equivalence_formatif_P

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Let $f \in \mathbb{Z} \times \mathbb{Z}$ be the function $f(n) = n\ div\ 5$, define the relation R as $(x, y) \in R$ if and only if $f(x) = f(y)$. Prove that R is an equivalence relation

A binary relation $R \subseteq S \times S$ is equivalence relation if it satisfies (R), (S), (T).

(R):
for all x $\in S : \{x|x = 5k, 5k + 1, 5k + 2, 5k + 3, 5k + 4.where\ k \in \mathbb{Z}\}$, we have $f(x) = \lfloor \frac{x}{5} \rfloor = f(x) \implies (x, x) \in R$
It is (R)
(S):
for all x,y $\in S : \{x|x = 5k, 5k + 1, 5k + 2, 5k + 3, 5k + 4.where\ k \in \mathbb{Z}\}$, we have $f(x) = \lfloor \frac{x}{5} \rfloor = \lfloor \frac{y}{5} \rfloor = f(y) \implies (x, y) \in R$
It is (S)

(T):
for all x,y,z $\in S : \{x|x = 5k, 5k + 1, 5k + 2, 5k + 3, 5k + 4.where\ k \in \mathbb{Z}\}$, we have $f(x) = \lfloor \frac{x}{5} \rfloor = \lfloor \frac{y}{5} \rfloor = f(y)$ and $f(y) = \lfloor \frac{y}{5} \rfloor = \lfloor \frac{z}{5} \rfloor = f(z) \implies f(x) = f(z) \implies (x, z) \in R$
It is (T)
Therefore R is an equivalence relation since it satisfies (R) (S) (T)