Series and Parallel Circuits

Shawn Lutch
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Abstract

In this lab, we were tasked with finding the equivalent resistance of four different circuits using Ohm's Law. We found that we were able to calculate a theoretical R_{eq} to within 10.5% of the measured R_{eq}. We calculated $R_{eq,1}$ within a difference of 10.48%, $R_{eq,2}$ within a difference of 2.55%, $R_{eq,3}$ within a difference of 3.81%, and $R_{eq,4}$ within a difference of 4.42%. These percent differences show that a small amount of systemic error affects our results.

Introduction

Ohm's Law (equation 1) describes the relationship between voltage, current, and resistance. Ohm's Law defines a direct proportionality between current I and voltage V, with a constant resistance R.

$$V = I \times R \tag{1}$$

A **resistor** is a device that obeys Ohm's Law and has some resistance R. A complex circuit that contains multiple resistors, each with its own value for R, has some **equivalent resistor**. The equivalent resistor is a resistor that would produce the same total current when the same total voltage is applied. The resistance of the equivalent resistor is referred to as the **equivalent resistance** of the circuit, denoted R_{eq}.

The equivalent resistor would obey Ohm's Law just as the actual resistors do. In a complex circuit that produces current I_{total} when a voltage V_{total} is applied, the equivalent resistance of the circuit can be calculated using a rearranged form of Ohm's Law. Using an ammeter and a fixed voltage source, we can use **equation 2** to calculate the equivalent resistance of the circuit.

$$R_{eq} = \frac{V_{total}}{I_{total}} \tag{2}$$

Without knowing the total voltage and total current of the circuit, we can still calculate the equivalent resistance of the circuit as long as we know the resistance of each resistor. We do this by performing calculations based on how the resistors are connected. For resistors connected in series, the resistances can simply be added together, as in **equation 3**. For resistors connected in parallel, we add the resistances as reciprocals, as in **equation 4**.

$$R_{eq} = \sum_{i=1}^{n} R_i \tag{3}$$

$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i} \tag{4}$$

In this lab, we demonstrate the ability to calculate equivalent resistance using two methods:

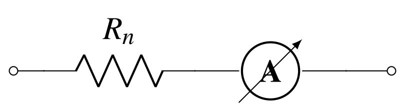
1. Using known values for V_{total} and I_{total} and equation 2
2. Using known values for $R_{1...n}$ and equations 3 and 4

Apparatus

- PASCO Capstone (data acquisition, display, analysis software)
- 850 Universal Interface
- AC/DC Electronics Laboratory
- Patch cords (x8)
- Resistors (x6)
  - 100 Ω (brown-black-brown-gold) resistors (x2)
  - 330 Ω (orange-orange-brown-gold) resistors (x2)
  - 560 Ω (green-blue-brown-gold) resistors (x2)

Experimental Procedure

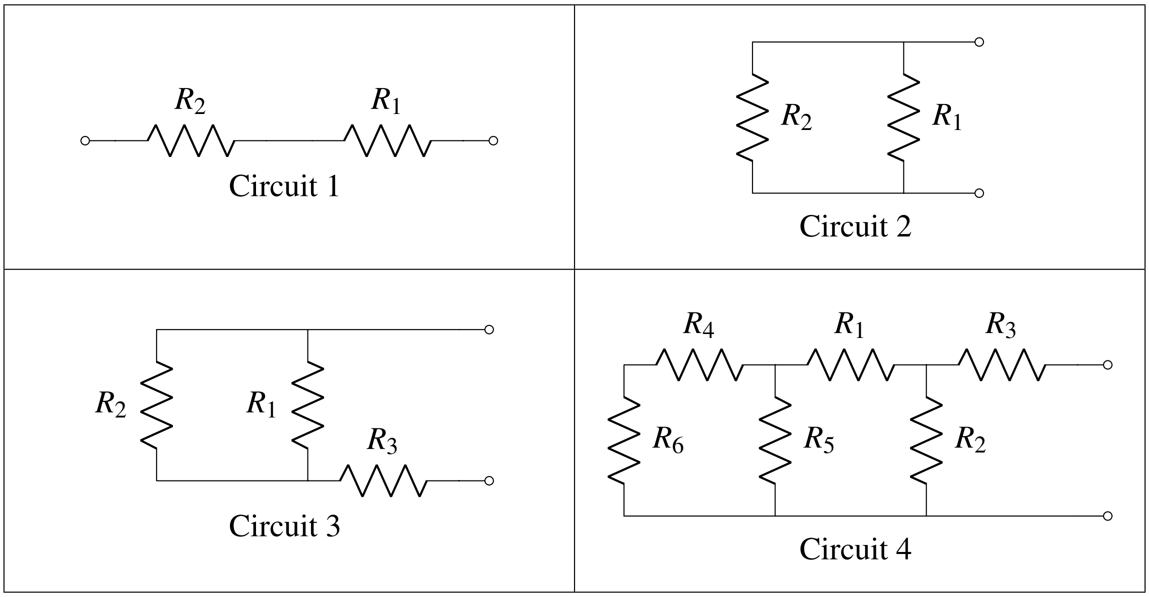
The precision of each resistor is $\pm 5\%$, as indicated by the gold band on each. For maximum precision in our values for each circuit's R_{eq}, we first needed to find the actual resistance of our six resistors. These numbers would be used later in our calculations for the theoretical R_{eq} of each circuit. The following circuit was used to calibrate each resistor $R_{1...6}$ by using the PASCO Capstone software to record the actual resistance to a precision of around $\pm 1\%$.



