Formatif task (HD): Show that $(A \cup B)^c = A^c \cap B^c$.

$(A \cup B)^c$

Restart

$= ((A \cup B) \cap \mathcal{U})^c$	Identity of ∩
$= ((A \cup B) \cap ((A^c \cap B^c) \cup (A^c \cap B^c)^c))^c$	Complement with ∪
$= (((A \cup B) \cap (A^c \cap B^c)) \cup ((A \cup B) \cap (A^c \cap B^c)^c))^c$	Distributivity of ∩ over ∪
$= (((A^c \cap B^c) \cap (A \cup B)) \cup ((A \cup B) \cap (A^c \cap B^c)^c))^c$	Commutativity of ∩
$=$	Commutativity of ∩

$(((A^c \cap B^c) \cap (A \cup B)) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	
=	Associativity of $\cap$
$((A^c \cap (B^c \cap (A \cup B))) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	
=	Distributivity of $\cap$ over $\cup$
$((A^c \cap ((B^c \cap A) \cup (B^c \cap B))) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	
=	Commutativity of $\cap$
$((A^c \cap ((B^c \cap A) \cup (B \cap B^c))) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	
=	Complement with $\cap$

$((A^c \cap ((B^c \cap A) \cup \emptyset)) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$

$= ((A^c \cap (B^c \cap A)) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	Identity of $\cup$
$= ((A^c \cap (A \cap B^c)) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	Commutativity of $\cap$
$= (((A^c \cap A) \cap B^c) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	Associativity of $\cap$
$= (((A \cap A^c) \cap B^c) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	Commutativity of $\cap$
$= ((\emptyset \cap B^c) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	Complement with $\cap$
$= (((B \cap B^c) \cap B^c) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	Complement with $\cap$
$= ((B \cap (B^c \cap B^c)) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	Associativity of $\cap$
$= ((B \cap B^c) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	Idempotence of $\cap$
$= (\emptyset \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	Complement with $\cap$
$=$	Complement with $\cap$

$(((A^c \cap B^c) \cap (A^c \cap B^c)^c) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	
$= (((A^c \cap B^c)^c \cap (A^c \cap B^c)) \cup ((A^c \cap B^c)^c \cap (A \cup B)))^c$	Commutativity of ∩
$= ((A^c \cap B^c)^c \cap ((A^c \cap B^c) \cup (A \cup B)))^c$	Distributivity of ∩ over ∪
$= ((A^c \cap B^c)^c \cap (((A^c \cap B^c) \cup A) \cup B))^c$	Associativity of ∪
$= ((A^c \cap B^c)^c \cap ((A \cup (A^c \cap B^c)) \cup B))^c$	Commutativity of ∪
$= ((A^c \cap B^c)^c \cap (((A \cup A^c) \cap (A \cup B^c)) \cup B))^c$	Distributivity of ∪ over ∩
$= ((A^c \cap B^c)^c \cap ((\mathcal{U} \cap (A \cup B^c)) \cup B))^c$	Complement with ∪
$= ((A^c \cap B^c)^c \cap (((A \cup B^c) \cap \mathcal{U}) \cup B))^c$	Commutativity of ∩
$= ((A^c \cap B^c)^c \cap ((A \cup B^c) \cup B))^c$	Identity of ∩

$= ((A^c \cap B^c)^c \cap (A \cup (B^c \cup B)))^c$	Associativity of $\cup$
$= ((A^c \cap B^c)^c \cap (A \cup (B \cup B^c)))^c$	Commutativity of $\cup$
$= ((A^c \cap B^c)^c \cap (A \cup \mathcal{U}))^c$	Complement with $\cup$
$= ((A^c \cap B^c)^c \cap (A \cup (A \cup A^c)))^c$	Complement with $\cup$
$= ((A^c \cap B^c)^c \cap ((A \cup A) \cup A^c))^c$	Associativity of $\cup$
$= ((A^c \cap B^c)^c \cap (A \cup A^c))^c$	Idempotence of $\cup$
$= ((A^c \cap B^c)^c \cap \mathcal{U})^c$	Complement with $\cup$
$= (A^c \cap B^c)^{cc}$	Identity of $\cap$
$= \boxed{A^c \cap B^c}$	Double complement

Edit