5.1_D

Haofan Xu

Oct 2023

Since all degree of vertices $deg(v_p)$ in Petersen graph is 3. And all degree of vertices $deg(v_k)$ in a complete graph K_5 is 4.
If the Petersen graph contains a subdivision of K_5
By using the Strategy 1 from the lecture, we can start with k_5 to perform the following operations to get the Petersen graph at the end:

1. Subdivide an edge: whatever which edge is subdivided by a new middle vertex, the degree of $deg(v_k)$ never change at all, keeping 4.
2. Adding a vertex: never decrease the degree of $deg(v_k)$.
3. Adding an edge: whatever we add an edge from new vertex to original vertex in K_5 or from new vertex to another new vertex we added, it never decreases the degree of $deg(v_k)$.

Hence the degree of $deg(v_k)$ never decrease to 3 as the all degree of vertices $deg(v_p)$ in Petersen graph, which makes a contradiction.$\Rightarrow$ it is impossible for the Petersen graph contains a subdivision of K_5
Proved.