

Problem Set 1

Applied Stats/Quant Methods 1

Due: September 30, 2024

Instructions

- Please show your work! You may lose points by simply writing in the answer. If the problem requires you to execute commands in R, please include the code you used to get your answers. Please also include the .R file that contains your code. If you are not sure if work needs to be shown for a particular problem, please ask.
- Your homework should be submitted electronically on GitHub.
- This problem set is due before 23:59 on Monday September 30, 2024. No late assignments will be accepted.

Question 1: Education

A school counselor was curious about the average of IQ of the students in her school and took a random sample of 25 students' IQ scores. The following is the data set:

```
1 y <- c(105, 69, 86, 100, 82, 111, 104, 110, 87, 108, 87, 90, 94, 113, 112, 98,  
      80, 97, 95, 111, 114, 89, 95, 126, 98)
```

1. Find a 90% confidence interval for the average student IQ in the school.

```
1 #####  
2 # here is answer number 1  
3  
4 # As the sample size is less than 30 (n=25), it's more appropriate to use  
5 # the t-distribution rather than the z-distribution.
```

```

6 # The t-distribution better reflects the uncertainty in sampling
  distribution
7 # for small sample sizes.
8 # Use t.test function to directly calculate the confidence interval
9 CI_result <- t.test(y, conf.level = 0.90, alternative = "two.sided", mu =
  0)
10
11 # Print the results
12 cat("90% Confidence Interval:", round(CI_result$conf.int[1], 2), "to",
  round(CI_result$conf.int[2], 2), "\n")
13
14 # Print mean for comparison
15 cat("Sample Mean:", round(mean(y), 2), "\n")
16
17 #here are my conclusions
18 # We are 90% confident that the true population mean IQ for students in
  this school
19 # falls between 93.96 to 102.92. The sample mean IQ is 98.44.

```

2. Next, the school counselor was curious whether the average student IQ in her school is higher than the average IQ score (100) among all the schools in the country.

Using the same sample, conduct the appropriate hypothesis test with $\alpha = 0.05$.

```

1 #####
2 # here is answer number 2
3
4 # Perform one-tailed t-test
5 test_result <- t.test(y, mu = 100, alternative = "greater")
6
7 # Print results
8 cat("Hypothesis Test Results:\n")
9 cat("t-statistic:", round(test_result$statistic, 4), "\n")
10 cat("p-value:", round(test_result$p.value, 4), "\n")
11 cat("95% Confidence Interval:\n")
12 cat("  Lower bound:", round(test_result$conf.int[1], 2), "\n")
13 cat("  Upper bound:", round(test_result$conf.int[2], 2), "\n")
14 cat("Sample mean:", round(test_result$estimate, 2), "\n")
15
16 #here are my conclusions
17 # The calculated t-statistic is -0.5957 and the p-value is 0.7215.
18 # Since the p-value (0.7215) is greater than our significance level
  (0.05),
19 # we fail to reject the null hypothesis. There is not enough statistical
20 # evidence to conclude that the average IQ in this school is
  significantly
21 # higher than 100 (the national average).
22 # The school counselor should interpret this result as indicating that
23 # the school's average IQ is not significantly different from the
  national
24 # average of 100, based on this sample.

```

Question 2: Political Economy

Researchers are curious about what affects the amount of money communities spend on addressing homelessness. The following variables constitute our data set about social welfare expenditures in the USA.

| | |
|--------|--|
| State | 50 states in US |
| Y | per capita expenditure on shelters/housing assistance in state |
| X1 | per capita personal income in state |
| X2 | Number of residents per 100,000 that are "financially insecure" in state |
| X3 | Number of people per thousand residing in urban areas in state |
| Region | 1=Northeast, 2= North Central, 3= South, 4=West |

Explore the expenditure data set and import data into R.

