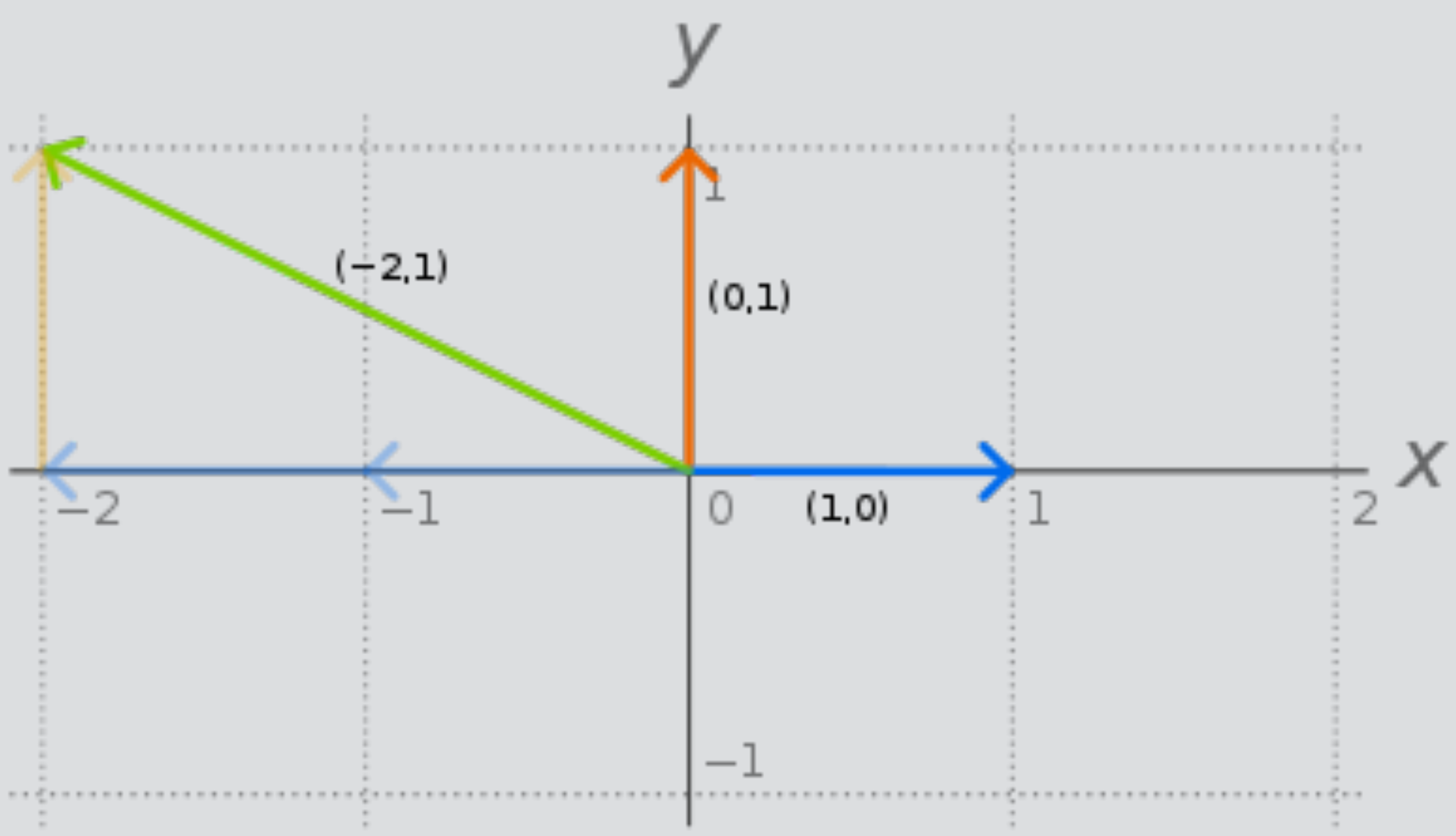
Linear Algebra Foundations Part 2



CS 3113

Multiplying transformation matrices

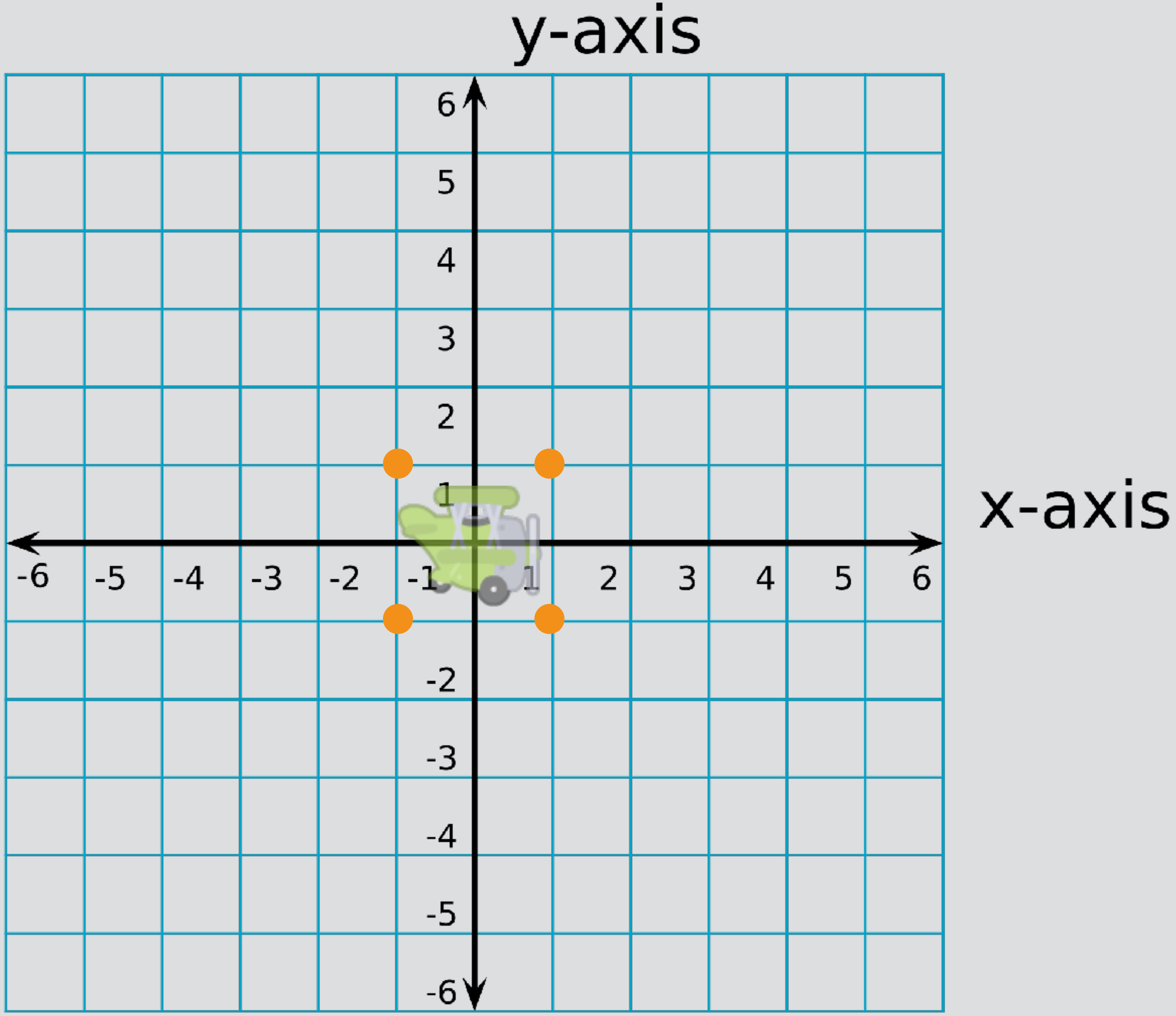
Multiplying transformation matrices combines their transformations.

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 5 \\ 0 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

FINAL MATRIX = SCALE MATRIX * ROTATE MATRIX * TRANSLATE MATRIX

Matrix multiplication is non-commutative!

