Formatif_HD_week1

Haofan Xu

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1 Proof

$Prove\ that\ if m =_{(n)} m',\ p =_{(n)} p'\ and\ q =_{(n)} q'\ then\ m + pq =_{(n)} m' + p'q'$
$According\ to\ the\ definition, we\ have:$
$m =_{(n)} p\ if\ n|(m - p)\ \ (a)$
$m = An + m',\ p = Bn + p',\ q = Cn + q'\ where\ A, B, C \in \mathbb{Z}\ (b)$
$\Rightarrow (m - m') = An,\ (p - p') = Bn,\ (q - q') = Cn\ (c)$
$So\ we\ just\ need\ to\ prove:\ n|(m + pq) - (m' + p'q')$

$$\begin{aligned}(m + pq) - (m' + p'q') &= (m - m') + (pq - p'q') \\ &= (m - m') + (pq - pq' + pq' - p'q') \\ &= (m - m') + p(q - q') + (p - p')q' \\ &= An + pCn + Bnq' \\ &= (a + pC + Bq')n\end{aligned} \tag{1}$$

$\Rightarrow n|(m + pq) - (m' + p'q')$
$\Rightarrow m + pq =_{(n)} m' + p'q'$