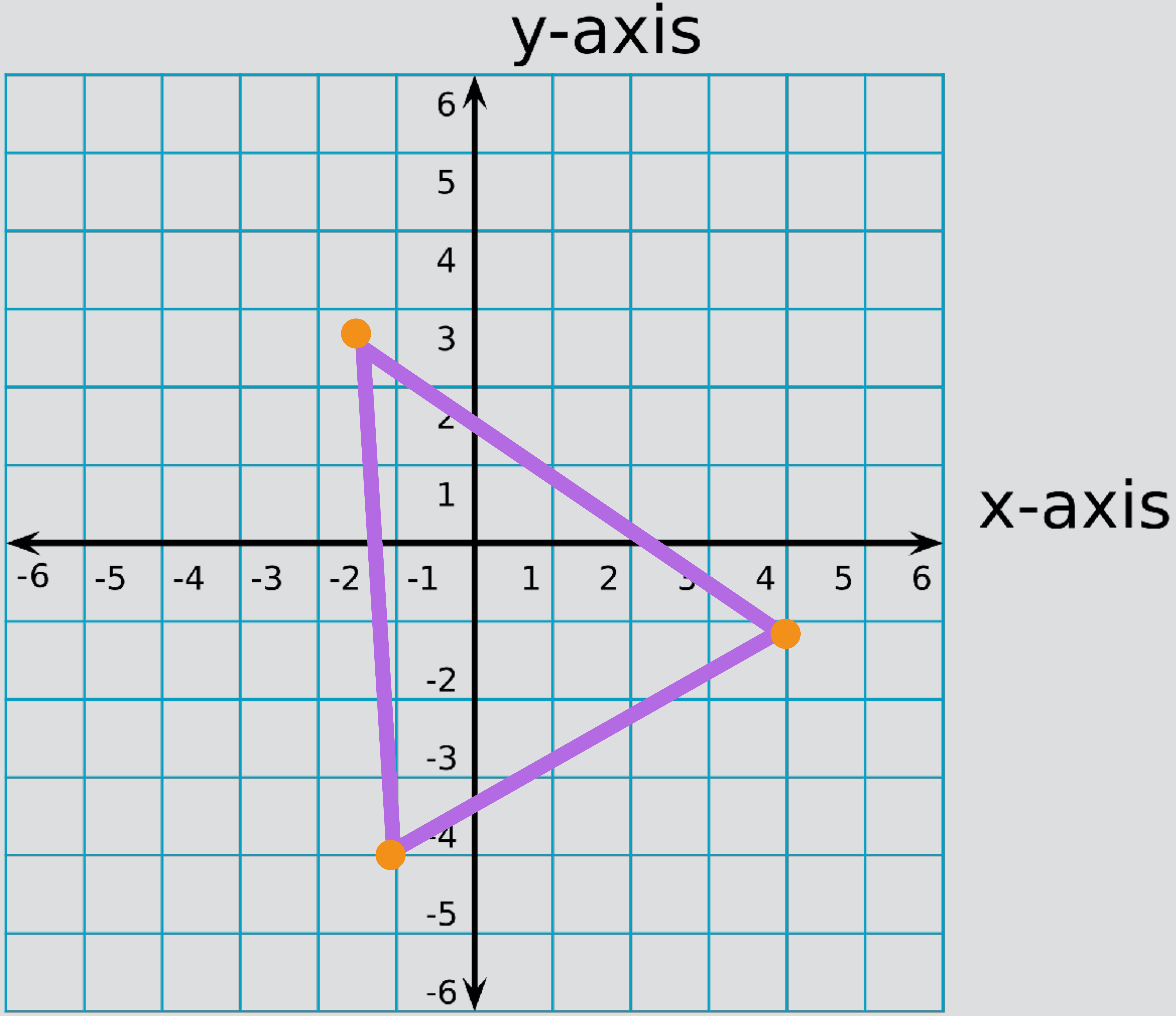
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Homogeneous coordinates

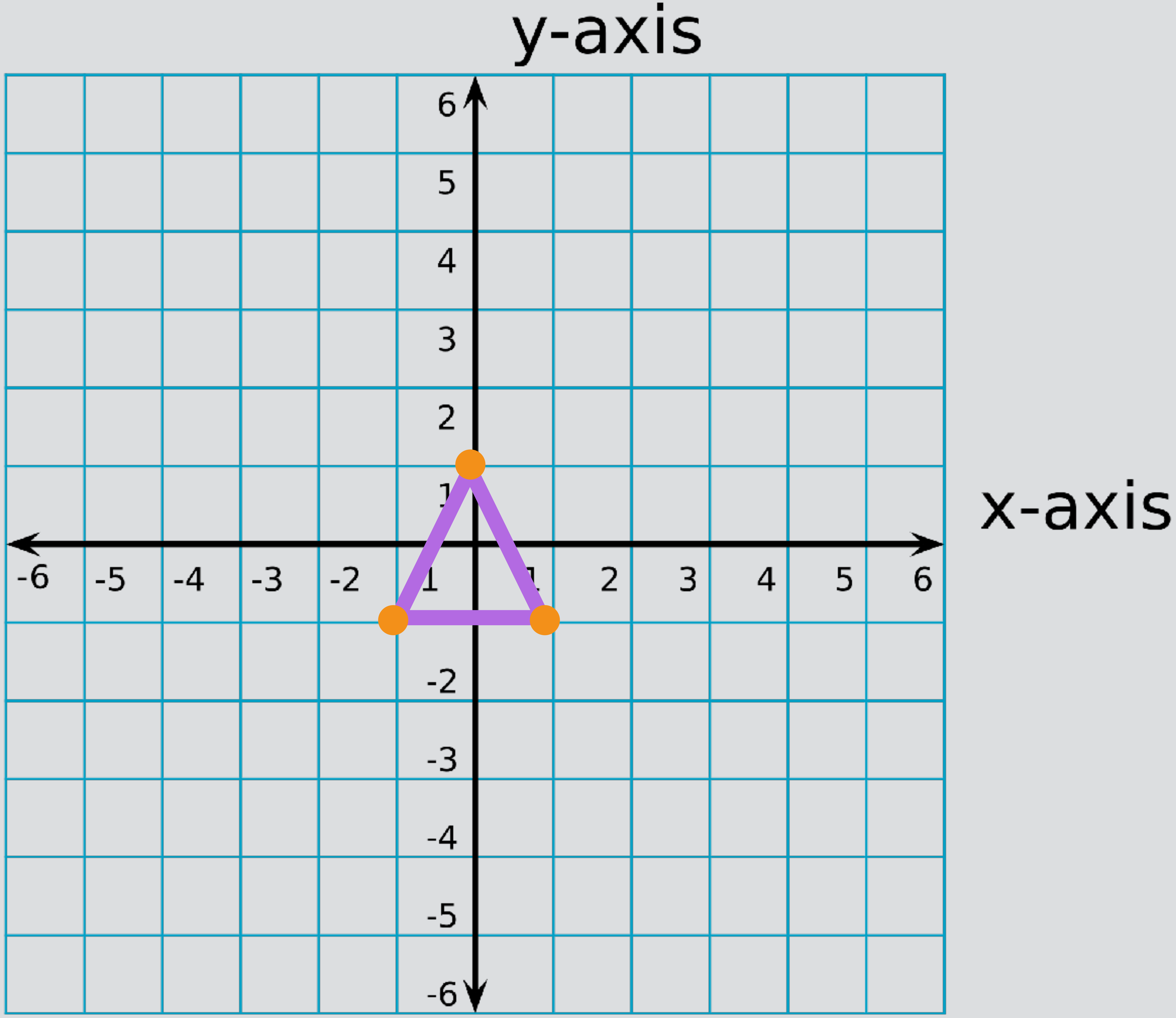
Affine transformations

