TeF 2 cv

Variační počet + 6.10 ✓

5.1
5.2
5.3
5.4
5.5
5.6
5.7
5.10

5.12
5.13

U5.3 + 5.15 (U5.3. jen druhá část bez důkazu Jacobiho Id.)

5.17

U5.6

5.19
5.20
5.21
5.24

U5.8 + 5.41 + 5.42
1/2 5.38
X 5.43

1. ZK. P.

U5.9
X U5.11 + 5.55
X 5.53

7.2
7.3
1/2 7.4
7.5 X dosazení do 7.4.
7.8
7.9
7.10
7.13
7.15
7.17
7.18 + 7.19
X 7.20
7.26
7.27
7.28
7.30
7.33
U7.6

U8.1
U8.2
U8.4 !
8.45
X 8.46 ??
8.50

2. ZK. P.

X U9.5B + 9.42
X 9.52
X 9.55
X U9.4

Podmínky pro zápočet

- max 3 neúčasti (1 lib. + 2 omluvené)
- alespoň 5 bodů ze zápočtových písemek
  1. záp. pís 6. cvičení (4 body)
  2. záp. pís 12. cvičení (4 body
- příprava zadaných příkladů

TEF 2

- příklady
- zápočtovky
- zkouška

5.1) $H=\sum p_j\dot q_j - L(q,\dot q,t)$; $\underbrace{p_i=\frac{\partial L}{\partial \dot q_i} \ ; \ \frac{d}{dt}\frac{\partial L}{\partial\dot q_i}-\frac{\partial L}{\partial q_i}=0}_{\dot p_i=\frac{\partial L}{\partial q_i}}$

$$\boxed{\frac{\partial H}{\partial q_j} = -\frac{\partial L}{\partial q_j} = -\dot p_j}$$

$$\boxed{\frac{\partial H}{\partial p_i}=\dot q_i}$$

5.2) $H=\frac{1}{2m}(p_x^2+p_y^2+p_z^2)$ ~~$+U(x,y,z)$~~ Kin. energie je homog. st. 2 rychlostí, potenciál není (volná částice) $\to H=E$

~~$p^2=p_x^2+p_y^2+p_z^2=p_r^2+r^2 p$~~ $(x,y,z)\to(r,\theta,\varphi)$

$|v|^2=\underbrace{\dot r^2+r^2\dot\theta^2+r^2\cos^2\theta\cdot\dot\varphi^2}$

$L=\frac12 m|v|^2$

$\Rightarrow p_r=\frac{\partial L}{\partial\dot r}=m\dot r$

$p_\theta=\frac{\partial L}{\partial\dot\theta}=mr^2\dot\theta$

$p_\varphi=\frac{\partial L}{\partial\dot\varphi}=mr^2\cos^2\theta\cdot\dot\varphi$

$$H=\frac1{2m}\left(p_r^2+\frac{p_\theta^2}{r^2}+\frac{p_\varphi^2}{r^2\cos^2\theta}\right)$$

$(x,y,z)\to(R,\varphi,z)$

$|v^2|=\dot R^2+R^2\dot\varphi^2+\dot z^2$ $\quad H=\frac1{2m}\left(p_R^2+\frac{p_\varphi^2}{R^2}+p_z^2\right)$

5.3) $H=\frac1{2m}(p_x^2+p_y^2+p_z^2)+U(x,y,z)$

$\dot p_i=-\frac{\partial U}{\partial x_i} \Longleftrightarrow \frac{d}{dt}(m\dot q_i)=F_i$ 1.+2. N.p.z. 3. N.p.z se týká více částic

$\dot q_i=\frac{p_i}{m}$

5.4) $H=\frac{p^2}{2m}+\frac12 kq^2$

5.5) $H=\frac1{2m}\left(p_r^2+\frac{p_\theta^2}{r^2}+\frac{p_\varphi^2}{r^2\cos^2\theta}\right)+U(r)$

$\dot p_r=-\frac{\partial H}{\partial r}=-\frac{\partial U}{\partial r}+2\left(p_\theta^2+\frac{p_\varphi^2}{\cos^2\theta}\right)\frac{1}{r^3}$

$\dot p_\theta=-\frac{\partial H}{\partial\theta}=\frac{p_\varphi^2}{r^2 2m}\frac{1}{\cos^3\theta}$

$\dot p_\varphi=-\frac{\partial H}{\partial\varphi}=0$

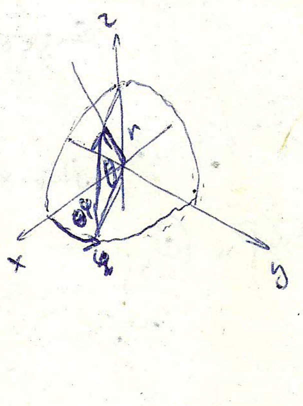
int. pohybu je p_φ ~~z-tová~~ ~~z-tová složka~~ z složka momentu hybnosti (φ je cykl. souř.)

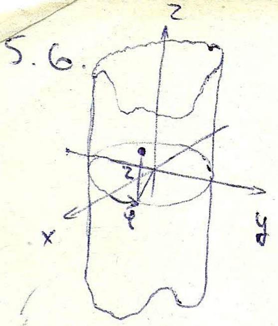
H (nezávisí na t)

$\dot r=\frac{p_r}{m}$

$\dot\theta=\frac{p_\theta}{mr^2}$

$\dot\varphi=\frac{p_\varphi}{mr^2\cdot\cos^2\theta}$



5.6. 

$$H=\frac{1}{2m}\left(\frac{p_\varphi^2}{R^2}+p_z^2\right)+\frac{1}{2}k\left(R^2+z^2\right)$$

$$\dot\varphi=\frac{\partial H}{\partial p_\varphi}=\frac{p_\varphi}{mR^2}\qquad \dot p_\varphi=-\frac{\partial H}{\partial \varphi}=0\ \Rightarrow\ \dot\varphi=\dot\varphi(0)\Rightarrow \underline{\varphi=\varphi(0)+\dot\varphi(0)\cdot t}$$

$$\dot z=\frac{p_z}{m}\qquad \dot p_z=-\frac{\partial H}{\partial z}=-kz\Rightarrow \ddot z+\frac{k}{m}z=0\Rightarrow \underline{z=A\cdot\sin\left(\sqrt{\tfrac{k}{m}}\,t+\xi\right)}$$

5.7

$$L=\frac{1}{2}m(\dot{\vec r})^2-q\left(\varphi(\vec r,t)-\dot{\vec r}\cdot\vec A(\vec r,t)\right)$$

$$\vec p=\frac{\partial L}{\partial\dot{\vec r}}=m\dot{\vec r}+q\cdot\vec A(\vec r,t)\qquad H=\vec p\cdot\dot{\vec r}-L=\frac{1}{2}m(\dot{\vec r})^2+q\varphi=\underline{\frac{1}{2}\cancel{m}\frac{(p-qA)^2}{m}+q\varphi}$$

5.10

$$L=\left[\frac{1}{2}m(\dot x+ax)^2-\frac{1}{2}m(b^2-a^2)x^2\right]e^{2at}$$

$$p=\frac{\partial L}{\partial\dot x}=m(\dot x+ax)e^{2at}\Rightarrow H=p\dot x-L=e^{2at}\left(\frac{1}{2}m(\dot x^2-a^2x^2)+\frac{1}{2}mx^2(b^2-a^2)\right)=$$

$$\cancel{=e^{2at}\frac{1}{2}\left[m^2(\dot x^2+2\dot x a x+a^2x^2)e^{2at}\right.}\quad \cancel{me^{2at}\ -m2\dot x a x-3ma^2}$$

$$\dot x=\frac{pe^{-2at}}{m}-ax$$

$$H=e^{2at}\left(\frac{1}{2}m\left(\frac{p^2e^{-4at}}{m^2}\cancel{-\frac{2pe^{-2at}ax}{m}}\cancel{-ax^2}\right)+\frac{1}{2}mx^2(b^2-a^2)\right)=$$

$$\underline{\underline{=\frac{1}{2m}p^2e^{-2at}-pax+\frac{e^{2at}}{2}mx^2(b^2-a^2)}}$$

$$\dot x=\frac{\partial H}{\partial p}=\frac{p}{m}e^{-2at}-ax\ \Rightarrow\ p=(\dot x+ax)me^{2at}\Rightarrow \dot p=(\ddot x+a\dot x)me^{2at}+(\dot x+ax)me^{2at}2a$$

$$\dot p=-\frac{\partial H}{\partial x}=+pa-e^{2at}m(b^2-a^2)x$$

$$me^{2at}(\ddot x+3a\dot x+2a^2x)=(\dot x+ax)mae^{2at}-e^{2at}m(b^2-a^2)x$$

$$\underline{\ddot x+2a\dot x+b^2x=0}$$

5.12) $\{e^{\alpha q},e^{\beta p}\}=\alpha e^{\alpha q}\beta e^{\beta p}=\alpha\beta e^{\alpha q+\beta p}$

5.13) $L_i=\varepsilon_{imn}x_mp_n\Rightarrow\{L_i,L_j\}=\frac{\partial L_i}{\partial x_k}\frac{\partial L_j}{\partial p_k}-\frac{\partial L_i}{\partial p_k}\frac{\partial L_j}{\partial x_k}=\varepsilon_{ikn}p_n\varepsilon_{jmk}x_m-\varepsilon_{imk}x_m\varepsilon_{jkn}p_n=$

