信号与图像处理基础

Frequency Filtering

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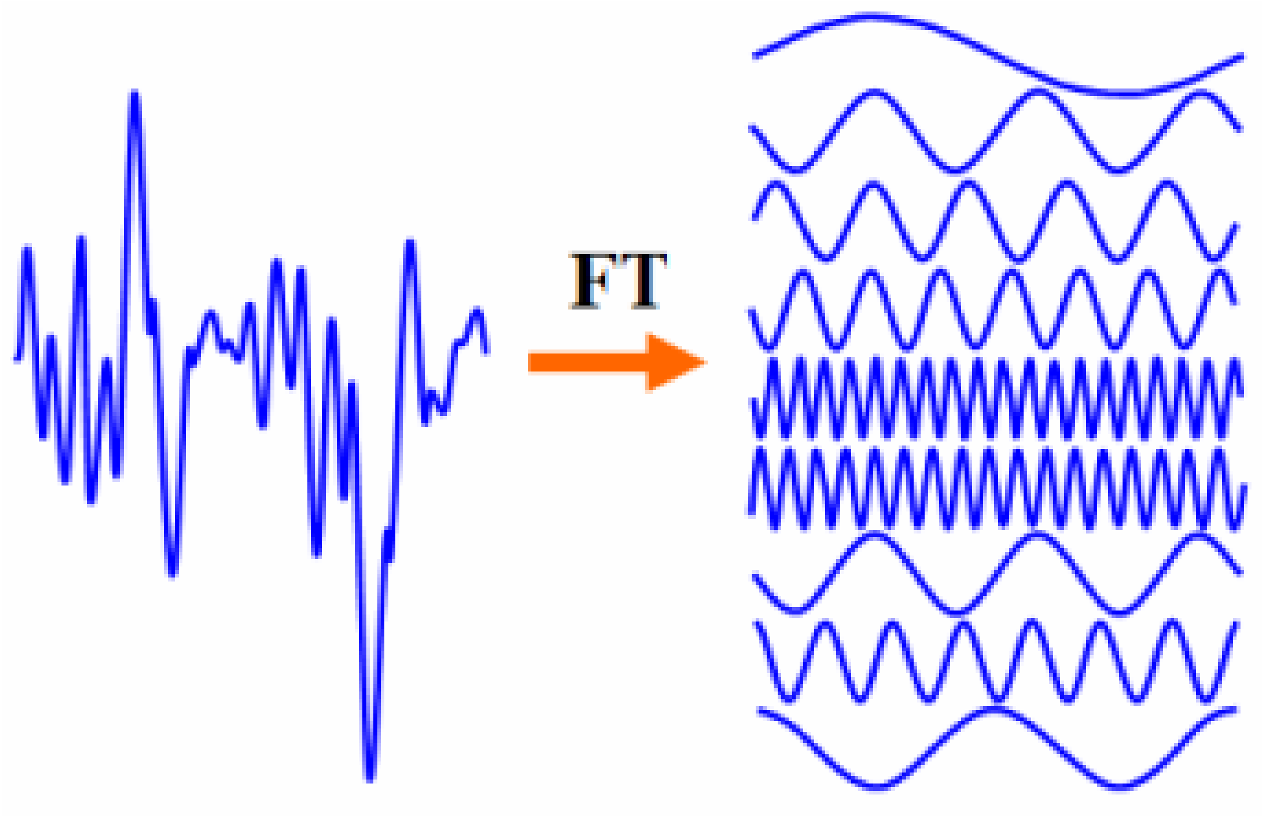
主要内容

- 绪论
- 低通滤波
- 高通滤波和带通滤波

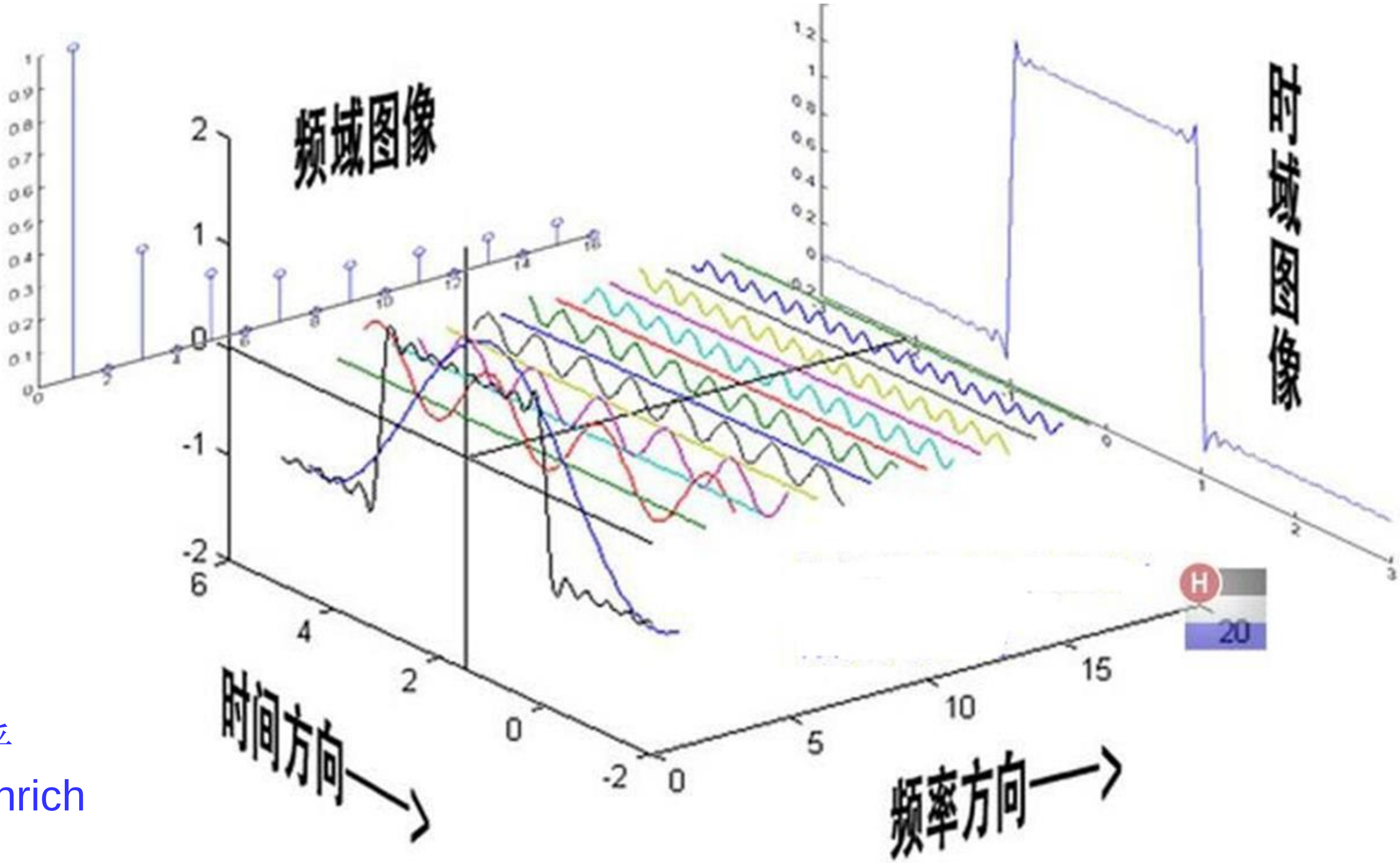
1. 绪论

傅立叶变换



1. 绪论

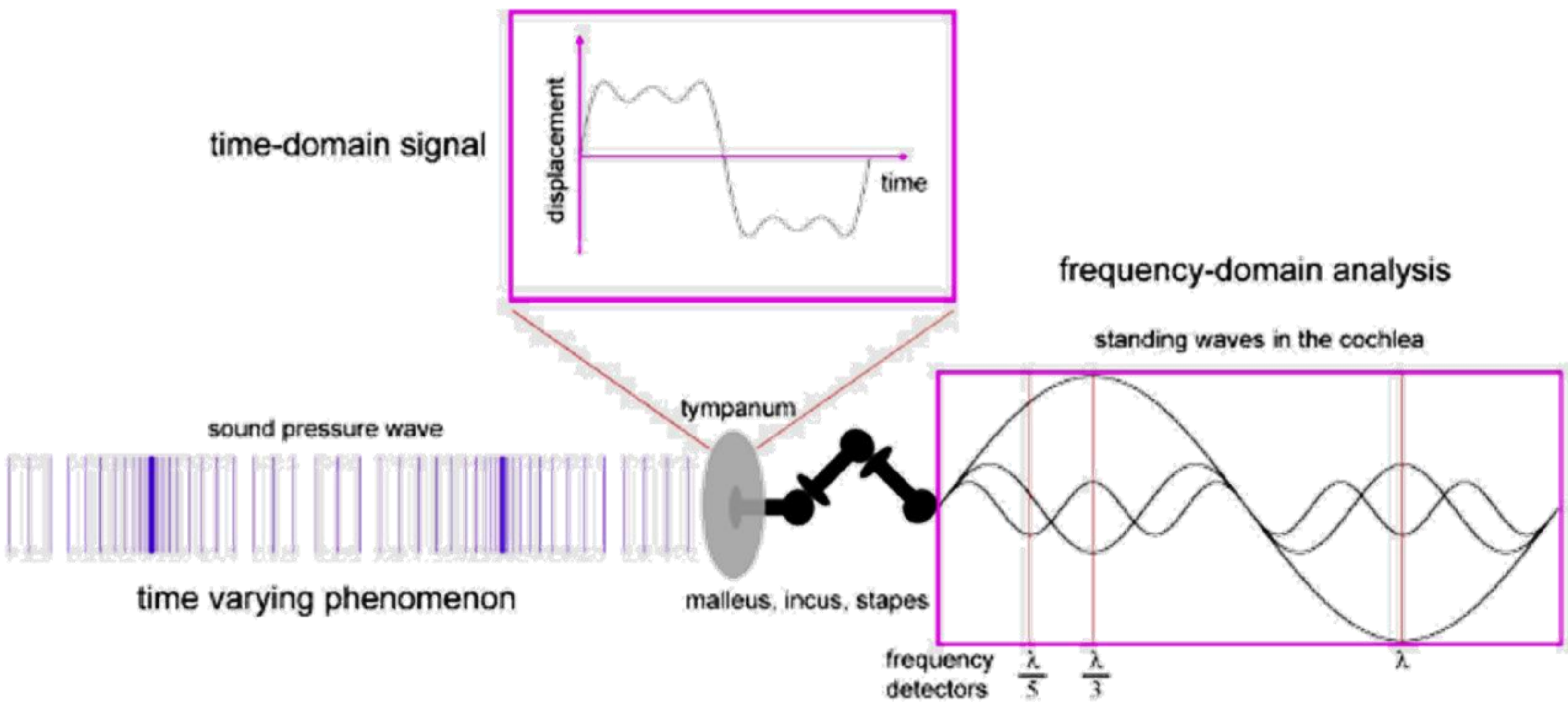
傅立叶变换



转载自知乎
作者：Heinrich

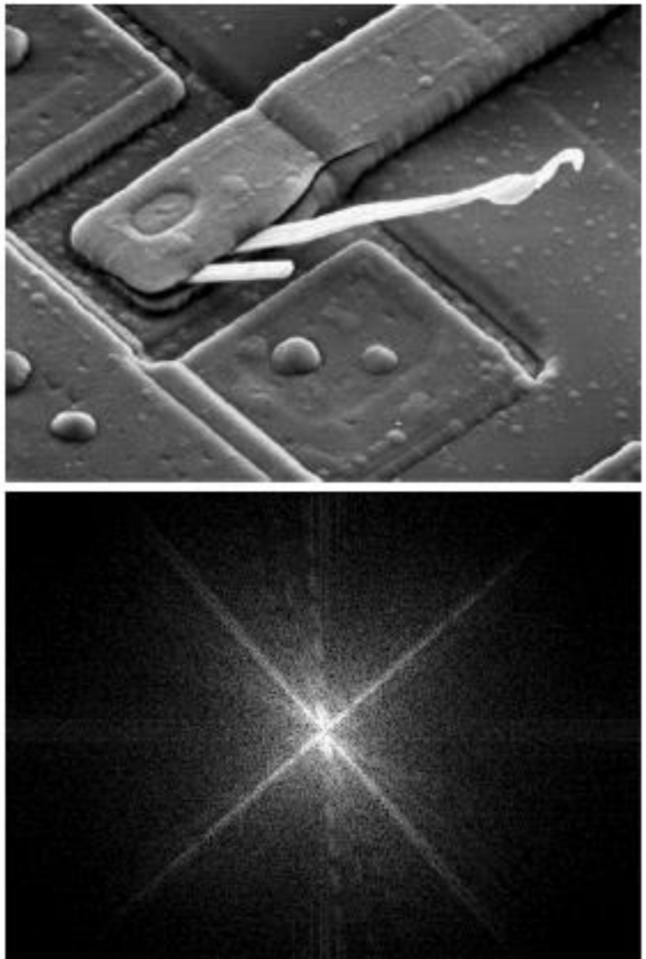
1. 绪论

频域分析


time-domain signal
displacement
time
frequency-domain analysis
standing waves in the cochlea
tympanum
sound pressure wave
time varying phenomenon
malleus, incus, stapes
frequency
detectors
$\frac{\lambda}{5}$
$\frac{\lambda}{3}$
λ

1. 绪论

图像傅立叶变换

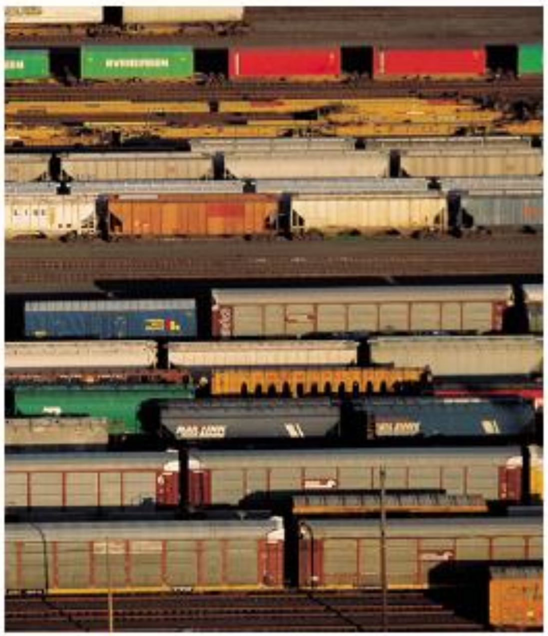


a
b

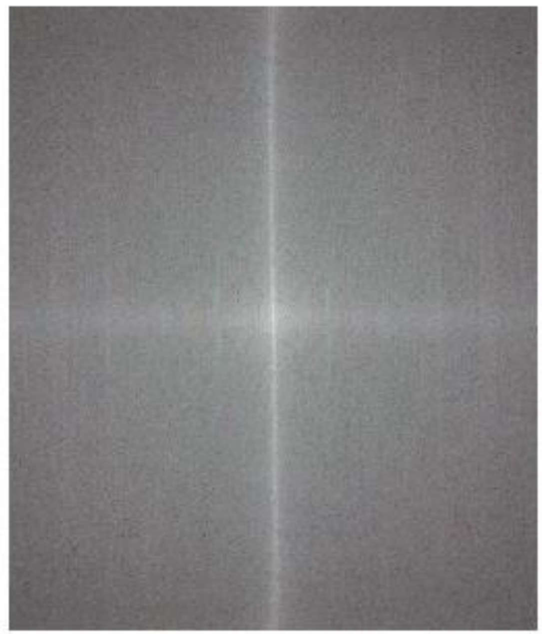
FIGURE 4.4 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