To account for electrical noise, which can cause error approaching 5 mA, we also calibrated the ammeter. We inserted the resistor with the closest resistance to 100Ω (which ended up being R_4) into the calibration circuit. Then, using the actual resistance value of R_4, we calculated the ideal current in a circuit powered by a range of DC voltages from 0V to 7V. We then measured and recorded the actual current of the circuit with each voltage, and notated a correction value that would be used later to account for electrical noise.

The lab manual gave us four circuit diagrams, from which we were to build the circuits. The print was small and heavily compressed, so it was difficult to read the R_n subscripts. We used the ideal resistance values given in the diagrams (not included in the reproductions on the next page) and process of elimination to figure out which resistor was supposed to be which.

Below are the four circuits, as laid out in the lab manual.



We used equations 3 and 4 to calculate the theoretical R_{eq} for each circuit. We found circuit 4 particularly troublesome, and it took some time to calculate its theoretical R_{eq}. Afterward, we built each of the circuits and measured the I_{total} of each circuit with a constant $V_{total} = 15$V DC. We used these values to calculate each circuit's experimental R_{eq}. We calculated the percent difference between theoretical and actual R_{eq} values for each circuit and called it a day.

We also discussed the difference in current and voltage between circuits in series and in parallel, as demonstrated with light bulbs. Those figures were not recorded and are not included in this lab report.

Data

Resistance Calibration

R_n	Ideal R (Ω)	Measured R (Ω)
R_1	330	315
R_2	560	535
R_3	100	95.7
R_4	100	96.8
R_5	560	537
R_6	330	314

Ammeter Calibration

Voltage (V)	Ideal I (mA)	Measured I (mA)	Correction (mA)
0	0	0.3	-0.3
1	10.3	10.6	-0.3
2	20.7	20.9	-0.2
3	31.0	31.0	0.0
4	41.3	41.3	0.0
5	51.7	51.6	0.1
6	62.0	62.1	-0.1
7	72.3	72.5	-0.2

Measured and Corrected Current

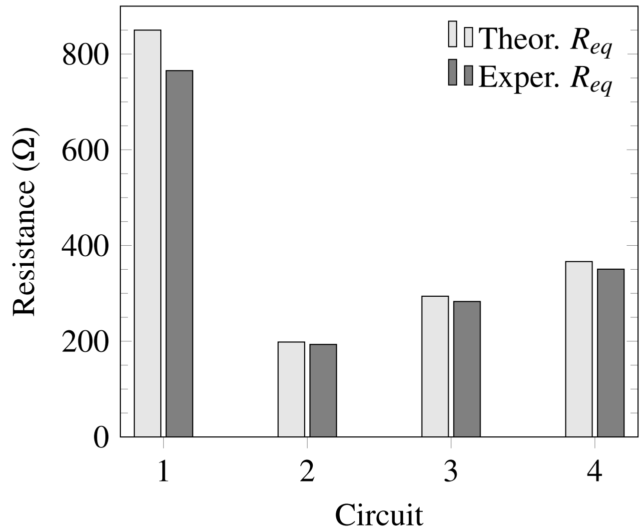
Circuit	Measured Current (A)	Corrected Current (A)
1	19.8	19.6
2	77.8	77.6
3	52.9	53.0
4	42.8	42.8

Calculations and Graphs

Calculated Resistances

Circuit	Theor. R_{eq} (Ω)	Exper. R_{eq} (Ω)	% Diff.
1	850	765	10.48
2	198.3	193.3	2.55
3	294.0	283.0	3.81
4	366.4	350.5	4.42

Resistance of Each Circuit



Discussion of Results and Error Analysis

Initially, we had used the wrong values for R_n in our theoretical R_{eq} calculations: we used the ideal resistances listed for each resistor, rather than the actual values obtained through calibration. This inflated our percent differences and was troubling, until we realized our mistake and revised our calculations.

In general, our calculated theoretical values were close to the measured experimental values within a reasonable margin of error. We found that we were able to calculate the theoretical R_{eq} with around $\pm 5\%$ difference from the experimental R_{eq}, with the exception of Circuit 1, which had a 10.48% difference between the two.

Currently, it is not clear why our theoretical R_{eq} varies so wildly from the experimental R_{eq} in Circuit 1, although the main source of error throughout the entire experiment was electrical noise from the voltage source. Although we corrected for discrepancies in current, electrical noise is not uniform, and is difficult to predict and correct for. In order to ensure the accuracy of our measured R_{eq} for Circuit 1, it would be advantageous to re-run the experiment. The theoretical R_{eq} was trivial to calculate since the circuit consisted of only two resistors in series, so it is odd that the measured R_{eq} should be so much lower.

It is possible that circuits were improperly built, though we built each circuit meticulously and ensured proper placement of resistors and patch cables.

Conclusion

The small differences in theoretical and measured R_{eq} show that both methods are sufficiently accurate for finding the equivalent resistance of a circuit in a theory environment. Errors can be reduced by reducing electrical noise and ensuring that circuits are properly built.