$$p = MR * MT * c$$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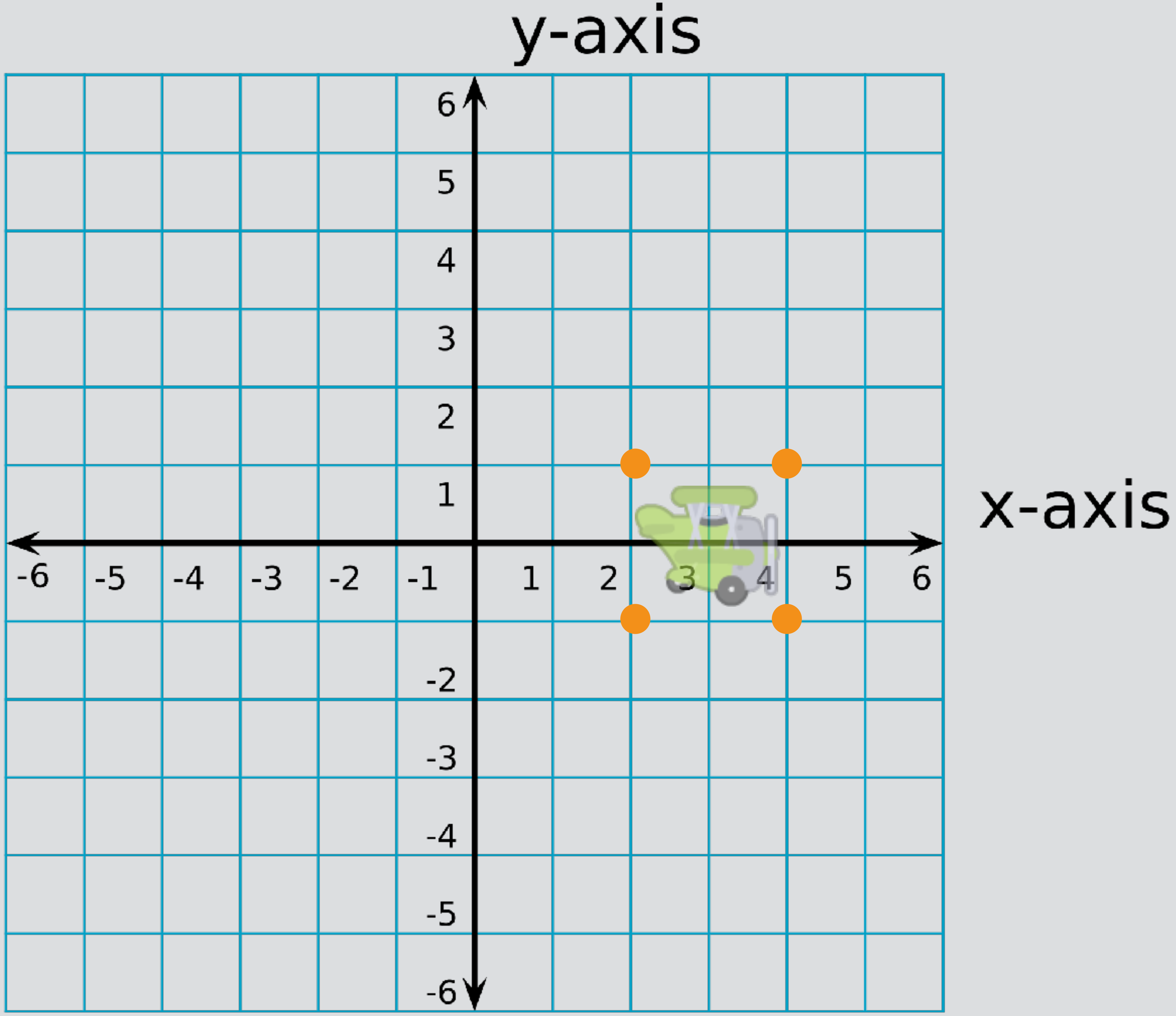
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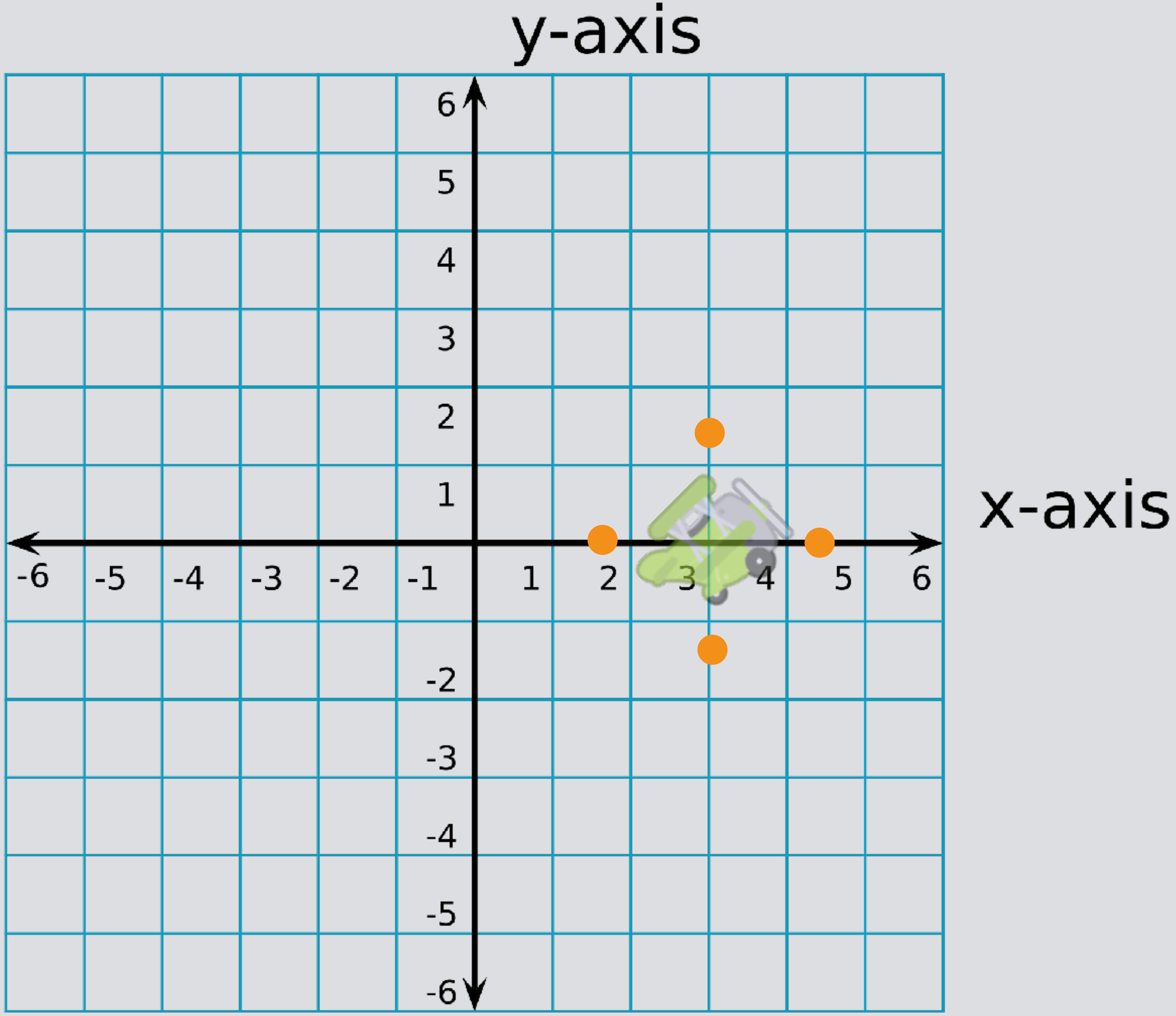
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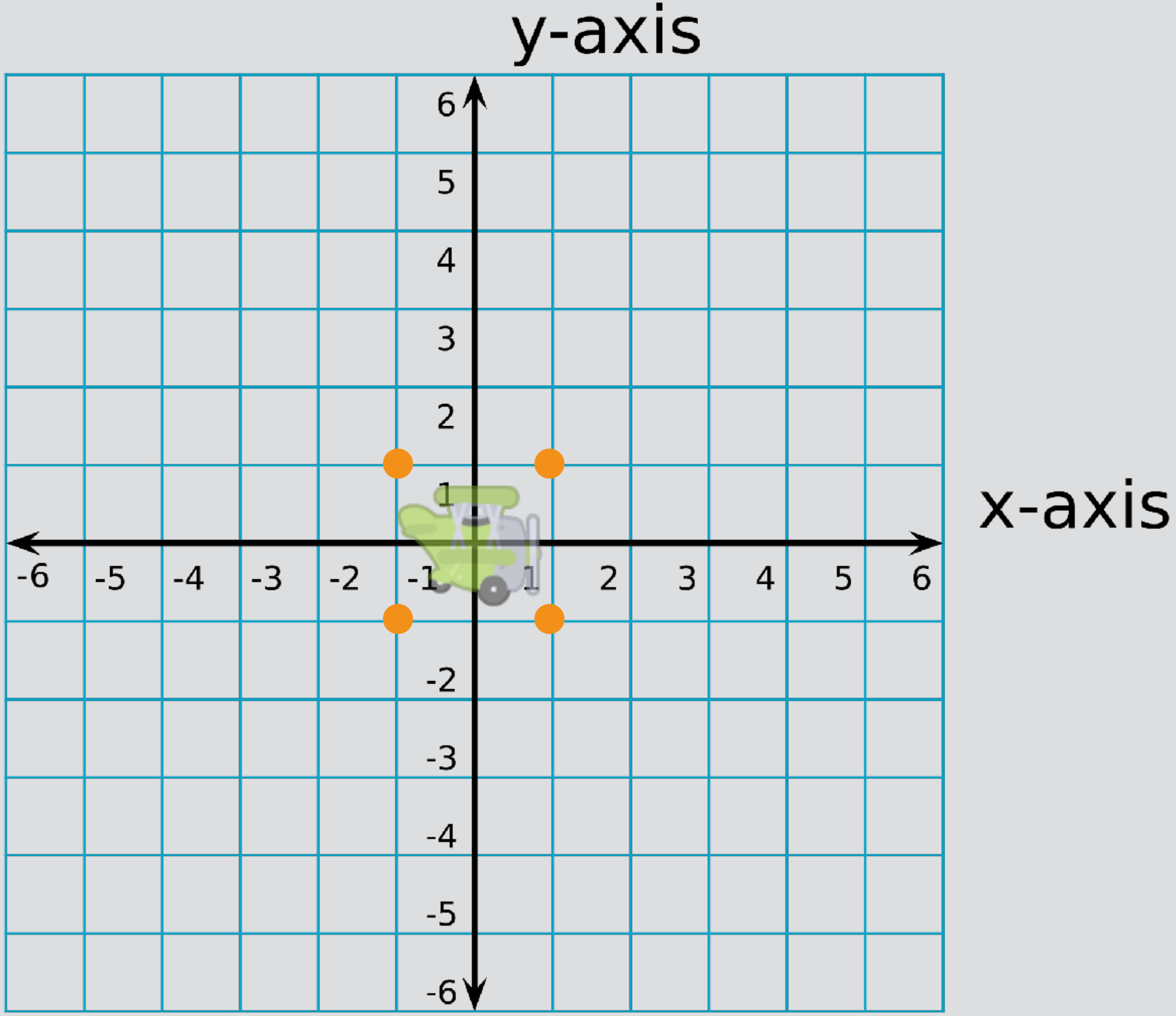
$$p = MT * MR * c$$



