Solution of Quiz 1

Problem 1:

a. Using the inclusion-exclusion principle, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Thus,

$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= \tfrac{1}{2} + \tfrac{2}{3} - \tfrac{5}{6} \\ &= \tfrac{1}{3}. \end{aligned}$$

b. No, since $A \cap B \neq \emptyset$.

c. We can write

$$\begin{aligned} C - (A \cup B) &= \Big(C \cup (A \cup B)\Big) - (A \cup B) \\ &= S - (A \cup B) && (\text{since } A \cup B \cup C = S) \\ &= (A \cup B)^c. \end{aligned}$$

Thus

$$\begin{aligned} P\big(C - (A \cup B)\big) &= P\big((A \cup B)^c\big) \\ &= 1 - P(A \cup B) \\ &= \tfrac{1}{6}. \end{aligned}$$

d. We have

$$P(C) = P(C \cap (A \cup B)) + P(C - (A \cup B)) = \frac{5}{12} + \frac{1}{6} = \frac{7}{12}.$$

Problem 2:

Let A be the event (set) of getting exactly k red balls. To find $P(A) = \frac{|A|}{|S|}$, we need to find $|A|$ and $|S|$. First, note that $|S| = \binom{100}{20}$. Next, to find $|A|$, we need to find out in how many ways we can choose k red balls and $20 - k$ green balls. Using the multiplication principle, we have

$$|A| = \binom{30}{k}\binom{70}{20-k}.$$

Thus, we have

$$P(A) = \frac{\binom{30}{k}\binom{70}{20-k}}{\binom{100}{20}}.$$