$\varepsilon_{ijk}\varepsilon_{lmk}=\delta_{il}\delta_{jm}-\delta_{im}\delta_{jl}$

$$=x_mp_n\Big(\underbrace{\varepsilon_{ikn}\varepsilon_{jmk}}_{\substack{\varepsilon_{nik}\varepsilon_{jmk}\\ \delta_{nj}\delta_{im}-\delta_{nm}\delta_{ij}}}-\underbrace{\varepsilon_{imk}\varepsilon_{jkn}}_{\substack{\varepsilon_{imk}\varepsilon_{njk}\\ \delta_{in}\delta_{mj}-\delta_{ij}\delta_{mn}}}\Big)=x_mp_n\Big(\underbrace{\delta_{nj}\delta_{im}}_{n=j\wedge i=m}-\underbrace{\delta_{in}\delta_{jm}}_{i=n\wedge j=m}\Big)=x_ip_j-x_jp_i=\underline{\varepsilon_{ijk}L_k}$$

$$\{L_i,p_j\}=\frac{\partial L_i}{\partial x_k}\underbrace{\frac{\partial p_j}{\partial p_k}}_{\delta_{jk}}=\frac{\partial L_i}{\partial x_j}=\varepsilon_{ijk}p_k$$

a z bilinearity a antisymetričnosti plyne platnost pro celé hybnost a moment

U5.3 Důkaz Poissonovy věty

(1) $\{F_1,\{F_2,F_3\}\} + \{F_2,\{F_3,F_1\}\} + \{F_3,\{F_1,F_2\}\} = 0$

Nechť A,B jsou int. p. $\Rightarrow \{A,H\} + \frac{\partial A}{\partial t} = 0 \ \wedge \ \{B,H\} + \frac{\partial B}{\partial t} = 0$

$$\{\{A,B\},H\} + \frac{\partial}{\partial t}\{A,B\} = \{\{A,B\},H\} + \{\frac{\partial A}{\partial t}, B\} + \{A, \frac{\partial B}{\partial t}\} =$$

$$= \{\{A,B\},H\} - \{\{A,H\},B\} - \{A,\{B,H\}\} = \cancel{\{\{A,B\},H\} + \{}$$

$$= \{H,\{B,A\}\} + \{B,\{A,H\}\} + \{A,\{H,B\}\} = 0 \Rightarrow \{A,B\} \text{ je i.p.}$$

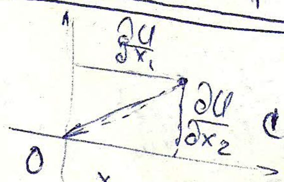
Q.E.D.

S.15) při S.13 $\Rightarrow \{L_1,L_2\} = L_3$ + Poissonova věta $\Rightarrow L_3$ je i.p.

S.17)
$$u = \frac{p_i p_i}{2m} - \Omega(x_1 p_2 - x_2 p_1) + U(r)$$
$$v = \frac{p_i p_i}{2m} + U(r)$$

$$\{u,v\} = \cancel{-\Omega p_2 \frac{p_1}{2m}} + \cancel{\Omega p_1 \frac{p_2}{2m}} + \cancel{\frac{\partial U}{\partial x_i}\frac{p_i}{m}} - \cancel{\frac{p_i}{m}\frac{\partial U}{\partial x_i}} - \Omega x_2 \frac{\partial U}{\partial x_1} + \Omega x_1 \frac{\partial U}{\partial x_2} =$$

$$= \left(\frac{\partial U}{\partial x_2} x_1 - \frac{\partial U}{\partial x_1} x_2\right)\Omega$$



centrální síla

$$\frac{x_1}{x_2} = \frac{\partial_1 U}{\partial_2 U} \Rightarrow \frac{\partial U}{\partial x_2} x_1 - \frac{\partial U}{\partial x_1} x_2 = 0$$

U. 5.6. Poincaréova věta o návratu

g je bijekce na omezené oblasti Ω a zachovává objem $\Rightarrow$

nemá ve smyslu skládání

$\Rightarrow (\forall \tilde{U} \subset \Omega, \text{okolí})(\exists x \in U)(\exists n \in \mathbb{N})(g^n(x) \in U)$

Dk.: $U, g(U), g(g(U)) = g^2(U), g^3(U), \ldots$

nemohou být disjunktní, pře mají stejný objem a je jich nekonečně mnoho $\Rightarrow$ měly by $+\infty_{\tilde{U}}$ objem (spor)

$\Rightarrow (\exists k,l;\ k>l)(\exists a \in \Omega)(a \in g^k \tilde{U} \cap g^l \tilde{U}) \Rightarrow a \in g^{k-l}(\tilde{U}) \cap \tilde{U}$

$\tilde{U} \Rightarrow x = a,\ n = k - l$

$\forall \tilde{U}$ najdu U (z bijekce)

5.19.

$Q_j = p_j \quad P_j = -q_j$

$$p_j dq_j - P_j dQ_j = p_j dq_j + q_j dp_j = d(q_j p_j)$$

$\Rightarrow$ vytv. funkce je $F = q_j p_j$

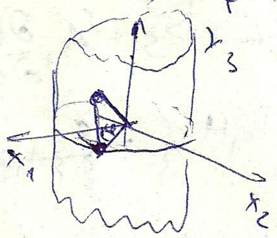
5.20.

$$F_2 = \sqrt{x_1^2 + x_2^2}\, P_R + \operatorname{arctg}\left(\frac{x_2}{x_1}\right) P_\varphi + x_3 P_z$$

$R = \frac{\partial F_2}{P_R} = \sqrt{x_1^2 + x_2^2}$

$\varphi = \operatorname{arctg}\frac{x_2}{x_1}$

$z = x_3$



5.21

$Q = \operatorname{arctg}\left(\sqrt{km}\,\frac{q}{p}\right)$

$P = \frac{1}{2}\left(\sqrt{km}\,q^2 + \frac{p^2}{\sqrt{km}}\right)$

$$dQ = \frac{\sqrt{km}}{1 + km\frac{q^2}{p^2}}\left(\frac{dq}{p} - \frac{q}{p^2}dp\right)$$

$$p\,dq - P\,dQ = p\,dq - \frac{1}{2}\frac{kmq^2 + p^2}{\sqrt{km}} \cancel{\sqrt{km}\,p^2} \frac{1}{p^2 + kmq^2}\frac{1}{p^2}(p\,dq - q\,dp) = \frac{1}{2}(p\,dq + q\,dp) = d\left(\frac{pq}{2}\right)$$

KANONICKÁ

$q = \frac{\operatorname{tg} Q}{\sqrt{km}} \cdot p \quad \Rightarrow \quad \cancel{2}P = \frac{\overbrace{(\operatorname{tg}^2 Q + 1)}^{\frac{1}{\cos^2 Q}} p^2}{2\sqrt{km}} \quad \Rightarrow \quad p = \sqrt{2P\sqrt{km}}\,\cos Q$

$q = \sqrt{\frac{2P}{\sqrt{km}}}\,\sin Q$

$$H' = \frac{\cancel{2}P}{\cancel{2}}\sqrt{\frac{k}{m}}\cos^2 Q + \frac{1}{\cancel{2}}\sqrt{\frac{k}{m}}\,\cancel{2}P\,\sin^2 Q = \sqrt{\frac{k}{m}}\,P$$

$\dot{Q} = \frac{\partial H}{\partial P} = \sqrt{\frac{k}{m}}$

$Q = \sqrt{\frac{k}{m}}\,t + Q(0)$ fáze ; $P = P(0)$ $\quad \dot{P} = 0$

~~súčin amplitúdy rýchlosti a polohy děleny 2~~ $\cancel{A^2\left(\frac{2P}{\omega}\right) \Rightarrow \frac{\omega A^2}{2} = P}$

5.24

$Q = q^\alpha \cos(\beta p)$

$P = q^\alpha \sin(\beta p)$

$$dQ = \alpha q^{\alpha-1}\cos(\beta p)\,dq - \beta q^\alpha \sin(\beta p)\,dp$$

$$p\,dq - P\,dQ = p\,dq - \frac{\alpha q^{2\alpha-1}}{2}\sin(2\beta p)\,dq + \beta q^{2\alpha}\sin^2(\beta p)\,dp \Rightarrow$$

$\int \sin^2(\beta p)\,dp = \frac{p}{2} - \frac{\sin(2\beta p)}{4\beta} + C$

$F = pq - \frac{q^{2\alpha}}{4}\sin(2\beta p) + C_1(p)$

$\wedge\ F = \frac{\beta}{2}pq^{2\alpha} - \frac{q^{2\alpha}}{4}\sin(2\beta p) + C_2(q) \quad \wedge$

$\Rightarrow \frac{\beta = 2}{\alpha = \frac{1}{2}}$

$F = pq - \frac{q}{4}\sin(4p) + \cancel{\text{konst.}}$

(odp. dosazením) $F_4 = \sqrt[\alpha]{\frac{P}{\sin(\beta p)}}\,(p - \sin(4p))$

U.58 Hamilton-Jacobi

$H = \frac{p_i p_i}{2m}$

$H\left(\frac{\partial S}{\partial x_j}\right) + \frac{\partial S}{\partial t} = 0$

$\frac{1}{2m}\sum\left(\frac{\partial S}{\partial x_j}\right)^2 + \frac{\partial S}{\partial t} = 0$ ... konstanty

Legendrovský, $X_i = \frac{\partial S_2}{\partial P_i} = \frac{P_i t}{m} + x_i$

$\Rightarrow P_i = (X_i - x_i)\frac{m}{t}$

$\ldots = -E \quad$ kde $E = \frac{P_j P_j}{2m} \quad \Rightarrow \quad S_2 = \frac{P_j P_j}{2m}t + \sum P_j x_j$

$S_1 = S_2 - P_i X_i = -\frac{P_i P_i}{2m}t = -\frac{m\sum(X_i - x_i)^2}{2t}$

5.41) $\frac{m}{2t}\sum(X_i-x_i)^2 = \text{const.}$ rozšiřující se sféra (rychlost závisí na energii which v const.)

a fixujeme poč. polohu

$$\frac{P_jP_j}{2m}t + \sum P_j x_j = \text{const.}$$ pohybující se rovina ve směru vekt. (P_1, P_2, P_3) (v const. jsou se poč. ~~podm.~~ poloha.)