An affine transformation matrix combines a linear transformation matrix with a translation using homogeneous coordinates.

Affine Identity

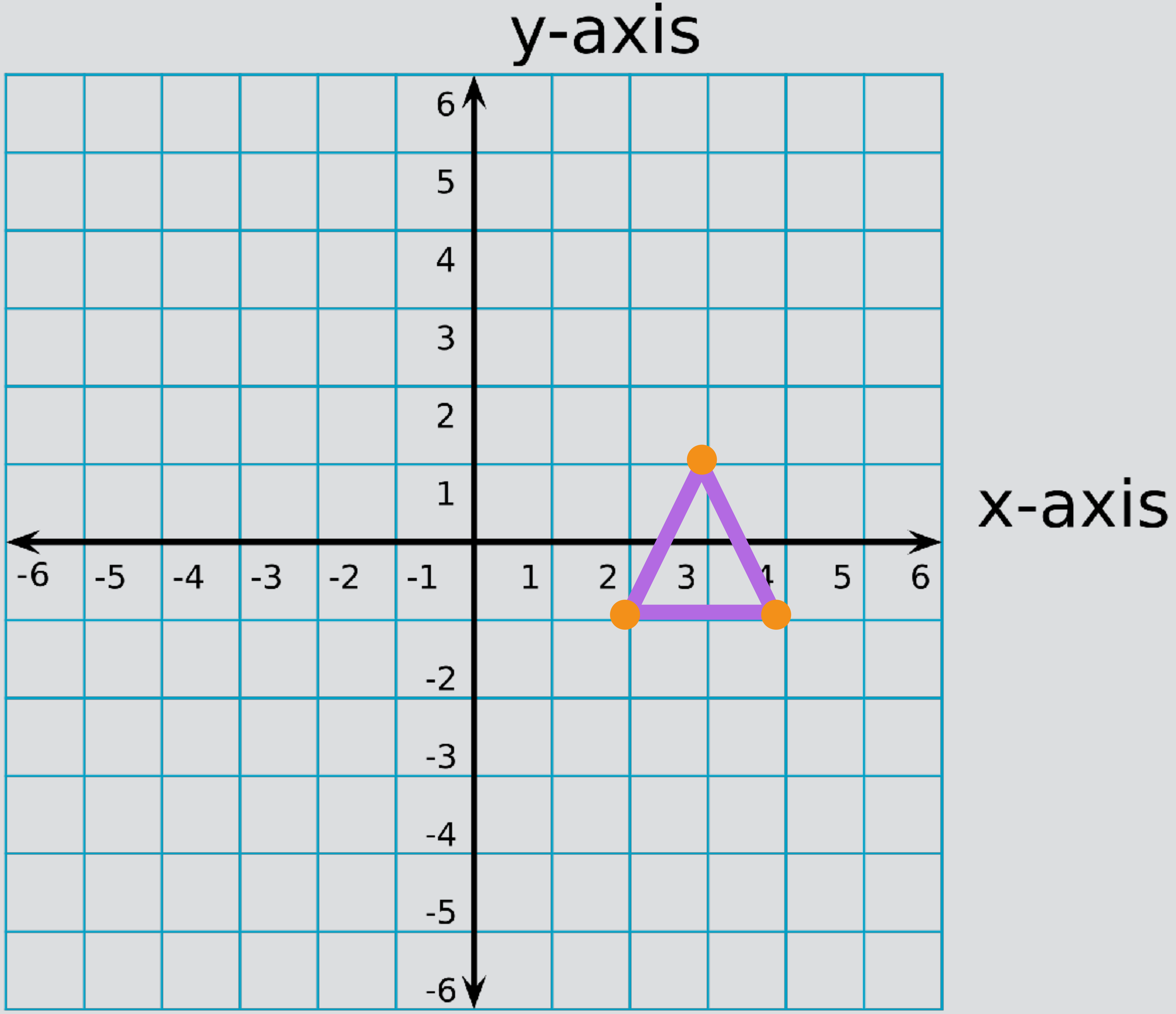
Linear part

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



