信号与图像处理基础

Basics of Digital Image Processing

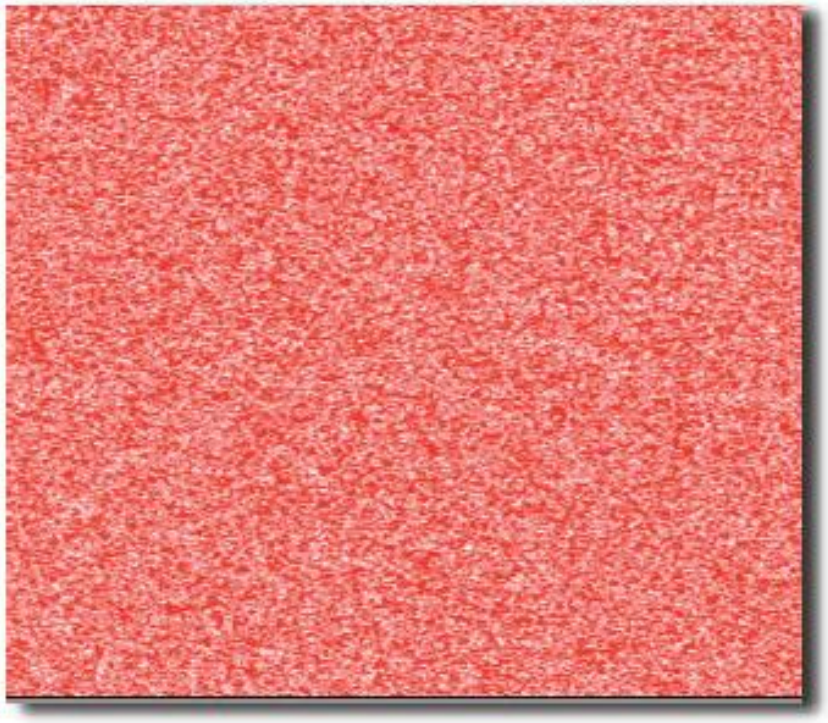
中国科学技术大学 自动化系
曹 洋

数字图像处理基础

- 图像结构
- 图像的空间变换
- 图像的亮度变换

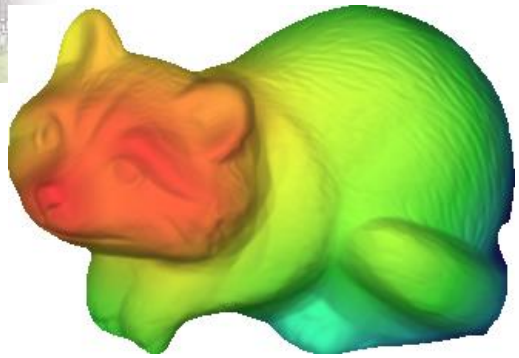
图像结构

- 图像结构



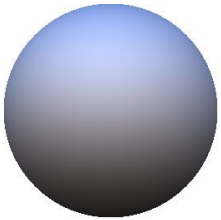
图像并不是一组随机像素的集合

图像结构



Z
表面形状

$S(Z, L)$
渐晕图



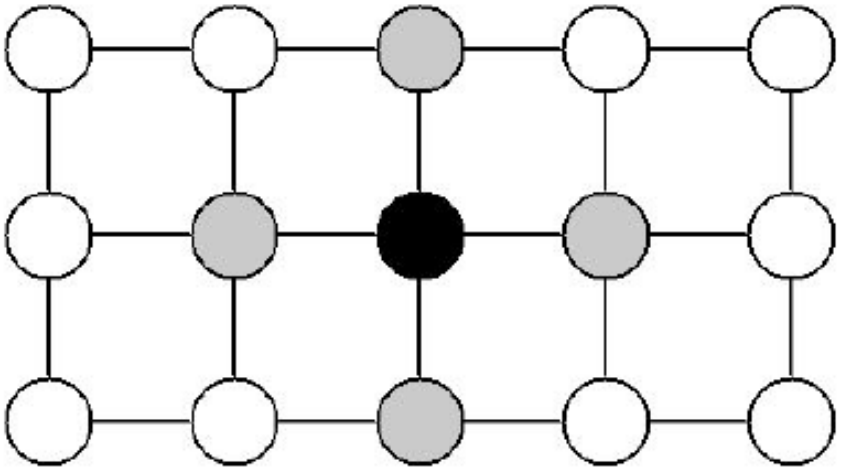
L
光照图

R
反射图

$I = R + S(Z, L)$
获取图像

图像结构

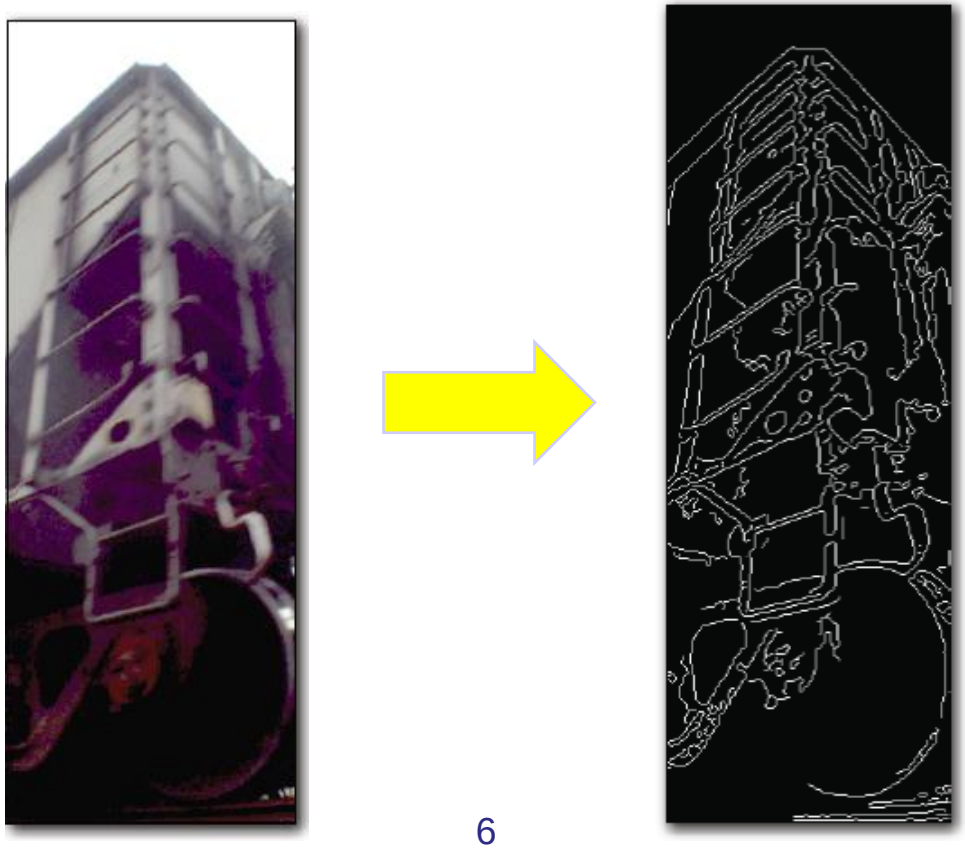
- **图像结构-邻域特性**



4-adjacency, 8-adjacency, m-adjacency

图像结构

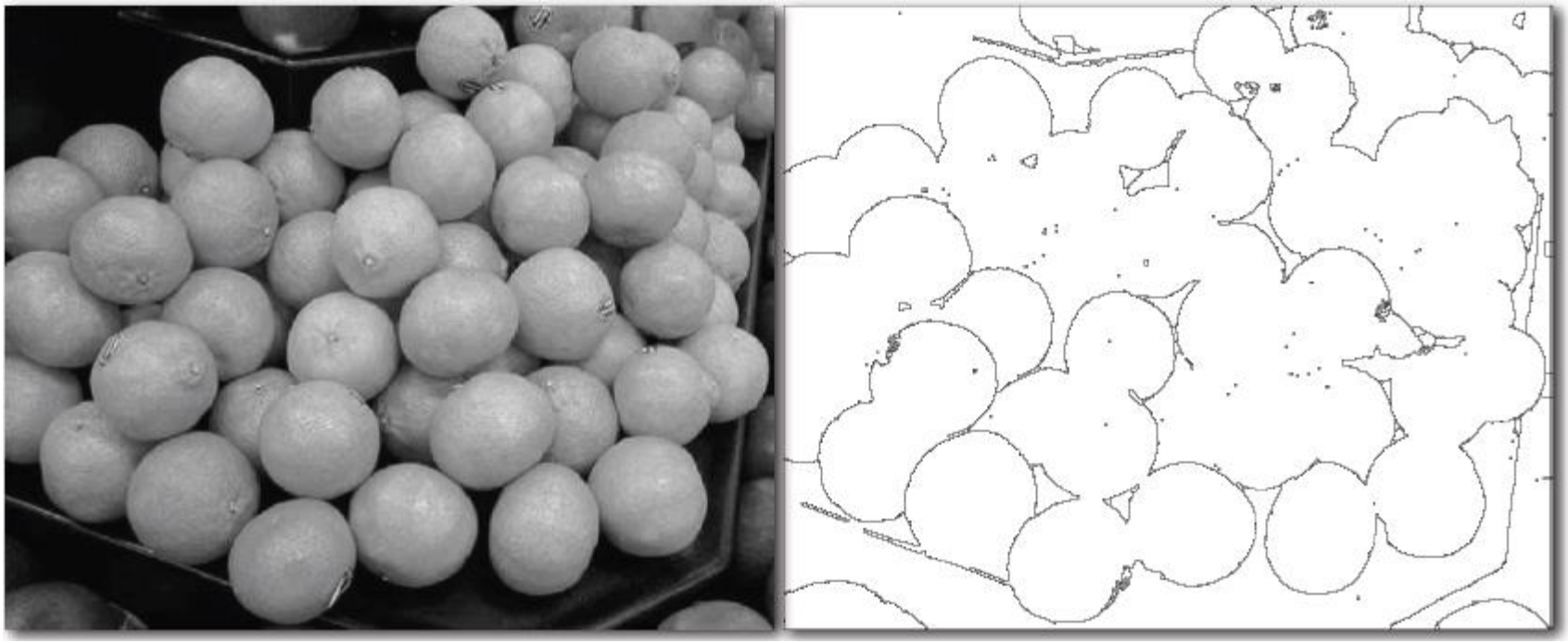
- 图像结构-边缘



Edge = Connected pixels with high gradient values

图像结构

- 图像结构-轮廓



Region = Connected set of pixels
Contour = Region boundary

图像结构

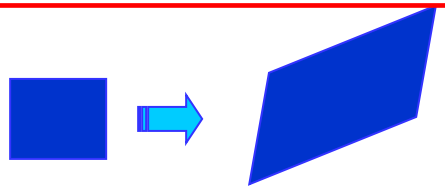
- 图像结构- 前景/背景

图像的空间变换

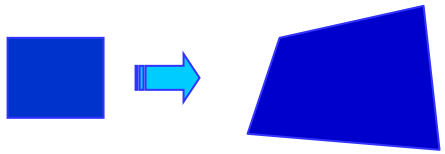
- 图像的空间变换

•Similarity
(translation,
scale, rotation)

•Affine



•Projective
(homography)



图像的空间变换

- 图像的相似性变换

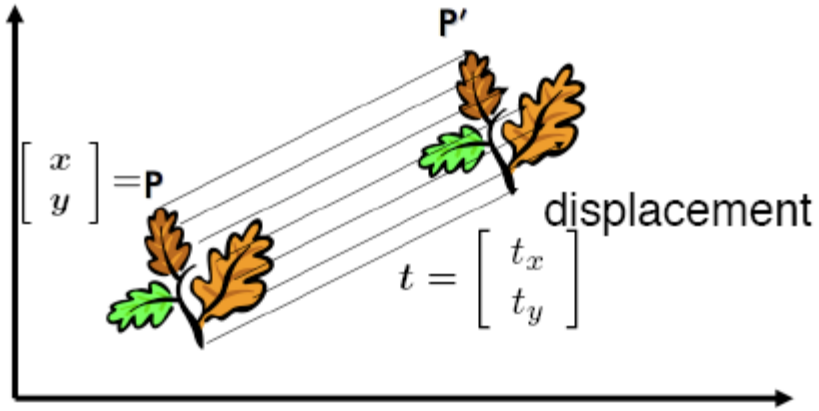
$$(x', y') = T\{(x, y)\}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = T \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous coordinates

图像的空间变换

- 图像的平移变换

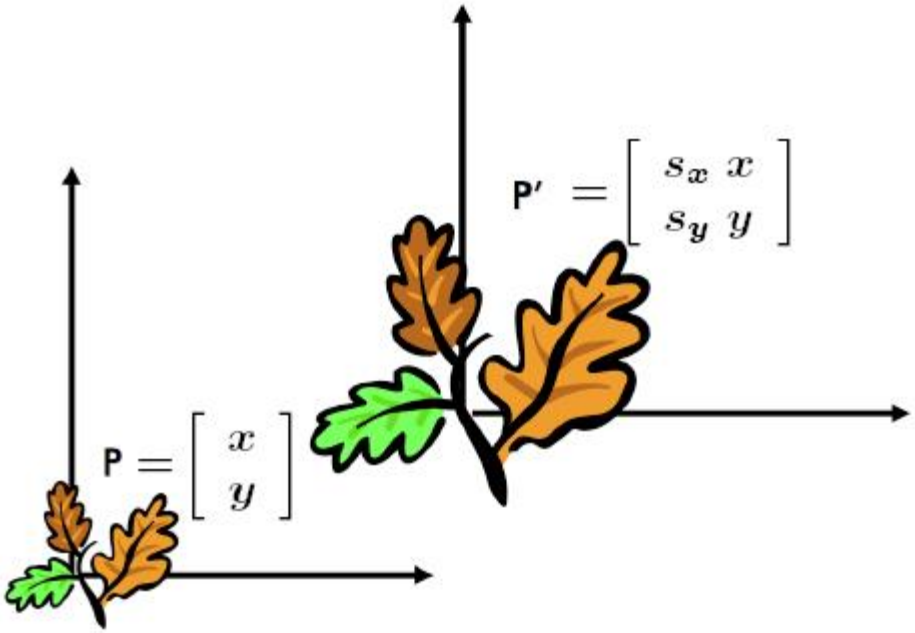


$$P' = P + \boldsymbol{t}$$

$$\Rightarrow \quad P' \rightarrow \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{translation matrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