5.42)

$x_i = v_i t + X_i$

$L = \sum \frac{1}{2} m \dot{x}_i^2$ $\quad S = \int_0^t \frac{1}{2} m v^2 dt = \frac{mv^2 t}{2} = \underline{\underline{\frac{m}{2t}\sum(x_i - X_i)^2}}$

5.38)

$H = \frac{p^2}{2m} + \frac{k}{2}q^2 \quad ; S(q, P, t)$

$$H\left(q, \frac{\partial S}{\partial q}\right) + \underbrace{\frac{\partial S}{\partial t}}_{-E} = 0 \implies \frac{kq^2}{2} + \frac{1}{2m}\left(\frac{\partial S}{\partial q}\right)^2 + \cancel{\frac{\partial S}{\partial t}} \cancel{0} = E$$

konz. soust

$$S_1(q) = \int \sqrt{2mE - kmq^2}\, dq$$

$$S = S_1(q) + \underbrace{f(P)}_{?} + Et$$

$\implies$ 5.43) ?

4.5.9) fázový prostor $\Omega \subset \mathbb{R}^{2s}$, $\frac{\partial H}{\partial t} = 0$

F_k je i.p.

nezávislost znamená že 2s rozm. vektory $\left(\frac{\partial F_k}{\partial q_i}, \frac{\partial F_k}{\partial p_i}\right)$ jsou LN $\implies$ H je i.p $\implies H(q_i, p_i) = \text{const.}$ omezuje fázovou trajektorii na varietu dim 2s-1. "energet. slupka"

$\implies$ r i.p. omezuje trajektorii

$r = 2s-1$ je trajektorie určena rovnicemi $F_1 = H = C_1, F_2 = C_2, \dots, F_{2s-1} = C_{2s-1}$ (dim 1) na 2s-r dim varietu $\implies$

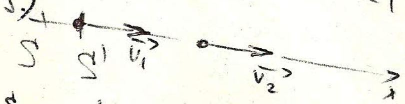
7.2.)

$\vec{r_V} = \frac{(\vec{r}\cdot\vec{V})}{|\vec{V}|^2}\vec{V}$ $\quad \vec{r}' = -\beta\gamma ct\frac{\vec{V}}{|\vec{V}|} + \gamma \vec{r_V} + \vec{r_{\perp V}} = -\gamma\vec{V}t + (\gamma-1)\frac{(\vec{r}\cdot\vec{V})}{|\vec{V}|^2}\vec{V} + \vec{r} = \vec{r} + \left(\frac{\gamma-1}{|\vec{V}|^2}(\vec{r}\cdot\vec{V}) - \gamma t\right)\vec{V}$

$\vec{r_{\perp V}} = \vec{r} - \vec{r_V}$

$t' = \frac{1}{c}\left(\gamma ct - \gamma\beta\frac{(\vec{r}\cdot\vec{V})}{|\vec{V}|}\right) = \gamma\left(t - \frac{\vec{r}\cdot\vec{V}}{c^2}\right)$

7.3.)



$S \to S'$ $\quad v_1' = 0$

$$v_2' = \frac{dx'}{dt'} = \frac{-\beta_1\gamma_1 c\,dt + \gamma_1 dx}{\gamma_1 dt - \frac{1}{c}\beta\gamma_1 dx} = \frac{-v_1 + v_2}{1 - \frac{v_1 v_2}{c^2}} = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}}$$

7.4.)

$d\vec{r}' = d\vec{r} + \left(\frac{\gamma-1}{|\vec{V}|^2}(d\vec{r}\cdot\vec{V}) - \gamma dt\right)\vec{V}$

$dt' = \gamma\left(dt - \frac{d\vec{r}\cdot\vec{V}}{c^2}\right)$

$$\vec{v}' = \frac{d\vec{r}'}{dt'} = \frac{\overbrace{\frac{d\vec{r}}{dt}}^{\vec{v}} + \left(\frac{\gamma-1}{|\vec{V}|^2}\vec{v}\cdot\vec{V} - \gamma\right)\vec{V}}{\gamma\left(1 - \frac{\vec{v}\cdot\vec{V}}{c^2}\right)} =$$

$$= \left(\frac{\vec{v}}{\gamma} - \vec{V} + \left(1 - \frac{1}{\gamma}\right)\frac{\vec{v}\cdot\vec{V}}{|\vec{V}|^2}\right)\left(1 - \frac{\vec{v}\cdot\vec{V}}{c^2}\right)^{-1}$$

Vecc $\gamma \approx 1$

$$\approx (\vec{v} - \vec{V})\left(1 - \frac{\vec{v}\cdot\vec{V}}{c^2}\right)^{-1} \approx \vec{v} - \vec{V} + \frac{\vec{v}\cdot\vec{V}}{c^2}\vec{v}$$

7.8. $S' \to S$ $\quad t = \gamma\tau + \cancel{\frac{v}{c^2}\gamma x}$ $\quad \frac{1}{\gamma} = \sqrt{1-0{,}99^2} \approx \sqrt{0{,}01 \cdot 2} \approx 0{,}141$

$\gamma \approx 7$

$v = 0{,}99c$

$c\tau = 660\,m$ $\quad \langle l \rangle = c\langle t \rangle = \gamma c \langle \tau \rangle \approx 4{,}6\,km$

$s = 30\,km$ $\quad \frac{s}{\langle l \rangle} \approx 6{,}5$ $\quad \left(\frac{1}{2}\right)^{\frac{s}{\Delta l}} \approx 0{,}015$

7.9)

$v = 0{,}99c \Rightarrow \gamma = 7$ $\quad$ a) $t = \frac{l}{v} \approx 1{,}67 \cdot 10^{-5}\,s$

$l = 5\,km$ $\quad$ b) $\tau = \frac{t}{\gamma} \approx 2 \cdot 10^{-6}\,s$

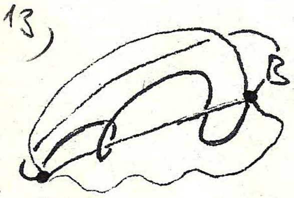
c) $\lambda = v\tau \approx 600\,m$

7.10) $1c = 3 \cdot 10^8 \frac{m}{s}$

$1y = 365 \cdot 24 \cdot 3{,}6 \cdot 10^3\,s = 31{,}5 \cdot 10^6\,s$ $\quad 1\,l.y. = 94{,}5 \cdot 10^{14}\,m$

$g = 9{,}81 \cdot \frac{31{,}5^2}{94{,}5} \cdot 10^{-14} \cdot 10^{12} \frac{l.y.}{y^2} = 1{,}03 \frac{l.y.}{y^2}$ $\quad c = 1 \frac{l.y.}{y}$ z def.

7.13)



$B\,(ct_2, \vec{r}_2)$

$A = (ct_1, \vec{r}_1)$

$(\Delta s)^2 > 0$

$\Delta t, \Delta \vec{r}$ v nějaké sous. (spojené s $\vec{r}_1, \vec{r}_2$)

$\vec{r}_2' - \vec{r}_1' = 0$

$(\Delta s)^2 = c^2\tau^2$ vl. s.

$(\Delta s)^2 = c^2t^2 - (\Delta\vec{r})^2$ soustava spojená s $\vec{r}_1, \vec{r}_2$

maximalizujeme $\Rightarrow$ minimalizujeme

$\vec{r}_2 + \left(\frac{\gamma-1}{|\vec{V}|^2}(\vec{r}_2 \cdot \vec{V}) - \gamma t_2\right)\vec{V} =$

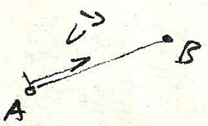
$= \vec{r}_1 + \left(\frac{\gamma-1}{|\vec{V}|^2}(\vec{r}_1 \cdot \vec{V}) - \gamma t_1\right)\vec{V}$

$\Rightarrow (\vec{r}_2 - \vec{r}_1) + \left[\frac{\gamma-1}{|\vec{V}|^2}(\vec{r}_2 - \vec{r}_1)\cdot\vec{V} - \gamma(t_2 - t_1)\right]\vec{V} = 0 \Rightarrow$

$\left[\gamma\Delta t - \frac{\gamma-1}{|\vec{V}|^2}\Delta\vec{r}\cdot\vec{V}\right]\vec{V} = k\,\Delta\vec{r}$

$|\Delta\vec{r}| = l$

$|\vec{V}| = V$ $\quad \Rightarrow \boxed{\vec{V} \parallel \Delta\vec{r}}$



$\gamma\Delta t - \frac{\gamma - \cancel{1}}{V^2} l \cdot V = \cancel{1}$

$\Delta t = \frac{\gamma l}{V} \Rightarrow \boxed{V = \frac{\gamma l}{\Delta t}}$

7.16 <u>Fizeaův pokus</u>

$$v_x = \frac{v_x' + V}{1 + \frac{V v_x}{c^2}} = (v_x' + V)\left(1 + \frac{V v_x}{c^2}\right)^{-1} \approx (v_x' + V)\left(1 - \frac{V v_x}{c^2}\right) = v_x' + V - \frac{V v_x^2}{c^2} - \underbrace{\frac{V^2 v_x}{c^2}}_{\approx 0}$$

ve směru

$V \ll c$

$$\frac{c}{n} = v_x \approx c$$

$$v = \frac{c}{n} + V - \frac{V}{n^2} = \frac{c}{n} + V\left(1 - \frac{1}{n^2}\right)$$

proti $-V$: $v = \frac{c}{n} - V\left(1 - \frac{1}{n^2}\right)$

7.17)

$$\operatorname{tg}\theta' = \frac{v_y\sqrt{1-\frac{V^2}{c^2}}}{v_x - V} = \frac{|\vec{v}_{xy}|\sin\theta \cdot \sqrt{1-\frac{V^2}{c^2}}}{|\vec{v}_{xy}|\cos\theta - V}$$