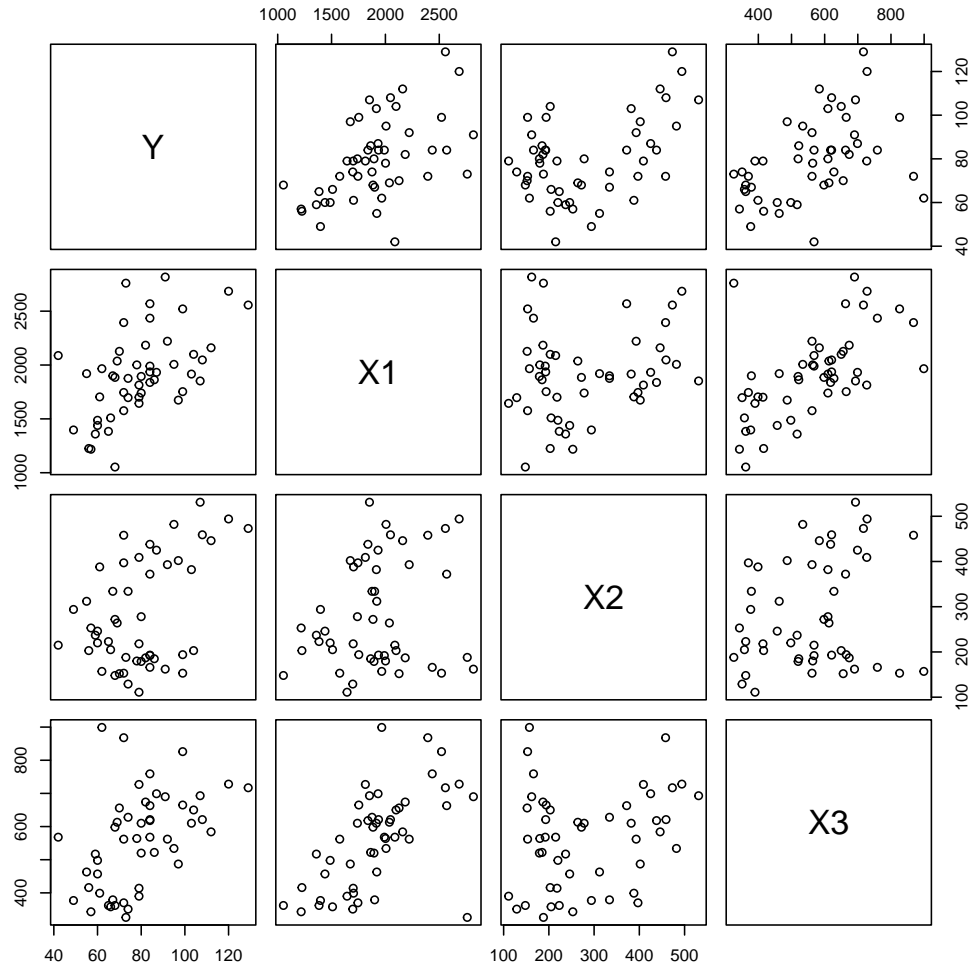
```
1 # read in expenditure data
2 expenditure <- read.table("https://raw.githubusercontent.com/ASDS-TCD/StatsI_Fall2024/main/datasets/expenditure.txt", header=T)
3 # explore the data
4 str(expenditure) # Examine the structure of the dataframe
5 summary(expenditure) # Get summary statistics for all variables
```

| STATE | Y | X1 | X2 | X3 | Region |
|----------|---------|--------------|---------------|---------------|--------------|
| Min. | : 42.00 | Min. :1053 | Min. :111.0 | Min. :326.0 | Min. :1.00 |
| 1st Qu.: | 67.25 | 1st Qu.:1698 | 1st Qu.:187.2 | 1st Qu.:426.2 | 1st Qu.:2.00 |
| Median : | 79.00 | Median :1897 | Median :241.5 | Median :568.0 | Median :3.00 |
| Mean : | 79.54 | Mean :1912 | Mean :281.8 | Mean :561.7 | Mean :2.66 |
| 3rd Qu.: | 90.00 | 3rd Qu.:2096 | 3rd Qu.:391.8 | 3rd Qu.:661.2 | 3rd Qu.:3.75 |
| Max. : | 129.00 | Max. :2817 | Max. :531.0 | Max. :899.0 | Max. :4.00 |