图像的空间变换

- 图像的尺度变换

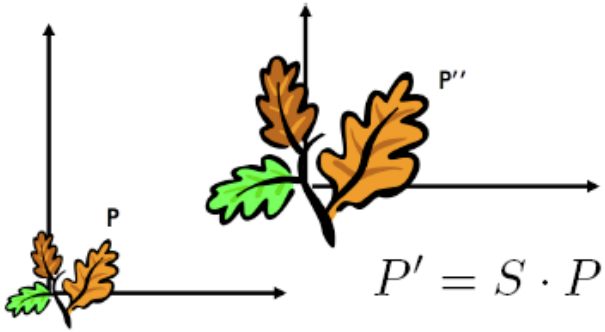


$$\begin{bmatrix} s_x\ x \\ s_y\ y \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{S} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

scaling matrix

图像的空间变换

- 图像的平移+尺度变换



Is the ordering important?

$$P' = S \cdot P$$

$$P'' = T \cdot P'$$

$$\Rightarrow \quad P'' = (T \cdot S) \cdot P$$

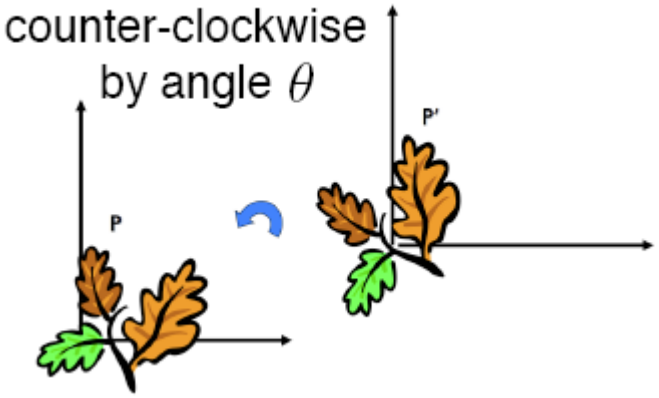
$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

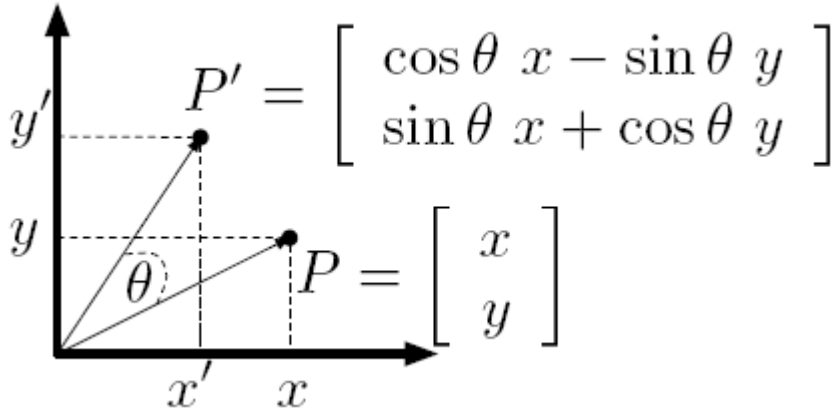
$$A = \begin{bmatrix} s_x & 0 & t_x \\ 0 & s_y & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

scaling + translation matrix

图像的空间变换

- **图像的旋转变换**





$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

rotation matrix

图像的空间变换

- **图像的旋转+平移+尺度变换**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{t}} \underbrace{\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{S} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

rotation matrix　translation matrix　scaling matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} R\,S & \boldsymbol{t} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

图像的空间变换

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

至少需要多少个点对才能保证求出矩阵T ？

图像的空间变换

$$\begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

$$x \cdot t_{11} + y \cdot t_{12} + 1 \cdot t_{13} + 0 \cdot t_{21} + 0 \cdot t_{22} + 0 \cdot t_{23} = x'$$

$$0 \cdot t_{11} + 0 \cdot t_{12} + 0 \cdot t_{13} + x \cdot t_{21} + y \cdot t_{22} + 1 \cdot t_{23} = y'$$

$$\begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix} \begin{bmatrix} t_{11} \\ t_{12} \\ t_{13} \\ t_{21} \\ t_{22} \\ t_{23} \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

图像的空间变换

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ \vdots & & & & & \\ x_N & y_N & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_N & y_N & 1 \end{bmatrix} \begin{bmatrix} t_{11} \\ t_{12} \\ t_{13} \\ t_{21} \\ t_{22} \\ t_{23} \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ \vdots \\ x'_N \\ y'_N \end{bmatrix}$$

图像的空间变换

$$\underbrace{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ \vdots & & & & & \\ x_N & y_N & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_N & y_N & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} t_{11} \\ t_{12} \\ t_{13} \\ t_{21} \\ t_{22} \\ t_{23} \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ \vdots \\ x'_N \\ y'_N \end{bmatrix}}_{B}$$

图像的空间变换

$$AX = B$$

$$\text{if } \det(A) \neq 0 \Rightarrow X = A^{-1}B$$

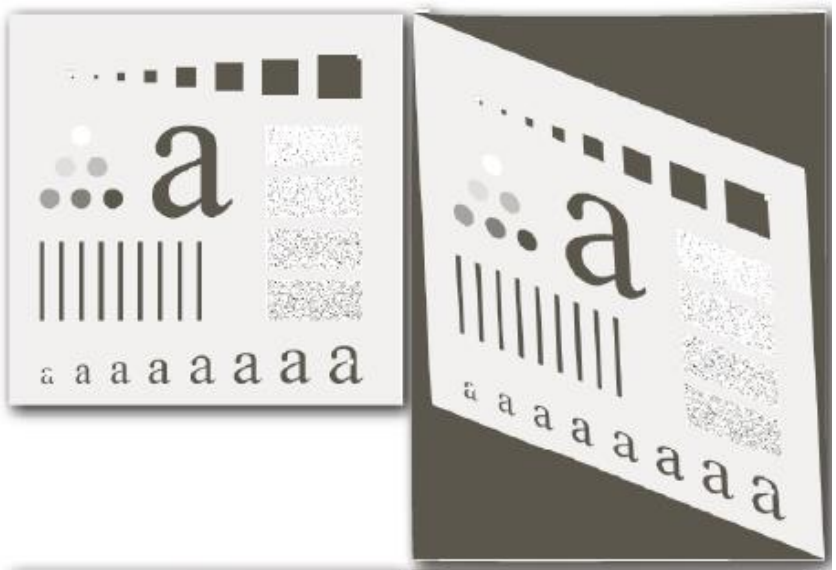
or

$$\text{if } \det(A^T A) \neq 0 \Rightarrow X = (A^T A)^{-1} A^T B$$

最小二乘法的矩阵形式

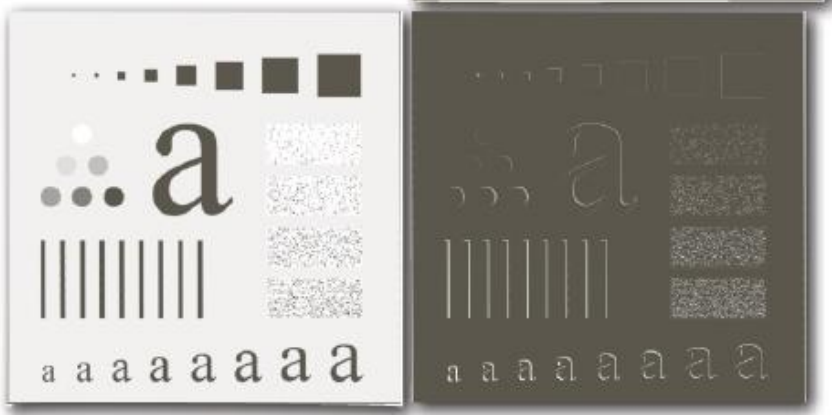
图像的空间变换

输入图像
（变换前）



输出图像
（变换后）

反变换图像



估计误差

图像的空间变换

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\left[\begin{array}{c\|c} \boldsymbol{I} & \boldsymbol{t} \end{array}\right]_{2\times3}$	2	orientation	
rigid (Euclidean)	$\left[\begin{array}{c\|c} \boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times3}$	3	lengths	
similarity	$\left[\begin{array}{c\|c} s\boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times3}$	4	angles	
affine	$\left[\begin{array}{c} \boldsymbol{A} \end{array}\right]_{2\times3}$	6	parallelism	
projective	$\left[\begin{array}{c} \tilde{\boldsymbol{H}} \end{array}\right]_{3\times3}$	8	straight lines	

图像的空间变换

- 基于图像插值的空间变换

original

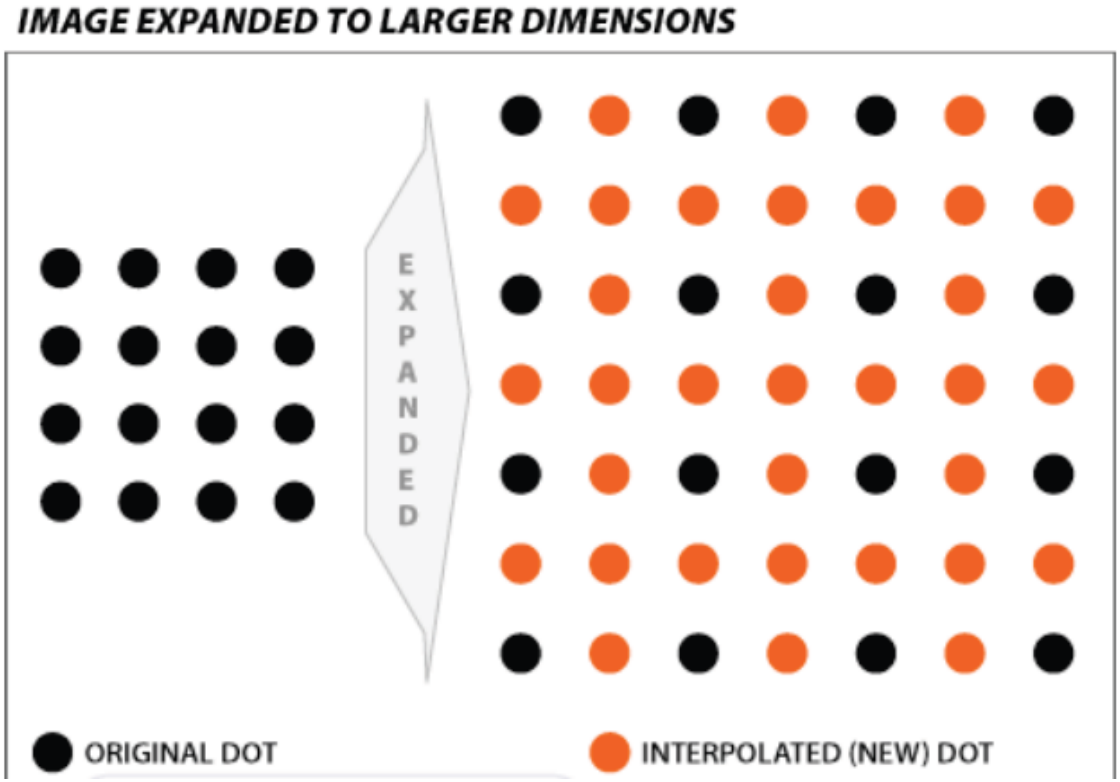
resampling

shrinking

zooming

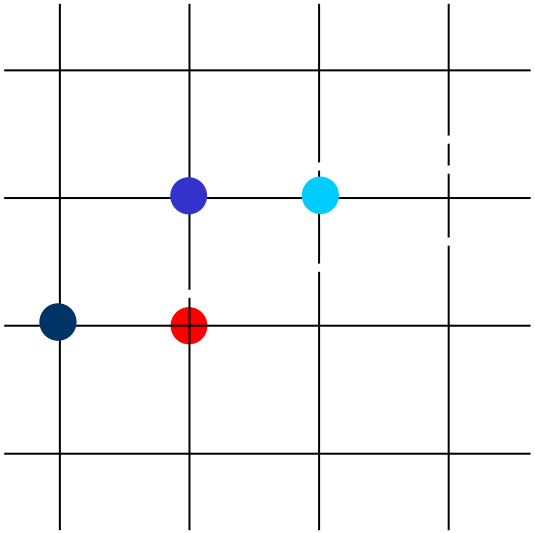
图像的空间变换

- 基于图像插值的空间变换

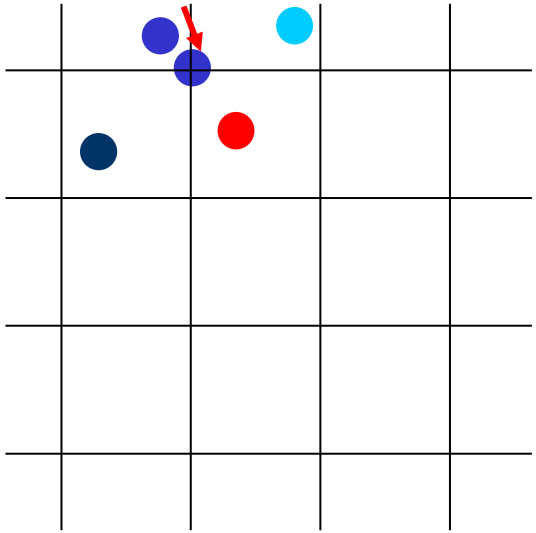


图像的空间变换

- 图像插值-最近邻插值



f(x', y')