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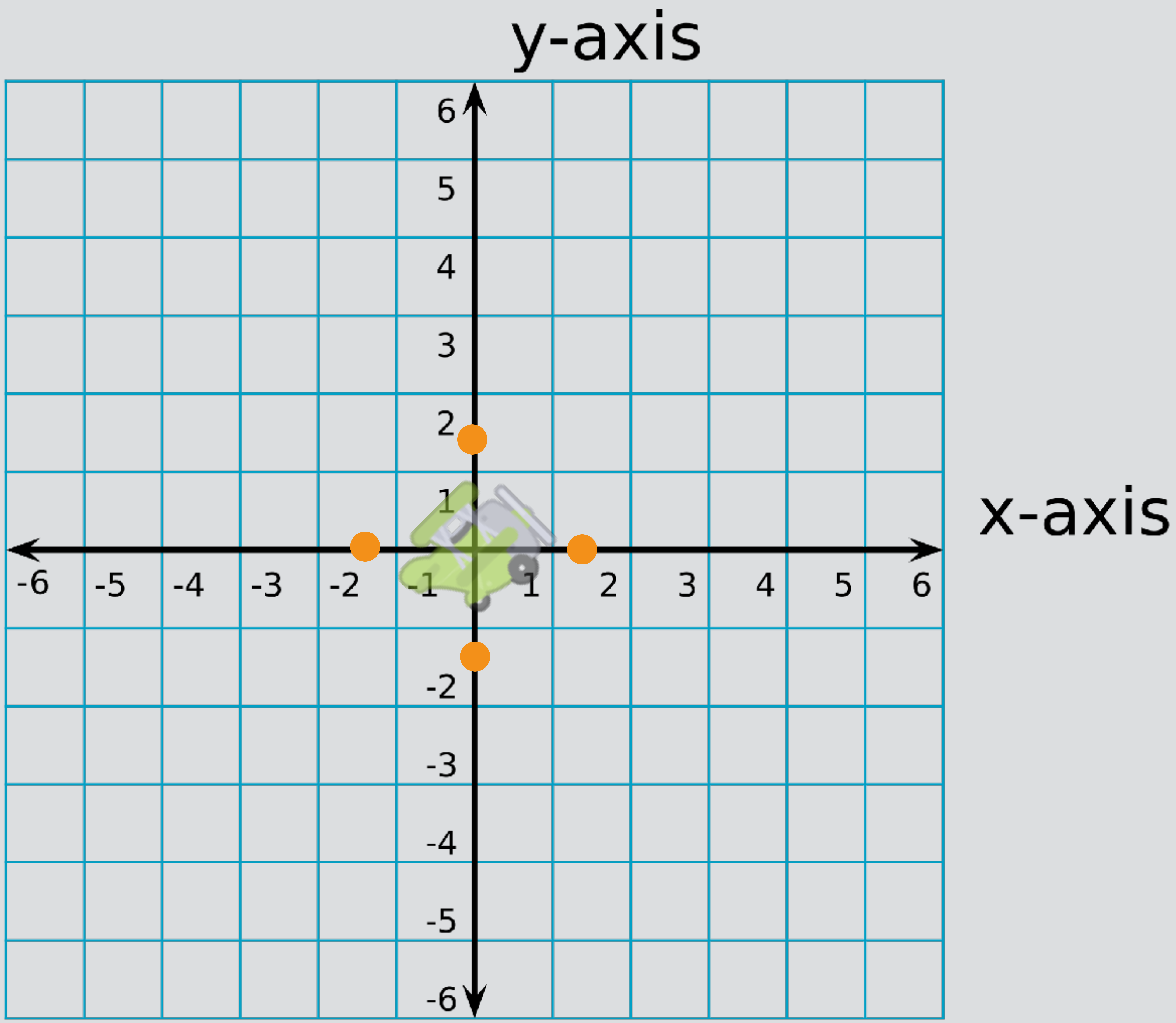
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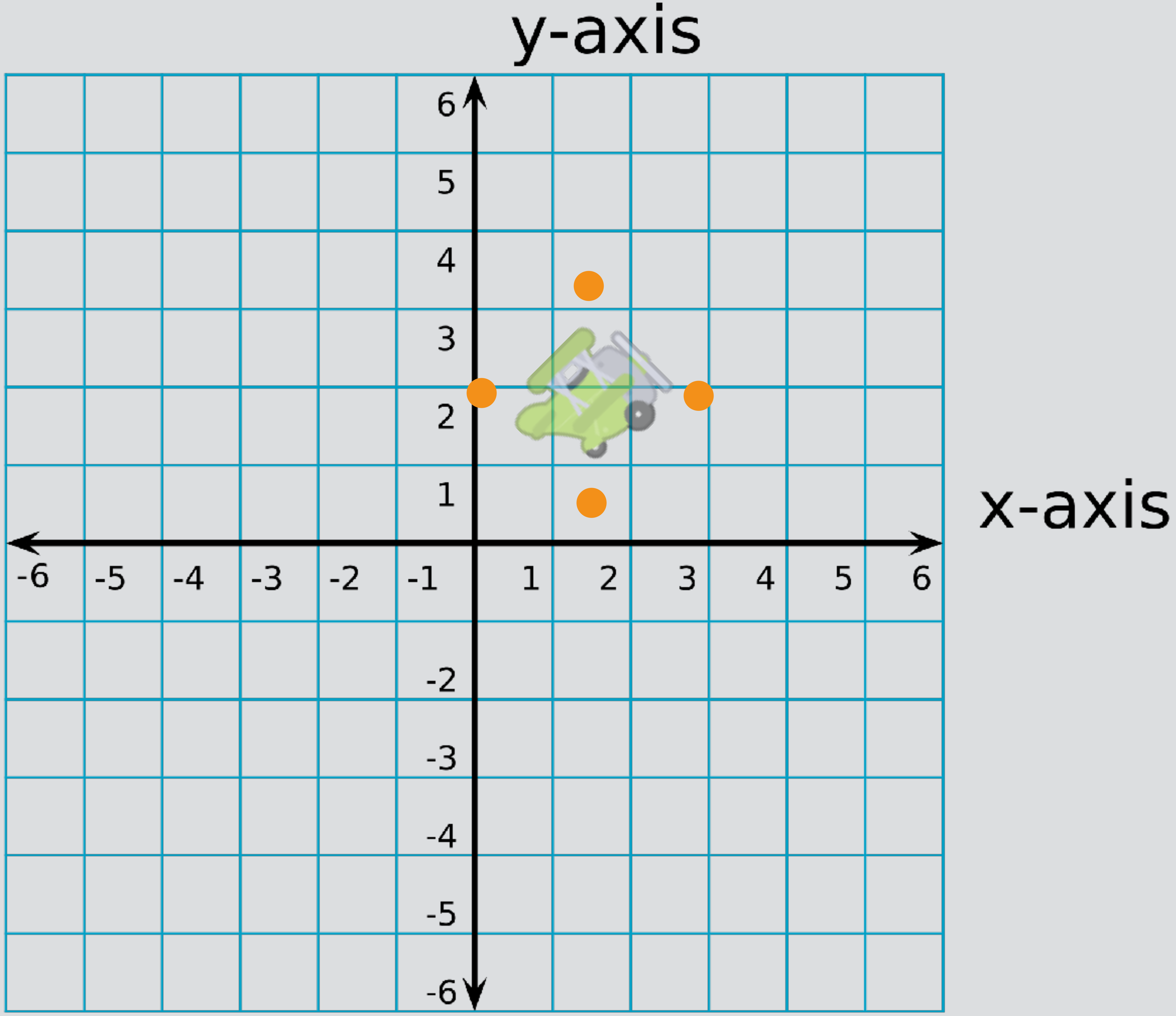
y-axis
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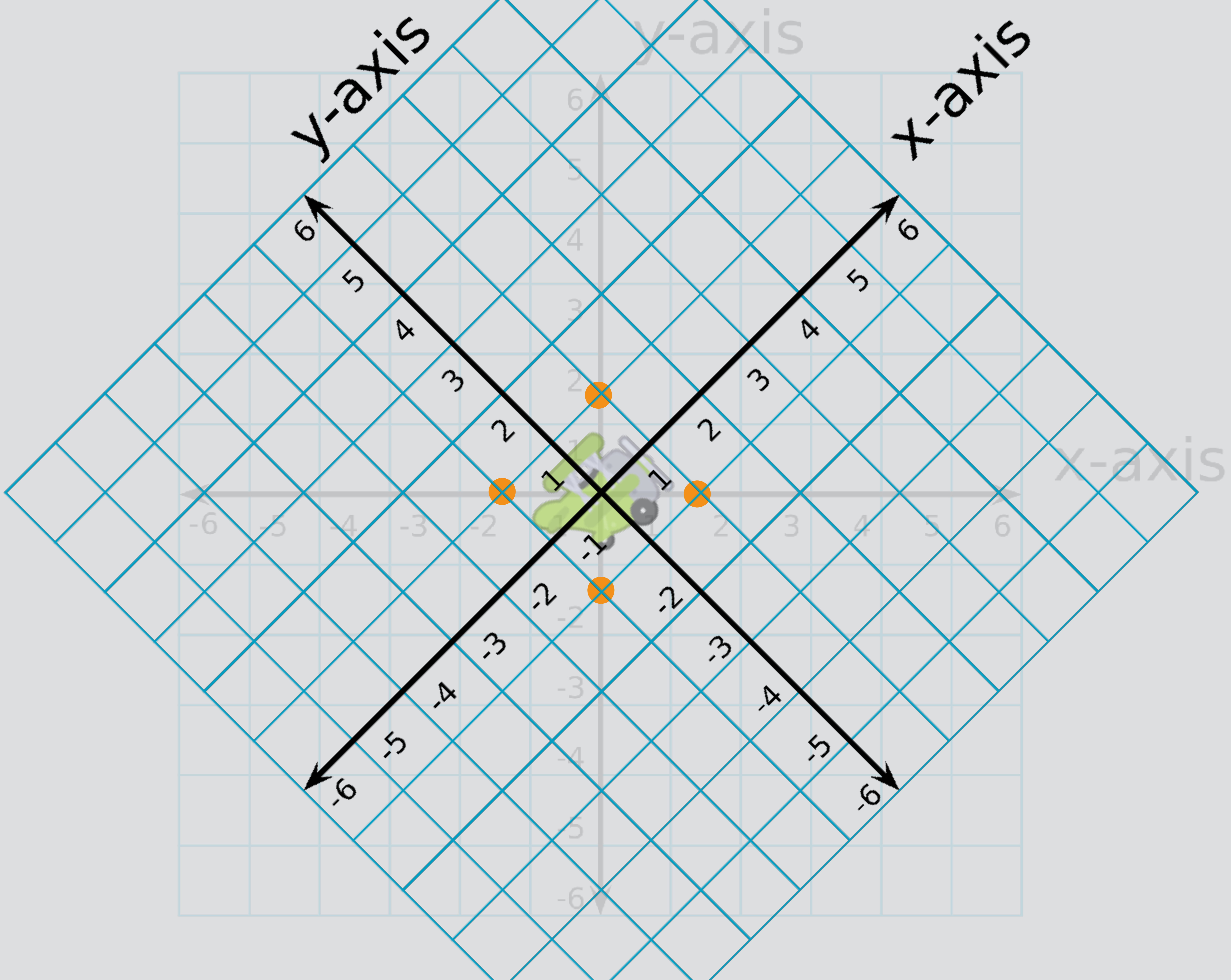


y-axis
x-axis

```
p = MT * MS * c
```

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 5 \\ 0 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 5 \\ 0 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 11 \\ 12 \\ 1 \end{bmatrix}$$

$$p = MS * MT * c$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 10 \\ 0 & 4 & 16 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 10 \\ 0 & 4 & 16 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 16 \\ 24 \\ 1 \end{bmatrix}$$

FINAL MATRIX = SCALE MATRIX * ROTATE MATRIX * TRANSLATE MATRIX

Reference frames

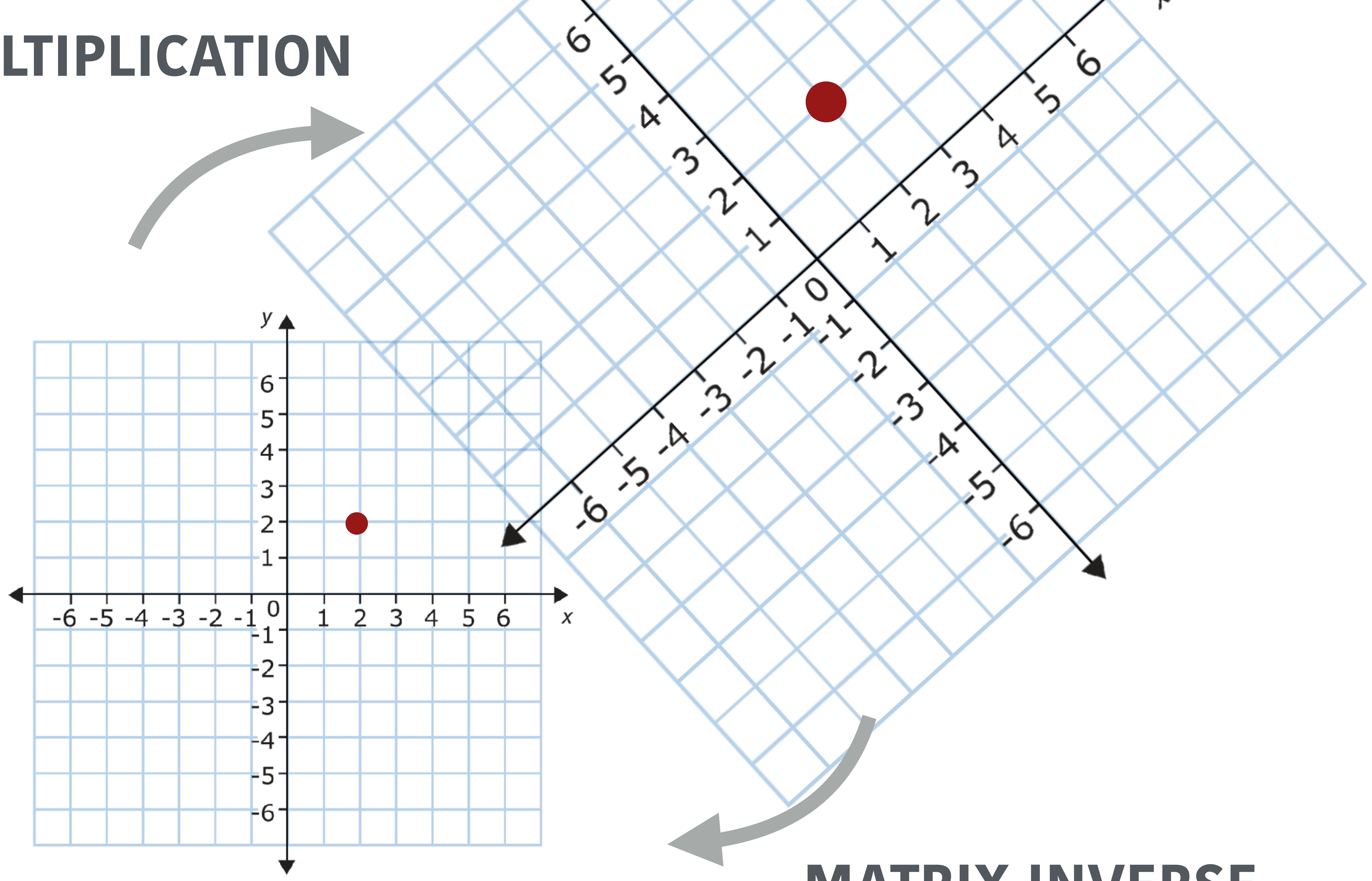
Inverse of a matrix.

