Taylor Series Approximation

To conclude in few words, Taylor Series Approximation is to fit smooth function using polynomial functions. The core idea behind it is “replace curve by straight lines”.

The mathematical definition is as follows:

$$f(x) = \sum_{n=1}^{N} \frac{\mathbf{f}^n(\mathbf{x}_0)}{n!}(\mathbf{x} - \mathbf{x_0})^n + \mathbf{R}_n(x)$$

where the $\mathbf{R}_n(x)$ is the error between the original function and the function expanded by Taylor Series Approximation, which is a high order infinitely small term of the expansion.

Theoretically, we can model any function using Taylor series approximation. The basic component of Taylor Series Approximation is power function. with high order, the plotted curve mostly just represents the function with high order, ignoring low order power function. x that are near zero in high order power functions, the curve is flat, but the function value goes straight up steeply. the higher order, the steeper.While function with lower power increases flatly.

When N == 0, Taylor Series Approximation becomes the Lagrange mean value Theorem.

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