Formatif_D_week1

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1 Proof

Prove that $if m =_{(n)} p$ *if and only if* $m\%n = p\%n$

" $\Rightarrow$ " :
if $m =_{(n)} p$ *is true*
$m = An + p,$ *where* $A \in \mathbb{Z}$
$\Rightarrow m - p = An$
Suppose $m\%n = r_1,\ p\%n = r_2$
$\Rightarrow m = A_1n + r_1,\ p = A_2n + r_2$ where $A_1, A_2, r_1, r_2 \in \mathbb{Z}$
$\Rightarrow m - p = (A_1n + r_1) - (A_2n + r_2) = (A_1 - A_2)n + (r_1 - r_2)$
$\Rightarrow A_1 - A_2 = A, r_1 - r_2 = 0$
$\Rightarrow r_1 = r_2$
$\Rightarrow m\%n = p\%n$
" $\Leftarrow$ " :
when $m\%n = p\%n$ *is true*
assume $m\%n = p\%n = r,$ *where* $r \in \mathbb{Z}$
$m = An + r,\ p = Bn + r$ *where* $A, B \in \mathbb{Z}$
then $m - p = An + r - (Bn + r) = (A - B)n,$ *since* $(A - B) \in \mathbb{Z}$
$\Rightarrow n|m - p \Rightarrow m =_{(n)} p$