Inverse of a matrix.

=

A matrix that “undoes” the transformation of the original matrix.

MATRIX MULTIPLICATION



MATRIX INVERSE

Inverse of a matrix.

- Scaled by 1/scale (determinant of the matrix)

Inverse of a matrix.

- Scaled by 1/scale (determinant of the matrix)

- Rotated by the transpose of the linear part of the matrix.

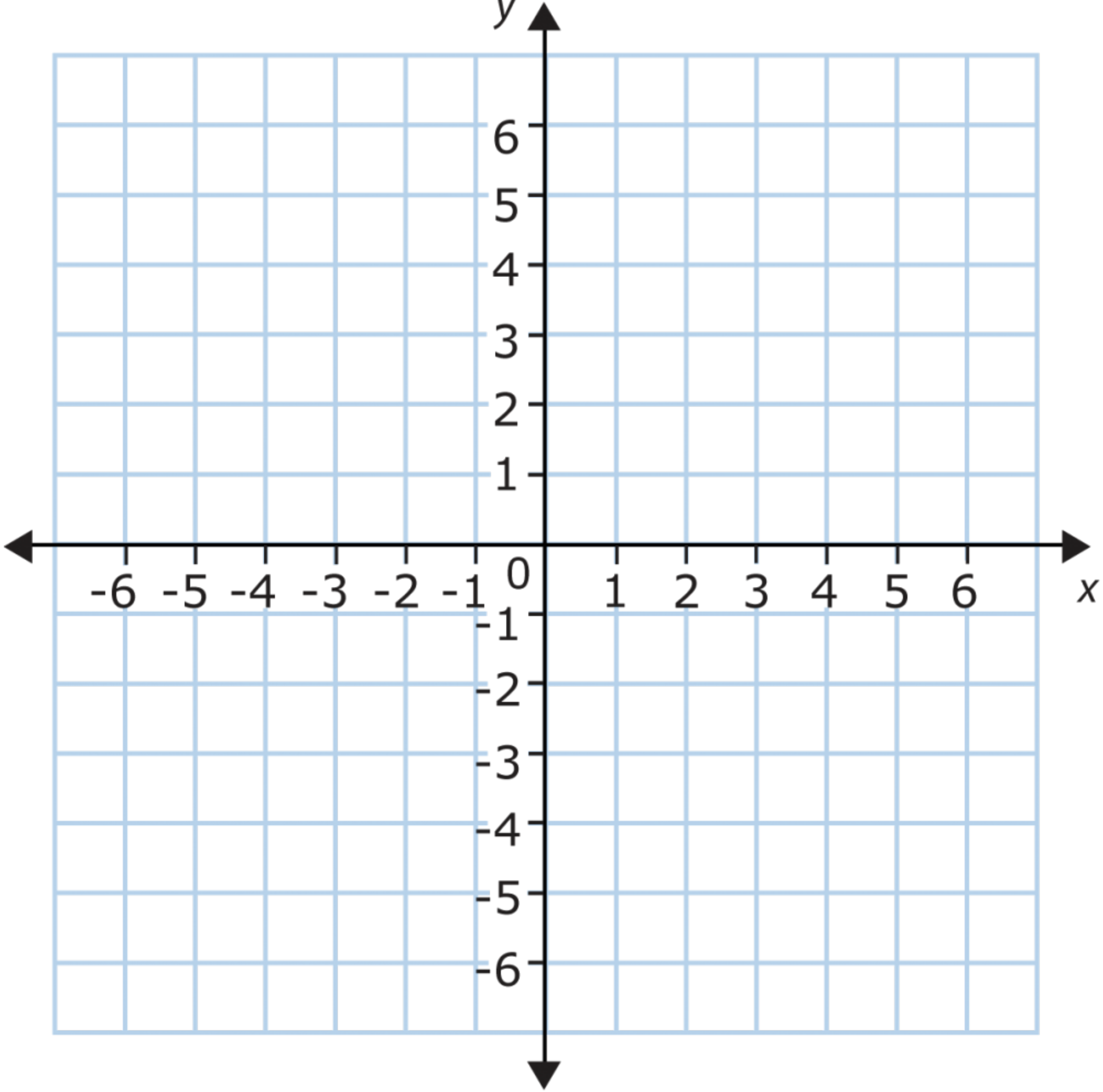
$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse of a matrix.

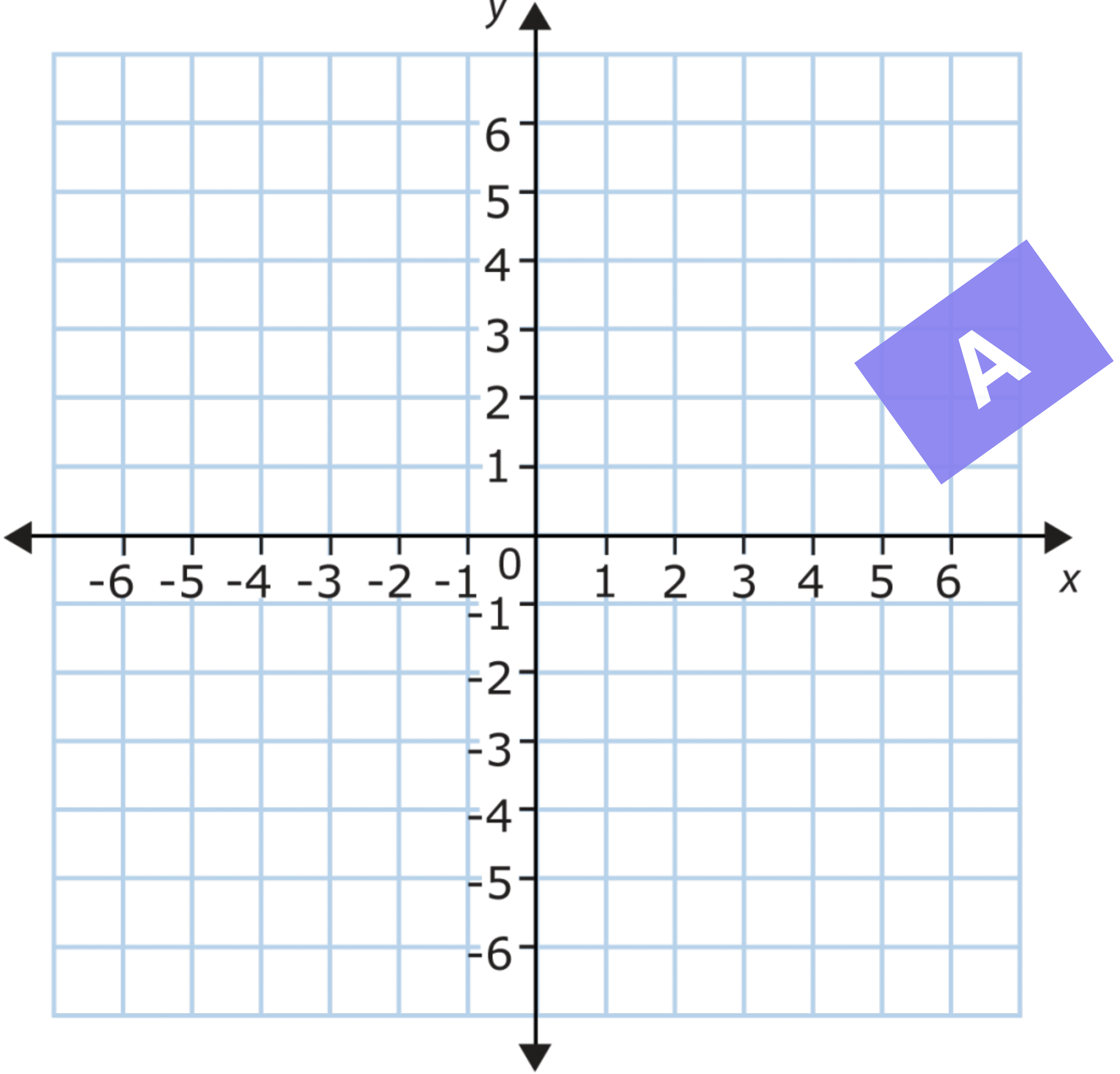
- Scaled by 1/scale (determinant of the matrix)
- Rotated by the transpose of the rotation.
- Translated by the translation * -1

Transforming between coordinate spaces.