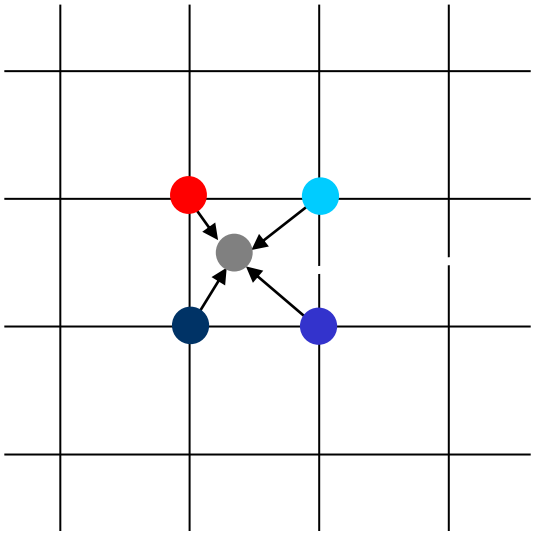


g(x, y)

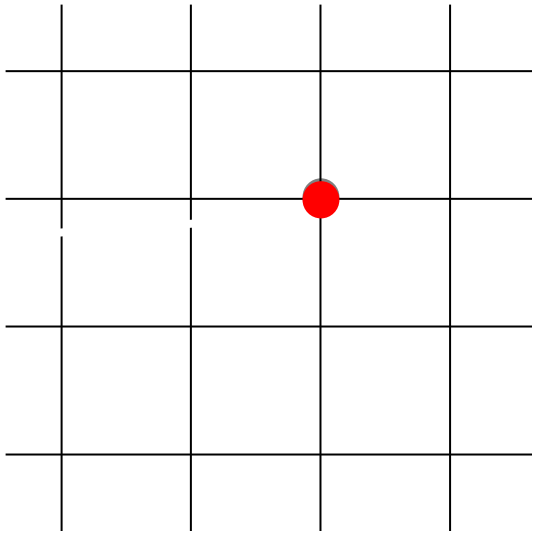
前向映射

图像的空间变换

- 图像插值-最近邻插值



f(x', y')



g(x, y)

后向映射

图像的空间变换

- **图像插值-双线性插值**

$$f(x,y) = ax + by + cxy + d$$

coefficients that need to be estimated

图像的空间变换

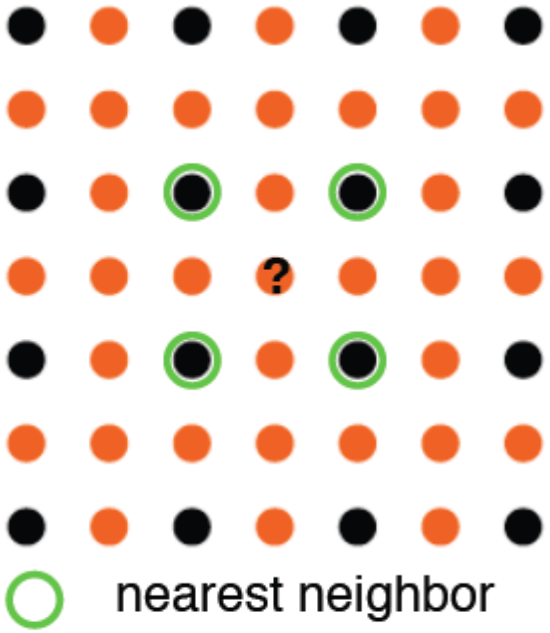
- 图像插值-双线性插值

$$f(x,y) = ax + by + cxy + d$$

coefficients that need to be estimated

图像的空间变换

- 图像插值-双线性插值

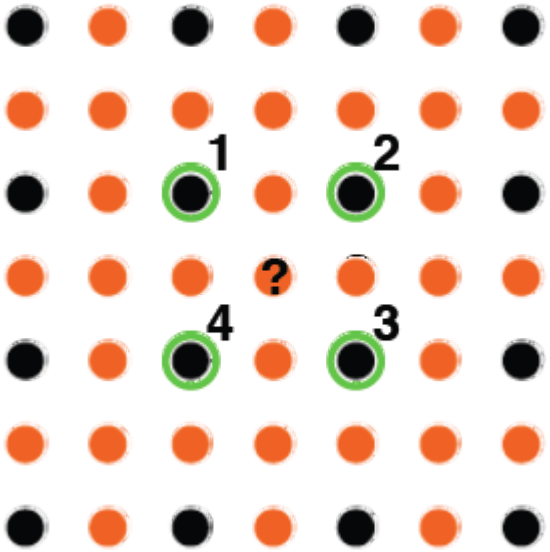


$$f(x,y) = ax + by + cxy + d$$

coefficients that need to be estimated

图像的空间变换

- 图像插值-双线性插值



$$f(x,y) = ax + by + cxy + d$$

coefficients that need to be estimated

$$ax_1 + by_1 + cx_1y_1 + d = f(x_1, y_1)$$

$$ax_2 + by_2 + cx_2y_2 + d = f(x_2, y_2)$$

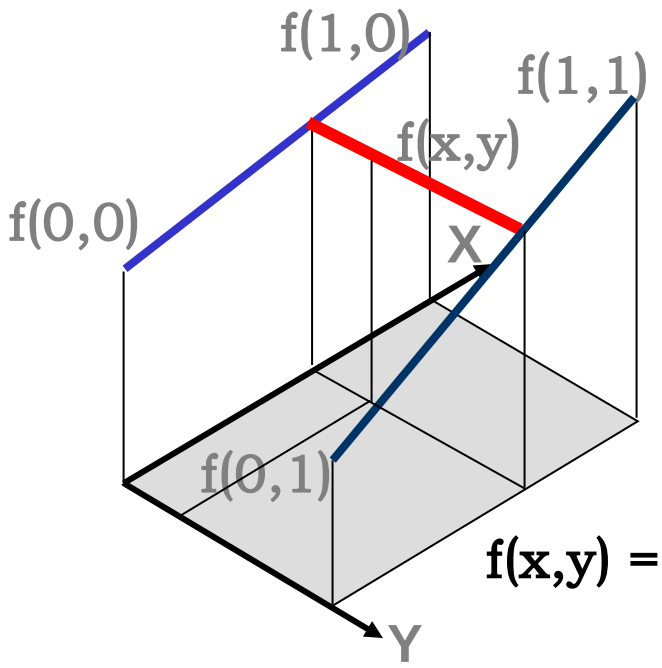
$$ax_3 + by_3 + cx_3y_3 + d = f(x_3, y_3)$$

$$ax_4 + by_4 + cx_4y_4 + d = f(x_4, y_4)$$

$$a,\ b,\ c,\ d$$

图像的空间变换

- 图像插值-双线性插值



$$f(x,0) = f(0,0)+x[f(1,0)-f(0,0)]$$
$$f(x,1) = f(0,1)+x[f(1,1)-f(0,1)]$$
$$f(x,y) = f(x,0)+y[f(x,1)-f(x,0)]$$

$$f(x,y) = [f(1,0)-f(0,0)]x+[f(0,1)-f(0,0)]y$$
$$+[f(1,1)+f(0,0)-f(0,1)-f(1,0)]xy+f(0,0)$$