Affine Translation

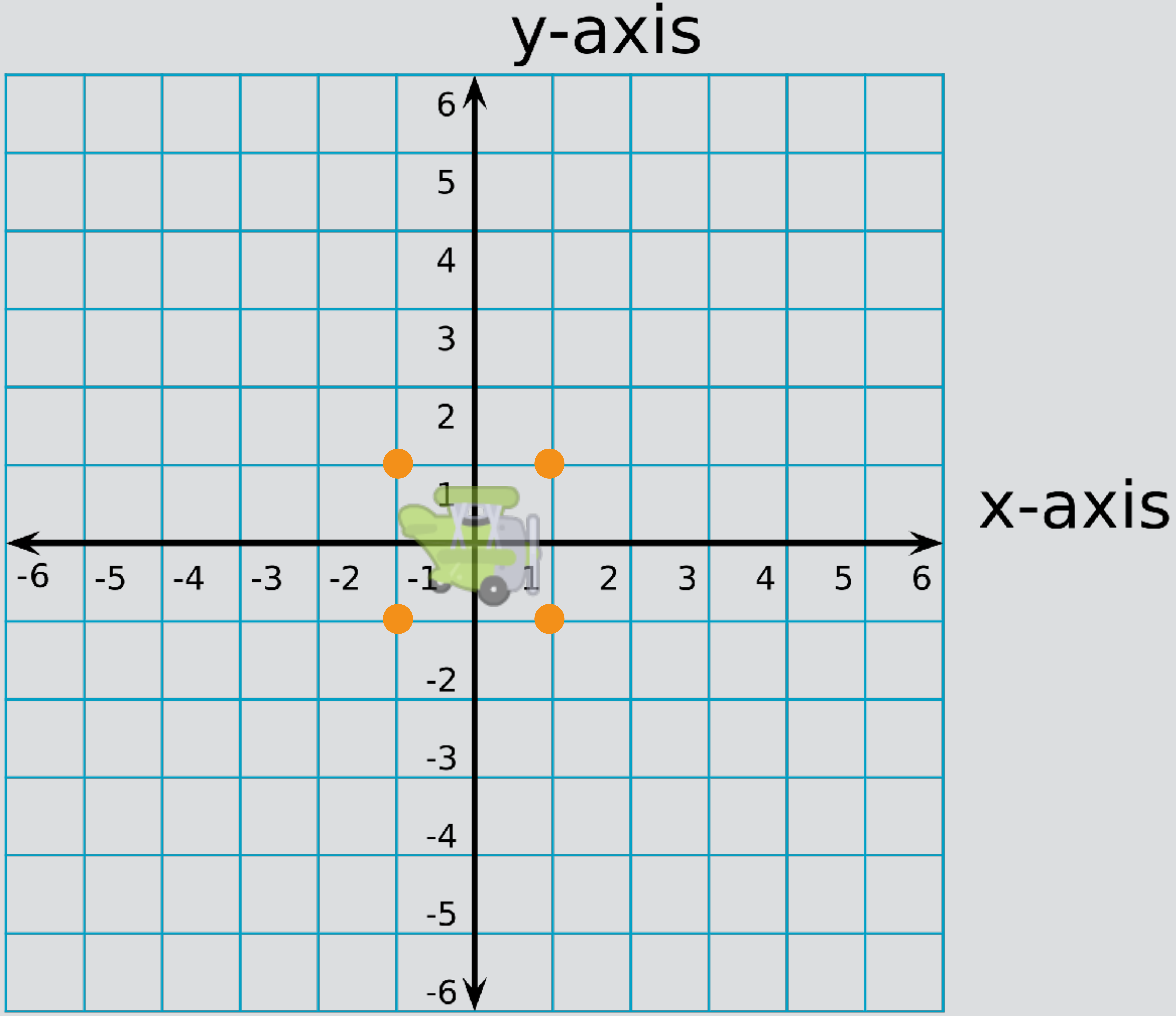
$$\begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



Multiplying transformation matrices

Matrix multiplication is non-commutative!

