CentraleSupélec
LSD
2016 - 2017

Proximal Denoising

To denoise an image, one can seek for a solution to the following optimization problem :

$$\underset{x\in\mathbb{R}^N}{\text{minimize}} \ \frac{1}{2}\|x-y\|^2 + f(Lx), \tag{1}$$

where $y \in \mathbb{R}^N$ is the vector corresponding to the observed image which is corrupted by noise, $L \in \mathbb{R}^{N\times N}$ is a suited orthogonal transform , and f is a separable penalty function of the form :

$$\big(\forall u = (u_i)_{1\le i\le N} \in \mathbb{R}^N\big) \qquad f(u) = \sum_{i=1}^{N} f_i(u_i)$$

where $(\forall i \in \{1,\dots,N\})$ $f_i \in \Gamma_0(\mathbb{R})$.

1. Show that this problem amounts to calculate the proximity operator of a function in $\Gamma_0(\mathbb{R}^N)$.
2. Check that this is also equivalent to calculate $L^{-1}\operatorname{prox}_f(Ly)$.
3. Download the florence.jpg image. Add to this image a white Gaussian noise with zero-mean and standard deviation equal to 30.
4. Compute the minimizer of (1) when L is a 2D orthonormal wavelet decomposition. Recall that such a decomposition generates a set of so-called approximation coefficients and sets of detail coefficients defined at different resolutions. Consider the case when

$$(\forall i \in \{1,\dots,N\}) \quad f_i = \begin{cases} 0 & \text{si } i \in \mathbb{K} \\ \varphi & \text{sinon,} \end{cases}$$

where $\mathbb{K}$ is the index set of approximation coefficients, $\varphi = \chi|\cdot|^q$ with $q \in \{1, 4/3, 3/2, 2, 3, 4\}$, and $\chi \in]0, +\infty[$ is a constant whose choice will be made in order to minimize the mean square estimation error.