图像的空间变换

- **最近邻插值 vs 双线性插值**

zoomed in images

nearest neighbor

bilinear

图像的空间变换

- **图像插值**

- Bilinear N = 1
- Bicubic N = 3

$$f(x,y) = \sum_{i=0}^{N}\sum_{j=0}^{N} a_{ij}x^i y^j$$

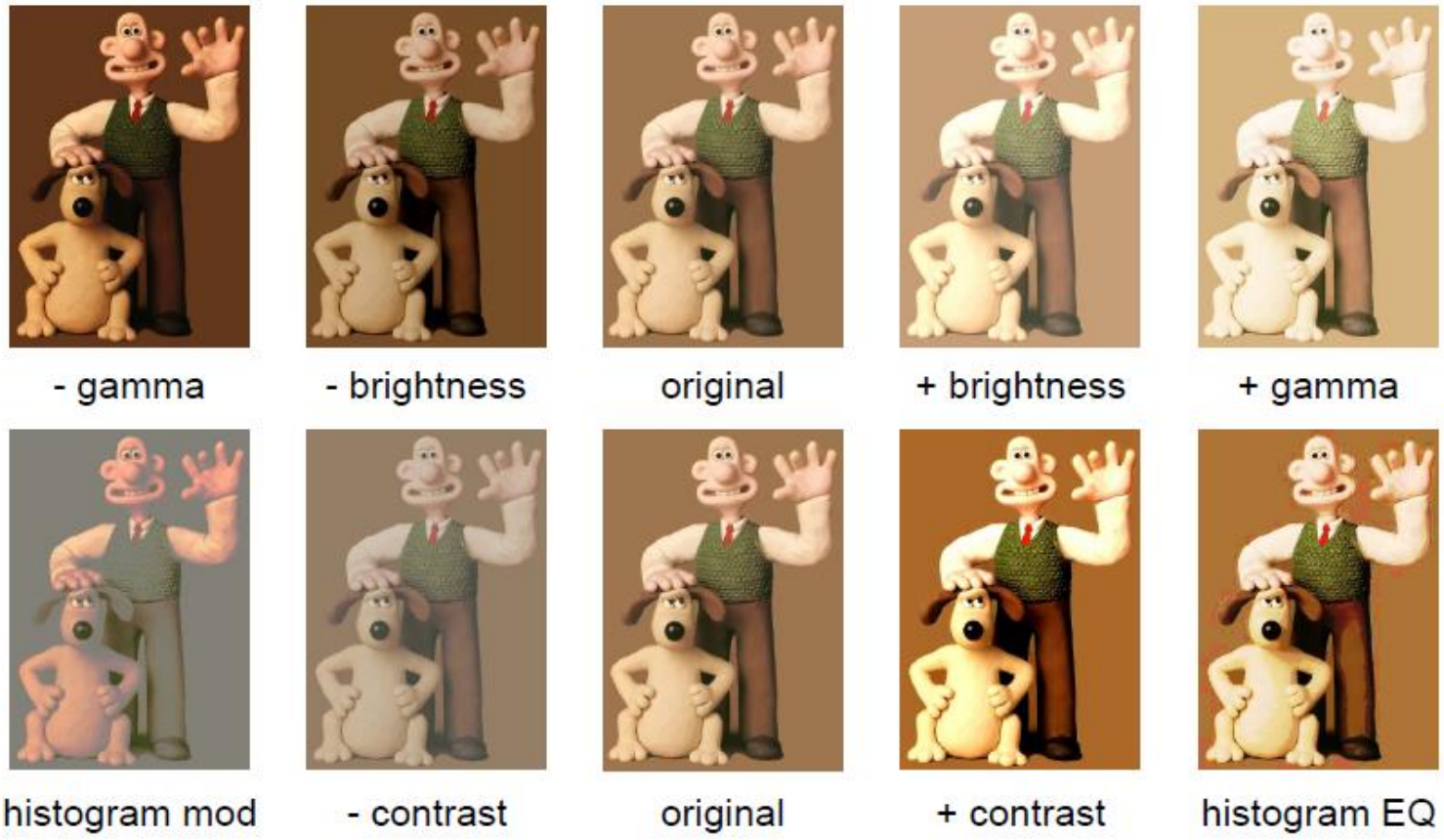
new locations

new locations

estimated from the known neighboring locations

图像的亮度变换

- 图像的亮度变换



图像的亮度变换

- **图像的亮度变换**

$$g(x,y) = T\{f(x,y)\}$$

transformed input

input

图像的亮度变换

- 亮度归一化（尺度变换）

$$g(x,y) = K\frac{f(x,y) - \min[f(x,y)]}{\max[f(x,y)] - \min[f(x,y)]}$$

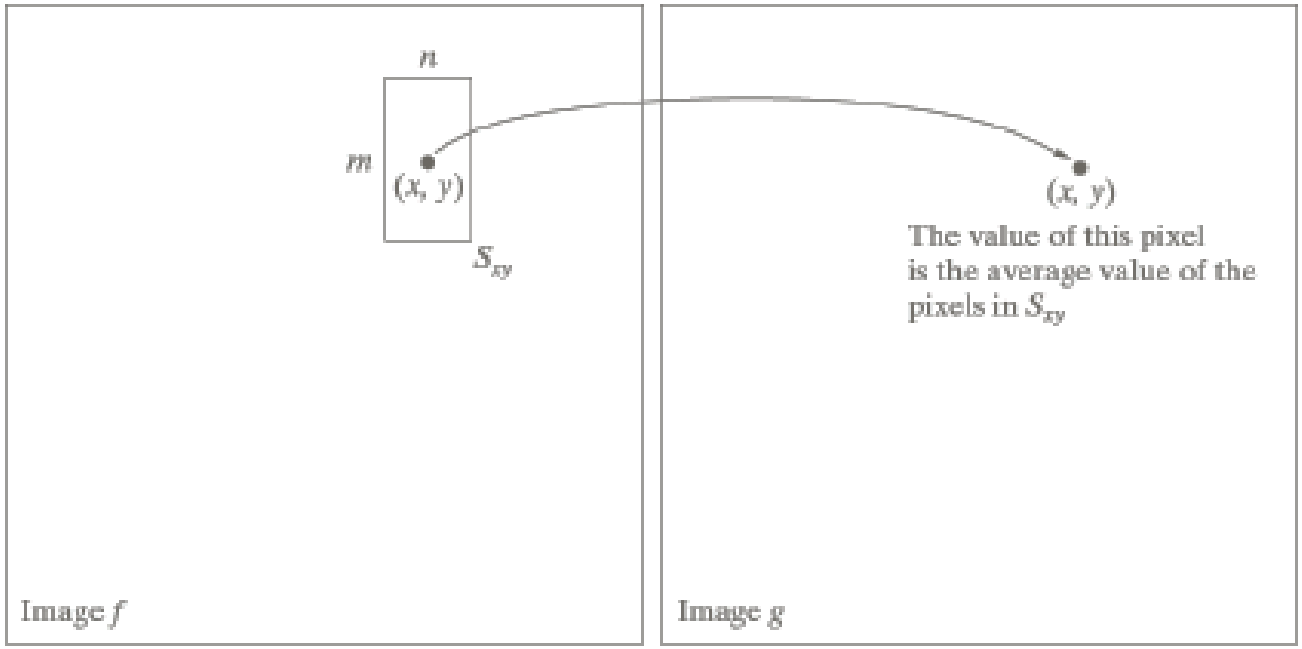
input

output

图像的亮度变换

- 亮度局部平均（平滑处理）

$$g(x,y) = \frac{1}{mn} \sum_{(x',y') \in N_{xy}} f(x',y')$$



图像的亮度变换

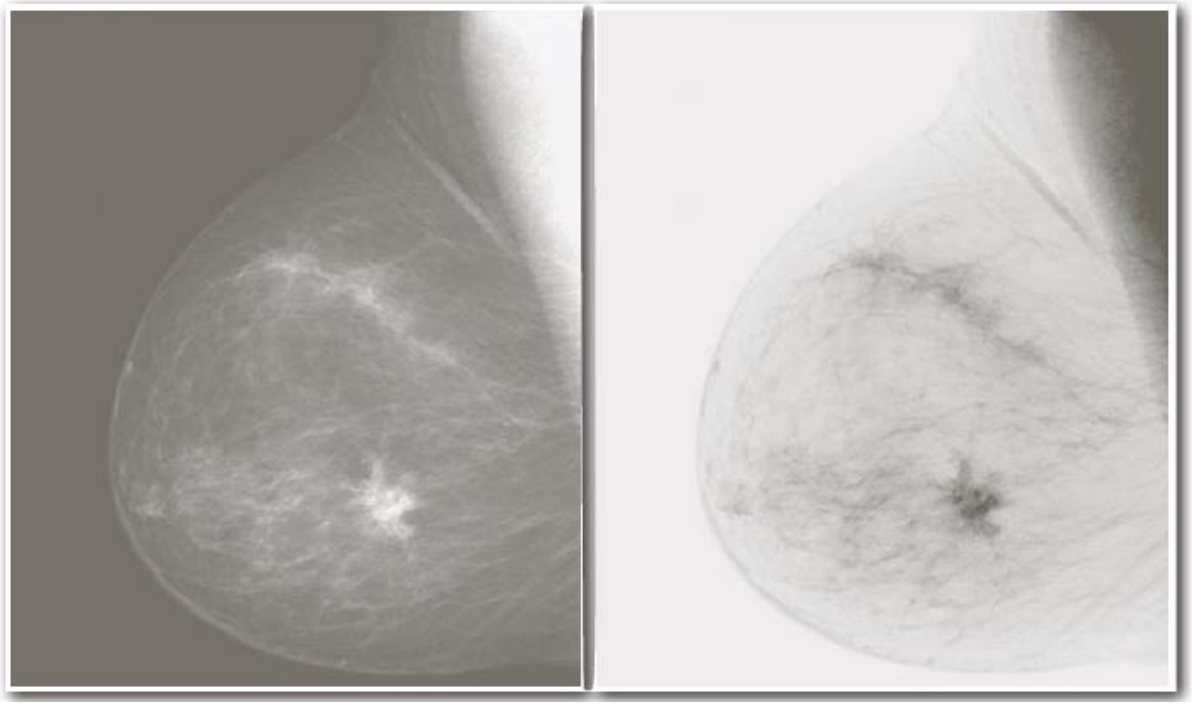
- **亮度局部平均（平滑处理）**

$$g(x,y)=\frac{1}{mn}\sum_{(x',y')\in N_{xy}} f(x',y')$$

图像的亮度变换

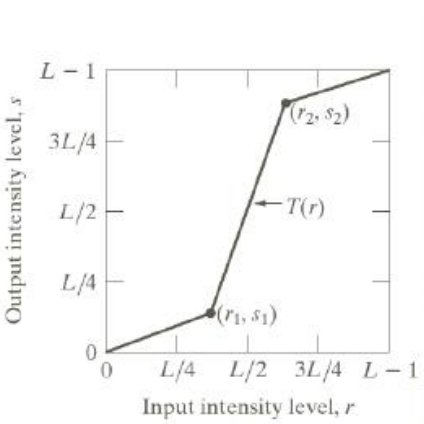
- 图像取反

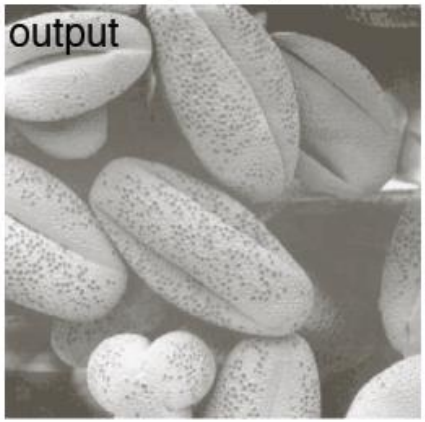
$$g(x,y) = L - 1 - f(x,y)$$

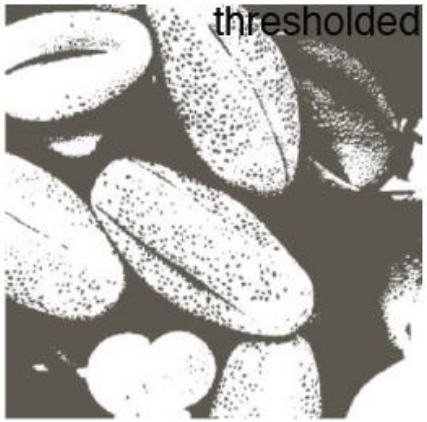


图像的亮度变换

- 图像亮度拉伸



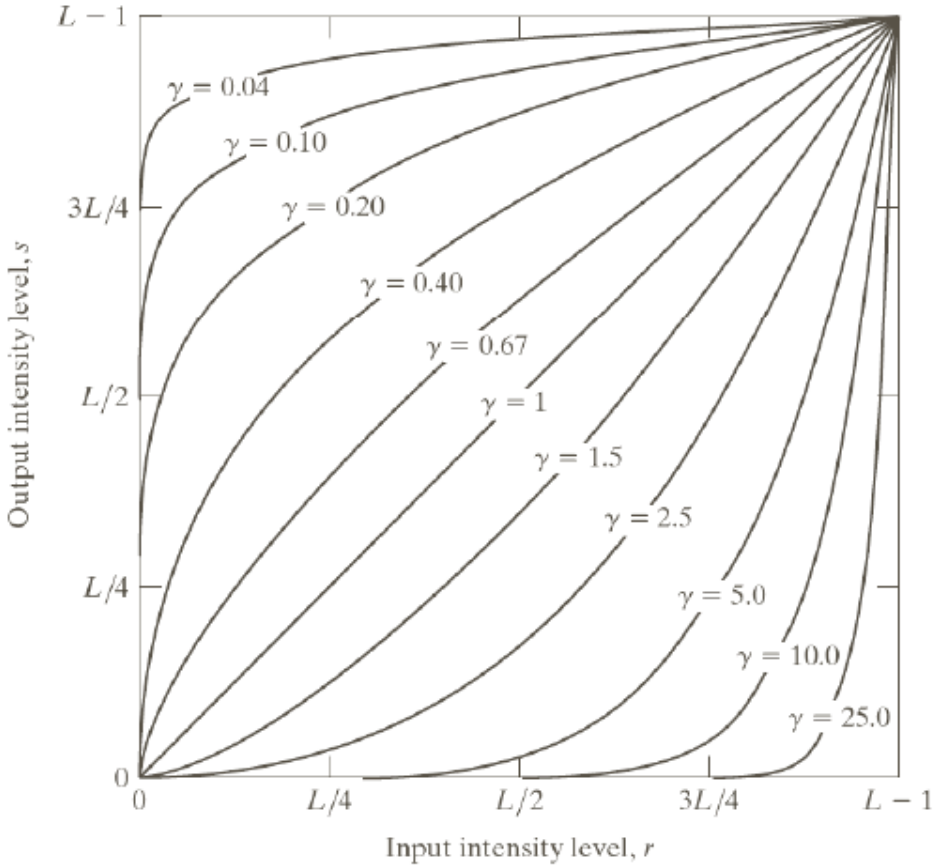




图像的亮度变换

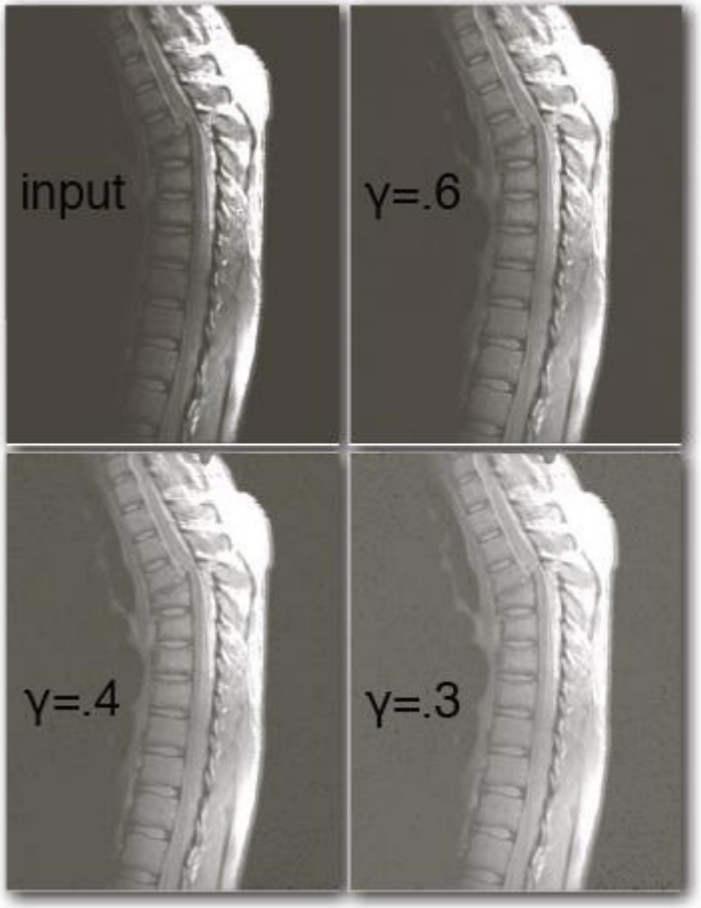
- 图像伽马校正

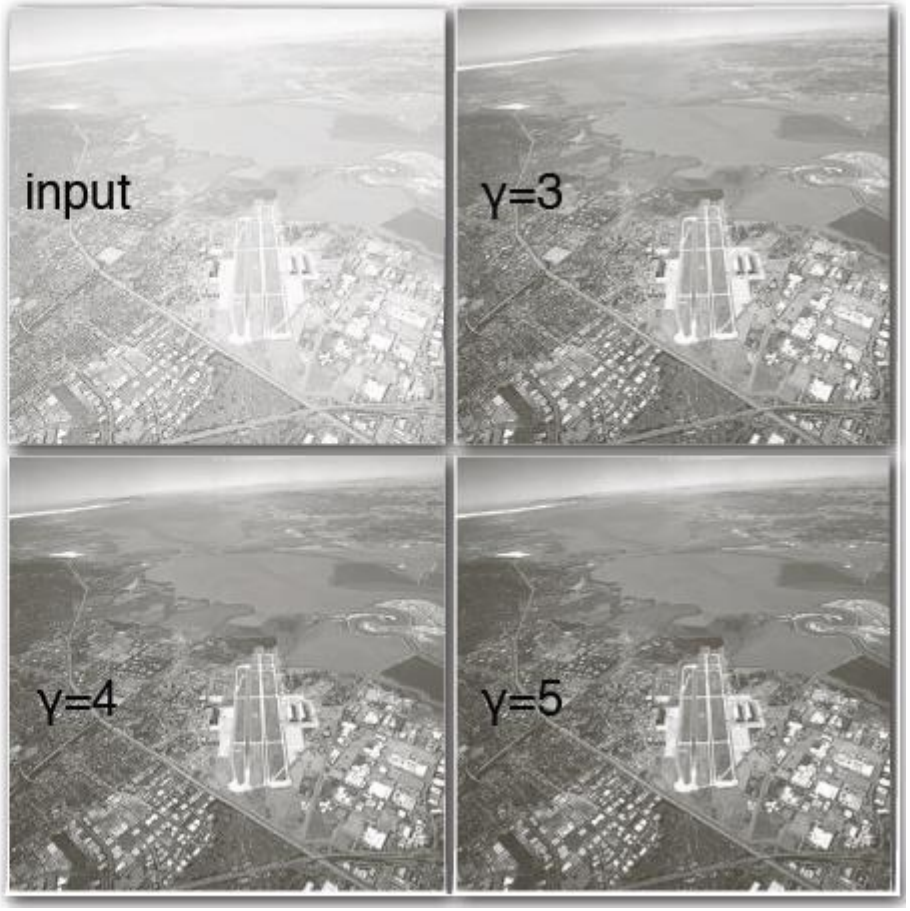
$$s = cr^{\gamma}$$



图像的亮度变换

- 图像伽马校正

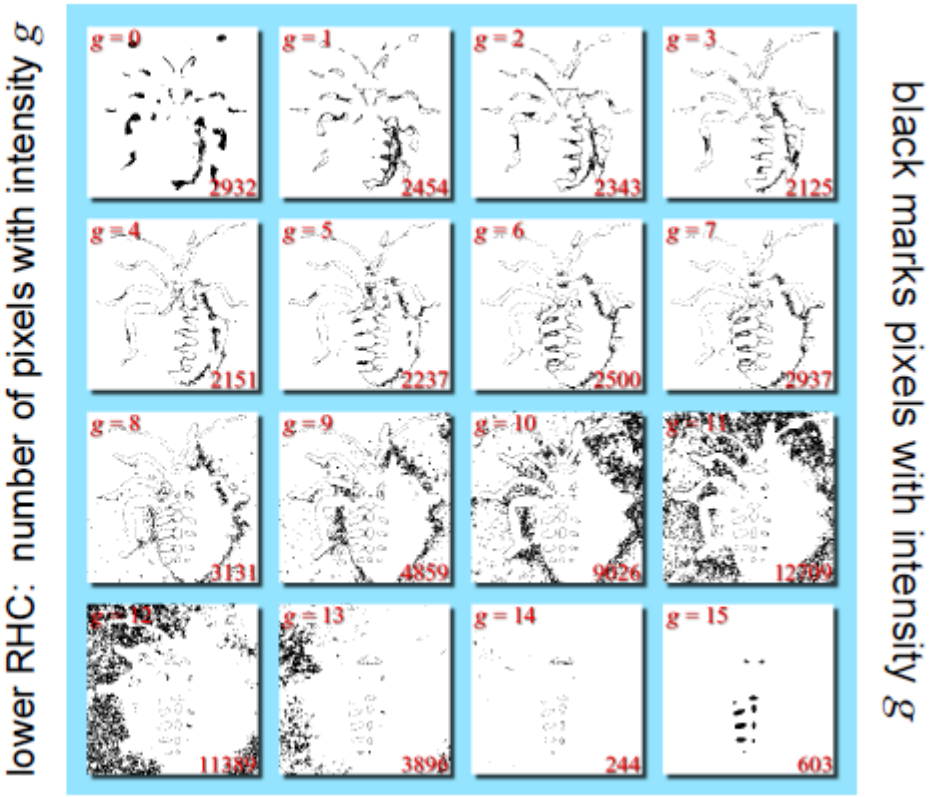