1. 绪论

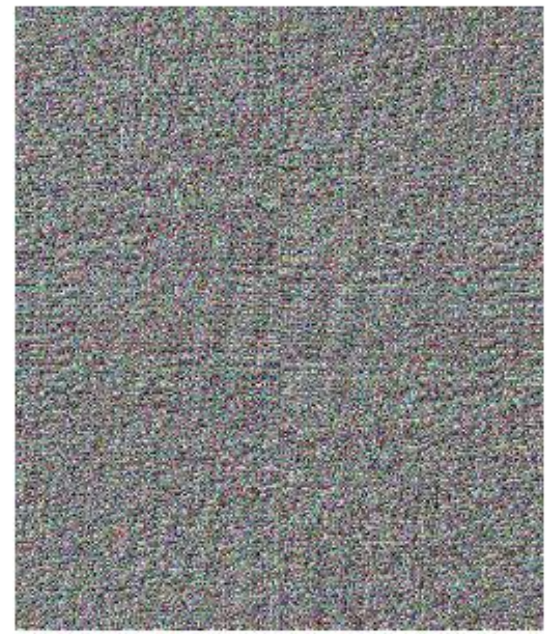
图像傅立叶变换



$\mathbf{I}$



$\log\{|\mathcal{F}\{\mathbf{I}\}|^2+1\}$

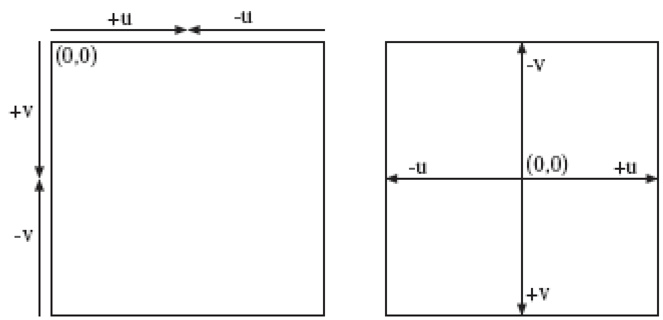


$\angle[\mathcal{F}\{\mathbf{I}\}]$

1. 绪论

图像傅立叶变换

为了便于分析和描述，需要对频谱进行中心化。

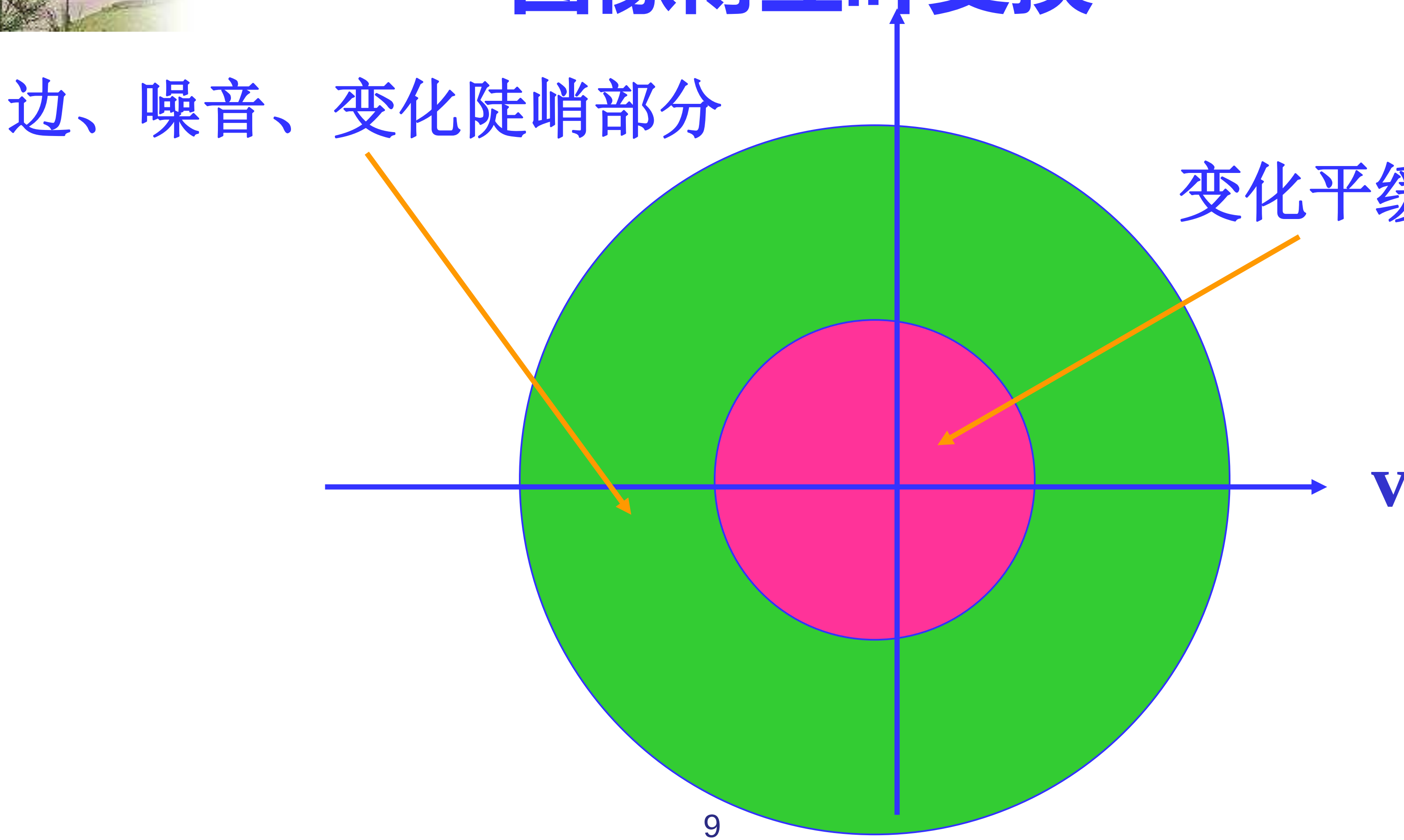


DFT storage　　Centered

用$(-1)^{x+y}$乘以输入图像来进行中心变换

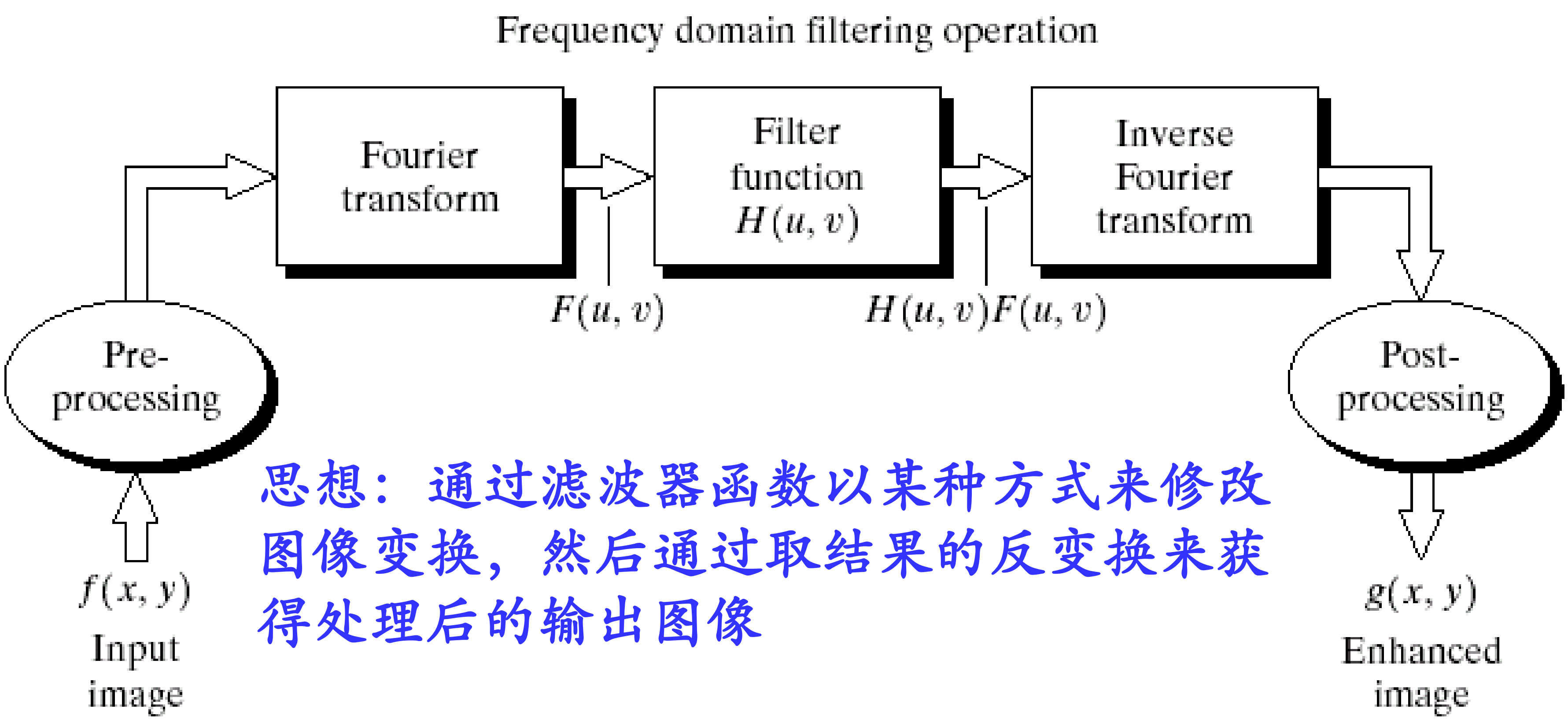
1. 绪论

图像傅立叶变换



1. 绪论

图像频域处理



思想：通过滤波器函数以某种方式来修改图像变换，然后通过取结果的反变换来获得处理后的输出图像

1. 绪论

图像频域处理基本步骤

1. 用$(-1)^{x+y}$乘以输入图像来进行中心变换。

$$f(x,y)(-1)^{x+y}$$

2. 计算图像的DFT，即F(u,v)
3. 计算 $H(u,v)F(u,v)$
4. 对上述进行反DFT变换；
5. 提取（4）的实部；
6. 用$(-1)^{x+y}$乘以(5)中的结果；

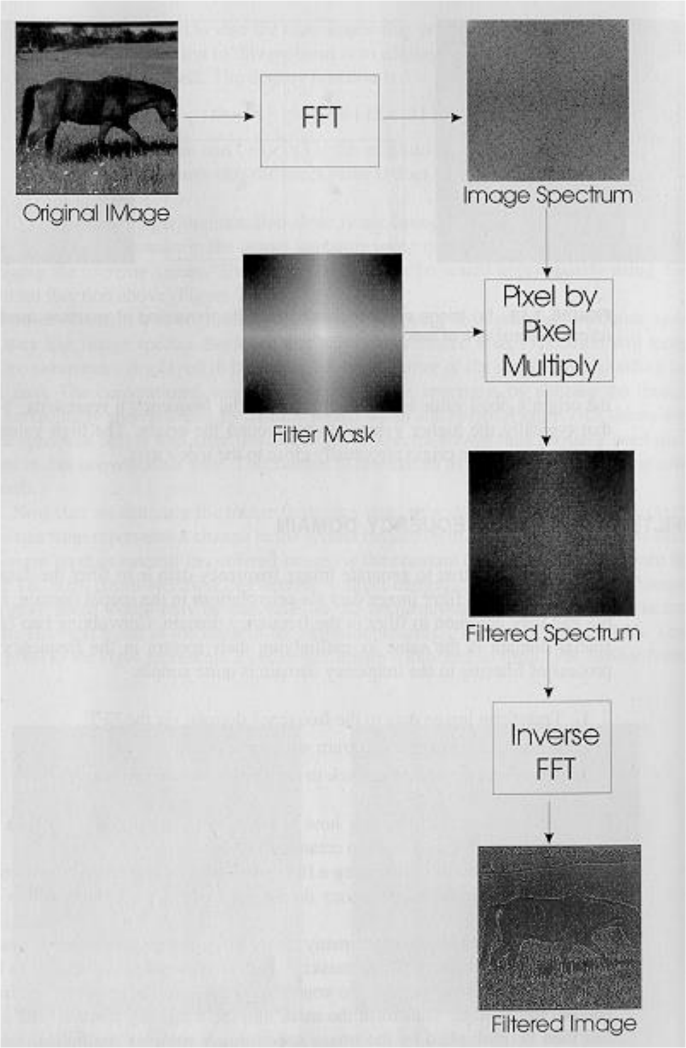
H(u,v)称为滤波器。

1. 绪论

$$f(x,y) * h(x,y) = g(x,y)$$

$$\Updownarrow$$

$$F(u,v)\,H(u,v) = G(u,v)$$

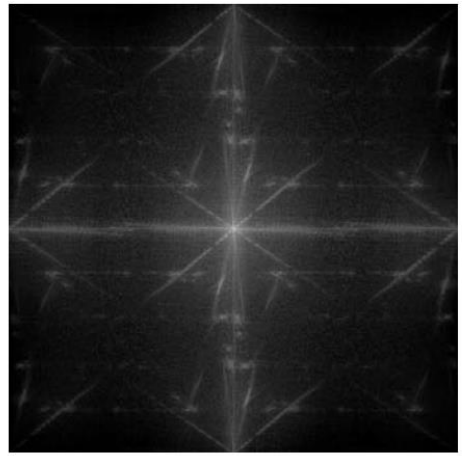


Case 1: H(u,v) is specified in the frequency domain.

Case 2: h(x,y) is specified in the spatial domain.

1. 绪论

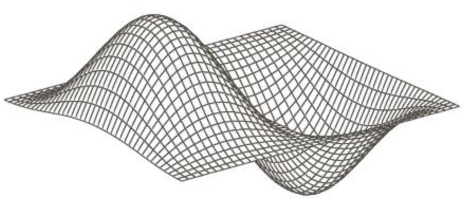
600 x 600

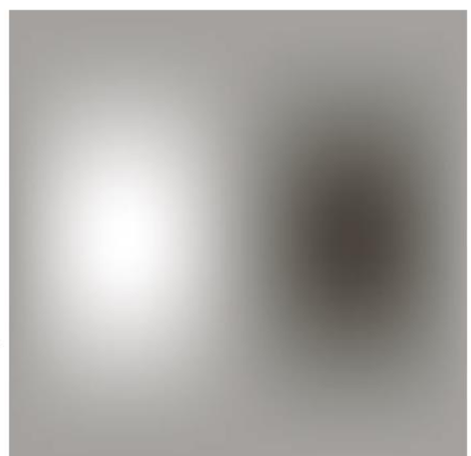


<u>Case 1:</u> F(u,v) is specified in the frequency domain.

−1	0	1
−2	0	2
−1	0	1

Sobel





<u>Case 2:</u> h(x,y) is specified in the spatial domain.

1. 绪论

对于一个空域滤波器h(x,y)，如何得到其在频域的函数 H(u,v)?

- If h(x,y) is given in the spatial domain (case 2), we can generate H(u,v) as follows:

    1.Form $h_p(x,y)$ by padding with zeroes.（零延拓）

    2. Multiply by $(-1)^{x+y}$ to center its spectrum.（中心化）

    3. Compute its DFT to obtain H(u,v)（离散傅里叶变换）

1. 绪论

图像频域处理

需要注意的是H(u)事实上是在复数域上的,因此

- 其幅值部分与图像各频率的幅值相乘，改变其频域内容
- 其相位部分使图像的相位平移
- 幅值调制（比较常见）
- 相位调制（比较少见）

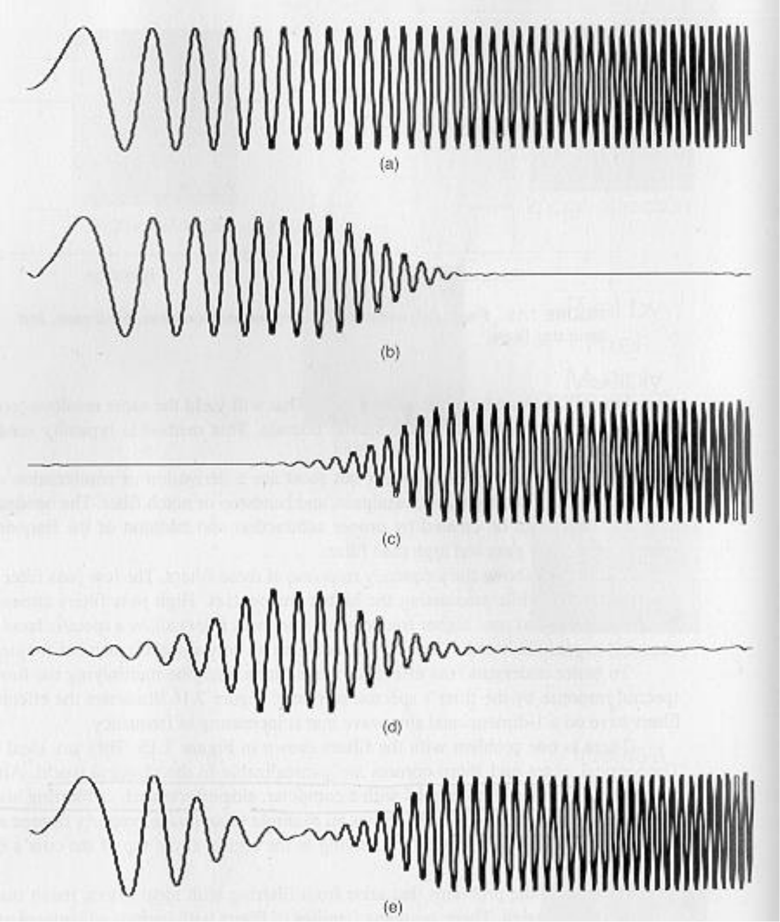
1. 绪论

图像频域处理

频域滤波方法分类

• 低通滤波	保留低频成分
• 高通滤波	保留高频成分
• 高频提升滤波	提升高频成分
• 带通滤波	保留中频成分
• 带阻滤波	阻断中频成分

1. 绪论



Original signal

Low-pass filtered

High-pass filtered

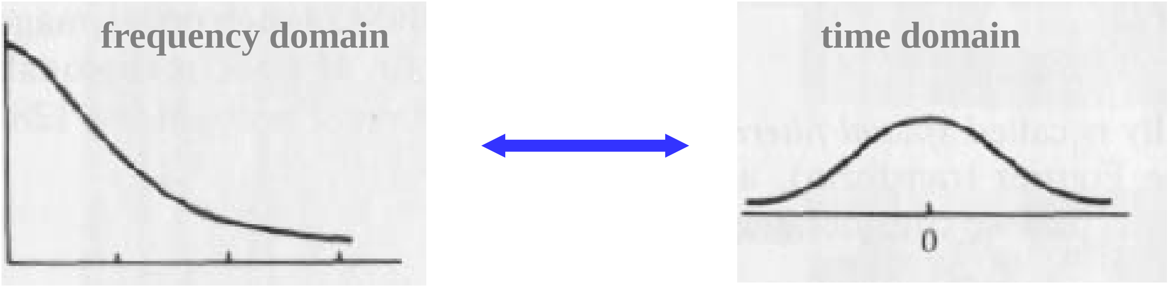
Band-pass filtered

Band-stop filtered

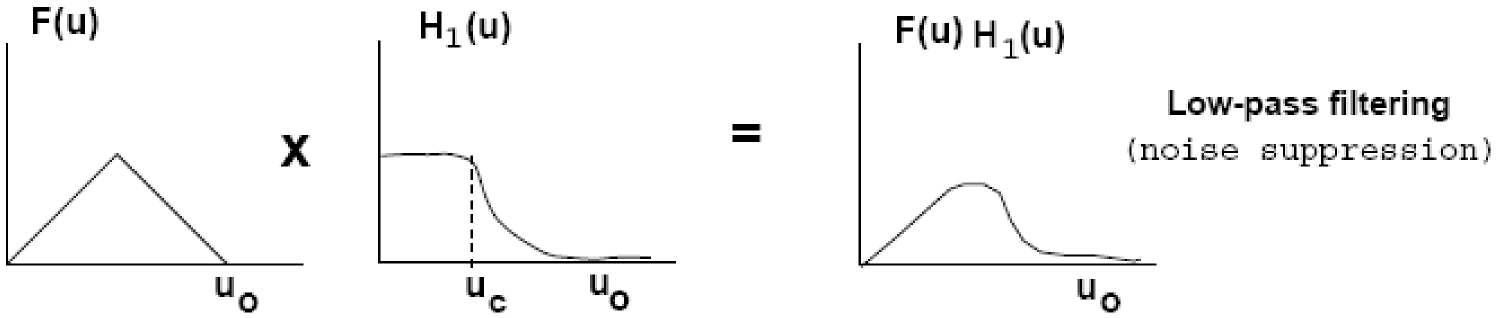
1. 绪论

低通滤波

- 保留低频成分- 通常用于平滑、去噪等任务中



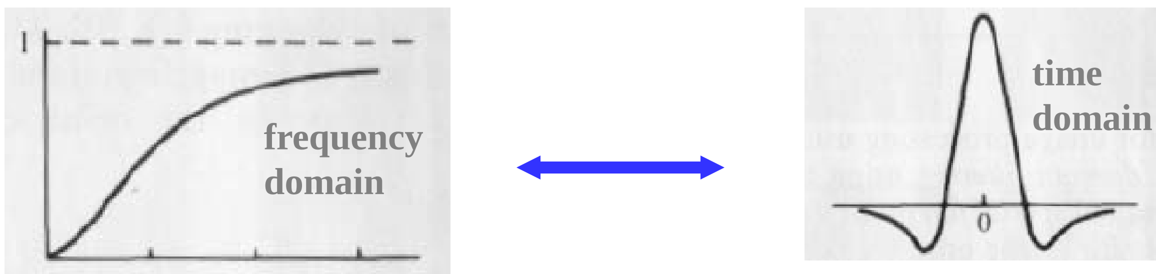
Example:



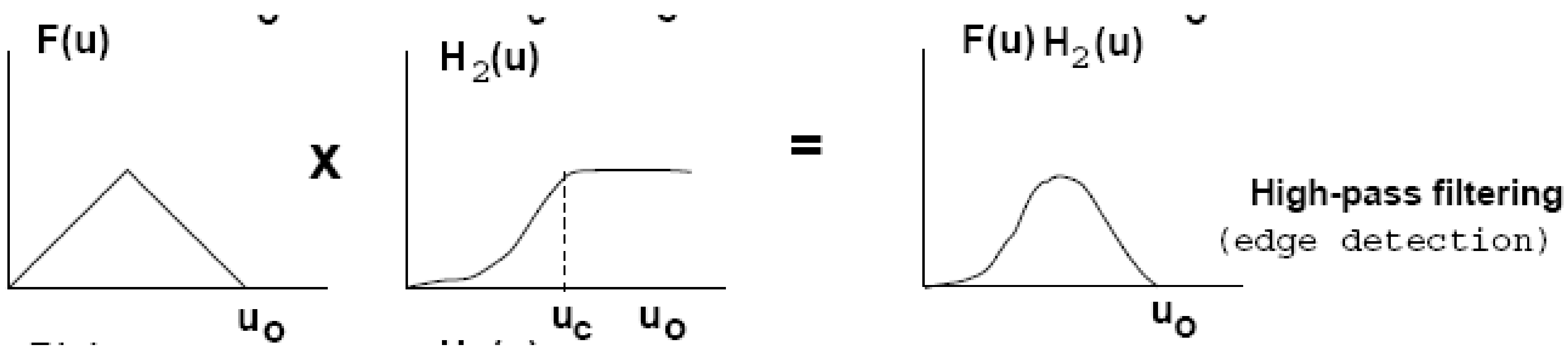
1. 绪论

高通滤波

- 保留高频成分- 通常用于边缘提取等任务中



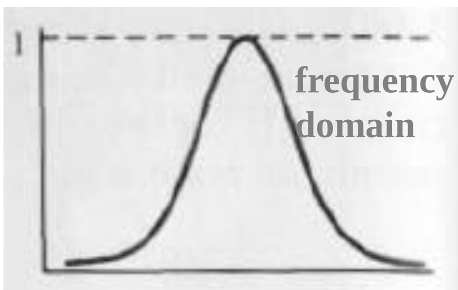
Example:

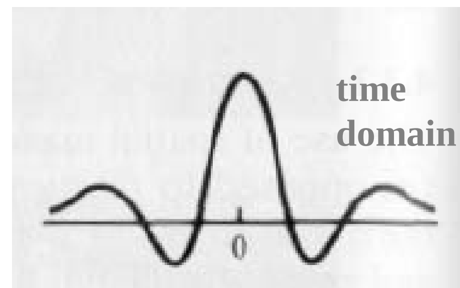


1. 绪论

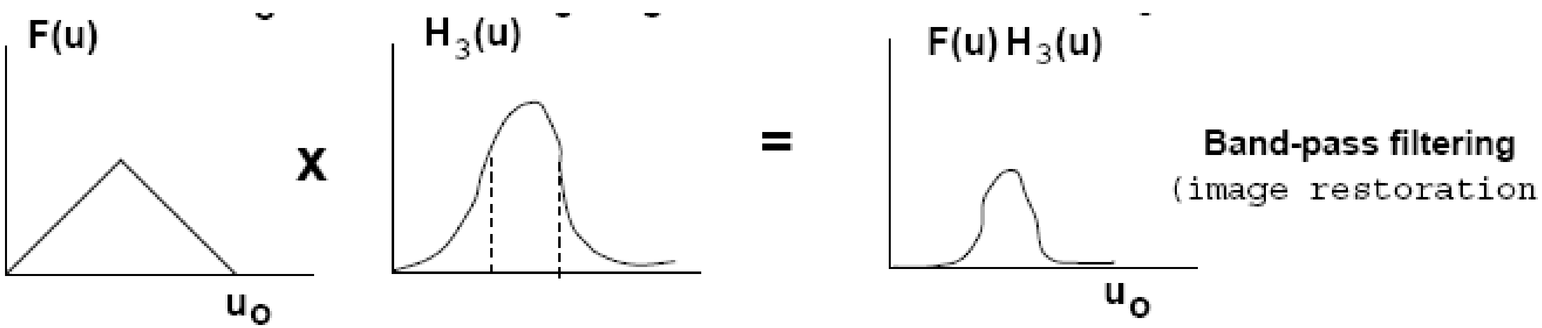
带通滤波

- 保留中频成分-通常用于纹理提取等任务中





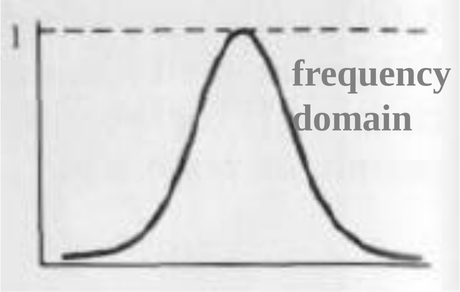
Example:

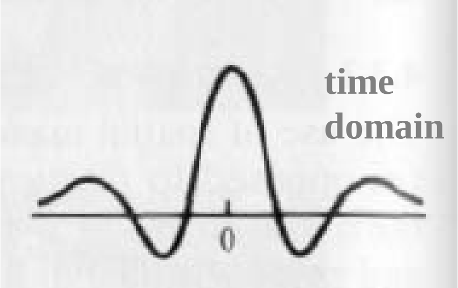


1. 绪论

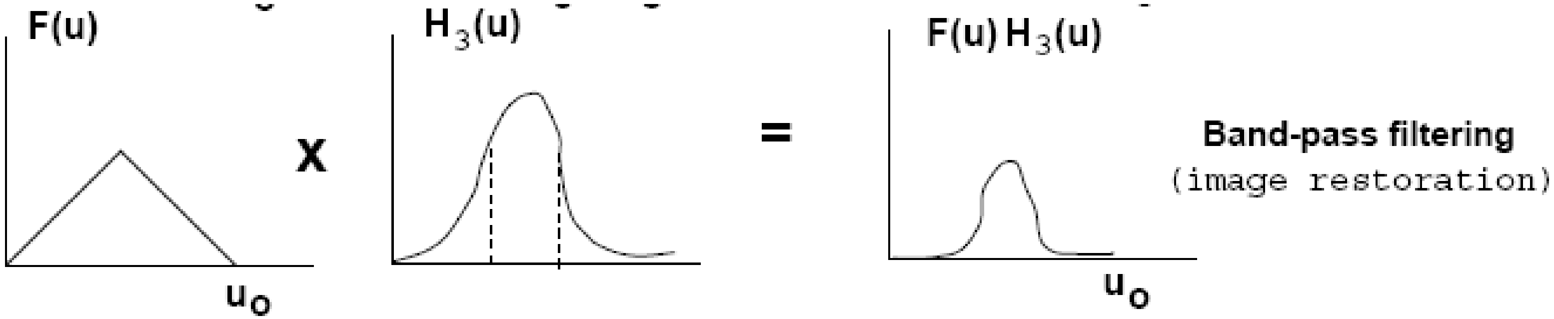
带通滤波

- 保留中频成分-通常用于纹理提取等任务中





Example:



1. 绪论

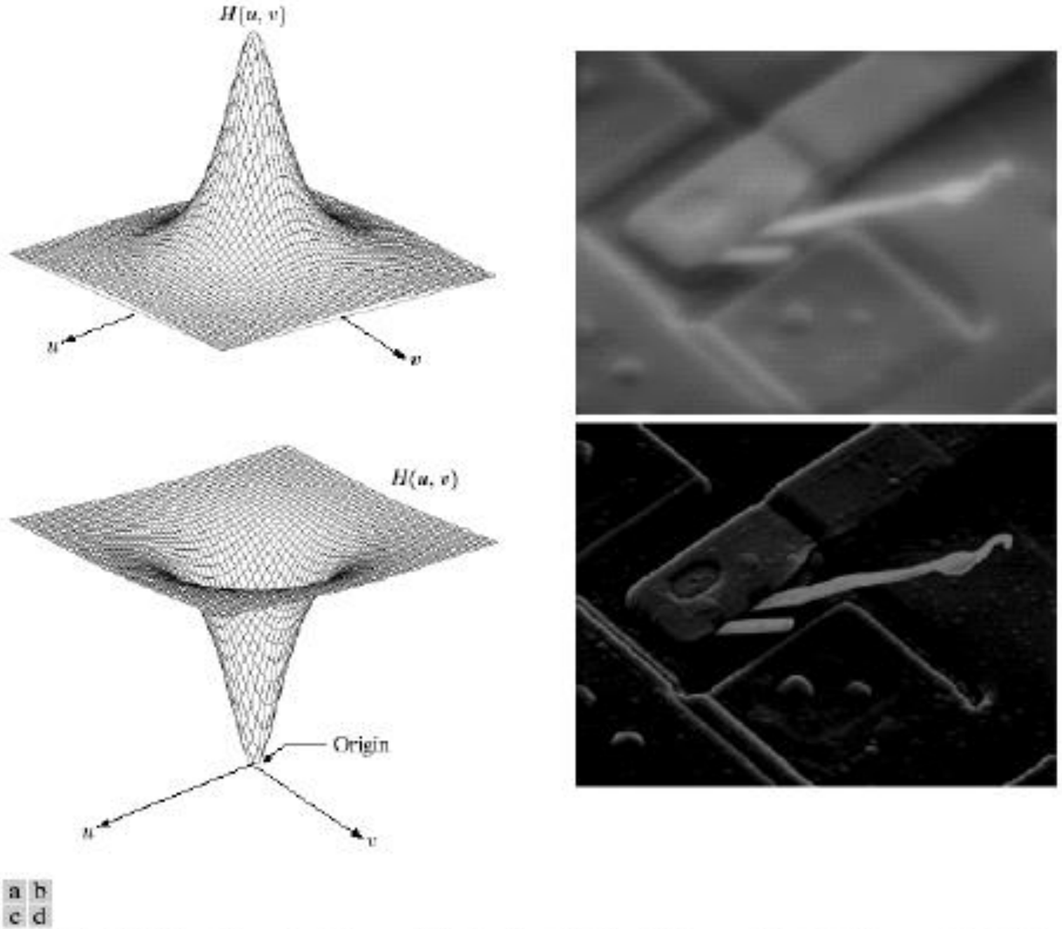
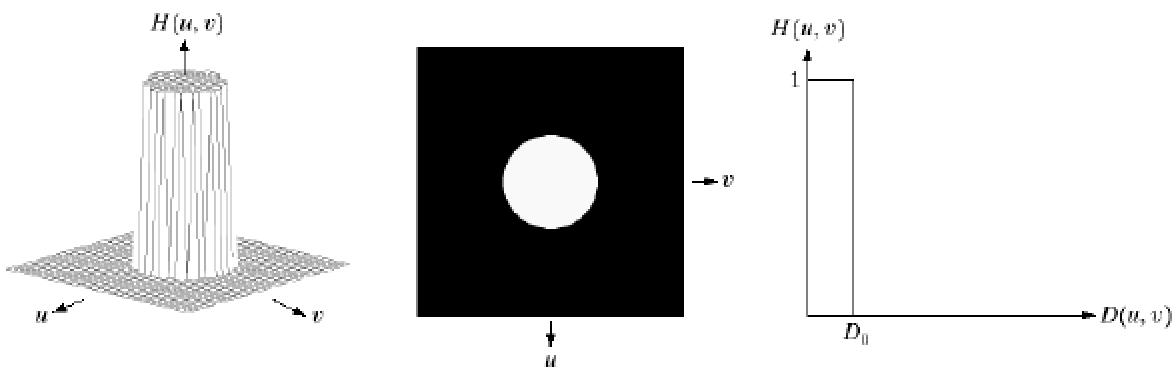


FIGURE 4.7 (a) A two-dimensional lowpass filter function. (b) Result of lowpass filtering the image in Fig. 4.4(a). (c) A two-dimensional highpass filter function. (d) Result of highpass filtering the image in Fig. 4.4(a).

2. 低通滤波

理想低通滤波

$$H(u,v)=\begin{cases}1 & D(u,v)\le D_0\\ 0 & D(u,v)>D_0\end{cases}$$

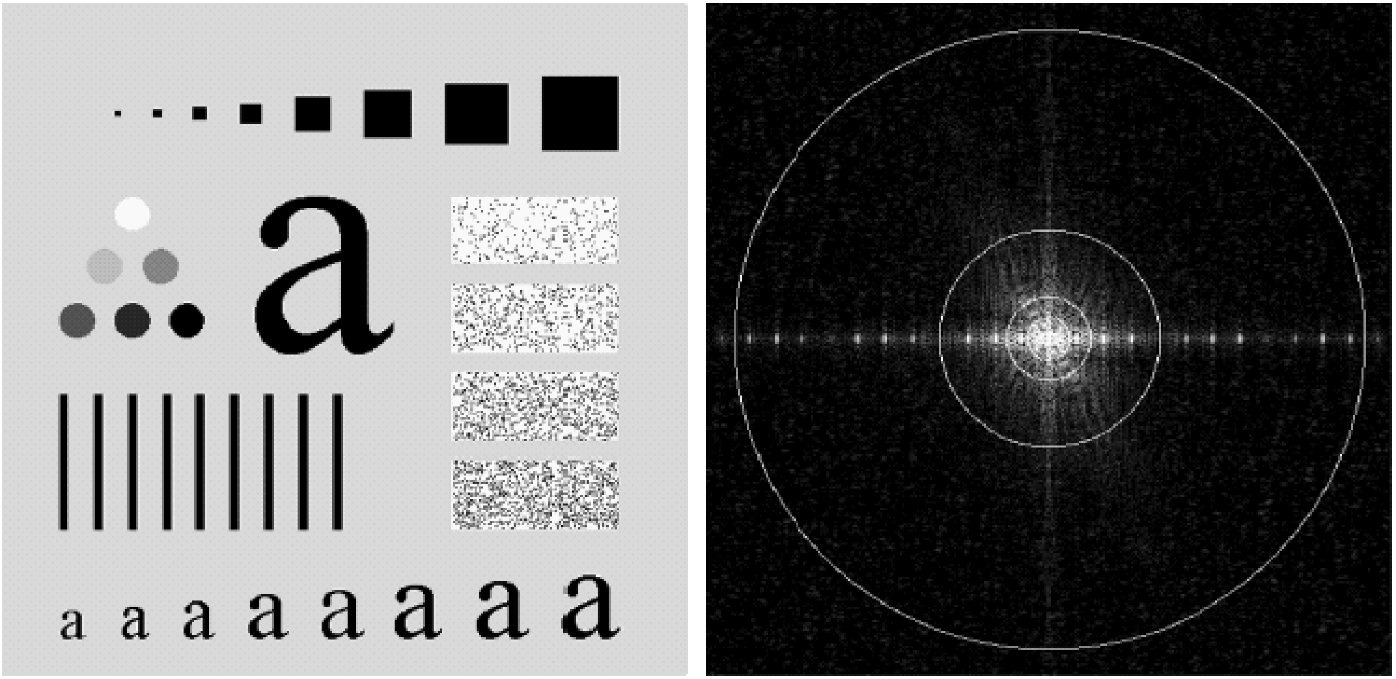


a b c

FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

2. 低通滤波

理想低通滤波



a b

FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

2. 低通滤波

频域图像分析

1. 特性研究需要具有相同截止频率加以比较，方法是用图像功率。
2. 定义：

$$P_T = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} p(u,v)$$

$$p(u,v) = |F(u,v)| = R^2(u,v) + I^2(u,v)$$

3. 如果变换被中心化，原点在图像的矩形中心，则半径为r圆包含的功率为：

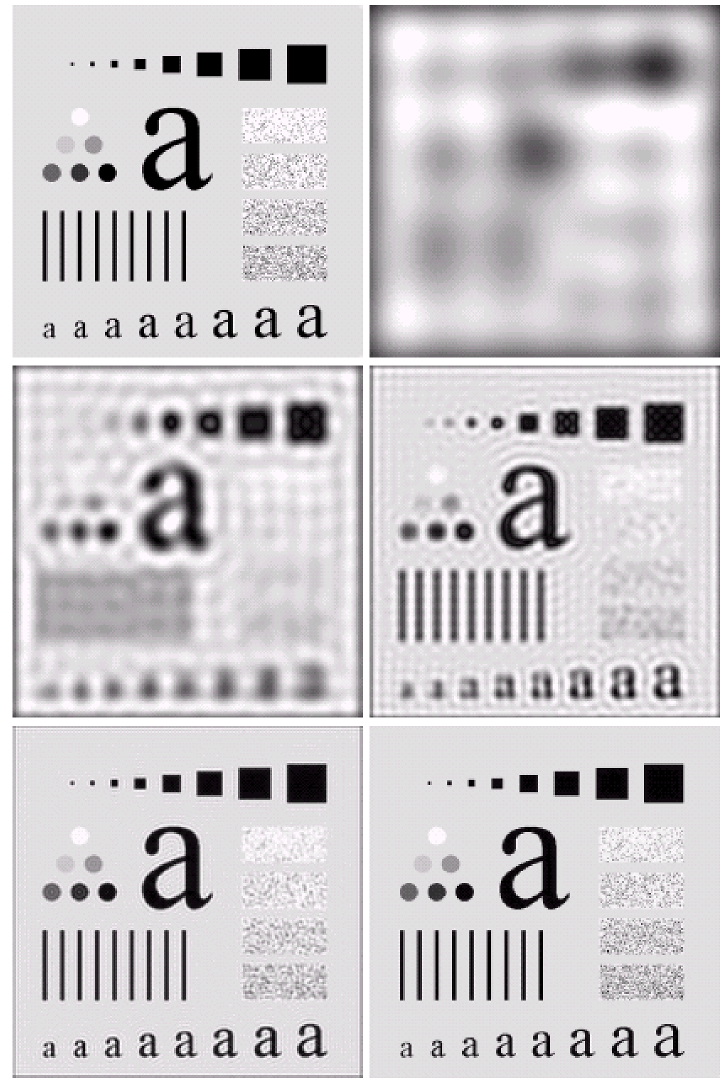
$$a = 100[\sum_{u}\sum_{v} p(u,v) / P_T(u,v)]$$