$$p = MR * MT * c$$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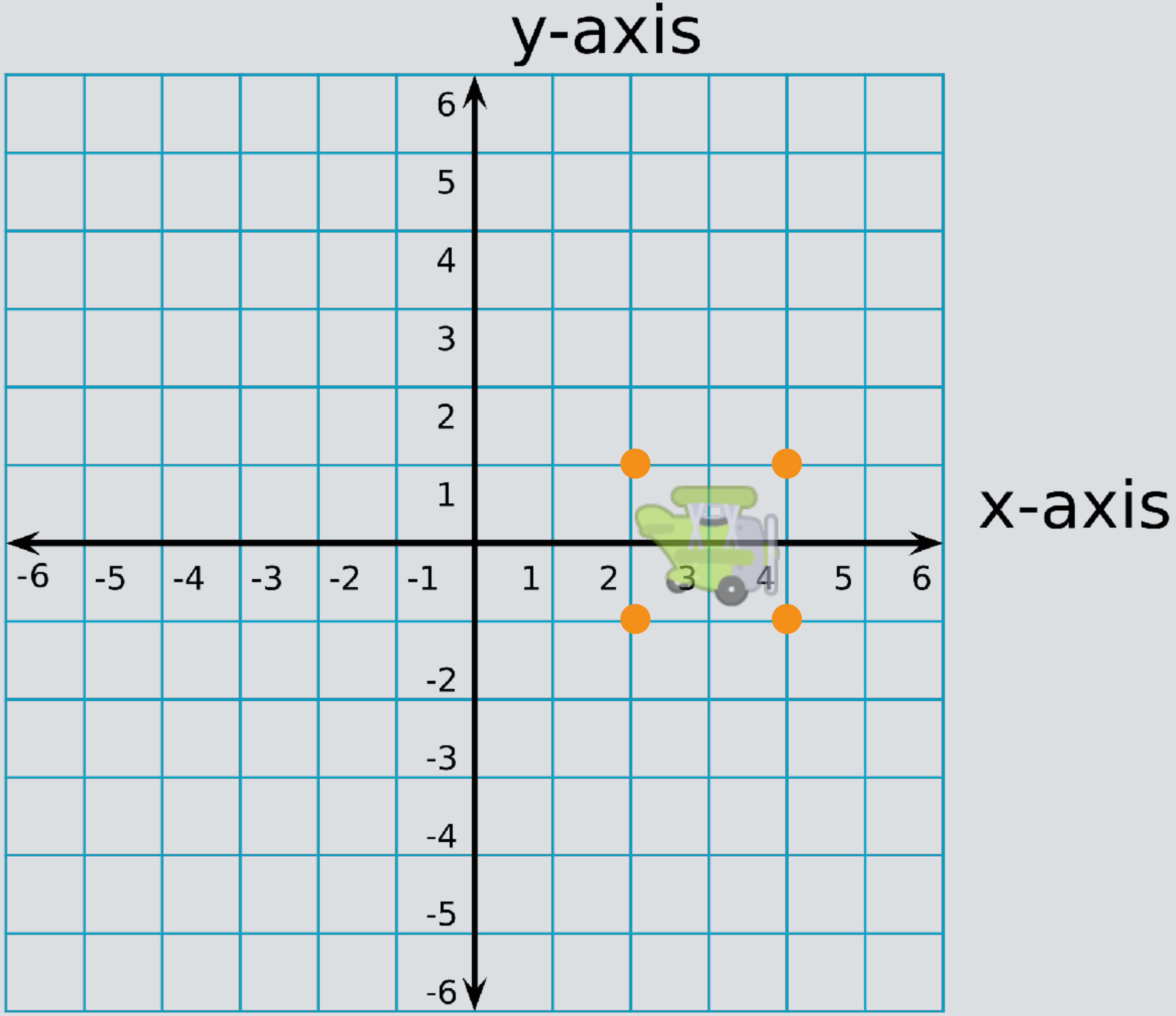
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