- Please plot the relationships among Y, X1, X2, and X3? What are the correlations among them (you just need to describe the graph and the relationships among them)?

```
1 #####
2 # here is answer number 1
3
4 #plot all of the bivariate relationships between the outcome and the
  predictors
5 #as well as the predictors themselves
6 #To visualize relationships between these variables
7 #we can create a scatter plot matrix using the pairs() function
8 pdf("plot_pairs.pdf")
9 pairs(expenditure[, c("Y", "X1", "X2", "X3")])
10 dev.off()
```

Figure 1: The Plot Relationships Among Y, X1, X2, X3



```

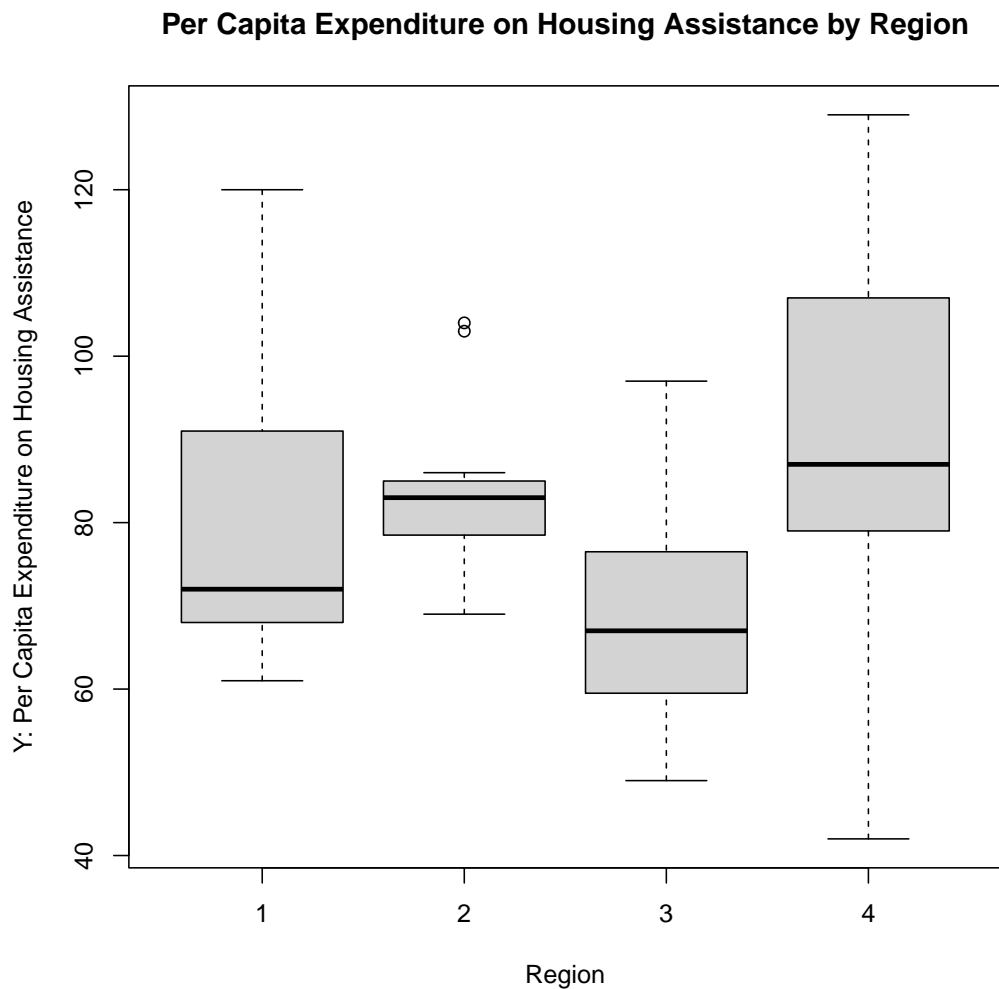
11 # Calculate correlation coefficients
12 cor(expenditure[,c("Y", "X1", "X2", "X3")])
13
14 #here are my conclusions
15 #Y and X1 show a strong positive correlation (r = 0.5317). The
    scatterplot clearly displays a positive linear relationship.
16 #Y and X2 have a moderate positive correlation (r = 0.4483). The
    scatterplot indicates a positive relationship, though not as strong as
    Y-X1.
17 #Y and X3 also exhibit a moderate positive correlation (r = 0.4637). The
    scatterplot shows a similar positive trend.
18 #X1 and X2 have a weak positive correlation (r = 0.2056). The scatterplot
    shows a more dispersed pattern with no clear linear trend.
19 #X1 and X3 demonstrate a strong positive correlation (r = 0.5925). This