图像的亮度变换

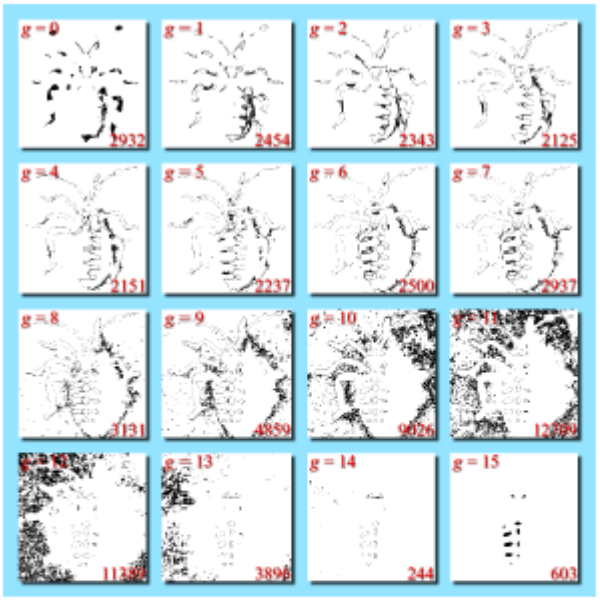
- 图像直方图

16-level (4-bit) image



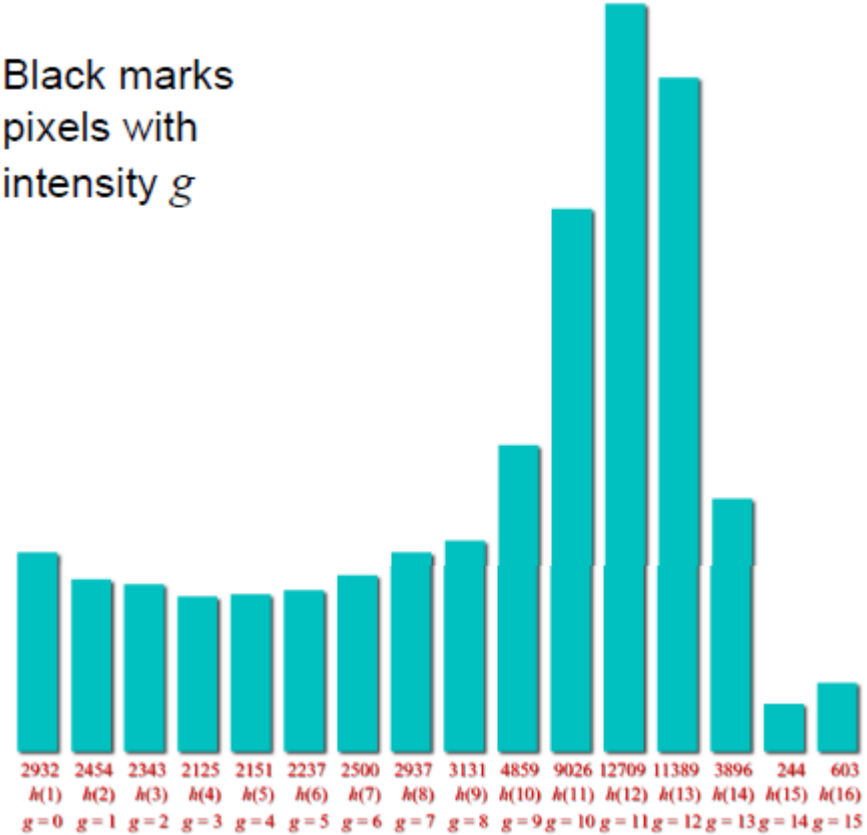
图像的亮度变换

- 图像直方图



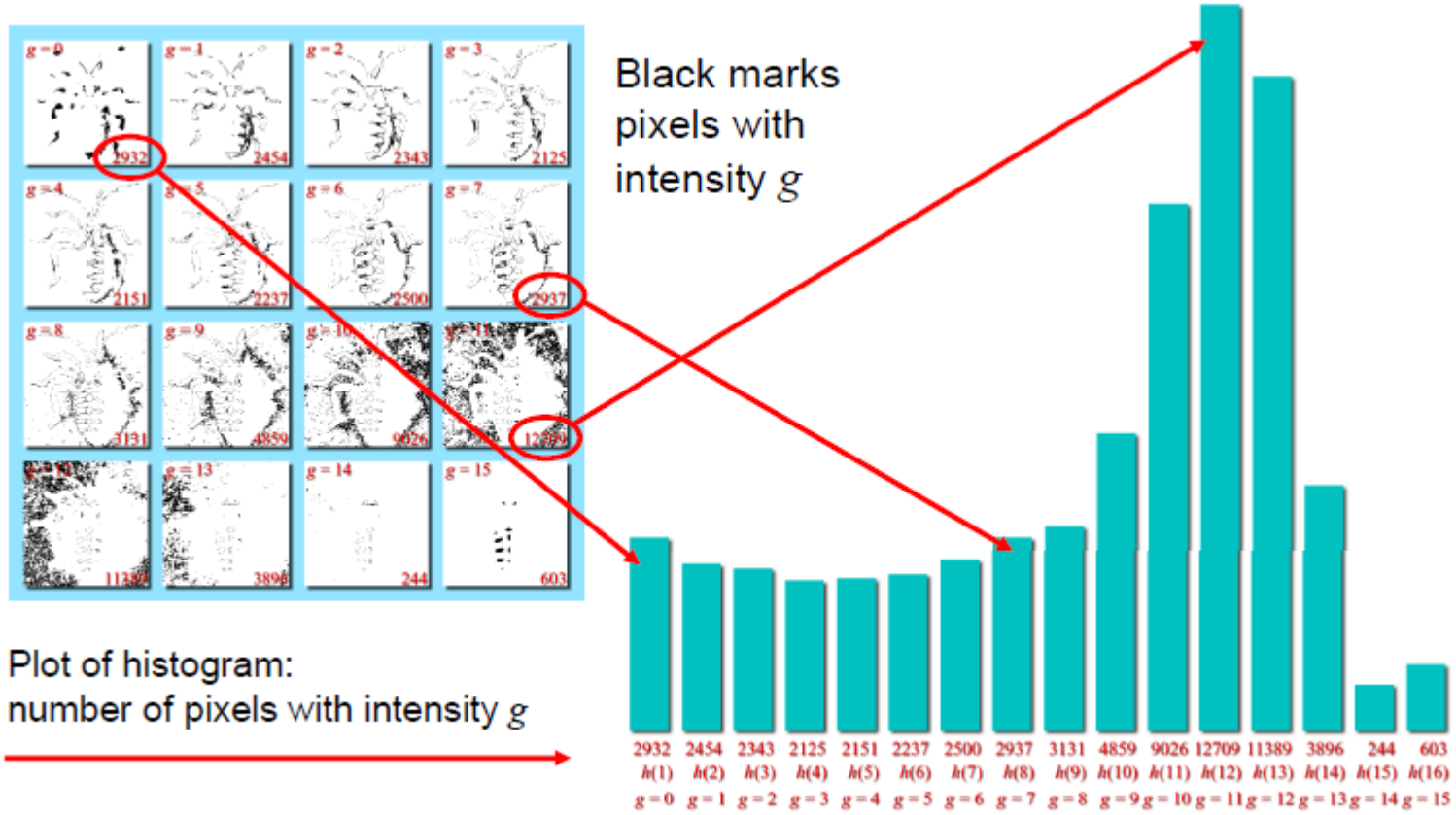
Plot of histogram:
number of pixels with intensity g

Black marks
pixels with
intensity g



图像的亮度变换

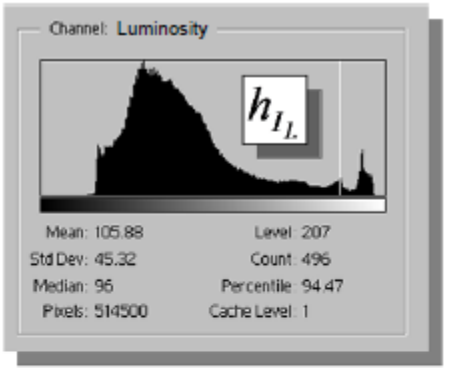
- 图像直方图

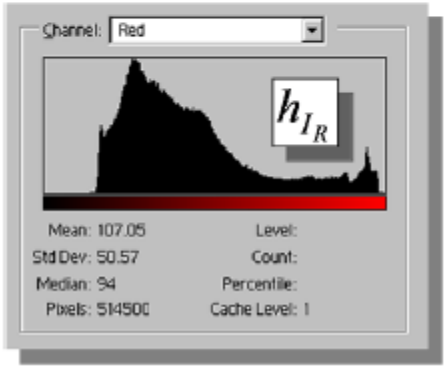


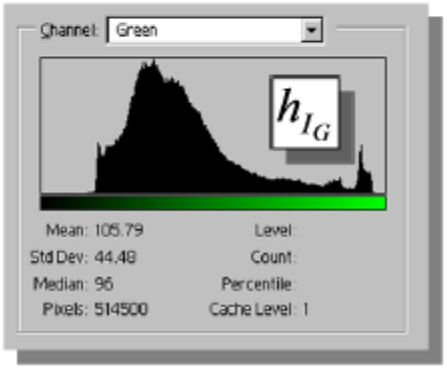
图像的亮度变换

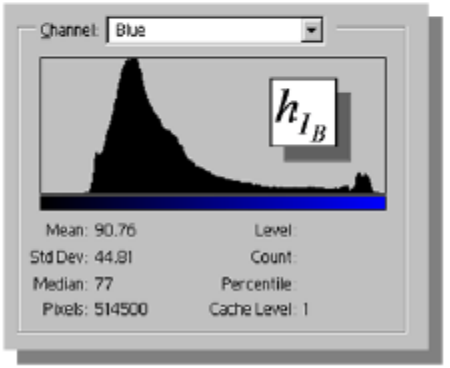
- 彩色图像直方图

There is one histo-gram per color band R, G, & B. Luminosity histogram is from 1 band = (R+G+B)/3









图像的亮度变换

- 直方图均衡化

input

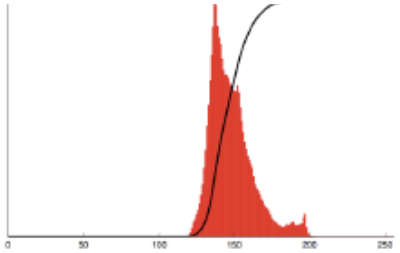
output

- 通过增加直方图分布区域，提高图像对比度
- 会产生**artifact**

图像的亮度变换

- 直方图均衡化

input



intensity-level histogram (red)
cumulative histogram (black)

output

$$h(r_i \le r < r_{i+1}) = \frac{n_i}{n}$$

n_i: number of pixels within bin *i*

bin *i*

n: total number of pixels

图像的亮度变换

- 直方图均衡化

input

output

$s = T(r)$

output intensity

input intensity

$p(s)$

$p(r)$

flat distribution of intensities

flat histogram of intensities

narrow histogram of intensities

narrow distribution of intensities

图像的亮度变换

- 直方图均衡化(PDF 和CDF)