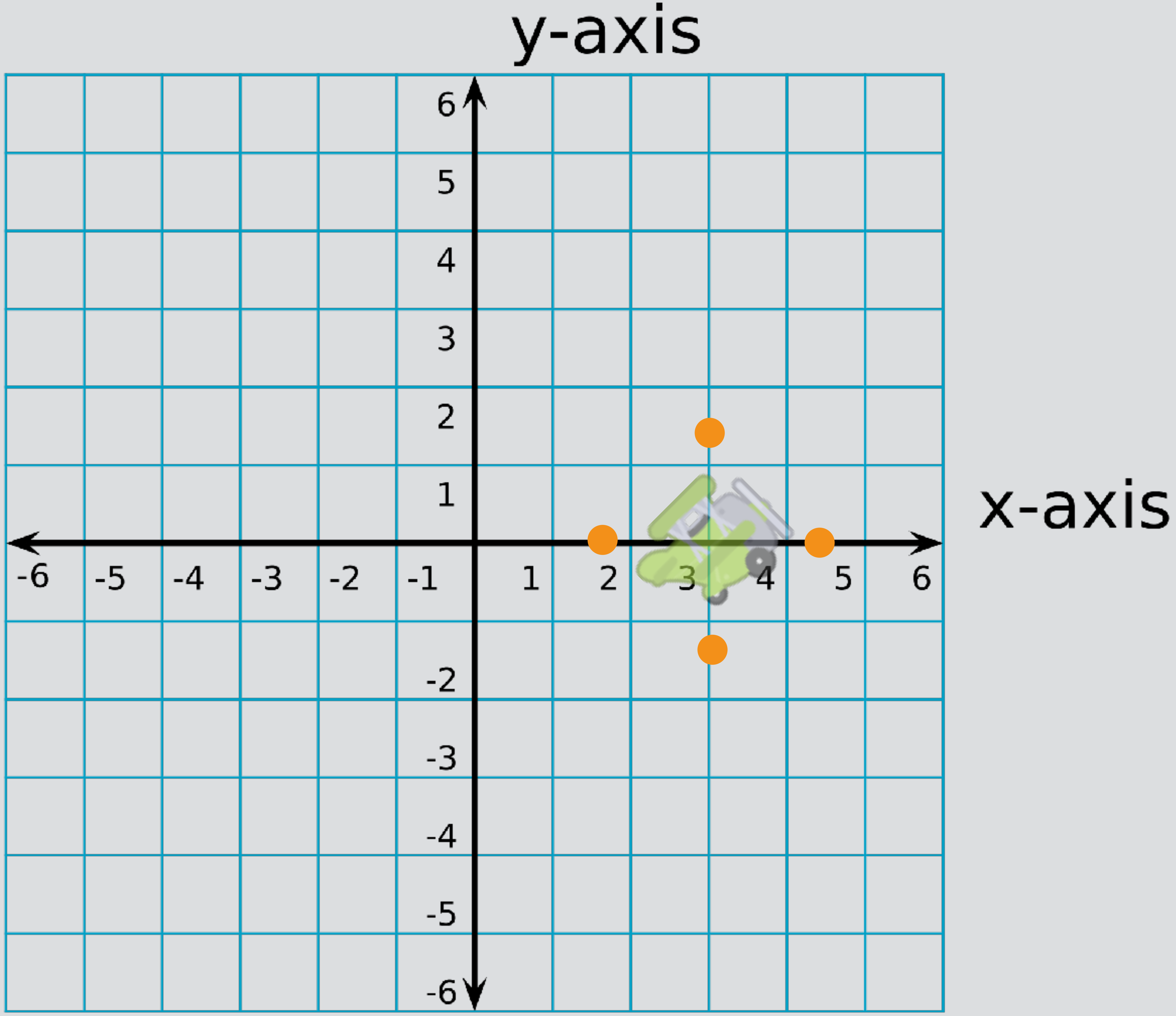
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```
p = MT * MR * c