7.18

$$\operatorname{tg}\theta = \frac{c \cdot \sin\theta'\sqrt{1-\beta^2}}{c \cdot \cos\theta' + V} = \frac{\sin\theta'\sqrt{1-\beta^2}}{\cos\theta' + \beta}$$

$$\operatorname{tg}\alpha = \frac{a}{b} \Rightarrow \sin\alpha = \frac{a}{\sqrt{a^2+b^2}} \quad \cos\alpha = \frac{b}{\sqrt{a^2+b^2}}$$

$$\sin\theta = \frac{\sin\theta'\sqrt{1-\beta^2}}{1 + \cos\theta' \cdot \beta}$$

$$\cos\theta = \frac{\cos\theta' + \beta}{1 + \cos\theta'\beta}$$

$$\text{"}\sqrt{a^2+b^2}\text{"} = \sqrt{\sin^2\theta'(1-\beta^2) + \cos^2\theta' + 2\cos\theta'\beta + \beta^2} = \sqrt{1 + \underbrace{\cos^2\theta'\beta^2}_{V \ll c \Rightarrow \approx 0} + 2\cos\theta'\beta} \approx$$

Taylor

$$\approx 1 + \cos\theta'\beta$$

$$\sin(\theta' - \theta) = \sin\theta'\cos\theta - \sin\theta\cos\theta' =$$

$$= \frac{\sin\theta' \cdot \cos\theta' + \beta\sin\theta' - \sin\theta'\cos\theta'\overbrace{\sqrt{1-\beta^2}}^{\beta \ll 1 \Rightarrow \approx 0}}{1 + \cos\theta'\beta} \approx \frac{\beta\sin\theta'}{1 + \cos\theta'\beta}$$

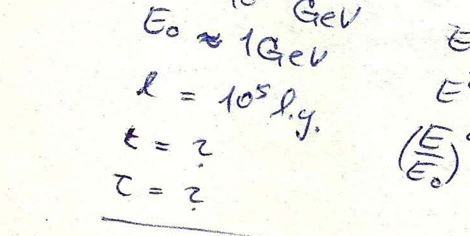
$$\sin\Delta\theta \approx \frac{\beta\sin\theta'}{1 + \cos\theta'\beta} \xrightarrow{\beta \ll 1} \Delta\theta \approx \beta \cdot \sin\theta' \cdot (1 + \cos\theta' \cdot \beta)^{-1} \approx$$

Taylor

$$\approx \beta \cdot \sin\theta' - \underbrace{\beta^2\sin\theta'\cos\theta'}_{\beta \ll 1 \Rightarrow \approx 0} = \beta \cdot \sin\theta'$$

$$\Delta\theta \approx \frac{V}{c}\sin\theta'$$

7.19) <u>Bradleyho aberace stálic</u>



$$\Delta\theta \approx \frac{V}{c} \cdot \sin\theta' \in (-20'', 20'')$$

$|v| = 30\,\frac{km}{s}$

7.26) $E = 10^{10}\,\text{GeV}$

$E_0 \approx 1\,\text{GeV}$

$l = 10^5\,\text{l.y.}$

$t = ?$

$\tau = ?$

$$E^2 = \vec{p}^{\,2}c^2 + m_0^2c^4$$

$$E^2 = m_0^2\gamma^2v^2c^2 + m_0^2c^4$$

$$\left(\frac{E}{E_0}\right)^2 = \underbrace{\gamma^2\beta^2 + 1}_{\underbrace{\frac{\beta^2}{1-\beta^2} + 1}_{\frac{1}{1-\beta^2}}} \Rightarrow \boxed{\frac{E_0^2}{E^2} = 1 - \beta^2}$$

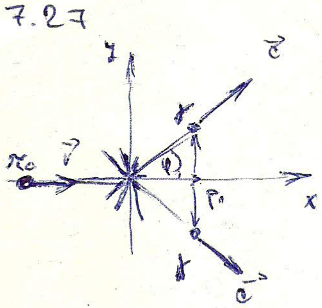
$$v = \sqrt{1 - \underbrace{\left(\frac{E_0}{E}\right)^2}_{\approx 10^{-20}}}\,c \approx c$$

$\underline{\underline{t = 10^5\,y}}$

$$l = l_0 \cdot \sqrt{1-\beta^2} = l_0\sqrt{\frac{E_0^2}{E^2}} = l_0\frac{E_0}{E} =$$

$$= 10^5 \cdot 10^{-10} \cdot 365 \cdot 24 \cdot 3600\ \text{l.s.} = \underline{\underline{315\,l.s}} \Rightarrow \underline{\underline{\tau = 315\,s}}$$

7.27



$$E_\pi = \frac{m_0 c^2}{\sqrt{1-\beta^2}}$$
$$2E_\gamma = 2m_\gamma c^2$$

z.z.E $\Rightarrow$ $2m_\gamma = \dfrac{m_0}{\sqrt{1-\beta^2}}$

$$p_\pi = \frac{m_0}{\sqrt{1-\beta^2}} v$$

z.z.h.

$$p_\pi = 2p_1$$

$$p_1 = m_\gamma \cdot c \cdot \cos\frac{\varphi}{2}$$

$$\frac{m_0}{\sqrt{1-\beta^2}} v = 2m_\gamma c \cdot \cos\frac{\varphi}{2}$$

$$\boxed{\cos\frac{\varphi}{2} = \frac{v}{c}}$$

7.28

$$p_\gamma^{\mu} = p_-^{\mu} + p_+^{\mu}$$

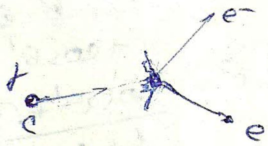
$$p_\gamma^{\mu} p_{\gamma\mu} = p_-^{\mu} p_{-\mu} + 2p_-^{\mu} p_{+\mu} + p_{+\mu} p_+^{\mu}$$

(1) $= m_{\gamma 0} c^2 = 0$

$$p_-^{\mu} p_{-\mu} = \underbrace{m_{e0}^2 c^2}_{>0}$$

$$p_+^{\mu} p_{+\mu} = \underbrace{m_{e+0}^2 c^2}_{>0}$$

$$p_-^{\mu} = \left(\frac{E}{c}, \vec{p}_-\right)$$
$$p_+^{\mu} = \left(\frac{E}{c}, \vec{p}_+\right)$$



$$p_-^{\mu} p_{+\mu} = g^{\mu\nu} p_{-\mu} p_{+\mu} = \frac{E^2}{c^2} - \vec{p}_- \vec{p}_+ = \underbrace{\frac{E^2}{c^2} + |\vec{p}_+|^2}_{>0}$$

v soustavě hm. středu e^-, e^+: $\vec{p}_- = -\vec{p}_+$

spor s (1)

7.30)

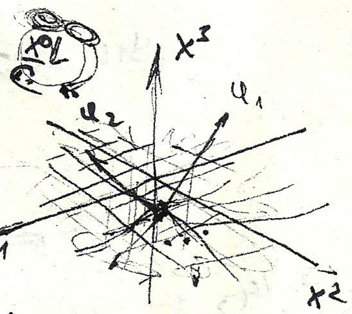
(m_0, q) ; $\vec{A} = (0, 0, A(x,y))$

$$\mathcal{L} = -m_0 c^2 \sqrt{1-\frac{v^2}{c^2}} - U \qquad U = q(E \mp \vec{v}\cdot\vec{A})$$

$$\mathcal{L} = -m_0 c^2 \sqrt{1-\frac{v^2}{c^2}} + q \cdot v_z \cdot A(x,y)$$

$\Rightarrow$ z je cykl. souř.

$$p_z = \frac{\partial \mathcal{L}}{\partial \dot{z}} = +m_0 c^2 \frac{1}{2} \frac{\frac{1}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} 2v_z + q \cdot A(x,y) = \frac{m_0 v_z}{\sqrt{1-\frac{v^2}{c^2}}} + q \cdot A(x,y)$$



7.33) $\varphi_t = \dfrac{\partial\varphi}{\partial t}$ $\qquad \dfrac{\partial}{\partial x_0}\dfrac{\partial}{\partial \varphi_{x_0}} = \dfrac{\partial}{\partial t}\dfrac{\partial}{\partial \varphi_{(ct)}} = \dfrac{\partial}{\partial t}\dfrac{\partial}{\partial \varphi_t}$

$$\boxed{\frac{\partial}{\partial x_\mu}\frac{\partial \mathcal{L}}{\partial \varphi_{a,x_\mu}} - \frac{\partial \mathcal{L}}{\partial \varphi_a} = 0}$$

x^μ jsou 4-souř., φ_a nezáv. veličiny pole

a) $(\varphi^4)_2$: $\mathcal{L} = \frac{1}{2}(\varphi_t^2 - \varphi_x^2) + \frac{1}{2}\mu^2\varphi^2 - \frac{1}{4}\lambda\varphi^4$

$$\frac{\partial}{\partial t}\varphi_t - \frac{\partial}{\partial x}\varphi_x - \mu^2\varphi + \lambda\varphi^3 = 0 \Rightarrow \varphi_{tt} - \varphi_{xx} = \mu^2\varphi - \lambda\varphi^3$$

b) sinus-Gordon $\mathcal{L} = \frac{1}{2}(\varphi_t^2 - \varphi_x^2) + (\cos\varphi - 1) \Rightarrow \varphi_{tt} - \varphi_{xx} + \sin\varphi = 0$

c) Born-Infeld $\mathcal{L} = \sqrt{1-\varphi_t^2+\varphi_x^2}$

$$\Rightarrow \frac{\partial}{\partial t}\frac{-\varphi_t}{\sqrt{1-\varphi_t^2+\varphi_x^2}} + \frac{\partial}{\partial x}\frac{\varphi_x}{\sqrt{1-\varphi_t^2+\varphi_x^2}} = 0 \Rightarrow$$

$$\Rightarrow \frac{-\sqrt{1-\varphi_t^2+\varphi_x^2}\,\varphi_{tt} + \varphi_t \frac{*)}{\sqrt{1-\varphi_t^2+\varphi_x^2}} + \varphi_{xx}\sqrt{\;} - \varphi_x \frac{\varphi_x\varphi_{xx} - \varphi_t\varphi_{xt}}{\sqrt{\;}}}{1-\varphi_t^2+\varphi_x^2} = 0$$

