《计算机辅助几何设计》作业5

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1　作业要求

1. 已知如图1的平面型值点，求作非均匀4阶（3次）B样条曲线，并用程序实现。要求：(1) 用红色圆圈显示型值点，用蓝色方块显示所求得的B样条的控制顶点 (2) 递交程序代码及作业报告 2. 将上述程序进行扩展，写一个通用的非均匀4阶插值B样条曲线的作图软件：用户在平面上用鼠标指定型值点以及型值点的类型（直线段、尖点、一阶连续等），软件自动生成插值的B样条曲线。(1) 用户界面可自行设计，要求界面友好，操作方便，各种条件设置灵活 (2) 递交程序代码、软件使用说明书及测试报告

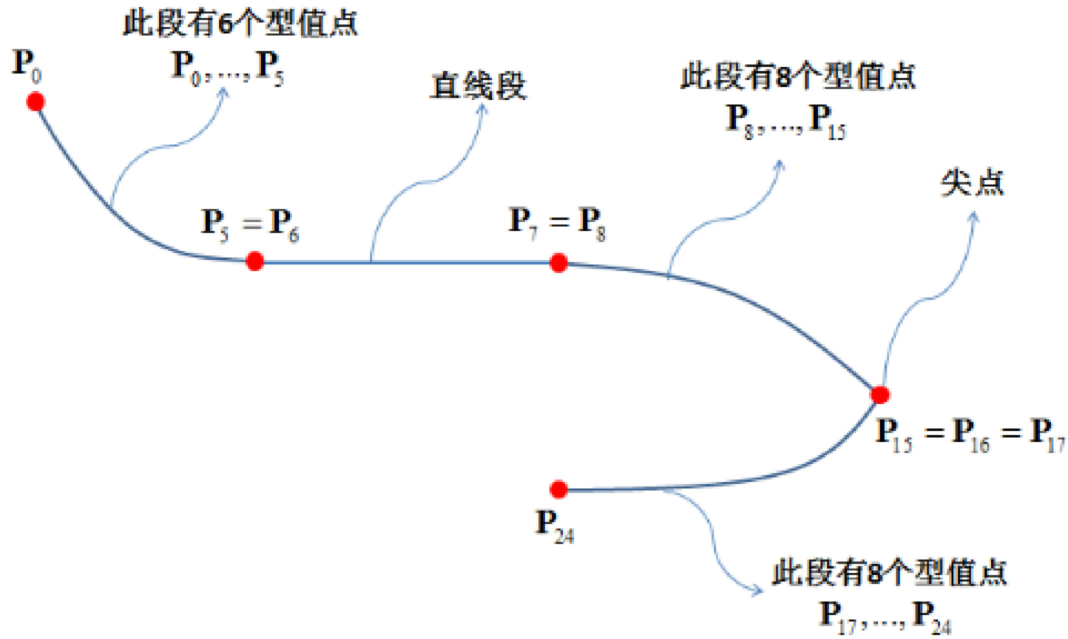

图 1:

2 算法解析

设用户输入型值点为$\{\boldsymbol{k}_i\}(k=0,1,\cdots,n)$，结向量为$T$,k阶单位B样条基函数为：

$$N_i^1(t)=\begin{cases}1, & t_i\le t<t_{i+1}\\ 0, & \text{otherwise}\end{cases} \tag{1}$$

$$N_i^k=\frac{t-t_i}{t_{i+k-1}-t_i}N_i^{k-1}(t)+\frac{t_{i+k}-t}{t_{i+k}-t_{i+1}}N_{i+1}^{k-1}(t)$$

核心是求得所有de Boor控制点，然后带入B样条曲线方程：

$$\boldsymbol{x}(t)=\sum_{i=0}^{n}N_{i,k}(t)\cdot\boldsymbol{d}_i \tag{2}$$

得到结果

2.1 C^2连续曲线设计

对于$n+1$个型值点，分段三次插值B样条曲线需要$n+3$个de Boor控制点，结向量选取为

$$\begin{aligned} T &= (t_0, t_1, t_2, \cdots, t_{n+2}, t_{n+3}, t_{n+4}, t_{n+5}, t_{n+6}) \\ &= (s_0, s_0, s_0, s_0, s_1, s_2, s_3 \cdots, s_{n-1}, s_n, s_n, s_n, s_n) \end{aligned} \tag{3}$$

