Formatif_C_week3

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Prove that any set S and any binary relation $R \subseteq S \times S : R; I = I; R = R$

Proof:
$For\ part\ R; I = R$
$" \Rightarrow " :$
Since we have $(R; I)$, by the definition of composition and identity relation:
$I \overset{def}{=} \{(x, x) : x \in S\} = \{(r_2, r_2) : r_2 \in S\}$
$R; I \overset{def}{=} \{r_1, r_2 \in S \times S\ there\ exists\ r_2 \in S,\ such\ that\ (r_1, r_2) \in R\ and\ (r_2, r_2) \in I\}$
$\Rightarrow (r_1, r_2) \in R, (r_2, r_2) \in I\ where\ r_1, r_2 \in S$
$\Rightarrow (r_1, r_2) \in R$
$\Rightarrow (R; I) \subseteq R$
$" \Leftarrow " :$
Since we have R, by the definition of composition and identity relation:
$R \overset{def}{=} \{(r_1, r_2) : r_1, r_2 \in S\}$
$\Rightarrow (r_1, r_2) \in R,\ where\ r_1, r_2 \in S$
Since $R; I \overset{def}{=} \{r_1, r_2 \in S \times S\ there\ exists\ r_2 \in S,\ such\ that\ (r_1, r_2) \in R\ and\ (r_2, r_2) \in I\}\ and\ I \overset{def}{=} \{(x, x) : x \in S\} = \{(r_2, r_2) : r_2 \in S\}$
$\Rightarrow (r_1, r_2) \in R; I$
$\Rightarrow R \subseteq R; I$
$\Rightarrow R; I = R$

$For\ part\ I; R = R$
$" \Rightarrow " :$
Since we have $(I; R)$, by the definition of composition and identity relation:
$I \overset{def}{=} \{(x, x) : x \in S\} = \{(r_1, r_1) : r_1 \in S\}$
$I; R \overset{def}{=} \{r_1, r_1 \in S \times S\ there\ exists\ r_1 \in S,\ such\ that\ (r_1, r_1) \in I\ and\ (r_1, r_2) \in R\}$
$\Rightarrow (r_1, r_1) \in I, (r_1, r_2) \in R\ where\ r_1, r_2 \in S$
$\Rightarrow (r_1, r_2) \in R$
$\Rightarrow (I; R) \subseteq R$

" $\Leftarrow$ " :

Since we have R, by the definition of composition and identity relation:

$R \stackrel{def}{=} \{(r_1, r_2) : r_1, r_2 \in S\}$

$\Rightarrow (r_1, r_2) \in R,\ where\ r_1, r_2 \in S$

Since $I; R \stackrel{def}{=} \{r_1, r_1 \in S \times S\ there\ exists\ r_1 \in S,\ such\ that\ (r_1, r_1) \in I\ and\ (r_1, r_2) \in R\}\ and\ I \stackrel{def}{=} \{(x, x) : x \in S\} = \{(r_1, r_1) : r_1 \in S\}$

$\Rightarrow (r_1, r_2) \in I; R$

$\Rightarrow R \subseteq I; R$

$\Rightarrow I; R = R$

$\Rightarrow R; I = I; R = R$