Question 1 *lost*

A ship's GPS system has broken down and is lost somewhere in a 2^Mkm by 2^Mkm patch of the pacific ocean. Your job is to find the 1km x 1km grid square that contains the ship. In order to find it, you can send a wide-beam signal to a (grid-aligned) rectangular region of the ocean and wait to see if they send a signal back saying that they received it. If the ship is within the rectangle you selected, the ship will always receive your signal, and if it is outside the rectangle then it won't receive your signal.

Sending wide-beam signals like this is expensive, so you want to use them as few times as you can. What is the least number of signals you need to send to guarantee that you can find the ship?

Question 2 *Blocks*

You are given n stacks of identical blocks. The ith stack contains a positive number of blocks, let us denote this as h_i. You are also able to move any number of blocks from the ith stack to the $(i+1)$th stack, as long as every stack always contains a positive number of blocks. You want to know if the sizes of the stacks can be made *strictly* increasing. For example $\langle 1,4,4,5\rangle$ is not strictly increasing, but by moving a block from pile $2 \to 3$ and $3 \to 4$, we can get $\langle 1,3,4,6\rangle$ which is strictly increasing.

Design an $O(n)$ algorithm that determines whether it is possible for the stacks to be made *strictly* increasing.

Question 3 *Drill*

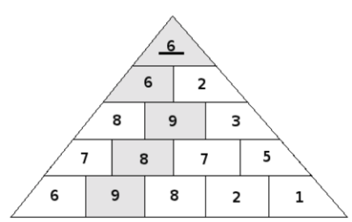
UNSW is doing a fire drill. It consists of n rooms and m corridors ($m \geq n-1$), where each corridor connects two different rooms. There are x students who must be moved from room 1 to room n. Your job is to divide the class of x students into several waves. Each wave will be released from room 1, and make their way through the corridors. To prevent overcrowding, each corridor has a limit l_i, which is the maximum number of students in a single wave who can use this corridor. Once all students in this wave have reached room n, the next wave of students will be released from room 1.

Design a polynomial time algorithm which determines the minimum number of waves that must be formed.

Question 4 *Triangle*

You are given a triangular grid of positive integers. The grid consists of n rows, the ith of which has i entries. For $1 \leq j \leq i \leq n$, let $T(i,j)$ denote the jth entry in row i.

Define a *route* to be any path that starts at the top entry and ends at any entry of the bottom row, with each step going either diagonally down to the left or diagonally down to the right. Your task is to find the largest sum of numbers that can be encountered on a route. For example, in the pictured triangular grid, the optimal route is indicated by grey cells, and so the answer is $6+6+9+8+9=38$.



Design a dynamic programming algorithm which solves the task in $O(n^2)$ time.

Question 5 *Family*

There are N disconnected towns, you live in town 1 and your family lives in town N. The government has a plan to build roads to connect the towns to each other: They have a list of E roads and each day they will build the next road on the list.

As soon as there are enough roads to connect a path from you to your family, you will visit them, but you want to know how long you need to wait.

Design an $O((N+E)logE)$ algorithm that determines how many days you need to wait before you can visit your family.

Question 6 *Lizard*

There is a rectangular grid with R rows and C columns. In row r and column c, there is a stone of height h_{rc}, which holds a_{rc} lizards. Both h_{rc} and a_{rc} are non-negative integers. If h_{rc} is zero, this denotes that there is no stone at (r, c) and hence a_{rc} is guaranteed to also be zero.

Each lizard can jump between two stones if they are separated a distance of at most d. In other words, it can jump from a stone at (r_1, c_1) to a stone at (r_2, c_2) (which may be occupied by any number of lizards) if $\sqrt{(r_1 - r_2)^2 + (c_1 - c_2)^2} \leq d$.

However, the stones are not stable, so whenever a lizard leaves a stone, the height of the stone is decreased by 1. If the new height of the stone is zero, there is no more stone at at (r, c). Any remaining lizards on this stone will drown, and lizards will no longer be able to jump onto this stone.

We want to help as many lizards as possible to escape the grid. A lizard escapes if it can jump between rocks, then take a jump of at most distance d to take them beyond the boundary of the grid.

Design a polynomial time algorithm to find the maximum number of lizards that can escape from the grid.

Question 7 *Digits*

You are given a positive integer n and a decimal digit k. Your task is to count the number of n-digit numbers (without leading zeros) in which the digit k appears an even number of times. Note that we consider 0 to be an even 1-digit number.

Design a dynamic programming algorithm which solves this problem and runs in $O(n)$ time.

Question 8 *Tasks*

Alice has n tasks to do, the ith of which is due by the day d_i. She can work on one task each day, starting from day 1, and each task takes one day to complete. Morever, Alice is a severe procrastinator and wants to accomplish every task as close as possible to its due date. If Alice finishes the ith task on day j, her rage will increase by $d_i - j$.

Design an $O(n \log n)$ algorithm that determines whether all tasks can be completed by their deadlines, and if so, outputs the minimum total rage that Alice can accumulate.