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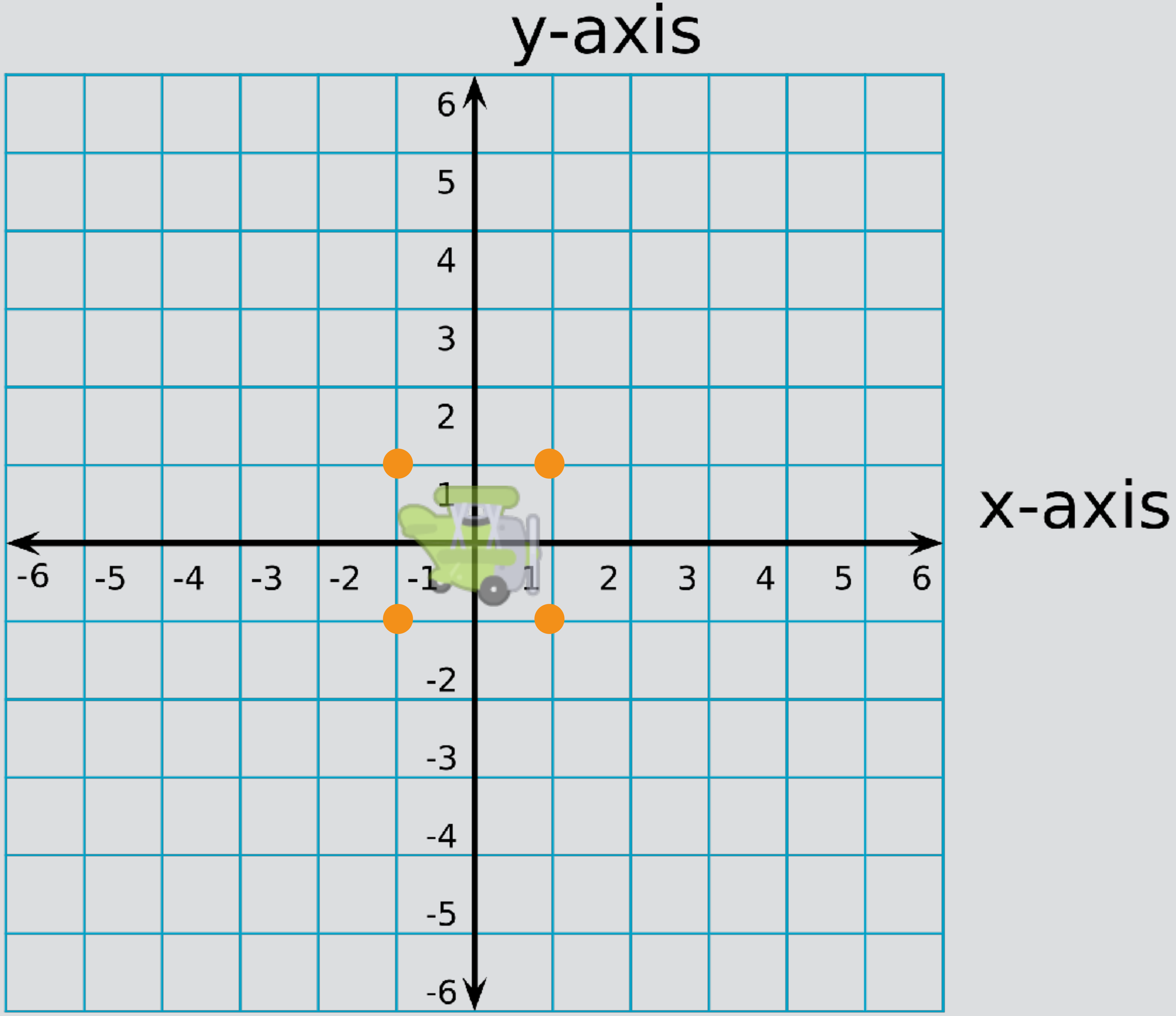
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