```

positive relationship is clearly visible in the scatterplot.
 20 #X2 and X3 show a weak positive correlation ($r = 0.2210$). The scatterplot displays a somewhat random distribution of points.

- Please plot the relationship between Y and *Region*? On average, which region has the highest per capita expenditure on housing assistance?

Figure 2: The Plot Relationships Between Y and Region



```
1 #####
2 # here is answer number 2
3
4 #Plot the relationship between Y and Region
5 #To visualize this relationship, we can use a boxplot
6 pdf("plot_boxplot.pdf")
```

```

7 boxplot(Y ~ Region, data = expenditure,
8         main = "Per Capita Expenditure on Housing Assistance by Region",
9         xlab = "Region",
10        ylab = "Y: Per Capita Expenditure on Housing Assistance")
11 dev.off()
12
13 #here are my conclusions
14 #On average, Region 4 (West) has the highest per capita expenditure on
    housing assistance.
15 #This can be seen from the box plot, where Region 4
16 #has the highest median (represented by the thick black line in the
    middle of the box)
17 #and the largest interquartile range.

```

- Please plot the relationship between Y and $X1$? Describe this graph and the relationship. Reproduce the above graph including one more variable *Region* and display different regions with different types of symbols and colors.

```

1 #####
2 # here is answer number 3
3
4 #Plot the relationship between Y and X1
5 pdf("plot_X1&Y.pdf")
6 plot(expenditure$X1, expenditure$Y,
7       xlab="X1: Per Capita Personal Income",
8       ylab="Y: Per Capita Expenditure on Housing Assistance",
9       main="Relationship between Income and Housing Assistance Expenditure
    ")
10 # Add regression line
11 abline(lm(Y ~ X1, data=expenditure), col="red")
12 dev.off()
13
14 # Calculate correlation coefficient
15 cor(expenditure$Y, expenditure$X1)
16
17 #install stargazer packages
18 install.packages("stargazer")
19 library(stargazer)
20
21 # Run and show regression analysis
22 regression1 <- lm(Y ~ X1, data=expenditure)
23
24 output_stargazer <- function(outputFile, ...) {
25   output <- capture.output(stargazer(...))
26   cat(paste(output, collapse = "\n"), "\n", file=outputFile, append=TRUE)
27 }
28 output_stargazer("regression_output1.tex", regression1)
29
30 #Plot the relationship between Y and X1 and region
31 #display different regions with different types of symbols and colors.

```

```

32 pdf("plot_X1&Y&region.pdf")
33 plot(expenditure$X1, expenditure$Y,
34       col = expenditure$Region,
35       pch = as.numeric(expenditure$Region), # Different symbol for each
region
36       xlab = "X1: Per Capita Personal Income",
37       ylab = "Y: Per Capita Expenditure on Housing Assistance",
38       main = "Relationship between Income and Housing Assistance
Expenditure")
39 # Add legend
40 legend("topright",
41       legend = c("Northeast", "North Central", "South", "West"),
42       col = 1:4,
43       pch = 1:4)
44 dev.off()
45
46 #here are my conclusions
47 #The relationship between Y (Per Capita Expenditure on Housing Assistance
)
48 #and X1 (Per Capita Personal Income) shows a positive correlation.
49
50 #The scatter plot displays a clear upward trend, with the
51 #red regression line indicating a positive relationship. As personal
income
52 #increases, there's a general tendency for housing assistance expenditure
to increase as well.
53
54 #The correlation coefficient between X1 and Y is 0.5317212, indicating a
moderate positive correlation.
55
56 #The regression coefficient for X1 is 0.025, which is statistically
significant
57 #at the 1% level (p<0.01). This means that for every $1 increase in per
capita
58 #personal income, we expect an average increase of $0.025 in per capita
housing assistance expenditure.
59
60 #The R value is 0.283, indicating that about 28.3% of the variation in
61 #housing assistance expenditure is explained by personal income.
62
63 #The F-statistic (18.920, p<0.01) suggests that the overall model is
statistically significant.