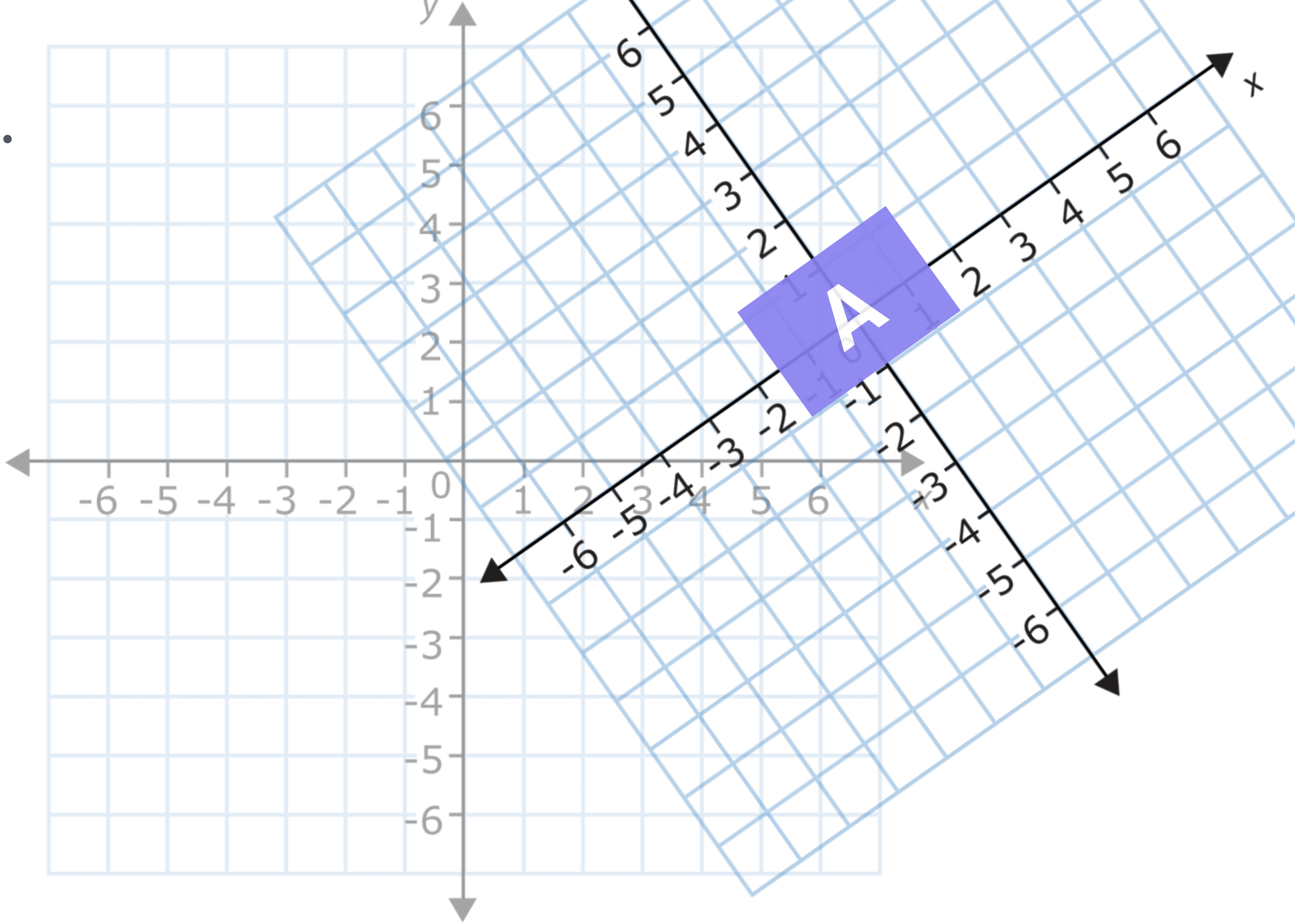
World space.



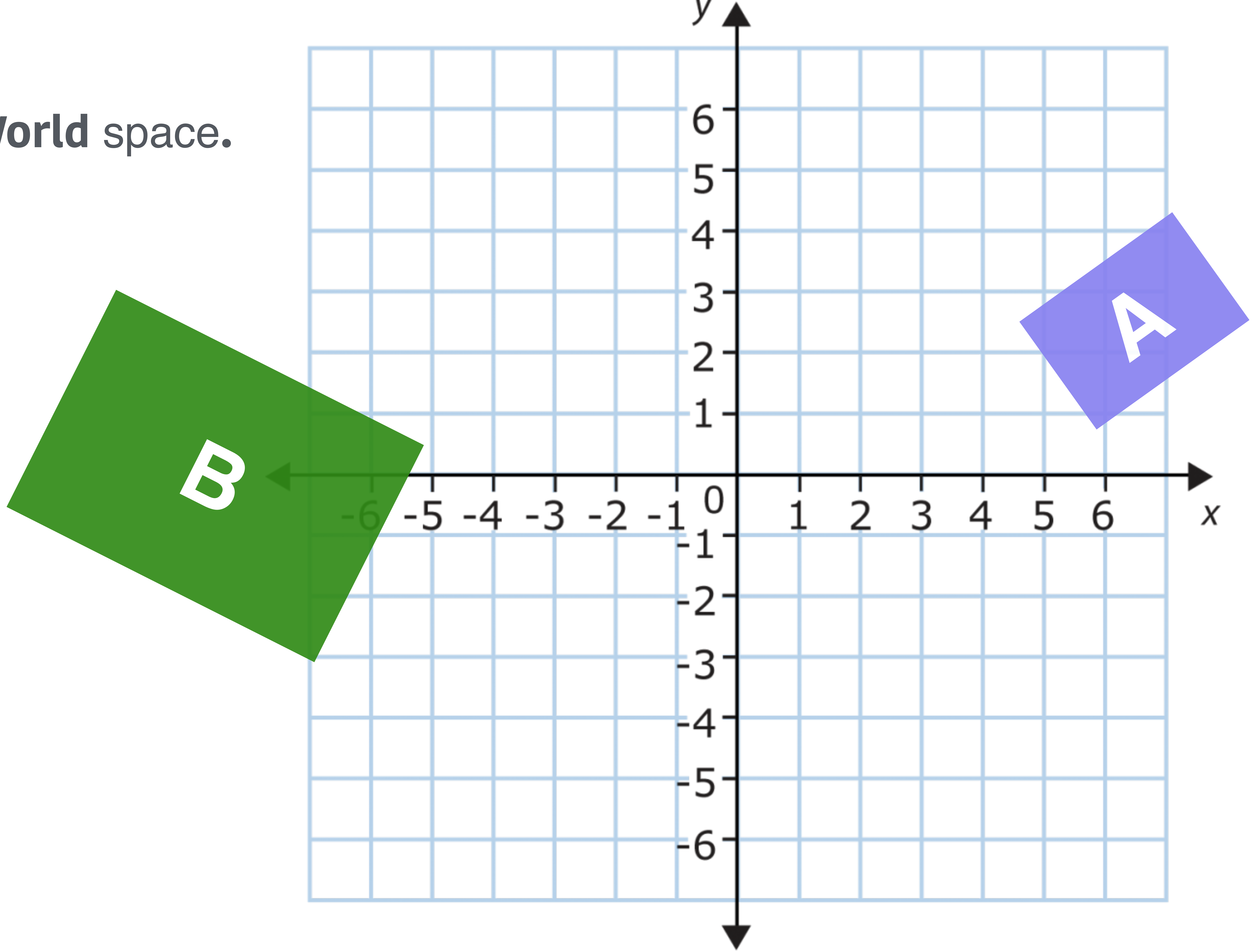
World space.

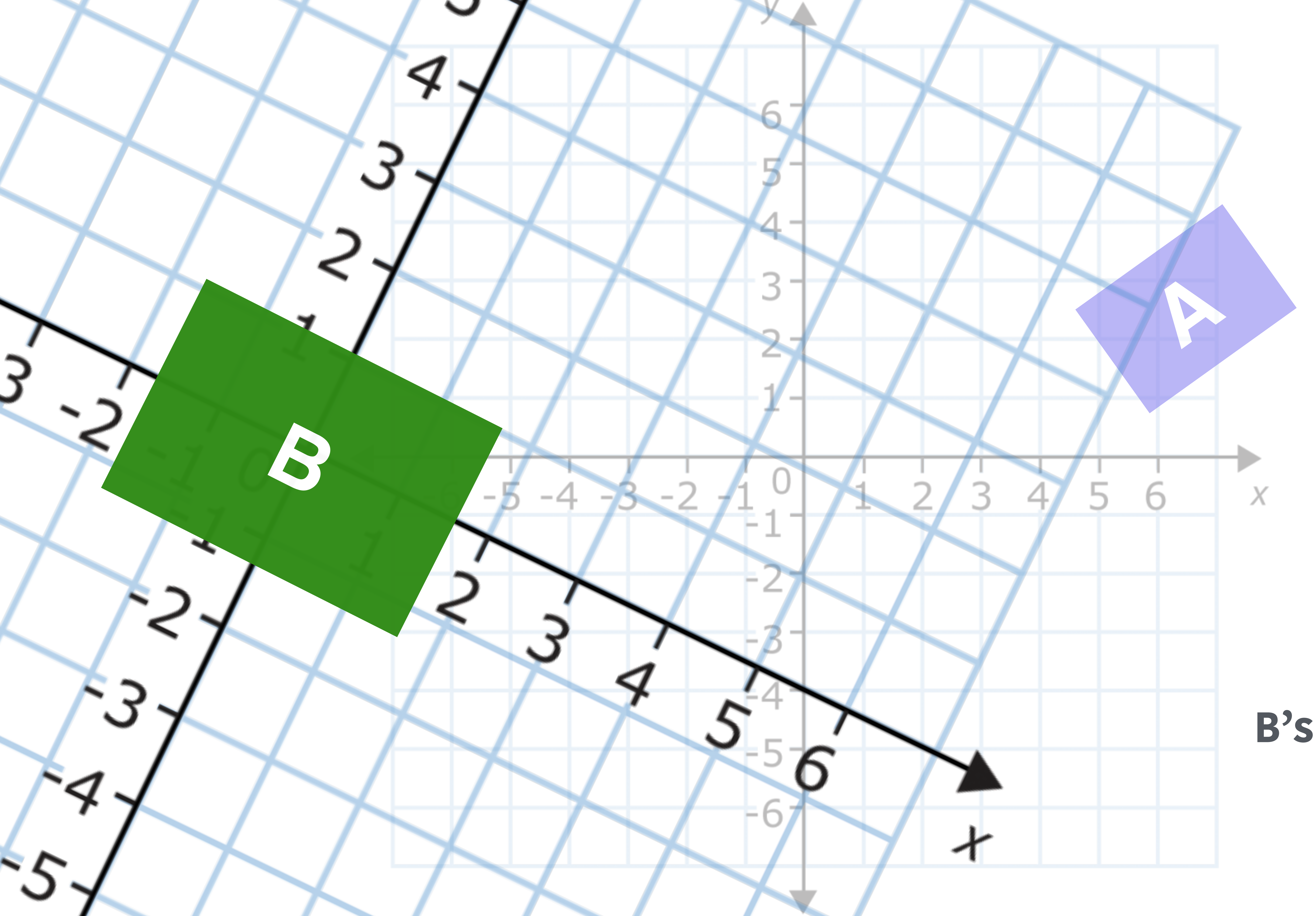


A’s object space.