2. 低通滤波

理想低通滤波

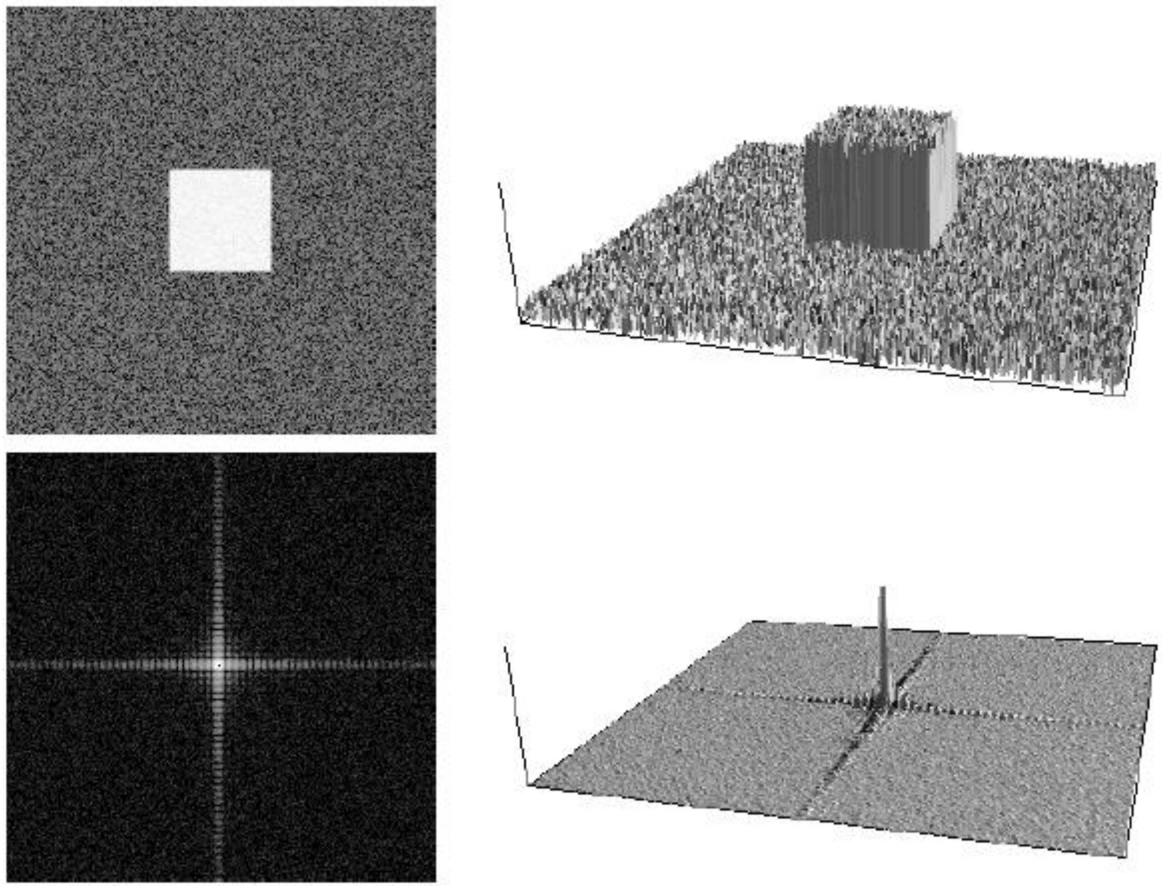
a. 原图;
b. 含92%图像功率;
c. 含94.6%图像功率;
d. 含96.4%图像功率;
e. 含98%图像功率;
f. 含99.5%图像功率:

被滤除的8%的功率中，含有多数尖锐的细节信息;



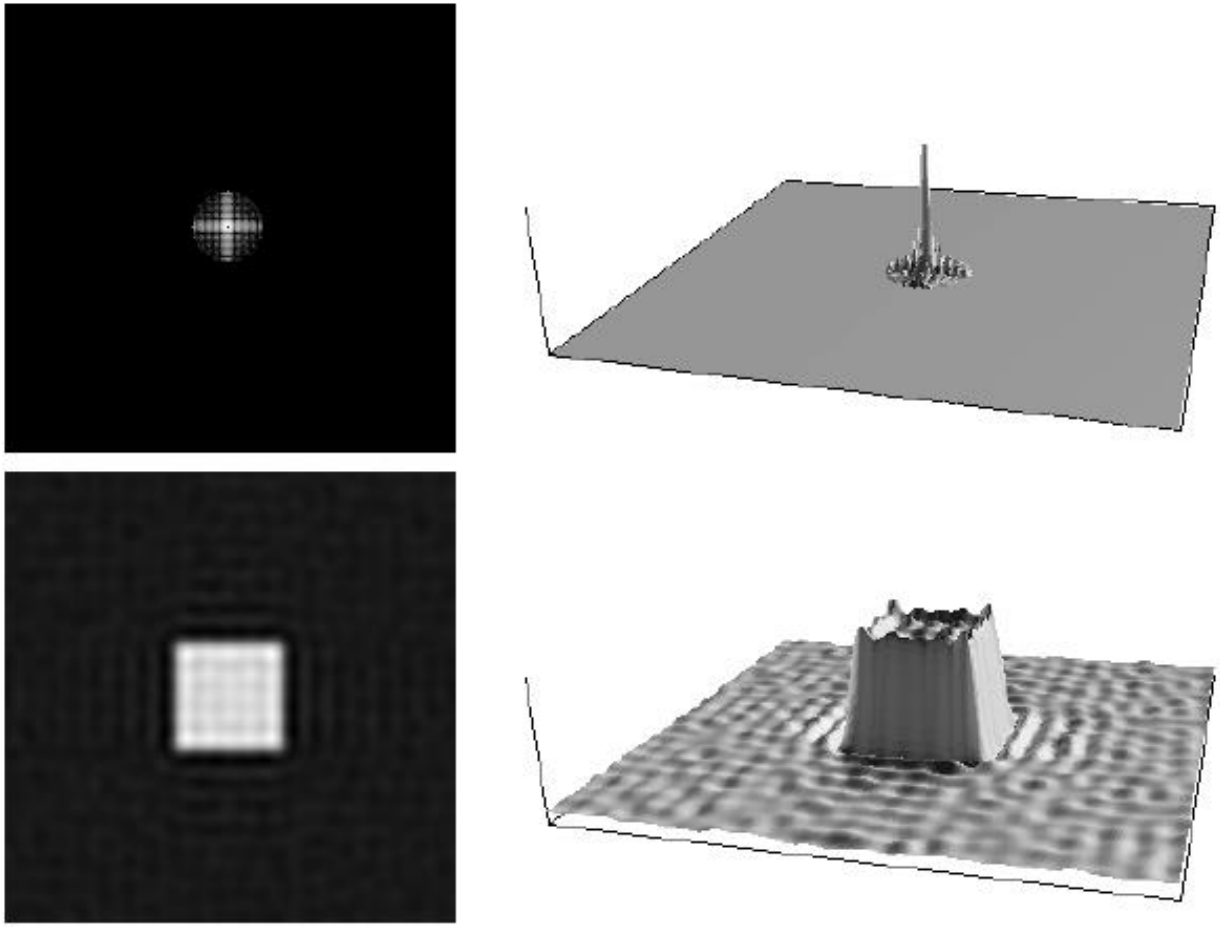
2. 低通滤波

理想低通滤波



2. 低通滤波

理想低通滤波和振铃效应



2. 低通滤波

理想低通滤波和振铃效应

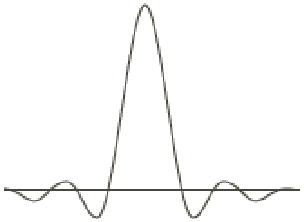
$$F(u,v)\,H(u,v)=G(u,v)$$

$$\Updownarrow$$

$$f(x,y)*h(x,y)=g(x,y)$$

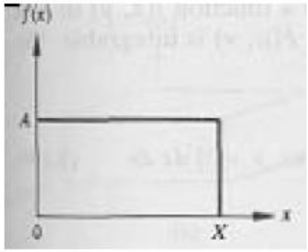
窗函数的傅里叶变换

time domain

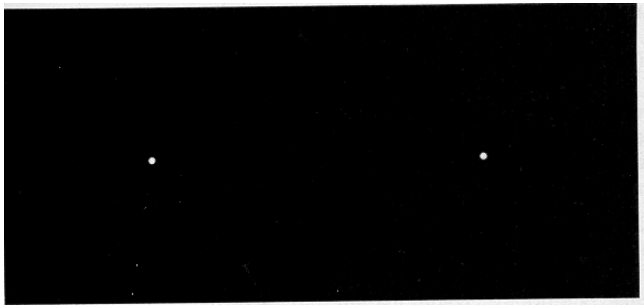


$\Leftrightarrow$

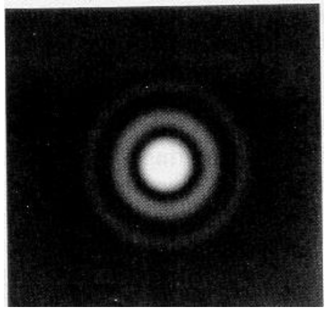
freq. domain



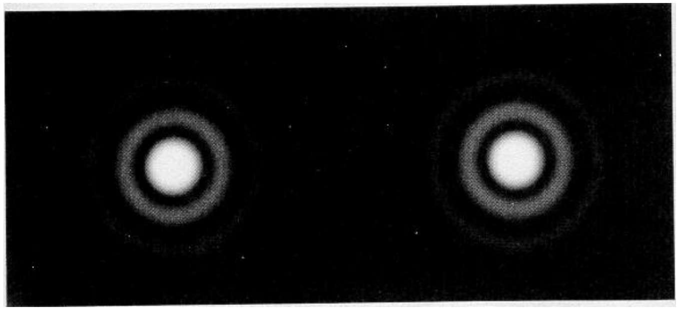
$$f(x,y)=\delta(x+x_0,y)+\delta(x-x_0,y)$$



$*$



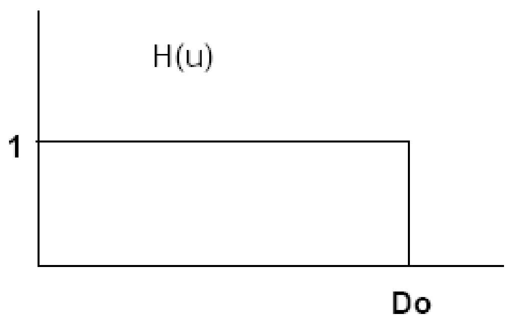
$=$

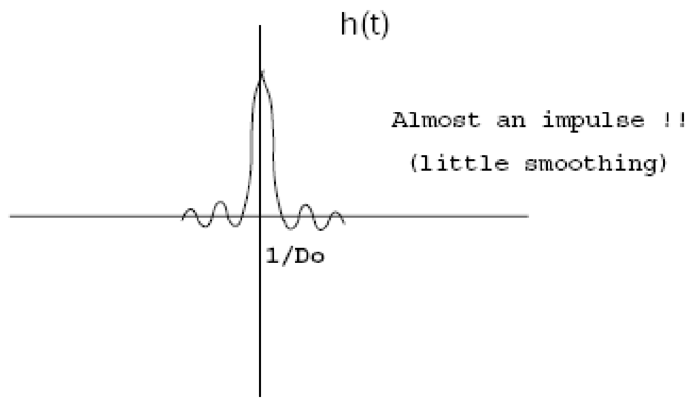


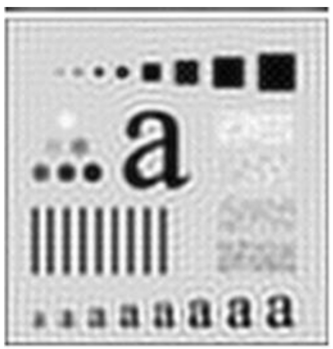
2. 低通滤波

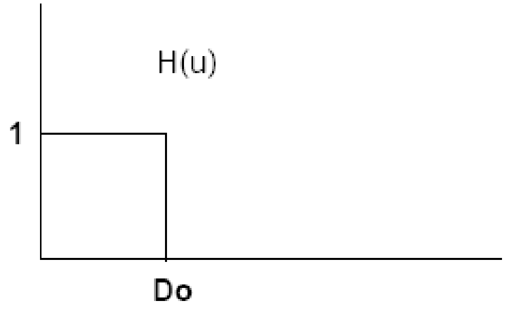
理想低通滤波和振铃效应

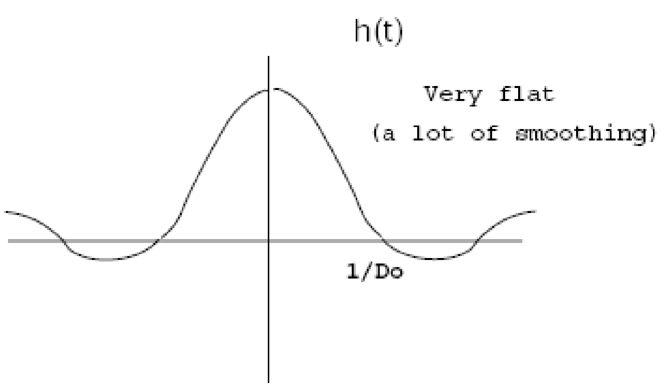
D_0 决定了平滑的效果







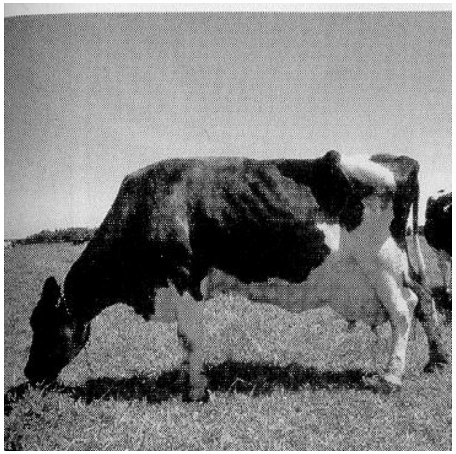


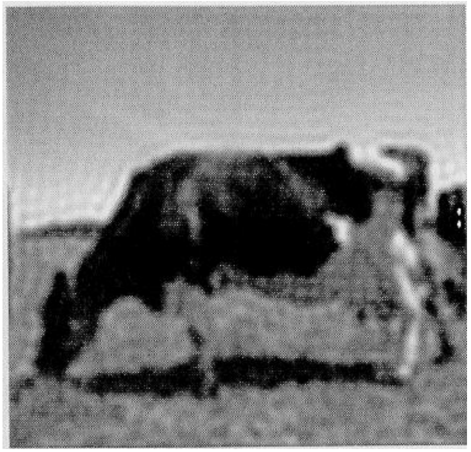


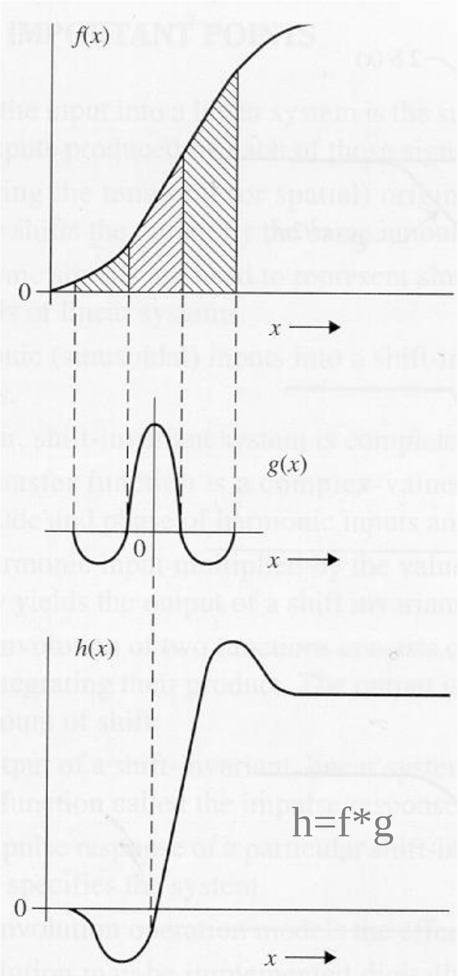
2. 低通滤波

理想低通滤波和振铃效应

- Sharp cutoff frequencies produce an overshoot of image features whose frequency is close to the cutoff frequencies **(ringing effect).**