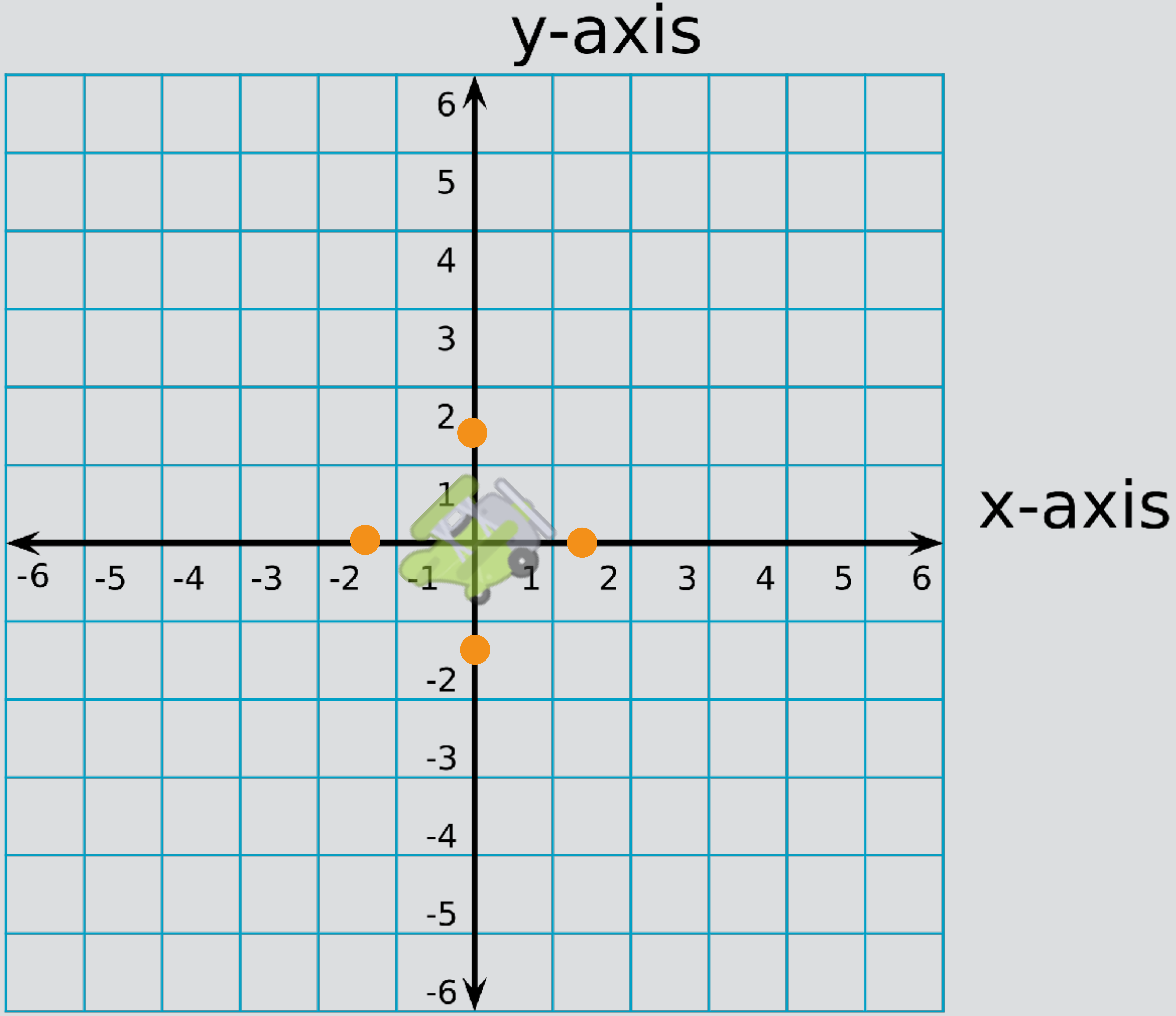
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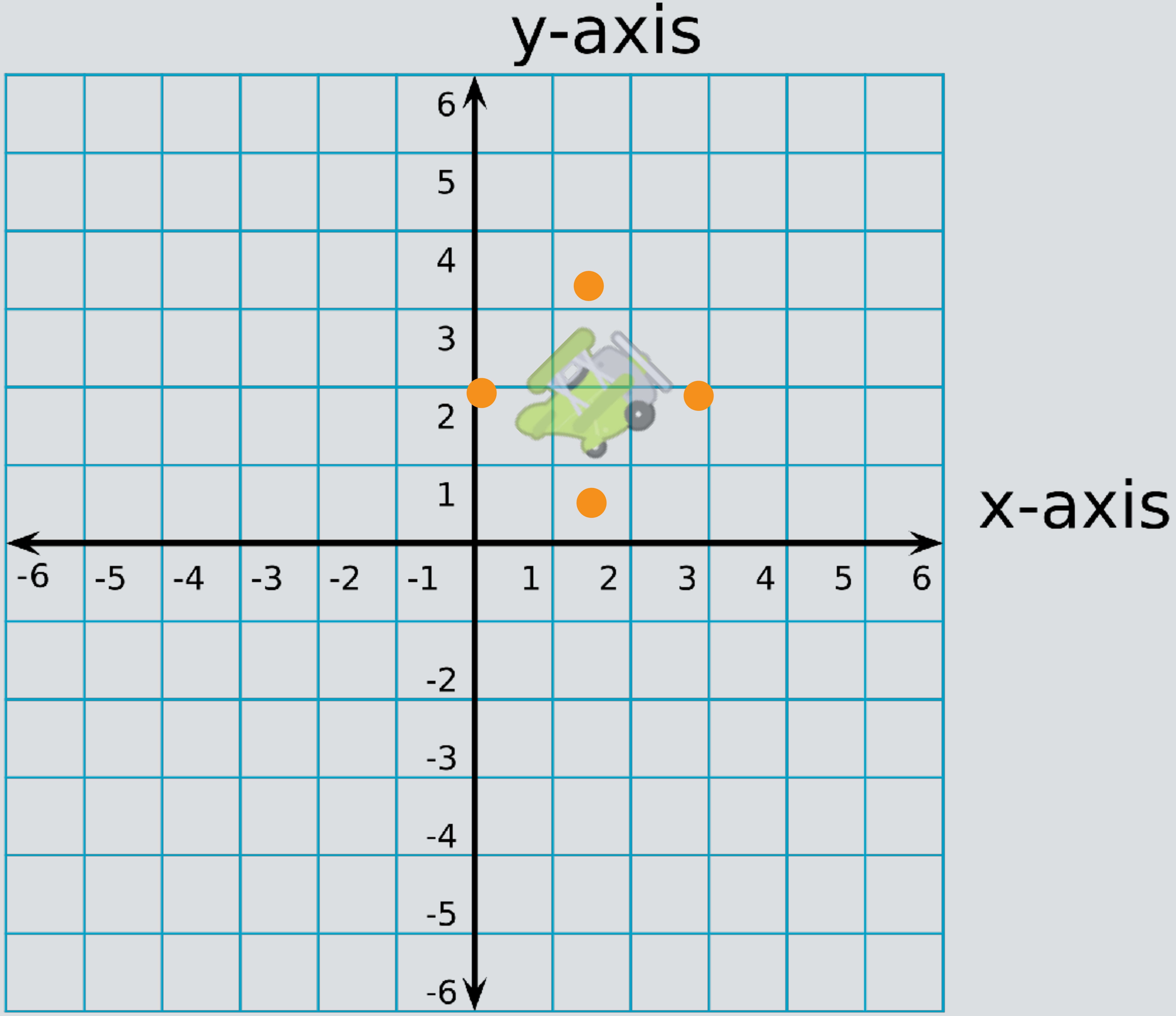