PDF：Probability Distribution Function 概率密度函数
CDF：Cumulative Distribution Function累积分布函数

$$P(X \le x_0) = \int_{-\infty}^{x_0} p_X(x)dx$$

cumulative distribution function

probability density function

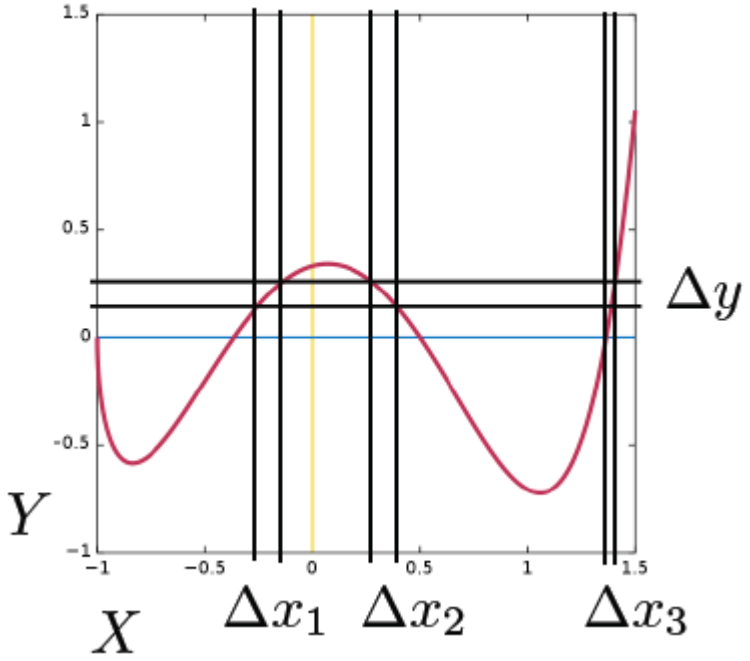
example: $p_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$ Gauss

图像的亮度变换

- 直方图均衡化(PDF 和CDF)

$X,\ Y$ random variables

$$\boldsymbol{Y = T(X)}$$



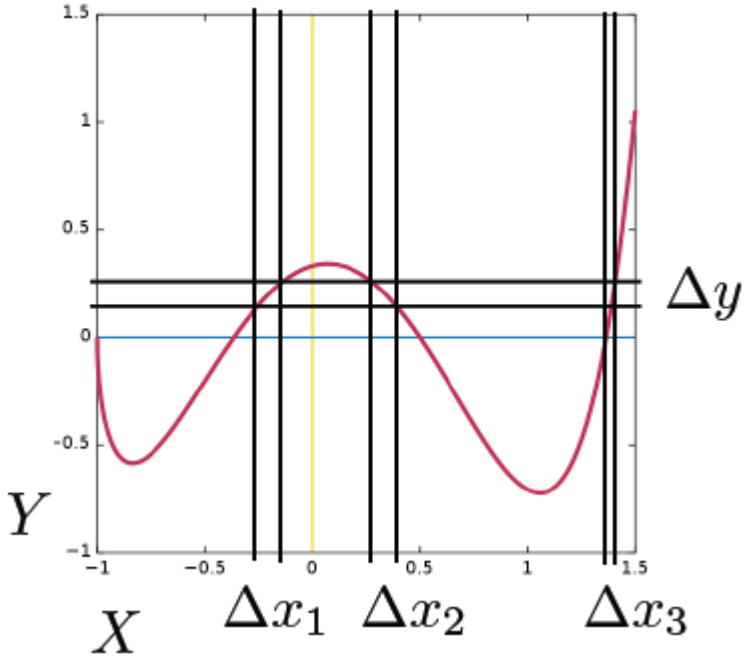
$$P_Y(y < Y \leq y + \Delta y) = \sum_i P_X(x_i < X \leq x_i + \Delta x_i)$$

图像的亮度变换

- 直方图均衡化(PDF 和CDF)

$X,\ Y$ random variables

$$\boldsymbol{Y = T(X)}$$



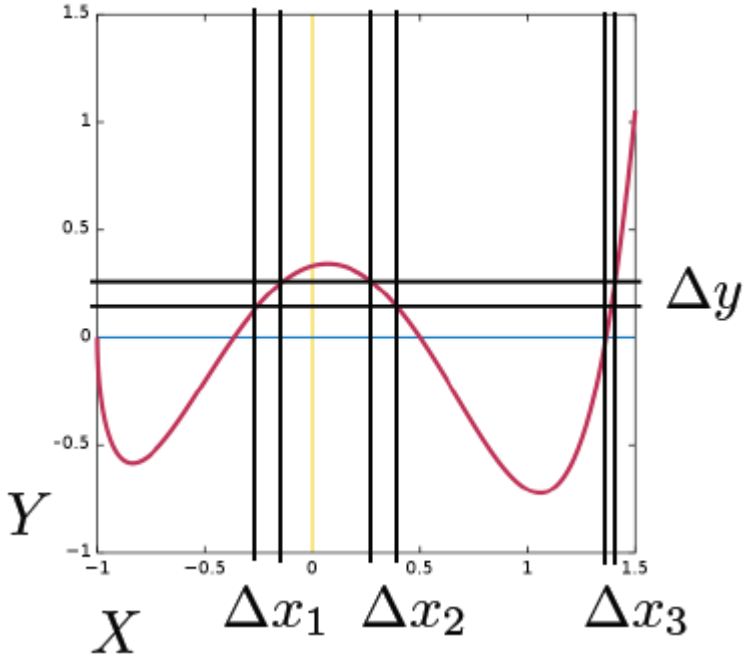
$$p_Y(y)dy = \sum_i p_X(x_i)dx$$

图像的亮度变换

- 直方图均衡化(PDF 和CDF)

$X,\ Y$ random variables

$$\boldsymbol{Y = T(X)}$$



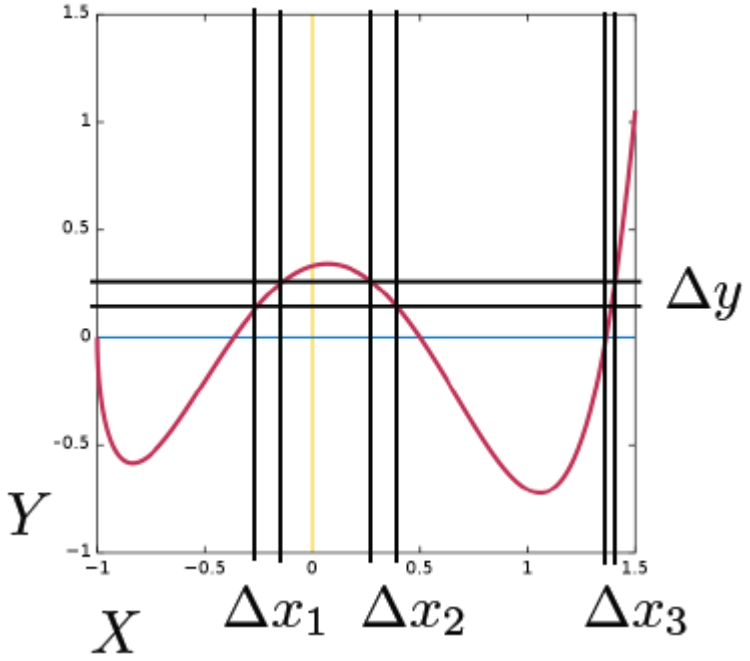
$$p_Y(y)dT(x) = \sum_i p_X(x_i)dx$$

图像的亮度变换

- 直方图均衡化(PDF 和CDF)

$X,\ Y$ random variables

$$\boldsymbol{Y = T(X)}$$



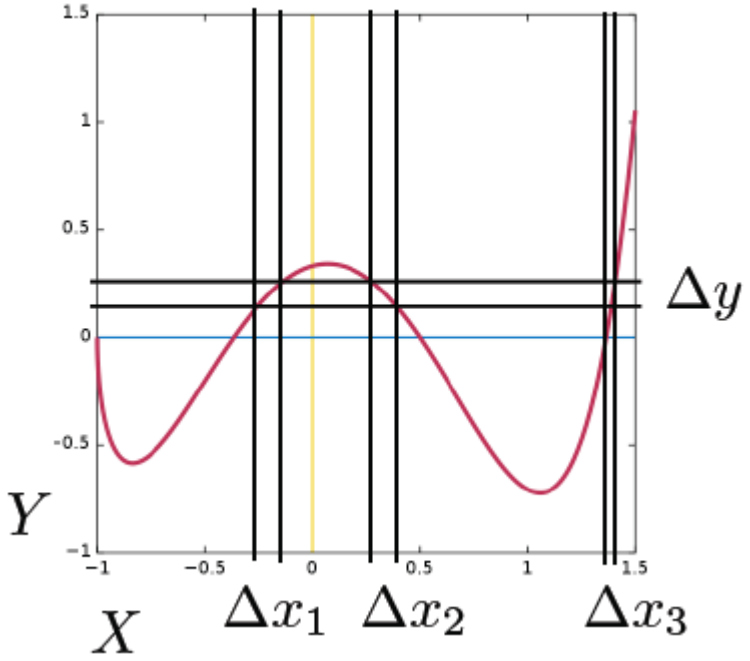
$$dT(x) = \sum_i \frac{p_X(x_i)}{p_Y(y)} dx$$

图像的亮度变换

- 直方图均衡化(PDF 和CDF)