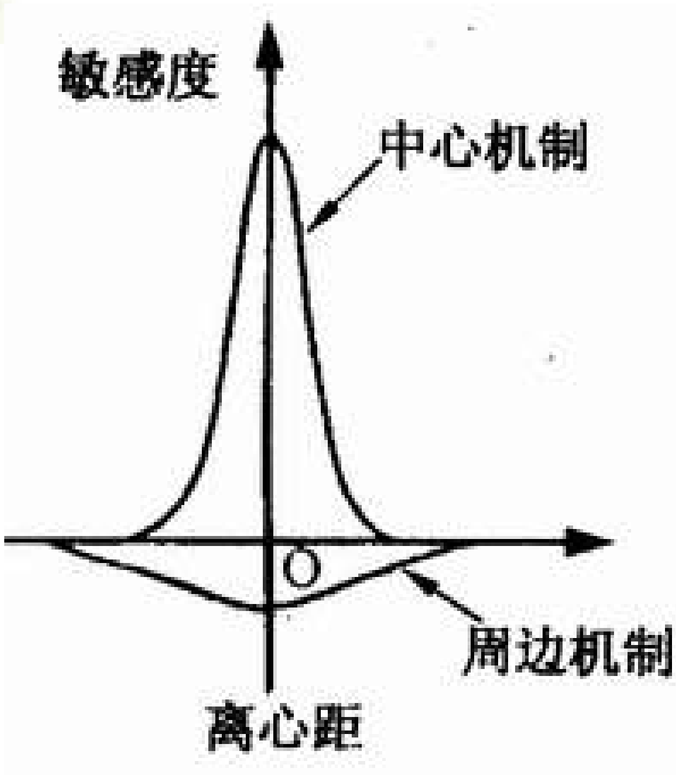




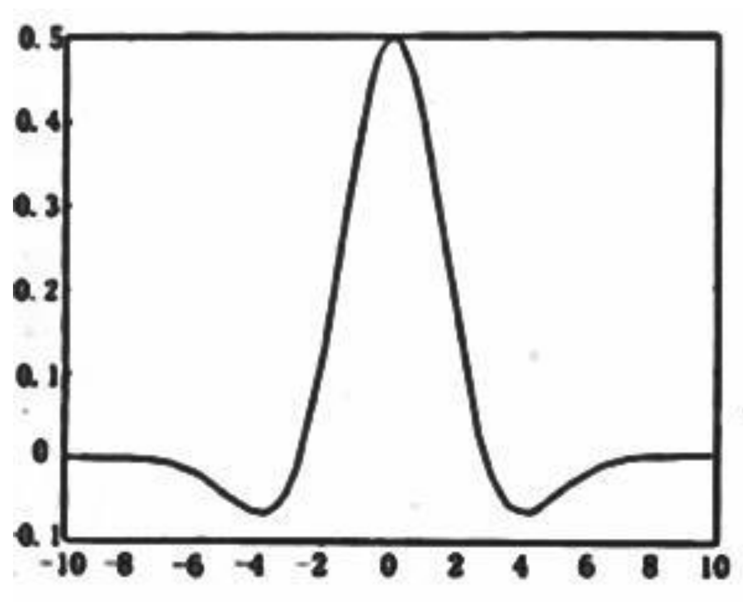


马赫带—指人们在明暗变化的边界，常常在亮区看到一条更亮的光带，而在暗区看到一条更暗的线条。这就是马赫带现象，马赫带不是由于刺激能量的分布，而是由于受到视觉”惰性”的影响

感受野同心圆拮抗式模型



（Rodieck, 1965）



墨西哥草帽算子

2. 低通滤波

其它低通滤波器

- Gaussian

$$H(u) = e^{-\frac{1}{2}u^2/{u_c}^2}$$

- Butterworth

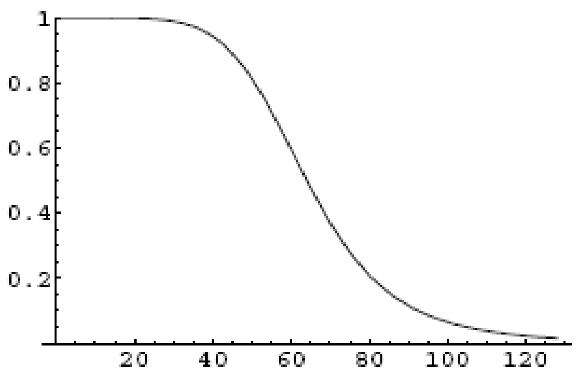
$$H(u) = \frac{1}{1 + (u^2/{u_c}^2)^n}$$