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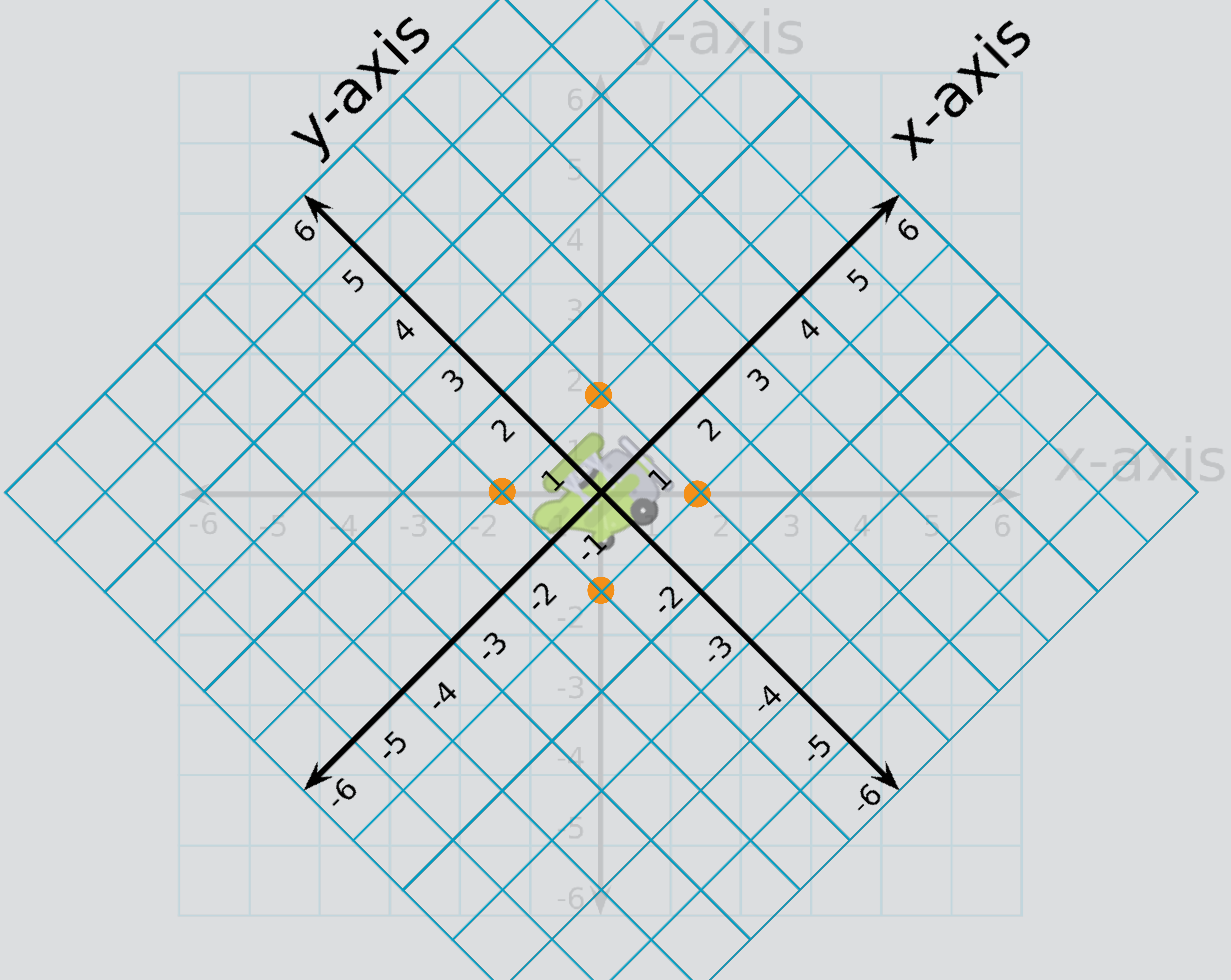

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