$X,\ Y$ random variables

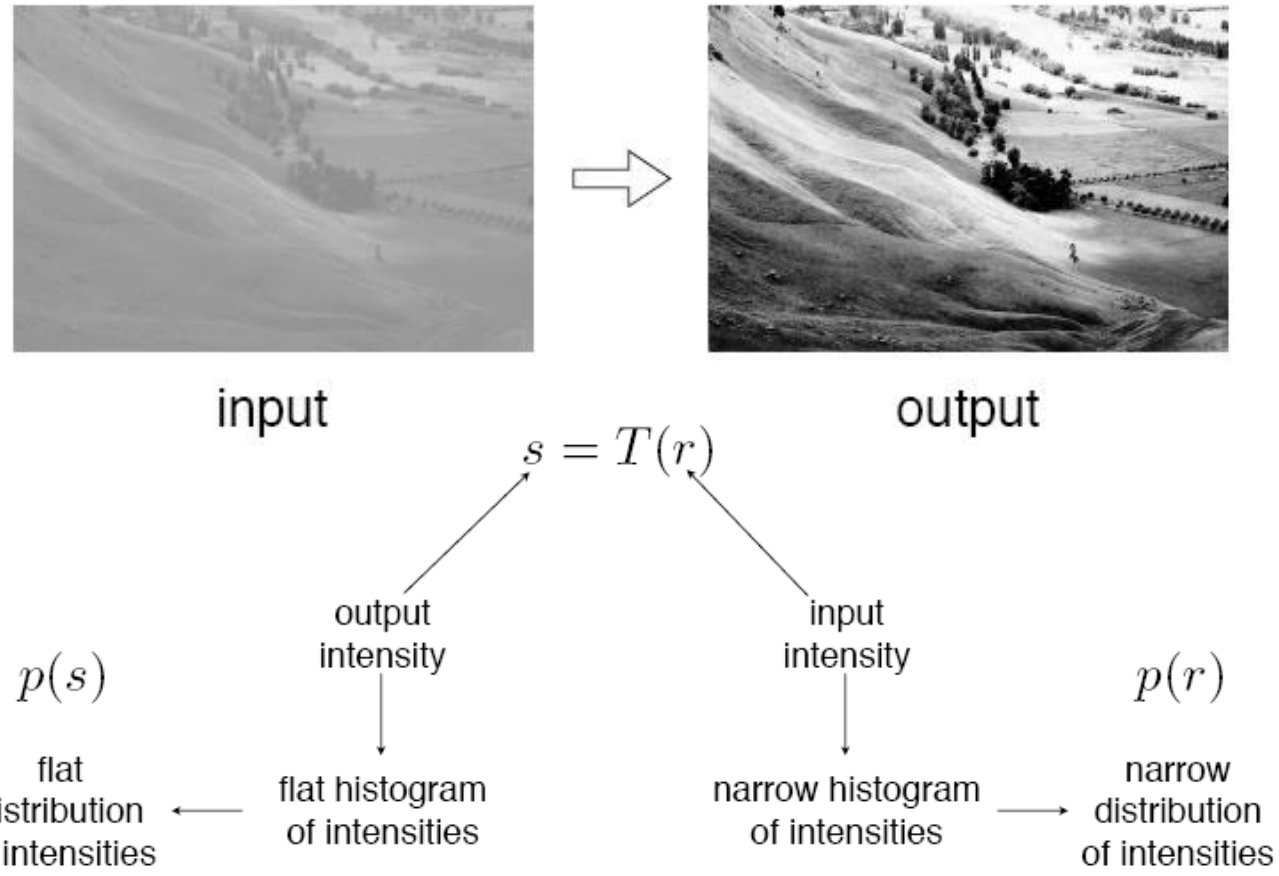
$$\boldsymbol{Y = T(X)}$$



$$T(x) = \int_{-\infty}^{x} \frac{p_X(w)}{p_Y(w)} dw$$

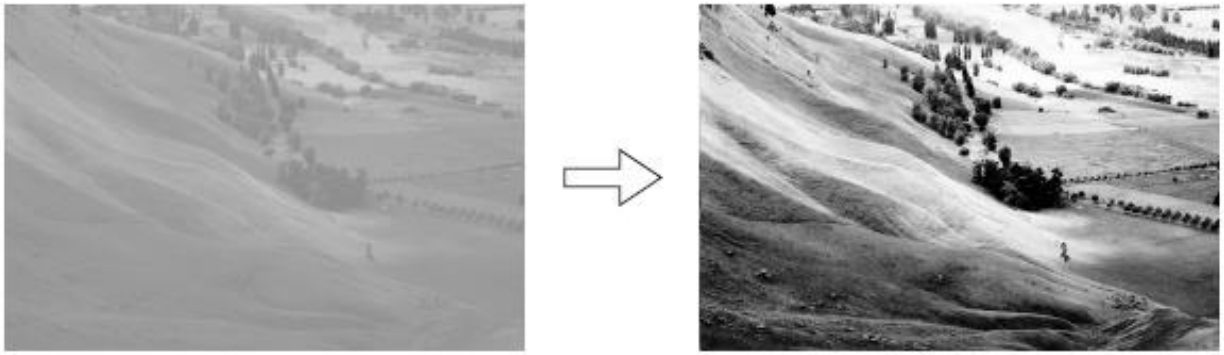
图像的亮度变换

- 直方图均衡化



图像的亮度变换

- 直方图均衡化



input output

$$s = T(r)$$

output intensity

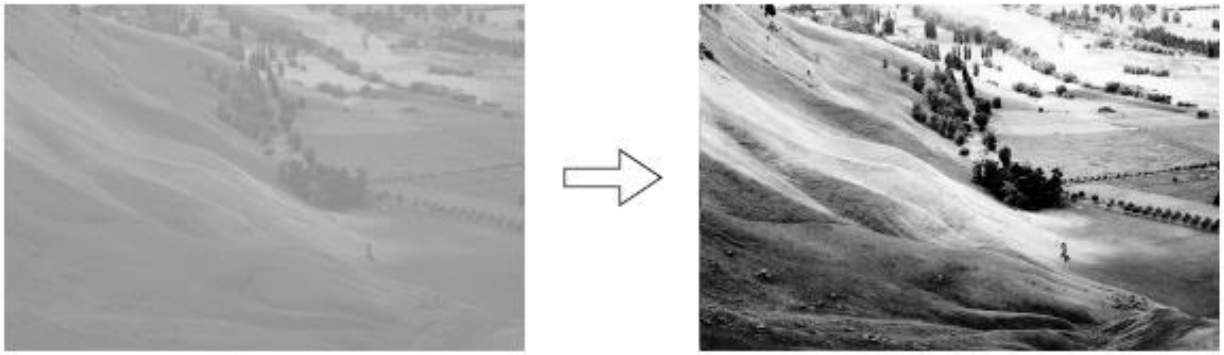
input intensity

monotone increasing function

$$s = T(r) = \int_{-\infty}^{r} \frac{p_r(w)}{p_s(w)} dw$$

图像的亮度变换

- 直方图均衡化



input

output

$s = T(r)$

output intensity

input intensity

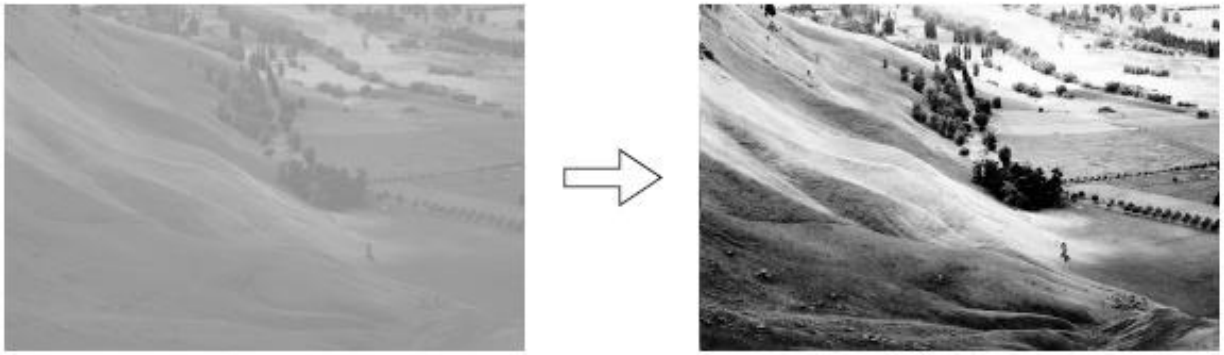
monotone increasing function

$$s = T(r) = \int_{-\infty}^{r} \frac{p_r(w)}{p_s(w)} dw$$

= 1

图像的亮度变换

- 直方图均衡化



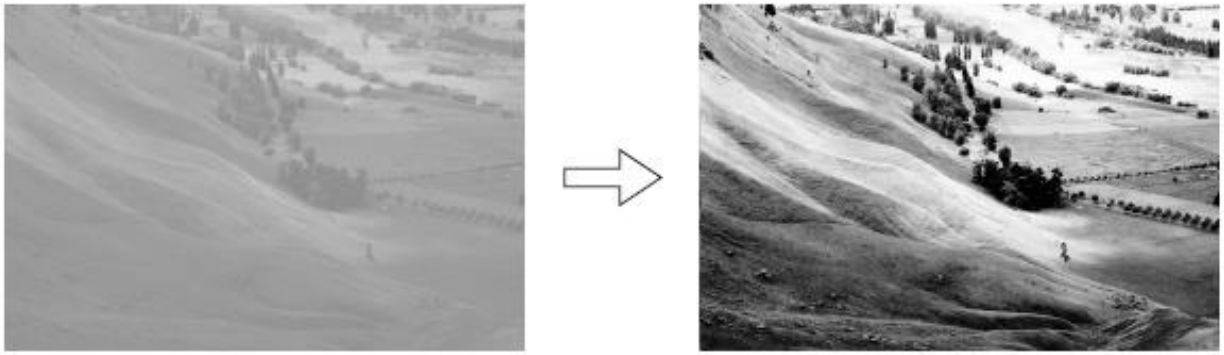
input　　output

$$s = T(r) = \int_{-\infty}^{r} p_r(w)dw$$

continuous pdf

图像的亮度变换

- 直方图均衡化



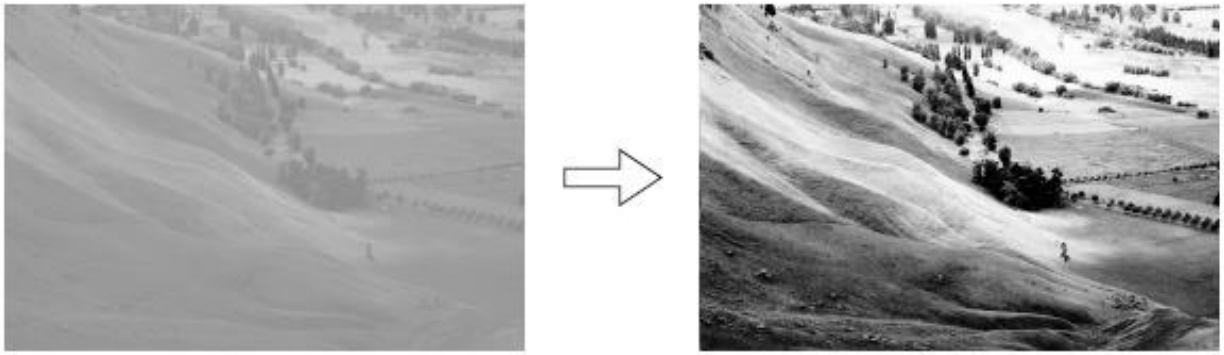
input　　output

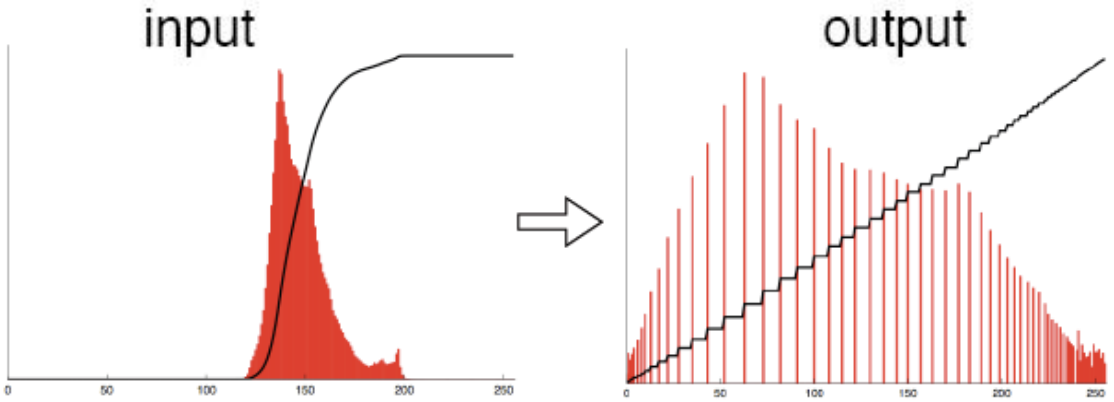
$$s = T(r) = \sum_{i=-\infty}^{r} P_r(i)$$

discrete probability mass

图像的亮度变换

- 直方图均衡化

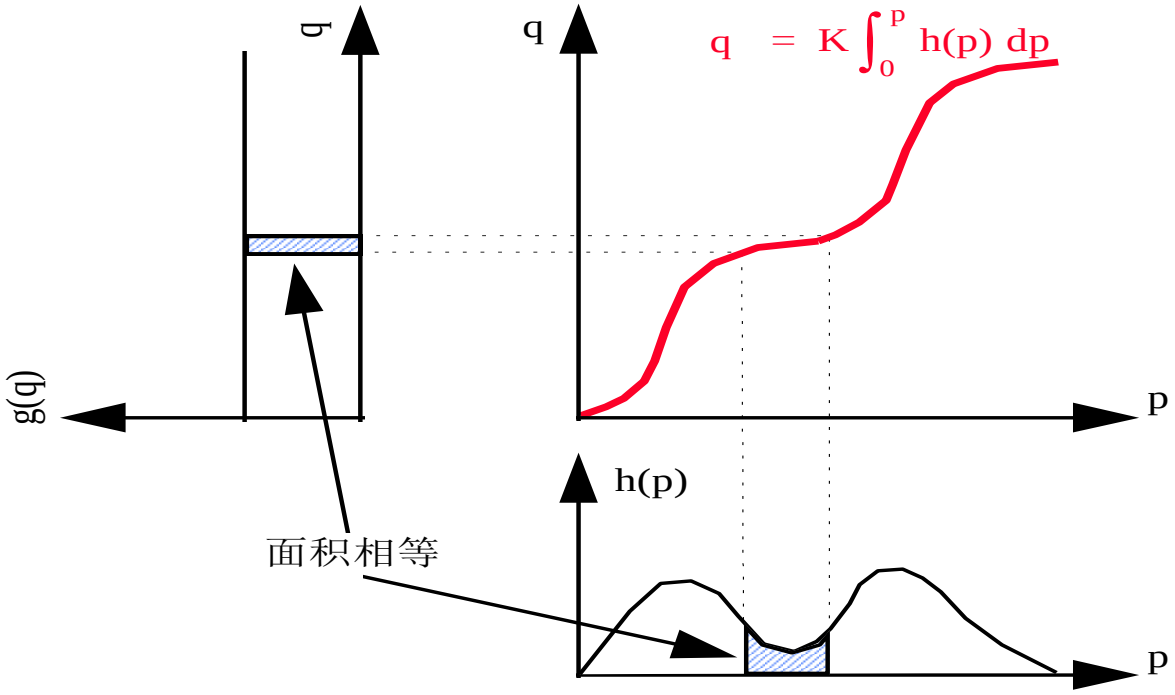




intensity-level histogram (red); cumulative histogram (black)

