NIA

4.(a) Let x^* be a unique maximum solution for g.

$\rightarrow x^*$ is a bitstring consisting only of 1-bits.

Let $x^{(t)}$ be solution generated in the t-th iteration $(t \in \mathbb{N}_0)$

We define a random variable X_t which is 1 if $x^{(t)}$ equals the optimal solution x^* and otherwise 0

↳ X_t is a Bernoulli distribution with $p = \frac{1}{2^n}$ because there are 2^n possible bitstrings (while only of them is the optimal solution)

If T is the smallest t for which $X_t = 1$ then T is a geometrically distributed random variable.

↳ The expected runtime of the RandomSearch algorithm is the expectation value $E(T)$ because after T steps we found the optimal solution and the algorithm stops.

$$E(T) = \frac{1}{p} = \frac{1}{\frac{1}{2^n}} = 2^n$$

4.(b) ~~Given $\mathcal{E}$ being the initial solution of $(1+1)$ EA~~ where

Let X count the number of 1-bits in the initial solution

Because the initial solution is uniformly randomly chosen:

$\rightarrow E(X) = \frac{n}{2}$

Further let $\bar{X}$ count the number of 0-bits in the initial solution.

$P(\bar{X} \leq \frac{n}{4}) = P(X \geq \frac{3n}{4})$

Because the probability of choosing less the 1-bits $P(X<0)$ is zero and $\frac{3n}{4} > 0$ assuming that we have at least one bit in our bitstring $(n > 0)$, we can apply Markov's Inequality:

$$P(\bar{X} \leq \frac{n}{4}) = P(X \geq \frac{3n}{4}) \leq \frac{E(X)}{\frac{3n}{4}} = \frac{4(\frac{n}{2})}{3n} = \frac{2}{3}$$

The upper bound on $P(\mathcal{E})$ is $\frac{2}{3}$.

Homework 3

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4) ~~a~~ e)
If $\mathcal{E}$ not occured, then we got the event $\overline{\mathcal{E}}$ where there are at least $\frac{n}{4}$ zero bits. In that case $g(x)$ returns „1". We won't go into a phase where $g(x)$ returns „0", because we only improve and the only way to improve is by getting the maximum with the all ones bitstring.

Since each bit flips with probability $\frac{1}{n}$ the best setting is with the least possible amount of zeros: $\frac{1}{4}n + 1$ zeros.

The probability of getting the desired bitstring is flipping all the zeros with $p = \frac{1}{n}$ and not flipping the rest with $1-p$:

$$P(\text{„improvement"}^{*}) = \left(\frac{1}{n}\right)^{\frac{1}{4}n+1} \cdot \left(1-\frac{1}{n}\right)^{n-\left(\frac{1}{4}n+1\right)}$$

$$\leq \left(\frac{1}{n}\right)^{\frac{1}{4}n}$$

This is geometrically distributed, therefor $E = \frac{1}{p}$:

$$E(\text{„all-ones"} \mid \overline{\mathcal{E}}) \geq \frac{1}{\left(\frac{1}{n}\right)^{\frac{1}{4}n}} = \Omega\left(n^{\frac{1}{4}n}\right)$$

* a stated before: an improvement results in the all-ones string

Homework 3

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4/d)

The expected runtime $E(T)$ is the sum of the runtime conditioned on $\mathcal{E}$ and $\overline{\mathcal{E}}$.

1. If $E(T/\mathcal{E}) \geq E(T|\overline{\mathcal{E}})$ then obviously
$E(T) = \Omega(n^{\frac{1}{4}n})$ (as shown in c): $E(T|\overline{\mathcal{E}}) = \Omega(n^{\frac{1}{4}n})$)

2. If $E(T/\mathcal{E}) < E(T/\overline{\mathcal{E}})$ then
$$E(T) = E(T|\mathcal{E}) \cdot Pr(\mathcal{E}) + Pr(\overline{\mathcal{E}})\,\Omega(n^{\frac{1}{4}n})$$
$E(T)$ gets small when $Pr(\overline{\mathcal{E}})$ is at its minimum, because $E(T|\mathcal{E}) < E(T/\overline{\mathcal{E}})$. In b) we showed
$$Pr(\mathcal{E}) \leq \frac{2}{3} \Rightarrow Pr(\overline{\mathcal{E}}) \geq \frac{1}{3}$$
Therefore:
$$E(T) = \frac{2}{3} E(T|\mathcal{E}) + \frac{1}{3}\Omega(n^{\frac{1}{4}n}) = \Omega(n^{\frac{1}{4}n})$$

Since RANDOMSEARCH has an expected runtime of 2^n it has a better expeted runtime than $(1+1)$ EA on g.