*) $-\varphi_t\varphi_{tt} + \varphi_x\varphi_{xt}$

$$\Rightarrow (-\varphi_{tt}+\varphi_{xx})(1-\varphi_t^2+\varphi_x^2) + \varphi_t(-\varphi_t\varphi_{tt} + \varphi_x\varphi_{xt}) - \varphi_x(\varphi_{xx}\varphi_x - \varphi_t\varphi_{xt})$$

$$\Rightarrow \cancel{(1-\varphi_t^2+\varphi_x^2)(\varphi_{tt}+\varphi_{xx}) - (\varphi_t^2+\varphi_x^2)} = 0$$

$$-\varphi_{tt}(1+\varphi_x^2) + \varphi_{xx}(1-\varphi_t^2) + 2\varphi_t\varphi_x\varphi_{xt} = 0$$

d) Korteweg–de Vries $\mathcal{L} = \frac{1}{2}\theta_x\theta_t + \frac{\alpha}{6}\theta_x^3 + \theta_x\psi_x + \frac{1}{2}\psi^2 = 0$

$$\frac{1}{2}\theta_{xt} + \frac{1}{2}\theta_{tx} + \frac{\alpha}{2}\theta_x\theta_{xx} + \psi_{xx} = 0 \Rightarrow \theta_{xt} + \alpha\theta_x\theta_{xx} + \psi_{xx} = 0$$

$\theta_{xx} = \psi$

$$\Rightarrow \theta_{xx} + \alpha\theta_x\theta_{xx} + \theta_{xt} = 0$$

e) ?

U 7.6

$$\frac{\partial k^{\alpha\mu}}{\partial x^{\alpha\mu}} = 0 \implies \underbrace{\frac{1}{c}\int_{\mathbb{R}^3} k^0 \, dV}_{K} = \text{const}$$

$$\int_\Omega \frac{\partial k^{\alpha\mu}}{\partial x^{\alpha\mu}} \, dV \overset{4\,\text{Gauss}}{=} \oint_{\partial\Omega} k^{\alpha\mu} \cdot d\vec{S}_4 = 0 \xrightarrow{R\to+\infty}$$ vymizí tok povrchem koule a zbyde časová hranice $\oint_{x^0\in(ct_1,ct_2)}$

$\Omega \equiv \{x^{\alpha\mu}\in\mathbb{R}^4 \mid (x^1)^2+(x^2)^2+(x^3)^2 = R^2 ;\ x^0\in(ct_1,ct_2)\}$

~~válce~~ koule ohr. v časovém int. (t_1,t_2)

$$\oint_{x_0=ct_2} k^{\alpha\mu} d\vec{S}_4^{(4)} + \oint_{x_0=ct_1} k^{\alpha\mu} d\vec{S}_4^{(1)} = \int_{\mathbb{R}^3, x^0=ct_2} k^0 d\tau - \int_{\mathbb{R}^3, x^0=ct_1} k^0 dV = 0 \xrightarrow{t_2\to t_1} \frac{dK}{dt} = 0$$

U 8.1 Fyzikální význam Maxwellových rovnic

$\operatorname{div}\vec{D} = \rho \implies \oint_S \vec{D}\,d\vec{S} = Q_{in S}$ vakuum $\operatorname{div}\vec{E} = \frac{\rho}{\varepsilon_0} \implies \oint_S \vec{E}\,d\vec{S} = \frac{Q_{in S}}{\varepsilon_0}$

$\operatorname{div}\vec{B} = 0 \iff$ nejsou magnet. monopóly $\implies \oint \vec{B}\cdot d\vec{S} = 0 \implies$ „co vteče, to vyteče"

$\operatorname{rot}\vec{H} = j + \frac{\partial B}{\partial t} \implies \oint_l \vec{H}\cdot d\vec{l} = J + J_D$; $\operatorname{rot}\vec{E} = -\frac{\partial\vec{B}}{\partial t} \implies \oint_l \vec{E}\cdot d\vec{l} = -\frac{\partial\Phi_B}{\partial t}$

Ampérův zákon

U 8.2 δ-fce

Elektron v $\vec{r}_0$ $\quad \rho(\vec{r}) = e\cdot\delta^3(\vec{r}-\vec{r}_0)$ $\quad \boxed{\delta^3(\vec{r}) = \delta(x)\delta(y)\delta(z)}$

Jednotkový proudový element v $\vec{r}_0$ směru $\vec{n}$ $\quad \vec{j}(\vec{r}) = \vec{n}\cdot j_0\cdot\delta^3(\vec{r}-\vec{r}_0)$

Pro soubor nábojů $\vec{r}_1,\dots,\vec{r}_\alpha$; $q_1,\dots,q_\alpha$; $v_1,\dots,v_\alpha$: $\rho(\vec{r}) = \sum_{\alpha\in\hat{n}} q_\alpha\,\delta^3(\vec{r}-\vec{r}_\alpha)$

$\vec{j}(\vec{r}) = \sum_{\alpha\in\hat{n}} q_\alpha \vec{v}_\alpha\,\delta^3(\vec{r}-\vec{r}_\alpha)$

U 8.4 Lorentzovy transformace elm. potenc. a polí

$A^{\alpha\mu} = \left(\frac{\varphi}{c}, \vec{A}\right)$; $(\alpha^{\alpha\mu}{}_\nu) = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$F_{\alpha\mu\nu} = \frac{\partial A_\nu}{\partial x^{\alpha\mu}} - \frac{\partial A_\mu}{\partial x^\nu}$; $k^{\alpha\mu} = eF^{\alpha\mu\nu} j_\nu$ Lorentzova síla

$F'^{\alpha\mu\nu} = \alpha^{\alpha\mu}{}_\rho\,\alpha^\nu{}_\sigma\,F^{\rho\sigma}$

8.45) $\vec{A}_1 = \left(-\frac{1}{2}By, \frac{1}{2}Bx, 0\right)$; $\vec{B} = \nabla\times\vec{A}_1 = (0,0,B)$; $\vec{A}_2 = (0,Bx,0)$; $\vec{B}_2 = \nabla\times\vec{A}_2 = (0,0,B)$

$\vec{A}_2 - \vec{A}_1 = \left(\frac{1}{2}By, \frac{1}{2}Bx, 0\right) = \nabla\left(\frac{1}{2}Bxy\right)$ $\quad$ CHYBA VE SKRIPTECH!

8.46) sf. $\quad R \quad \varphi \quad \theta$

A_1

$A_2 \quad z$ $\quad$ nemění s výsledkem

$A_\nu = \frac{1}{2}B\sqrt{x^2+y^2}$

8.50) $A^{\alpha\mu} = \left(\frac{k}{c}\frac{q}{|\vec{r}|}, \vec{0}\right)$ $\quad \beta = -\frac{v}{c}$ $\quad \alpha = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

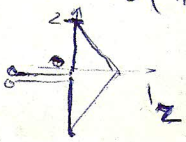
$$A'^{\alpha\mu} = \left(\gamma\frac{k}{c}\frac{qe}{\sqrt{(x-vt)^2+y^2+z^2}}, -\frac{v\gamma}{c^2}\frac{e}{\ }\right)$$

CHYBA VE SKRIPTECH!

4 9.5. $l \ll \frac{\lambda}{2\pi}$ KRÁTKY DIPÓL – DIPÓLOVÁ ANTÉNA

$I(z,t) = I(z) . \cos(\omega_1 t)$

$I(z) = I_0 \left(1 - \frac{|z|}{\frac{l}{2}}\right)$



lin. hustota náboje

τ : $\frac{d\tau}{dt} = -\mathrm{div}\,\vec{j} = \frac{dI(z)}{dz} \cos \omega t$

$\vec{\mu} = (0, 0, \mu_z)$

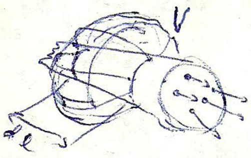
$\tau = \mathrm{sgn}(z) \frac{2 I_0}{\omega l} \sin(\omega t)$

$\mu_z(t) = \int_{-\frac{l}{2}}^{\frac{l}{2}} z \cdot \tau \cdot dz = \left[\frac{2 I_0}{\omega l} \sin(\omega t) \frac{z^2}{2}\right]_0^{\frac{l}{2}} = \frac{I_0}{2\omega l} \sin \omega t$

$dW = \frac{1}{Z_0} \vec{E}^2 R^2 d\Omega = \frac{I_0^2}{4\omega l^2} \frac{1}{Z_0} \frac{4\pi}{R^2} R^2$

9.42

$\mathrm{div}\,\vec{j} = -\frac{\partial \rho}{\partial t}$



$\oint_S \vec{j}\, d\vec{S} = -\frac{dQ}{dt}$

$-\frac{\partial \rho}{\partial t} = \mathrm{div}\,\vec{j} \quad / . \,dS$

$\boxed{-\frac{\partial \tau}{\partial t} = \frac{\partial I}{\partial z}}$

ZÁPOČTOVKA

Roman Pavelka

$\frac{1}{2}$ b.