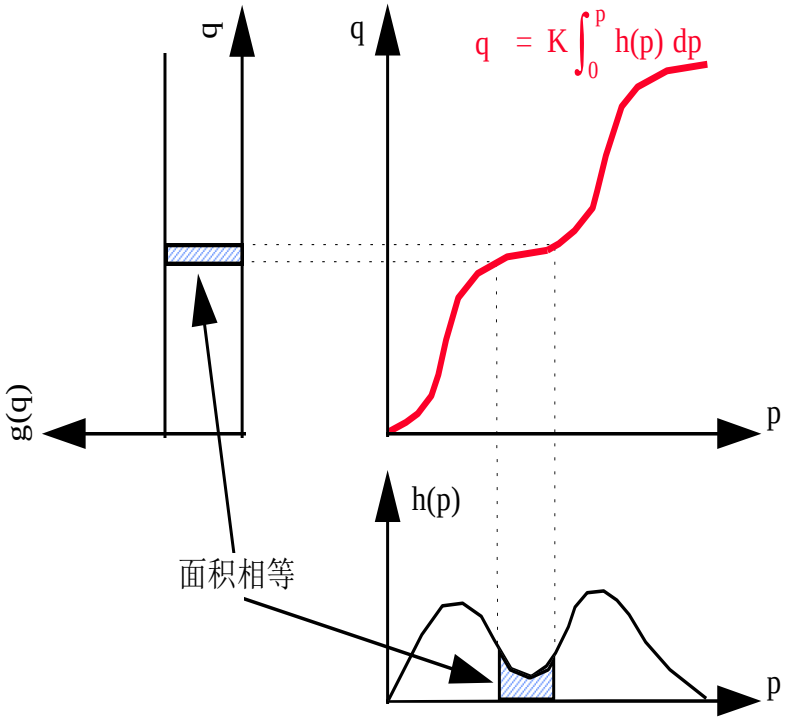
图像的亮度变换

- 直方图均衡化



图像的亮度变换

- 直方图均衡化



用p表示输入图像灰度、q表示变换后的输出图像灰度，则根据对均衡化处理的要求，变换后的图像灰度分布g(q)应为

$$g(q) = \frac{N/K}{N} = \frac{1}{K}$$

这里，K是图像的量化灰度级数，而N为图像中像素的总数目。因此，N/K为图像中灰度级为q的像素数目。

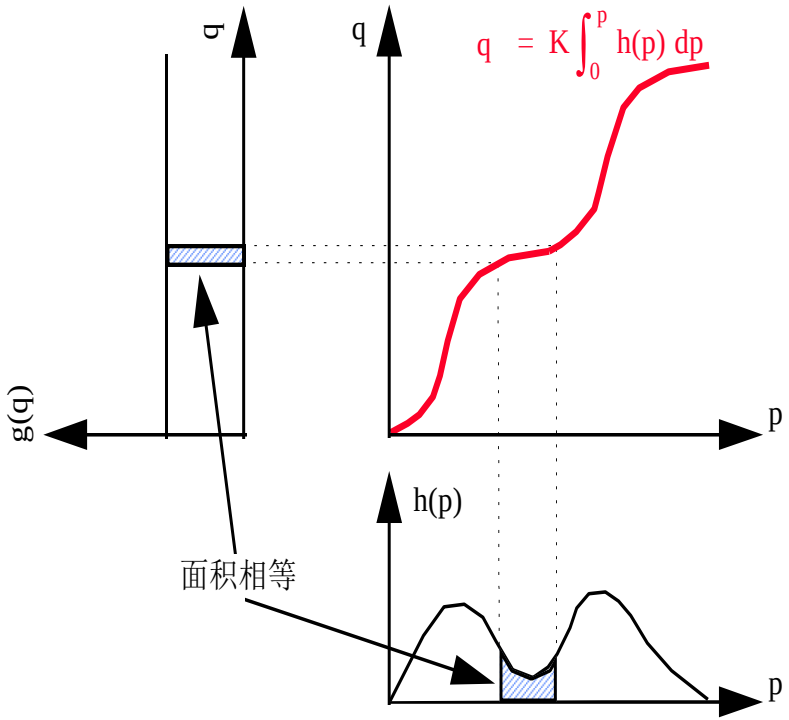
根据h(p)和g(q)的分布可确定输出灰度q和输入灰度p之间的映射关系。

设所需的灰度变换关系为

$$q = f(p)$$

图像的亮度变换

- 直方图均衡化



对灰度变换式两端求微分，有

$$dq = f'(p)dp$$

即

$$f'(p) = \frac{dq}{dp}$$

这里，dp和dq分别表示输入灰度和输出灰度的微分区间。由于要求对输入图像进行灰度均衡化变换前后，其相应的微分区间dp和dq中所包含的像素数应相等，故有

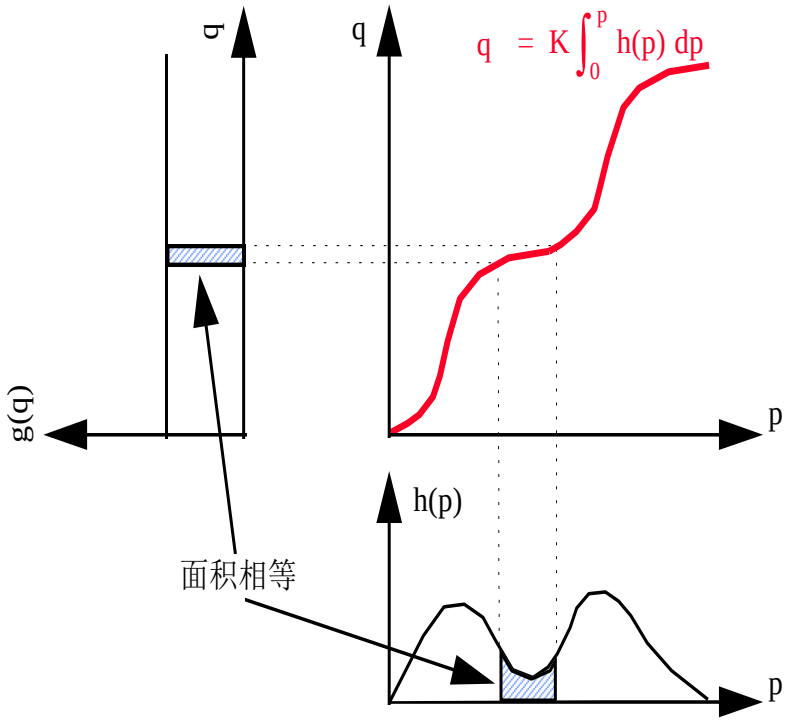
$$h(p)dp = g(q)dq$$

或

$$\frac{dq}{dp} = \frac{h(p)}{g(q)}$$

图像的亮度变换

- 直方图均衡化



因此，有

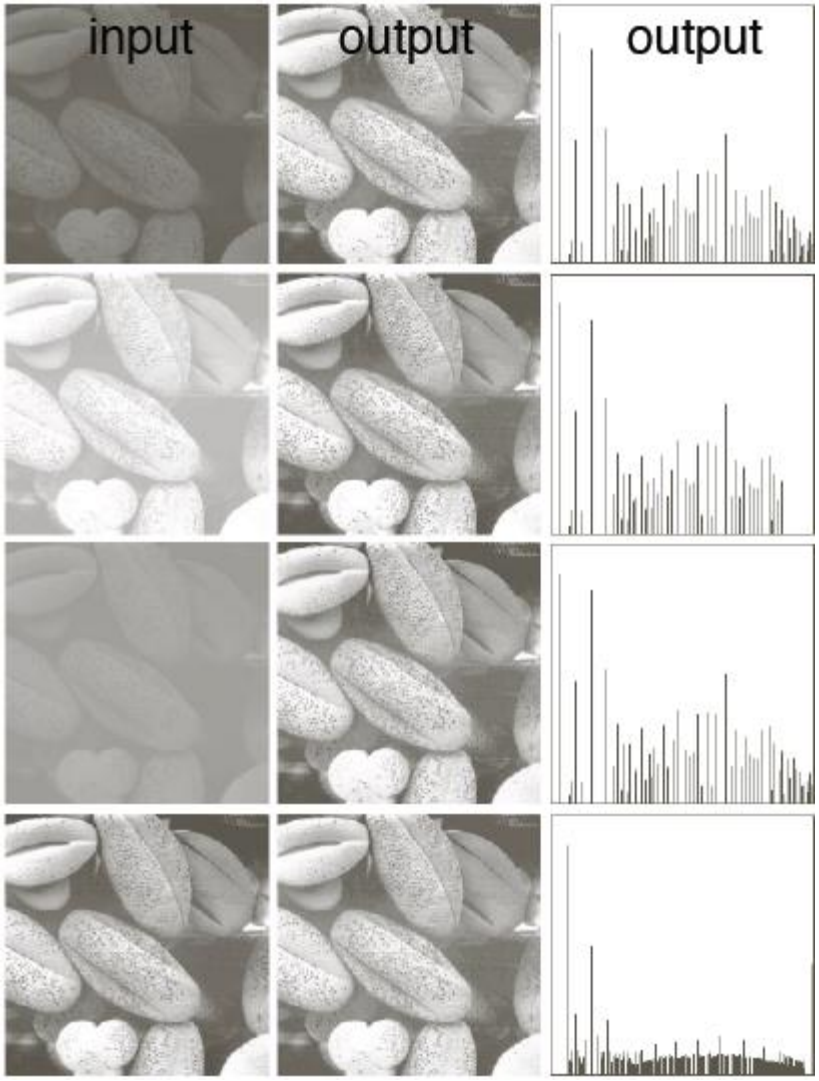
$$f'(p)=\frac{dq}{dp}=\frac{h(p)}{g(q)}=Kh(p)$$

求上式关于p的积分，有

$$f(p)=K\int_0^p h(p)dp$$

由于h(p)为正值函数，所以，f(p)是单值的、并且是单调增加的函数，满足所需要的变换特性。

实例: 直方图均衡化



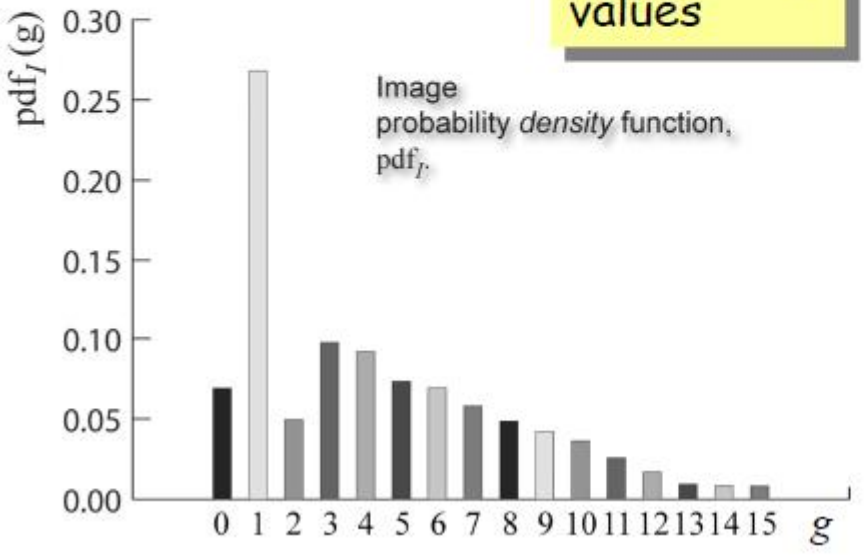
图像的亮度变换

- 直方图规定化

Example: Histogram Matching

Image pdf

Image with 16 intensity values

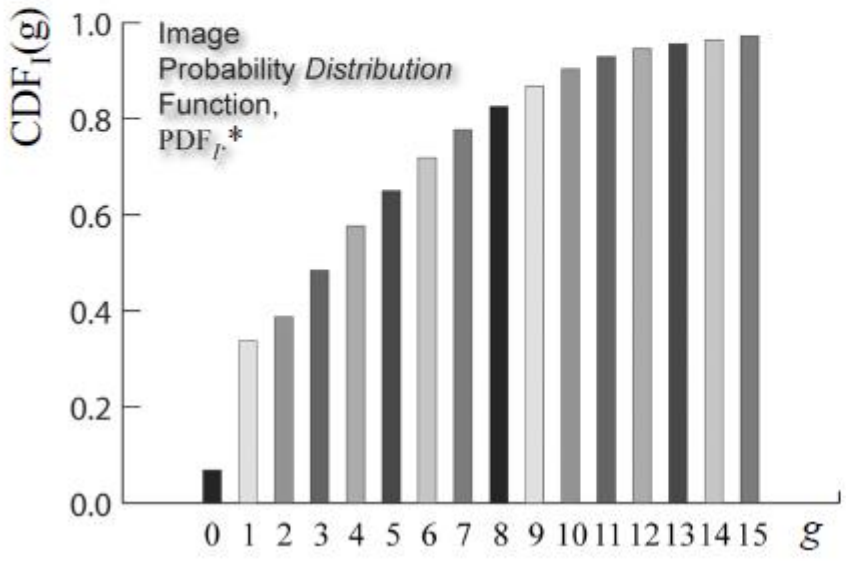


图像的亮度变换

- 直方图规定化

Example: Histogram Matching

Image CDF



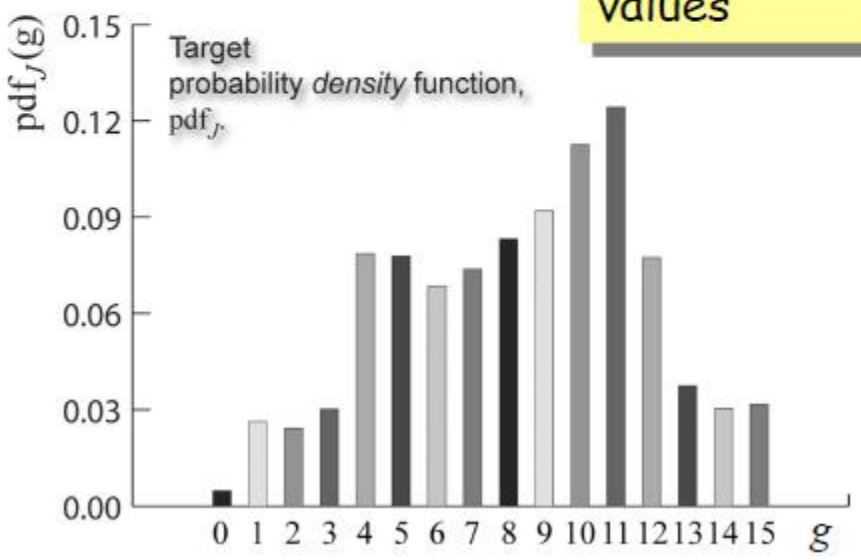
图像的亮度变换

- 直方图规定化

Example: Histogram Matching

Target pdf

Target with 16 intensity values

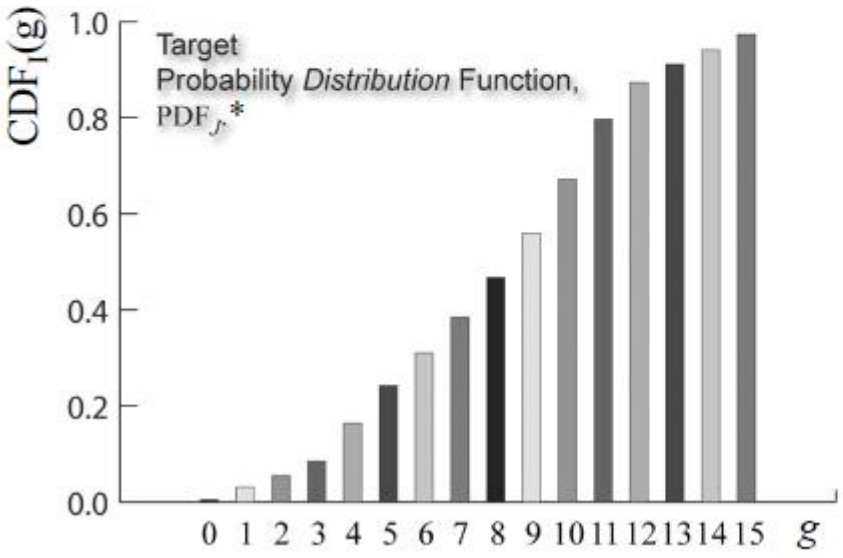


图像的亮度变换

- 直方图规定化

Example: Histogram Matching

Target CDF



图像的亮度变换

- 直方图规定化

original

target

remapped

图像的亮度变换

- 直方图规定化

original

target

luminosity remapped