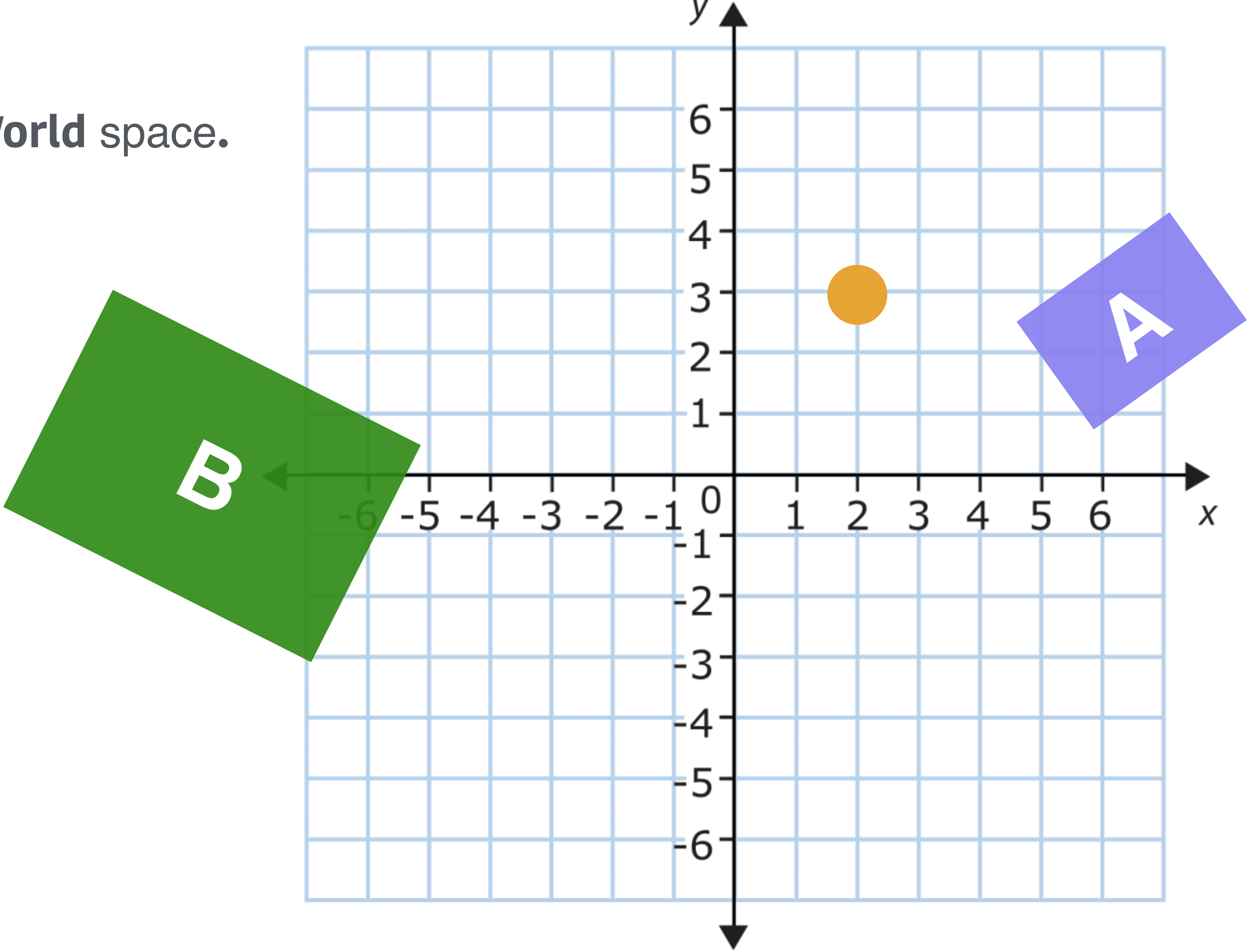
World space.



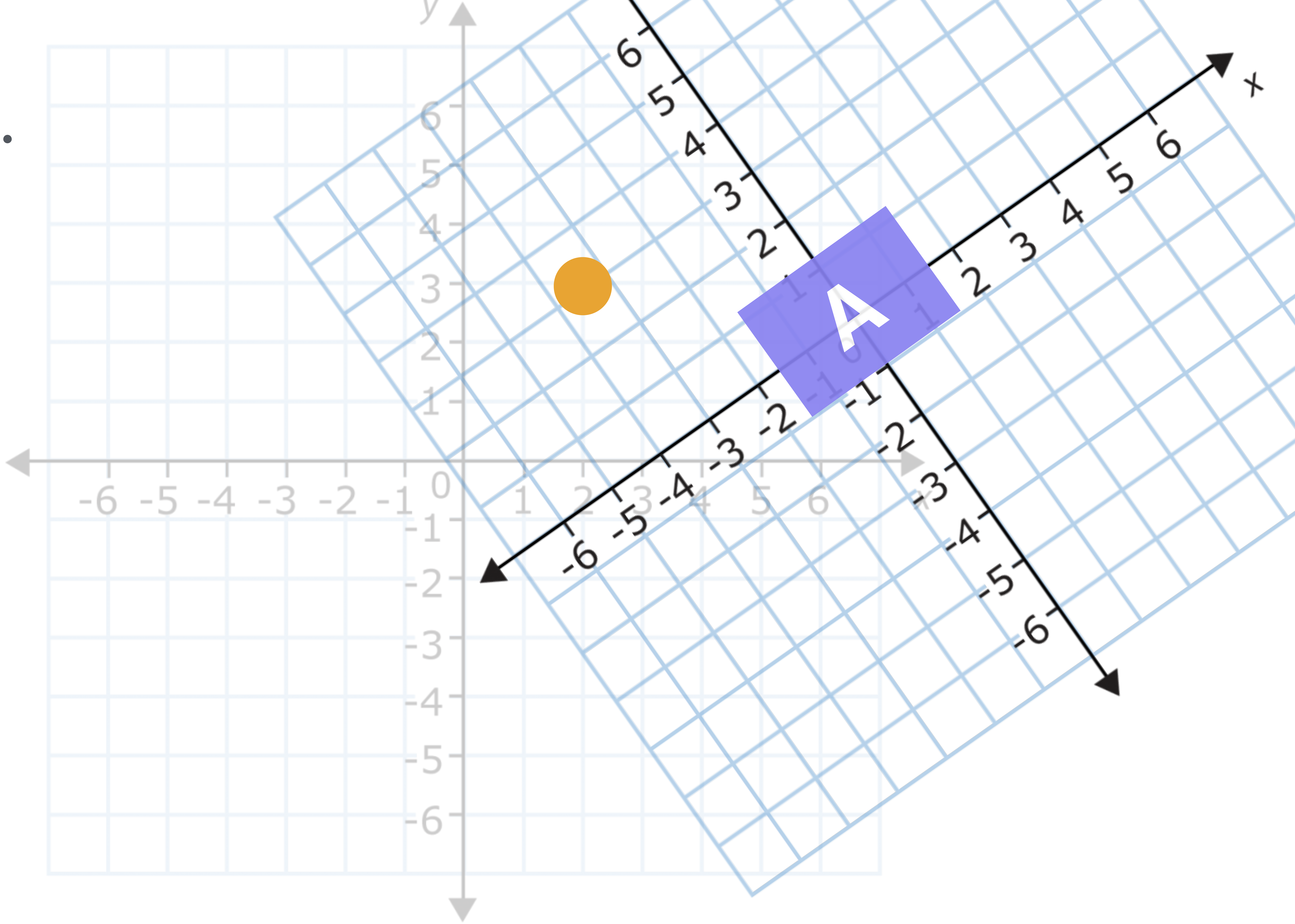


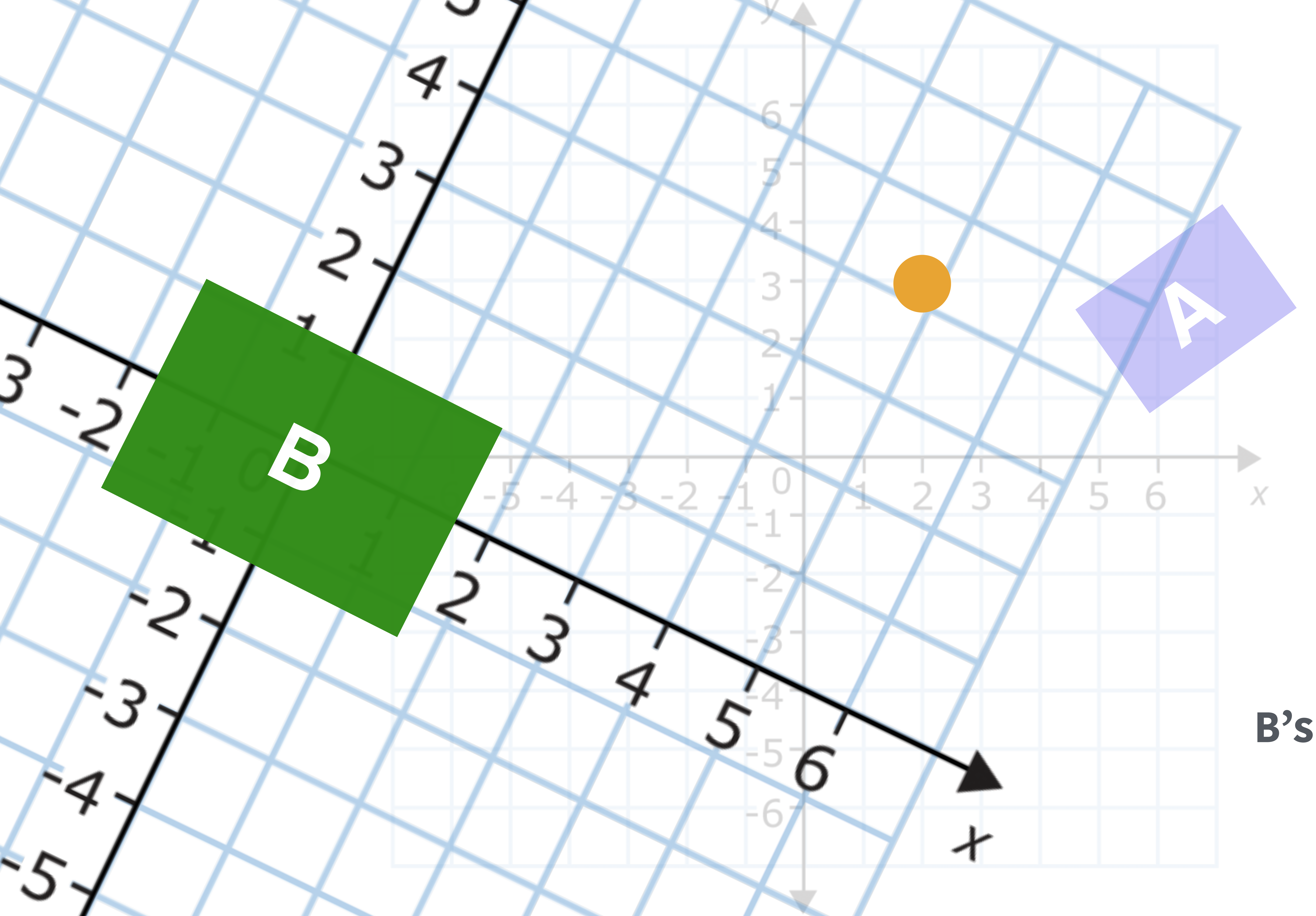
B’s object space.

World space.