1) Lorentzova transformace $S' \to S$ 4-potenciálu odp. homog. mag. poli $\vec{B}' = (0,0,B)$

$$\varphi' = 0$$
$$\vec{A}' = \left(-\tfrac{1}{2}By', \tfrac{1}{2}Bx', 0\right)$$

a) prověříme, zda A odp. $\vec{B}$

$$\nabla \times \vec{A}' = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_{x'} & \partial_{y'} & \partial_{z'} \\ -\frac{1}{2}By' & \frac{1}{2}Bx' & 0 \end{vmatrix} = \left(0, 0, \tfrac{1}{2}B + \tfrac{1}{2}B\right) = B'$$

b) matice ~~...~~ transformace $\boxed{\beta = -\frac{v}{c}}$; $\gamma = (1+\beta^2)^{-\frac{1}{2}}$

$$A = \begin{pmatrix} \gamma & -\beta\gamma & 0 & \\ -\beta\gamma & \gamma & & \\ & & 1 & 0 \\ 0 & & 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} \varphi \\ \vec{A} \end{pmatrix} = A \cdot \begin{pmatrix} 0 \\ -\frac{1}{2}By' \\ -\frac{1}{2}Bx' \\ 0 \end{pmatrix} = \left\{ -\frac{1}{2}B \left[y' \begin{pmatrix} -\beta\gamma \\ \gamma \\ 0 \\ 0 \end{pmatrix} + x' \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right] \right\} =$$

$$= -\frac{1}{2}B \begin{pmatrix} -\beta\gamma y' \\ \gamma y' \\ x' \\ 0 \end{pmatrix} = \begin{pmatrix} \dfrac{-\frac{1}{2}B\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} y \\ \dfrac{-\frac{1}{2}By'}{\sqrt{1-\frac{v^2}{c^2}}} \\ -\frac{1}{2}Bx' \\ 0 \end{pmatrix}$$

✓ + $y' = y$
$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

1 bod

2) $p^\mu \Rightarrow m_0, \vec{v}$

$$p^0 = \frac{E}{c}$$

a) $(p^0 c)^2 = \sum_{\mu=1}^{3} p^{\mu} p_{\mu} c^2 + m_0^2 c^{4}$

$$~~E = \sum_{\mu=1}^{3} p^{\mu} p_{\mu} + m_0^2 c^2~~$$

$$~~m_0 = \sqrt{\frac{1}{c^2}\sum_{1}^{3}(p^{\mu})^2} = \frac{1}{c}\sqrt{\sum_{\mu=1}^{3}(p^{\mu})^2}~~$$

$$E^2 = \vec{p}^2 c^2 + m_0^2 c^4$$

$$m_0 = \frac{1}{c}\sqrt{g^{\mu\nu} p_{\mu} p_{\nu}}$$ ✓

$$g^{\mu\nu} = \begin{pmatrix} 1 & \vec{0}^T \\ \vec{0} & -E \end{pmatrix}$$

b) $|\vec{v}|$

$$\vec{p} = \gamma m_0 \vec{v}$$

$$m^2 c^4 = \gamma^2 m_0^2 |\vec{v}|^2 c^2 + m_0^2 c^4$$

$$\gamma^2 \cancel{m_0^2} c^2 = \gamma^2 \cancel{m_0^2} |\vec{v}|^2 + m_0^2 c^2$$

$$~~\vec{v} = \sqrt{c^2 - \frac{c^2}{\gamma^2}} = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2~~$$

$$v = \frac{c|\vec{p}|}{p^0}$$

$$c^2 = |\vec{v}|^2 + \frac{c^2}{\gamma^2} \Rightarrow \cancel{c^2} = |\vec{v}|^2 + c^2\left(1 - \frac{v^2}{c^2}\right) \Rightarrow |\vec{v}|^2 = |\vec{v}|^2$$

2b) $E^2 = (\vec{p})^2 c^2 + m_0^2 c^4$

$c^2 p^{0\,2} = \gamma^2 m_0^2 \cdot |\vec{v}|^2 c^2 + m_0^2 c^{\cancel{4}\,2}$

$$\underbrace{\gamma^2 |\vec{v}|^2}_{\frac{v^2}{1-\frac{v^2}{c^2}}} = \frac{p^{0\,2}}{m_0^2} - c^2 = \frac{p^{0\,2}\,c^2}{g^{\sigma\mu\nu} p_{\sigma\mu} p_\nu} - c^2$$

$$\frac{v^2}{1-\frac{v^2}{c^2}} = \underbrace{c^2\left(\frac{p^{0\,2}}{g^{\sigma\mu\nu} p_{\sigma\mu} p_\nu} - 1\right)}_{\alpha}$$

$v^2 = \alpha - \alpha \frac{v^2}{c^2}$

$v^2 c^2 = \alpha c^2 - \alpha v^2$

$v^2 = \frac{\alpha c^2}{\alpha^2 + c^2}$

$$v = \frac{c^2 \sqrt{\frac{p^{0\,2}}{g^{\sigma\mu\nu} p_{\sigma\mu} p_\nu} - 1}}{\sqrt{c^4\left(\frac{p^{0\,2}}{g^{\sigma\mu\nu} p_{\sigma\mu} p_\nu} - 1\right)^2 + c^4}} = 1$$

$$= \sqrt{p_0^2 - g^{\sigma\mu\nu} p_{\sigma\mu} p_\nu}$$

---

3) 1/2 b

$\beta = -\frac{v_2}{c}$ $\quad \gamma = (1+\beta^2)^{-\frac{1}{2}}$ $\quad A = \begin{pmatrix} +\gamma & -\beta\gamma & 0 \\ -\beta\gamma & \gamma & 0 \\ 0 & & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \end{pmatrix}$

a) $v_1' = \frac{dx'}{dt'} = \frac{d(-\beta ct + x)}{d\left(\frac{1}{c}(tc - \beta x)\right)} = \frac{-\beta c + v_1}{\frac{1}{c} - \frac{\beta}{c} v_1} = \frac{v_2 + v_1}{1 + \frac{v_1 v_2}{c^2}}$

$v_2' = 0$

$$\Rightarrow v_{rel} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

b) τ_2 $v S_2$ $\quad \tau_2 = \tau_2$ $\quad \frac{\tau_2}{\tau_1} = \frac{1}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{c}{\sqrt{c^2 - v_1^2}}$

τ_1 $v S$ $\quad \tau_1 = \tau_2 \sqrt{1 - \frac{v_{rel}^2}{c^2}}$

---

4) $\mathcal{L} = \frac{1}{2}(\varphi_x^2 + 2\varphi_x \varphi_t \cos\varphi - \varphi_t^2) + \sin\varphi.$

$t \cdot c = x^0$

$\varphi_t = \frac{1}{c} \varphi_{x_0}$

$\frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \varphi_t} + \frac{\partial}{\partial x} \frac{\partial \mathcal{L}}{\partial \varphi_x} - \frac{\partial \mathcal{L}}{\partial \varphi} = 0 \Rightarrow \partial t$

$\frac{\partial}{\partial x^0} \frac{\partial}{\partial \varphi_{x_0}} = \frac{\partial}{\partial ct} \frac{\partial}{\partial \frac{1}{c}\varphi_t}$

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4) 1 bod

$$\frac{\partial}{\partial x^0}\frac{\partial}{\partial \varphi_{x^0}} = \frac{\partial}{\partial t}\frac{\partial}{\partial \varphi_t}$$

$$\frac{\partial}{\partial t}(\varphi_x \cdot \cos\varphi - \varphi_t) + \frac{\partial}{\partial x}(\varphi_t \cdot \cos\varphi + \varphi_x) + \varphi_x\varphi_t \sin\varphi - \cos\varphi = 0$$

$$\varphi_{x,t}\cos\varphi + \varphi_x\cdot(-\sin\varphi)\varphi_t - \varphi_{t,t} + \varphi_{tx}\cos\varphi \cancel{- \varphi_t \sin\varphi\, \varphi_x} +$$

$$+ \varphi_{x,x} + \cancel{\varphi_x\varphi_t \sin\varphi} - \cos\varphi = 0$$

$$2\varphi_{t,x}\cos\varphi + \varphi_{x,x} - \varphi_{t,t} - \varphi_x\varphi_t\sin\varphi - \cos\varphi = 0$$

---

ad 2 b)

$$(\vec{p})^2 = \gamma^2 m_0^2 |\vec{v}|^2$$

$$\frac{(\vec{p})^2}{m_0^2} = \frac{v^2}{1-\frac{v^2}{c^2}} \Rightarrow \frac{(\vec{p})^2}{m_0^2}(c^2 - v^2) = c^2 v^2$$

$$v^2 = \frac{\frac{(\vec{p})^2}{m_0^2}c^2}{\frac{(\vec{p})^2}{m_0^2} + c^2} \Rightarrow |\vec{v}| = \frac{\sqrt{(\vec{p})^2}\,\frac{c}{\cancel{m_0}}}{\sqrt{\vec{p}^2 + m_0^2 c^2}} =$$

$$= \frac{|\vec{p}|c}{\sqrt{\vec{p}^2 + g^{\mu\nu}p_\mu p_\nu}} = \frac{|\vec{p}|c}{\frac{E}{c}} = \frac{\vec{p}}{E}c^2 \qquad \frac{\cancel{kg}\,\frac{m}{\cancel{s}}\,m^2}{\cancel{kg}\,\frac{m^2}{s^2}\,\cancel{s}}$$

ZÁPOČTOVKA

$\vec{r}\times /\ m\ddot{\vec{r}} = e\dot{\vec{r}}\times\vec{B}$ Roman Pavelka

$\vec{B}=(0,0,B)$ $\frac{d}{dt}(m\vec{r}\times\dot{\vec{r}})=e\vec{r}\times(\dot{\vec{r}}\times\vec{B})$ $\frac{1}{2}B\begin{pmatrix}\vec{i} & \vec{j} & \vec{k}\\ \dot{x} & \dot{y} & \dot{z}\\ -y & x & 0\end{pmatrix} = (-\dot{z}x, -\dot{z}y, \dot{x}x+\dot{y}y)$