```

Figure 3: The Plot Relationships Between Y and X1

Relationship between Income and Housing Assistance Expenditure

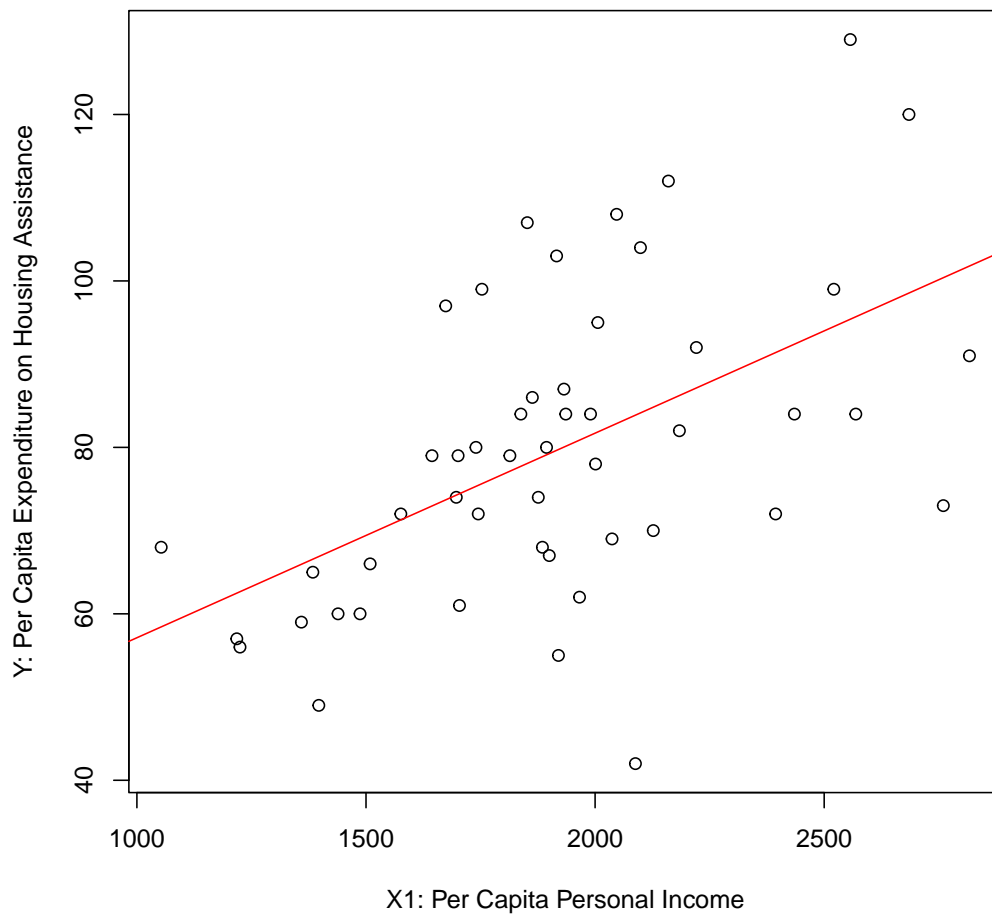


Table 1

| <i>Dependent variable:</i> | |
|----------------------------|-----------------------------|
| | Y |
| X1 | 0.025*** (0.006) |
| Constant | 32.546*** (11.034) |
| Observations | 50 |
| R ² | 0.283 |
| Adjusted R ² | 0.268 |
| Residual Std. Error | 15.836 (df = 48) |
| F Statistic | 18.920*** (df = 1; 48) |
| <i>Note:</i> | *p<0.1; **p<0.05; ***p<0.01 |

Figure 4: The Plot Relationships Between Y and X1 and Region

Relationship between Income and Housing Assistance Expenditure