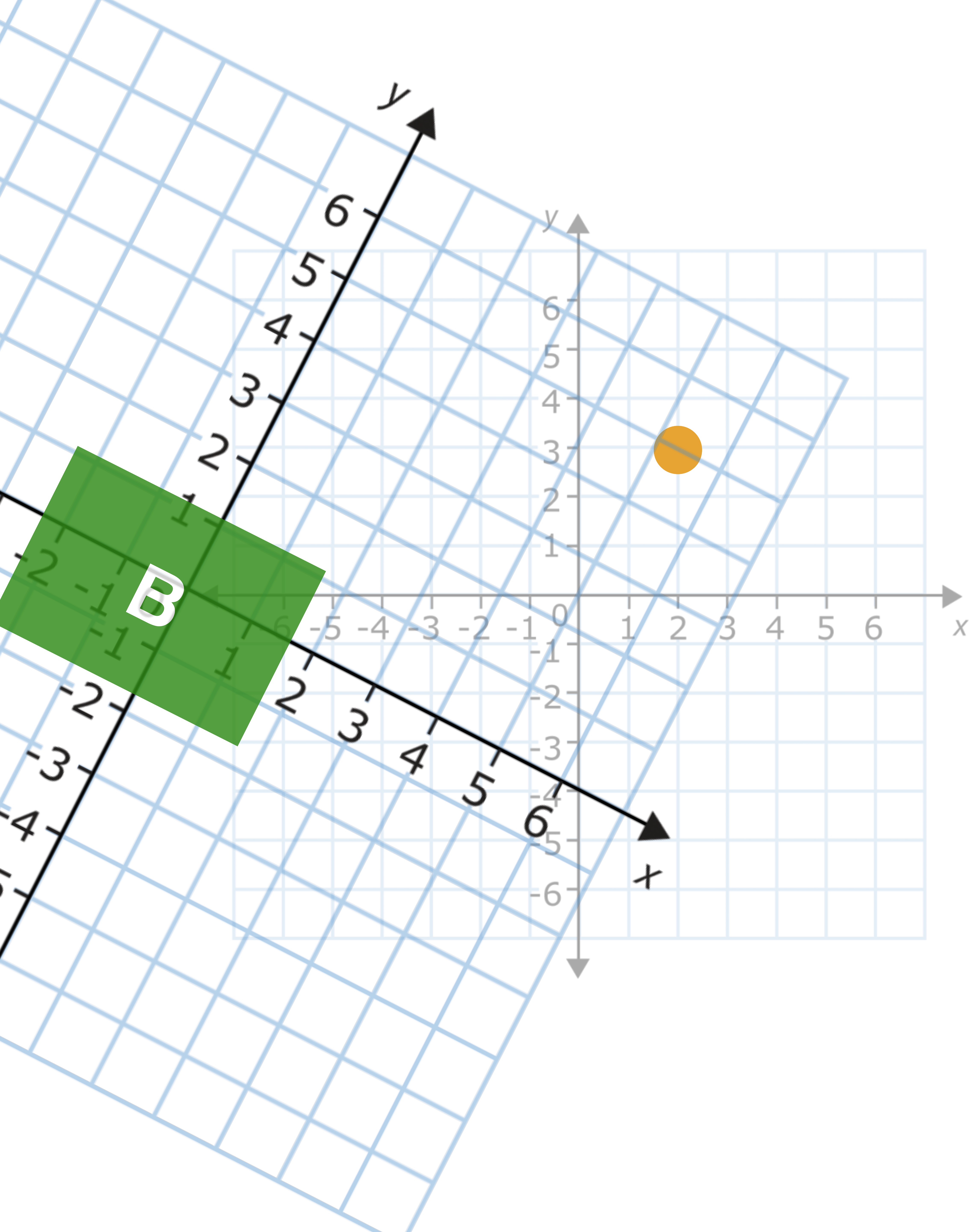


A’s object space.

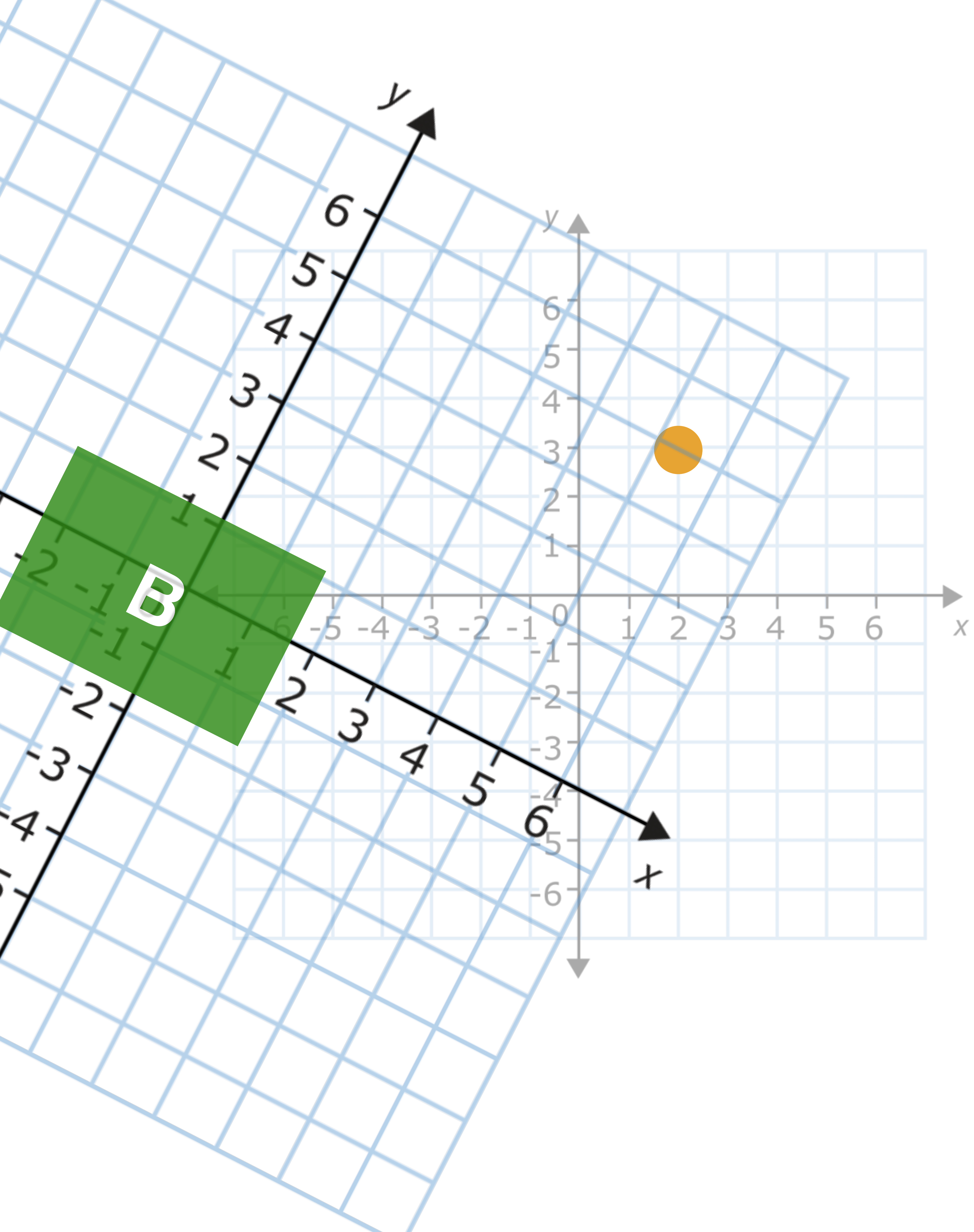




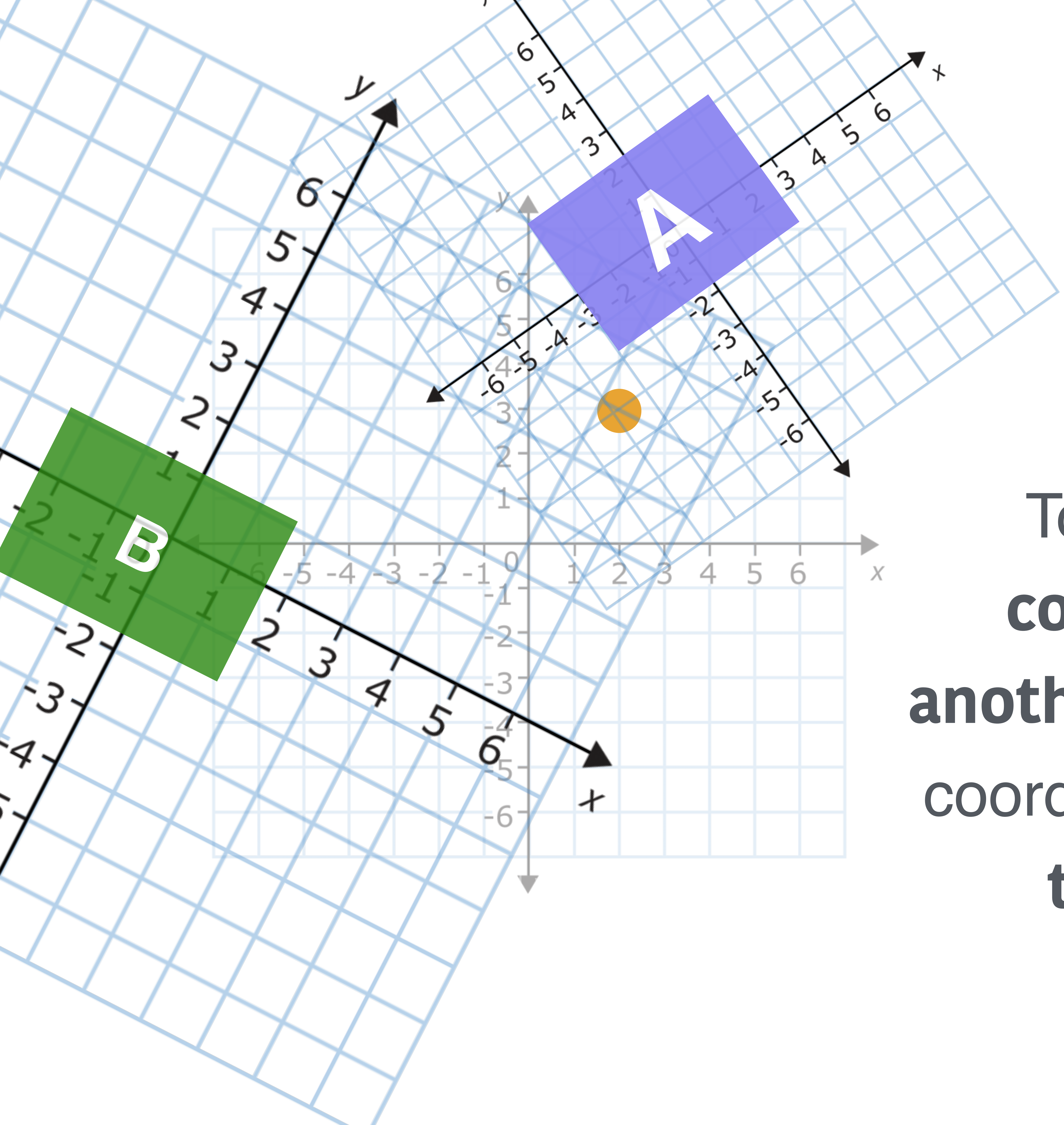
B's object space.



To express an **object space** coordinate vector in terms of **world space**, multiply the vector by that object's **transform matrix**.