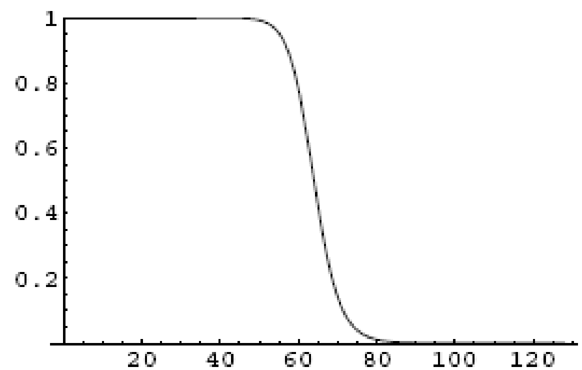
抑制振铃效应 vs 平滑滤波性能

2. 低通滤波

巴特沃斯低通滤波器

$$H(u)=\frac{1}{1+(u^2/u_c{}^2)^n}$$





Uc：截止频率，控制频率的截断位置；

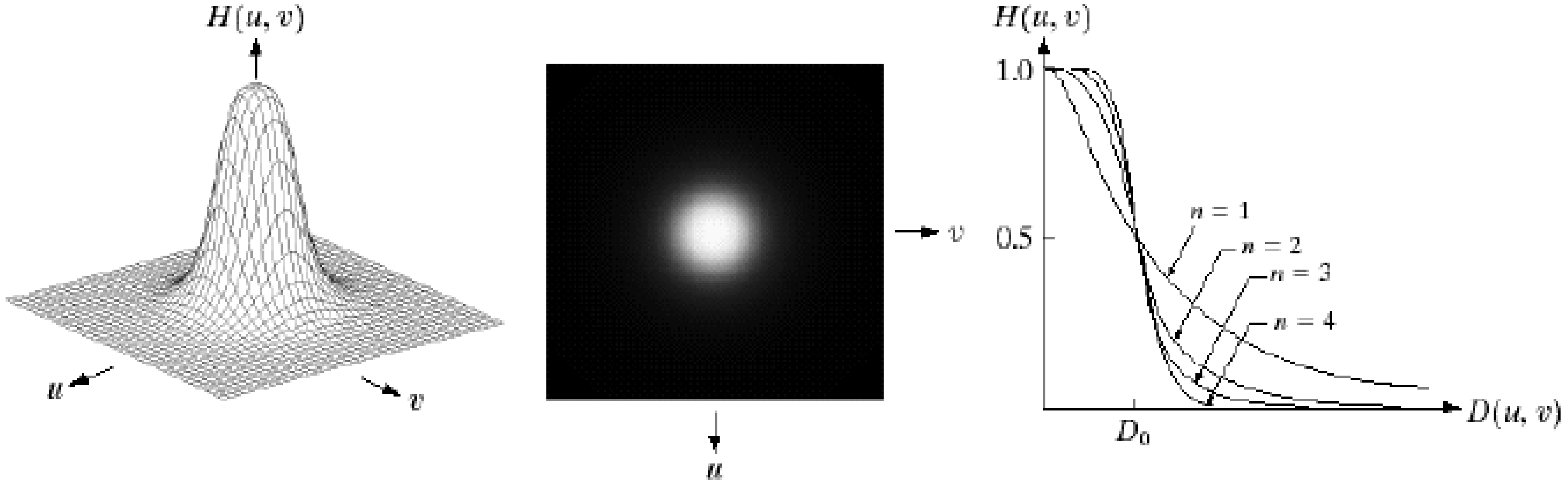
n：滤波器阶次，控制截断性能；

n越大，平滑性能越好，但会产生振铃；

n越小，平滑性能越弱，但振铃效应不明显。

2. 低通滤波

巴特沃斯低通滤波器

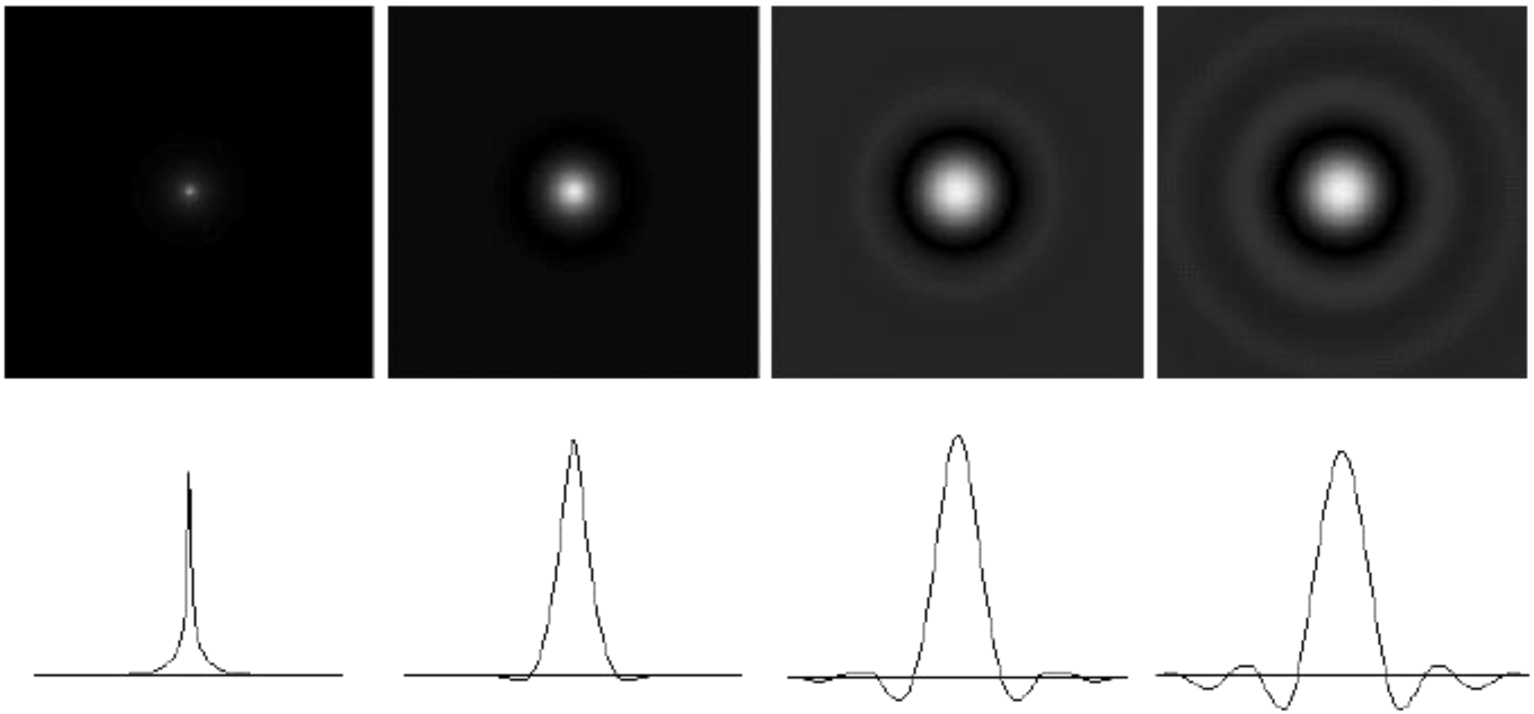


a b c

FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

2. 低通滤波

巴特沃斯低通滤波器



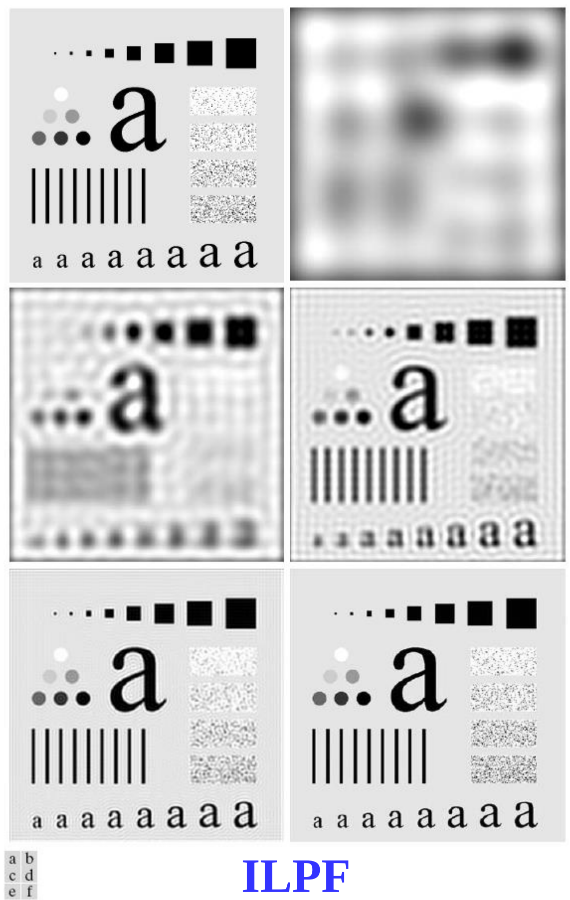
a b c d

FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

2. 低通滤波

巴特沃斯低通滤波器 vs 理想低通滤波器

D_0=10, 30, 60, 160, 460



ILPF

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

BLPF

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

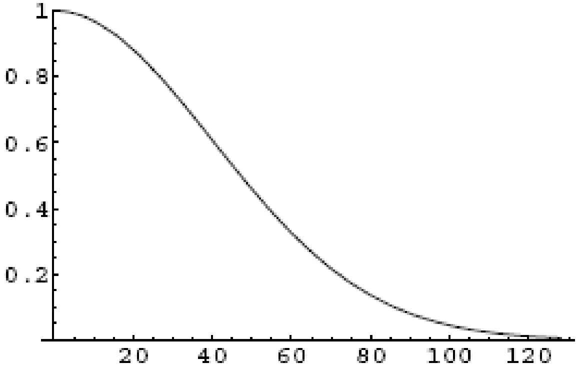
D_0=10, 30, 60, 160, 460

n=2

2. 低通滤波

高斯低通滤波器

$$H(u) = e^{-\frac{1}{2}u^2/u_c{}^2}$$

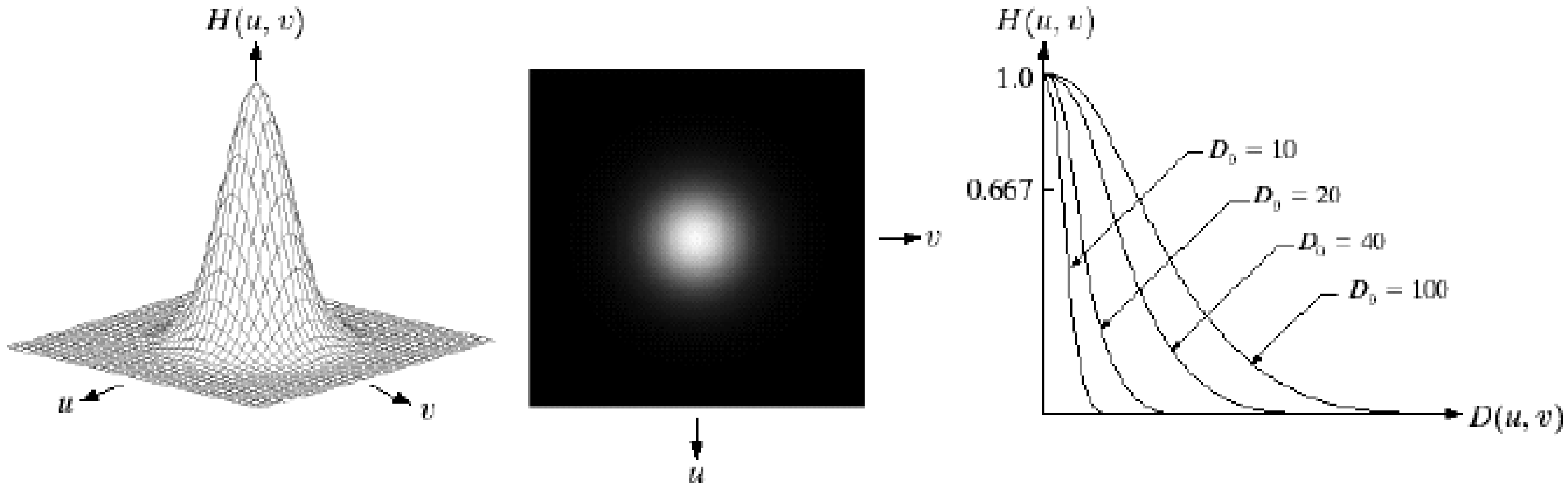


Uc：高斯滤波函数的方差；

Uc的意义与空域高斯滤波相同

2. 低通滤波

高斯低通滤波器



a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

2. 低通濾波

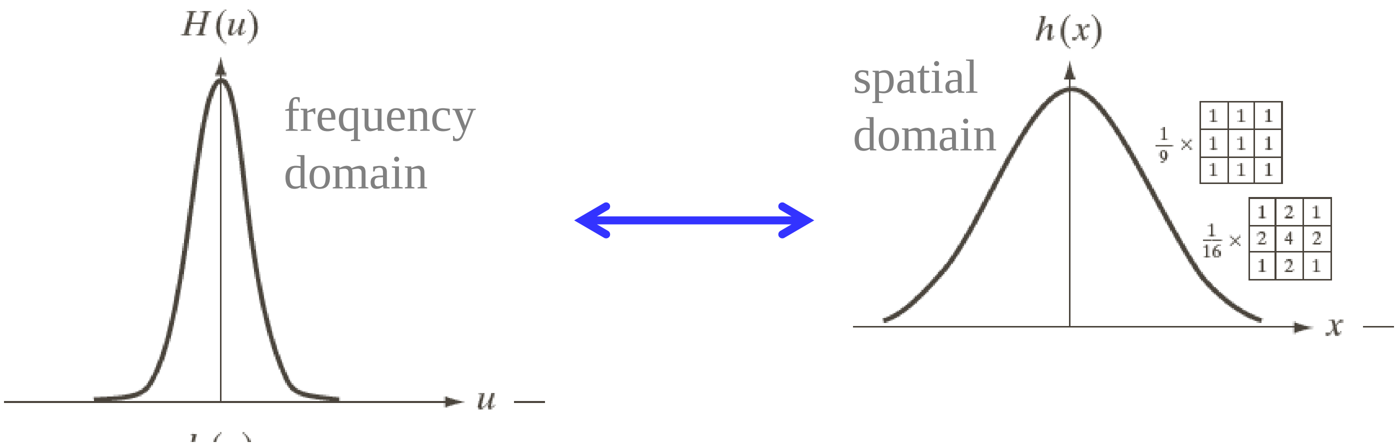
高斯低通濾波器

Let H(u) denote the 1-D frequency domain Gaussian filter

$$H(u) = Ae^{-u^2/2\sigma^2}$$

The corresponding filter in the spatial domain

$$h(x) = \sqrt{2\pi}\sigma A e^{-2\pi^2\sigma^2x^2}$$



2. 低通滤波

高斯低通滤波器

a 为原图
b, c, d, e, f分别为截止频率点半径为5, 15, 30, 80, 230示例结果。

没有出现振铃，图像平滑；但是，平滑的效果较BPLF差些。

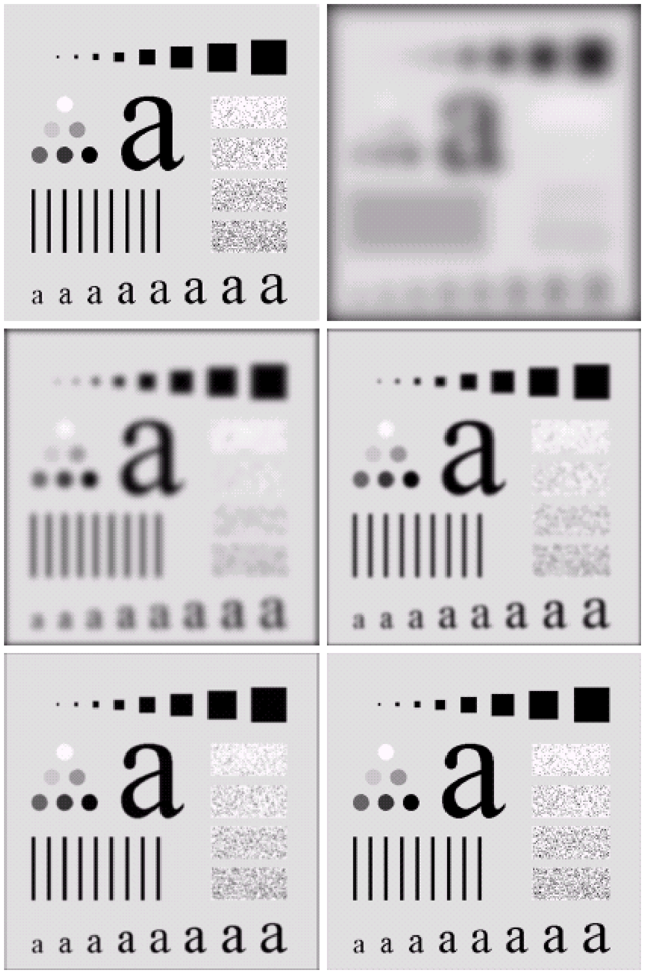
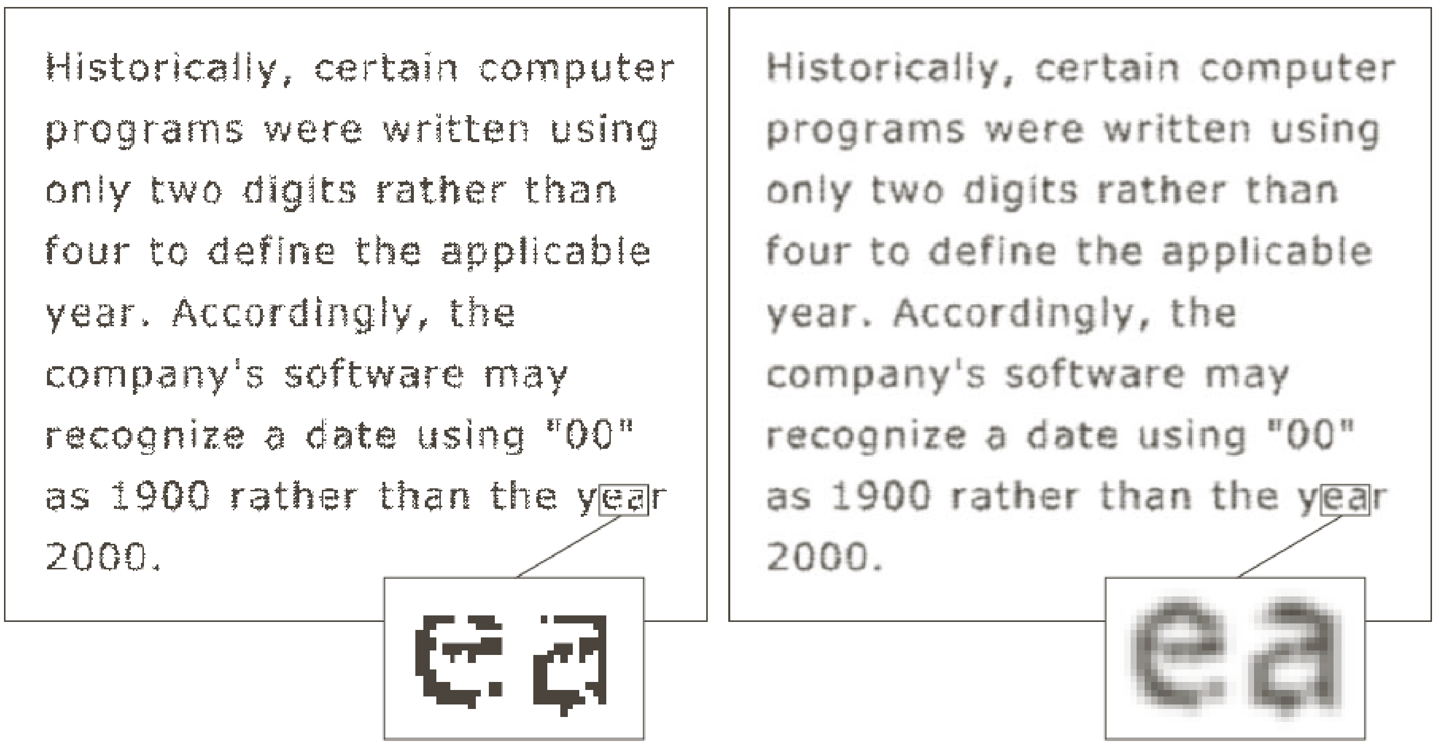


FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

a b
c d
e f

2. 低通滤波

高斯低通滤波器的应用



a b

FIGURE 4.49 (a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

2. 低通滤波

高斯低通滤波器的应用

a b c

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

3. 高通滤波与带通滤波

高通滤波的作用

- 去模糊（Deblurring）
- 逆卷积（Deconvolution）
- 锐化（Sharpening）
- 边缘提取（Edge Detection ）

消除模糊，但是放大噪声

3. 高通滤波与带通滤波

高通滤波器

Ideal: $$H(u) = \begin{cases} 1 & \text{if } u > u_c \\ 0 & \text{otherwise} \end{cases}$$

Gaussian: $$H(u) = 1 - e^{-\frac{1}{2}u^2/{u_c}^2}$$

Butterworth: $$H(u) = \frac{1}{1+\left({u_c}^2/u^2\right)^n}$$

3. 高通滤波与带通滤波

高通滤波

Ideal

Butterworth

Gaussian

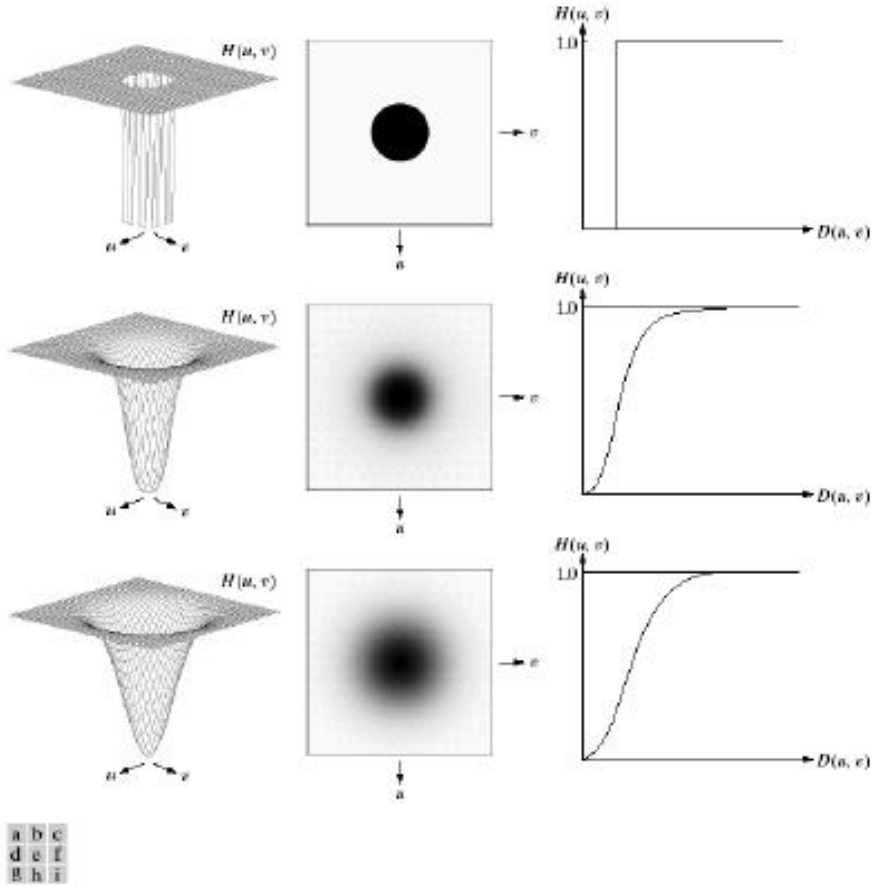
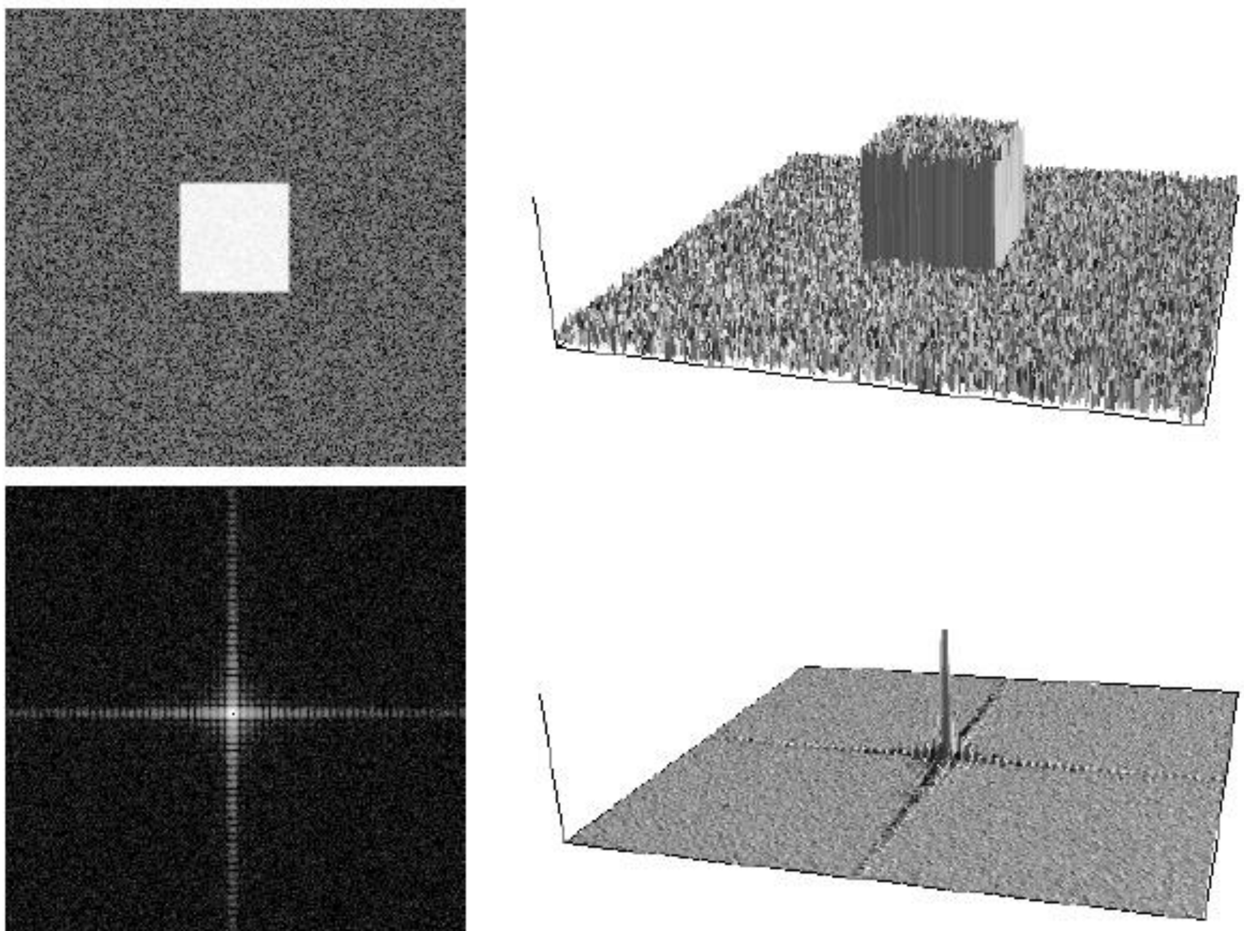


FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

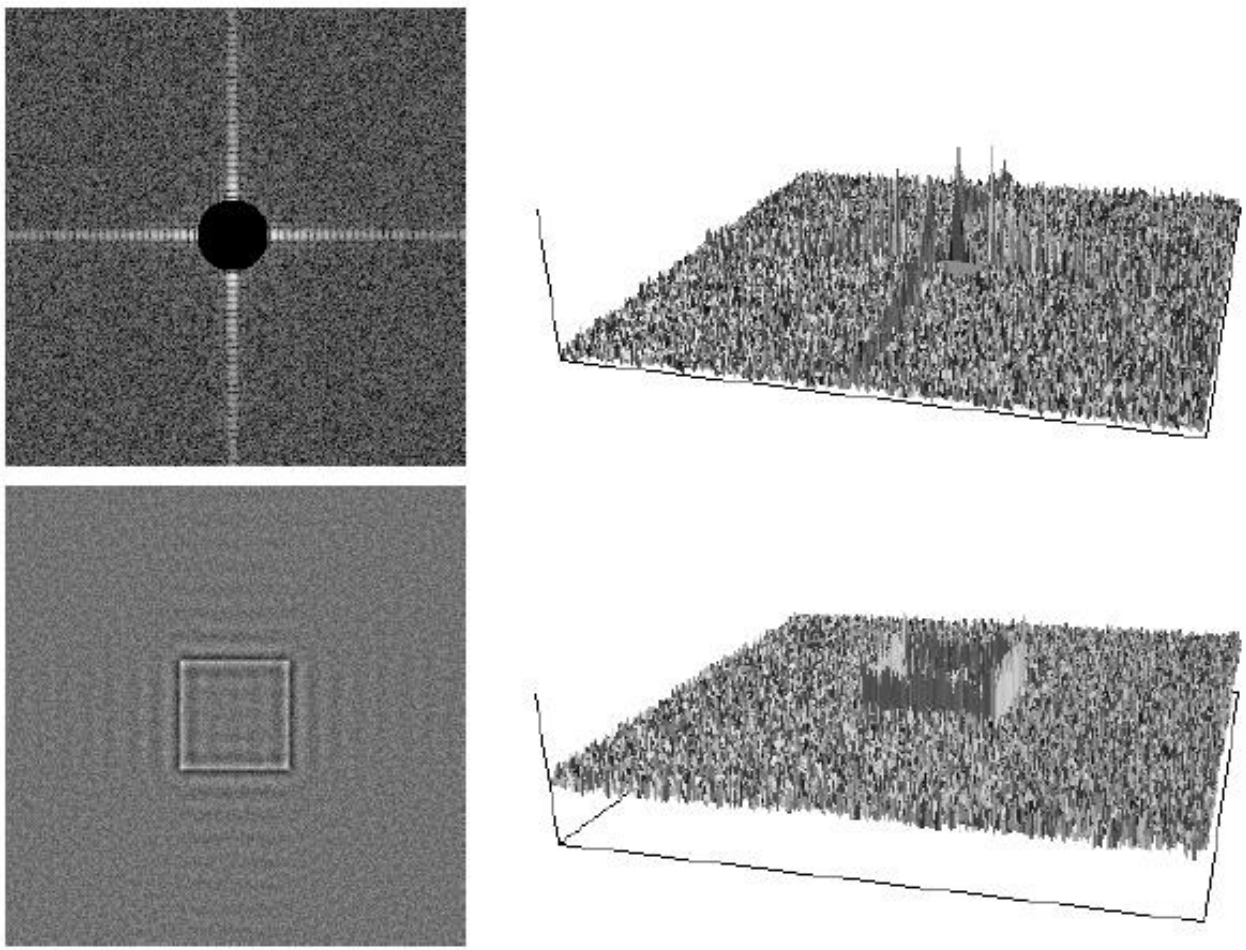
3. 高通滤波与带通滤波

理想高通滤波



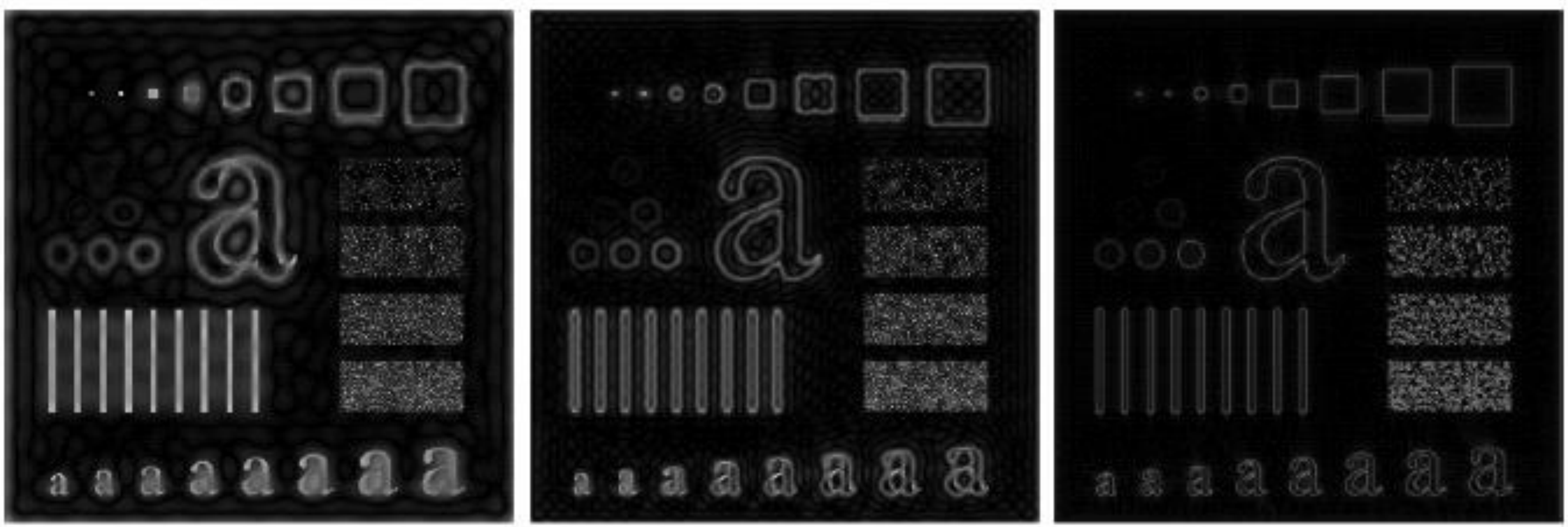
3. 高通滤波与带通滤波

理想高通滤波



3. 高通滤波与带通滤波

理想高通滤波



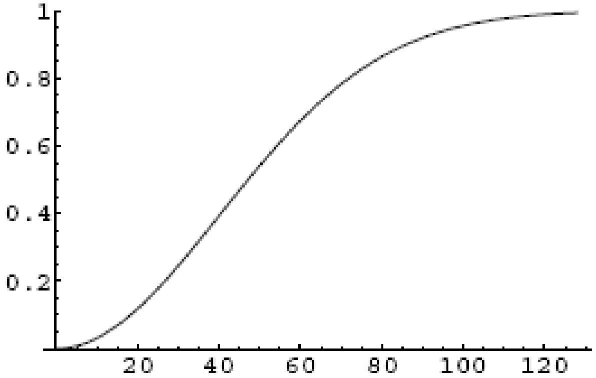
a b c

FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

3. 高通滤波与带通滤波

高斯高通滤波

$$H(u) = 1 - e^{-\frac{1}{2}u^2/{u_c}^2}$$



3. 高通滤波与带通滤波

高斯高通滤波

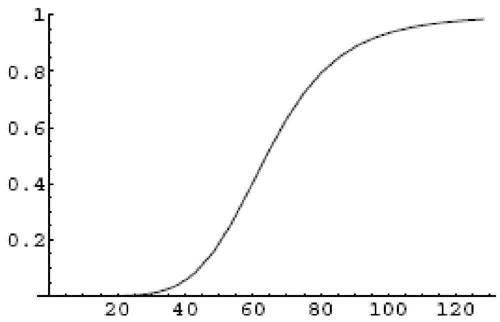
a b c

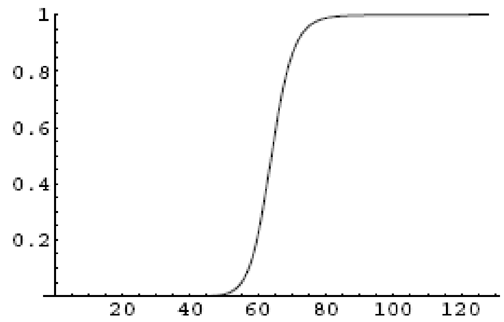
FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.

3. 高通滤波与带通滤波

巴特沃斯高通滤波

$$H(u) = \frac{1}{1 + \left(u_c^2 / u^2\right)^n}$$



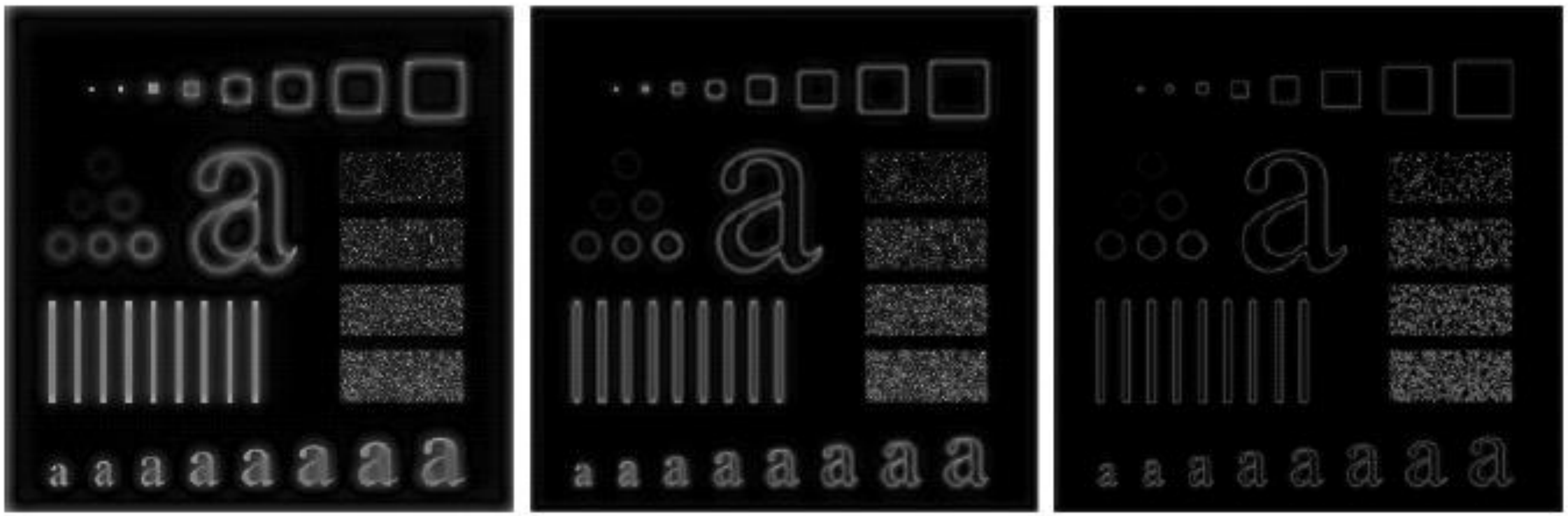


Uc：截止频率，控制频率的截断位置；

n：滤波器阶次，控制截断性能；

3. 高通滤波与带通滤波

巴特沃斯高通滤波



a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

3. 高通滤波与带通滤波

高通滤波的空域表示

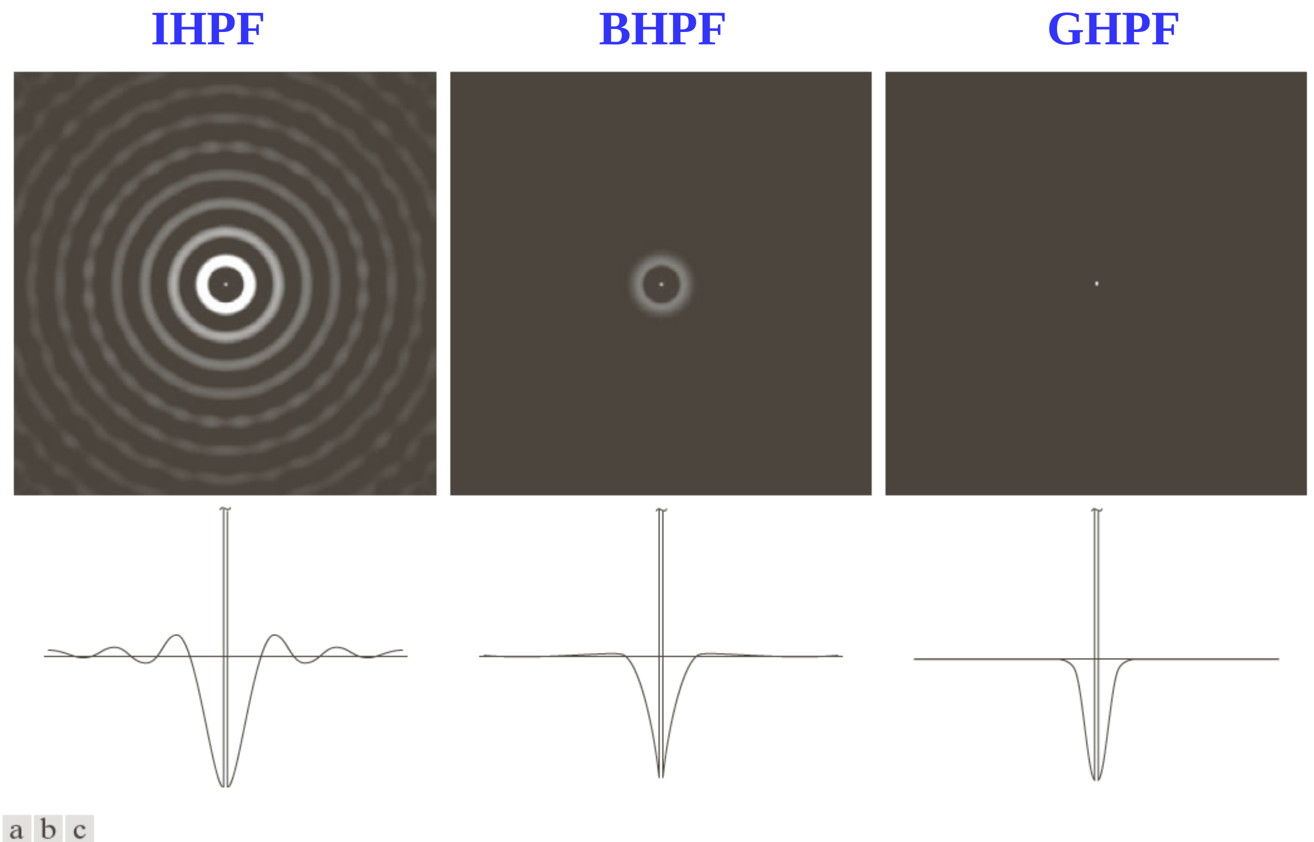


FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

3. 高通滤波与带通滤波

理想高通滤波 vs 高斯高通滤波

IHPF

D_0=30,60,160

a b c

FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30, 60$, and 160.

BHPF

D_0=30,60,160

n=2

a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60$, and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

3. 高通滤波与带通滤波

巴特沃斯高通滤波 vs 高斯高通滤波

BHPF

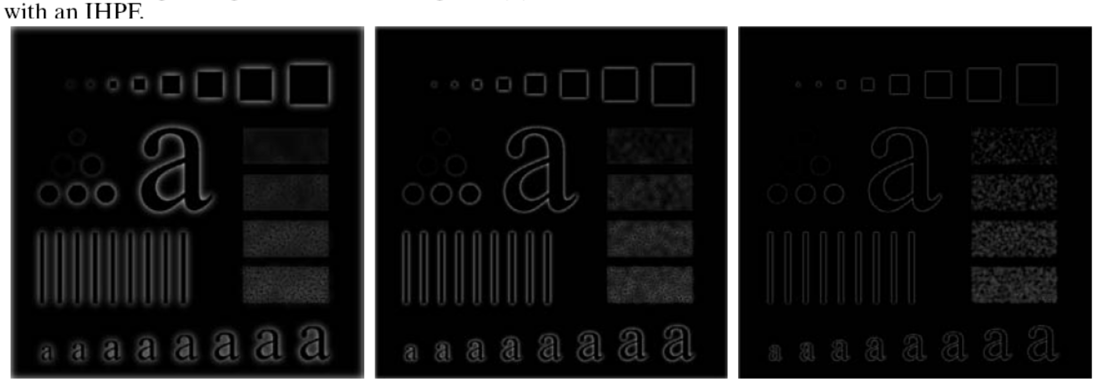
D_0=30,60,160

n=2

a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60$, and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

GHPF



D_0=30,60,160

a b c

FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30, 60$, and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

3. 高通滤波与带通滤波

高通滤波应用

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

3. 高通滤波与带通滤波

高通滤波：LOG

Let $H(u)$ denote the difference of Gaussian filter

$$H(u) = Ae^{-u^2/2\sigma_1^2} - Be^{-u^2/2\sigma_2^2}$$

with $A \geq B$ and $\sigma_1 \geq \sigma_2$

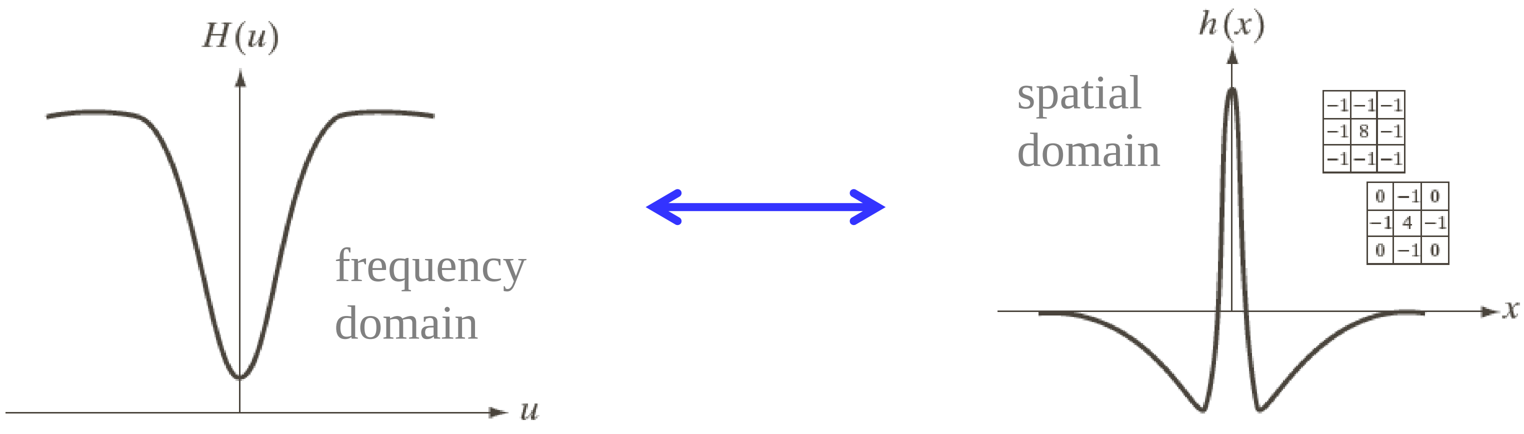
The corresponding filter in the spatial domain

$$h(x) = \sqrt{2\pi}\sigma_1 Ae^{-2\pi^2\sigma_1^2x^2} - \sqrt{2\pi}\sigma_2 Ae^{-2\pi^2\sigma_2^2x^2}$$

This is a high-pass filter!

3. 高通滤波与带通滤波

高通滤波：LOG



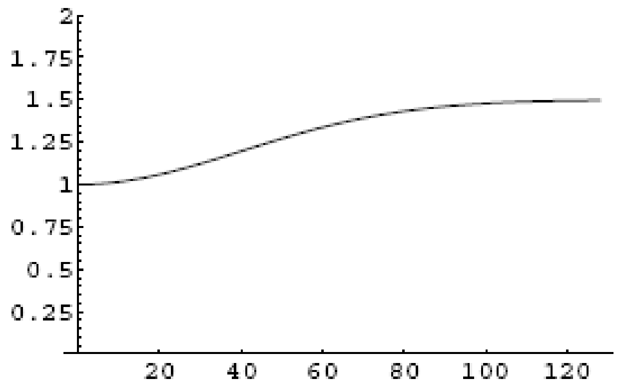
High-pass filter!

