functions_formatif_P

Haofan Xu

Oct 2023

By the given condition: $((\frac{a}{b}, \frac{c}{d}), \frac{e}{f}) \in R$ if and only if $(ad + bc)f = ebd$
$\implies \frac{ad+bc}{bd} = \frac{e}{f}$ $(b, d, f \neq 0$ since $\frac{a}{b}, \frac{c}{d}, \frac{e}{f} \in \mathbb{Q})$
suppose $\frac{a'}{b'} = \frac{a}{b}$
$\implies a' = ka, b' = kb$ where $k \in \mathbb{Z}$ since $\frac{a}{b} \in \mathbb{Q}$
if $(\frac{a}{b}, \frac{c}{d}) = \frac{ad+bc}{bd} \in S$
Then $((\frac{a}{b}, \frac{c}{d}), \frac{e}{f}) \in R \Leftrightarrow \frac{ad+bc}{bd} = \frac{e}{f}$
we have $(\frac{a'}{b'}, \frac{c}{d}) = \frac{a'd+b'c}{b'd} = \frac{kad+kbc}{kbd} = \frac{ad+bc}{bd} \in S$
So $((\frac{a'}{b'}, \frac{c}{d}), \frac{e}{f}) \in R \Leftrightarrow \frac{ad+bc}{bd} = \frac{e}{f}$
Therefore
for all $(\frac{a}{b}, \frac{c}{d}) \in S$, there is an exact one $\frac{e}{f} \in T$ such that $((\frac{a}{b}, \frac{c}{d}), \frac{e}{f}) \in R$
$\implies$ R is Fun and Tot