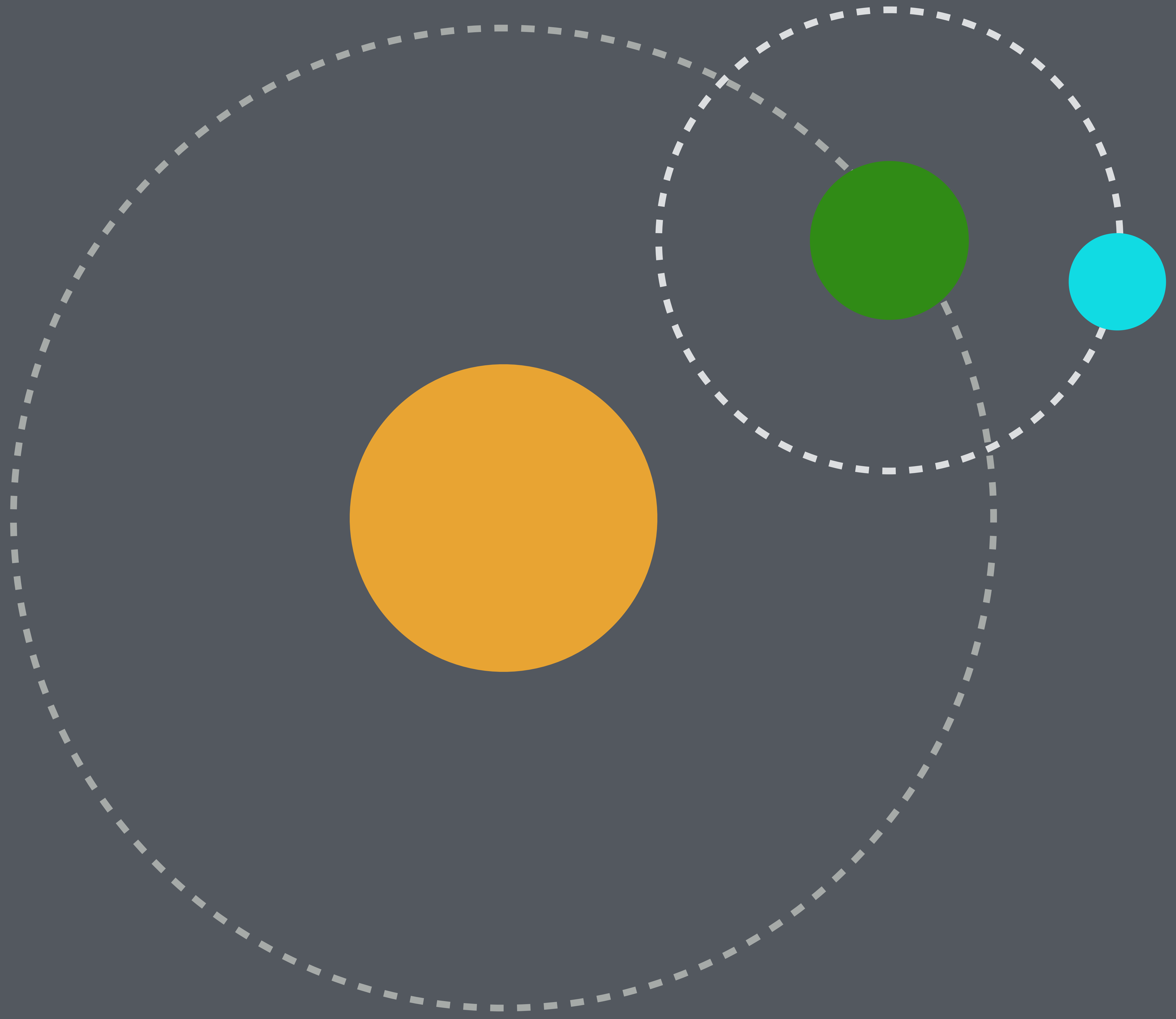


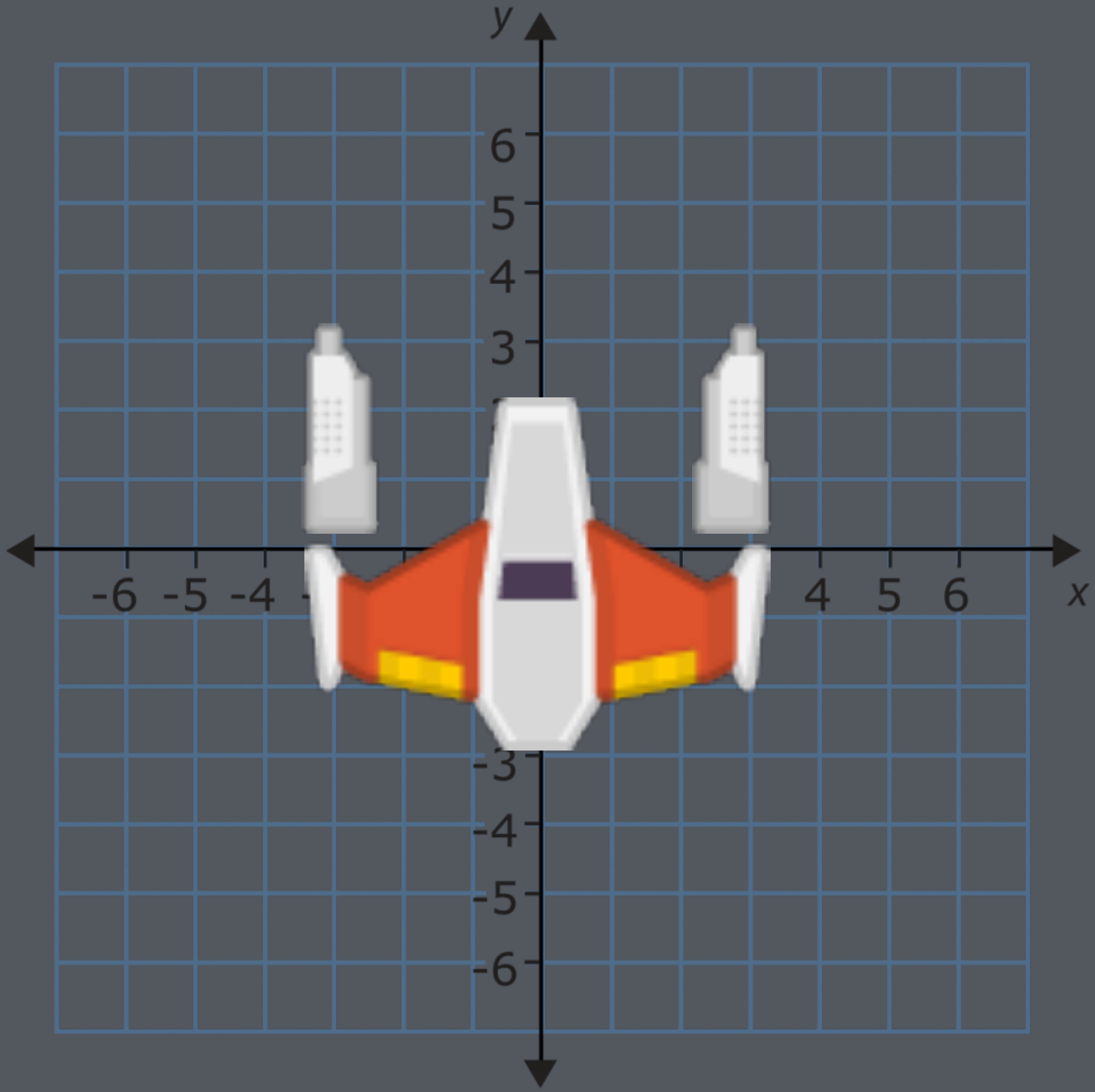
To express a **world space** coordinate vector in terms of an entity’s **object space**, multiply the vector by the **inverse** of that object’s transform matrix.



To express an **object space coordinate** vector in terms of **another object's space**, convert the coordinate **to world space**, then **to the other object's space**.

Frame hierarchies

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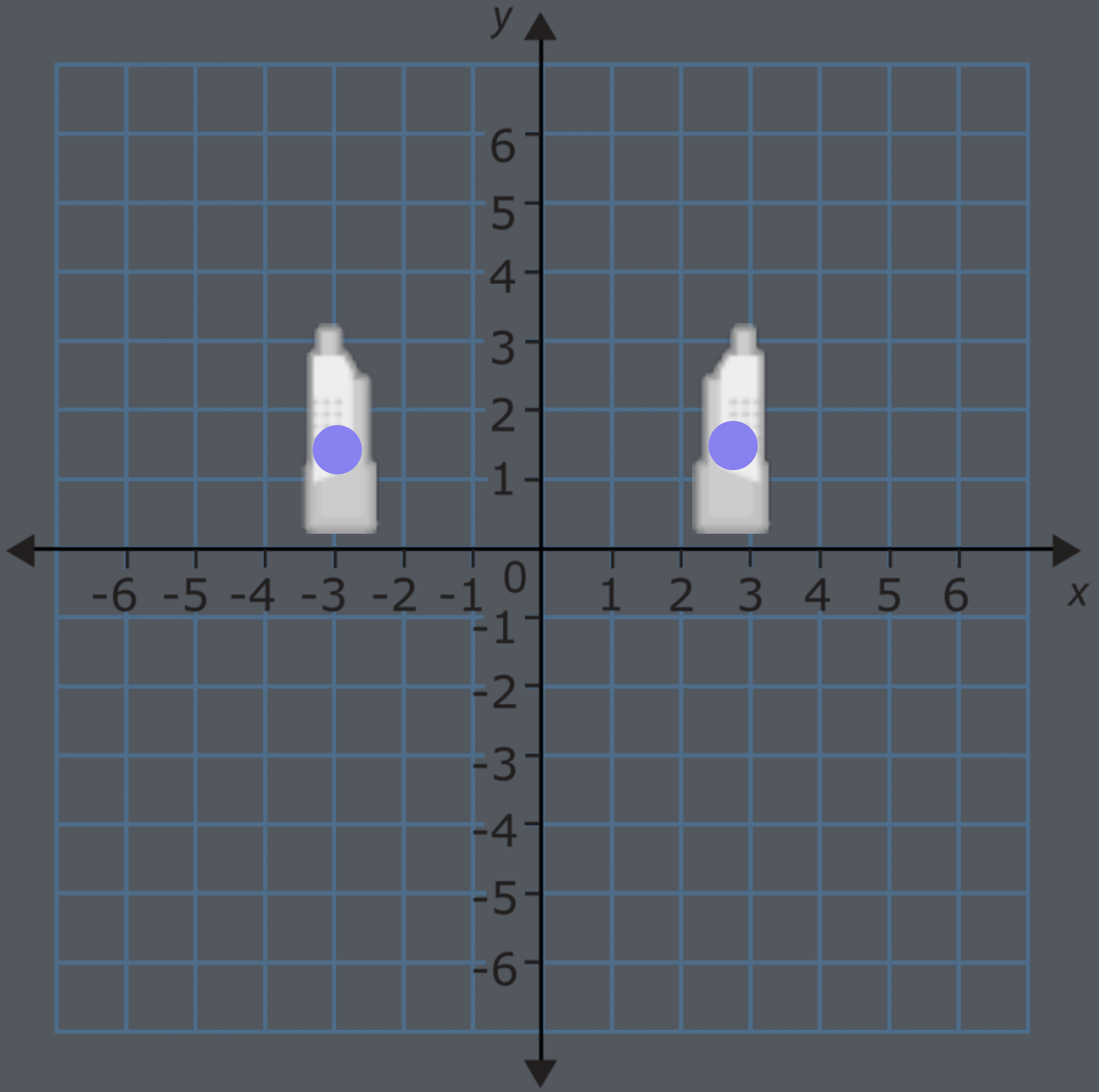
4
3
-3
-4
-5
-6
-6 -5 -4
4 5 6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x
0
-6 -5 -4 -3 -2 -1
1 2 3 4 5 6
6 5 4 3 2 1
-1 -2 -3 -4 -5 -6