~~$\frac{1}{2}B(0,0,\dot{x}x-\dot{y}y)$~~

(1) 1 bod

$\vec{B}=\nabla\times\vec{A}$ $\longrightarrow$ $\vec{A}=\left(-\frac{1}{2}By, \frac{1}{2}Bx, 0\right)$

$\nabla\times\vec{A} = \frac{1}{2}B\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ -y & x & 0\end{vmatrix} = \frac{1}{2}B(0,0,1+1) = (0,0,B)$ ✓

(2) 1 bod

$-q(\varphi-\vec{v}\cdot\vec{A})$

$L(\vec{r},\dot{\vec{r}}) = \frac{1}{2}m\dot{\vec{r}}^2 + q(\dot{\vec{r}}\cdot\vec{A}) = \frac{1}{2}m\dot{\vec{r}}^2 - q\left[\cancel{\ldots}\right]$

~~zob. hybnost $\vec{P}=\frac{\partial L}{\partial\dot{\vec{r}}}$~~ ~~$L=\frac{1}{2}m(\dot{\vec{r}})^2-q(-\dot{z}x, -\dot{z}y$~~

$$L = \frac{1}{2}m\dot{\vec{r}}^2 + q\left(-\frac{1}{2}By\dot{x}+\frac{1}{2}Bx\dot{y}\right)$$ cylindr.

(3) 1 bod

$P_x = \frac{\partial L}{\partial\dot{x}} = m\dot{x} \mp \frac{1}{2}Bqy \longrightarrow \dot{x} = \frac{P_x \pm \frac{1}{2}Bqy}{m}$

$P_y = \frac{\partial L}{\partial\dot{y}} = m\dot{y} \pm \frac{1}{2}Bqx$ $\quad \dot{y} = \frac{P_y \mp \frac{1}{2}Bqx}{m}$

$P_z = \frac{\partial L}{\partial\dot{z}} = m\dot{z}$

$H = P_x\dot{x}+P_y\dot{y}+P_z\dot{z}-L = \frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2)+\frac{1}{2}Bqy\dot{x}-\frac{1}{2}Bqx\dot{y} + q\left(-\frac{1}{2}By\dot{x}+\frac{1}{2}Bx\dot{y}\right) =$

$= \frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2)$

$= \frac{1}{2m}\left(\left(P_x\pm\frac{1}{2}Bqy\right)^2+\left(P_y\mp\frac{1}{2}Bqx\right)^2+P_z^2\right) =$

$= \frac{1}{2m}\left[P_x^2+P_y^2+P_z^2 + Bq(yP_x+xP_y)+\frac{1}{4}B^2q^2(x^2+y^2)\right]$

(4) H se zachovává $\left(\{H,H\}=0 \wedge \frac{\partial H}{\partial t}=0\right)$

p_z (z je cyklická souřadnice)

~~q~~ q náboj se zachovává, m taky (nerelativistický případ)

$$\vec{l}=\vec{r}\times\vec{p}=(yp_z-zp_y,\ zp_x-xp_z,\ xp_y-yp_x)$$

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$\{l_z,H\}=\frac{\partial l_z}{\partial x_i}\frac{\partial H}{\partial p_{x_i}}-\frac{\partial l_z}{\partial p_{x_i}}\frac{\partial H}{\partial x_i}=p_y\left(\frac{p_x}{m}-\frac{Bqy}{2m}\right)-p_x\left(\frac{p_y}{m}+\frac{Bqx}{2m}\right)$$

$$-(-y)\cdot\left[\frac{Bqp_y}{2m}+\frac{B^2q^2x}{4m}\right]-x\left[\frac{-Bqp_x}{2m}+\frac{1}{4}\frac{B^2q^2y}{2m}\right]=$$

$$=\cancel{\frac{B^2q^2xy}{4m}}\ \frac{1}{4}\frac{B^2q^2}{4m}(x+y)$$

1 bod

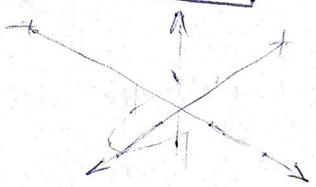
(5) $P=\frac{2p}{3\sqrt{q}}$; $Q=q^{3/2}$ $\Rightarrow dQ=\frac{3}{2}\sqrt{q}\,dq$

$$PdQ-pdq=\frac{2p}{3\sqrt{q}}\frac{3}{2}\sqrt{q}\,dq-p\,dq=0$$

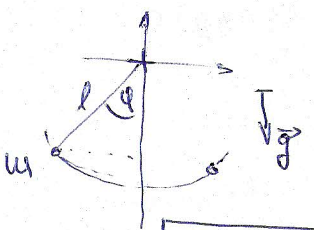
nechť $\frac{\partial H}{\partial t}=0$, pak Hamilton~~ián~~ova funkce je vytvořující funkcí $(\dot{H}=0)$

ZKOUŠKA ~~[illegible]~~!

Roman PAVELKA



1) Hamiltonova fce matematického kyvadla



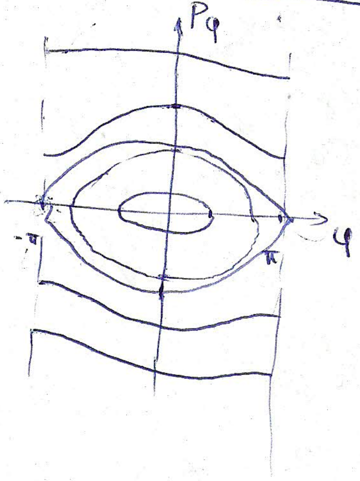
$$\mathcal{L}' = \frac{1}{2} m l^2 \dot\varphi^2 - m(1-\cos\varphi) l g$$

$$\mathcal{L} = \frac{1}{2} m l^2 \dot\varphi^2 + mlg\cos\varphi$$

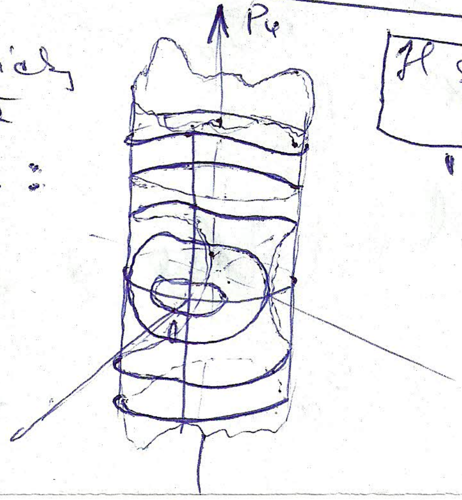
$$p_\varphi = \frac{\partial \mathcal{L}}{\dot\varphi} = m l^2 \dot\varphi \implies \mathcal{H} = \frac{1}{2} m l^2 \dot\varphi^2 - mlg\cos\varphi$$

$$\boxed{\mathcal{H}(\varphi, p_\varphi) = \frac{1}{2} \frac{p_\varphi^2}{m l^2} - lg\cos\varphi}$$

$$\boxed{\dot\varphi = \frac{p_\varphi}{m l^2} \quad ; \quad \dot p_\varphi = -mlg\sin\varphi}$$



topologicky správně $S_1 \times \mathbb{R}$:



$\boxed{\mathcal{H} \text{ se zachovává} \quad p ~~[illegible]~~ = A + B\cos\varphi}$

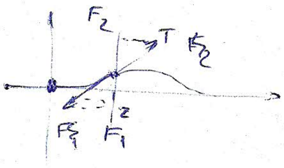
"energetické slupky"

2) $\cancel{\mathcal{L} = \frac{1}{2}\rho \dot\psi^2(z,t) - \frac{1}{2}T \psi_z^2(z,t)}$

$\frac{J}{m}$ $\quad \frac{kg}{m}\frac{m^2}{s^2} = \frac{J}{m}$ $\quad N\, m^2 = Jm$

$$\boxed{\frac{1}{2}\rho\, \psi_t^2(z,t) - \frac{1}{2} T \psi_z^2(z,t)}$$

$\frac{kg}{m}\frac{m^2}{s^2} = \frac{J}{m}$ $\quad N \frac{m^2}{m^2} = \frac{J}{m}$



$F = F_2 - F_1$

$$\frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \psi_t} + \frac{\partial}{\partial z}\frac{\partial \mathcal{L}}{\partial \psi_z} - \frac{\partial \mathcal{L}}{\partial \psi} = 0$$

$$\rho\, \psi_{tt} - T\psi_{zz} = 0$$

$$\boxed{\psi_{tt} = \frac{T}{\rho}\psi_{zz}}$$

$\frac{N}{\frac{kg}{m}} = \frac{m^2}{s^2}$

řešíme d'Alambertovou substitucí

$$\xi = x - vt$$

$$\eta = x + vt \implies \boxed{v = \sqrt{\frac{T}{\rho}}}$$

3) $p^{\sigma\mu} = \left(\frac{E}{c}, \vec{p}\right)$ $\quad \vec{p} = m\vec{v} = \gamma m_0 \vec{v}$; $p^{\sigma\mu} p_{\sigma\mu} = \frac{E^2}{c^2} - \gamma^2 m_0^2 v^2 = m_0^2 c^2$

4) m_0 je aditivní pro neinteragující částice
při vzájemné interakci se započítává vazebná energie