3. 高通滤波与带通滤波

高频提升滤波

$$H(u) = 1 + \alpha\, HP(u)$$

- $HP(u)$ is a high-pass filter
- α controls how much to boost the higher frequencies



3. 高通滤波与带通滤波

高频提升滤波

Unsharp Masking:

$$g_{mask}(x,y) = f(x,y) - f_{LP}(x,y)$$

Highboost filtering:
(alternative definition)

$$\begin{aligned} g(x,y) &= f(x,y) + kg_{mask}(x,y) = f(x,y) + k(f(x,y) - f_{LP}(x,y)) \\ &= f(x,y) + kf_{HP}(x,y) \end{aligned}$$

previous definition: $g(x,y) = (A-1)f(x,y) + f_{HP}(x,y)$

Frequency domain:

$$f_{LP}(x,y) = f(x,y) * h_{LP}(x,y)$$

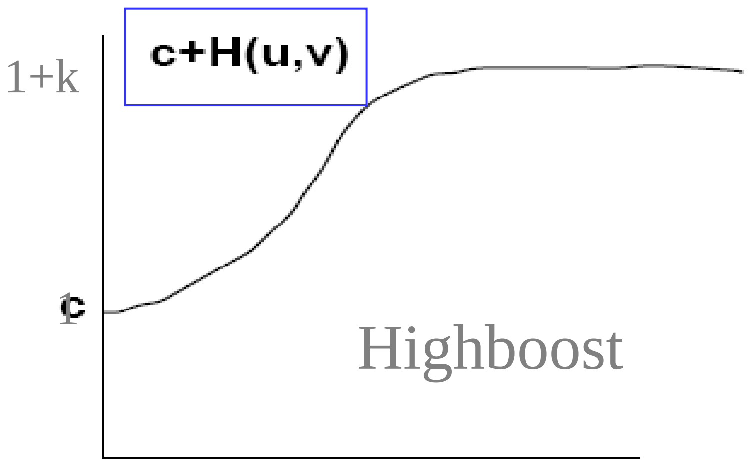
$$\mathbf{F}(f_{LP}(x,y)) = F(u,v)H_{LP}(x,y)$$

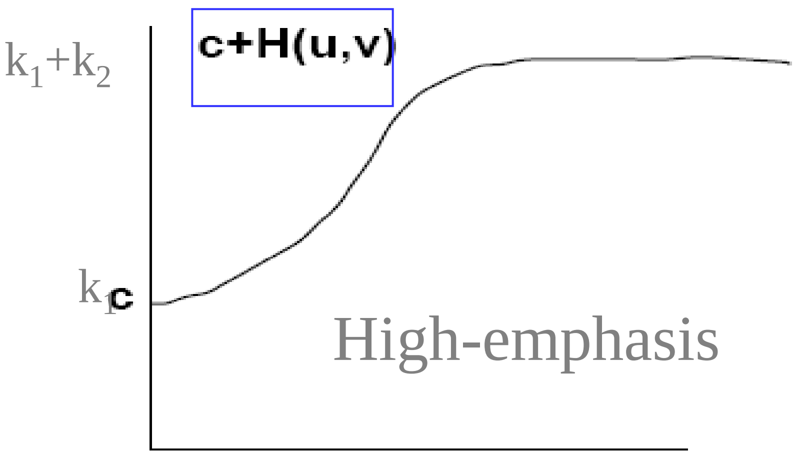
3. 高通滤波与带通滤波

高频提升滤波

$$g(x,y)=\mathbf{F}^{-1}((1+kH_{HP}(u,v))F(u,v))$$
$$k\geq 0$$

$$g(x,y)=\mathbf{F}^{-1}((k_1+k_2H_{HP}(u,v))F(u,v))$$
$$k_1\geq 0,\ k_2\geq 0$$





3. 高通滤波与带通滤波

高频提升滤波的应用

D_0=40

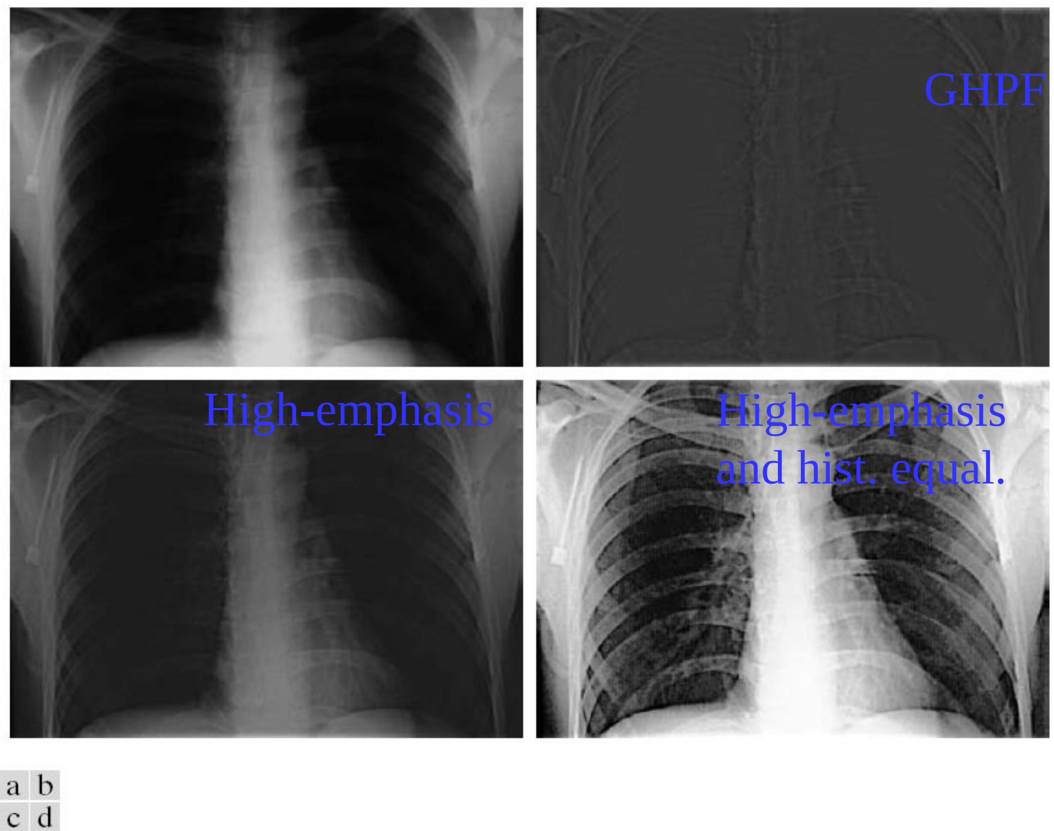


FIGURE 4.59 (a) A chest X-ray image. (b) Result of highpass filtering with a Gaussian filter. (c) Result of high-frequency-emphasis filtering using the same filter. (d) Result of performing histogram equalization on (c). (Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

High-Frequency Emphasis filtering Using Gaussian filter

k_1=0.5, k_2=0.75

3. 高通滤波与带通滤波

高频提升滤波的应用

$$H(u,v)=(\gamma_H-\gamma_L)\left[1-e^{-c\left[(u^2+v^2)/D_0^2\right]}\right]+\gamma_L$$

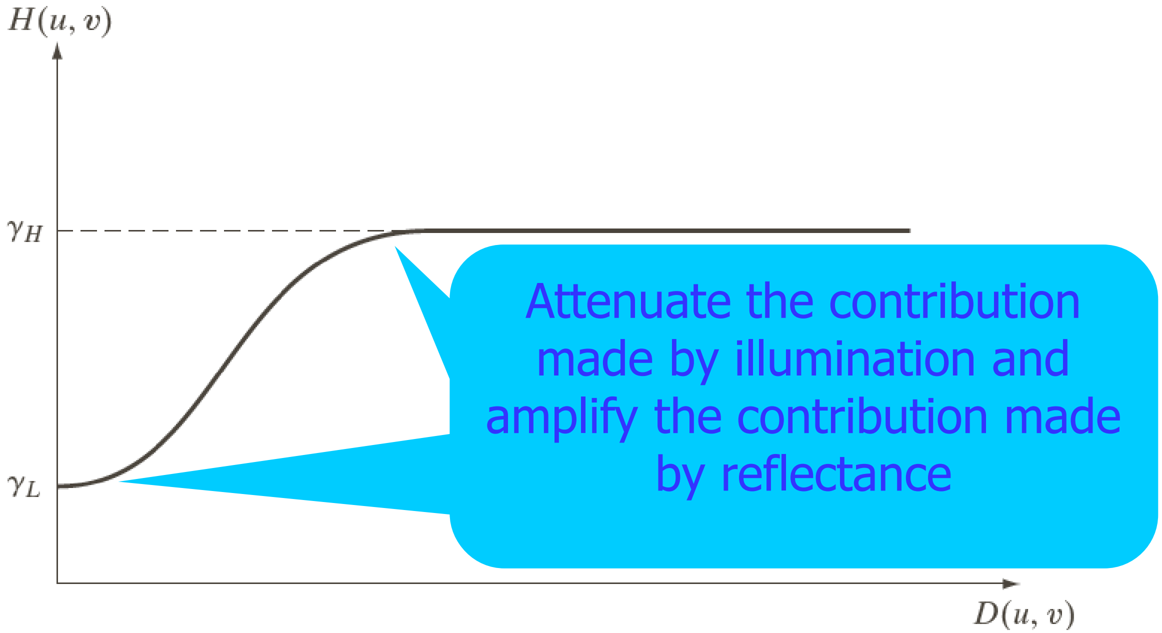


FIGURE 4.61 Radial cross section of a circularly symmetric homomorphic filter function. The vertical axis is at the center of the frequency rectangle and $D(u, v)$ is the distance from the center.

3. 高通滤波与带通滤波

带通滤波

Tradeoff: Blurring vs. Noise

- Low-pass reduces noise but accentuates blurring
- High-pass reduces blurring but accentuates noise

A compromise:

Band-boost filtering boosts certain midrange freqencies and partially corrects for blurring, but does not boost the very high (most noise corrupted) frequencies.

3. 高通滤波与带通滤波

同态滤波（本质图像，Retinex颜色恒常性）

- 如何从图像中去除光照变换所带来的影响是图像处理领域的经典问题
  - 提升图像中高频部分
  - 衰减低频部分但是要保留细节

3. 高通滤波与带通滤波

同态滤波

- Consider the following model of image formation:

$$f(x,y) = i(x,y)\, r(x,y)$$

i(x,y): illumination
r(x,y): reflection

- In general, the illumination component i(x,y) varies **slowly** and affects **low** frequencies mostly.
- In general, the reflection component r(x,y) varies **faster** and affects **high** frequencies mostly.

<u>Idea:</u> separate low frequencies due to i(x,y)
from high frequencies due to r(x,y)

3. 高通滤波与带通滤波

同态滤波

- Low and high frequencies from **i(x,y)** and **r(x,y)** are mixed together.

$$f(x,y) = i(x,y)\, r(x,y) \quad \Rightarrow \quad F(u,v) = I(u,v) * R(u,v)$$

- When applying filtering, it is difficult to handle low/high frequencies separately.

$$F(u,v)H(u,v) = [I(u,v) * R(u,v)]H(u,v)$$

3. 高通滤波与带通滤波

同态滤波

Can we separate them?

- Idea:

Take the ln( ) of $f(x,y) = i(x,y)\,r(x,y)$

$$ln(f(x,y)) = ln(i(x,y)) + ln(r(x,y))$$

3. 高通滤波与带通滤波

同态滤波的基本步骤

(1) Take $ln(f(x,y)) = ln(i(x,y)) + ln(r(x,y))$

(2) Apply FT: $F(ln(f(x,y))) = F(ln(i(x,y))) + F(ln(r(x,y)))$

$$Z(u,v) = Illum(u,v) + Refl(u,v)$$

(3) Apply H(u,v)

$$Z(u,v)H(u,v) = Illum(u,v)H(u,v) + Refl(u,v)H(u,v)$$

3. 高通滤波与带通滤波

同态滤波的基本步骤

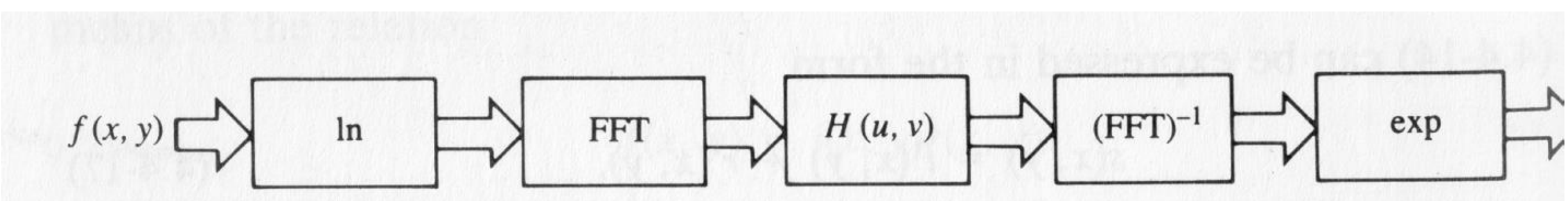
(4) Take Inverse FT:

$$F^{-1}(Z(u,v)H(u,v)) = F^{-1}(Illum(u,v)H(u,v)) + F^{-1}(Refl(u,v)H(u,v))$$

$$s(x,y) = i'(x,y) + r'(x,y)$$

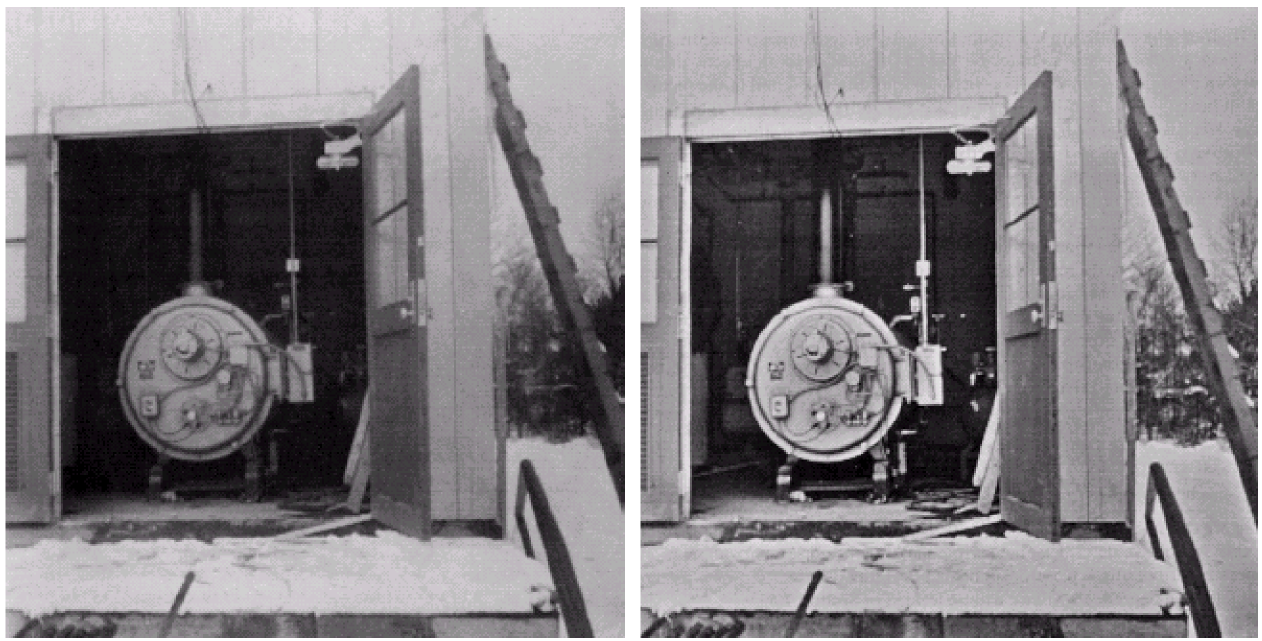
(5) Take exp( ) or

$$e^{s(x,y)} = e^{i'(x,y)} e^{r'(x,y)} \qquad g(x,y) = i_0(x,y) r_0(x,y)$$



3. 高通滤波与带通滤波

同态滤波的应用

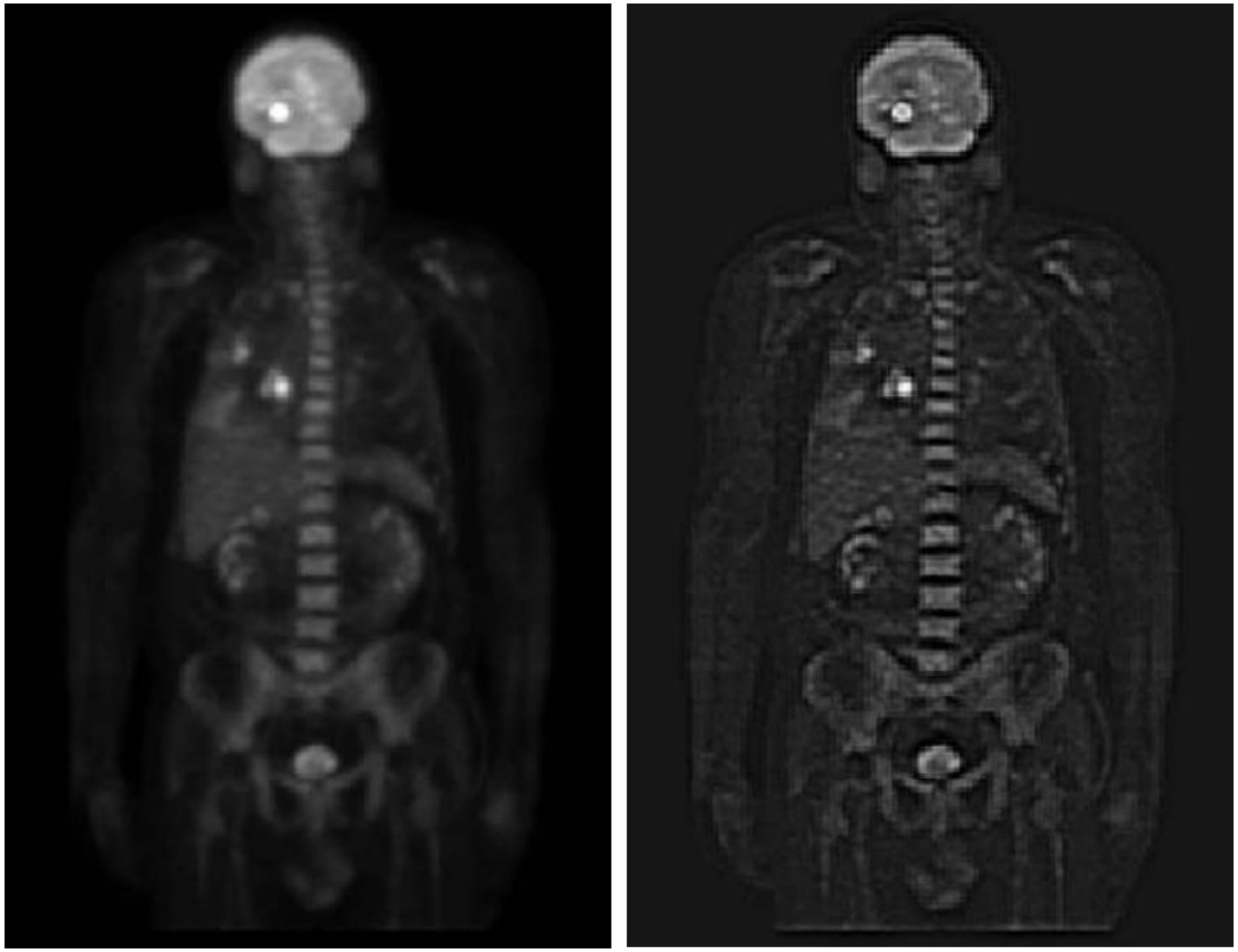


a b

FIGURE 4.33 (a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)

3. 高通滤波与带通滤波

同态滤波的应用



a b

FIGURE 4.62 (a) Full body PET scan. (b) Image enhanced using homomorphic filtering. (Original image courtesy of Dr. Michael E. Casey, CTI PET Systems.)

$$\gamma_L = 0.25$$

$$\gamma_H = 2$$

$$c = 1$$

$$D_0 = 80$$

3. 高通滤波与带通滤波

同态滤波的应用

3. 高通滤波与带通滤波

同态滤波的应用

3. 高通滤波与带通滤波

同态滤波的应用