其中结序列$\{s_0, s_1, \cdots, s_{n-1}, s_n\}$选为自然数序列$\{0, 1, 2, \cdots, n\}$

插值条件为：

$$\begin{aligned} &\boldsymbol{x}(s_0) = \boldsymbol{k}_0 = \boldsymbol{d}_0 \\ &\boldsymbol{x}(s_i) = \boldsymbol{k}_i = N_{i+1,4}(s_i)\boldsymbol{d}_{i+1} + N_{i+2,4}(s_i)\boldsymbol{d}_{i+2} \\ &\qquad \text{for } i = 1, \cdots, n-1 \\ &\boldsymbol{x}(s_n) = \boldsymbol{k}_n = \boldsymbol{d}_{n+2} \end{aligned} \tag{4}$$

使用自然边界条件：

$$\begin{aligned} \ddot{\boldsymbol{x}}(s_0) = 0 \Leftrightarrow \frac{\boldsymbol{d}_2 - \boldsymbol{d}_1}{s_2 - s_0} = \frac{\boldsymbol{d}_1 - \boldsymbol{d}_0}{s_1 - s_0} \\ \ddot{\boldsymbol{x}}(s_n) = 0 \Leftrightarrow \frac{\boldsymbol{d}_{n+2} - \boldsymbol{d}_{n+1}}{s_n - s_{n-1}} = \frac{\boldsymbol{d}_{n+1} - \boldsymbol{d}_n}{s_n - s_{n-2}} \end{aligned} \tag{5}$$

可以组装成对角系统方程进行求解：

$$
\begin{pmatrix}
1 & & & & & & & \\
\alpha_0 & \beta_0 & \gamma_0 & & & & & \\
& \alpha_1 & \beta_1 & \gamma_1 & & & & \\
& & & \ddots & & & & \\
& & & & \alpha_{n-1} & \beta_{n-1} & \gamma_{n-1} & \\
& & & & & \alpha_n & \beta_n & \gamma_n \\
& & & & & & & 1
\end{pmatrix}
\begin{pmatrix}
\boldsymbol{d}_0 \\ \boldsymbol{d}_1 \\ \boldsymbol{d}_2 \\ \vdots \\ \boldsymbol{d}_n \\ \boldsymbol{d}_{n+1} \\ \boldsymbol{d}_{n+2}
\end{pmatrix}
=
\begin{pmatrix}
\boldsymbol{k}_0 \\ \mathbf{0} \\ \boldsymbol{k}_1 \\ \vdots \\ \boldsymbol{k}_{n-1} \\ \mathbf{0} \\ \boldsymbol{k}_n
\end{pmatrix}
\tag{6}
$$

其中，

$$
\begin{aligned}
&\alpha_0 = s_2 - s_0 \\
&\beta_0 = -(s_2 - s_0) - (s_1 - s_0) \\
&\gamma_0 = s_1 - s_0 \\
&\alpha_n = s_n - s_{n-1} \\
&\beta_n = -(s_n - s_{n-1}) - (s_n - s_{n-2}) \\
&\gamma_n = s_n - s_{n-2} \\
&\alpha_i = N_{i,4}(s_i) \\
&\beta_i = N_{i+1,4}(s_i) \\
&\gamma_i = N_{i+2,4}(s_i) \\
&\text{for } i = 1, \cdots, n-1
\end{aligned}
\tag{7}
$$

实现效果：

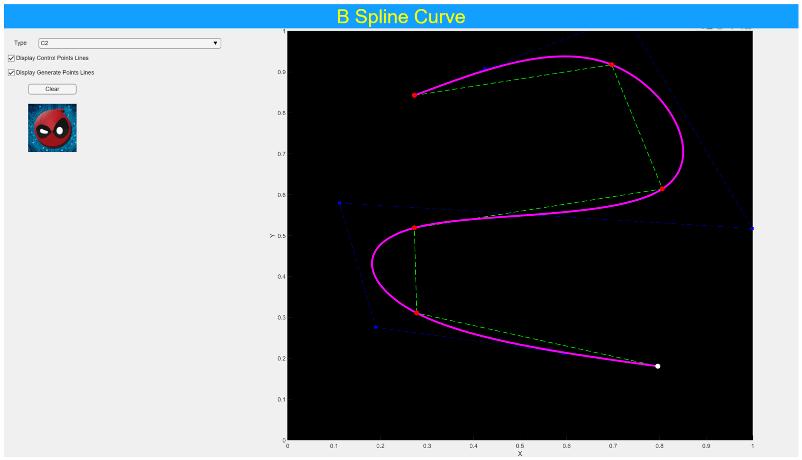

图 2:

2.2 直线段设计

设用户输入型值点为$\{\boldsymbol{k}_i\}(k=0,1,\cdots,n)$，要使得分段三次插值B样条曲线表现为直线段，需要有重叠的de Boor控制点，结向量选取为

$$\begin{aligned} T &= (t_0, t_1, t_2, \cdots, t_{n+2}, t_{n+3}, t_{n+4}, t_{n+5}, t_{n+6}) \\ &= (s_0, s_0, s_0, s_0, s_0, s_1, s_1, s_2, s_2 \cdots, s_{n-1}, s_{n-1}, s_n, s_n, s_n, s_n, s_n) \end{aligned} \tag{8}$$

其中结序列$\{s_0, s_1, \cdots, s_{n-1}, s_n\}$选为自然数序列$\{0, 1, 2, \cdots, n\}$

插值条件为：

$$\begin{aligned}
\boldsymbol{x}(s_0) &= \boldsymbol{k}_0 = \boldsymbol{d}_0 = \boldsymbol{d}_2 \\
\boldsymbol{x}(s_i) &= \boldsymbol{k}_i = \boldsymbol{d}_{2i} \\
\boldsymbol{x}(s_i) &= \boldsymbol{k}_i = \boldsymbol{d}_{2i+1} \\
&\text{for } i = 1, \cdots, n-1 \\
\boldsymbol{x}(s_n) &= \boldsymbol{k}_n = \boldsymbol{d}_{2n+2} = \boldsymbol{d}_{2n+4}
\end{aligned} \tag{9}$$

边界条件为：

$$\begin{aligned}
&\ddot{\boldsymbol{x}}(s_0) = 0 \Leftrightarrow \frac{\boldsymbol{d}_2 - \boldsymbol{d}_1}{s_2 - s_0} = \frac{\boldsymbol{d}_1 - \boldsymbol{d}_0}{s_1 - s_0} \Leftrightarrow 2\boldsymbol{d}_0 - 3\boldsymbol{d}_1 + \boldsymbol{d}_1 = 0 \\
&\ddot{\boldsymbol{x}}(s_n) = 0 \Leftrightarrow \frac{\boldsymbol{d}_{2n+3} - \boldsymbol{d}_{2n+2}}{s_n - s_{n-1}} = \frac{\boldsymbol{d}_{2n+2} - \boldsymbol{d}_{2n+1}}{s_n - s_{n-2}} \Leftrightarrow \boldsymbol{d}_{2n+1} - 3\boldsymbol{d}_{2n+2} + 2\boldsymbol{d}_{2n+3} = 0
\end{aligned} \tag{10}$$

可以组装成对角系统方程进行求解：

$$\begin{pmatrix}
1 & & & & & & & & \\
2 & -3 & 1 & & & & & & \\
 & & 0 & 1 & 0 & & & & \\
 & & & 0 & 1 & 0 & & & \\
 & & & & & \ddots & & & \\
 & & & & & 0 & 1 & 0 & \\
 & & & & & & 1 & -3 & 2 \\
 & & & & & & & & 1
\end{pmatrix}
\begin{pmatrix}
\boldsymbol{d}_0 \\ \boldsymbol{d}_1 \\ \boldsymbol{d}_2 \\ \boldsymbol{d}_3 \\ \vdots \\ \boldsymbol{d}_{2n+2} \\ \boldsymbol{d}_{2n+3} \\ \boldsymbol{d}_{2n+4}
\end{pmatrix}
=
\begin{pmatrix}
\boldsymbol{k}_0 \\ \mathbf{0} \\ \boldsymbol{k}_1 \\ \boldsymbol{k}_1 \\ \vdots \\ \boldsymbol{k}_{n-1} \\ \mathbf{0} \\ \boldsymbol{k}_n
\end{pmatrix} \tag{11}$$

实验效果：

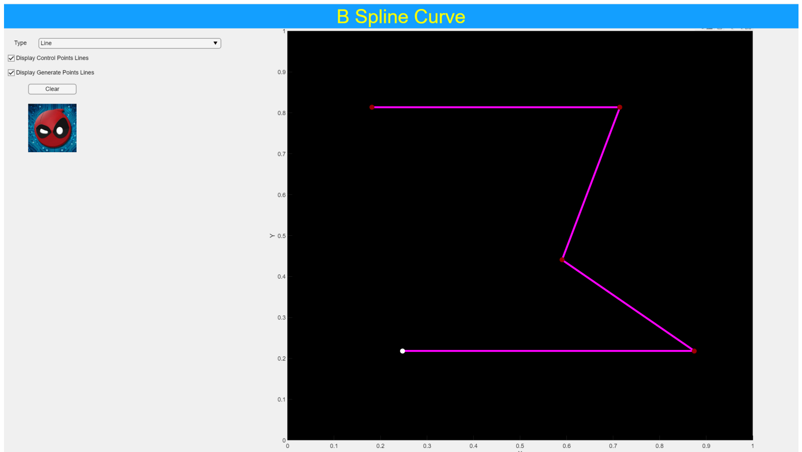

图 3:

2.3 尖点设计

设用户输入型值点为$\{\boldsymbol{k}_i\}(k=0,1,\cdots,n)$,结向量选取为

$$
\begin{aligned}
T &= (t_0, t_1, t_2, \cdots, t_{n+2}, t_{n+3}, t_{n+4}, t_{n+5}, t_{n+6}) \\
&= (s_0, s_0, s_0, s_0, s_0, s_0, s_1, s_1, s_1, s_2, s_2, s_2 \cdots, s_{n-1}, s_{n-1}, s_{n-1}, s_n, s_n, s_n, s_n, s_n, s_n)
\end{aligned} \tag{12}
$$

其中结序列$\{s_0, s_1, \cdots, s_{n-1}, s_n\}$选为自然数序列$\{0,1,2,\cdots,n\}$ 插值条件:

$$
\begin{aligned}
&\boldsymbol{x}(s_0) = \boldsymbol{k}_0 = \boldsymbol{d}_0 = \boldsymbol{d}_2 = \boldsymbol{d}_3 \\
&\ddot{\boldsymbol{x}}(s_i) = 0 \Leftrightarrow \frac{\boldsymbol{d}_{3i+1} - \boldsymbol{d}_{3i}}{s_{i+2} - s_i} = \frac{\boldsymbol{d}_{3i} - \boldsymbol{d}_{3i-1}}{s_{i+1} - s_i} \Leftrightarrow 2\boldsymbol{d}_{3i-1} - 3\boldsymbol{d}_{3i} + \boldsymbol{d}_{3i+1} = 0 \\
&\ddot{\boldsymbol{x}}(s_i) = 0 \Leftrightarrow \frac{\boldsymbol{d}_{3i+3} - \boldsymbol{d}_{3i+2}}{s_{i+1} - s_i} = \frac{\boldsymbol{d}_{3i+2} - \boldsymbol{d}_{3i+1}}{s_{i+1} - s_{i-1}} \Leftrightarrow \boldsymbol{d}_{3i+1} - 3\boldsymbol{d}_{3i+2} + 2\boldsymbol{d}_{3i+3} = 0 \\
&\qquad \text{for } i = 1, \cdots, n-1 \\
&\boldsymbol{x}(s_n) = \boldsymbol{k}_n = \boldsymbol{d}_{3n+1} = \boldsymbol{d}_{3n+2} = \boldsymbol{d}_{3n+4}
\end{aligned} \tag{13}
$$

边界条件为：

$$
\begin{aligned}
\ddot{\boldsymbol{x}}(s_0) = 0 &\Leftrightarrow \frac{\boldsymbol{d}_2 - \boldsymbol{d}_1}{s_2 - s_0} = \frac{\boldsymbol{d}_1 - \boldsymbol{d}_0}{s_1 - s_0} \Leftrightarrow 2\boldsymbol{d}_0 - 3\boldsymbol{d}_1 + \boldsymbol{d}_1 = 0 \\
\ddot{\boldsymbol{x}}(s_n) = 0 &\Leftrightarrow \frac{\boldsymbol{d}_{3n+4} - \boldsymbol{d}_{3n+3}}{s_n - s_{n-1}} = \frac{\boldsymbol{d}_{3n+3} - \boldsymbol{d}_{3n+2}}{s_n - s_{n-2}} \Leftrightarrow \boldsymbol{d}_{3n+2} - 3\boldsymbol{d}_{3n+3} + 2\boldsymbol{d}_{3n+4} = 0
\end{aligned}
\tag{14}
$$

可以组装成对角系统方程进行求解：

$$
A\boldsymbol{d} = \boldsymbol{k} \tag{15}
$$

实验效果：

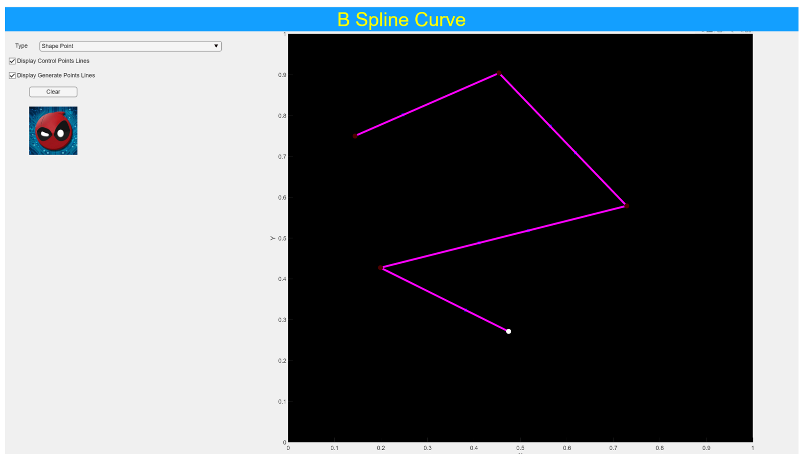

图 4:

2.4 分段设计

在能够全局表示三种不同类型曲线之后，只需将各种曲线的边界条件和插值条件连接起来即可，注意不同曲线的衔接部分需要带入边界条件。同时结向量序列其中结序列$\{s_0, s_1, \cdots, s_{n-1}, s_n\}$选为自然数序列$\{0, 1, 2, \cdots, n\}$，结序列的重数反映结点的类型，重数为1为$C^2$连续点，重数为2为直线连续点，重数为3为尖点连续

点。例如，结序列$\{s_0^{(1)}, s_1^{(2)}, s_2^{(3)}\}$对应结向量：

$$
\begin{aligned}
T &= (t_0, t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}) \\
&= (s_0, s_0, s_0, s_0, s_1, s_1, s_3, s_3, s_3, s_3, s_3, s_3)
\end{aligned}
\tag{16}
$$

最终同样可以将各个条件组装为对角线性系统方程进行求解

实验效果： 其中红色点颜色由亮到暗依次为：C^2连续点、直线点、尖点

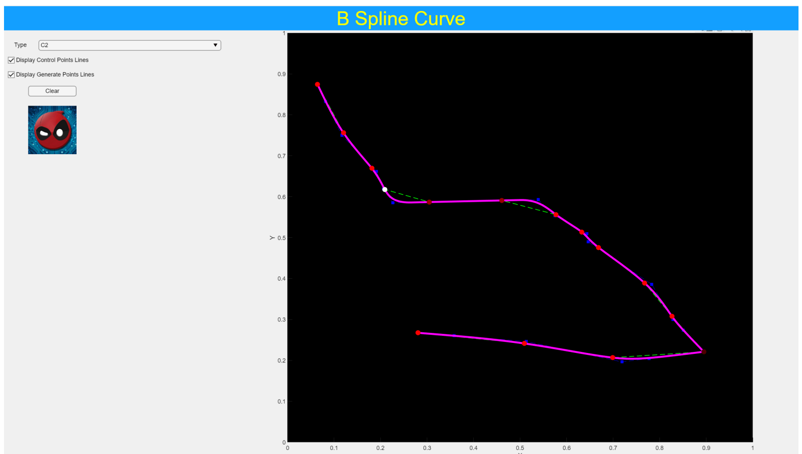

图 5:

3 软件操作说明

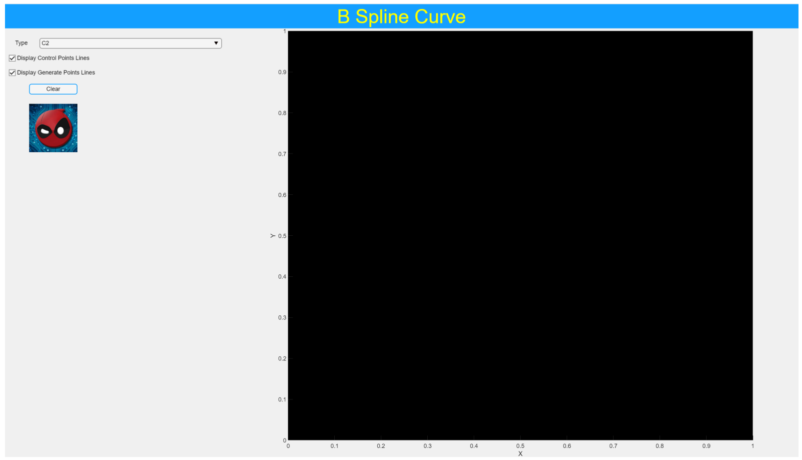

图 6:

- 左边下拉条可以选择下一个添加点的类型
- 左边勾选框可以选择是否显示de Boor点以及型值点连线
- 在坐标区中单击鼠标左键直接添加新点，添加后将立即显示生成的曲线
- 对已有的点单击鼠标左键将选定该点
- 按住选定点进行拖动可以进行顶点的编辑