5) $F^{\sigma\mu\nu} = \begin{pmatrix} 0 & \frac{E_1}{c} & \frac{E_2}{c} & \frac{E_3}{c} \\ -\frac{E_1}{c} & 0 & -B_3 & B_2 \\ -\frac{E_2}{c} & B_3 & 0 & -B_1 \\ -\frac{E_3}{c} & -B_2 & +B_1 & 0 \end{pmatrix}$; $F_{\sigma\mu\nu} = \begin{pmatrix} 0 & -\frac{E_1}{c} & -\frac{E_2}{c} & -\frac{E_3}{c} \\ \frac{E_1}{c} & 0 & B_3 & -B_2 \\ \frac{E_2}{c} & -B_3 & 0 & B_1 \\ \frac{E_3}{c} & B_2 & -B_1 & 0 \end{pmatrix}$

$F'^{\mu\nu} = \alpha^{\mu}{}_{\rho}\, \alpha^{\nu}{}_{\sigma}\, F^{\rho\sigma} \Rightarrow$

$\alpha^{\mu}{}_{\nu} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$E'_x = F'^{01} = \boxed{\alpha^{0}{}_{\rho}\, \alpha^{1}{}_{\sigma}}\, F^{\rho\sigma} =$

$\rho \in \{0,1\}$
$\sigma \in \{0,1\}$
$(\rho,\sigma) \in \{(0,1),(1,0)\}$

$= \gamma^2 \cdot \left(-\frac{E_1}{c}\right) + \beta^2\gamma^2 \frac{E_1}{c} =$

$= \underbrace{\gamma^2}_{\frac{1}{(1-\beta^2)}} (\beta^2 - 1)\frac{E}{c} = -(\beta^2-1)\frac{E}{c}$

6) elektromagnetické pole jako zaření
- řešení ~~elm.~~ M. rovnic bez zdrojů
- energie je odnášena pryč
- d'Alambertovo řešení M. rovnic

7) $\Delta\varphi(|\vec{r}|) = \operatorname{div} \operatorname{grad} \varphi(r) = \operatorname{div}\left[\frac{\partial\varphi}{\partial r}\nabla r\right] = \operatorname{div} \frac{\partial\varphi}{\partial r}\frac{\vec{r}}{r} =$

$\frac{\partial\sqrt{x^2+y^2+z^2}}{\partial x} = \frac{x}{\sqrt{x^2+y^2+z^2}}$

$= \frac{\partial^2\varphi}{\partial r^2} + \frac{\partial\varphi}{\partial r}\frac{3r^2 - r^2}{r^3} = \frac{\partial^2\varphi}{\partial r^2} + \frac{\partial\varphi}{\partial r}\frac{2}{r}$

$\frac{\partial}{\partial x}\left(\frac{\partial\varphi}{\partial r}\cdot\frac{x}{r}\right) = \frac{\partial^2\varphi}{\partial r^2}\frac{x}{\sqrt{x^2+y^2+z^2}}\frac{x}{r} + \frac{\partial\varphi}{\partial r}\frac{r^2-x^2}{r^3}$

$\frac{\partial}{\partial x}\frac{x}{r} = \frac{r - x\frac{x}{\sqrt{x^2+y^2+z^2}}}{r^2} = \frac{r - \frac{x^2}{r}}{r^2} = \frac{r^2 - x^2}{r^3}$

2.2. E v elm. poli

$\frac{W}{m^3}$ $\quad \frac{J}{m^3 s} = \frac{W}{m^3}$

platí rovnice kontinuity $\vec{j}\vec{E}$+

$$\boxed{\begin{aligned} \operatorname{div} \vec{S} &= -\frac{\partial \varepsilon}{\partial t} \\ \oint_{\partial V} \vec{S} \cdot d\vec{f} &= -\frac{dE_{inV}}{dt} \end{aligned}}$$

$d\vec{f}$

V

$\vec{S} = \vec{E} \times \vec{H}$ $\quad$ W/m² $\quad \frac{V}{m}$ $\frac{A}{m}$

$[\vec{S}] = \frac{W}{m^2}$

$B = \mu_0 H$

$$\begin{aligned} \operatorname{div} \vec{E} &= \frac{\rho}{\varepsilon_0} && (1) \\ \operatorname{div} \vec{B} &= 0 && (2) \\ \operatorname{rot} \vec{E} &= -\frac{\partial B}{\partial t} && (3) \\ \operatorname{rot} \vec{B} &= \mu_0 \vec{j} + \frac{\partial \vec{D}}{\partial t} && (4) \end{aligned}$$

$$-\frac{d}{dt}\int w\, dV = \int \vec{j}\vec{E}\, dV + \int \operatorname{div} \vec{S}\, dV$$

=> ža elm. vlna

$$\frac{N}{mC} = \frac{C}{[\varepsilon_0]} \Rightarrow [\varepsilon_0] = \frac{mC^2}{N}$$

$$\begin{aligned} \operatorname{div} \vec{E} &= 0 \\ \operatorname{div} \vec{B} &= 0 \\ \operatorname{rot} \vec{E} &= -\frac{\partial B}{\partial t} \\ \operatorname{rot} \vec{B} &= \frac{\partial \vec{E}}{\partial t} \end{aligned}$$

$\vec{B} = \nabla \times \vec{A} \Rightarrow (2)$ ✓

$\operatorname{div} \vec{E} =$

$$\frac{N}{mC} = \frac{[B]}{s} \qquad \frac{N}{m^2 A} = [\mu_0] A$$

$$[B] = \frac{N}{mA}$$

$$\vec{j}\cdot\vec{E} = -\frac{\partial w}{\partial t} - \operatorname{div} \vec{S}$$

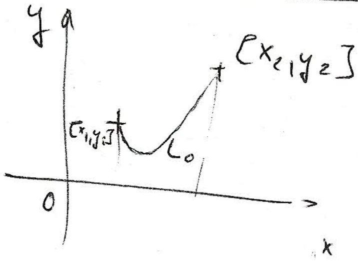
$\vec{E} \times \vec{H}$

$$\frac{mC^2}{N}\frac{N}{C}\frac{N}{mA} \cdot \frac{N m C s}{mC} = \frac{Nm}{m} s = \frac{J}{m} s =$$

$$E \quad \frac{Ns}{mC} = \frac{[B]}{s}$$

$$\frac{Ns}{mC}\frac{N}{C}\frac{mC^2}{N} = \frac{Nm}{m} s = \frac{J}{m} s = \frac{W}{m}$$

6.10 Řetězovka



hledáme $f: x \to y$ na $\langle x_1, x_2 \rangle$, že:

(1) $f(x_1) = y_1 \wedge f(x_2) = y_2$

funkcionál

(2) $L[f] = \int_{x_1}^{x_2} \sqrt{1+y'^2}\, dx \equiv L_0$

3) funkc. lin. hustota $dl = \sqrt{dx^2 + dy^2} = dx \cdot \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$E_p[f] = g \cdot \varsigma \int_{x_1}^{x_2} y \cdot \sqrt{1+y'^2}\, dx$

$E_p = \int_{x_1}^{x_2} dm \cdot g \cdot y = \int_{x_1}^{x_2} \underbrace{\varsigma \cdot dl}_{dm}\, g \cdot y =$

má v f extremálu $\Longleftrightarrow$ $\widetilde{E}_p[f] = \int_{x_1}^{x_2} y \cdot \sqrt{1+y'^2}\, dx$ má v f extr.

Děláme extremálu na varietě křivek (definovanou podm. 2)

$\Rightarrow$ Lagrangeovy multiplikátory

$$\Lambda[f] = \widetilde{E}_p[f] - \lambda(L[f] - L_0) = \int_{x_1}^{x_2} \underbrace{\sqrt{1+y'^2}\,(y-\lambda)}_{F(y,y')}\, dx - L_0$$

F nezávisí na x $\Rightarrow$ fígl :)

z variač. počtu: v f extremála $\Lambda \Leftrightarrow$

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\frac{\partial F}{\partial y'} = 0 \quad | \cdot y'$$

$$\underbrace{\frac{\partial F}{\partial y} y' + \frac{\partial F}{\partial y'} y''}_{\frac{dF}{dx}} - \underbrace{\left(\frac{\partial F}{\partial y'} y'' + \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) \cdot y'\right)}_{\frac{d}{dx}\left(\frac{\partial F}{\partial y'} y'\right)} = 0 \qquad + \frac{\partial F}{\partial y'} y'' - \frac{\partial F}{\partial y'} y''$$

$$\frac{d}{dx}\left(F - \frac{\partial F}{\partial y'} \cdot y'\right) = 0$$

$$F - \frac{\partial F}{\partial y'} \cdot y' = C$$

$$(y-\lambda)\left[\sqrt{1+y'^2} - y' \frac{1 \cdot \cancel{2} y'}{\cancel{2}\sqrt{1+y'^2}}\right] = C$$

$$(y-\lambda)\left[\frac{1 + \cancel{y'^2} - \cancel{y'^2}}{\sqrt{1+y'^2}}\right] = C$$

$$\sqrt{1+y'^2} = \frac{1}{C}(y-\lambda)$$

$$y' = \sqrt{\frac{(y-\lambda)^2}{C^2} - 1} \qquad u = \frac{y-\lambda}{C} \qquad u' = \frac{y'}{C}$$

$$C \cdot u' = \sqrt{u^2 - 1}$$

$$\frac{u'}{\sqrt{u^2-1}} = \frac{1}{C}$$

$$\int \frac{1}{\sqrt{u^2-1}} \frac{du}{\cancel{dx}} \cancel{dx} = \int \frac{1}{C} dx$$

$$C \operatorname{argcosh} \frac{y-\lambda}{C} = x + k$$

$$\frac{y-\lambda}{C} = \cosh \frac{x+k}{C} \Rightarrow$$

$$\boxed{y = C \cdot \cosh\left(\frac{x+k}{C}\right) + \lambda}$$

C, k, λ se určí z:

$y(x_1) = y_1$

$y(x_2) = y_2$

$\int_{x_1}^{x_2} \sqrt{1+y'^2}\, dx = L$