

**Task (pass).**

We are taught that to add two fractions we do the following:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}.$$

However, it is not immediately obvious that if we use a different *representation* of  $\frac{a}{b}$  that we will necessarily get the same answer. That is, if  $\frac{a'}{b'} = \frac{a}{b}$  is it necessarily the case that:

$$\frac{ad + bc}{bd} = \frac{a'd + b'c}{b'd}?$$

What we need to show is that the  $+$  operation is a **well-defined function**.

More formally, let  $R \subseteq \mathbb{Q}^2 \times \mathbb{Q}$  be the binary relation defined as:

$$\left( \left( \frac{a}{b}, \frac{c}{d} \right), \frac{e}{f} \right) \in R \text{ if and only if } (ad + bc)f = bde$$

Show that  $R$  satisfies (Fun) and (Tot).